Quarkonia Spectral function at finite temperature ($\mu_B = 0$)

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HotQCD Collaboration

arXiv:2505.11313 [hep-lat]

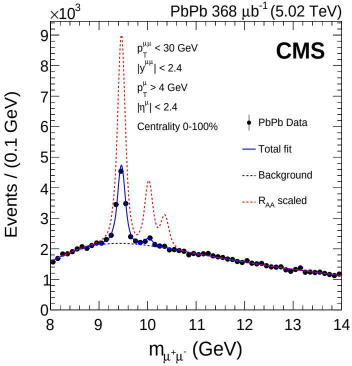
- Quarkonia (bound states of heavy $q\bar{q}$), are an important probe to study QGP.

- In medium properties encoded in the spectral functions.

- Di-lepton rate

$$\frac{d\Gamma_{\mu+\mu-}}{d^4Q} \sim \frac{e^2}{Q^2} n_b \, \rho_V(Q)$$

- For current, $J_\Gamma(\vec{x}, t) = \bar{\psi}(\vec{x}, t)\Gamma\psi(\vec{x}, t)$



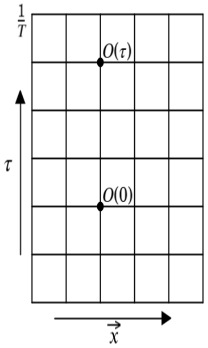
CMS Collaboration, PLB 790 (2019) 270

$$\rho_\Gamma(\omega, \vec{k}) = \int dt \, d^3\vec{x} \exp\left[i(\vec{k}.\vec{x} - \omega t)\right] \langle [J_\Gamma(\vec{x}, t), J_\Gamma(0, 0)] \rangle_T$$

- Euclidean-correlation function $G^E(\tau, \vec{k}) = \int \exp\left(i\vec{k}.\vec{x}\right)\langle J_\Gamma(\vec{x}, \tau) J_\Gamma(0, 0)\rangle$ where $J(\vec{x}, t) = \bar{\psi}(\vec{x}, \tau)\Gamma\psi(\vec{x}, \tau)$

$$G^E_\Gamma(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi}\rho_\Gamma(\omega, \vec{k})\frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

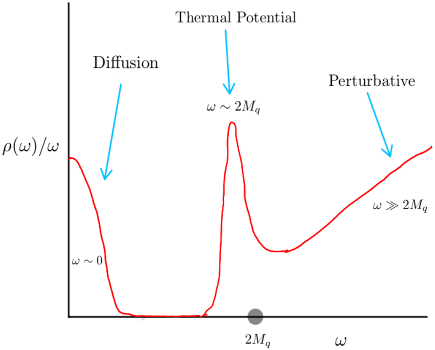
- Numerically ill-posed problem. Small number of data points and statistical errors.



We computed for charm and bottom correlation function on the lattice with $m_\pi \sim 320$ MeV.
We consider $\boldsymbol{\Gamma = \gamma_5, \vec{k} = \vec{0}}$.
$T \sim 1.2T_c(N_\tau = 32), 1.3T_c(N_\tau = 28), 1.62T_c(N_\tau = 24)$ $T_c = 180$ MeV.
$L = 2.7$ fm

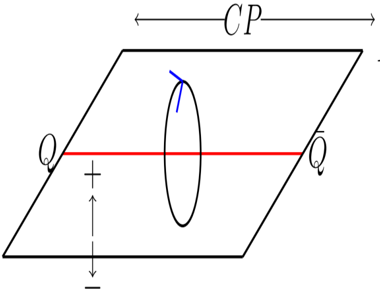
- $\omega \gg 2M$
  Thermal effects are suppressed. Vacuum perturbation theory will work.
- $\omega \sim 2M$
  Thermal effects are important. Spectral function needs to be calculated using thermal potential.
- $\omega \ll 2M$
  For the pseudoscalar channel, the spectral weights are exponentially suppressed.

  For the vector channel, there is a contribution around $\omega \sim 0$ due to transport.



Thermal Potential

$$|\phi_n^{\mathrm{H}}(\vec{r},t)\rangle = \underbrace{\bar{\psi}(-\vec{r}/2,t)\,U(-\vec{r}/2,\vec{0})\,H\,U(\vec{0},\vec{r}/2)\,\psi(\vec{r}/2,t)}_{O_H(r,t)}|n\rangle$$



$$L = 0,1,2\,(\Sigma,\Pi,\Delta)$$

$$CP = +,-\,(g,u)$$

$$P_x = +,-$$

$H = 1 \quad (\Sigma_g^+)$ — Standard Cornell Potential

$H = B_z \quad (\Sigma_u^-)$
$H = B_+ = B_x + iB_y \quad (\Pi_u)$
} Hybrid Potential

$$C^H(r,t) = \frac{1}{Z}\sum_n e^{-\beta E_n}\left\langle \phi_n^H(r,t) \middle| \phi_n^H(r,0)\right\rangle$$

- $M_q \to \infty,\ C^H(t;r) \propto \langle W^H(r,t)\rangle_T$
- Static potential:

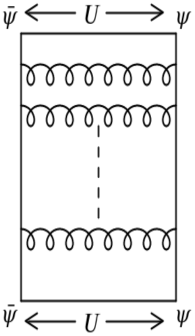
$$V^H(r) = i \lim_{t\to\infty} \frac{\partial \log W^H(r,t)}{\partial t}$$

- Non-perturbative formulation:

$$W_E^H(r,\tau) \ = \int_{-\infty}^{\infty} d\omega\, \rho^H(\omega, T)\exp(-\omega\,\tau)$$

$$W^H(r,it) \ = \int_{-\infty}^{\infty} d\omega\, \rho^H(\omega, T)\exp(-i\omega\, t)$$

A. Rothkopf et al., PRL. 108 (2012) 162001



Potential at zero temperature

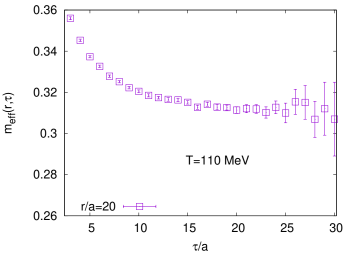
- From first principle at large τ: $W_E^H(r,\tau) = \exp(-V_H \tau)$
- $i \lim_{t\to\infty} \frac{\partial \log W^M(r,t)}{\partial t}$ exisits trivially,
- The potential can be extracted from the plateau of effective mass

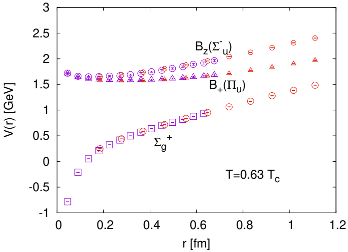
$$a\, m_{eff}^H(r,\tau) = \log\left(\frac{W_E^H(r,\tau)}{W_E^H(r,\tau+a)}\right)$$

(standard technique).

- In LO

$$V_{H=B_z/B_+}(r) = \frac{\alpha}{6r}$$
$$V_{H=1}(r) = -\frac{4\alpha}{3r}$$

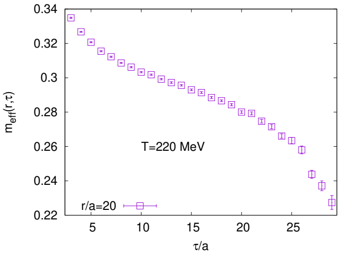




DB and S. Datta, PRD 103, 014512
S.Capitani et al, PRD 99, 034502

Effective Mass

- The limit $i \lim_{t\to\infty} \frac{\partial \log W^M(r,\tau\to it)}{\partial t}$ becomes non-trivial - no pure exponential decay.
- Needs further physics input.
- Let's look at the τ structure of HTL because of the existence of the limit.
M. Laine et al., JHEP 0703:054



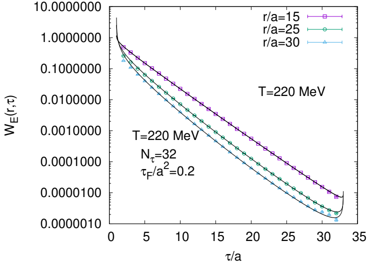
$$\log W(r,\tau) = -V_{\rm re}(r)\,\tau - \int_{-\infty}^{\infty} dq_0\,\sigma(r,q_0)\,\left[e^{q_0\tau} + e^{q_0(\beta-\tau)}\right]$$

$$V_T^{\rm re}(r) = \boxed{-\frac{g^2}{4\pi} C_F \left[m_D + \frac{e^{-m_D r}}{r}\right]} \quad \Rightarrow \text{Color Screening}$$

$$V_T^{\rm im}(r) = \boxed{\frac{g^2}{4\pi} C_F\, T \int_0^\infty \frac{z\,dz}{(z^2+1)^2}\left[1 - \frac{\sin(z m_D r)}{z m_D r}\right]} \quad \Rightarrow \text{Landau Damping}$$

$$\log W(r,\tau) = -V_{re}(r)\tau - \int_{-\infty}^{\infty} du\, \sigma(r,u) \left[e^{u\tau} + e^{u(\beta-\tau)}\right] + \ldots$$

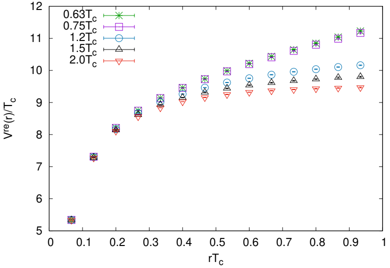
- $i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t}$ finite $\Rightarrow \lim_{u\to 0} \sigma(r,u) \sim \frac{1}{u^2}$
- $\sigma(r,u) = n_B(u) \left[\frac{V_{im}}{u} + c_1 u + c_3 u^3 + \ldots\right]$

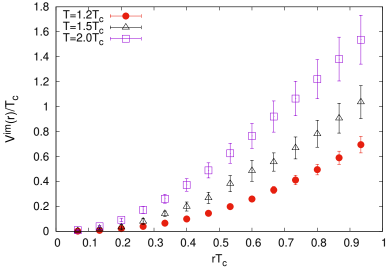


$$W(r,\tau) = A \exp\left[-V_{re}(r)\tau - \frac{\beta V_{im}(r)}{\pi} \log\left(\sin\left(\frac{\pi\tau}{\beta}\right)\right) + \ldots\right]$$

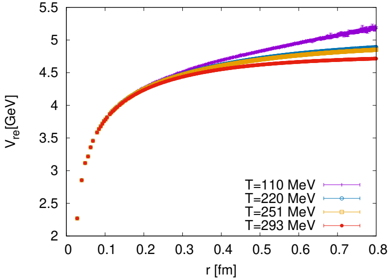
$\chi^2/\text{ndf} \sim 1$ for all distances

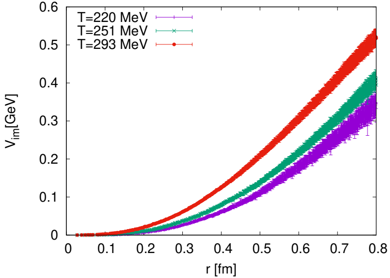
D. Bala and S. Datta, Phys. Rev. D 101, 034507



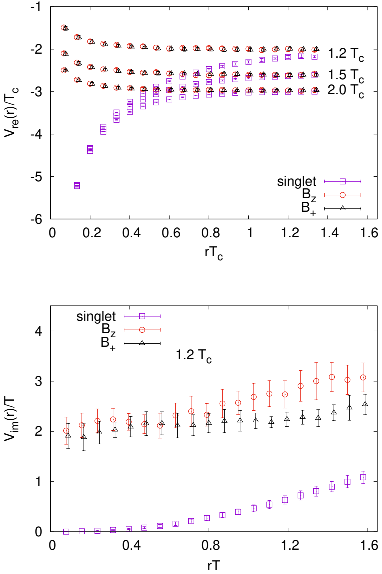


D. Bala and S. Datta, PRD 101, 034507



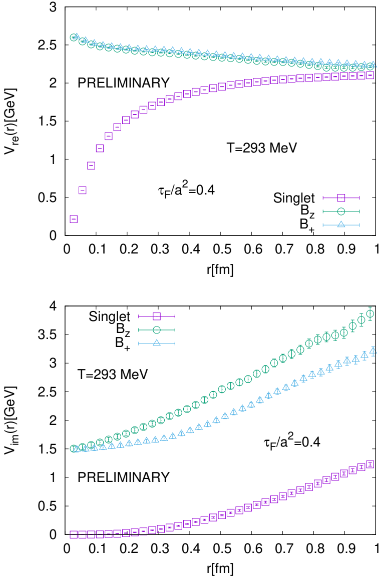


D. Bala, O. Kaczmarek et al., arXiv:2505.11313 [hep-lat]



$N_f = 0$

DB and S. Datta, PRD 103, 014512



$N_f = 2 + 1$

DB, S. Datta, O. Kaczmarek (work in progress)

Quarkonia Spectral function

Quarkonia Spectral function PS channel

$$C_{>}^{PS}(t) = \int d^3\vec{x}\, \langle \bar{\psi}(t,x)\gamma_5\ \psi(t,x)\bar{\psi}(0,\vec{0})\gamma_5\, U\, \psi(0,\vec{0})\rangle_T$$

$\Big\downarrow$ point split

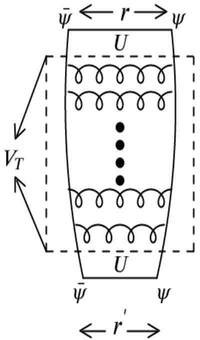
$$C_{>}^{PS}(t;\vec{r},\vec{r}') = \int d^3\vec{x}\, \langle \bar{\psi}(t,x+\frac{\vec{r}}{2})\gamma_5\, U\, \psi(t,x-\frac{\vec{r}}{2})\bar{\psi}(0,-\frac{\vec{r}'}{2})\gamma_5\, U\, \psi(0,-\frac{\vec{r}'}{2})\rangle_T$$

$M_Q \gg \Lambda_{QCD}, T$. Expansion in leading order inverse quark mass leads to, M.Laine et al, JHEP 0703:054

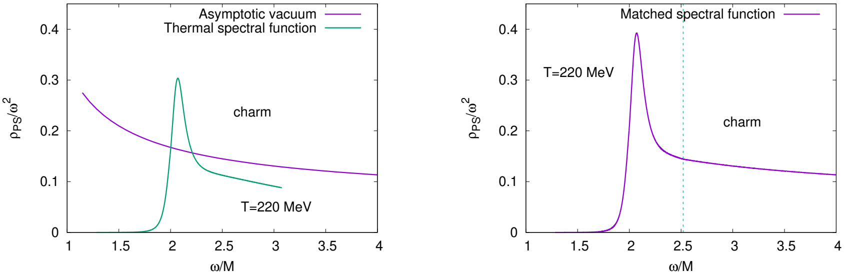
$$\left\{ i\partial_t - \left[2M + V_T(r) - \frac{\nabla_{\vec{r}}^2}{M} \right] \right\} C_{>}(t;\vec{r},\vec{r'}) = 0$$

where V_T is defined in static limit. with $C_{>}(0;\vec{r},\vec{r'}) = \delta^3(\vec{r}-\vec{r}')$

$$\rho_p(\omega) \propto \lim_{r\to 0, r'\to 0} \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\ C_{>}(t;\vec{r},\vec{r'})$$



$$\rho_{PS}(\omega) = A_0\, \rho^{T}_{PS}(\omega)\theta(\omega_0 - \omega) + \rho^{T=0}_{PS}(\omega)\theta(\omega - \omega_0)$$

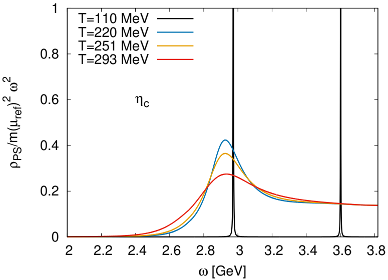


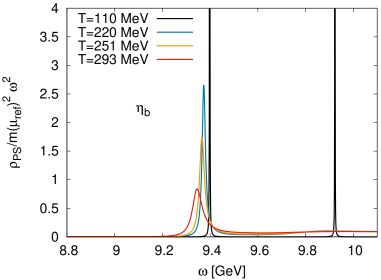
- $A_0 \sim 1 \qquad \omega_0 \sim 2.7\, M$

Similar spectral function using perturbative potential.

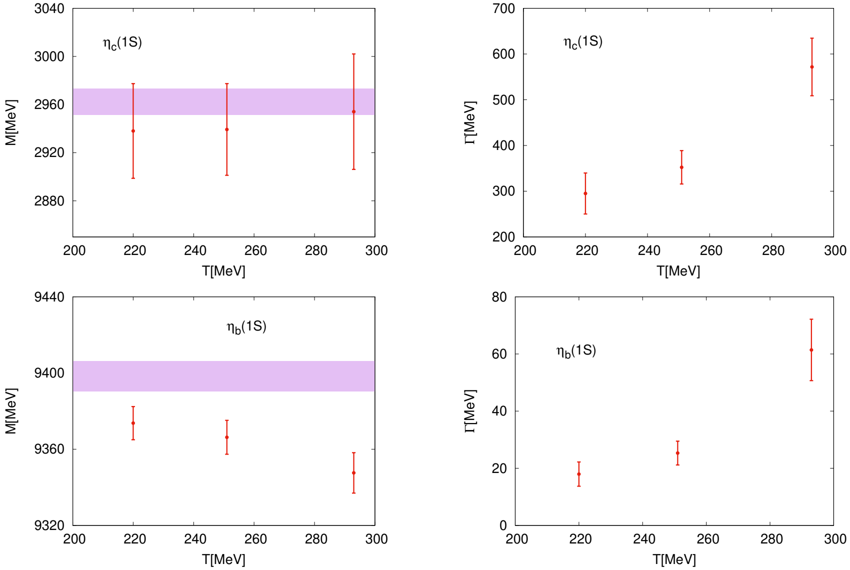
$N_f = 0$ M. Laine et al, JHEP11 (2017) 206

$N_f = 3$ DB, O.Kaczmarek et al, hep-lat,arXiv:2505.11313





- (1S) state for bottom disappear much after T_c ($T_c = 180 MeV$)
- Significant thermal effects on charmonium state.

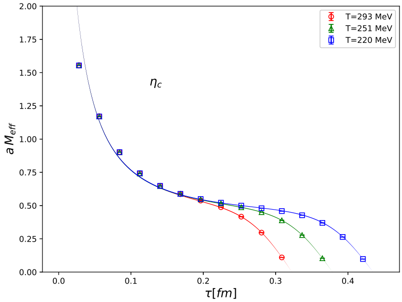


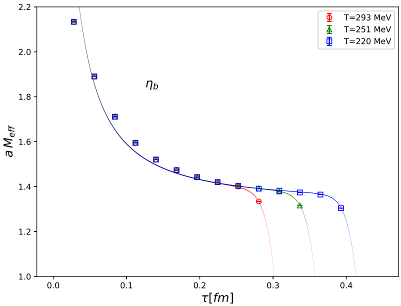
- $\Gamma_c(1S) \gg \Gamma_b(1S)$

Consistency check with Lattice

$$G^E_{PS}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_{PS}(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

$$m_{eff}(\tau_i) = \log\left(\frac{G^E_{PS}(\tau_i)}{G^E_{PS}(\tau_{i+1})}\right)$$



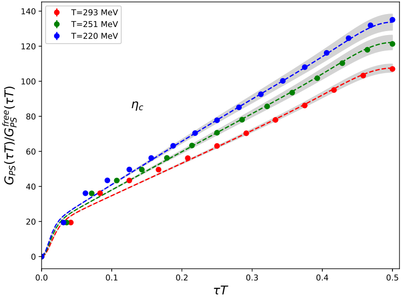


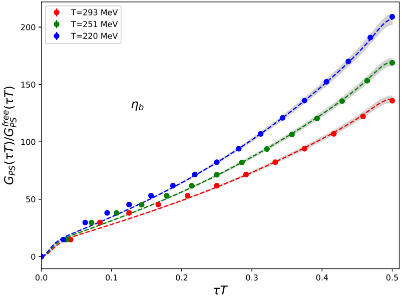
Consistent with lattice data.

Consistency check with Lattice

$$\rho_{PS}^{model}(\omega, A) = A\,\rho_{PS}(\omega)$$

$$G_{PS}^{E}(\tau, A) = \int_0^\infty \frac{d\omega}{\pi}\rho_{PS}^{model}(\omega, A)\frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

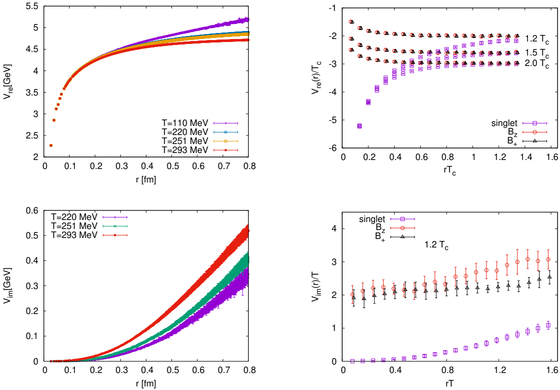




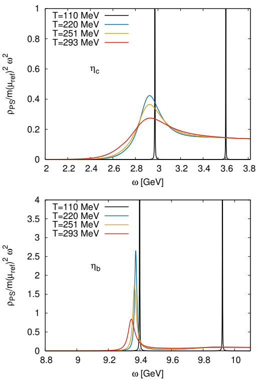
- These spectral functions indeed describe the lattice correlator .

Summary

Thermal Potential



Spectral Function



Color screening is supported by Lattice QCD

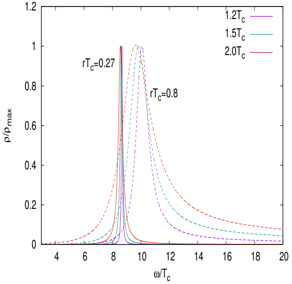
Hybrid quarkonia are unlikey to form

Large medium modification of charm states

HTL Inspired Spectral Function: $\omega \sim V_{re}$:

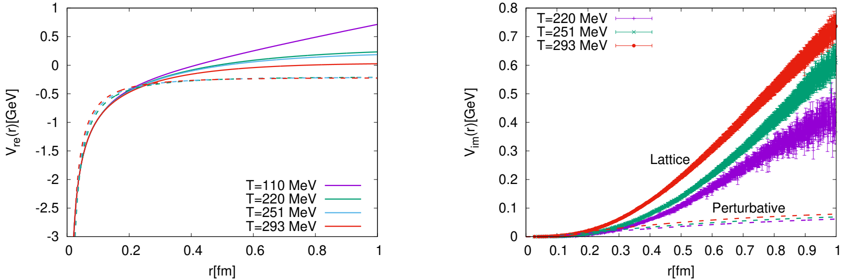
$$\rho_{\text{low}}(r;\omega \sim V_{re}) \approx \sqrt{\frac{2}{\pi}} \frac{V_{\text{im}}}{(V_{\text{re}}-\omega)^2 + V_{\text{im}}^2}$$

$$\approx \exp(-(V_{re}-\omega)/T)$$



$$\approx \frac{1}{(\omega - V_{re})^{\alpha}}$$

DB and S. Datta, PRD 103, 014512   DB, O. Kaczmarek et al., PRD 105, 054513

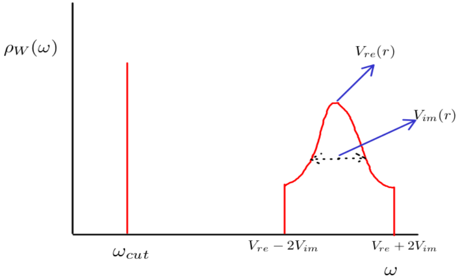


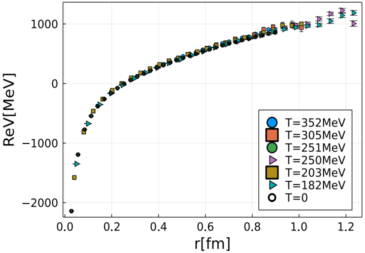
$$V_T^{re}(r) = -\frac{g^2}{4\pi} C_F \left[m_d + \frac{\exp(-m_d r)}{r} \right]$$

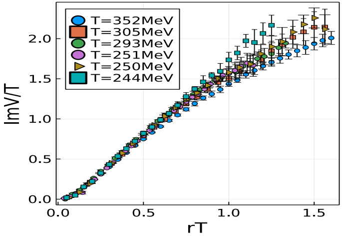
$$V_T^{im}(r) = \frac{g^2}{4\pi} C_F T \int_0^{\infty} \frac{z dz}{(z^2+1)^2} \left[1 - \frac{\sin(z m_d r)}{z m_d r} \right]$$

- Non-perturbative thermal potential is very much different from the perturbative potential.

Cut-Lorentzian







- **Ad-hoc cut-off applied on the Lorentzian.**
- **Existence of low-ω delta function.**

$$V(r) = i \lim_{t\to\infty} \frac{\partial \log W^M(r,t)}{\partial t} = \omega_{cut}!$$

R. Larsen et al., PRD. 109, 074504

- **Unstable fit.**