Quark Model Approach to Quarkonia

David Blaschke (University of Wroclaw & HZDR/CASUS Görlitz)

1. Quarkonia in the Medium: Dissociation by Quark Exchange
2. Chemical Freeze-out in the QCD Phase Diagram
3. Density Functional for Quark Matter in Hybrid Neutron Stars
4. Inhomogeneous Big Bang Cosmology: PBH & heavy elements
5. Join HIC+Astro Communities: COST Action „Bridges“

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2021/43/P/ST2/03319
CASUS Center for Advanced Systems Understanding
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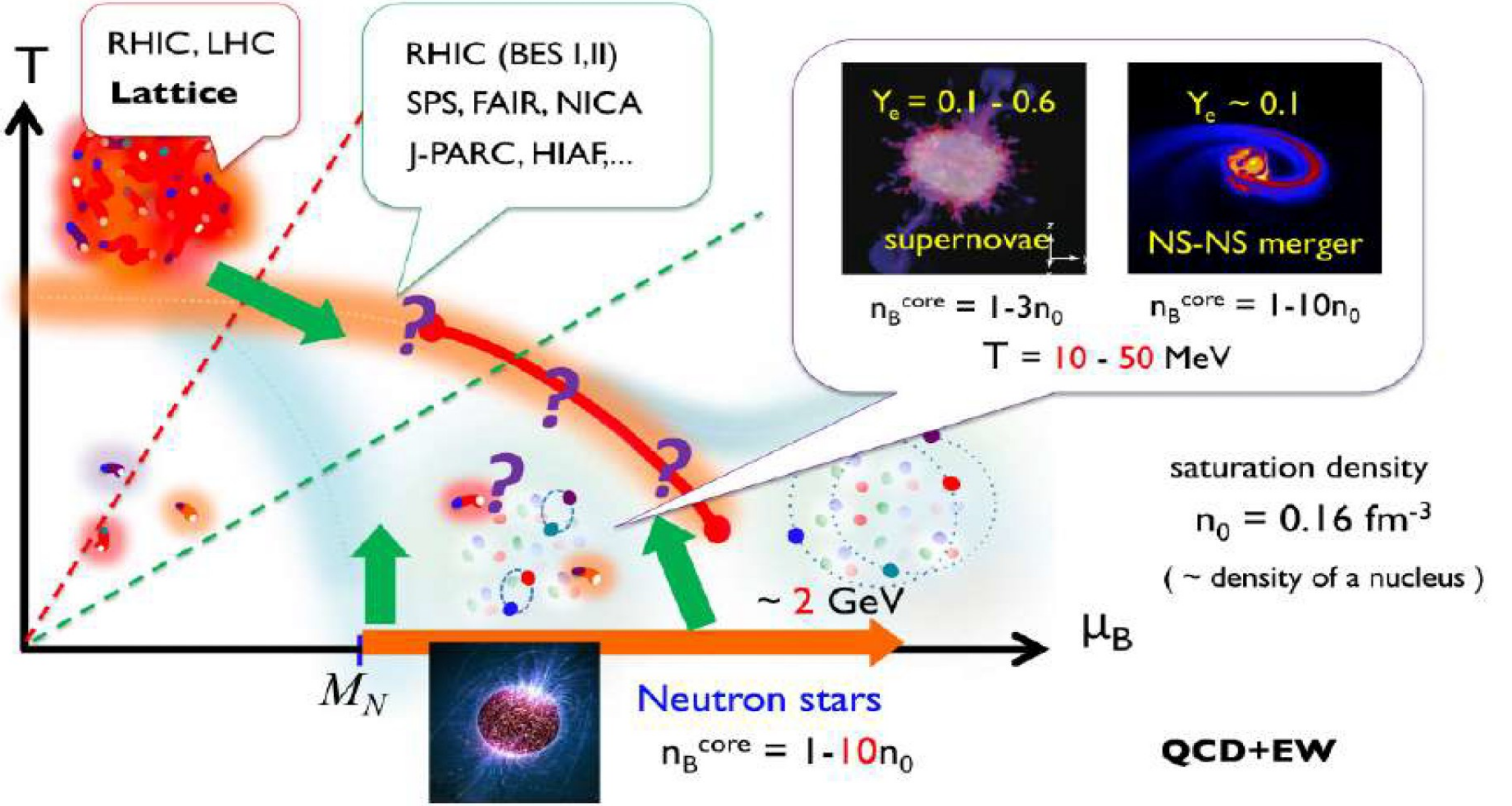
Fondazione Bruno Kessler
ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

Penetrating Probes of Hot High-mu_B Matter: Theory Meets Experiment

Bound and continuum states in strongly correlated plasmas

QCD phase diagram:



From: T. Kojo, "Delineating the properties of neutron star matter in cold, dense QCD", PoS Lattice2019, 244

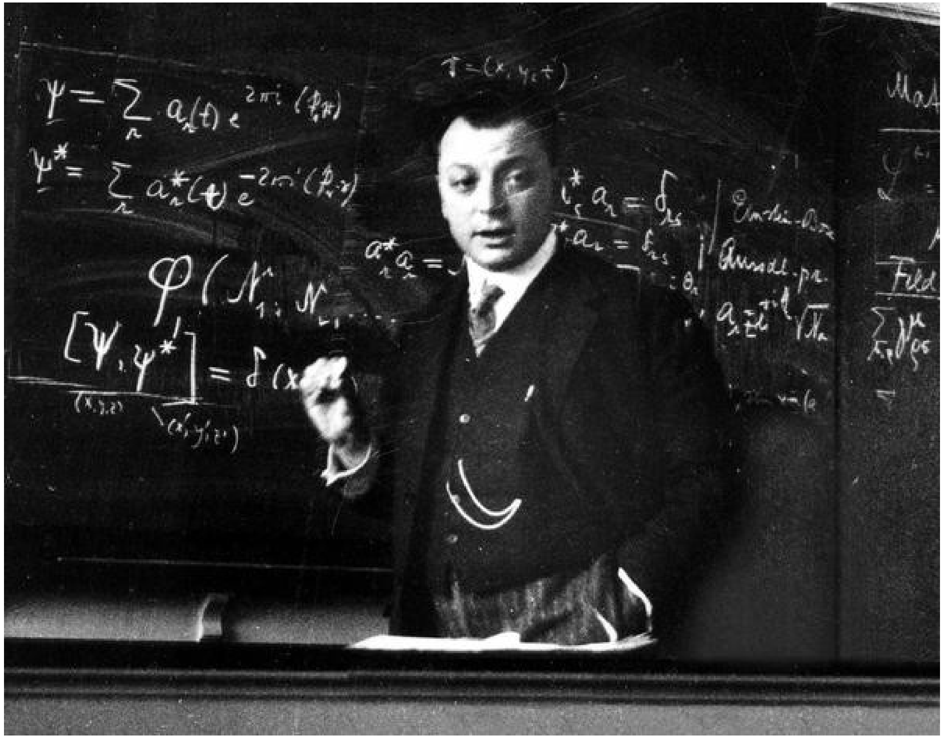
100 years of the Pauli Exclusion Principle

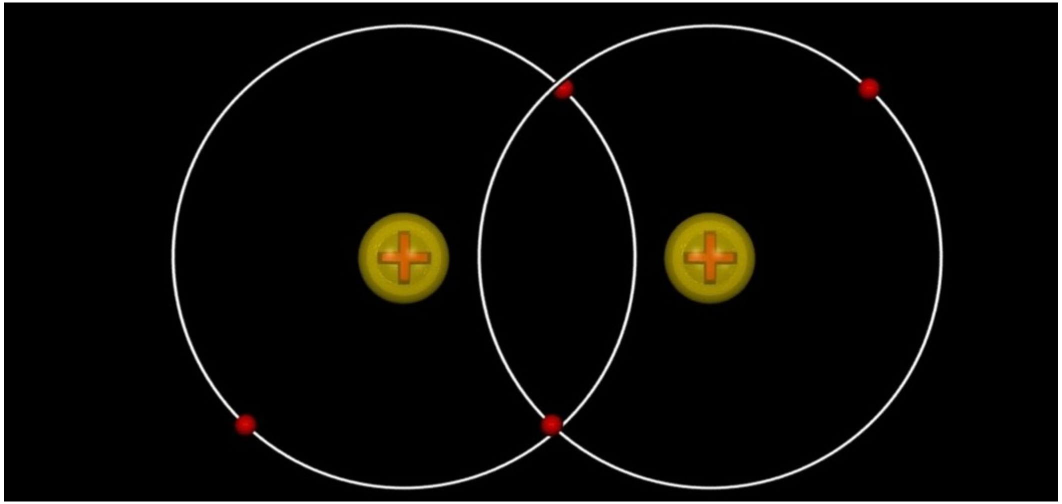
Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren.

Von **W. Pauli** jr. in Hamburg.

(Eingegangen am 16. Januar 1925.)

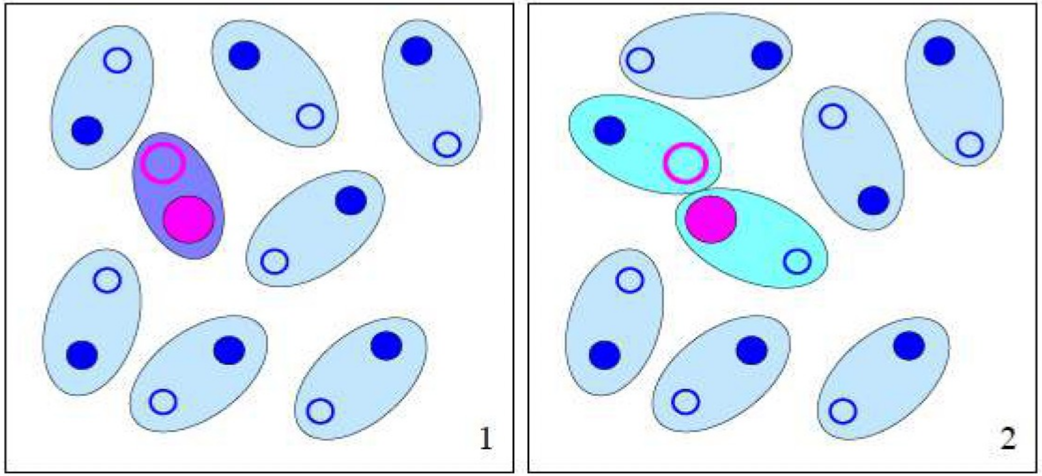
Zeitschrift für Physik. Bd. XXXI.

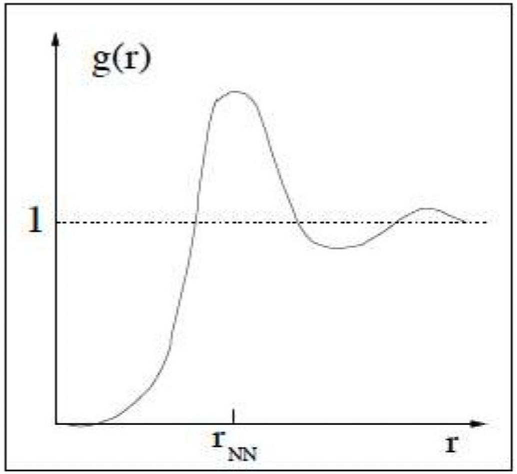




Bound and continuum states in strongly correlated plasmas

The Picture: String-flip (Rearrangement) $\Longleftrightarrow$ Pair correlation





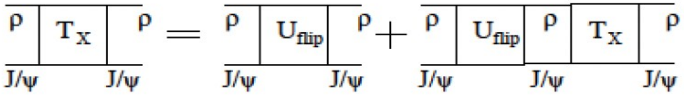
Horowitz et al. PRD (1985), D.B. et al. PLB (1985), Röpke, Blaschke, Schulz, PRD (1986)

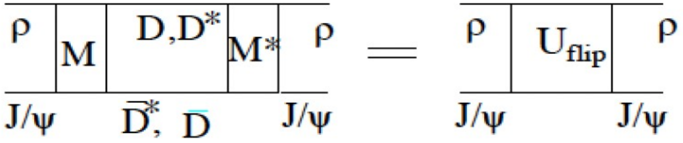
Thoma,[hep-ph/0509154] Gelman et al., PRC 74 (2006)

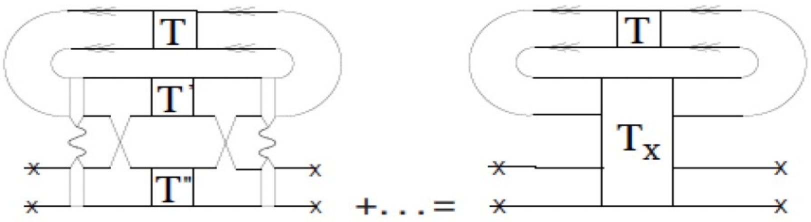
- Strong correlations present: hadronic spectral functions above T_c (lattice QCD)
- Finite width due to rearrangement collisions (higher order correlations)
- Liquid-like pair correlation function (nearest neighbor peak)

Bound and continuum states in strongly correlated plasmas

Close to T_c a resonant J/ψ - ρ interaction gives a contribution to the plasma Hamiltonian which could lead to a “pocket” in the effective interaction potential ...



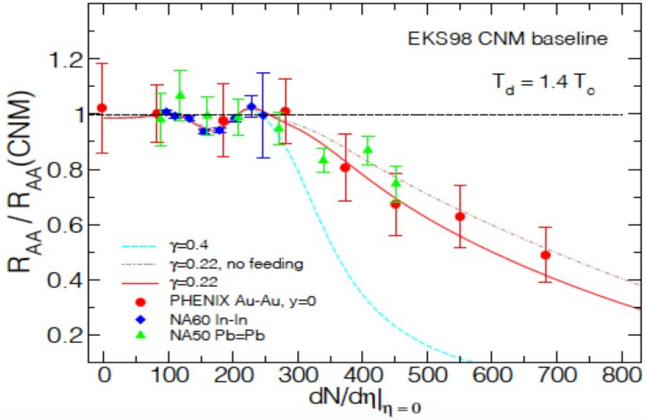




High density of ρ-like states in the medium is required for this contribution to be sizeable.

A “dip” in the NA60 In+In data for J/psi suppression →

A fact which was largely ignored by theorists!



C. Peña, D.B., arxiv:1302.0831

Nucl. Phys. A 927 (2014) 1

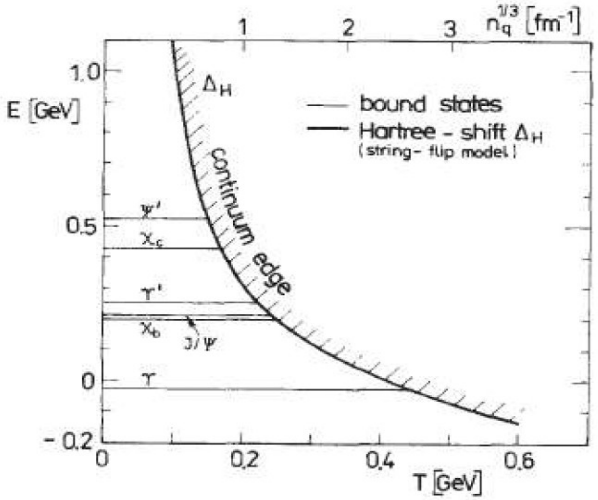
Kinetic equation for quarkonium in dense hadronic matter

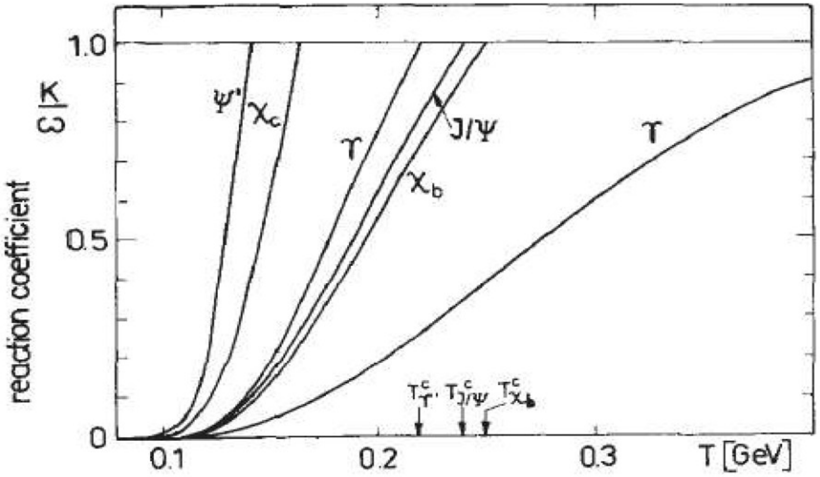
String-flip model for quarkonia suppression

Bare potential $V(r) = \sigma r - \alpha_{\text{eff}}/r$ only acts within a sphere of nearest neighbors (saturation of color interaction), i.e. with probability $c(r) = n_q/3 \exp(-4\pi r^3/9)$. Results in Hartree shift of continuum edge

$$\Delta^H = \int d^3r\, V(r)c(r) = (4\pi/9)^{-1/3}\Gamma(4/3)\sigma/n_q^{1/3} - (4\pi/9)^{1/3}\Gamma(2/3)\alpha_{\text{eff}} n_q^{1/3}$$

Law of mass action: $n_{\bar{Q}Q}/(n_{\bar{Q}}n_Q) = (\Lambda_Q^3/3\sqrt{2})\exp[-(E_{\bar{Q}Q} - 2m_Q - \Delta^H)/T]$
reaction coefficient: $k_{\bar{Q}Q+\bar{q}q\leftrightarrow Q\bar{q}+\bar{Q}q} \propto \omega \exp(-A/T)$, $A = 2m_Q + \Delta^H - E_{\bar{Q}Q}$

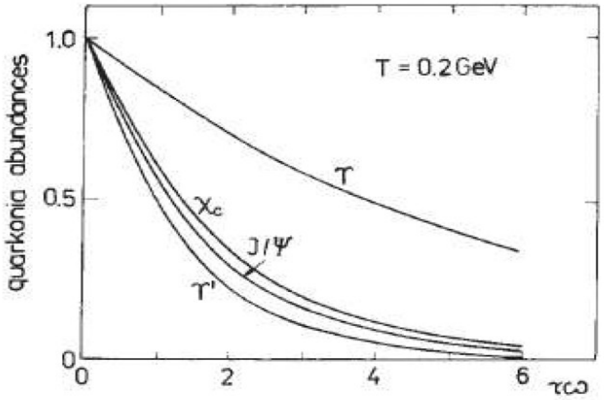




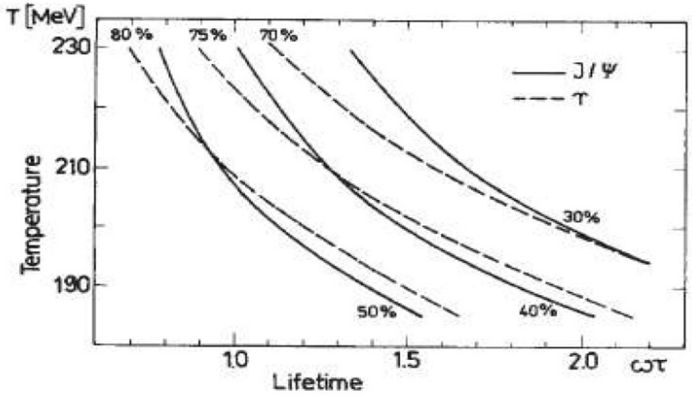
[Röpke, DB, Schulz, PLB **202**, 479 (1988)]

Kinetic equation for quarkonium in dense hadronic matter

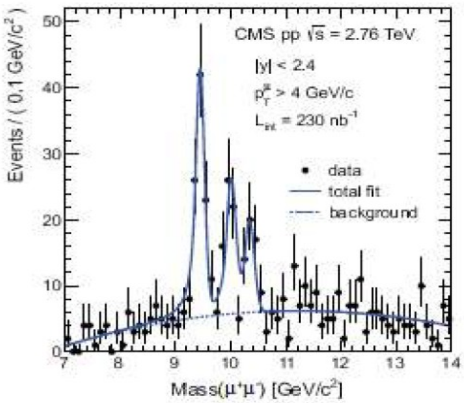
"Boiling-off" of Quarkonia

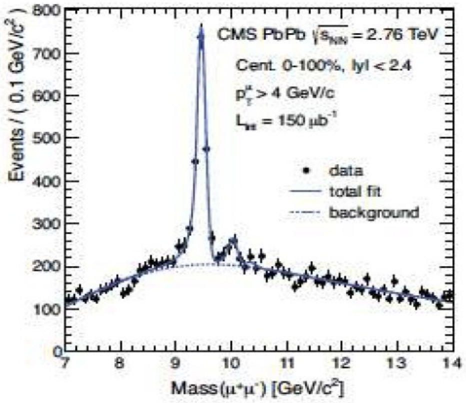


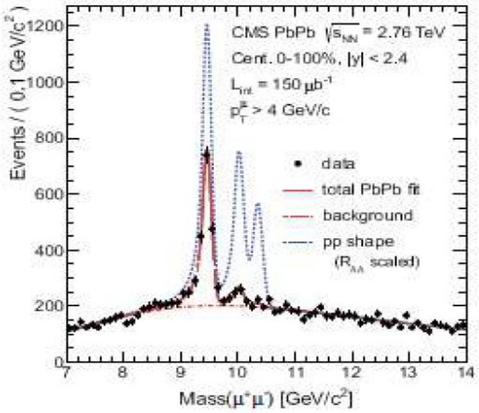
Relative suppression of Quarkonia



Bottomonium suppression at LHC (CMS collaboration, preliminary)

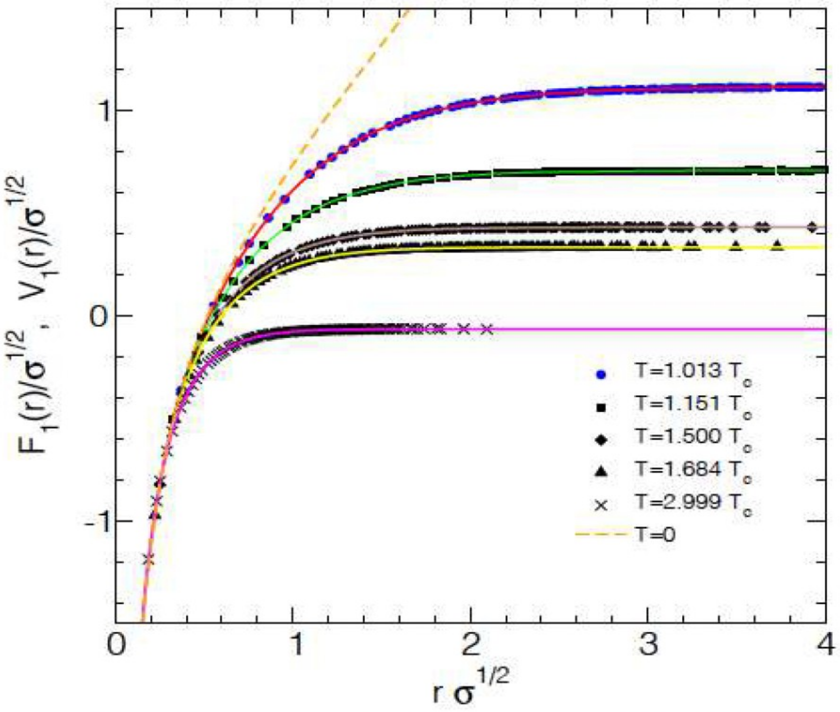






Kinetic equation for quarkonium in dense hadronic matter

Heavy quark potential at finite T from Lattice QCD



Blaschke, Kaczmarek, Laermann, Yudichev, EPJC 43, 81 (2005); [hep-ph/0505053]

Color-singlet free energy F_1 in quenched QCD

$$\langle \mathrm{Tr}[L(0)L^\dagger(r)]\rangle = \exp[-F_1(r)/T]$$

Long- and short- range parts

$$F_1(r,T) = F_{1,\mathrm{long}}(r,T) + V_{1,\mathrm{short}}(r)e^{-(\mu(T)r)^2}$$

$$F_{1,\mathrm{long}}(r,T) = \text{'screened' confinement pot.}$$

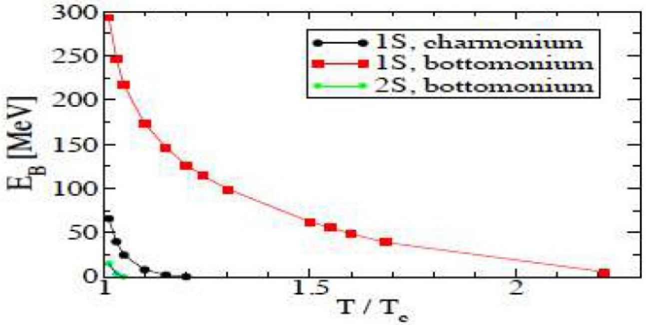
$$V_{1,\mathrm{short}}(r) = -\frac{4}{3}\frac{\alpha(r)}{r},\ \alpha(r) = \text{running coupl.} \quad (1)$$

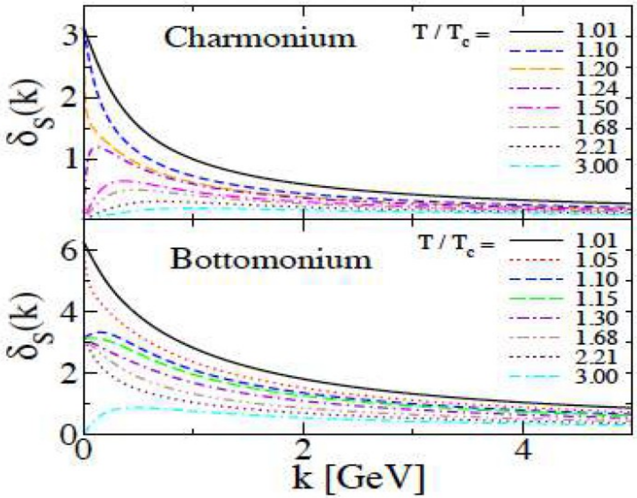
Quarkonium ($Q\bar{Q}$)	1S	1P$_1$	2S
Charmonium ($c\bar{c}$)	J/ψ(3097)	χ_{c1}(3510)	ψ' (3686)
Bottomonium ($b\bar{b}$)	Υ (9460)	χ_{b1} (9892)	Υ' (10023)

In-medium potential $\Longrightarrow$ Schrödinger Eqn.
$\Longrightarrow$ Bound/scatt. states $\Longrightarrow$ Mott effect

Kinetic equation for quarkonium in dense hadronic matter

Schrödinger equation: bound and scattering states





Quarkonia bound states at finite T:

$$[-\nabla^2/m_Q + V_{\text{eff}}(r,T)]\psi(r,T) = E_B(T)\psi(r,T)$$

Binding energy vanishes $E_B(T_{\text{Mott}}) = 0$: Mott effect

Scattering states:

$$\frac{d\delta_S(k,r,T)}{dr} = -\frac{m_Q V_{\text{eff}}}{k}\sin(kr + \delta_S(k,r,T))$$

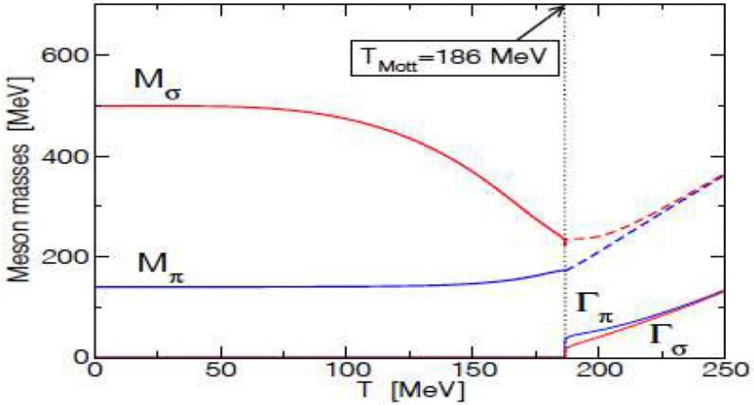
Levinson theorem:

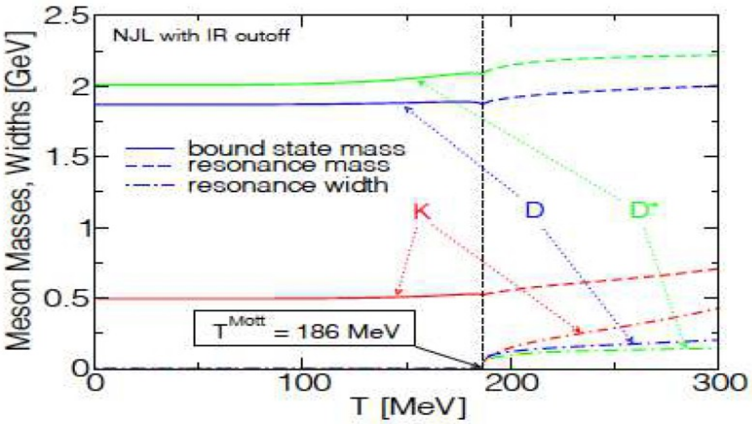
Phase shift at threshold jumps by π when bound state $\to$ resonance at $T = T_{\text{Mott}}$ (Mott effect)

Blaschke, Kaczmarek, Laermann, Yudichev
EPJC 43, 81 (2005); [hep-ph/0505053]

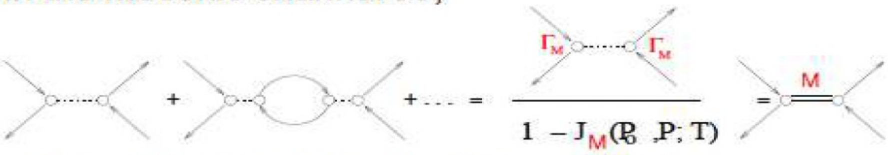
Kinetic equation for quarkonium in dense hadronic matter

Mott effect for mesons in a hot medium: NJL model primer





RPA-type resummation of quark-antiquark scattering in the mesonic channel M,



defines Meson propagator ($J_M = 2G\Pi_M$)

$$D_M(P_0, P; T) \sim [1 - J_M(P_0, P; T)]^{-1},$$

by the complex polarization function J_M
→ Breit-Wigner type spectral function

$$\mathcal{A}_M(P_0, P; T) = \frac{1}{\pi}\mathrm{Im}\, D_M(P_0, P; T) \sim \frac{1}{\pi}\frac{\Gamma_M(T)M_M(T)}{(s - M_M^2(T))^2 + \Gamma_M^2(T)M_M^2(T)}$$

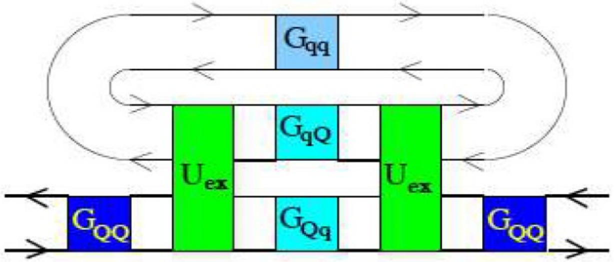
For $T < T_{\mathrm{Mott}}$: $\Gamma \to 0$, i.e. bound state
$\mathcal{A}_M(P_0, P; T) = \delta(s - M_M^2(T))$

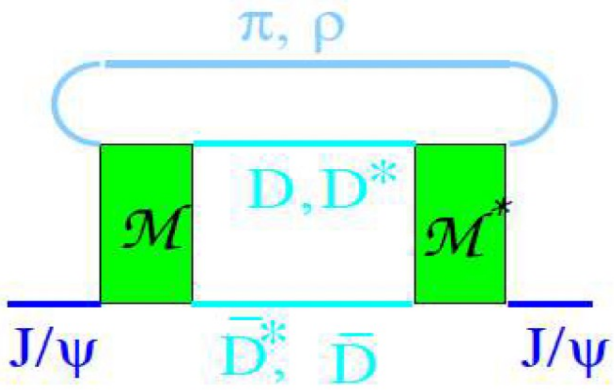
Light meson sector:
Blaschke, Burau, Volkov, Yudichev: EPJA **11** (2001) 319

Charm meson sector:
Blaschke, Burau, Kalinovsky, Yudichev,
Prog. Theor. Phys. Suppl. **149** (2003) 182

Kinetic equation for quarkonium in dense hadronic matter

Quantum kinetic approach to quarkonium breakup [Kadanoff-Baym]





$$\tau^{-1}(p) = \Gamma(p) = \Sigma^>(p) \mp \Sigma^<(p)$$

$$\Sigma^{\gtrless}(p,\omega) = \int_{p'}\int_{p_1}\int_{p_2}(2\pi)^4\delta_{p,p';p_1,p_2}\,|\mathcal{M}|^2\, G_\pi^{\lessgtr}(p')\; G_{D_1}^{\gtrless}(p_1)\; G_{D_2}^{\gtrless}(p_2)$$

$G_h^>(p) = [1 \pm f_h(p)]A_h(p)$ and $G_h^<(p) = f_h(p)A_h(p)$

low density approximation for the final states

$f_D(p) \approx 0 \;\;\Rightarrow\;\; \Sigma^<(p) \approx 0$

$$\tau^{-1}(p) = \int_{p'}\int_{p_2}\int_{p_2}(2\pi)^4\delta_{p,p';p_1,p_2}\,|\mathcal{M}|^2\, f_\pi(p')\; A_\pi(p')\; A_{D_1}(p_1)\; A_{D_2}(p_2)$$

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\frac{|\mathcal{M}(s,t)|^2}{\lambda(s,M_\psi^2,s')}\,,$$

$$\tau^{-1}(p) = \int\frac{d^3\mathrm{p}'}{(2\pi)^3}\int ds' \quad f_\pi(\mathrm{p}',s')\; A_\pi(s')v_{\mathrm{rel}}\;\sigma^*(s)$$

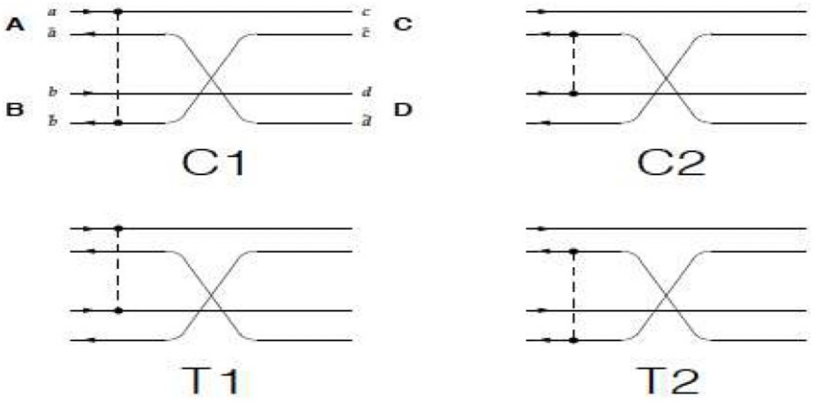
In-medium breakup cross section

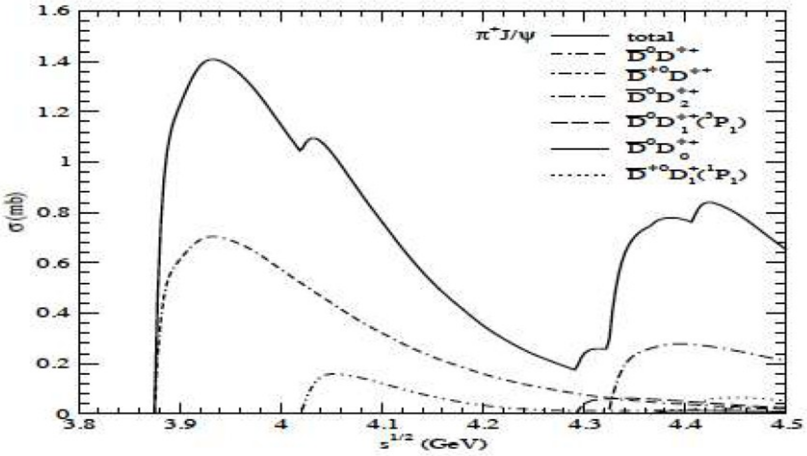
$$\sigma^*(s) = \int ds_1\; ds_2\; A_{D_1}(s_1)\; A_{D_2}(s_2)\;\sigma(s;s_1,s_2)$$

Medium effects in spectral functions A_h and $\sigma(s;s_1,s_2)$

Kinetic equation for quarkonium in dense hadronic matter

Quark rearrangement: Born diagrams of quark exchange in meson-meson interaction





Short history:

- Quark (+gluon) exchange model of short-range NN int.
  Holinde, PLB 118 (1982) 266; ...
- Born approx. to quark exchange in meson-meson scatt.
  Barnes, Swanson: PRD 46 (1992) 131
- Appl. to Charmonium dissociation: $J/\psi + \pi \rightarrow D + \bar{D}, \ldots$
  Martins, D.B., Quack: PRC 51 (1995) 2723
- Extension to other light mesons and excited charmonia
  Barnes, Swanson, Wong, Xu: PRD 68 (2003) 014903

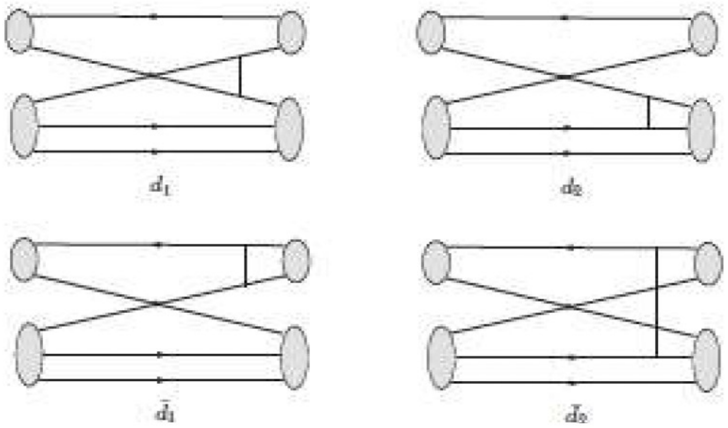
(C)apture Diagrams:
→ interaction can be absorbed into the 'ladder' of a meson
(T)ransfer Diagrams:
→ interaction between quarks from different mesons

Comments:

- Post-prior ambiguity for capture diagrams
- Interaction in capture and transfer diagrams different?
- Chiral symmetry restoration: problem in HFS term

Kinetic equation for quarkonium in dense hadronic matter

Quark rearrangement: Born diagrams of quark exchange in meson-baryon interaction



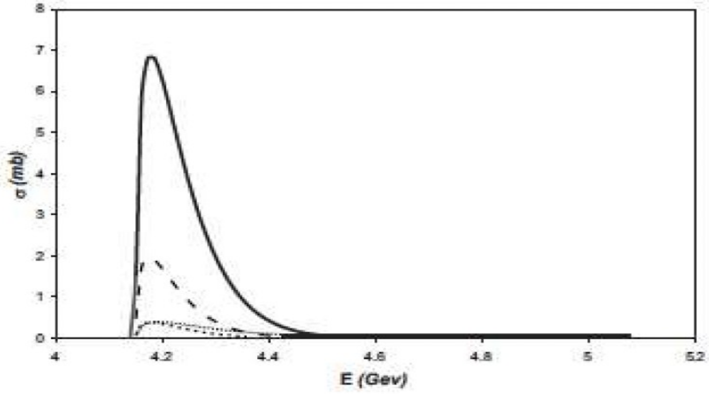
Hilbert, Black, Barnes, Swanson, nucl-th/0701087

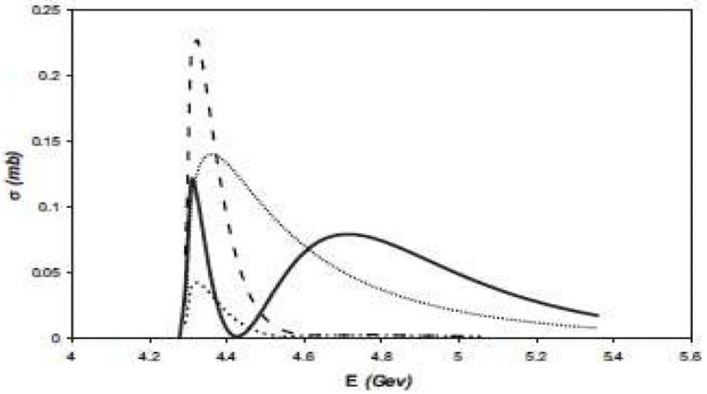
Example processes:

$$J/\psi p; \eta_c p \to \bar{D}^0\Lambda_c^+; \bar{D}^0\Sigma_c^+; \bar{D}^{0*}\Lambda_c^+; \bar{D}^{0*}\Sigma_c^+; D^-\Sigma_c^{++}; D^{-*}\Sigma_c^{++}$$

$$J/\psi n; \eta_c n \to \bar{D}^0\Sigma_c^0; \bar{D}^{0*}\Sigma_c^0; D^-\Sigma_c^+; D^{-*}\Sigma_c^+; D^-\Lambda_c^+; D^{-*}\Lambda_c^+$$

$$[J/\psi p]_{3/2} \to \bar{D}^{0*}\Sigma_{c\,3/2}^+; D^{-*}\Sigma_{c\,3/2}^{++}; \bar{D}^0\Sigma_{c\,3/2}^+; D^-\Sigma_{c\,3/2}^{++}$$

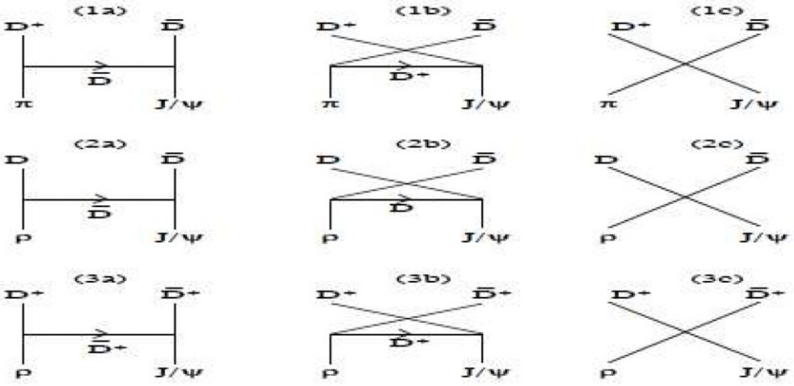


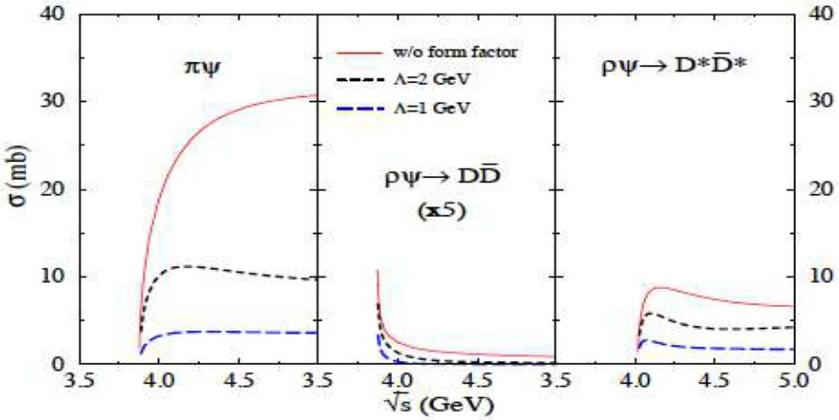


: $J/\psi p \to \bar{D}^0\Lambda_c^+$ (left) $J/\psi p \to \bar{D}^{0*}\Lambda_c^+$ (right). Curves are: total cross section (solid), hyperfine (dotted), linear (dashed),

Kinetic equation for quarkonium in dense hadronic matter

Quark rearrangement: chiral Lagrangian approach





Short history:

- Meson exchange model for NN interaction
  Yukawa(1935); Walecka, Ann. Phys. 83 (1974) 491; ...
- Application to charmonium diss: J/ψ+$\pi, \rho \to D + \bar{D}, \ldots$
  Matinyan, Müller, PRC 63 (1998) 2994
- Inclusion of formfactors for the meson-hadron vertices
  Haglin, PRC 61 (2000) 031902
  Lin, Ko, PRC 62 (2000) 034903
  Oh, Song, Lee, PRC 63 (2001) 034901
  D.B., Grigorian, Kalinovsky, hep-ph/0808.1705

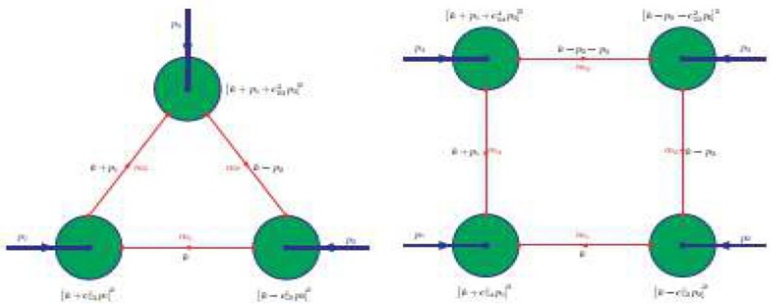
Meson exchange Diagrams:
→ Transfer diagrams: mesonic 'ladder' replaced by Born term
Contact Diagrams:
→ Capture diagrams: BS eq. at quark-meson vertex

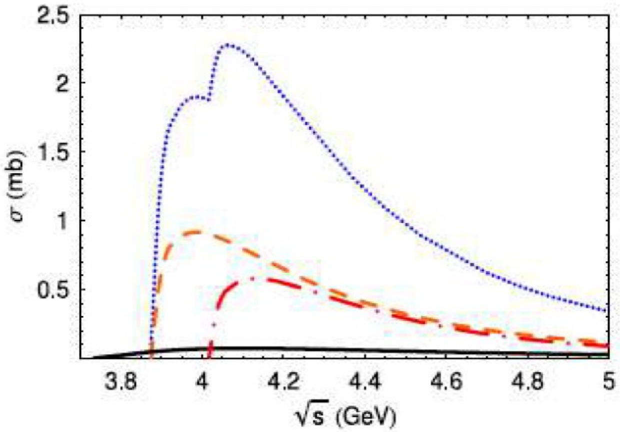
Comments:

- Formfactors ad hoc, not part of the χL approach
- Quark substructure effects absent, or hidden in FF
- Finite T, μ (and momentum-) behavior of vertices ?

Kinetic equation for quarkonium in dense hadronic matter

Quark rearrangement: relativistic quark model (DSE inspired)





Short history:

- Dyson-Schwinger approach to hadronic processes
  Roberts, Williams, PPNP 33 (1994) 477
- Application to D-mesons
  Ivanov, Kalinovsky, Roberts, PRD 60 (1999) 034018
- Calculation of $J/\psi + \pi \rightarrow D + \bar{D}$
  D.B., Burau, Ivanov, Kalinovsky, Tandy, hep-ph/0002047
  Ivanov, Körner, Santorelli, PRD 70 (2004) 014005
  Bourque, Gale, PRC 80 (2009) 015204

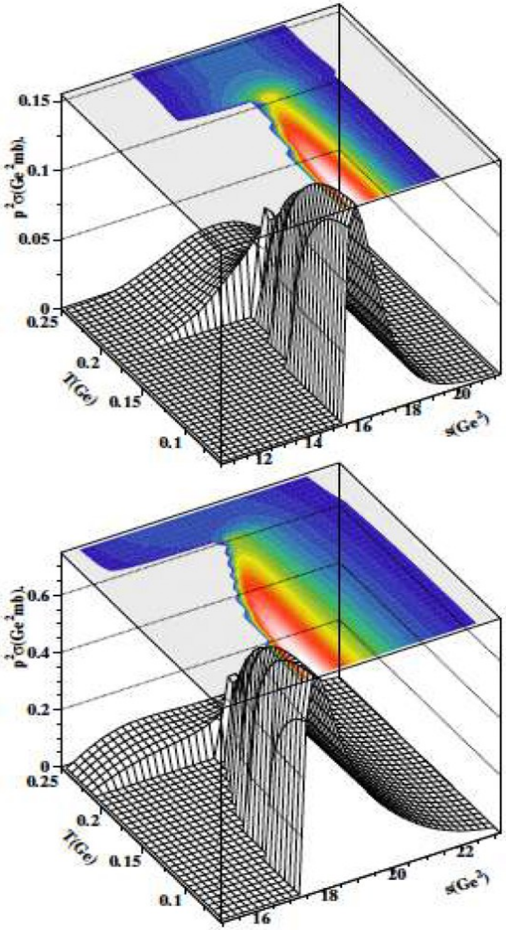
(Double) Triangle Diagrams:
$\rightarrow$ Meson exchange $\rightarrow$ Transfer diagrams
Box Diagrams:
$\rightarrow$ Contact Diagrams $\rightarrow$ Capture diagrams

Comments:

- Post-prior problem solved: covariant, chiral quark model
- Quark substructure effects in triangle and box diagrams
- BS amplitudes and quark propagators encode
  - Chiral restoration/ deconfinement
  - Mott effect: bound state dissociation

Kinetic equation for quarkonium in dense hadronic matter

In-medium J/psi breakup by pion and rho-meson impact



Approximation: $\sigma(s; s_1, s_2) \approx \sigma^{\mathrm{vac}}(s; s_1, s_2)$

Variety of models exists for $\sigma^{\mathrm{vac}}(s; s_1, s_2)$, use a relativistic one
Blaschke, et al. Heavy Ion Phys. **18** (2003) 49;
Ivanov, et al. PRD **70** (2004) 014005
Spectral function for D-mesons as Breit-Wigner

$$A_h(s) = \frac{1}{\pi} \frac{\Gamma_h(T)\, M_h(T)}{(s - M_h^2(T))^2 + \Gamma_h^2(T) M_h^2(T)} \longrightarrow \delta(s - M_h^2)$$

resonance ⇐ Mott-effect ⇐ bound state

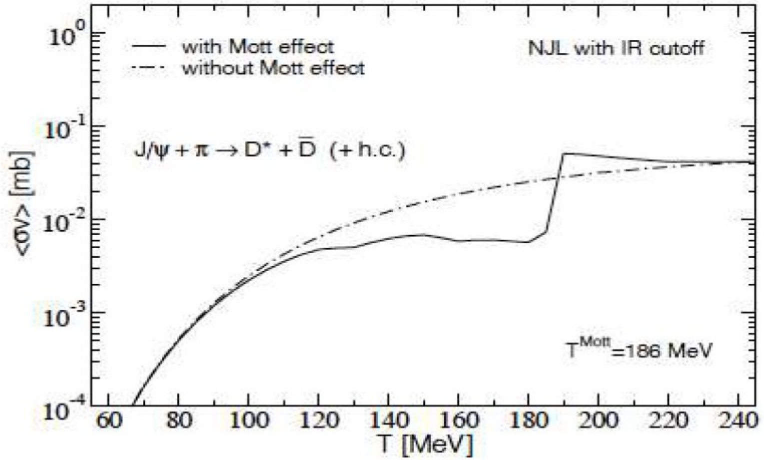
See NJL model calculations at finite temperature,
Blaschke et al.: Eur. Phys. J. **A 11** (2001) 319
Hüfner et al.: Nucl. Phys. **A 606** (1996) 260
Blaschke et al.: Nucl. Phys. **A 592** (1995) 561
Behaviour above the Mott temperature ($T \sim T_h^{\mathrm{Mott}}$)

$$\Gamma_h(T) \sim (T - T_h^{\mathrm{Mott}})^{1/2}\ \Theta(T - T_h^{\mathrm{Mott}}),$$
$$M_h(T) = M_h(T_h^{\mathrm{Mott}}) + 0.5\, \Gamma_h(T)$$

NJL model with IR cutoff: $T_h^{\mathrm{Mott}} = 186$ MeV universal

Kinetic equation for quarkonium in dense hadronic matter

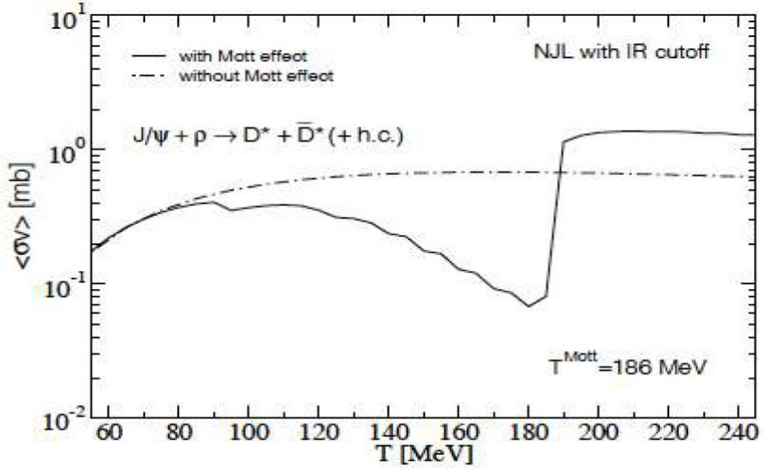
J/psi dissociation rate in a pi/rho meson resonance gas



Dissociation rate for a J/ψ at rest in a hot resonance gas $(h = \pi, \rho)$

$$\begin{aligned}\tau^{-1}(T) &= \tau_\pi^{-1}(T) + \tau_\rho^{-1}(T)\\ \tau_h^{-1}(T) &= \int \frac{d^3p}{(2\pi)^3} \int ds' A_h(s';T) f_h(p,s';T) j_h(p,s') \sigma_h^*(s;T)\\ &= \langle \sigma_h^* v_{\rm rel} \rangle n_h(T)\ ,\end{aligned}$$

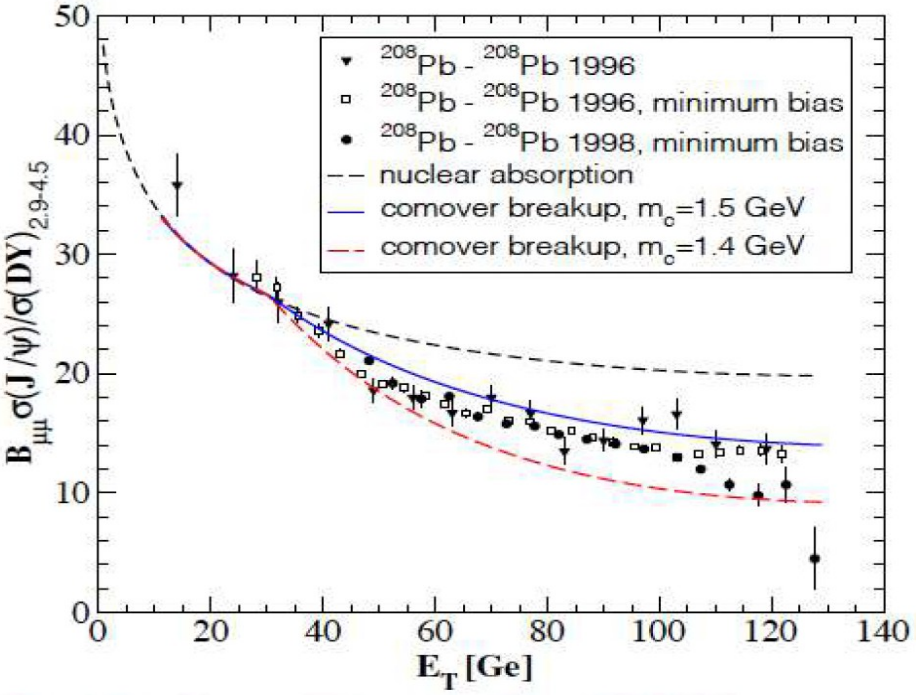
$$f_h(p,s;T) = g_h \{\exp[(\sqrt{p^2+s}-\mu)/T]-1\}^{-1}$$
$$s(p,s') = s' + M_\psi^2 + 2M_\psi\sqrt{p^2+s'}$$



- Masses slightly rising below $T^{\rm Mott}$
  $\Rightarrow$ reduction of breakup rate
- Mott-effect for intermediate states at $T^{\rm Mott}$
  $\Rightarrow$ breakup enhancement - "subthreshold" process
- Structure in the breakup rate at $T = T^{\rm Mott}$
- Additional J/ψ absorption channel opens
  $\Rightarrow$ "anomalous" suppression

Kinetic equation for quarkonium in dense hadronic matter

"Anomalous" J/psi suppression at CERN SPS



Blaschke, Burau, Kalinovsky, Proc. HQP-5, Dubna (2000); [nucl-th/0006071]

Modified Glauber model calculation

Wong, PRL76 (1996) 196;

Martins, Blaschke, Proc. HQP-4; [hep-ph/9802250]

$$S(E_T) = S_N(E_T)\exp\left[-\int_{t_0}^{t_f} dt\,\tau^{-1}(n(t))\right] = S_N(E_T)\exp\left[\int_{n_0(E_T)}^{n_f} dn < \sigma^* v_{\rm rel} >\right]$$

Nucl. abs: $S_N(E_T) = 18 + 36\ \exp(-0.26\sqrt{E_T})$

Longitudinal expansion: $n(t) = n_0(E_T)t_0/t$

Impact parameter representation of $n_0(E_T)$:

$E_T(b)/\mathrm{MeV} = 130 - b/\mathrm{fm}$

$n_0(b)/\mathrm{fm}^{-3} = 1.2\sqrt{1-(b/10.8\ \mathrm{fm})^2}$.

Threshold: Mott effect for D-Mesons

More detailed description: additional resonances, gain processes (D-fusion), HIC simulation

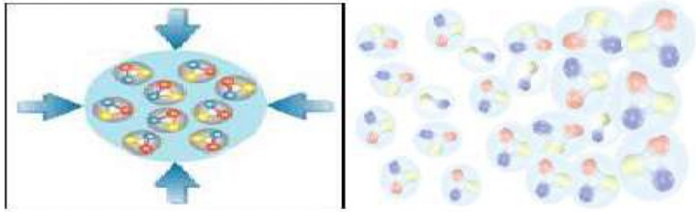
Grandchamp et al., PL B523 (2001); NP A709 (2002) 415; PRL 92 (2004) 212301; J. Phys. G 30 (2004) S1355.

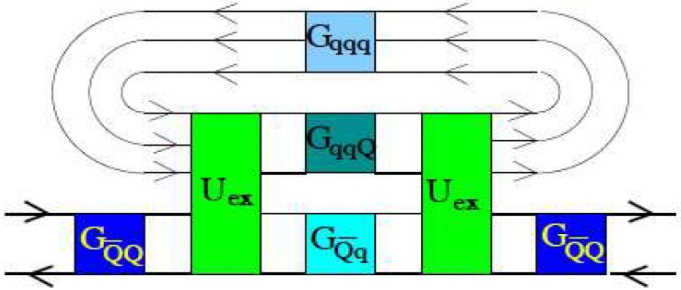
Kinetic equation for quarkonium in dense hadronic matter

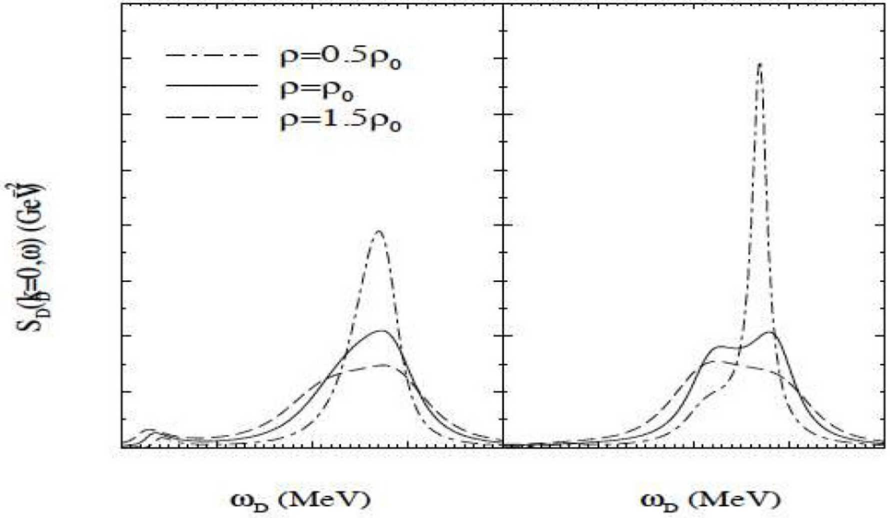
Charm and charmonium production at FAIR - CBM

J/ψ dissociation process in dense baryonic matter at FAIR-CBM: spectral functions for open charm hadrons (D-meson, Λ_c) are essential inputs! $\Longleftarrow$

D-meson spectral function in cold dense nuclear matter from a G-matrix approach $\downarrow$



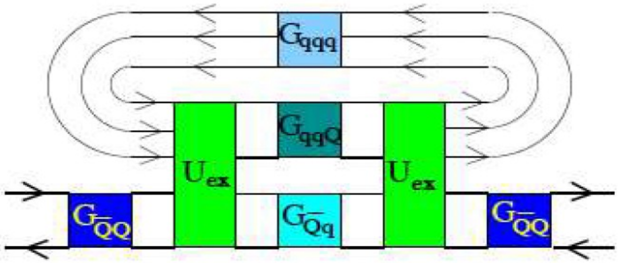


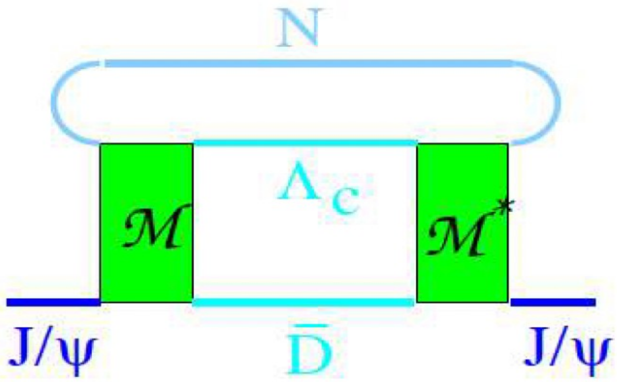


Tolos et al., EPJC (2005); nucl-th/0501151

Kinetic equation for quarkonium in dense hadronic matter

Quantum kinetics of J/psi suppression at CBM (high μ_B)

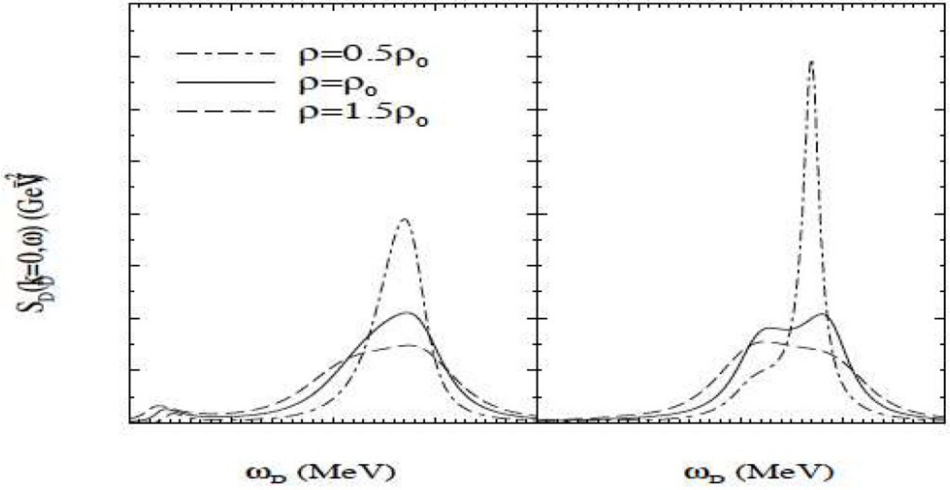




Charmonium lifetime in a dense nuclear medium ($f_D \approx 0$)

$$\tau^{-1}(p) = \int_{p'} \int_{p_2} \int_{p_2} (2\pi)^4 \delta_{p,p';p_1,p_2} |\mathcal{M}|^2 f_N(p')\, A_N(p')\, A_\Lambda(p_1)\, A_D(p_2)$$

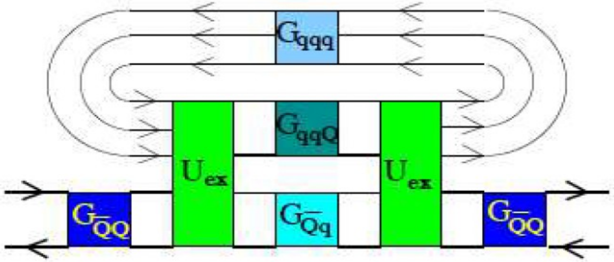
Medium effects in hadronic spectral functions A_h and $\sigma(s; s_1, s_2)$
D-meson spectral function in cold dense nuclear matter from a G-matrix approach ↓ (N, Λ_c similar)



Tolos et al., EPJC (2005); PRC 80, 065202 (2009)

Kinetic equation for quarkonium in dense hadronic matter

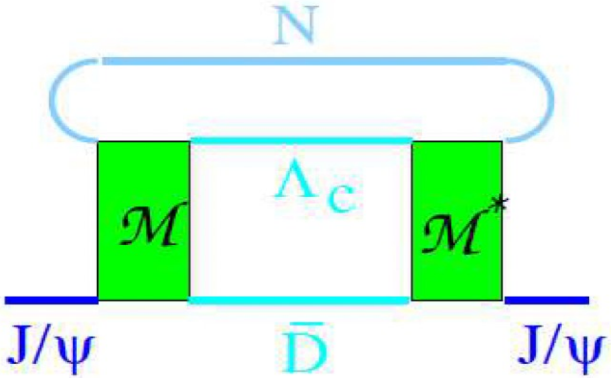
Quantum kinetics of J/psi suppression at CBM (high μ_B)

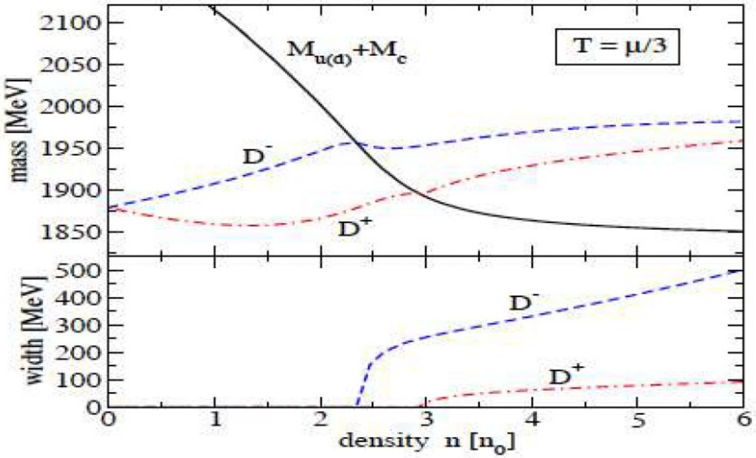


Charmonium lifetime in a dense nuclear medium ($f_D \approx 0$)

$$\tau^{-1}(p) = \int_{p'} \int_{p_2} \int_{p_2} (2\pi)^4 \delta_{p,p';p_1,p_2} |\mathcal{M}|^2 f_N(p')\, A_N(p')\, A_\Lambda(p_1)\, A_D(p_2)$$

Medium effects in hadronic spectral functions A_h and $\sigma(s; s_1, s_2)$
D-meson spectral function in hot, dense quark matter from a NJL model approach ↓ (N, Λ_c similar)

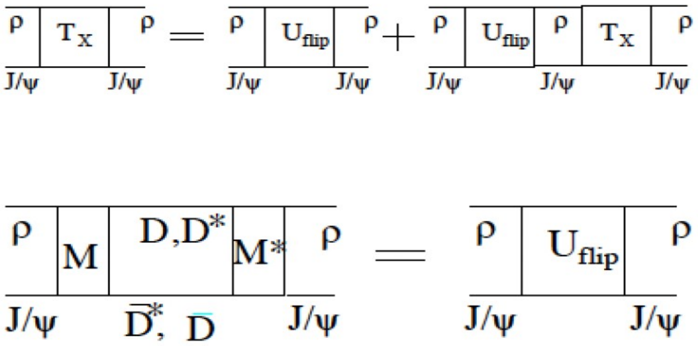


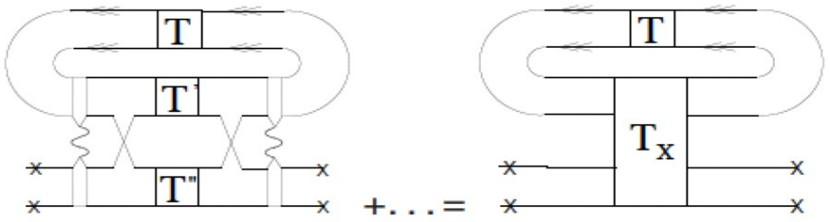


D.B., P. Costa, Yu. Kalinovsky, arxiv:1107.2913

Bound and continuum states in strongly correlated plasmas

Close to T_c a resonant J/ψ - ρ interaction gives a contribution to the plasma Hamiltonian which could lead to a "pocket" in the effective interaction potential ...

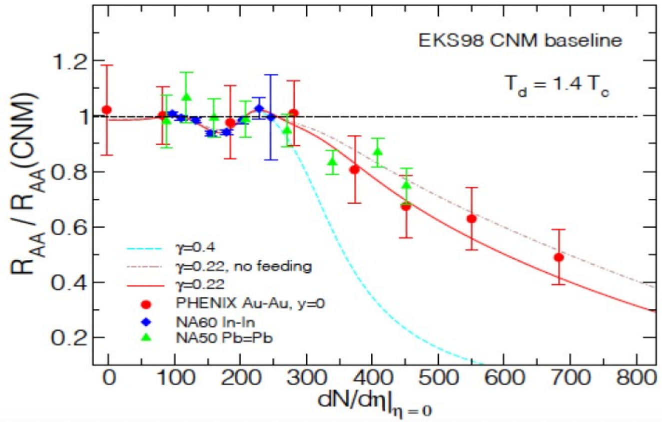




High density of ρ-like states in the medium is required for this contribution to be sizeable.

A "dip" in the NA60 In+In data for J/psi suppression →

A fact which was largely ignored by theorists!

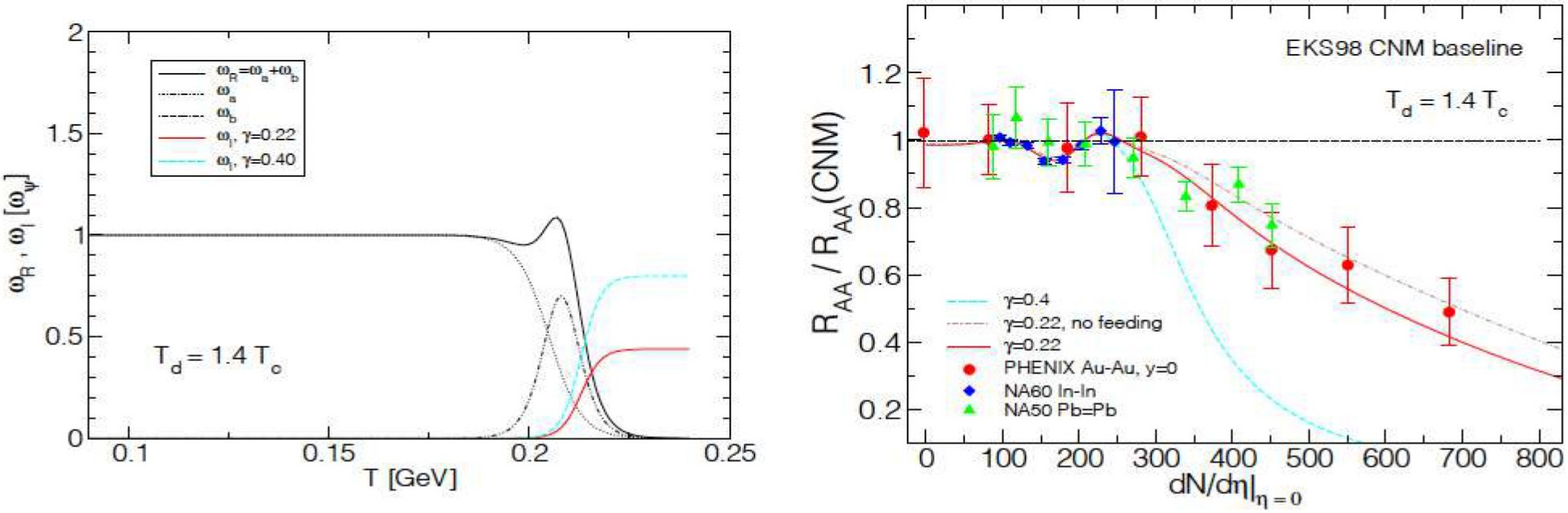


C. Peña, D.B., arxiv:1302.0831

Nucl. Phys. A 927 (2014) 1

Kinetic equation for quarkonium in dense hadronic matter

The NA60 In-In "dip" – a hint for subtle correlations?



D.B., C. Peña, Nucl. Phys. Proc. Suppl. 214 (2011) 137; arxiv:1106.2519

II. Freeze-out in the QCD Phase Diagram: „Inverse“ Mott Effect

Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

Povh-Huefner law behaviour for quark exchange between hadrons

PHYSICAL REVIEW C VOLUME 51, NUMBER 5 MAY 1995

Quark exchange model for charmonium dissociation in hot hadronic matter

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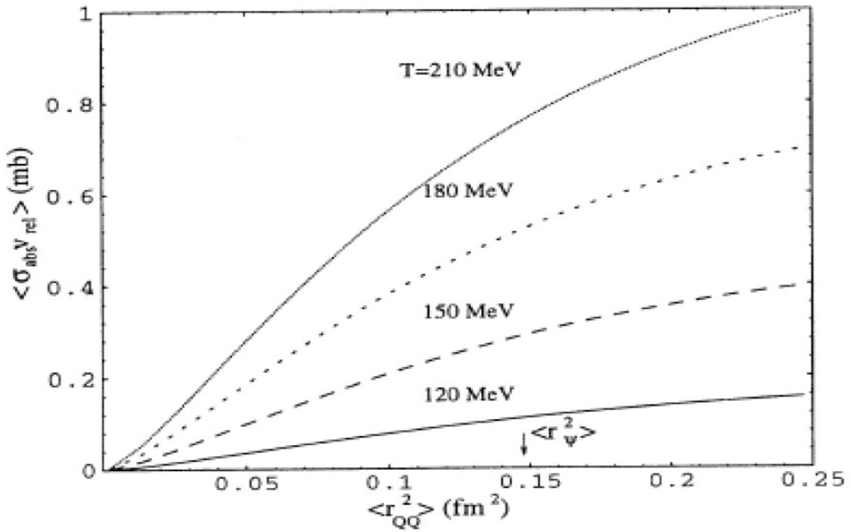
E. Quack‡
Gesellschaft für Schwerionenforschung mbH, Postfach 11 05 52, D-64220 Darmstadt, Germany
(Received 15 November 1994)



Povh-Hüfner Law

$$\langle \sigma_{\rm abs} v_{\rm rel} \rangle \propto \langle r^2 \rangle_{Q\bar{Q}} \langle r^2 \rangle_{q\bar{q}}$$

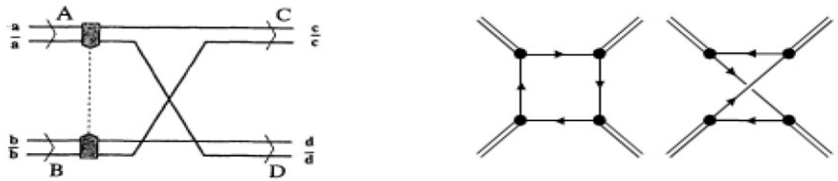
Flavor exchange processes

$$\phi + \pi \to K + \bar{K},$$

$$K^- + p \to \Lambda + X,$$
$$K^+ + p \to \Lambda + X,$$

Nonrelativistic → rel. quark loop integrals

$M_{fi} =$



Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

The basic idea: Localization of (certain) multiquark states ("cluster") = hadronization;
Reverse process = delocalization by quark exchange between hadrons

Freeze-out criterion:

$$\tau_{exp}(T,\mu) = \tau_{coll}(T,\mu)$$

$$\tau_{coll}^{-1}(T,\mu) = \sum_{i,j}\sigma_{ij} n_j$$

Povh-Huefner law, PRC 46 (1992) 990

$$\sigma_{ij} = \lambda \langle r_i^2\rangle \langle r_j^2\rangle$$

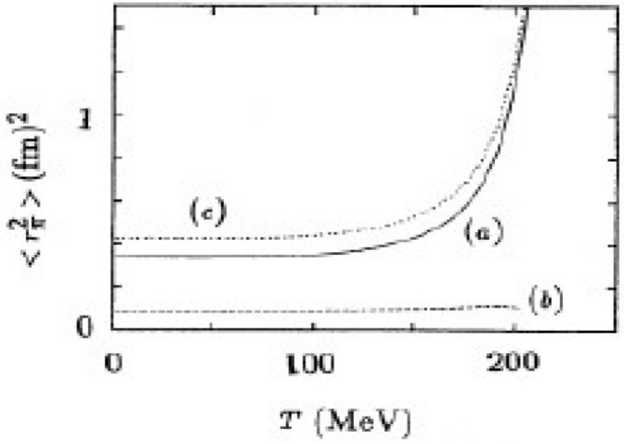
$$r_\pi^2(T,\mu) = \frac{3}{4\pi^2} f_\pi^{-2}(T,\mu)$$

$$f_\pi^2(T,\mu) = -m_0 \langle \bar{q}q\rangle_{T,\mu}/M_\pi^2$$

$$r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q}|\langle\bar{q}q\rangle_{T,\mu}|^{-1}$$

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_{MF}\left[1 - \frac{T^2}{8f_\pi^2(T,\mu)} - \frac{\sigma_N n_{s,N}(T,\mu)}{M_\pi^2 f_\pi^2(T,\mu)}\right]$$

Hippe & Klevansky, PRC 52 (1995) 2172



Mott-Anderson localization model for chemical freeze-out

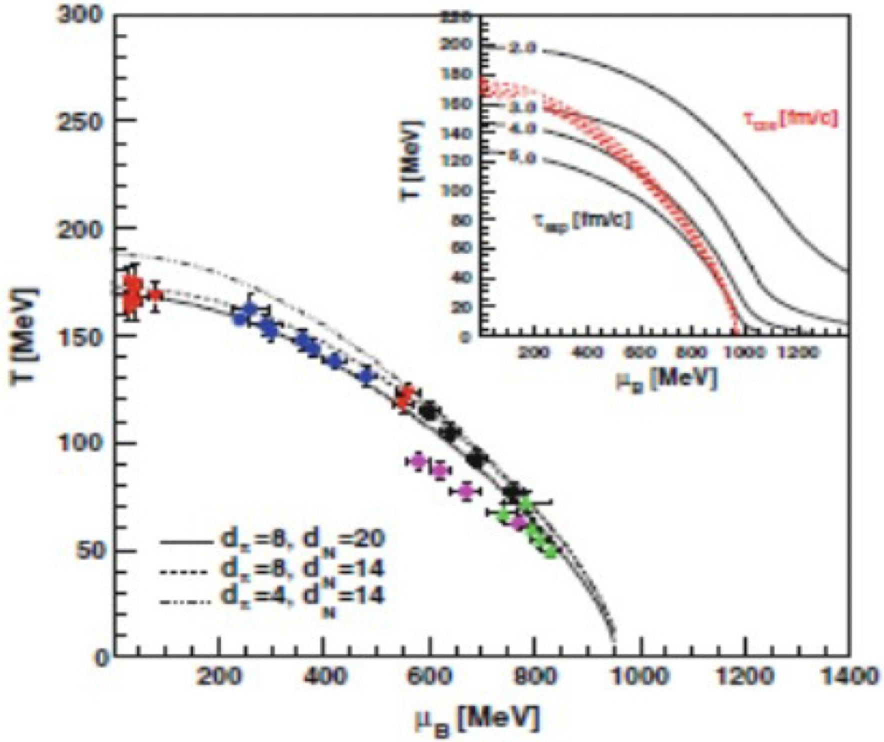
DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

Model results:

$$\tau_{exp}(T, \mu) = \tau_{coll}(T, \mu)$$

Collision time strongly T, mu dependent !

Schematic resonance gas: dπ pions, dN nucleons

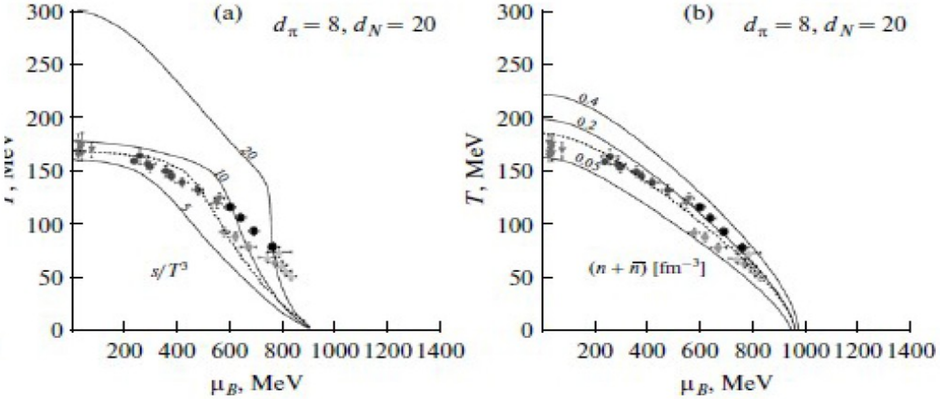


Expansion time scale from entropy conservation:

$$s(T, \mu)\ V(\tau_{exp}) = \text{const}$$

$$\tau_{exp}(T, \mu) = a s^{-1/3}(T, \mu),$$

Thermodynamics consistent with phenomenological Freeze-out rules:



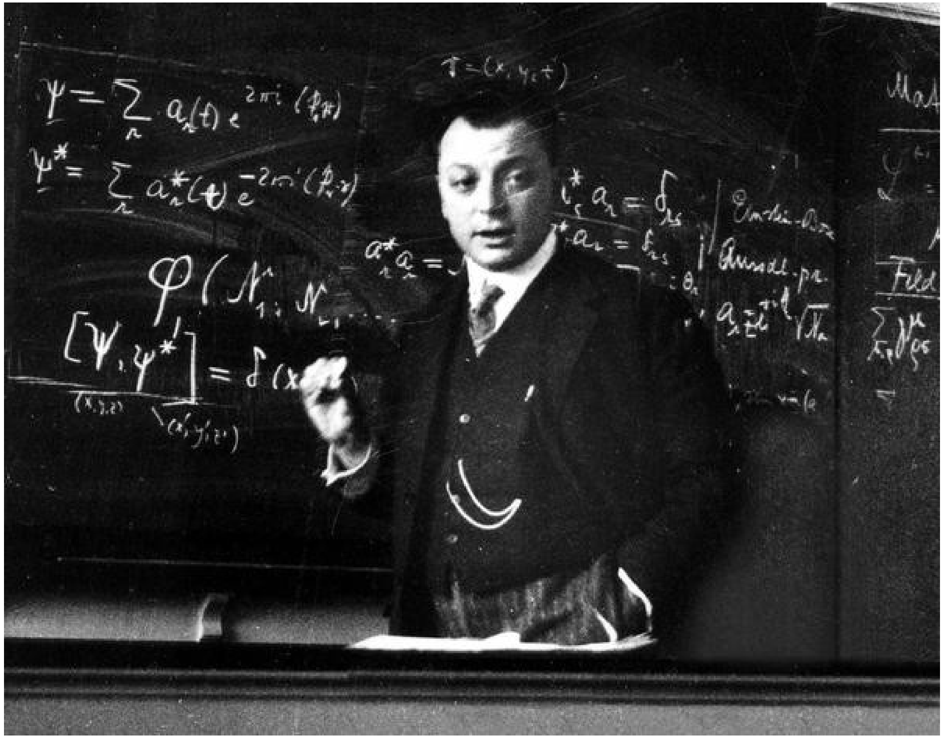
100 years of the Pauli Exclusion Principle

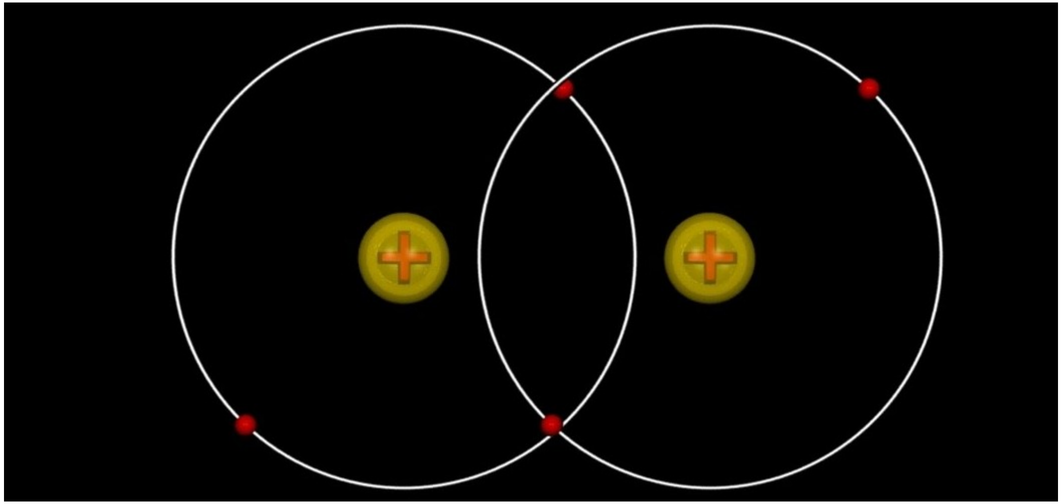
Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren.

Von **W. Pauli** jr. in Hamburg.

(Eingegangen am 16. Januar 1925.)

Zeitschrift für Physik. Bd. XXXI.





Cluster virial expansion for quark-hadron matter

The cluster decomposition of the thermodynamic potential is given as

$$\Omega_{\rm total}(T,\mu,\phi,\bar{\phi}) = \Omega_{PNJL}(T,\mu,\phi,\bar{\phi}) + \Omega_{\rm pert}(T,\mu,\phi,\bar{\phi}) + \Omega_{MHRG}(T,\mu,\phi,\bar{\phi}),$$

where the first two terms describe the quark and gluon degrees of freedom via the mean–field thermodynamic potential for quark matter in a gluon background field $\mathcal{U}$

$$\Omega_{PNJL}(T,\mu,\phi,\bar{\phi}) = \Omega_Q(T,\mu,\phi,\bar{\phi}) + \mathcal{U}(T,\phi,\bar{\phi})$$

with a perturbative correction $\Omega_{\rm pert}(T,\mu,\phi,\bar{\phi})$.
The Mott-Hadron-Resonance-Gas (MHRG) part for the multi-quark clusters is

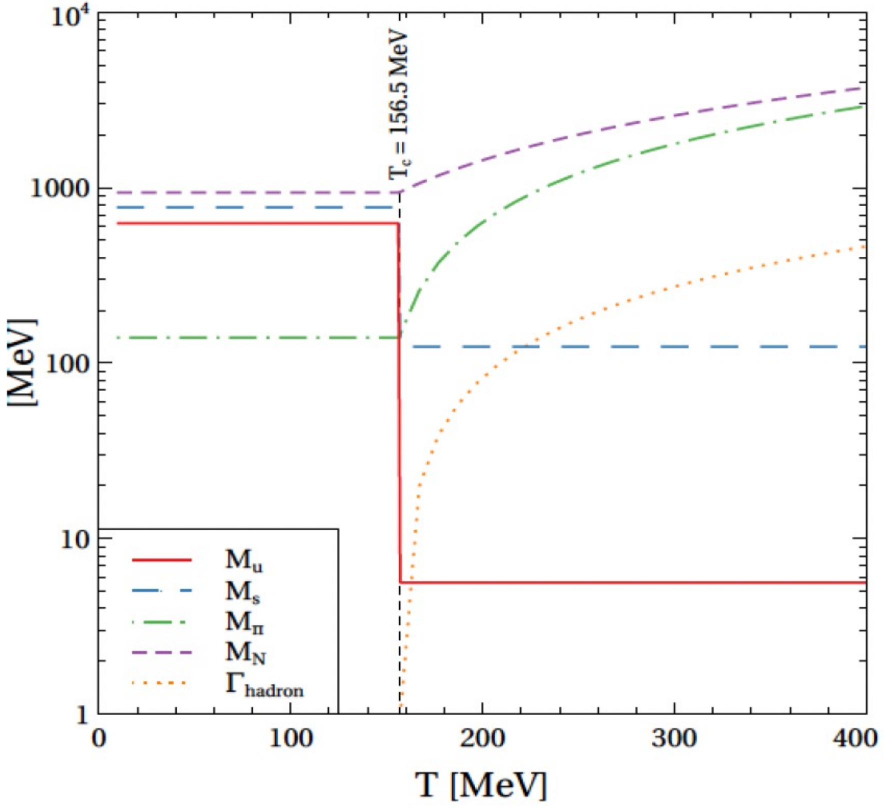
$$\Omega_{MHRG}(T,\mu,\phi,\bar{\phi}) = \sum_{i=M,B,\ldots} \Omega_i(T,\mu,\phi,\bar{\phi}),$$

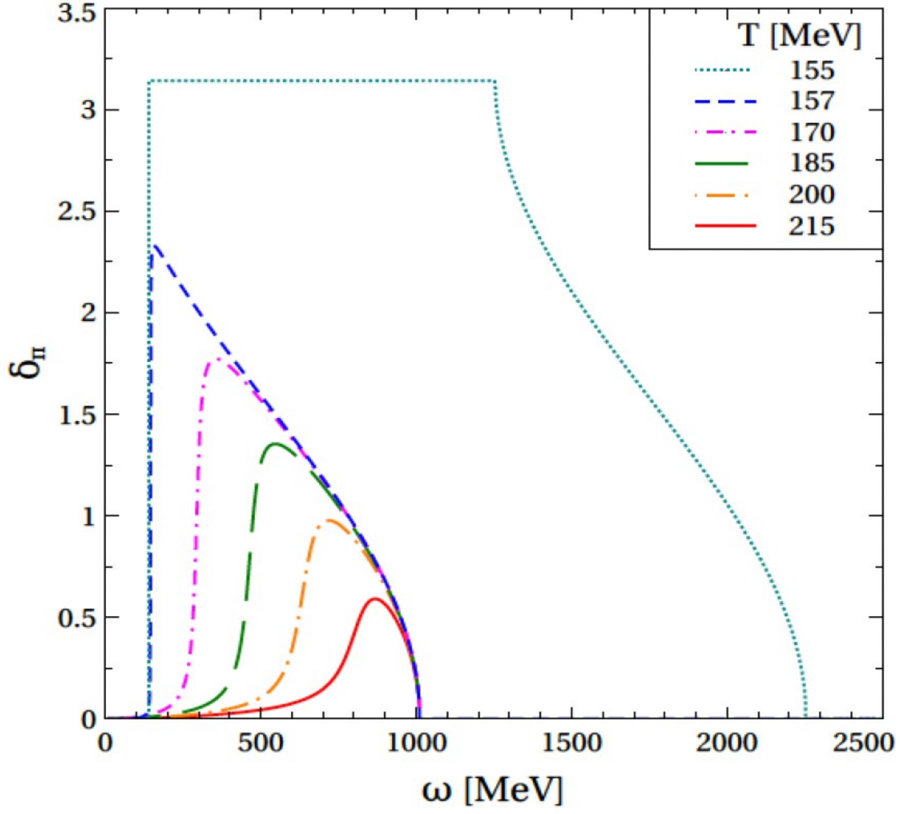
where the multi–quark states are described by the GBU formula:

$$\begin{aligned} n &= -\frac{\partial\Omega}{\partial\mu} = \sum_a a\, n_a(T,\mu) \\ &= \sum_a a\, d_a \int \frac{d\omega}{\pi} \int \frac{d^3q}{(2\pi)^3} \left\{ f_\phi^{(a),+} - \left[f_\phi^{(a),-} \right]^* \right\} 2\sin^2\delta_a(\omega,q) \frac{\partial\delta_a(\omega,q)}{\partial\omega} \,, \end{aligned}$$

where d_i is the degeneracy factor, a is the number of valence quarks in the cluster an $f_\phi^{(a),+}$, $\left[f_\phi^{(a),-} \right]^*$ are the Polyakov-loop modified distribution functions.
Analogous for the entropy density $s = -\partial\Omega/\partial T$.

Inputs: mass spectrum & phase shifts (models)





D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14

Inputs: mass spectrum (Particle Data Tables)

Mesons

PDG mesons	d_i	$M_{\rm PDG}$ [MeV]	M_i [MeV]	$M^<_{{\rm th},i}$ [MeV]	$M^>_{{\rm th},i}$ [MeV]
π^+/π^0	3	140	140	1254	11.2
K^+/K^0	4	494	494	1397	129.6
η	1	548	878	1349	90.1
ρ^+/ρ^0	9	775	783	1254	11.2
ω	9	783	783	1254	11.2
K^{*+}/K^{*0}	12	895	806*)	2651	140.8
η'	1	960	878	1349	90.1
a_0	3	980	1095*)	2508	22.4
f_0	1	980	1095*)	2508	22.4
ϕ	3	1020	1069	1540	248
..					
$\pi_2(1880)$	15	1895	1095*)	2508	22.4
$f_2(1950)$	5	1944	1095*)	2508	22.4
$a_4(2040)$	27	1996	1095*)	2508	22.4
$f_2(2010)$	5	2011	1095*)	2508	22.4
$f_4(2050)$	9	2018	1095*)	2508	22.4
$K^*_4(2045)$	36	2045	1238*)	2651	140.8
$\phi(2170)$	3	2175	1381*)	2794	259.2
$f_2(2300)$	5	2297	1095*)	2508	22.4
$f_2(2340)$	5	2339	1095*)	2508	22.4

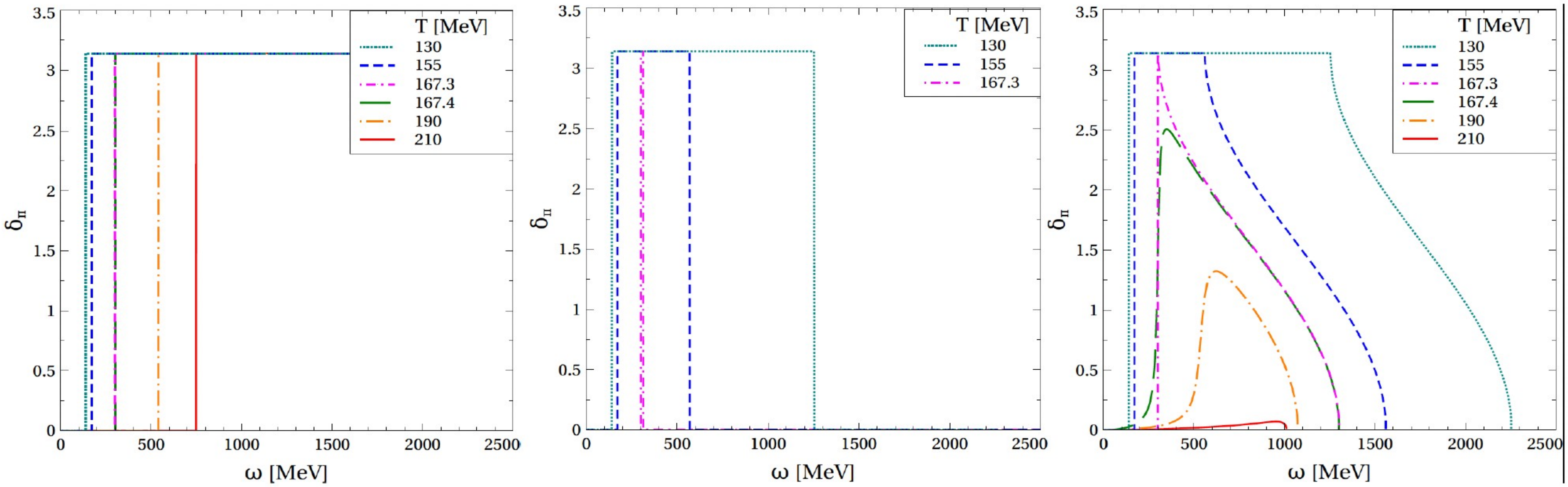
Baryons

PDG baryons	d_i	$M_{\rm PDG}$ [MeV]	M_i [MeV]	$M^<_{{\rm th},i}$ [MeV]	$M^>_{{\rm th},i}$ [MeV]
n/p	4	939	939	1881	16.8
Λ	2	1116	1082	2024	135.2
Σ	6	1193	1082	2024	135.2
Δ	16	1232	1251**)	3135	28
Ξ^0	2	1315	1225	2167	253.6
Ξ^-	2	1322	1225	2167	253.6
$\Sigma(1385)$	6	1385	1394**)	3278	146.4
$\Lambda(1405)$	2	1405	1394**)	3278	146.4
$N(1440)$	4	1440	1251**)	3135	28
..					
$N(2195)$	36	2220	1251**)	3135	28
$\Sigma(2250)$	6	2250	1394**)	3278	146.4
$\Omega^-(2250)$	2	2252	1680**)	3564	383.2
$N(2250)$	20	2275	1251**)	3135	28
$\Lambda(2350)$	10	2350	1394**)	3278	146.4
$\Delta(2420)$	48	2420	1251**)	3135	28
$N(2600)$	24	2600	1251**)	3135	28

… and colored clusters (model) !

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14

Inputs for the phase shifts (models)

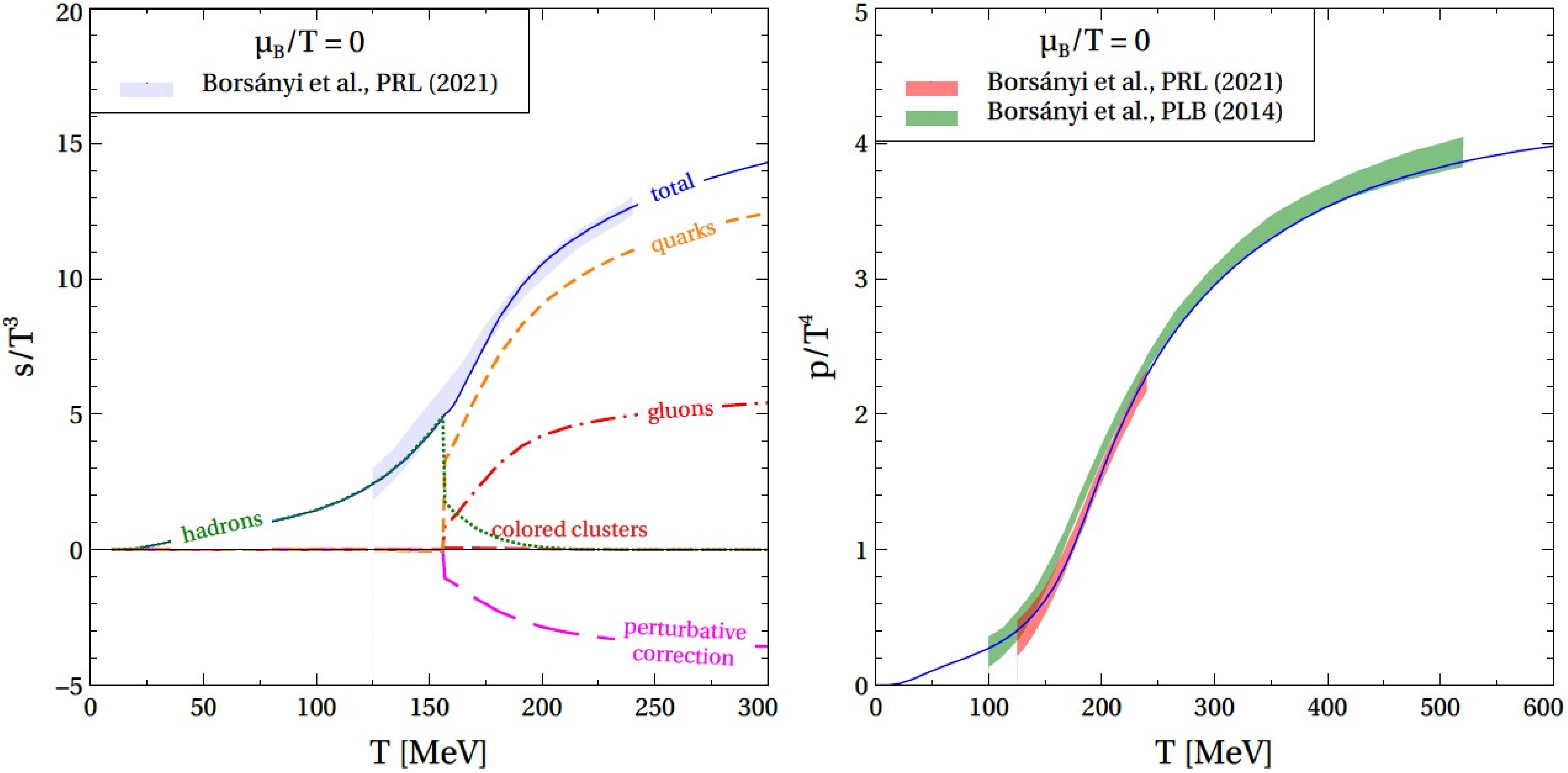


Step-up (SU) model → Hadron Resonance Gas

Step-up-step-down model

Step-up-continuum model → Mott Hadron Resonance Gas (MHRG)

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14



$\mu_B/T = 0$
Borsányi et al., PRL (2021)
total
quarks
gluons
hadrons
colored clusters
perturbative correction
s/T^3
T [MeV]
$\mu_B/T = 0$
Borsányi et al., PRL (2021)
Borsányi et al., PLB (2014)
p/T^4
T [MeV]

„Sudden Switch“ from HRG to QGP

Chemical Freeze-out: „inverse“ Mott effect – hadron localization = collapse of the wave function

$$H_{\rm exp}(T_{{\rm cf},i}) = \tau_i^{-1}(T_{{\rm cf},i})$$

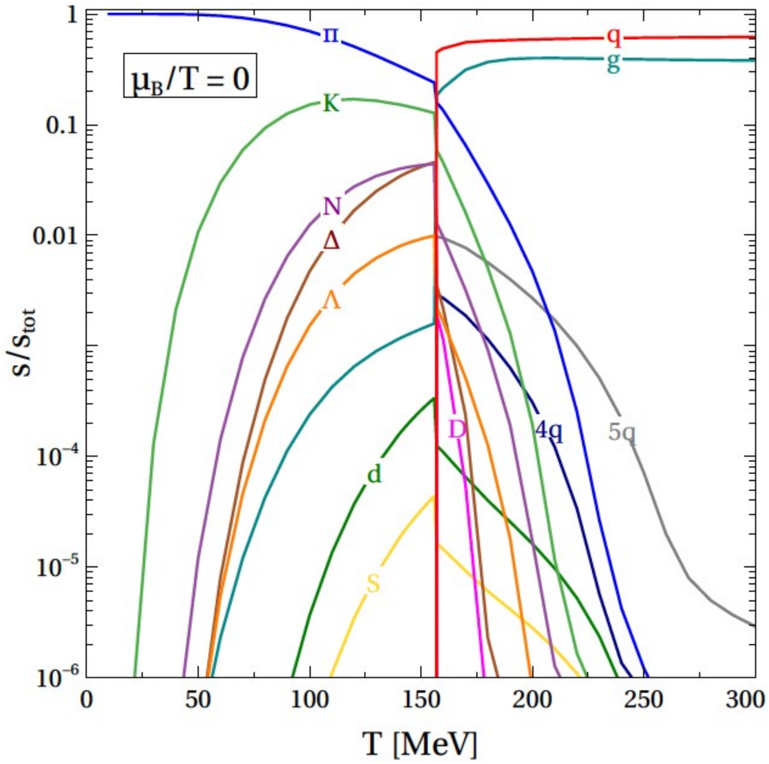
$$H_{\rm exp} = \frac{1}{\tau_{\rm exp}} = \frac{s^{1/3}}{a},$$

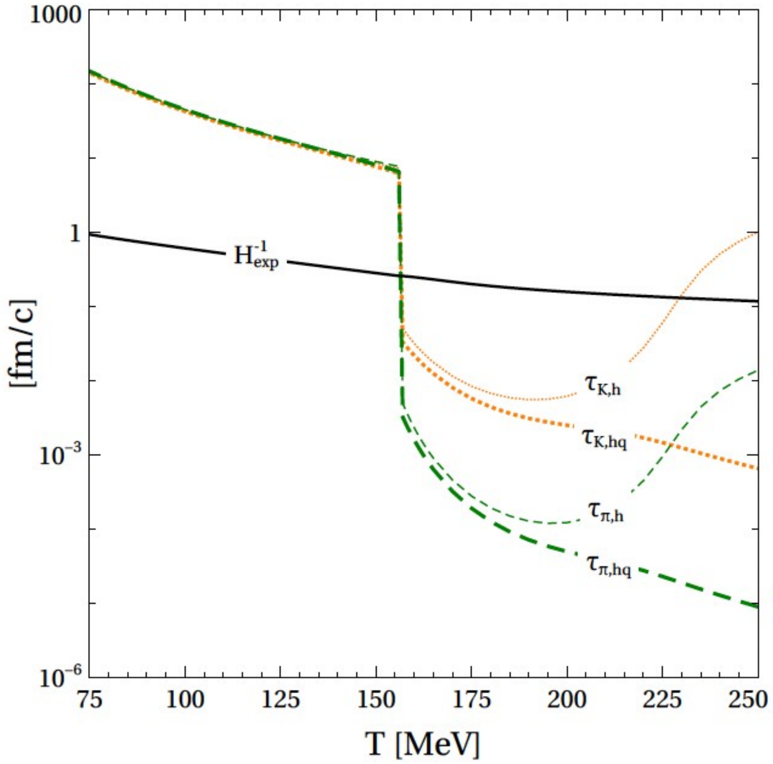
$$\tau_i^{-1} = \sum_i \sigma_{ij} v_{\rm rel} n_j,$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle,$$

$$\langle r^2 \rangle_\pi^{1/2} \sim |T - T_{{\rm Mott},\pi}|^{-1/2}$$

$$\langle r^2 \rangle_\pi \sim E_{B,\pi}(T)^{-1/2}$$





arXiv:2507.10497

Exploring the QCD Phase Diagram

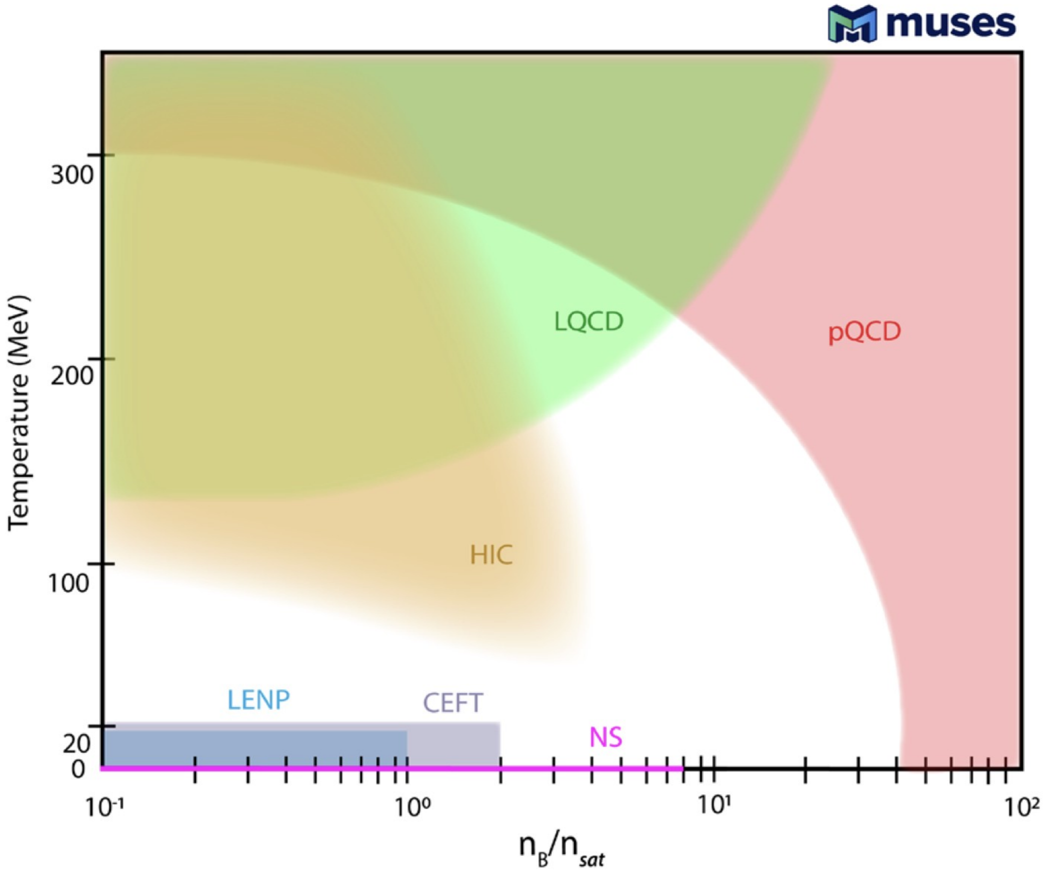
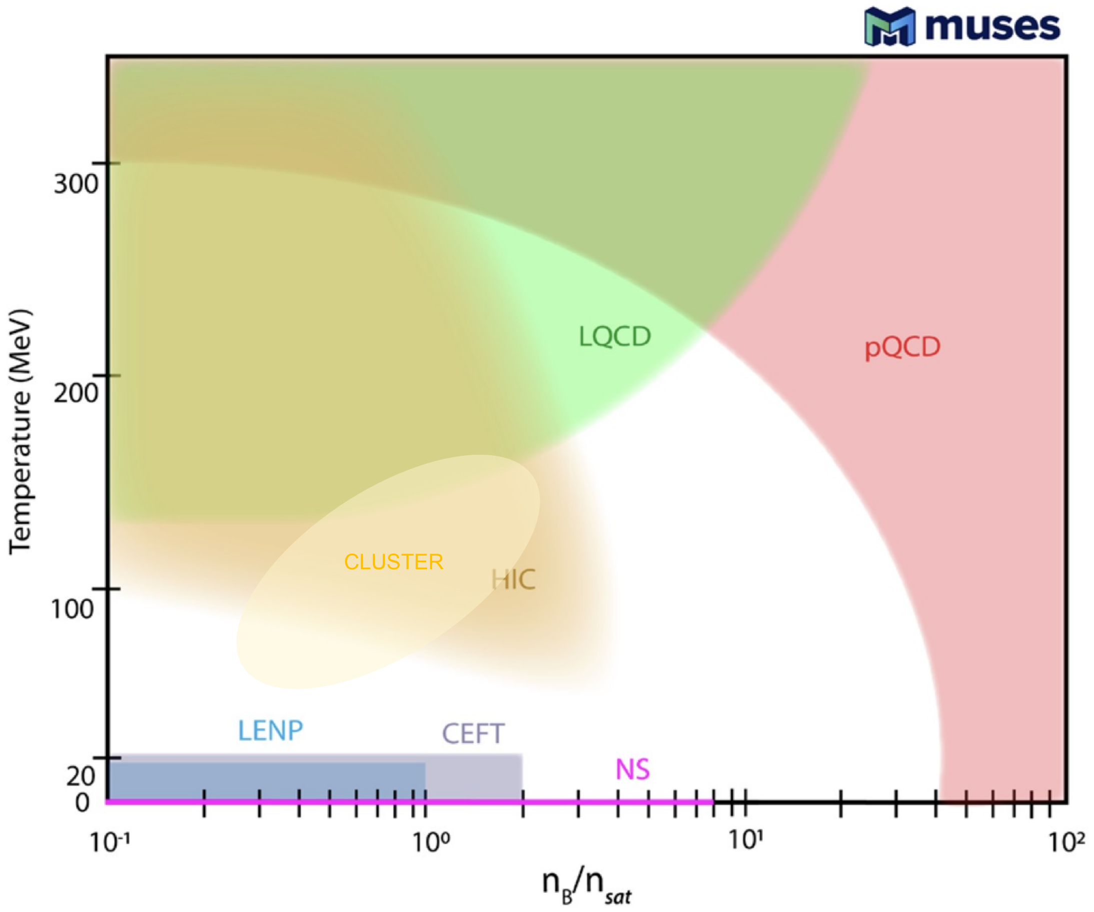


Fig. 1 Regions of the QCD phase diagram where constraints from heavy-ion collisions (HIC), lattice QCD (LQCD), perturbative QCD (pQCD), low-energy heavy-ion collisions (LENP), chiral effective field theory (χEFT), and astrophysics (neutron stars, NS) are available

Living Reviews in Relativity (2024)27:3
https://doi.org/10.1007/s41114-024-00049-6

Exploring the QCD Phase Diagram

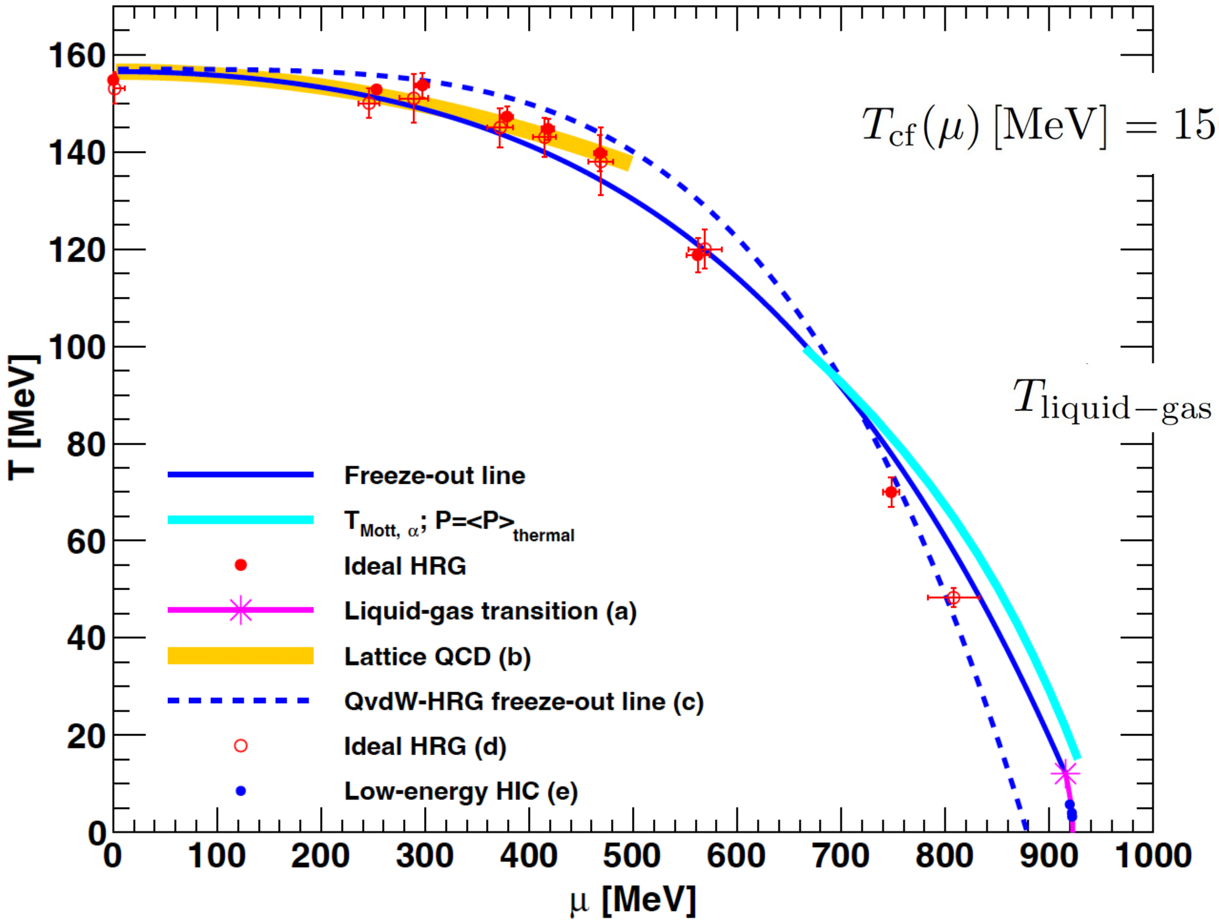


This Talk:

Chemical Freeze-Out @ T ~ 20 – 100 MeV

- Statistical Model Fit, T – μ diagram
- CFO in the T – n diagram
- Mott dissociation for light clusters
- CFO as inverse Mott dissociation
- Summary & Outlook

Statistical Model Fit for CFO, T – μ Diagram



The freeze-out line (blue):

$$T_{\rm cf}(\mu)\,[\mathrm{MeV}] = 156.5 - 76.68\,(\mu\,[\mathrm{GeV}])^2 - 139.7\,(\mu\,[\mathrm{GeV}])^4$$

interpolates between lattice QCD chiral restoration crossover (yellow) and CEP of the liquid-gas transition

$$T_{\rm liquid-gas} = 12.1\,\mathrm{MeV},\ \mu_{\rm liquid-gas} = 915.61\,\mathrm{MeV}$$

Mott dissociation line for alphas is well correlated in baryon-dominant region

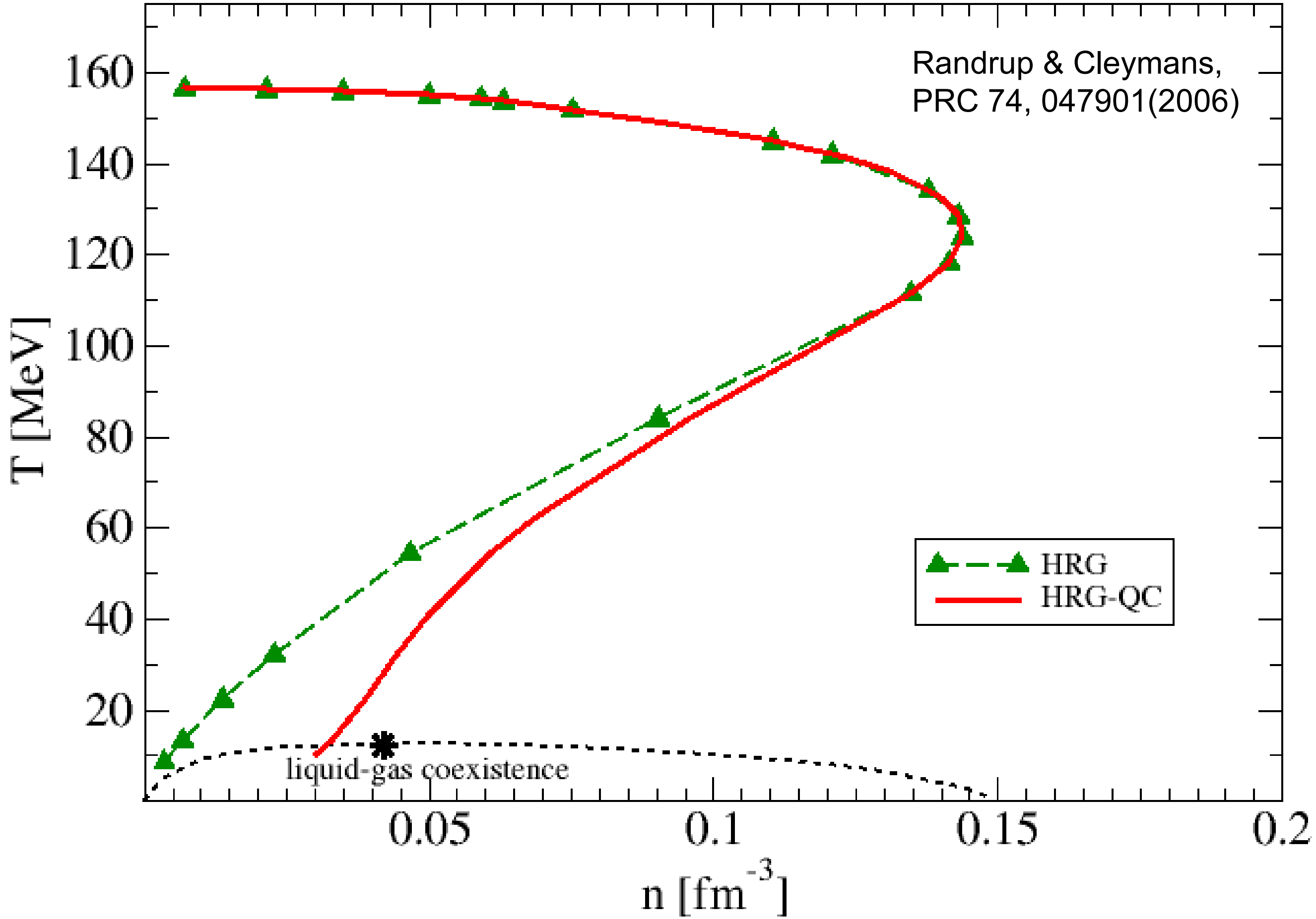
(a) Typel et al., PRC 81, 015803 (2010)

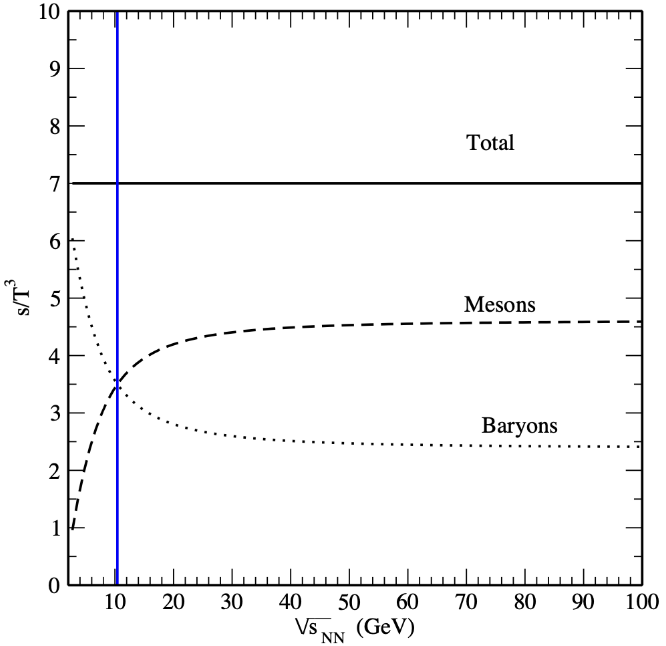
(c) Poberezhnyuk et al., PRC 100, 054904 (2019)

(e) Natowitz et al., PRL 104, 202501 (2010)

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

Statistical Model Fit for CFO, T – n Diagram

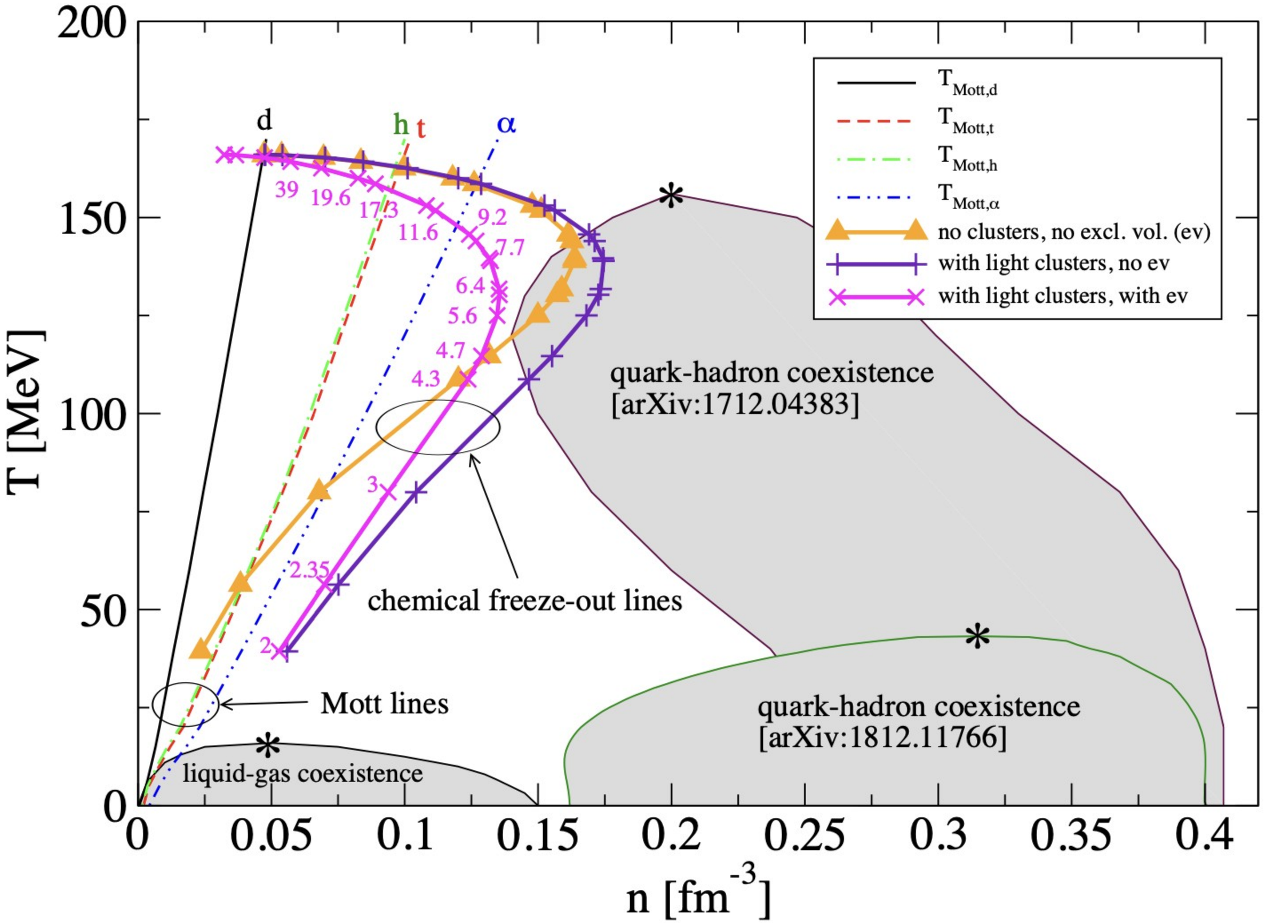




Transition meson- to baryon-dominance at T ~ 140 MeV

Andronic et al., NPA 837, 65 (2010)

Statistical Model Fit for CFO, T – n Diagram



Correlation of CFO line with chiral restoration/ deconfinement gets lost at T < 140 MeV

Is CFO in the baryon-dominated region correlated with Mott lines for dissociation of light clusters?

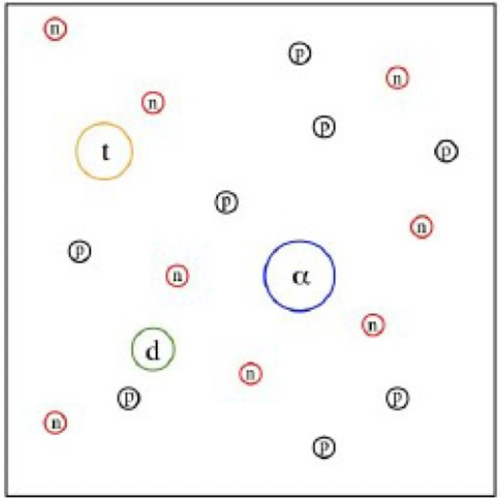
Blaschke et al., Springer Proc. Phys. 250, 183 (2020); arXiv:2001.02156 (SQM 2019)

Mott dissociation for bound states in a plasma

Chemical picture:

Ideal mixture of reacting components

Mass action law

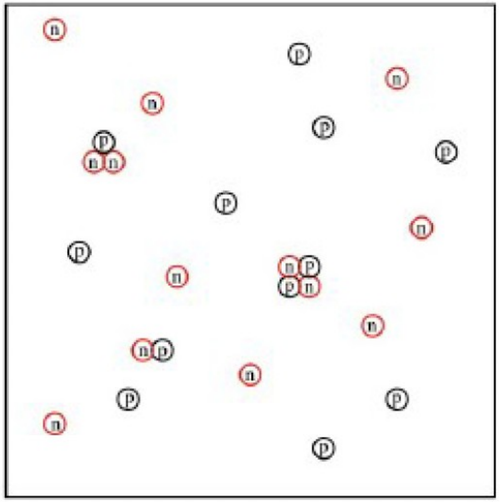


Interaction between the components
internal structure: Pauli principle

Physical picture:

"elementary" constituents

and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Mott dissociation for bound states in a plasma

Effective wave equation for deuterons in nuclear matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

Add self-energy

Pauli-blocking

$$= E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Thouless criterion

$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Mott dissociation for bound states in a plasma

Effective wave equation for deuterons in nuclear matter

[Derivation of a „Plasma Hamiltonian" from a Bethe-Goldstone Eq. for two-particle states]

In-medium two-particle wave equation in mean-field approximation

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$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

[R. Zimmermann et al. (5-men-work), Phys. Stat. Sol. (b) 90 (1978) 175]

Thouless criterion

$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

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Mott dissociation for clusters in nuclear matter

Nuclear Physics **A379** (1982) 536–552

PARTICLE CLUSTERING AND MOTT TRANSITIONS IN NUCLEAR MATTER AT FINITE TEMPERATURE
(I). Method and general aspects

G. RÖPKE

Sektion Physik, Wilhelm-Pieck-University, Rostock, GDR

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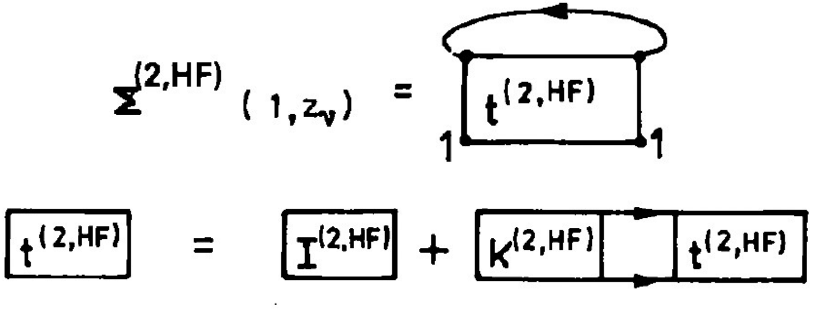

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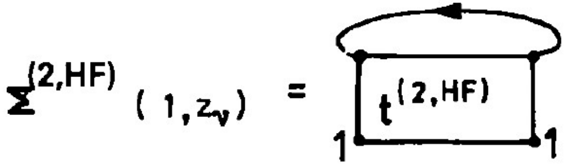

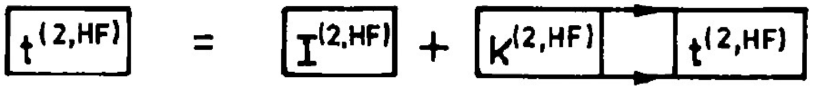

$$t^{(2,\mathrm{HF})}(1234,\Omega_\lambda)=\sum_\alpha \frac{\phi_\alpha^{\mathrm{HF}}(12)\phi_\alpha^{\mathrm{HF}}(34)}{E_\alpha^{\mathrm{HF}}-\hbar\Omega_\lambda}(1+\tfrac{1}{2}(f(1)+f(2)+f(3)+f(4)))$$
$$\times(\hbar\Omega_\lambda-E(1)-\Delta^{\mathrm{HF}}(1)-E(2)-\Delta^{\mathrm{HF}}(2))$$
$$\times(E_\alpha^{\mathrm{HF}}-E(3)-\Delta^{\mathrm{HF}}(3)-E(4)-\Delta^{\mathrm{HF}}(4)),$$

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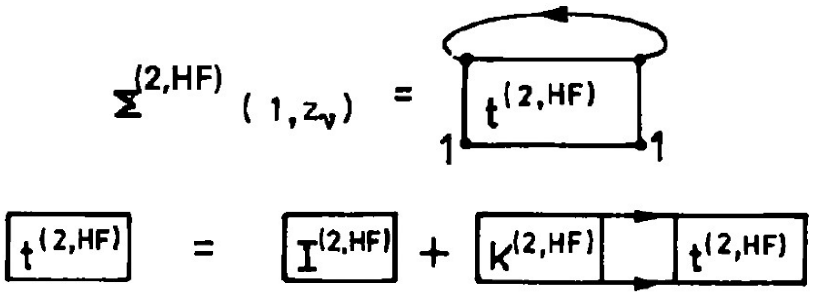

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$$\{E(1)+E(2)-E_\alpha^{\mathrm{HF}}\}\phi_\alpha^{\mathrm{HF}}(12)+\sum_{1'2'}V(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')$$
$$=\sum_{1'2'}\{\tfrac{1}{2}(f(1)+f(2)+f(1')+f(2'))V(12,1'2')$$
$$-(\Delta^{\mathrm{HF}}(1)+\Delta^{\mathrm{HF}}(2))\,\delta_{11'}\,\delta_{22'}\}\phi_\alpha^{\mathrm{HF}}(1'2')$$
$$=-\sum_{1'2'}H^{(2,\mathrm{HF})}_{\mathrm{nucl.matter}}(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2').$$

Mott dissociation for clusters in nuclear matter

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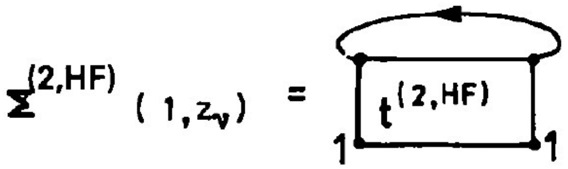
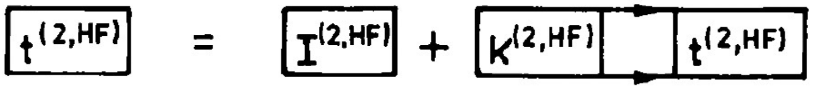
$$\Delta E_\alpha^{\mathrm{HF}} = \sum_{121'2'} \phi_\alpha^0(12)\phi_\alpha^{0*}(1'2')H^{(2,\mathrm{HF})}_{\mathrm{nucl.matter}}(121'2')\,,$$





$$t^{(2,\mathrm{HF})}(1234,\Omega_\lambda) = \sum_\alpha \frac{\phi_\alpha^{\mathrm{HF}}(12)\phi_\alpha^{\mathrm{HF}}(34)}{E_\alpha^{\mathrm{HF}} - \hbar\Omega_\lambda}(1+\tfrac{1}{2}(f(1)+f(2)+f(3)+f(4)))$$
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$$\times(E_\alpha^{\mathrm{HF}} - E(3) - \Delta^{\mathrm{HF}}(3) - E(4) - \Delta^{\mathrm{HF}}(4))\,,$$

$$\{E(1)+E(2)-E_\alpha^{\mathrm{HF}}\}\phi_\alpha^{\mathrm{HF}}(12) + \sum_{1'2'} V(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')$$
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$$-(\Delta^{\mathrm{HF}}(1)+\Delta^{\mathrm{HF}}(2))\,\delta_{11'}\,\delta_{22'}\}\phi_\alpha^{\mathrm{HF}}(1'2')$$
$$= -\sum_{1'2'} H^{(2,\mathrm{HF})}_{\mathrm{nucl.matter}}(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')\,.$$

Mott dissociation for clusters in nuclear matter

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(I). Method and general aspects

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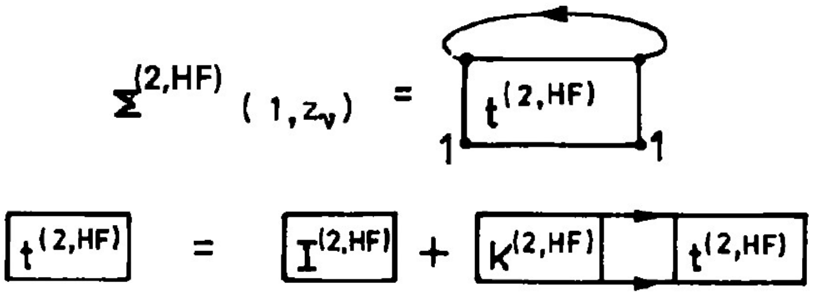
$$\Delta E_{d,P}^{\rm HF} = \tfrac{3}{2}t_0\rho_{\rm nucl} - \sqrt{2}t_0\rho_{\rm nucl}(1+x_0)\left(1+\frac{\pi}{\alpha^2\Lambda^2}\right)^{-3/2}\exp\left[-\frac{P^2}{16\alpha^2}\left(1+\frac{\pi}{\Lambda^2\alpha^2}\right)^{-1}\right]$$
$$= 2\Delta^{\rm HF}(0) + \Delta_{d,P}^{\rm Pauli}\rho_{\rm nucl}\,, \qquad (2.$$



$$t^{(2,\rm HF)}(1234,\Omega_\lambda) = \sum_\alpha \frac{\phi_\alpha^{\rm HF}(12)\phi_\alpha^{\rm HF}(34)}{E_\alpha^{\rm HF} - \hbar\Omega_\lambda}(1+\tfrac{1}{2}(f(1)+f(2)+f(3)+f(4)))$$
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Mott dissociation for clusters in nuclear matter

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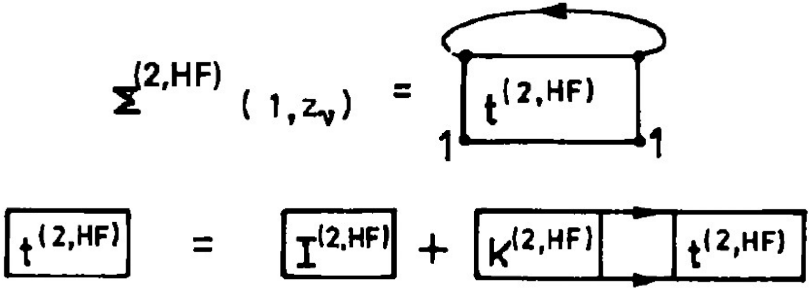
$$= 2\Delta^{HF}(0) + \Delta^{\text{Pauli}}_{d,P}\rho_{\text{nucl}} ,$$

$$\alpha = \hbar(|E_d^0|/M)^{1/2} = 0.23\ \text{fm}^{-1} .$$

$$\Lambda = (2\pi\beta\hbar^2/M)^{1/2}$$

$$\rho^{(0)}_{\text{nucl}}(\beta,\mu) = \frac{4}{\Lambda^3}\exp(\beta\mu) ,\quad t_0 = -1057.3\ \text{MeV}\cdot\text{fm}^3 \text{ and } x_0 = 0.56 .$$



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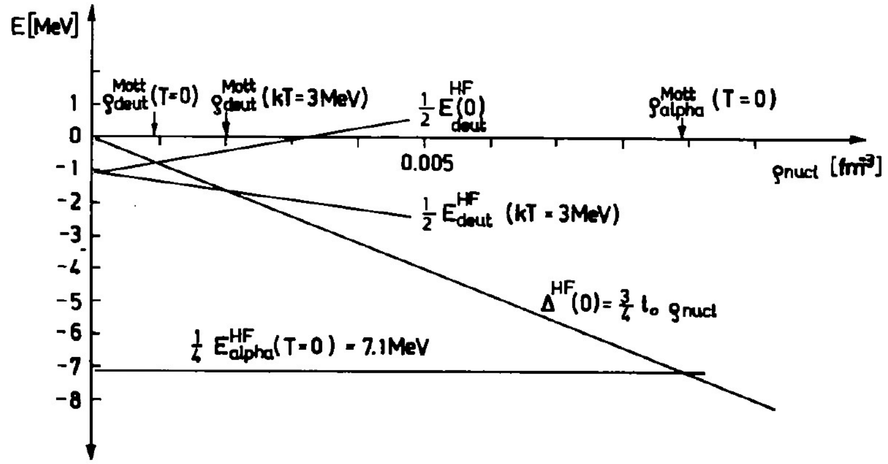
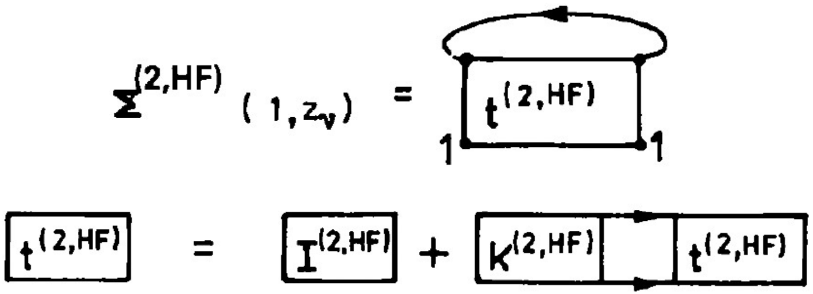
Mott dissociation for clusters in nuclear matter





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$$\Lambda=(2\pi\beta\hbar^2/M)^{1/2}$$

$$\rho^{(0)}_{\mathrm{nucl}}(\beta,\mu)=\frac{4}{\Lambda^3}\exp(\beta\mu),\quad t_0=-1057.3\ \mathrm{MeV}\cdot\mathrm{fm}^3\ \text{ and }\ x_0=0.56.$$

Mott dissociation for clusters in nuclear matter

Present state-of-the-art:

G. Röpke, Nuclear matter equation of state including two-, three-, and four-nucleon correlations, Phys. Rev. C 92 (5) (2015) 054001, https://doi.org/10.1103/PhysRevC.92.054001, arXiv:1411.4593.

$$E_{A,\nu}(P) = E^0_{A,\nu}(P) + \Delta E^{\rm SE}_{A,\nu}(P) + \Delta E^{\rm Pauli}_{A,\nu}(P) + \Delta E^{\rm Coulomb}_{A,\nu}(P).$$

The cluster binding energies $E^{\rm bind}_{A,\nu}(P;T,n_B,Y_p,T_{\rm eff})$ are defined as

$$E^{\rm bind}_{A,\nu}(P;T,n_B,Y_p) = -[E_{A,\nu}(P;T,n_B,Y_p) - E^{\rm cont}_{A,\nu}(P;T,n_B,Y_p)]$$

$$E^{\rm cont}_{A,\nu}(P;T,n_B,Y_p) = N E_n(P/A;T,n_B,Y_p) + Z E_p(P/A;T,n_B,Y_p)$$

the in-medium dispersion relations for nucleons ($\tau = n, p$) are defined as

$$E_\tau(p;T,n_B,Y_p) = \sqrt{[m_\tau c^2 - S(T,n_B,Y_p)]^2 + \hbar^2 c^2 p^2} + V_\tau(T,n_B,Y_p) - m_\tau c^2$$

$$S_i(T,n_B,Y_p) = (4463 - 6.610\,T - 0.1703\delta^2 + 4.112\delta^4)n_B \times \frac{1 + c_1 n_b + c_2 n_B^2}{1 + c_3 n_b + c_4 n_B^2},$$

$$V_p(T,n_B,Y_p) = (3403 + 0.000052\,T - 486.6\,\delta - 2.420\,\delta^2)n_B \times \frac{1 + d_1 n_b + d_2 n_B^2}{1 + d_3 n_b + d_4 n_B^2}$$

Scalar and vector mean fields from RDF EoS DD2
S. Typel et al., Phys. Rev. C 81, 015803 (2010)

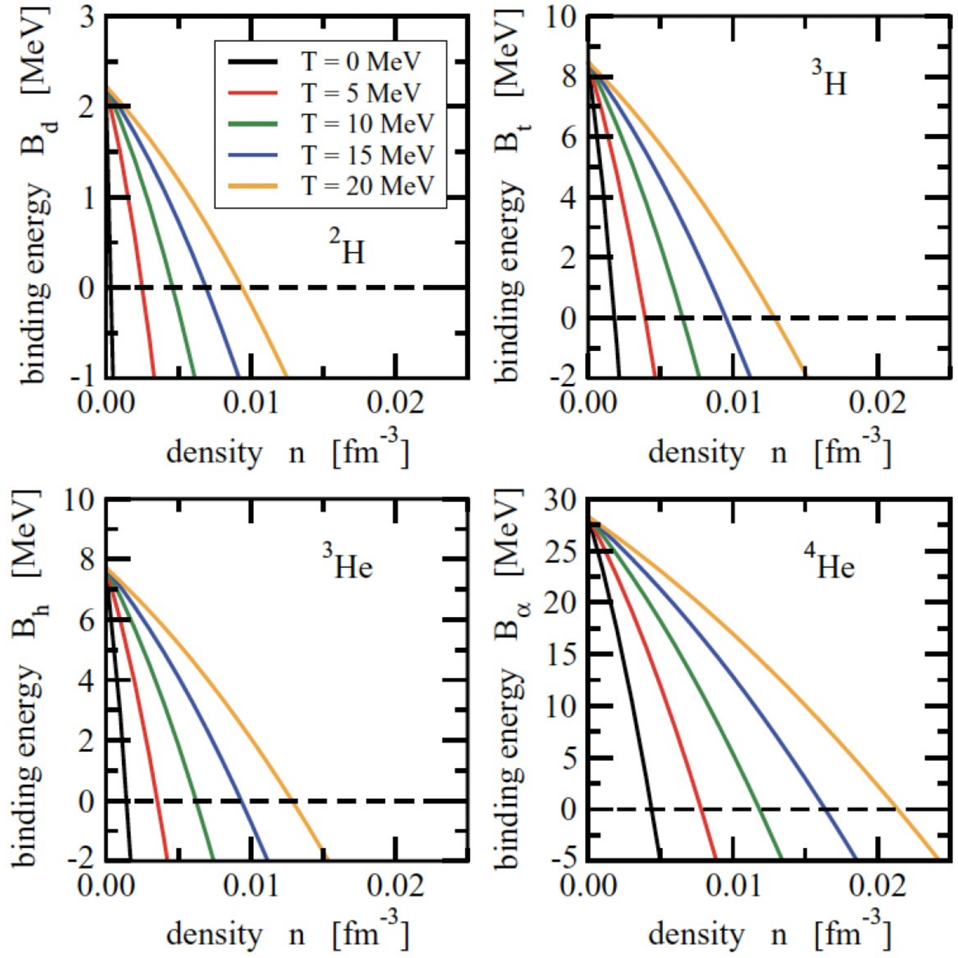
Parameter fit provided by G. Röpke et al.,
Phys. Part. Nucl. Lett. 15, 225 (2018)

$$c_1 = 20.56 - 0.04099\,T - 0.3394\,\delta^2 + 0.9972\,\delta^4\,,$$
$$c_2 = 15.98 + 0.8664\,T - 2.020\,\delta^2 - 3.018\,\delta^4\,,$$
$$c_3 = 24.27 - 0.07417\,T - 0.5427\,\delta^2 + 1.196\,\delta^4\,,$$
$$c_4 = 114.6 + 1.350\,T + 2.674\,\delta^2 + 0.7268\,\delta^4\,,$$

$$d_1 = 0.6629 - 0.006142\,T - 1.141\,\delta - 0.7176\,\delta^2,$$
$$d_2 = 10.7780 + 0.004432\,T + 0.8020\,\delta + 0.4576\,\delta^2,$$
$$d_3 = 3.433 + 0.000104\,T - 1.549\,\delta - 0.3360\,\delta^2,$$
$$d_4 = 23.01 - 0.03302\,T - 5.923\,\delta + 0.05090\,\delta^2,$$

with the isospin asymmetry $\delta = (1 - 2Y_p)$

Binding energies for light clusters in T – n plane



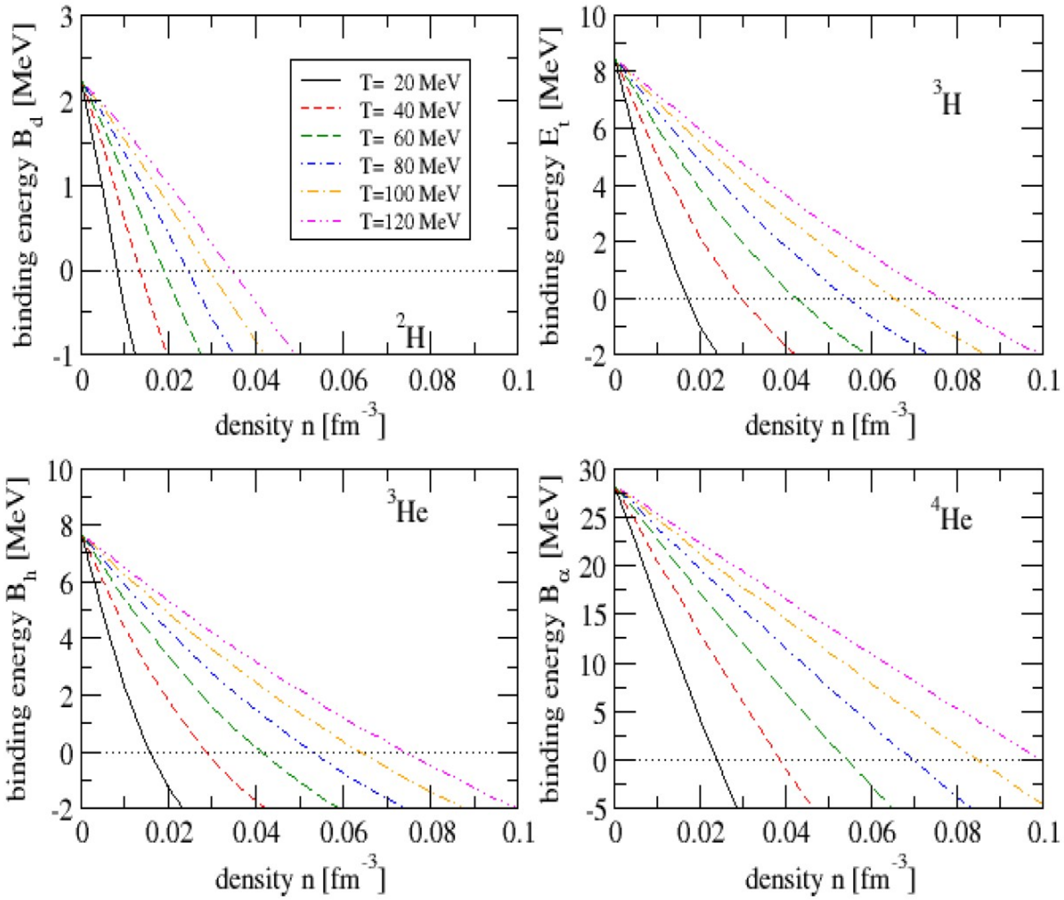
Vanishing binding energies
Indicate Mott effect for the
Light clusters!

Mott-lines in the T-μ plane
can be extracted, where the
Binding energy vanishes

Here lower temperatures:
0 < T[MeV] < 20

S. Typel et al., PRC 81,
015803 (2010)

Binding energies for light clusters in T – n plane

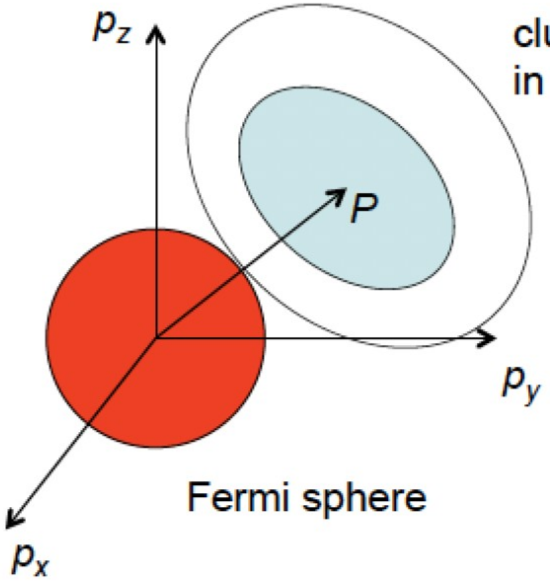


Mott-lines in the T-μ plane can be extracted, where The binding energy vanishes

Here higher temperatures:

20 < T[MeV] < 120

Pauli blocking: phase space occupation



cluster wave function (deuteron, alpha,…) in momentum space

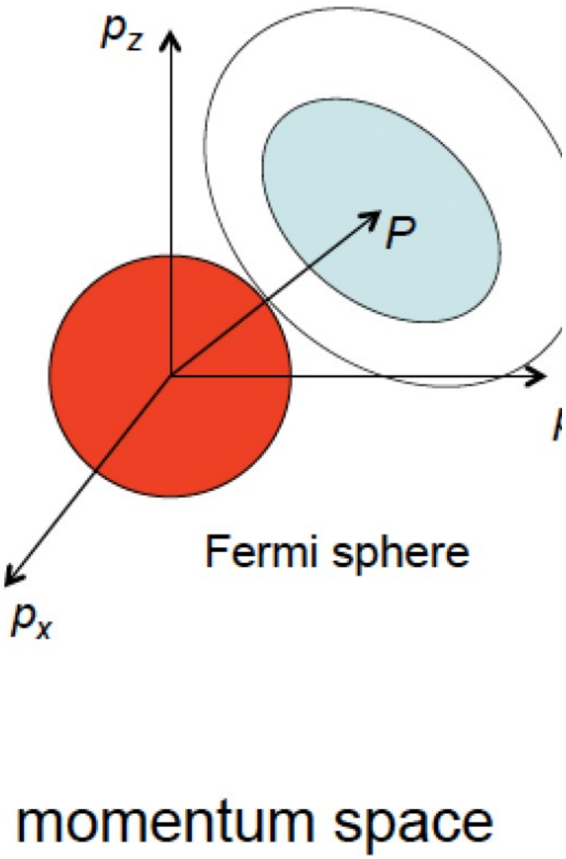
P - center of mass momentum

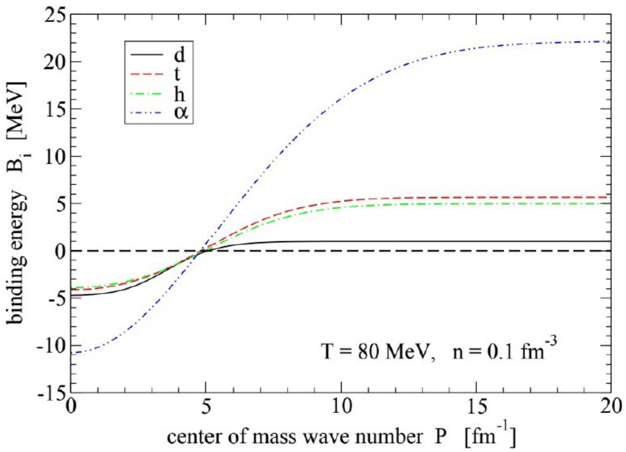
The Fermi sphere is forbidden, deformation of the cluster wave function in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction (disintegration of the bound state).

Momentum-dependent binding energies

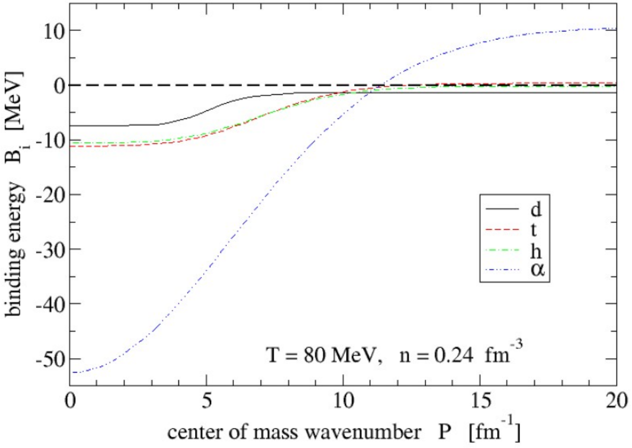


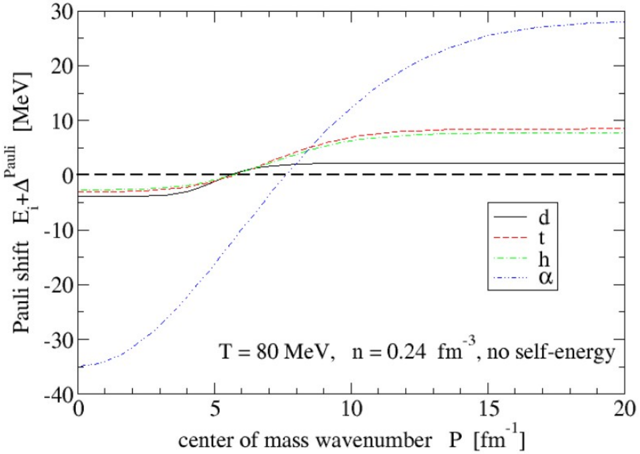


The light clusters that underwent a Mott Dissociation for low momenta become "resurrected" at high momenta relative to the medium !

The minimal momentum where this Occurs is called "Mott momentum";
It depends on temperature and density

Binding energies without selfenergy shift, Only Pauli blocking shift accounted for

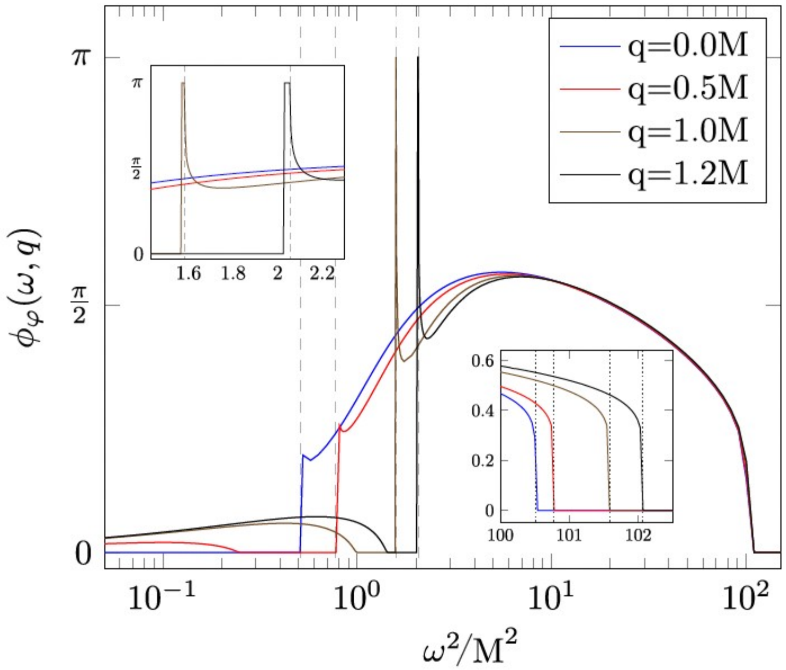


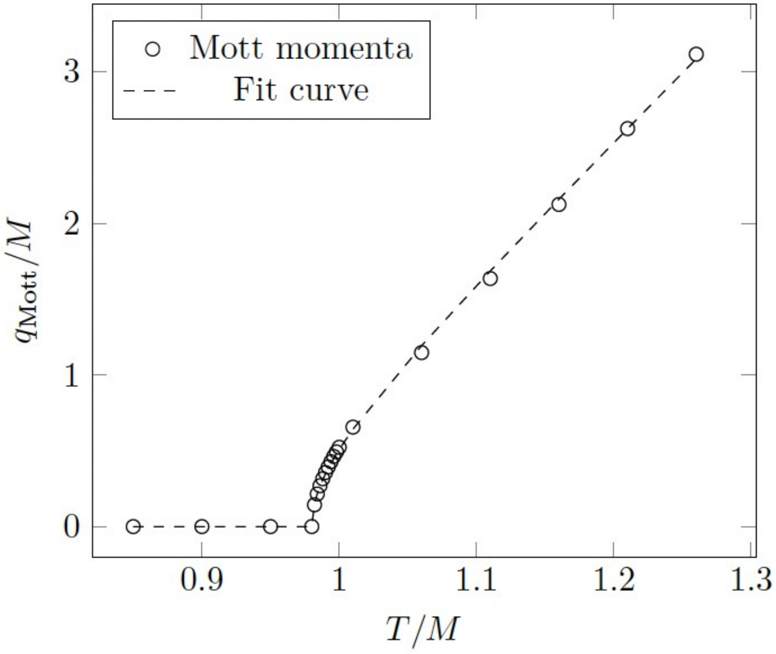


Mott momentum for bound states in matter

Based on the Pauli principle (100th anniversary), the Mott momentum is a general effect for bound states in matter, e.g. excitons in graphene!

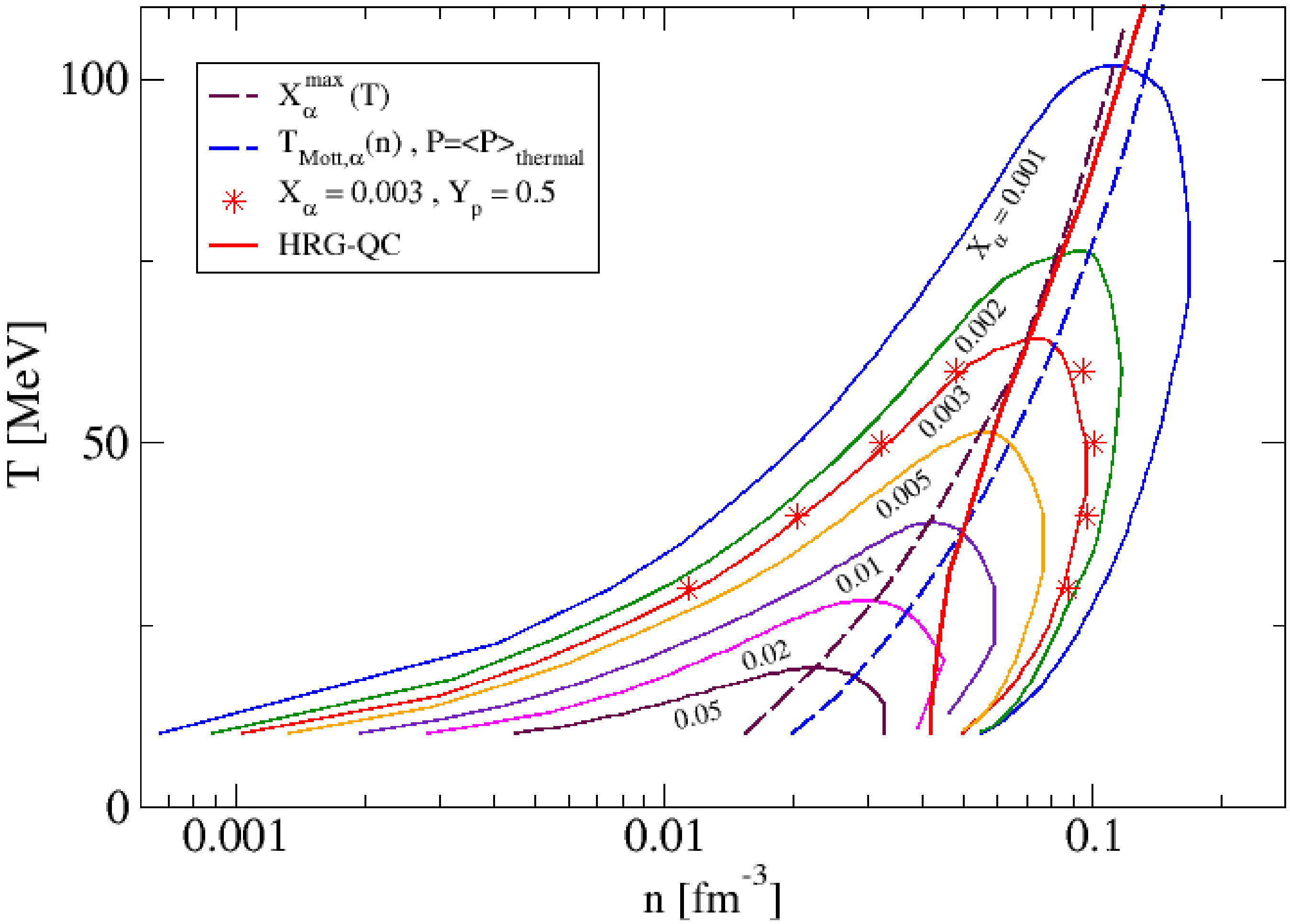
Biplab Mahato, D.B., D. Ebert,
Beth-Uhlenbeck equation for the thermodynamics of fluctuations in a generalized 2+1D Gross-Neveu model (in preparation)





See also: Pauli potential effects on the tetraquark spectrum [Morgan Kuchta, Master Thesis, University of Wroclaw (2024)]

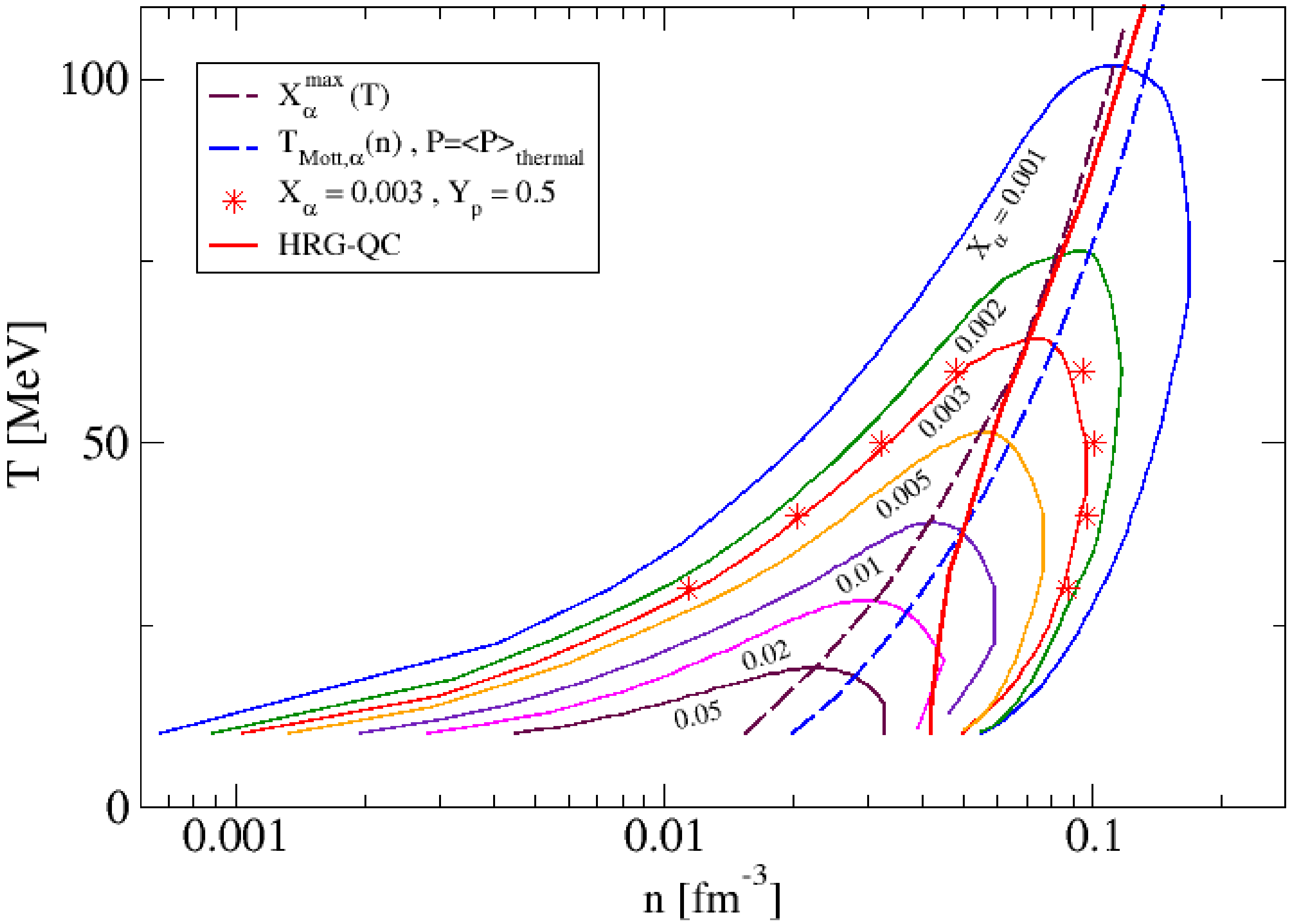
CFO in the Temperature – density plane



The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

CFO in the Temperature – density plane



The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

Pauli blocking →

→ Mott dissociation

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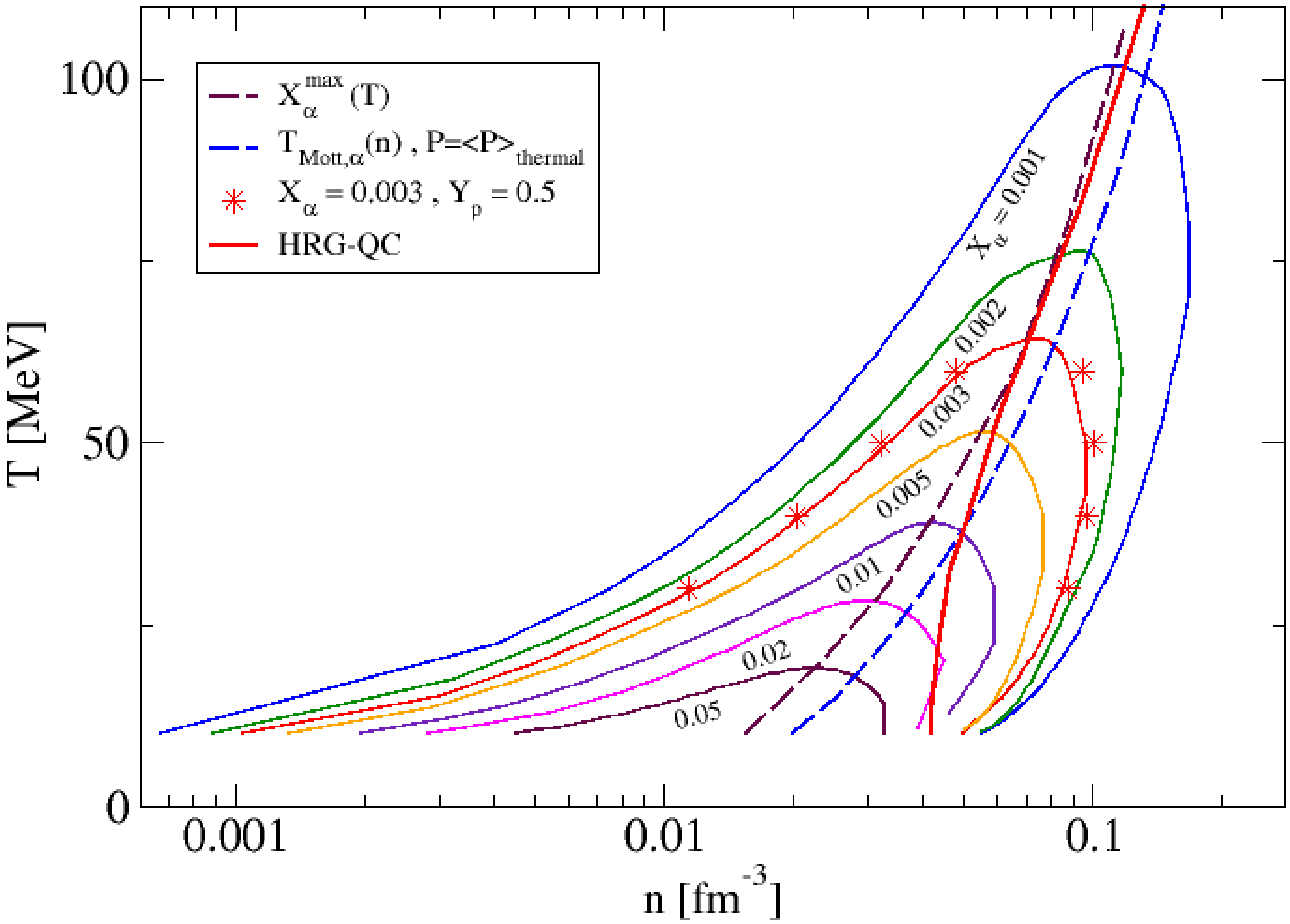
CFO in the Temperature – density plane



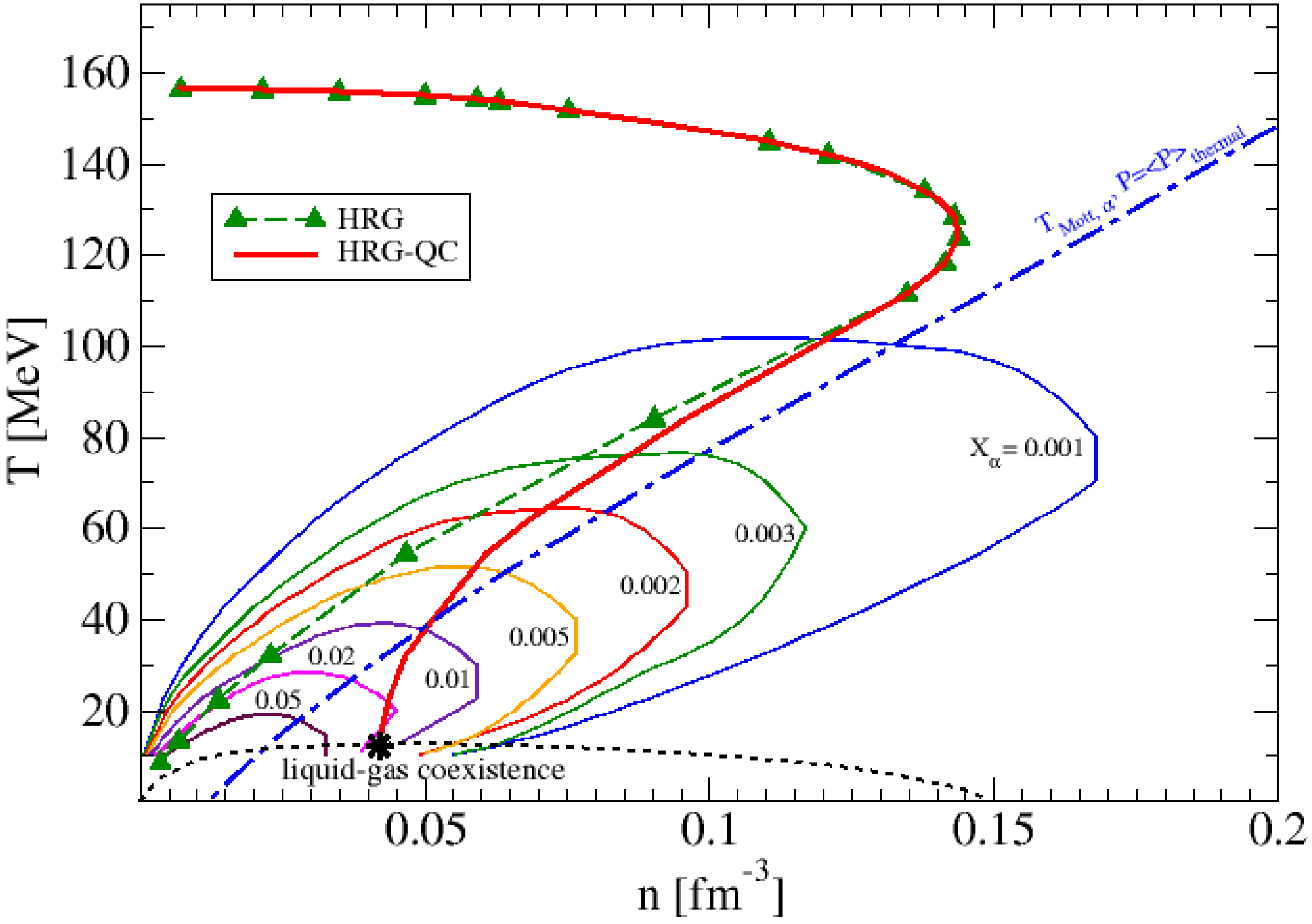
The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

Pauli blocking →
→ Mott dissociation

Mott-line for alpha clusters (equivalent to the line of maximum alpha fraction) is well correlated with the Chemical Freeze-Out (CFO) line

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

CFO in the Temperature – density plane



Main result:

Chemical freeze-out may be interpreted as „inverse" Mott transition:

Strong localization effect of nucleon-nucleon correlations in bound states (clusters) entails freeze out of the nuclear composition

„collapse of wave function"

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

III. Density Functional for Quark Matter in Hybrid Neutron Stars

Relativistic density functional for quark matter

What is new?

O. Ivanytskyi & D.B., Phys. Rev. D 105 (2022) 114042

Interaction $\mathcal{U} = D_0 \left[(1+\alpha)\langle\overline{q}q\rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^{\varkappa}$

- **Parameters**

  D_0 - dimensionfull coupling, controls interaction strength

  α - dimensionless constant, controls vacuum quark mass

  $\langle\overline{q}q\rangle_0$ - χ-condensate in vacuum (introduced for the sake of convenience)

$\varkappa = 1/3$ $\Downarrow$ motivated by String Flip model

$\varkappa = 1$ $\Downarrow$ Nambu–Jona-Lasinio model

$$\mathcal{U}_{SFM} \propto \langle q^+ q\rangle^{2/3}$$

$$\Sigma_{SFM} = \frac{\partial\mathcal{U}_{SFM}}{\partial\langle q^+ q\rangle} \propto \langle q^+ q\rangle^{-1/3} \propto separation$$

- **Dimensionality**

$$[\mathcal{U}] = energy^4 \quad [\overline{q}q] = energy^3 \quad \Rightarrow \quad [D_0]_{\varkappa=1/3} = energy^2 = [string\ \ tension]$$

self energy = string tension × separation $\Rightarrow$ **confinement**

Relativistic density functional for quark matter
Expansion around mean fields

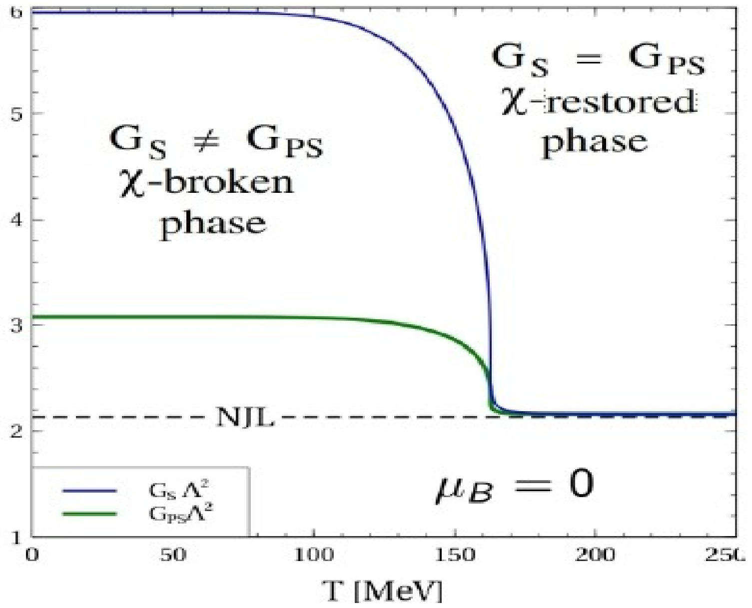
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{th}\ \text{order}} + \underbrace{(\bar{q}q - \langle\bar{q}q\rangle)\,\Sigma_S}_{1^{st}\ \text{order}} - \underbrace{G_S\,(\bar{q}q - \langle\bar{q}q\rangle)^2 - G_{PS}\,(\bar{q}i\vec{\tau}\gamma_5 q)^2}_{2^{nd}\ \text{order}} + \ldots$$

- **Mean-field scalar self-energy**

$$\Sigma_S = \frac{\partial\mathcal{U}_{MF}}{\partial\langle\bar{q}q\rangle}$$

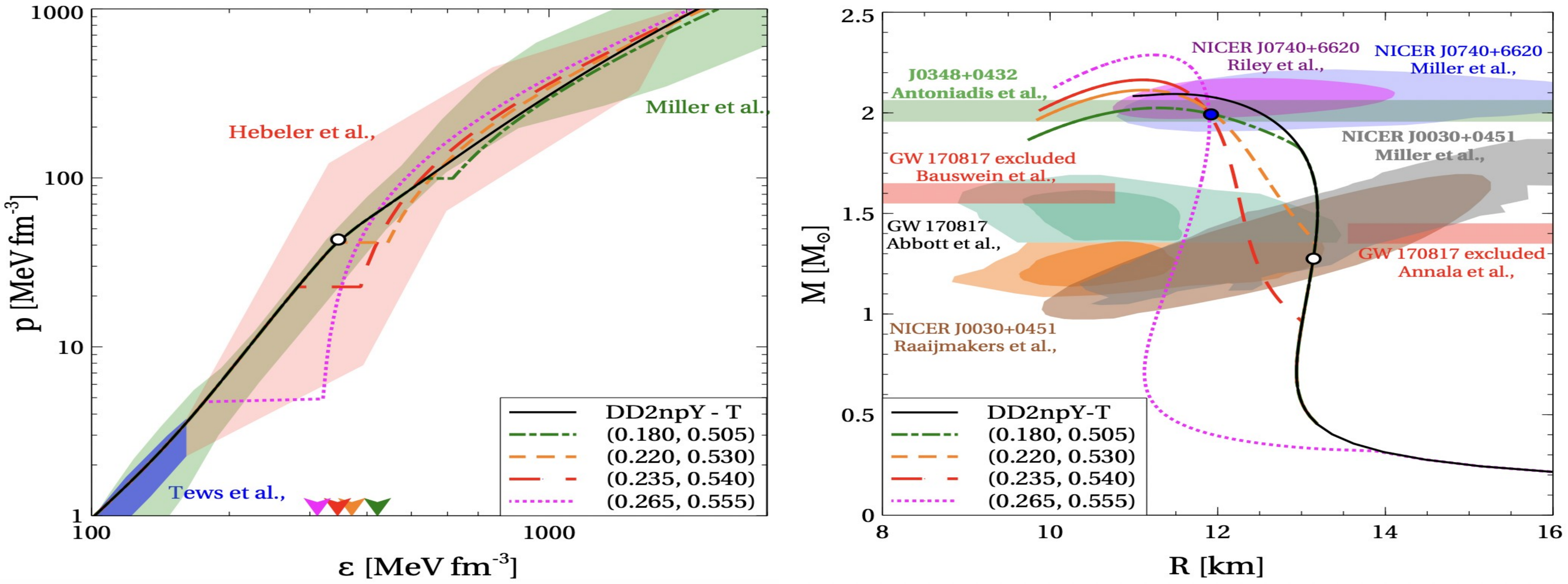
- **Effective medium dependent couplings**

$$G_S = -\frac{1}{2}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\bar{q}q\rangle^2}, \qquad G_{PS} = -\frac{1}{6}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\bar{q}i\vec{\tau}\gamma_5 q\rangle^2}$$



Relativistic density functional for quark matter

EOS and Mass-radius diagram for hybrid neutron stars



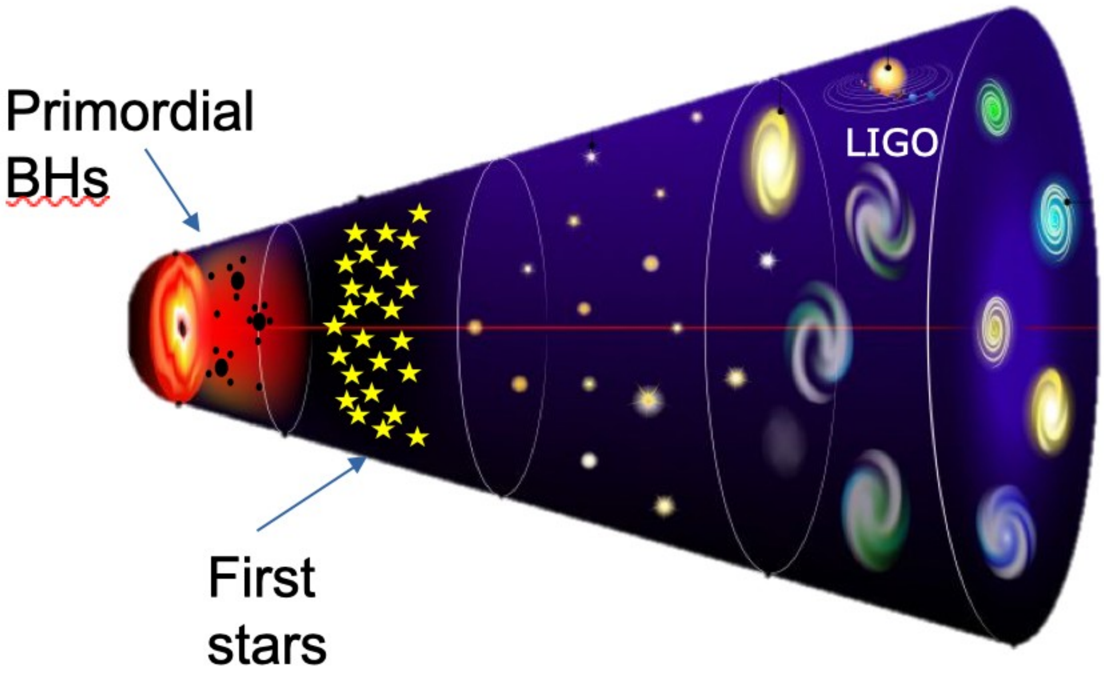
Observational constraints prefer early onset of deconfinement

IV. Primordial Black-Hole Formation and heavy r-Process Element Synthesis from the Cosmological QCD Transition

Mael Gonin et al., EPJA 61 (2025) 170; arXiv:2505.05463

PBH Formation and CFO of heavy elements?

JWST results – primordial black holes !

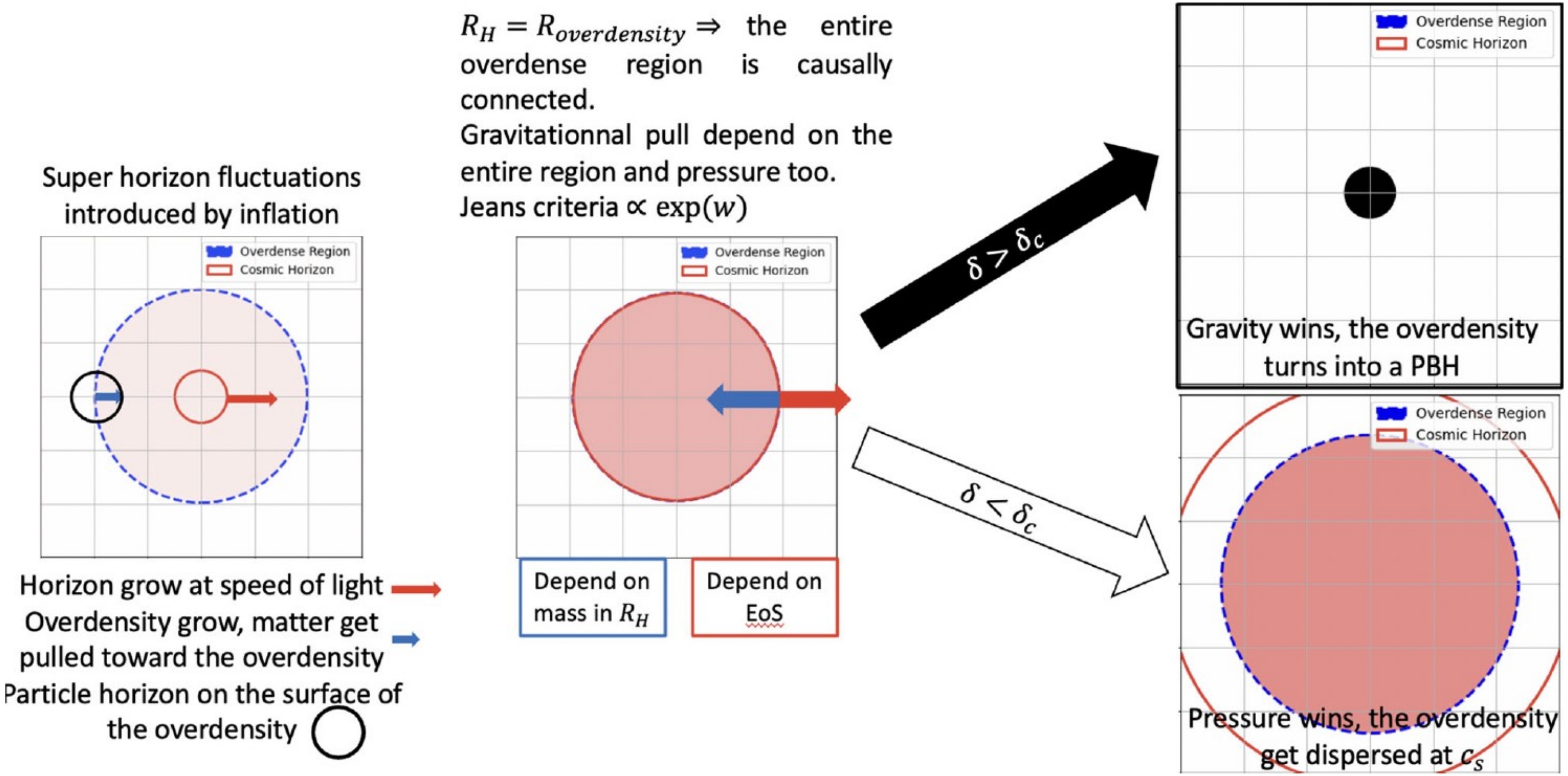


Talk at University of Wroclaw
by Günther Hasinger,
Founding director of the
German Centre for Astrophysics
In Görlitz:

DZA

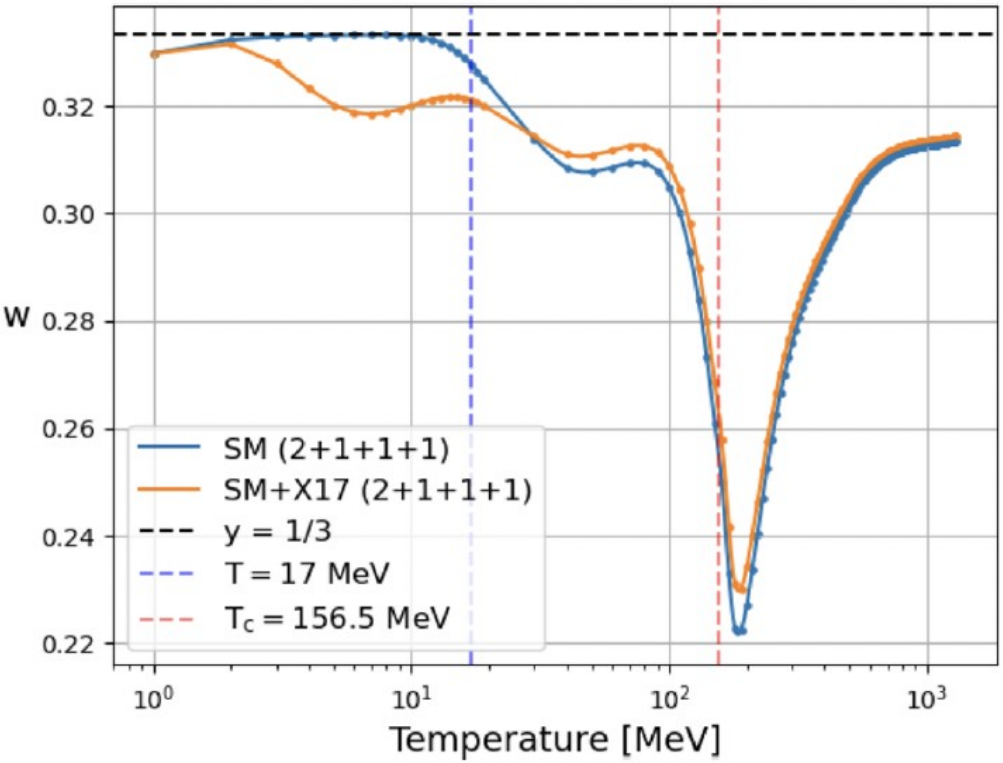
QCD hadronization transition plays key role plays for PBH formation !

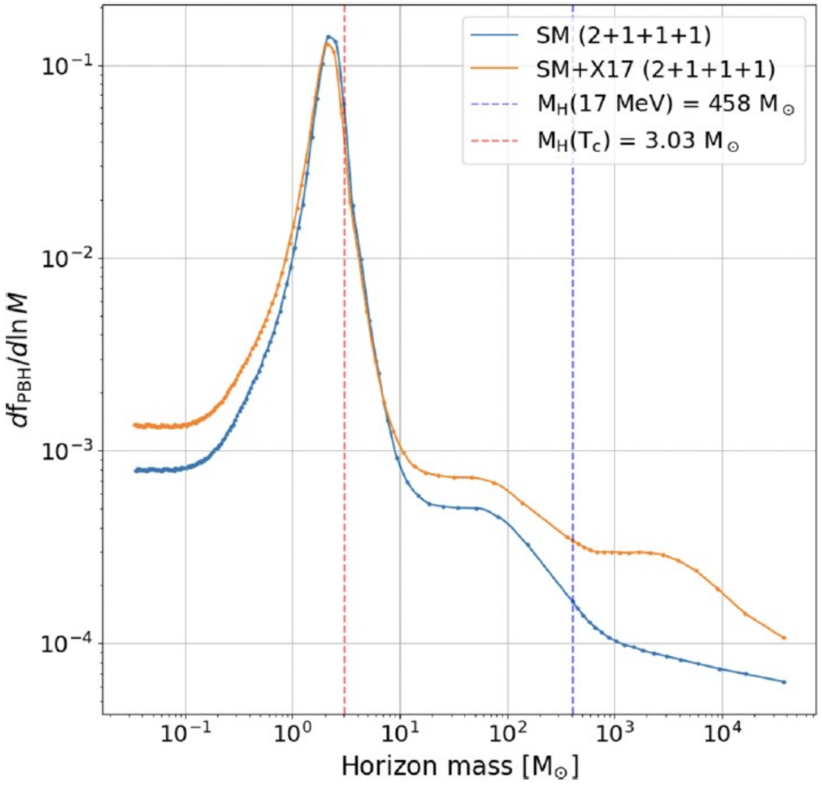
PBH Formation and CFO of heavy elements?



Two paths @ QCD hadronization: 1) PBH formation & 2) Nuclear droplets !

PBH Formation and CFO of heavy elements?

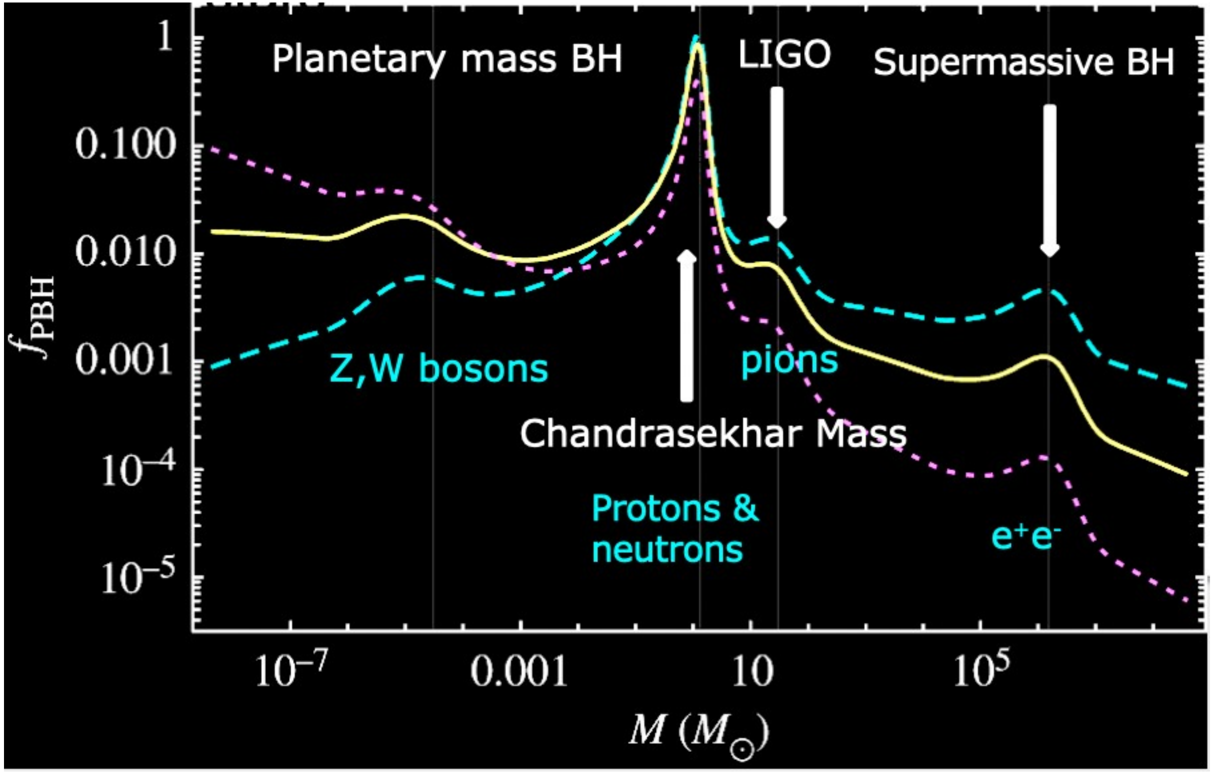




QCD hadronization transition plays key role plays for PBH formation !

PBH Formation and CFO of heavy elements?

JWST results – primordial black holes !



Different peaks correspond to different particles created at the early universe phase transitions and the corresponding reduction in the sound velocity.

BH mass corresponds to the horizon size at each time.

Only requirement is enough fluctuation power in a volume fraction of 10^{-9} of the early Universe.

Carr, Clesse, García-Bellido 2019

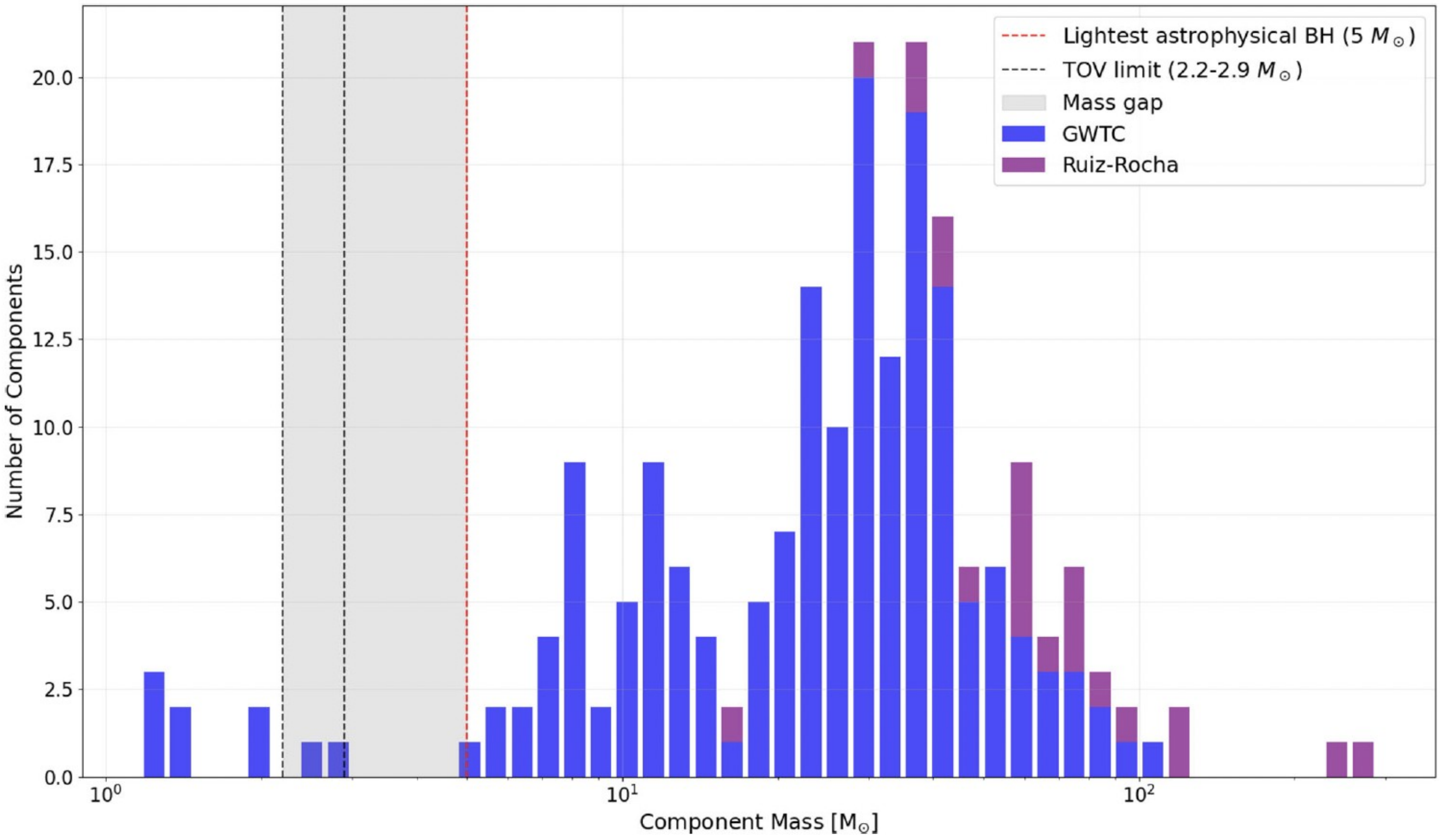
QCD hadronization transition plays key role plays for PBH formation !

PBH Formation and CFO of heavy elements?

LIGO BBH Merger Mass Distribution

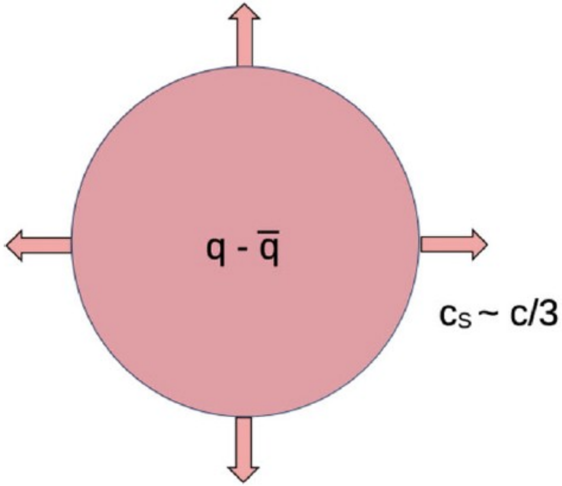
Crazy conjecture:

Peak mass “measures“ Cosmic QCD Transition TemperatureT_H~50 MeV ?

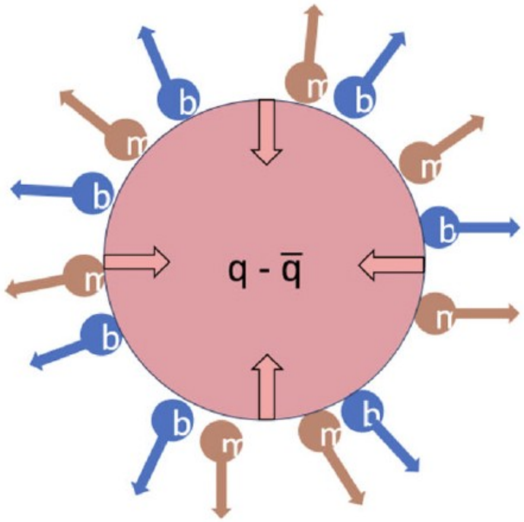


PBH Formation and CFO of heavy elements?

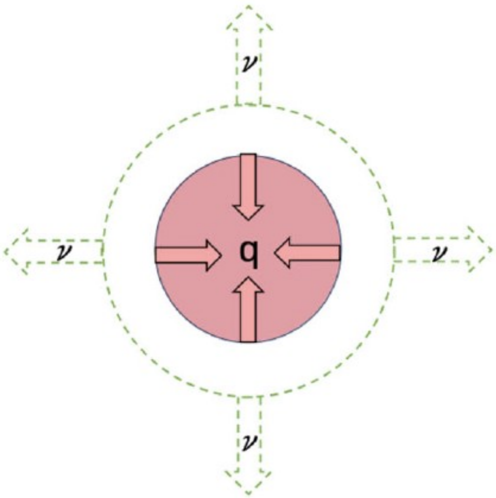
In this scenario the shrinking stops when the energy density gets to protons energy density.
This scenario requires $T > T_{QCD}$. It can be applied for pre-QCD transition failed collapse.



At first the overdensity expands at the speed of sound



Following Bjerrum-Bohr et al. 2012 and 2014. The quark plasma droplet shrinks by emitting baryons marked with b and mesons marked with m.



Following Witten 1984, the surplus quarks could be distilled by quark-antiquark annihilation to neutrinos being radiated off the glob.

PBH Formation and CFO of heavy elements?

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Cosmic separation of phases

Edward Witten*
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

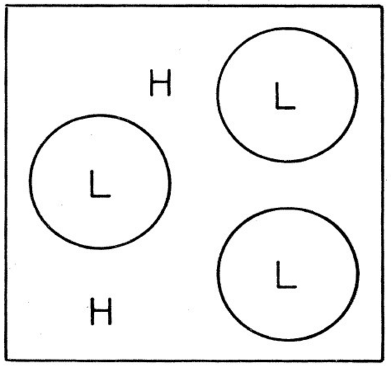


FIG. 1. Isolated expanding bubbles of low-temperature phase in the high-temperature phase.

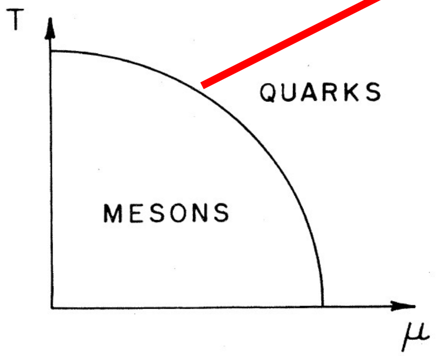
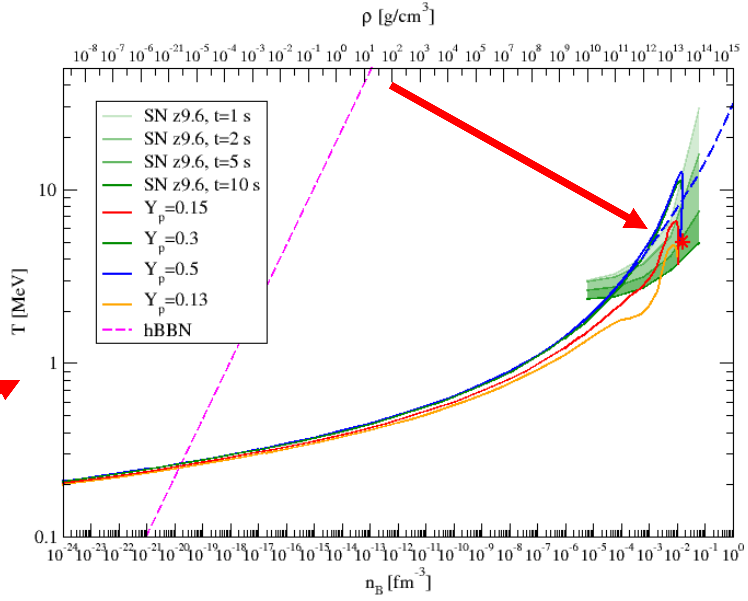


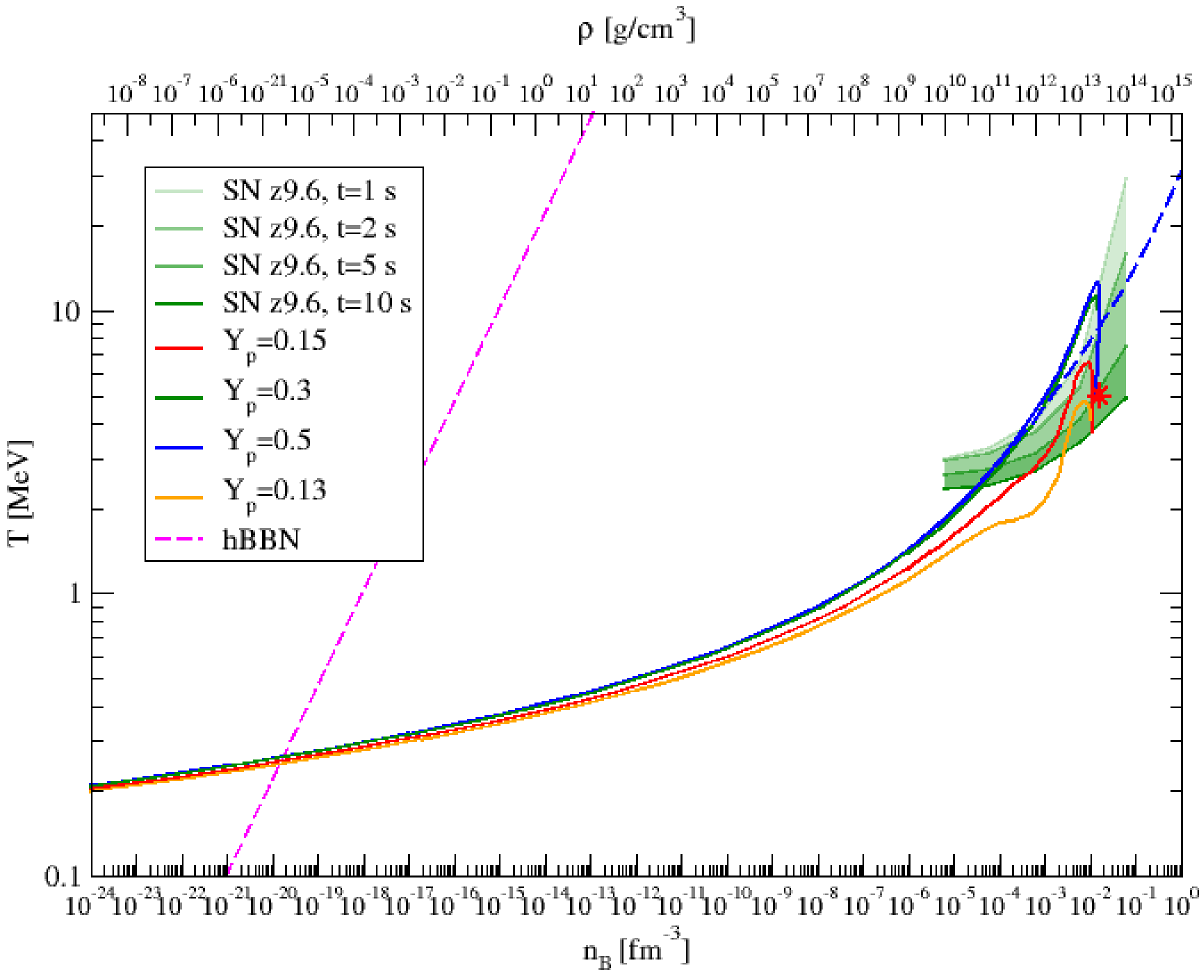
FIG. 4. A sketch of the coexistence temperature for quark matter of chemical potential μ coexisting with the meson-baryon phase of $\mu=0$. What is shown is the temperature, as a function of μ, at which the two phases exert equal pressure.



Accumulated mass fraction vs. mass number Â for solar element abundances compared with freeze-out model after neutron evaporation for T=5 MeV, μ_n=940.317 MeV, μ_p= 845.069 MeV

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

PBH Formation and CFO of heavy elements?



Can the primordial evolution of the Universe lead to these freeze-out Parameters (red star):

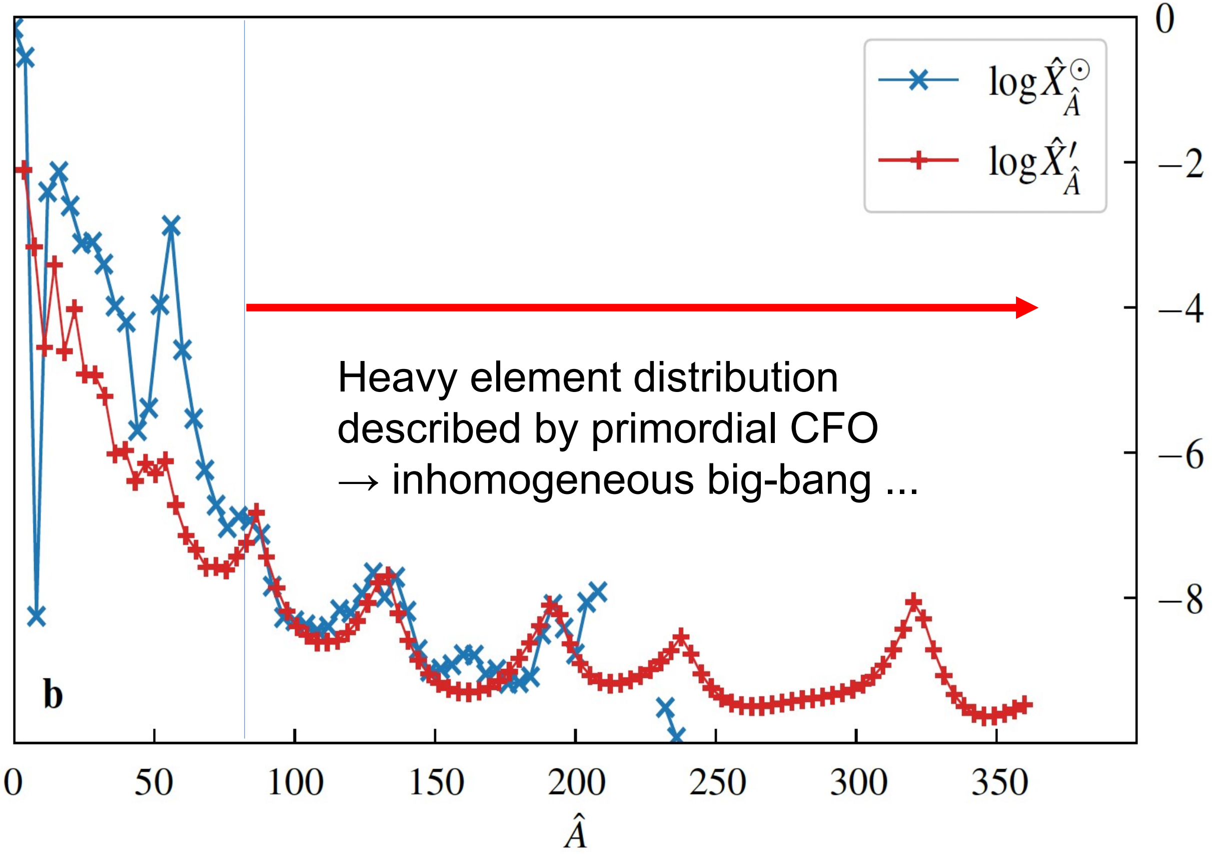
T=5 MeV,
μ_n=940.317 MeV,
μ_p= 845.069 MeV

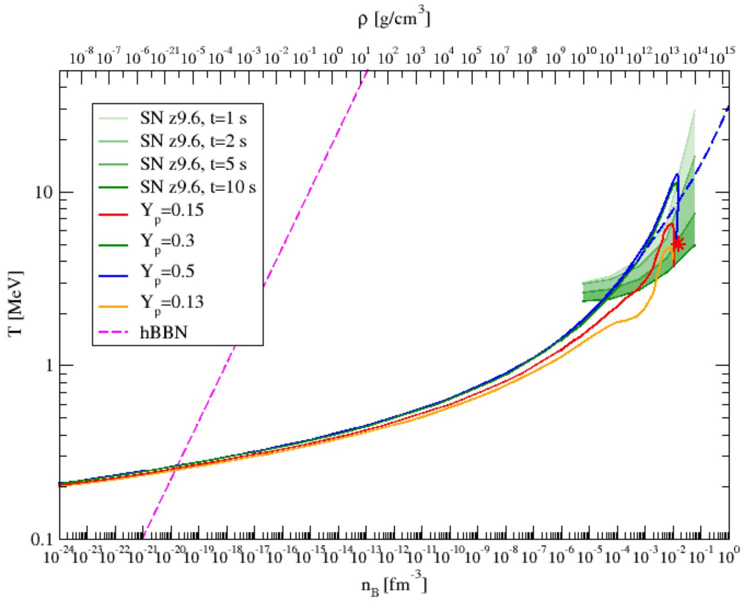
Maybe inhomogeneous Big Bang?

The freeze-out point lies in the domain of supernova explosions and binary neutron star mergers

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

PBH Formation and CFO of heavy elements?





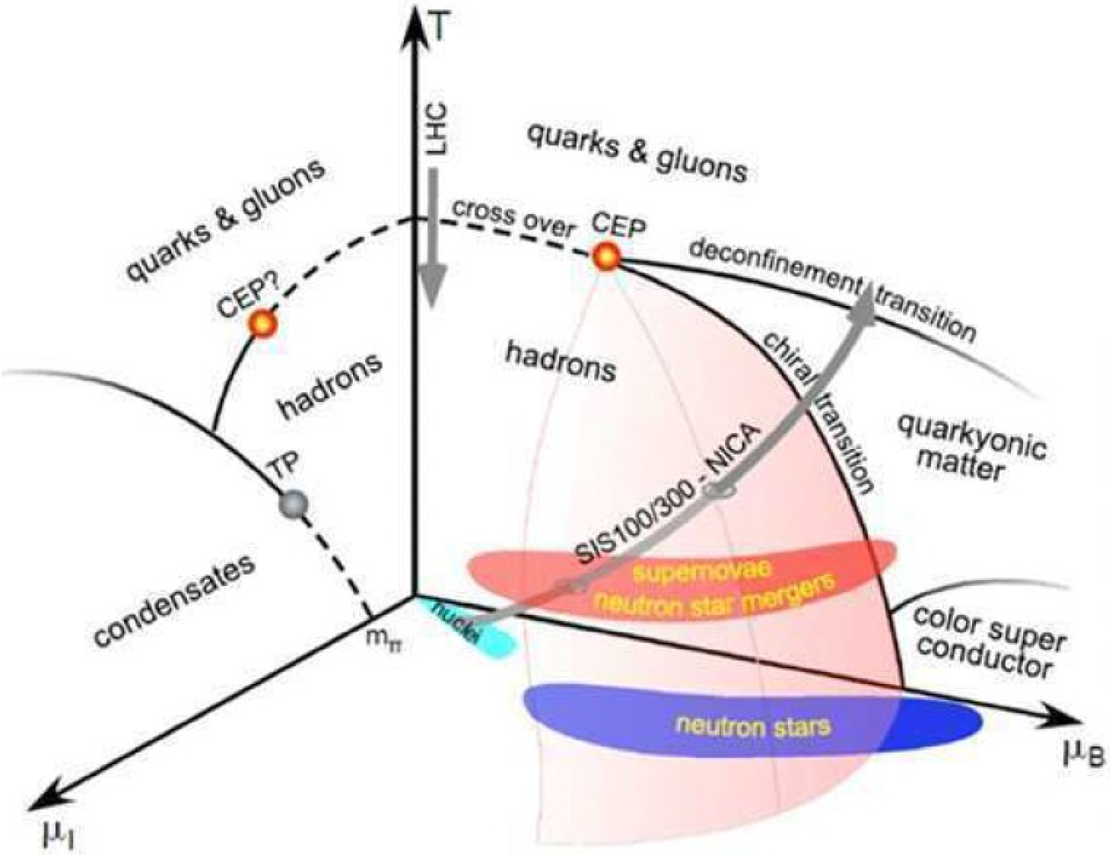
Accumulated mass fraction vs. mass number Â for solar element abundances compared with freeze-out model after neutron evaporation for T=5 MeV, μ_n=940.317 MeV, μ_p= 845.069 MeV

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

V. Join Communities in COST Action: BRIDGES

Proposal: Join HIC + Astro Communities = BRIDGES!

EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

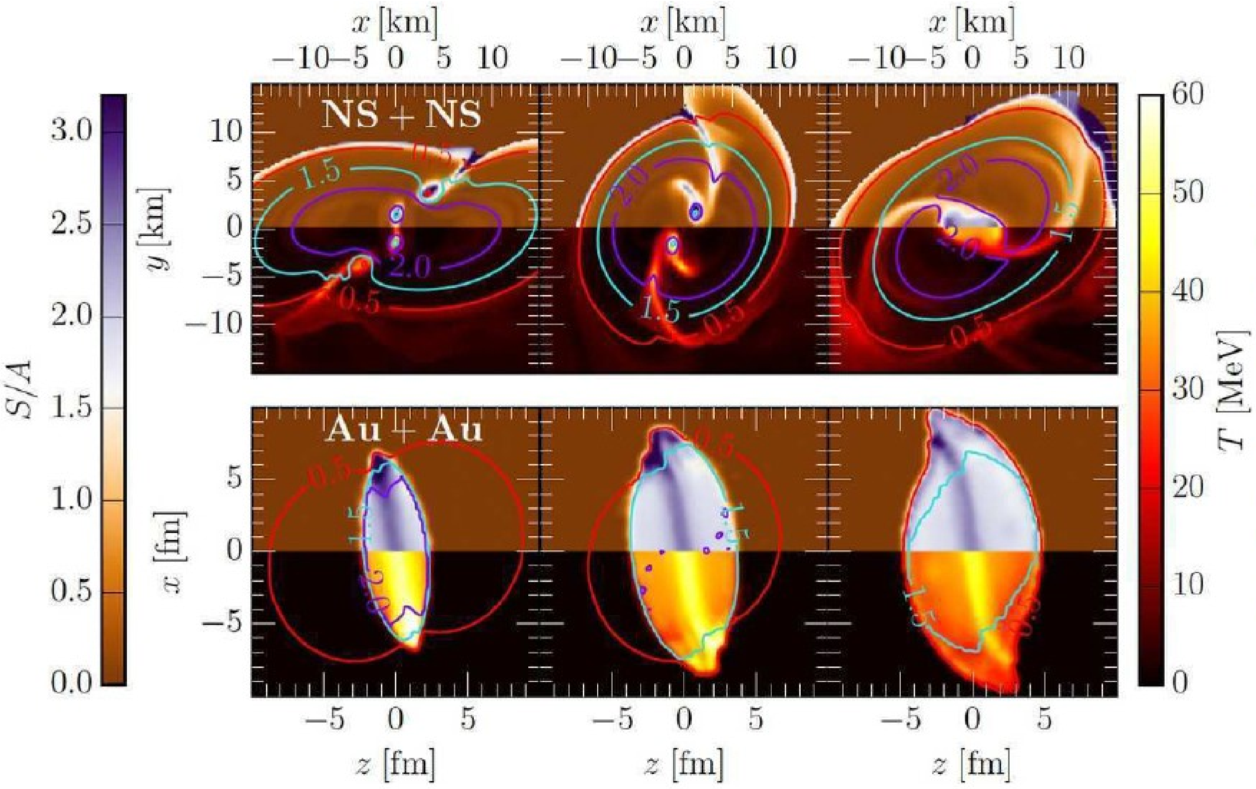


COST Action Proposal: Explore dense QCD Phase Diagram together !

Figure 1: Artistic view of the QCD phase diagram in the three-dimensional space of temperature T, and the chemical potentials μ_B and μ_I conjugate to baryon number and isospin asymmetry. The rose hypersurface indicates a first order phase transition between hadronic matter and QGP which affects neutron stars, their mergers and the FAIR-CBM HIC experiment labelled SIS100.

Proposal: Join HIC + Astro Communities = BRIDGES!

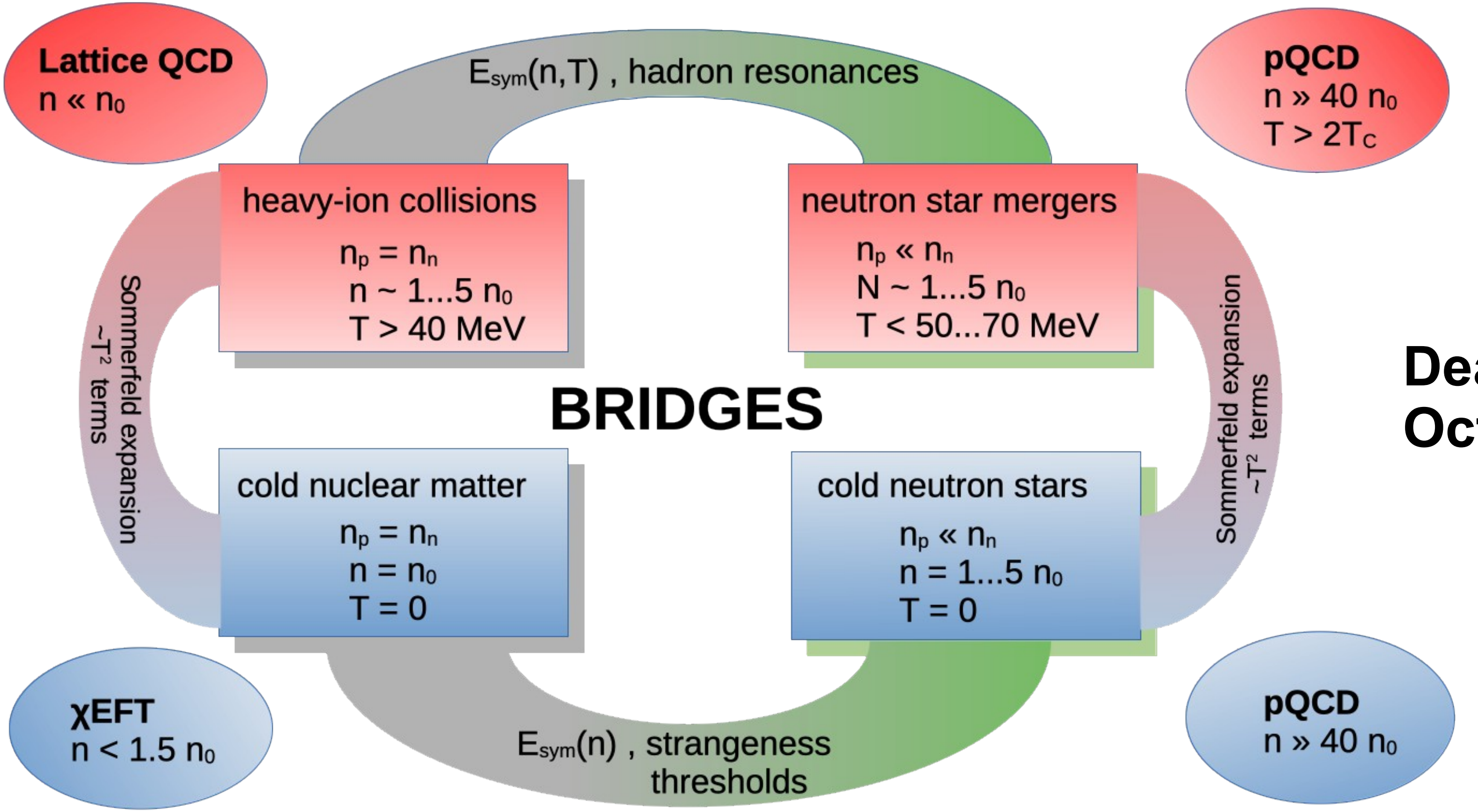
EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY



COST Action Proposal: Explore dense QCD Phase Diagram together !

Figure 2: Similarities in the distribution of temperature (entropy per baryon) in the lower (upper) colormaps. The snapshots are taken at t = -2, 0, +3 ms (t = -5, 0, +5 fm/c) before and after the NS+NS merger with total mass 2.6 M_{sun} in the upper panel (the full overlap of the Au+Au nuclear collision at E_{Lab}=450 A MeV in the lower panel). Taken from [24]

Join HIC + Astro Communities!



Deadline: October 2025

International Conference on Quark Confinement and the Hadron Spectrum (QCHS 2026) 29.6. - 4.7.2026, Wroclaw, Poland

David Blaschke (Uni Wroclaw & HZDR/CASUS),
Nora Brambilla (TU Munich),
Pior Surowka (Wroclaw Univ. Science & Technology)

https://indico.cern.ch/event/1531304

XVIIth Conference on Quark Confinement and the Hadron Spectrum (QCHS 2026)

29 June 2026 to 4 July 2026
Wrocław University of Science and Technology Congress Centre
Europe/Warsaw timezone

Enter your search term

- Overview
- Scientific Programme
- Committees
- Venue

Inaugurated in 1994 in Como, Italy, this series of conferences has become an important forum for scientists working on strong interactions, stimulating exchanges among theorists and experimentalists as well as across related fields.

The aim of the conference is to bring together people working on strong interactions from different approaches, ranging from lattice QCD to perturbative QCD, from models of the QCD vacuum to QCD phenomenology and experiments, from effective theories to physics beyond the Standard Model.

The scope of the conference also includes the interface between QCD, nuclear physics and astrophysics, and the wider landscape of strongly coupled physics. In particular, the conference will focus on the fruitful interactions and mutual benefits between QCD and the physics of condensed matter and strongly correlated systems.

The seventeenth edition of this conference series will be held at the Grunwaldzki Campus of the Wrocław University of Science and Technology, Poland, between June 29th and July 4th, 2026.

Starts 29 Jun 2026, 08:00
Ends 4 Jul 2026, 20:00
Europe/Warsaw

Wrocław University of Science and Technology Congress Centre
ul. Janiszewskiego 8
50-372 Wrocław
Poland
Go to map

David Blaschke
Nora Brambilla
Piotr Surówka

There are no materials yet.

https://indico.cern.ch/event/1531304 **(under construction)**

Local Organizing Committee (LOC)

David Blaschke (UWr & HZDR/CASUS Görlitz, co-chair)
Nora Brambilla (TU Munich, co-chair)
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Tobias Fischer (WUST)
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Gabriel Wlazlowski (Politechnika Warszawska) ...

Scientific Programme / Parallel Sessions

Vacuum Structure and Confinement	-	Session A
Light Quarks	-	Session B
Heavy Quarks	-	Session C
Deconfinement	-	Session D
QCD and New Physics	-	Session E
Nuclear and Astro-Particle Physics	-	Session F
Strongly-Coupled Theories and Dark Matter	-	Session G
Statistical Methods for Physics Analysis	-	Session H

Conference Venue:

Campus Grunwaldzki of
Wroclaw University of Science Technology

https://indico.cern.ch/event/1531304 (under construction, bookmark the website !)

Looking forward to meeting you again in Wroclaw in Summer 2026

For those who cannot wait until 2026 ...

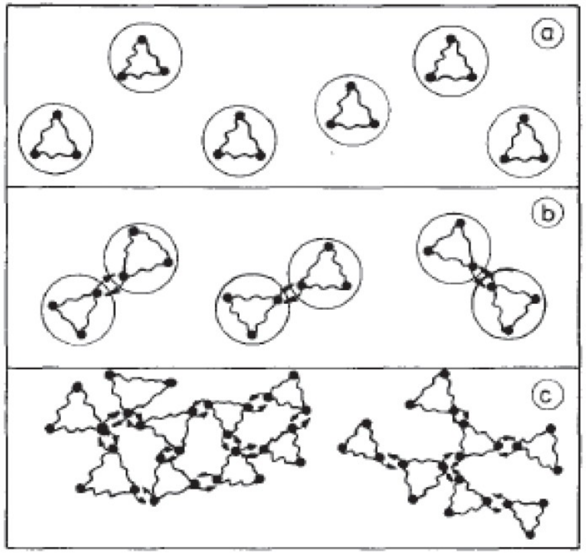
The Modern Physics of Compact Stars and Relativistic Gravity 2025

23.–26. Sept. 2025
Yerevan, Armenia

https://indico.global/event/13917

Nuclear Symmetry Energy from Quark Pauli Blocking ?

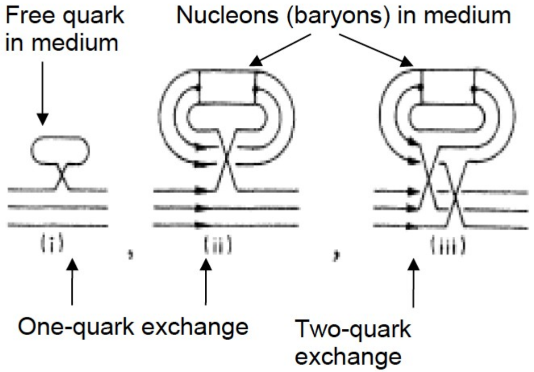
1. Nuclear Symmetry Energy from Quark Pauli Blocking ?



a) Low density: Fermi gas of nucleons (baryons)

b) ~ saturation: Quark exchange interaction and Pauli blocking among nucleons (baryons)

c) high density: Quark cluster matter (string-flip model ...)

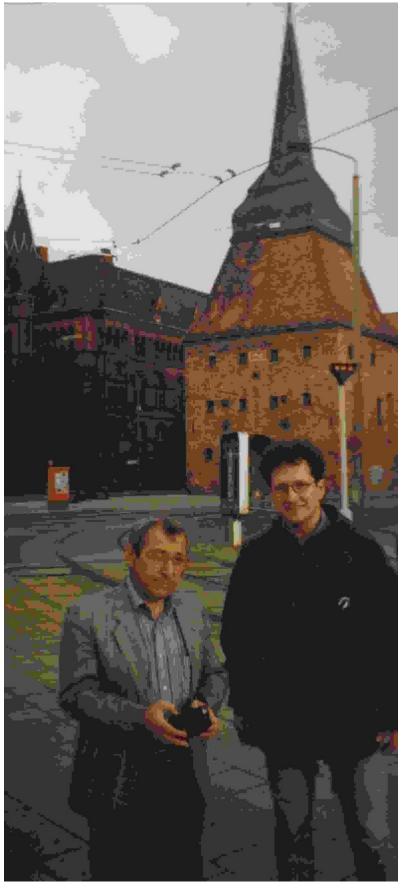


379 (1987); Roepke, DB, Schulz, PRD 34, 3499 (1986)

Pauli blocking shift (Oscillator model)

$$\Delta E_{\tau P}^{\text{Pauli}}(P_{F,n}, P_{F,p}) = \sum_{\tau'=n,p} \sum_{\alpha=1,2} c_{\tau\tau'}^{(\alpha)} W_\alpha(P, P_{F,\tau'}) \,,$$

α=1,2 … one-quark or two-quark exchange



Rostock visit 1990-ies

1. Nuclear Symmetry Energy from Quark Pauli Blocking ?

particles

1. Symmetric nuclear matter (SNM), for which $P_{F,n} = P_{F,p} = P_F$ and

$$\Delta E^{\text{Pauli}}_{nP_F}(P_F, P_F) = c^{(1)}_{nn} W_1(\lambda_1 P_F) + c^{(2)}_{nn} W_2(\lambda_2 P_F) + c^{(1)}_{np} W_1(\lambda_1 P_F) + c^{(2)}_{np} W_2(\lambda_2 P_F).$$

2. Pure neutron matter (PNM), for which $P_{F,p} = 0$, $P_{F,n} = P_F$ and

$$\Delta E^{\text{Pauli}}_{nP_F}(P_F, P_F) = c^{(1)}_{nn} W_1(\lambda_1 P_F) + c^{(2)}_{nn} W_2(\lambda_2 P_F) .$$

$$W_\alpha(x_{\alpha,F}) = W_\alpha(x_{\alpha,F}, x_{\alpha,F}) = \overline{W}_\alpha P(x_{\alpha,F}) ,$$

Exact:

$$P(x) = 12\sqrt{\pi}\,\mathrm{erf}(2x) + \frac{1}{x}\left[\mathrm{e}^{-4x^2}(11 - 4x^2) - 11\right] ,$$

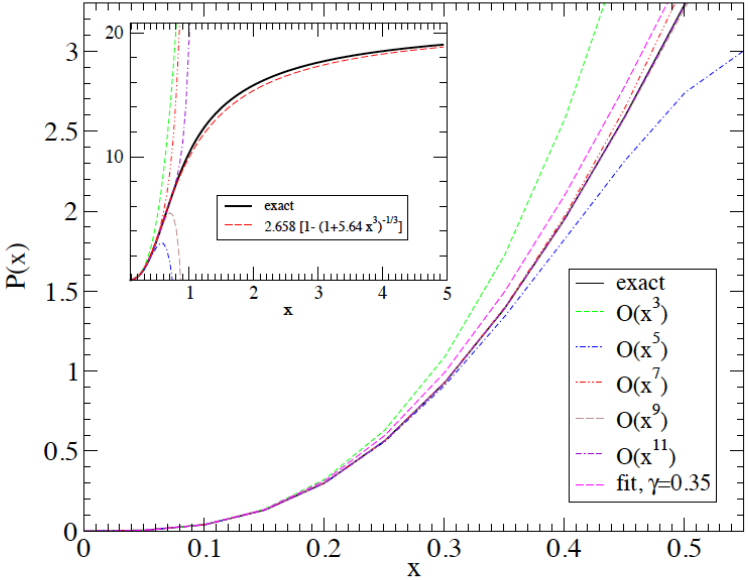
Expansion:

$$P(x) = 40x^3 - \frac{1088}{15}x^5 + \frac{608}{7}x^7 - \frac{3584}{45}x^9 + \frac{4436}{99}x^{11} - \mathcal{O}(x^{13}) .$$

Fit formula:

$$P(x) \approx 12\sqrt{\pi}\left[1 - \left(1 + \frac{10}{3\sqrt{\pi}\gamma}x^3\right)^{-\gamma}\right] , \gamma = 0.35,$$

(density functional)



Particles **2020**, *3*, 477–499; doi:10.3390/particles3020033

1. Nuclear Symmetry Energy from Quark Pauli Blocking ?

From energy shift to EoS: RMF scheme a la Linear Walecka (LW)+Quark Exchange (ex)

$$P = \frac{1}{8\pi^2}\sum_{\tau=n,p}\left[-E_\tau^* m_\tau^{*2} P_{F,\tau} + \frac{2}{3}E_\tau^* P_{F,\tau}^3 + m_\tau^{*4}\log\left(\frac{E_\tau^* + P_{F,\tau}}{m_\tau^*}\right)\right] + \frac{1}{2}G_\omega n^2 - \frac{1}{2}G_\sigma n_s^2 + P_{\rm ex},$$

$$\varepsilon = \frac{1}{8\pi^2}\sum_{\tau=n,p}\left[2\,E_\tau^{*3} P_{F,\tau} - E_\tau^* m_\tau^{*2} P_{F,\tau} - m_\tau^{*4}\log\left(\frac{E_\tau^* + P_{F,\tau}}{m_\tau^*}\right)\right] + \frac{1}{2}G_\omega n^2 + \frac{1}{2}G_\sigma n_s^2 + \varepsilon_{\rm ex},$$

$$n_{s,\tau} = \frac{m_\tau^*}{2\pi^2}\left[E_\tau^* P_{F,\tau} - m_\tau^{*2}\log\left(\frac{E_\tau^* + P_{F,\tau}}{m_\tau^*}\right)\right],$$

$$E_\tau^* = \sqrt{m_\tau^{*2} + P_{F,\tau}^2}$$

$$n_\tau = \frac{P_{F,\tau}^3}{3\pi^2},$$

$$m_\tau^* = m_\tau - G_\sigma n_{s,\tau},$$

$$\mu_\tau = E_\tau^* + G_\omega n_\tau + \mu_{{\rm ex},\tau}.$$

$$\mu_{{\rm ex},\tau} = \Delta_\tau(n,x) = \Delta E^{\rm Pauli}_{\tau P_{F,\tau}}(P_{F,n}, P_{F,p}),$$

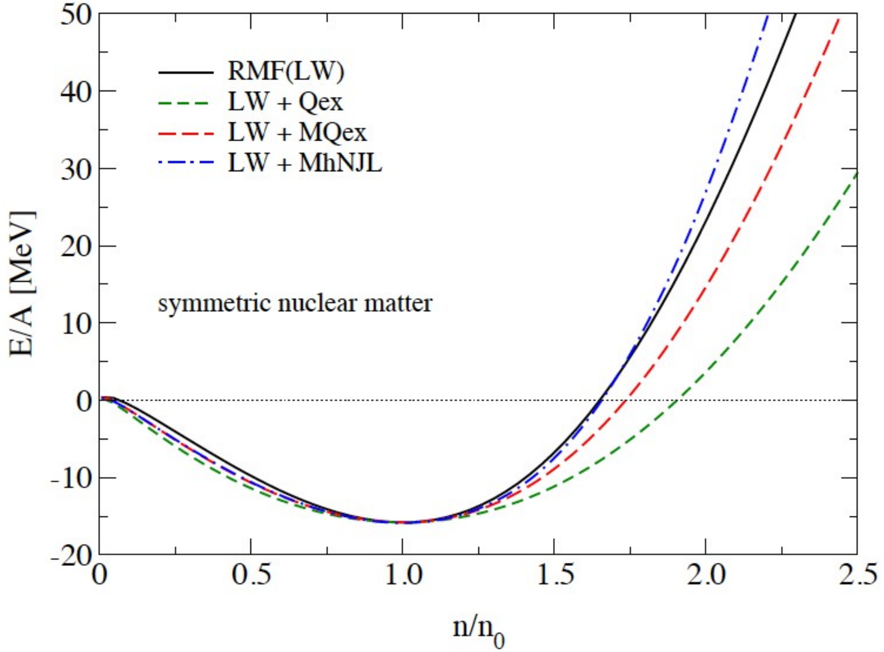
$$\varepsilon_{\rm ex} = \int_0^n dn'\{x\Delta_p(n',x) + (1-x)\Delta_n(n',x)\},$$

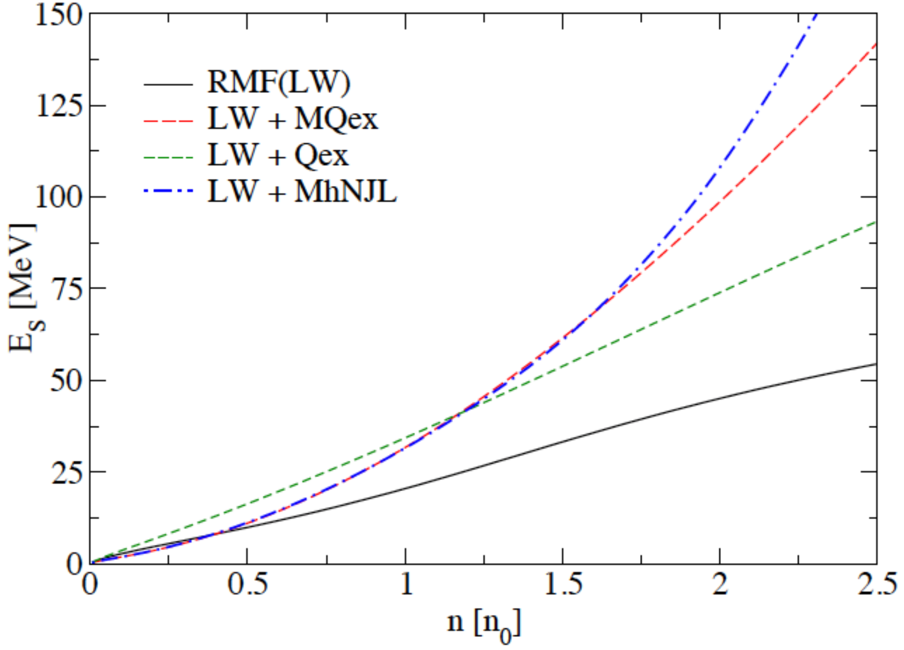
$$P_{\rm ex} = \sum_{\tau=n,p}\mu_{{\rm ex},\tau}\, n_\tau - \varepsilon_{\rm ex},$$

where n_p (n_n) denotes the proton (neutron) density and $x = n_p/n$ is the proton fraction.

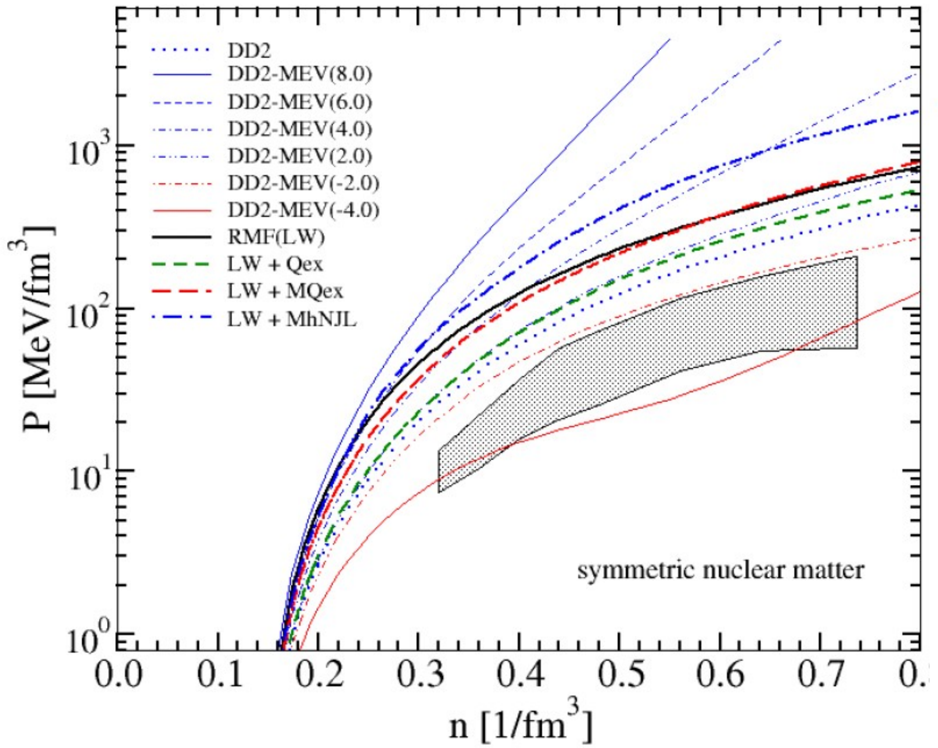
1. Nuclear Symmetry Energy from Quark Pauli Blocking ?

	$(g_\omega/m_\omega)^2$ [fm^2]	$(g_\sigma/m_\sigma)^2$ [fm^2]	K [MeV]	E_s [MeV]	$R_{1.4}$ [km]
RMF (LW)	11.6582	15.2883	608.874	21.58	13.22
LW + Qex	6.11035	9.91197	331.958	32.04	13.70
LW + MQex	8.59170	13.29118	481.713	34.12	14.40
LW + MhNJL	9.25683	13.9474	582.831	31.55	14.29

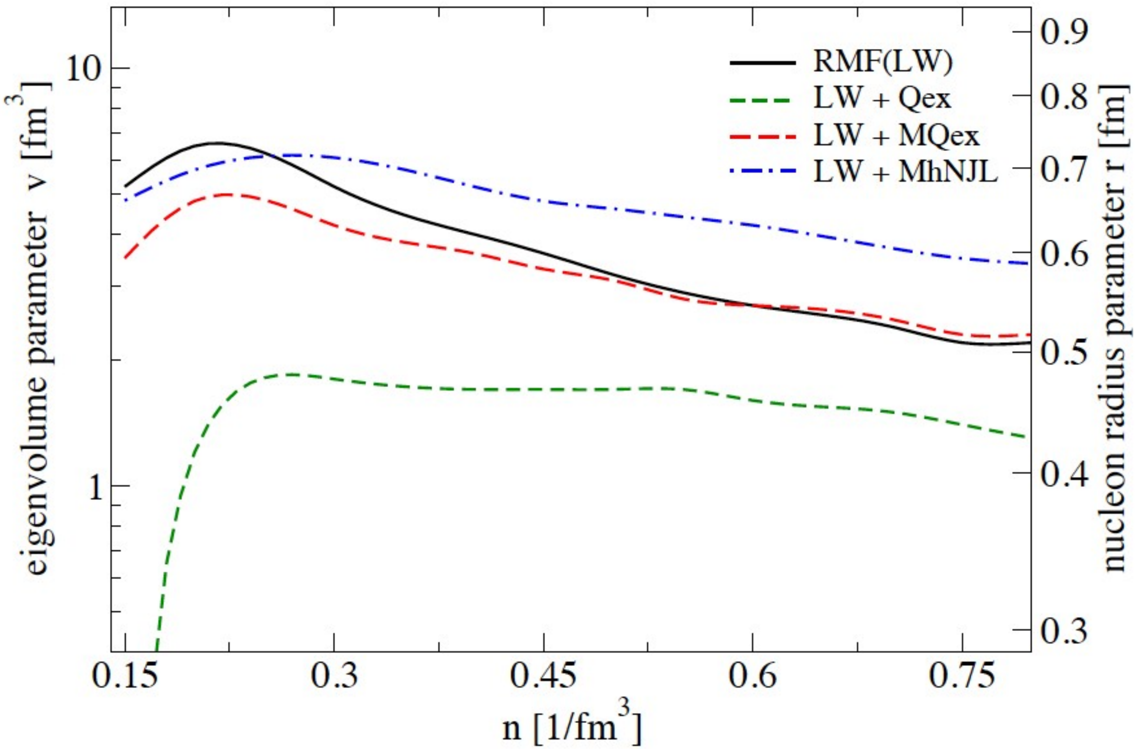




1. Nuclear Symmetry Energy from Quark Pauli Blocking ?



Quark Pauli blocking EoS compared to the DD2 EoS with excluded volume v [Typel, EPJA52, 16 (2016)]



Density dependence of nucleonic eigenvolume param. v that would reproduce the quark Pauli blocking EoS for the DD2 EoS with excl. vol. [Typel, EPJA52, 16 (2016)]