Electric Conductivity of QCD matter

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based on Phys. Lett. B 837, 137647 (2023)
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and work in progress with Tobias Bruschke and Andreas Kirchner

Penetrating Probes of Hot High-μ_B Matter: Theory meets Experiment
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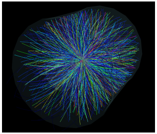
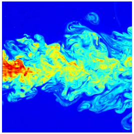
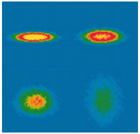
Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
    - thermodynamic equation of state $p(T,\mu)$
    - shear + bulk viscosity $\eta(T,\mu)$, $\zeta(T,\mu)$
    - heat conductivity $\kappa(T,\mu)$, $\ldots$
    - relaxation times, ...
    - electrical conductivity $\sigma(T,\mu)$
- fixed by **microscopic** properties encoded in Lagrangian $\mathscr{L}_{\mathrm{QCD}}$

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for energy density ϵ
- equation for fluid velocity u^μ
- equation for particle number density or charge density n

Need **further evolution equations** [e.g Israel & Stewart]

- equation for shear stress $\pi^{\mu\nu}$
- equation for bulk viscous pressure π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \ldots + \pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu$$

- equation for diffusion current ν^μ
- non-hydrodynamic degrees of freedom are needed for relativistic causality!

Electric current

- quarks carry electric charge
- electromagnetic current in fluid rest frame $u^\mu = (1,0,0,0)$

$$J^\mu = (n, \mathbf{J})$$

- conservation law for electromagnetic current

$$\nabla_\mu J^\mu = \frac{\partial}{\partial t} n + \boldsymbol{\nabla} \cdot \mathbf{J} = 0$$

- supplemented by evolution equation for diffusion current

$$\mathbf{J} + \tau \frac{\partial}{\partial t} \mathbf{J} = \sigma \mathbf{E} - D \boldsymbol{\nabla} n$$

- electric conductivity σ
- diffusion coefficient $D = \sigma/\chi$
- charge susceptibility $\chi = (\partial n/\partial \mu)|_T$
- relaxation time τ constrained by causality

$$\tau > D = \frac{\sigma}{\chi}$$

Spectral function from fluid dynamics

- retarded response

$$\delta J^\mu(x) = \int_y G_R^{\mu\nu}(x-y)\,\delta A_\nu(y)$$

- inverting equations of motion yields

$$G_R^{00}(\omega,\mathbf{p}) = \frac{i\sigma\mathbf{p}^2}{\omega - i\tau\omega^2 + iD\mathbf{p}^2}$$

$$G_R^{0j}(\omega,\mathbf{p}) = G_R^{j0}(\omega,\mathbf{p}) = \frac{i\sigma\omega p^j}{\omega - i\tau\omega^2 + iD\mathbf{p}^2}$$

$$G_R^{jk}(\omega,\mathbf{p}) = \frac{i\sigma\omega\delta^{jk}}{1 - i\tau\omega} + \frac{D\sigma\omega p^j p^k}{[\omega - i\tau\omega^2 + iD\mathbf{p}^2][1 - i\tau\omega]}$$

- spectral function

$$\rho(\omega,\mathbf{p}) = \mathrm{Im}\, G_{R\,\mu}^{\mu}(\omega,\mathbf{p})$$

$$= \frac{\sigma\omega(\omega^2 - \mathbf{p}^2)}{(\tau\omega^2 - D\mathbf{p}^2)^2 + \omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2 + 1}$$

Photon production rate in local thermal equilibrium

- photon production rate per unit volume and time

$$p^0 \frac{dR}{d^3p} = \frac{1}{(2\pi)^3} n_{\mathsf{B}}(\omega)\rho(\omega),$$

- electromagnetic spectral function $\rho(\omega)$
- frequency in the fluid rest frame

$$\omega = -u_\mu p^\mu$$

- Bose-Einstein distribution factor

$$n_{\mathsf{B}}(\omega) = \frac{1}{e^{\omega/T} - 1}$$

Dilepton production rate in local thermal equilibrium

- thermal dilepton production rate per unit volume and time

$$\frac{dR}{d^4p} = \frac{\alpha}{12\pi^4} \frac{1}{M^2} n_{\mathsf{B}}(\omega)\, \rho(\omega, M) \times \left(1 + \frac{2m^2}{M^2}\right) \sqrt{1 - \frac{4m^2}{M^2}}\, \Theta(M^2 - 4m^2),$$

- momentum of the dilepton pair $p^\mu = p_1^\mu + p_2^\mu$
- lepton mass m
- electromagnetic fine structure constant $\alpha = e^2/(4\pi)$

Electric conductivity

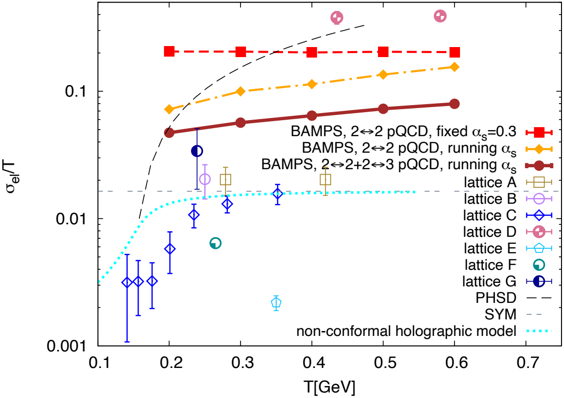
- Kubo relation for electric conductivity

$$\sigma = \frac{1}{2} \lim_{\omega \to 0} \frac{1}{\omega} \rho(\omega, \mathbf{p}) \Big|_{\mathbf{p}^2 = \omega^2} = \frac{1}{3} \lim_{\omega \to 0} \frac{1}{\omega} \rho(\omega, \mathbf{p}) \Big|_{\mathbf{p} = 0}$$

- small frequency limit either at $\mathbf{p}^2 = \omega^2$ or at $\mathbf{p} = 0$
- ratio ρ/ω has transport peak at small frequency

Predictions of electrical conductivity

- many predictions of electric conductivity in the literature
- perturbative predictions [Arnold, Moore & Yaffe] $0.19 < \sigma/T < 2$
- lattice estimates vary
- would be great to have some experimental constraints



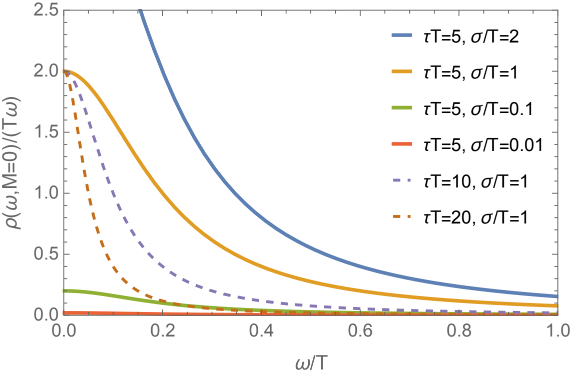
[figure compiled by Greif et al. (2014)]

Electric current spectral function

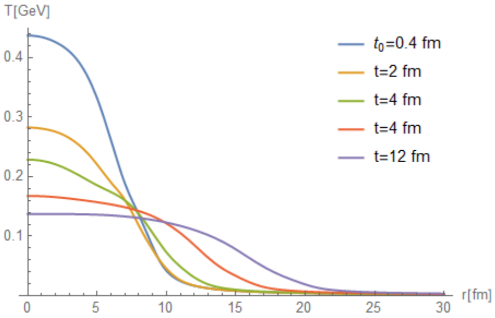
- from equations of motion we find the spectral function

$$\rho(\omega, \mathbf{p}) = \frac{\sigma\omega(\omega^2 - \mathbf{p}^2)}{(\tau\omega^2 - D\mathbf{p}^2)^2 + \omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2 + 1}.$$

- height of peak proportional to conductivity
- decay governed by width $\sim 1/\tau$

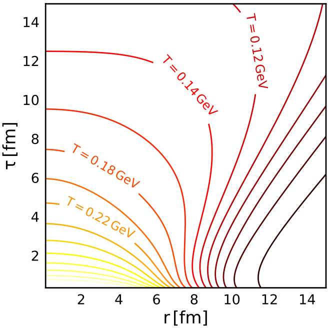


Fluid dynamics



- integrate over the QGP fire ball using $T(r,t)$ and $u(r,t)$ from FluiduM [Floerchinger, Grossi, Jeon (2019)]
- Pb-Pb-collisions at $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV
- centrality class 0-5%

Freeze-out surface



- kinetic freeze out surface: hypersurface after which particle momenta don't change any more
- integrate photon and dielectorn production rate up to this freeze-out surface
- electromagnetic currents freeze in, no radiation afterwards
- take here $T_{\mathsf{fo}} = 140$ MeV

Decay contributions

- calculate also photons from resonance decays with FastReso
  [Mazeliauskas, Floerchinger, Grossi, Teaney, EPJC 79, 284 (2019)]
- Cooper-Frye with resonance decays

$$E_p \frac{dN_a}{d^3 p} = -\frac{1}{(2\pi)^3} \int d\Sigma_\mu \, g_a^\mu(x, p), \qquad g_b^\mu(x, p) = \int_q D_b^a(p, q) f_a(x, q) q^\mu$$

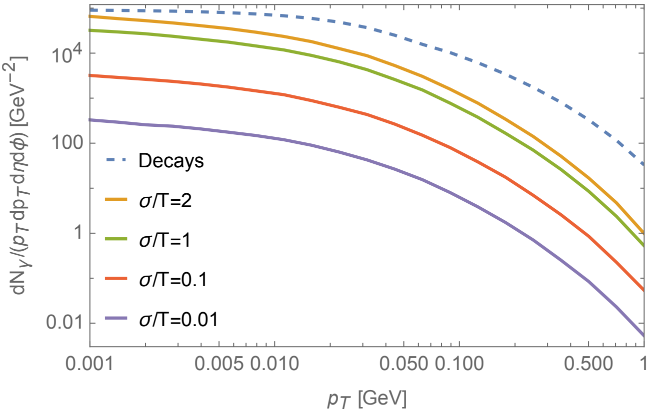
- decay map relates spectra before and after resonance decays

$$E_p \frac{dN_b}{d^3 p} = \int_q D_b^a(p, q) \; E_q \frac{dN_a}{d^3 q}$$

- dielectron from resonances calculated with PYTHIA

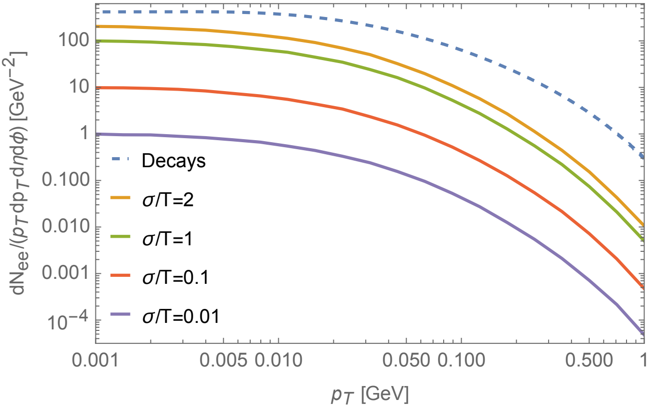
Photon spectrum

- transverse momentum spectrum of photons
- photons from hadronic resonance decays also shown



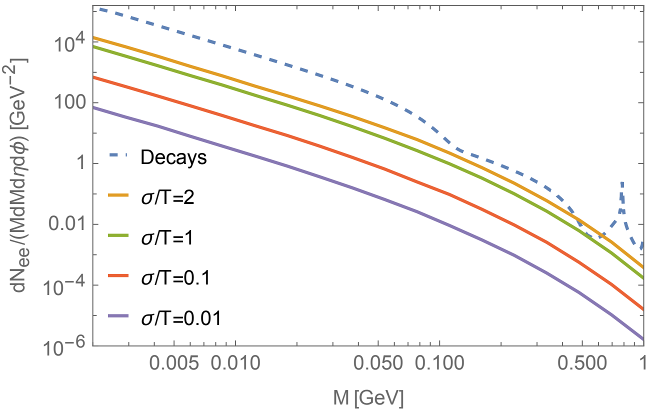
Dielectron spectrum

- transverse momentum spectrum of electron-positon pairs
- dielectrons from hadronic resonance decays also shown



Dielectron mass spectrum

- invariant mass spectrum of electron-positon pairs
- dielectrons from hadronic resonance decays also shown



How to deal with resonance decays?

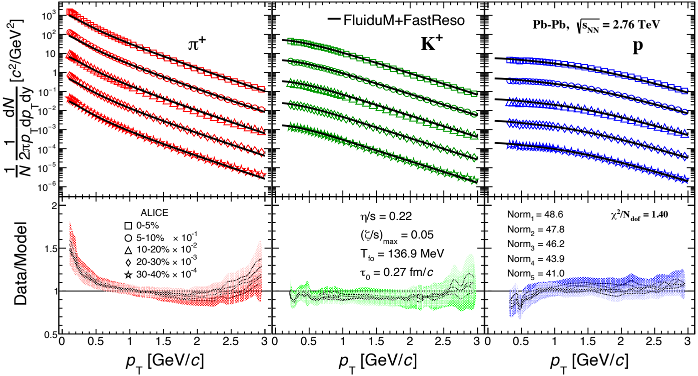
- for dielectrons it helps to accept only pairs at $M > 100$ MeV to reduce the decay background
- for photons one could use Hanbury-Brown-Twiss interferometry to disentangle contributions from resonance decays and thermal photons
- could one use Hanbury-Brown-Twiss methods also for dielectrons?

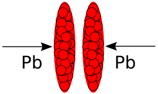
Conclusions 1

- electric current spectral function at small frequencies and momenta determined by fluid dynamics
- electric conductivity can be constrained experimentally
- background from resonance decays must be subtracted (e. g. with Hanbury-Brown-Twiss method)

Particle production at the Large Hadron Collider

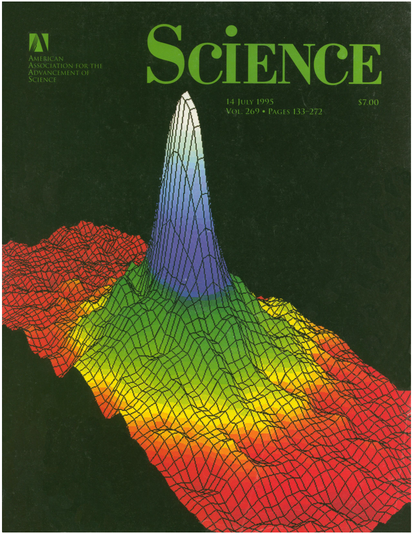
[Devetak, Dubla, Floerchinger, Grossi, Masciocchi, Mazeliauskas & Selyuzhenkov, JHEP 06(2020)044]





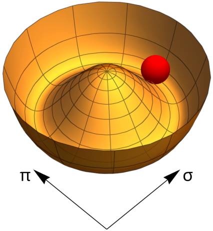
- data are very precise now - high quality theory development needed
- data show excess of pions at low momentum
- also Xe-Xe and at RHsIC [Lu, Kavak, Dubla, Masciocchi, Selyuzhenkov (2025)]

Bose-Einstein condensation in cold atomic gases



[M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, *Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor*, Science 269, 198 (1995)]

Chiral condensate



- QCD with (almost) massless quarks has (approximate) chiral symmetry

$$\mathsf{SU}(N_f)_L \times \mathsf{SU}(N_f)_R \times \mathsf{U}(1)_B$$

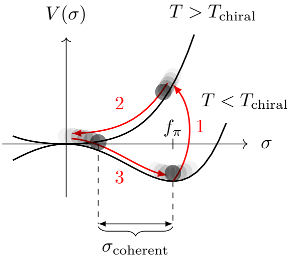
- spontaneously broken to $\mathsf{SU}(N_f)_V \times \mathsf{U}(1)_B$ by vacuum expectation value

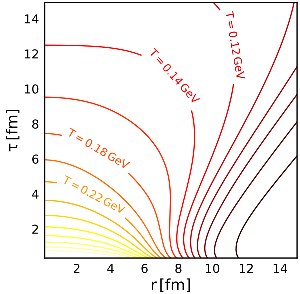
$$\langle \sigma(x) \rangle = f_\pi \approx 93 \text{ MeV}$$

- chiral symmetry restoration at

$$T > T_c \approx 155 \text{ MeV}$$

Coherent field





- chiral condensate σ could still be displaced from vacuum value at kinetic freeze-out
- *displaced* vacuum defines *coherent state*
- here *coherent state* of $\sigma/f_0(500)$

  [Pelaez, *From controversy to precision on the sigma meson: A review on the status of the non-ordinary $f_0(500)$ resonance*, Phys. Rep. 658, 1–111 (2016)]
- different from *disoriented chiral condensate*

  [A. A. Anselm (1989), J. D. Bjorken (1991), ...]

Coherent particle production

- evolution of coherent field with local source

$$[(\partial/\partial t)^2 - \boldsymbol{\nabla}^2 + m^2][\sigma(t,\mathbf{x}) - f_\pi] = J(t,\mathbf{x})$$

- resulting momentum distribution

$$E_{\mathbf{p}} \frac{dN}{d^3p} = \frac{1}{2} \frac{J(\mathbf{p})^* J(\mathbf{p})}{(2\pi)^3}$$

fom Fourier-transformed source

$$J(\mathbf{p}) = \int dt d^3x \, J(t,\mathbf{x}) e^{iE_{\mathbf{p}} t - i\mathbf{p}\mathbf{x}}$$

- can also be obtained from integral over freeze-out surface

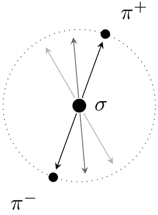
$$J(\mathbf{p}) = \int_\Sigma d\Sigma^\mu j_\mu(t,\mathbf{x},\mathbf{p})$$

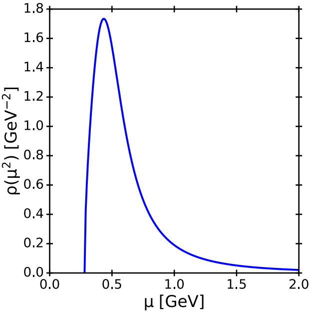
with current

$$j_\mu(t,\mathbf{x},\mathbf{p}) = [\partial_\mu \sigma(t,\mathbf{x}) + ip_\mu[\sigma(t,\mathbf{x}) - f_\pi]] \, e^{iE_{\mathbf{p}} t - i\mathbf{p}\mathbf{x}}$$

- need model for $\sigma(t,\mathbf{x})$ on freeze-out surface

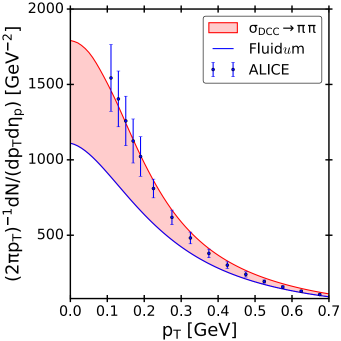
Resonance decay





- main decay channel $\sigma/f_0(500) \to \pi^+\pi^-$
- take invariant mass distribution from spectral function (Sill parametrization)

Charged pion transverse momentum spectrum



[T. Bruschke, Master thesis, Uni Jena (2025)]

- precise form of $\sigma(x)$ on freeze-out surface not important
- comparison to data shown for $\sqrt{s_{\rm NN}} = 2.76$ TeV PbPb collisions at LHC, centrality class $0 \ldots 5\%$
- chiral condensate at freeze-out assumed to be $\sigma \approx 0.25\, f_\pi$

Conclusions 2

- chiral condensate could deviate from vacuum value on freeze-out surface
- shows up as coherent $\sigma/f_0(500)$ field
- decay to charged pions $\sigma/f_0(500) \rightarrow \pi^+\pi^-$
- could explain low p_T pion excess in experimental data