ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

PENETRATING PROBES OF HOT HIGH–μ_B MATTER:
THEORY MEETS EXPERIMENT

ECT* Workshop, 21-25 July 2025

Probing the phase-coexistence region with leptons

Jørgen Randrup

Lawrence Berkeley National Laboratory

ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

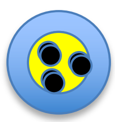
PENETRATING PROBES OF HOT HIGH–μ_B MATTER:
THEORY MEETS EXPERIMENT

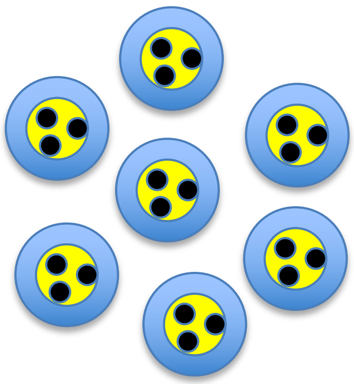
ECT* Workshop, 21-25 July 2025

Probing the phase-coexistence region with leptons?

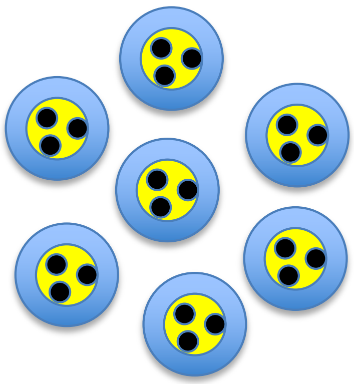
Jørgen Randrup

Lawrence Berkeley National Laboratory

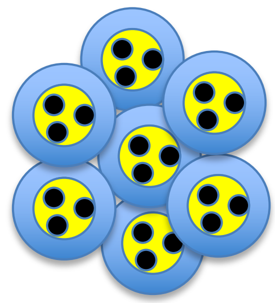




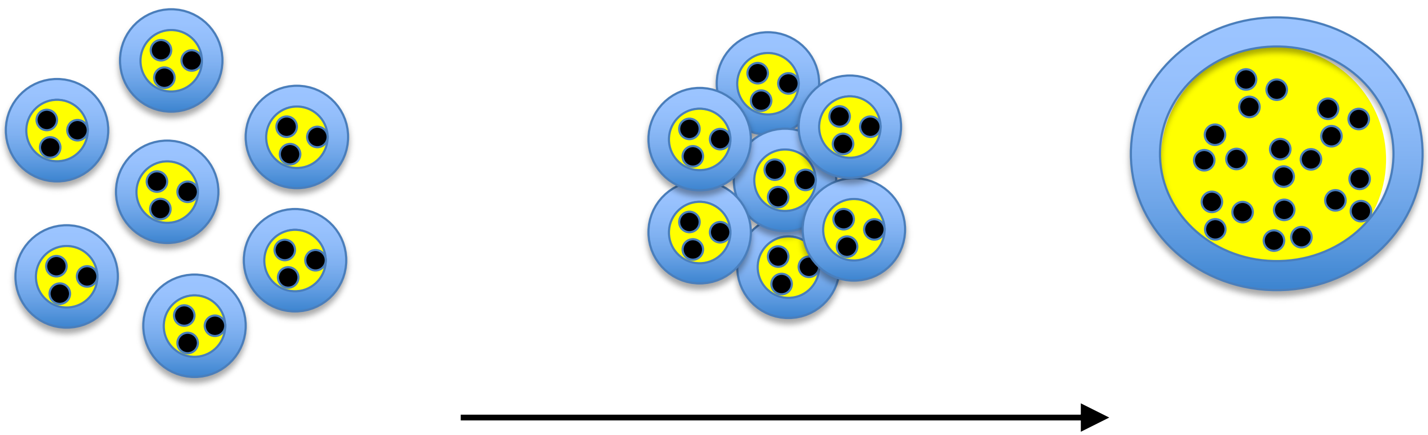
Hadron gas



Dilute: hadron gas



Dense: hadrons?



Density ρ

CONFINED PHASE

Hadron Gas

$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$

DECONFINED PHASE

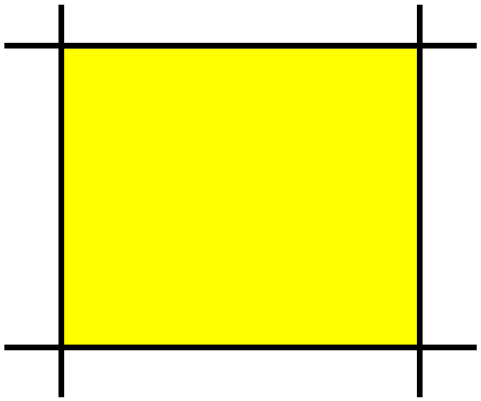
Quark Gluon Plasma

$$p^Q = p_g + p_q + p_{\bar{q}} - B$$

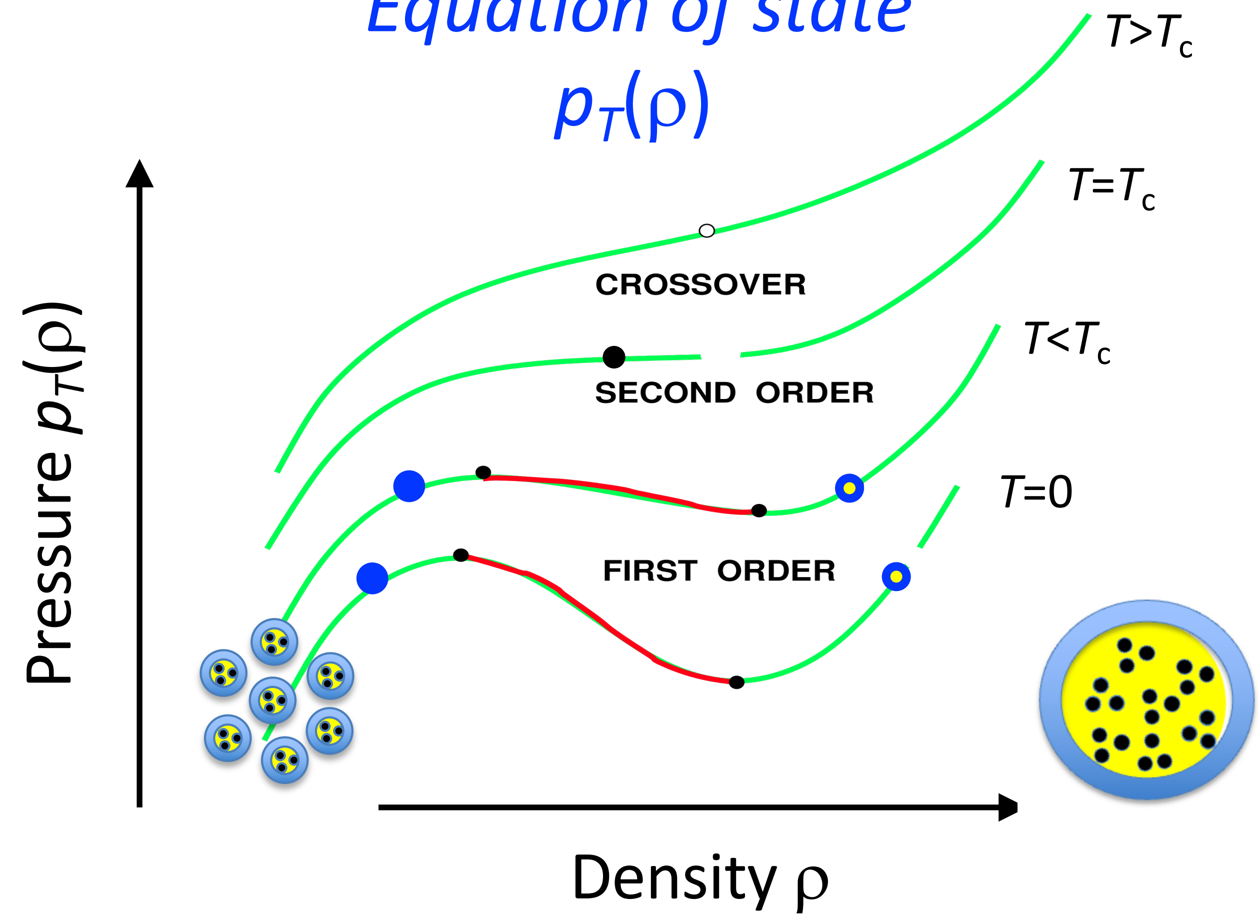
Equation of state

The pressure in *uniform* matter at *equilibrium*

$p(\varepsilon,\rho)$ $p_T(\rho)$ $(\, p_{T\mu} \,)$



Equation of state
$p_T(\rho)$

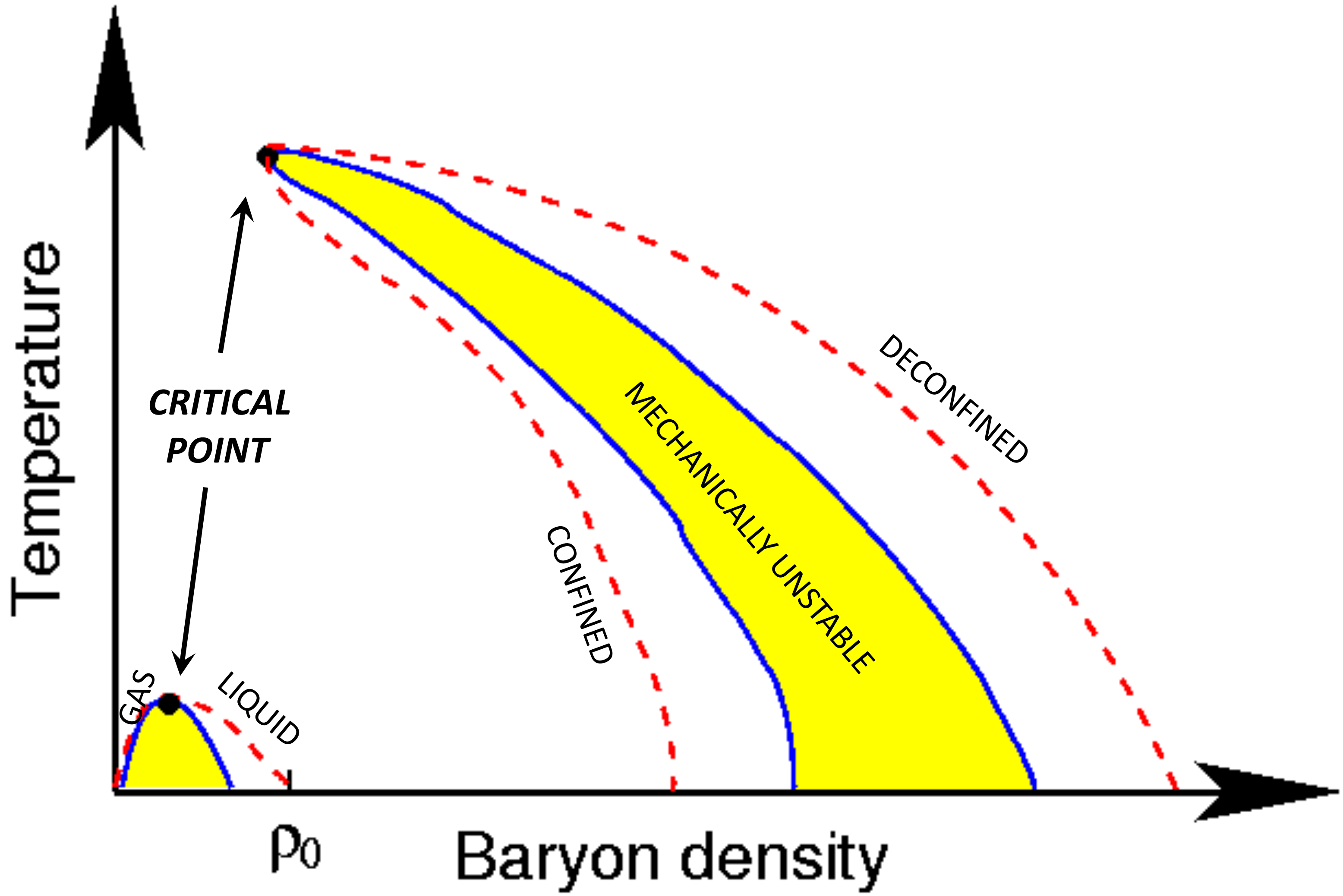


$T > T_c$:
Phase
CHANGE

$T < T_c$:
Phase
TRANSITION

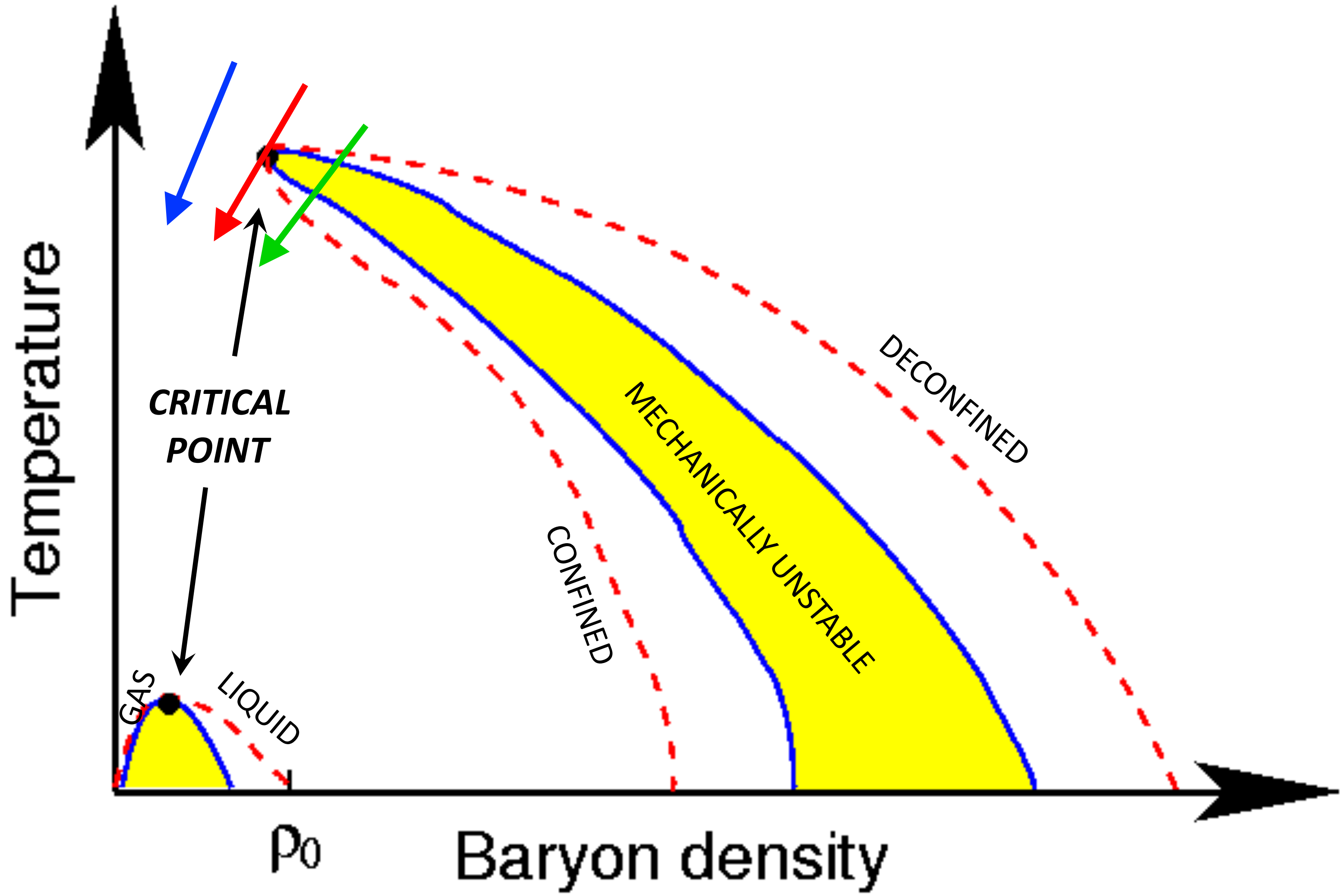
$T < T_c$: Phase ***COEXISTENCE***: same (T, μ, p)

Phase diagram of strongly interacting matter



Recent analysis of neutron star data shows that a phase transition is unlikely to appear below compressions of ≈5,
L. Brandes & W. Weise, Phys. Rev. D **111,** 034005 (2025)

Phase diagram of strongly interacting matter

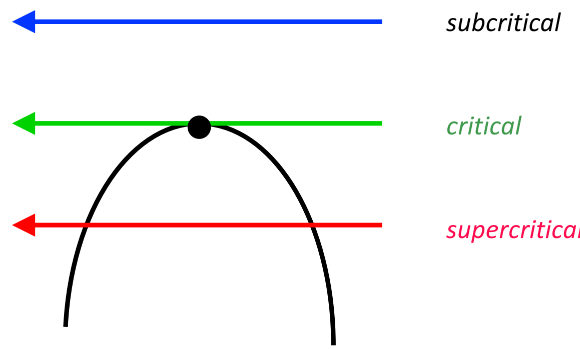


Collective dynamics near a critical point

Amplitude evolution:

$$\frac{d}{dt}A_\nu(t) \;=\; -i\omega_\nu A_\nu(t) \;+\; B_\nu(t)$$

$$\omega_\nu = \epsilon_\nu + i\gamma_\nu$$



Markovian noise:

$$\prec B_\nu(t)\, B_\mu(t')^* \succ \;=\; 2\mathcal{D}_{\nu\mu}\,\delta(t-t')$$

Correlation function:

$$\sigma_{\nu\mu}(t_1,t_2) \;\equiv \prec A_\nu(t_1)\, A_\mu(t_2)^* \succ$$

Evolution:

seed — *feedback*

$$\frac{d}{dt}\sigma_{\nu\mu}(t) \;=\; 2\mathcal{D}_{\nu\mu} \;-\; i(\omega_\nu - \omega_\mu^*)\sigma_{\nu\mu}$$

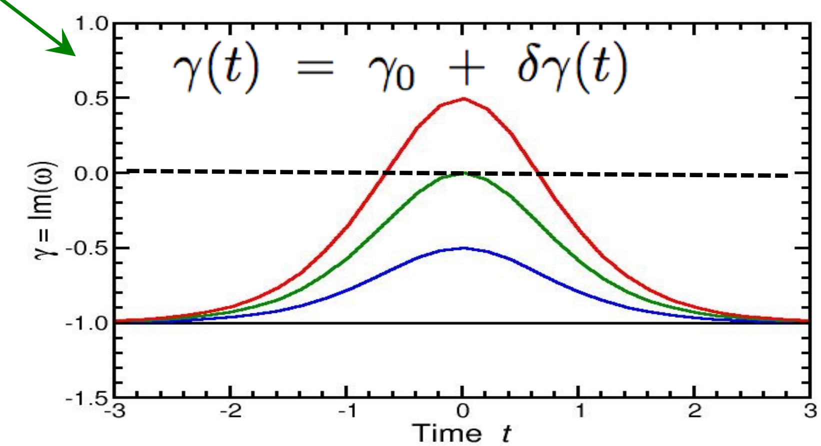
Variance of a single mode:

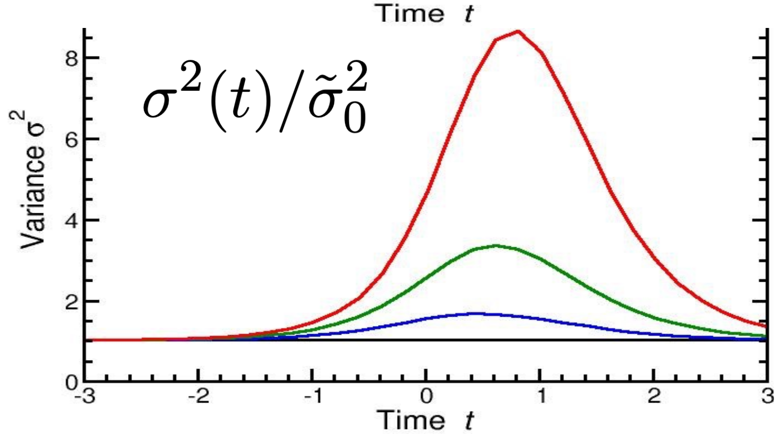
$$\frac{d}{dt}\sigma_\nu^2 \;=\; 2\mathcal{D}_\nu \;+\; 2\gamma_\nu\sigma_\nu^2$$

$$\tilde{\sigma}^2 = -\mathcal{D}_\nu/\gamma_\nu$$

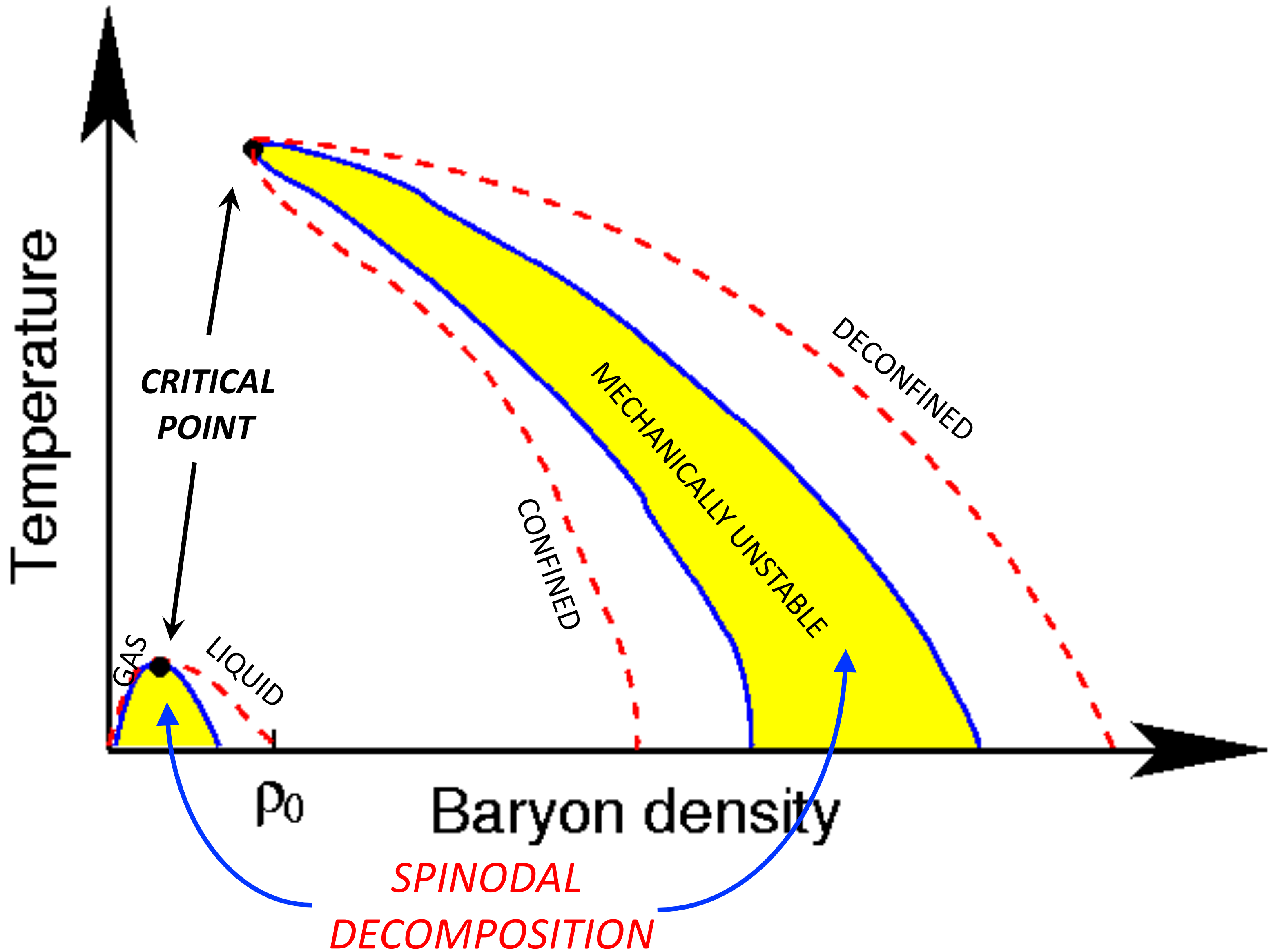
$$\Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

=> $$\sigma_\nu^2(t) \;=\; \left[2\mathcal{D}_\nu \int_0^t \mathrm{e}^{-2\Gamma_\nu(t')}dt' + \sigma_0^2\right]\mathrm{e}^{2\Gamma_\nu(t)}$$

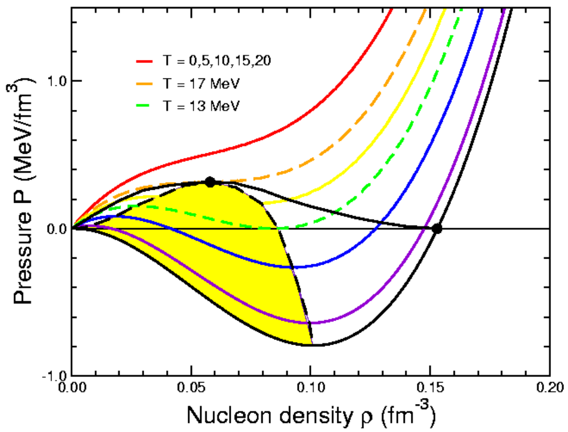




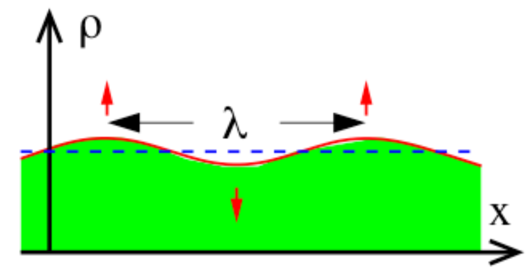
Phase diagram of strongly interacting matter



Spinodal decomposition in nuclear matter

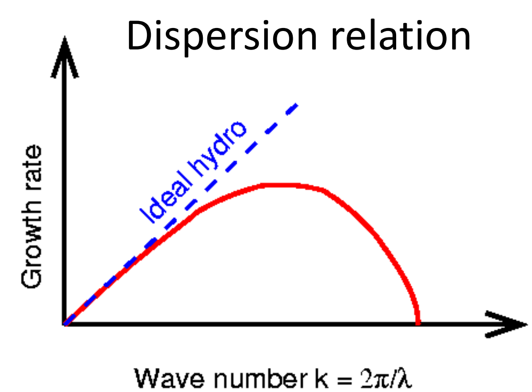


In the spinodal region, density undulations are amplified:

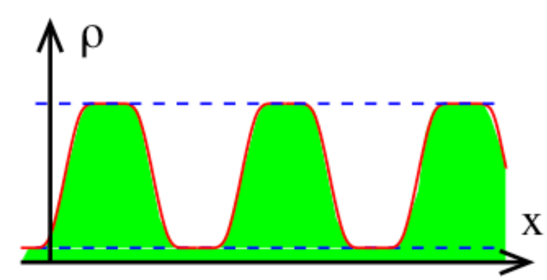


Long-wavelength undulations grow slowly
(it takes time to relocate the matter)

Short-wavelength undulations grow slowly
(they are hardly felt due to finite range)

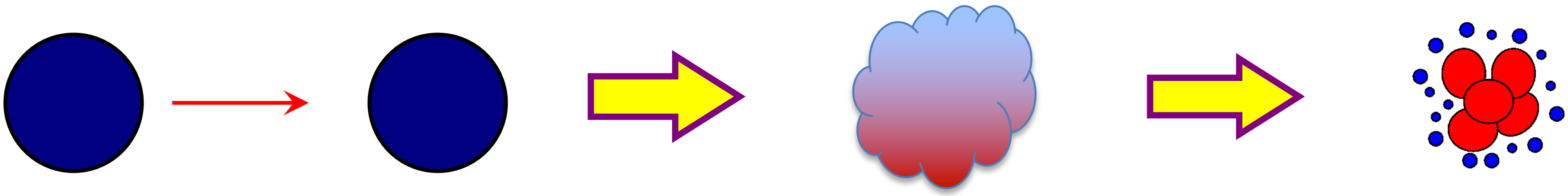


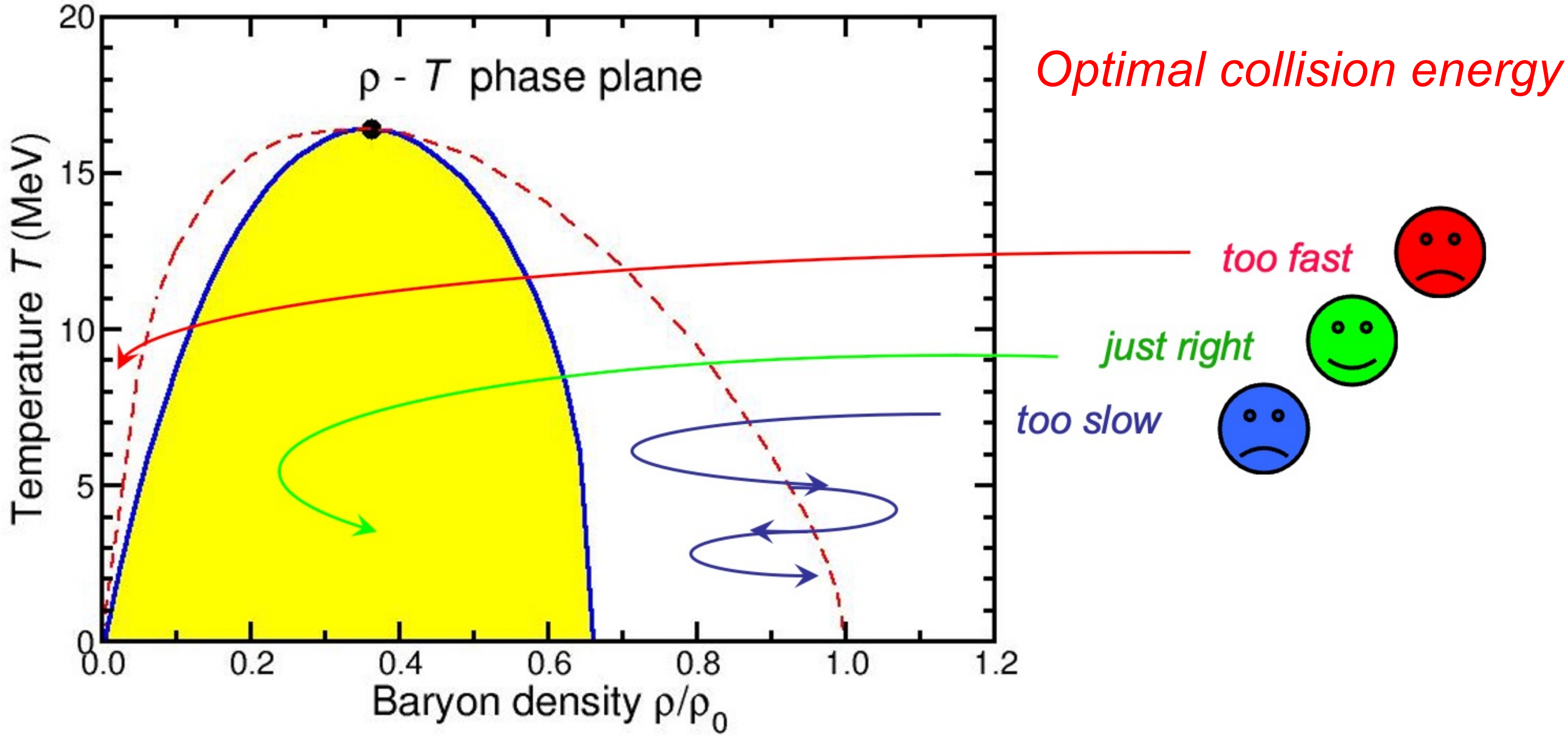
=> There is an *optimal wavelength* that grows faster than all others



Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
Physics Reports **389** (2004) 263

Nuclear spinodal fragmentation



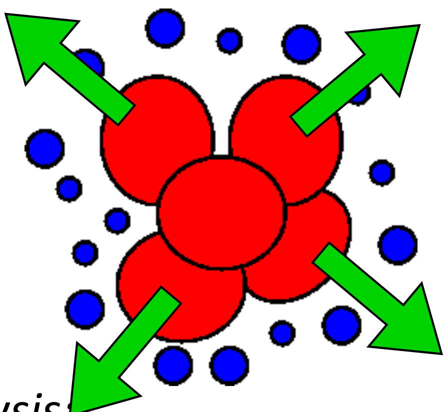


Nuclear spinodal fragmentation

32 MeV/*A* Xe + Sn (*b*=0)

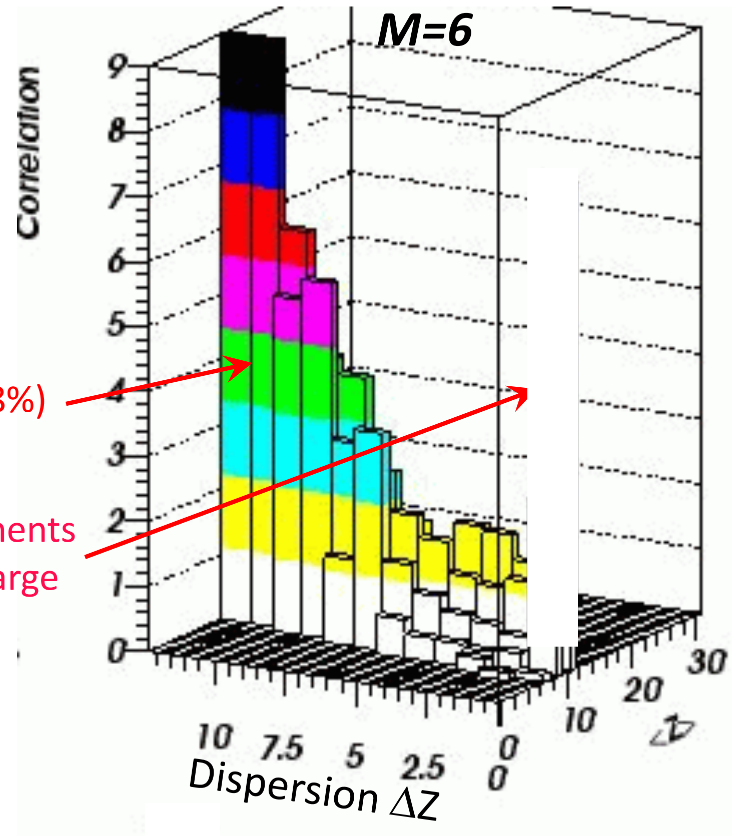
Theory (*Boltzmann-Langevin*):

The collision produces a compressed system whose subsequent expansion brings its bulk into the spinodal region where it condenses into several fragments having the *same* size



Statistical distribution (≈98%)

In these events, all 6 fragments have (nearly) the same charge

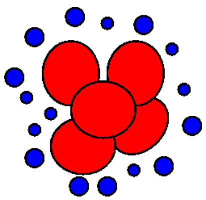


Analysis:

For each event having *M* fragments, calculate mean fragment charge <Z> and the charge *dispersion* ΔZ

→ LEGO plot of (<Z>, ΔZ)

Nuclear spinodal fragmentation *OCCURS!*



32 MeV/*A* Xe + Sn (*b*=0):

select events with *M* IMFs

Bin wrt $\begin{cases} \langle Z \rangle : & \text{average IMF charge} \\ \Delta Z : & \text{dispersion in IMF charge} \end{cases}$

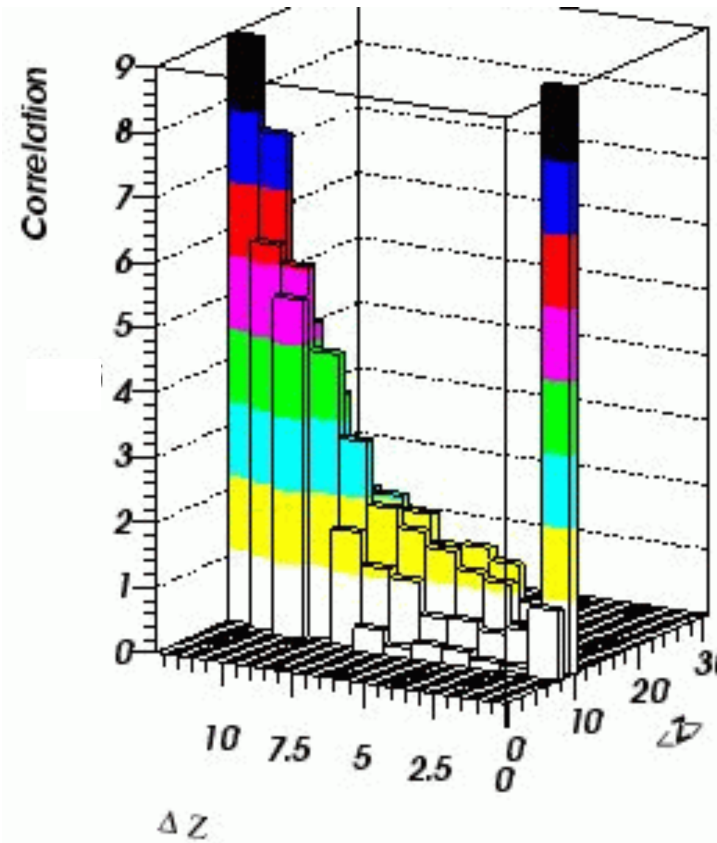
Experiment (*INDRA @ GANIL*)

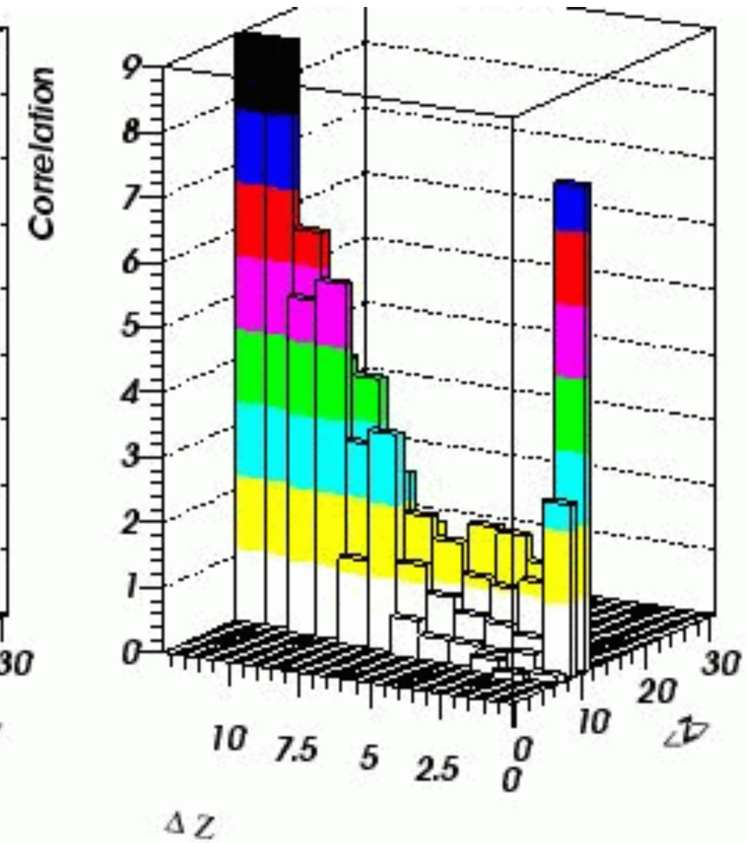
B. Borderie *et al*, PRL **86** (2001) 3252

Theory (*Boltzmann-Langevin*)

Ph. Chomaz *et al*, PRL 73 (1994) 3512

M=6:





Nuclear spinodal fragmentation

OCCURS!

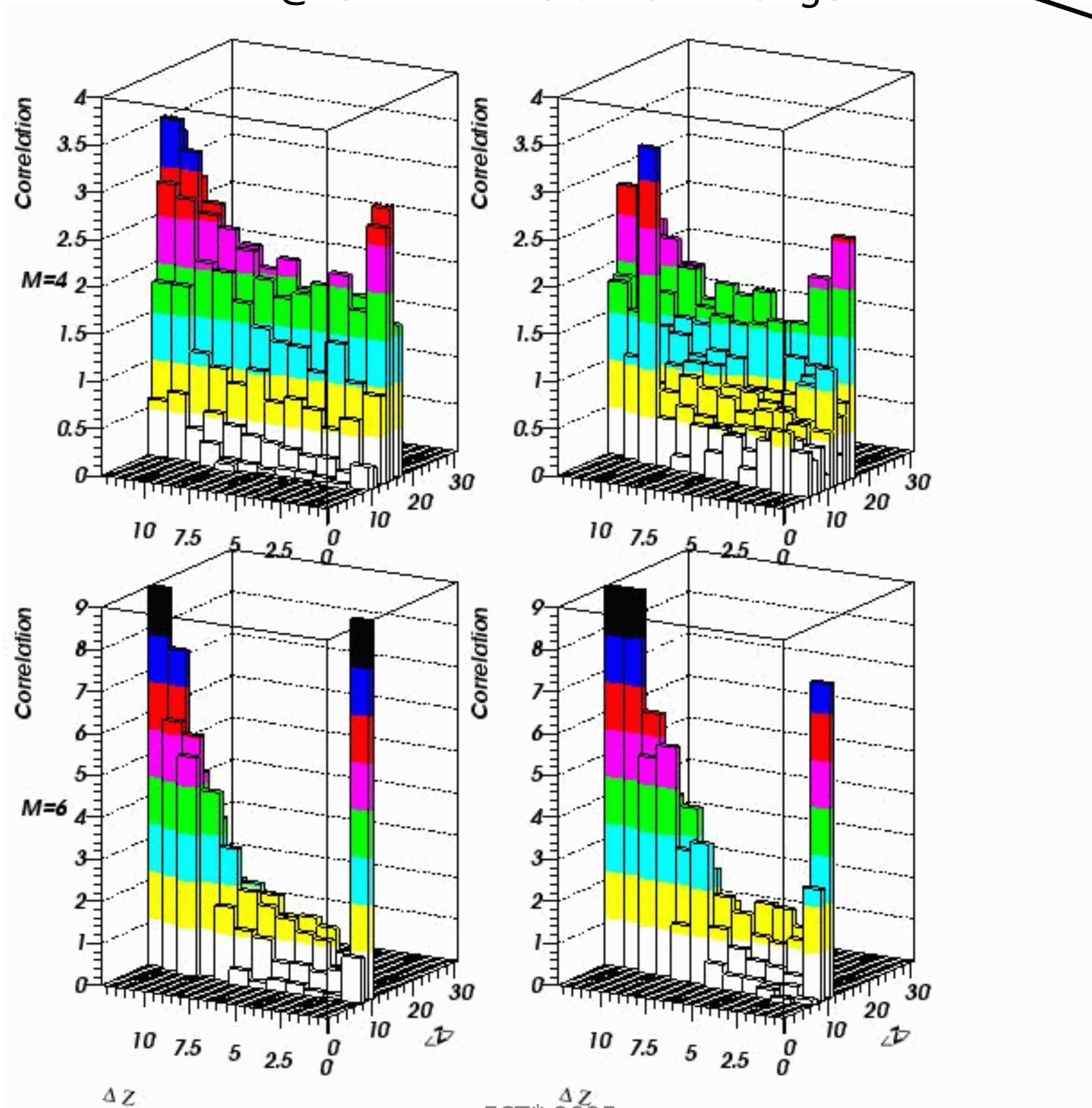
Experiment	**Theory**
INDRA @ GANIL	*Boltzmann-Langevin*

B. Borderie *et al*, Phys. Rev. Lett. 86 (2001) 3252

Ph. Chomaz *et al*, Phys. Rev. Lett. 73 (1994) 3512

M=4:

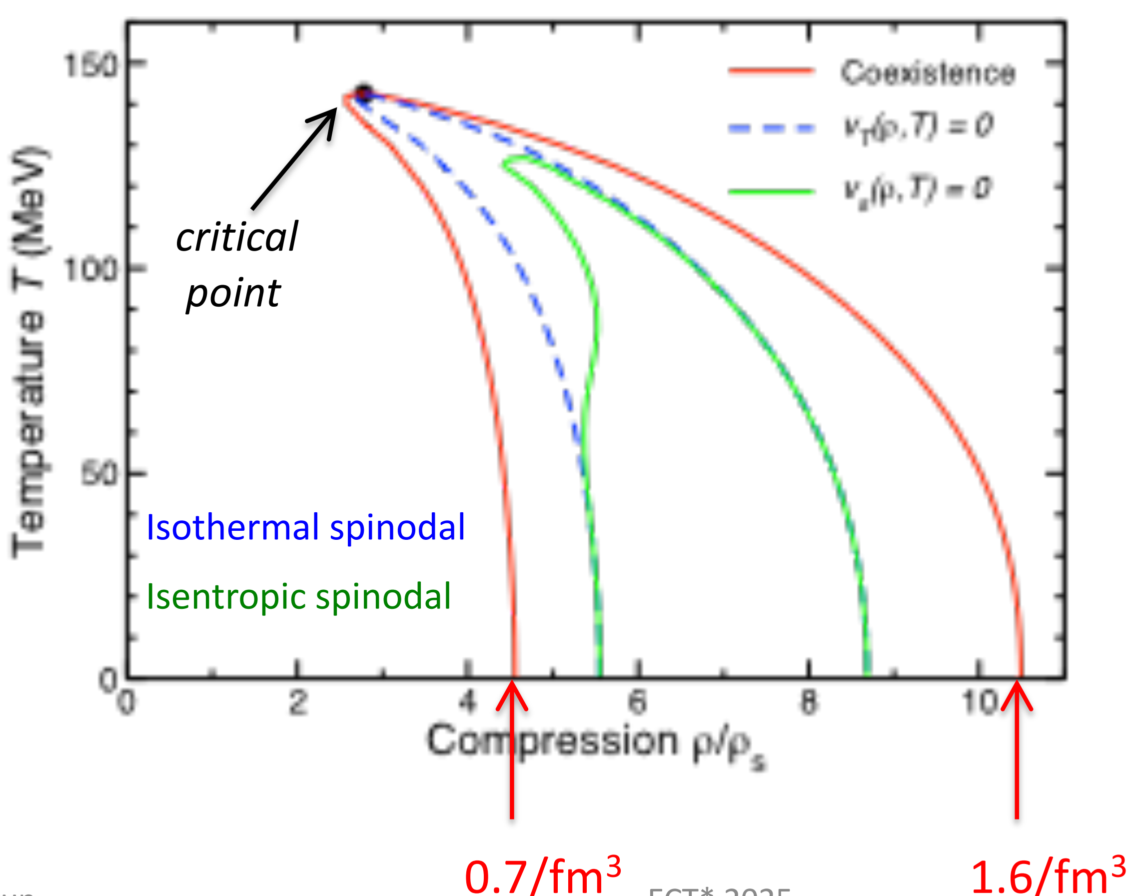
M=6:



Is the spinodal mechanism a useful tool for probing the deconfinement phase transition?

Construct a plausible *equation of state* $p_0(\varepsilon,\rho)$:
Interpolate between hadron gas and QGP

J Randrup,
PRC **82**, 034902 (2010)



Is the spinodal mechanism a useful tool for probing the deconfinement phase transition?

Construct a plausible *equation of state* $p_0(\varepsilon,\rho)$:

Interpolate between hadron gas and QGP

J Randrup,
PRC **82**, 034902 (2010)

Introduce *finite-range correction*:

Equation of state: Finite range

Free energy density for uniform matter: $f_0(\rho, T)$

But we need to treat non-uniform systems: $\rho(\boldsymbol{r}), \;\; T(\boldsymbol{r})$

Local density approximation: $f(\boldsymbol{r}) \;\doteq\; f_0(\rho(\boldsymbol{r}), T(\boldsymbol{r}))$

Total free energy: $F[\rho(\boldsymbol{r}), T(\boldsymbol{r})] \;=\; \int d^3\boldsymbol{r}\, f(\boldsymbol{r})$

The local density approximation implies:

$F_T($  $)$ = $F_T($  $)$

No good! => Finite range *must* be taken into account

Is the spinodal mechanism a useful tool for probing the deconfinement phase transition?

Construct a plausible *equation of state* $p_0(\varepsilon,\rho)$:

Interpolate between hadron gas and QGP

J Randrup,
PRC **82**, 034902 (2010)

Introduce *finite-range correction*:

$$p(\boldsymbol{r}) \;\approx\; p_0(\varepsilon(\boldsymbol{r}),\rho(\boldsymbol{r})) \;-\; C\rho_0\nabla^2\rho(\boldsymbol{r})$$

=> Interface tension, spinodal clumping

J. Randrup,
PRC **79**, 054911 (2009)

MB Pinto, V Koch, J Randrup,
PRC **86,** 025023 (2012)

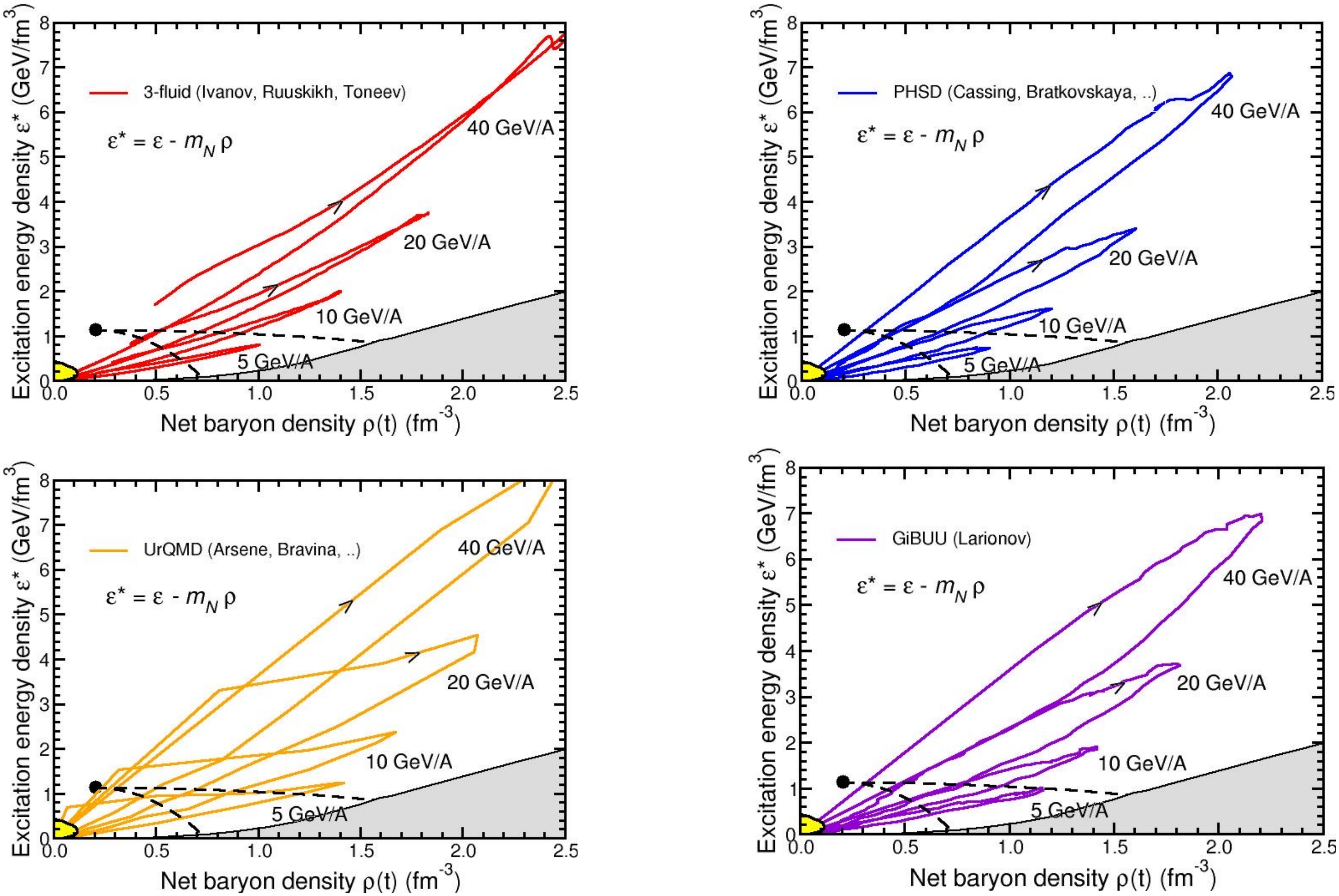
Carry out *dynamical simulations*:

3D relativistic fluid dynamics (Jan Steinheimer)

Which collision energy?

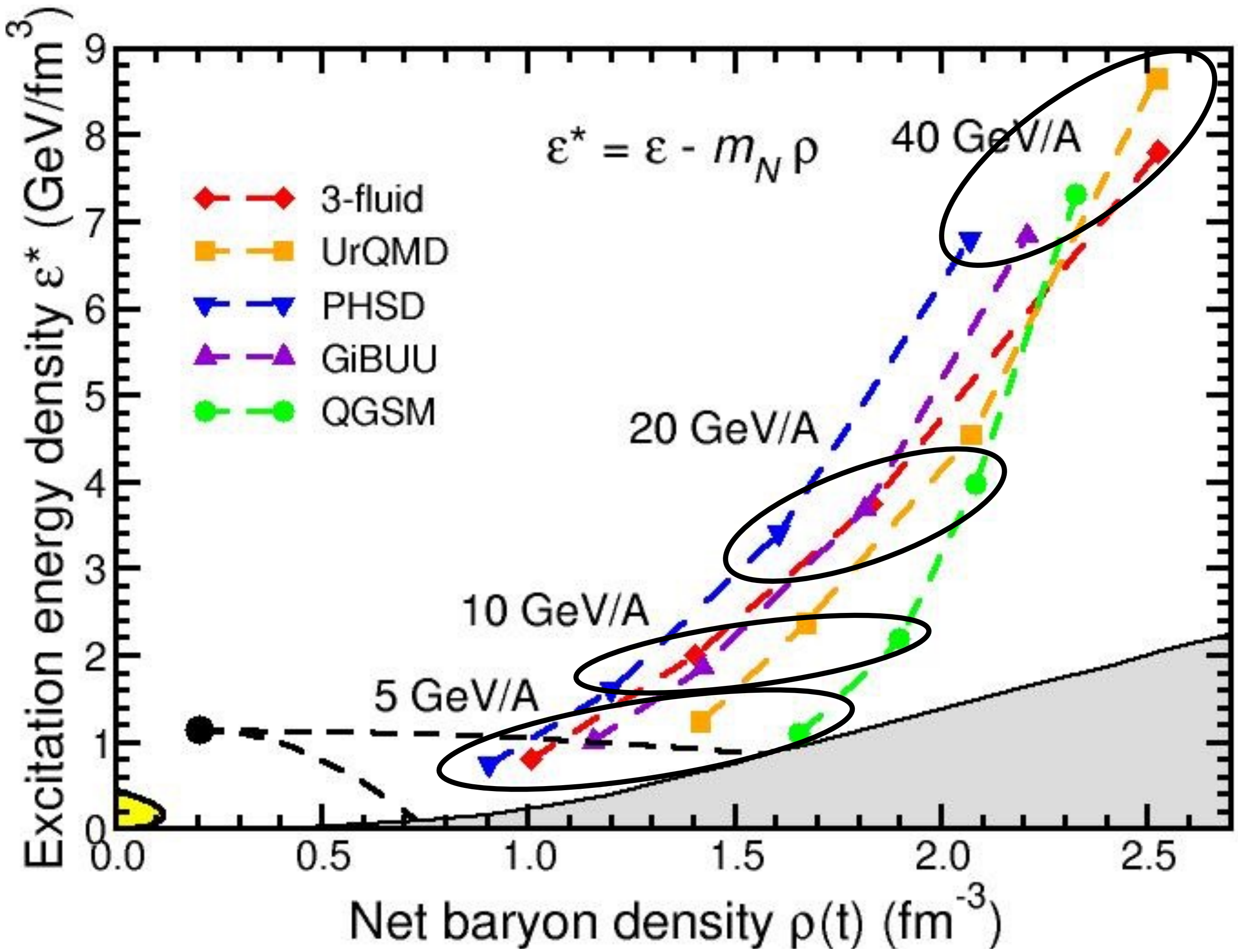
Dynamical phase trajectories ($\rho(t),\varepsilon^*(t)$)

with various dynamical models



Arsene, Bravina, Cassing, Ivanov, Larionov, Randrup, Russkikh, Toneev, Zeeb, Zschiesche, PRC **75** (2007) 034902

Maximum compression



Arsene, Bravina, Cassing, Ivanov, Larionov, Randrup, Russkikh, Toneev, Zeeb, Zschiesche, PRC **75** (2007) 034902

Is the spinodal mechanism a useful tool for probing the deconfinement phase transition?

Construct a plausible *equation of state* $p_0(\varepsilon,\rho)$:
Interpolate between hadron gas and QGP

J Randrup,
PRC **82**, 034902 (2010)

Introduce *finite-range correction*:

$$p(\boldsymbol{r}) \approx p_0(\varepsilon(\boldsymbol{r}), \rho(\boldsymbol{r})) - C\rho_0\nabla^2\rho(\boldsymbol{r})$$

=> Interface tension, spinodal clumping

J. Randrup,
PRC **79**, 054911 (2009)

MB Pinto, V Koch, J Randrup,
PRC **86,** 025023 (2012)

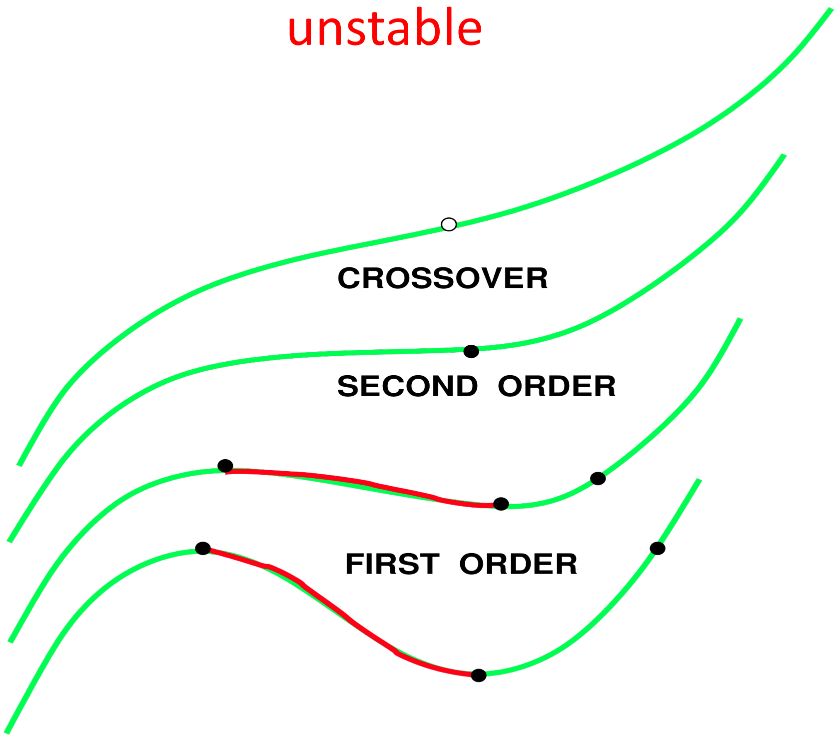
Carry out *dynamical simulations*:
3D relativistic fluid dynamics (Jan Steinheimer)

J. Steinheimer & J. Randrup,
PRL **109**, 212301 (2012);
PRC **87**, 054903 (2013)

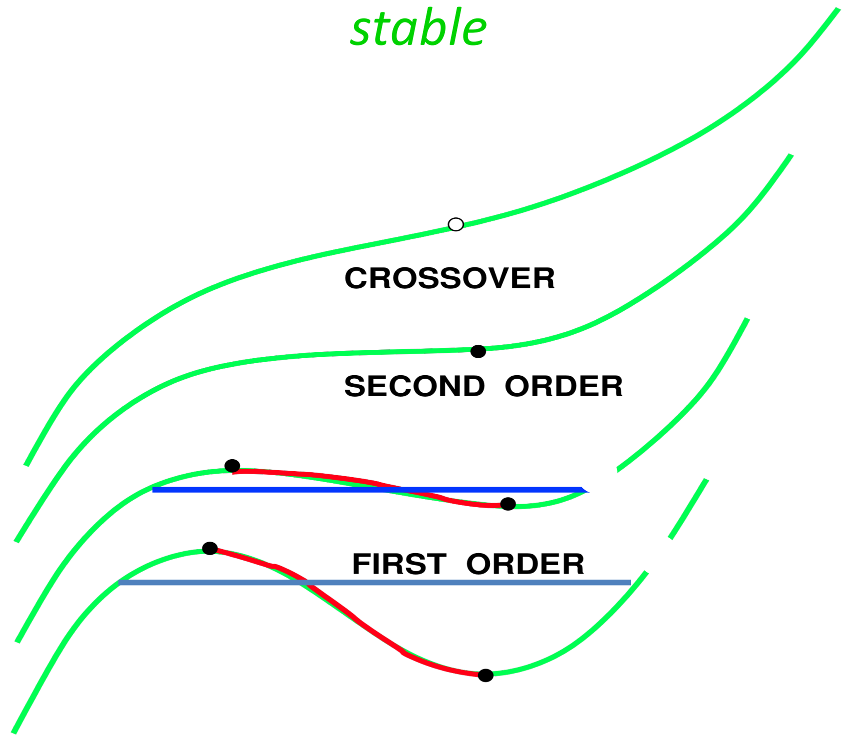
Does phase separation (clumping) occur?

If so, what are the observable effects?

Actual two-phase EoS:
unstable



Maxwell partner EoS:
stable

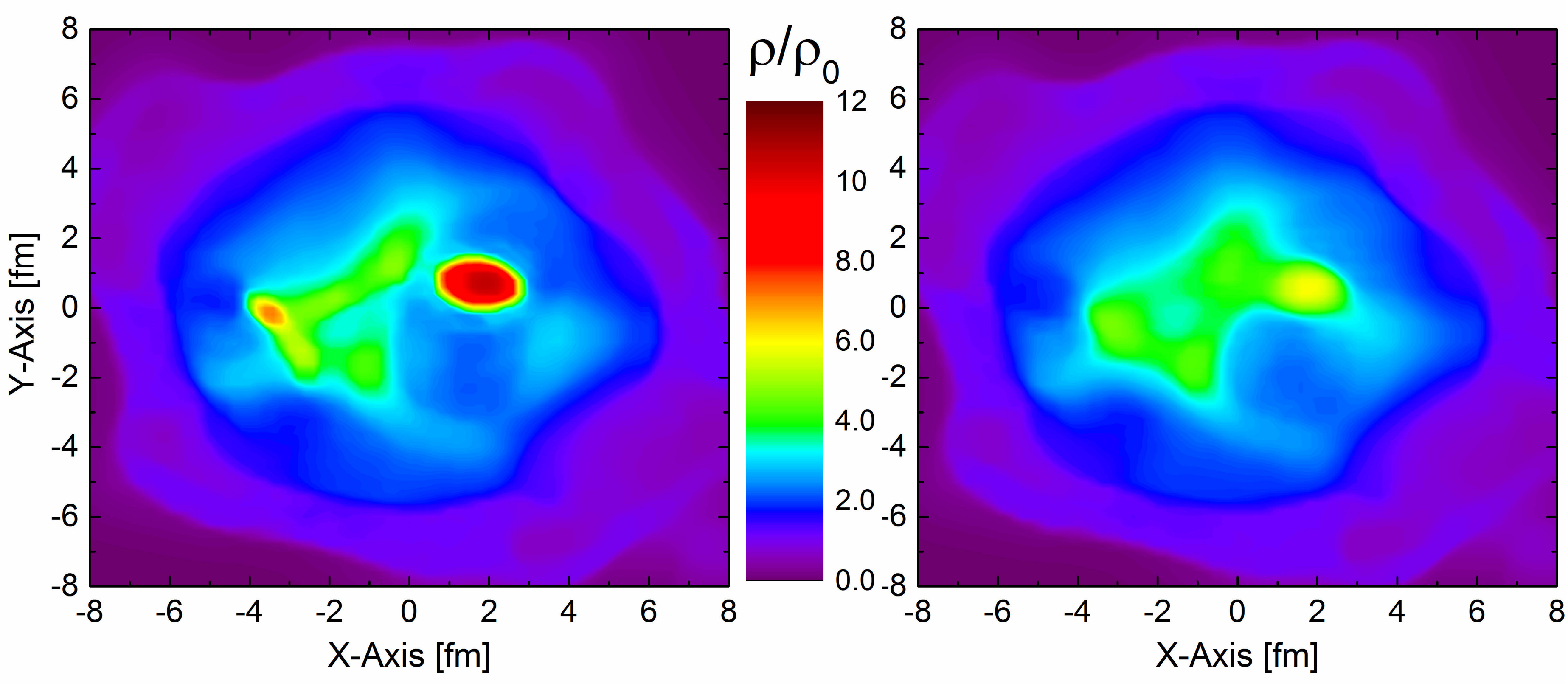


3 *A* GeV Pb + Pb (*b*=0)

Identical initial conditions

Two-phase EoS: *unstable*

Maxwell EoS: *stable*



J. Steinheimer & J. Randrup,
Phys Rev Lett **109**, 212301 (2012);
Phys Rev C **87**, 054903 (2013)

Is the spinodal mechanism a useful tool for probing the deconfinement phase transition?

Does phase separation (clumping) occur?

Yes, within an optimal energy range

J. Steinheimer & J. Randrup,
Phys Rev Lett **109**, 212301 (2012);
Phys Rev C **87**, 054903 (2013)

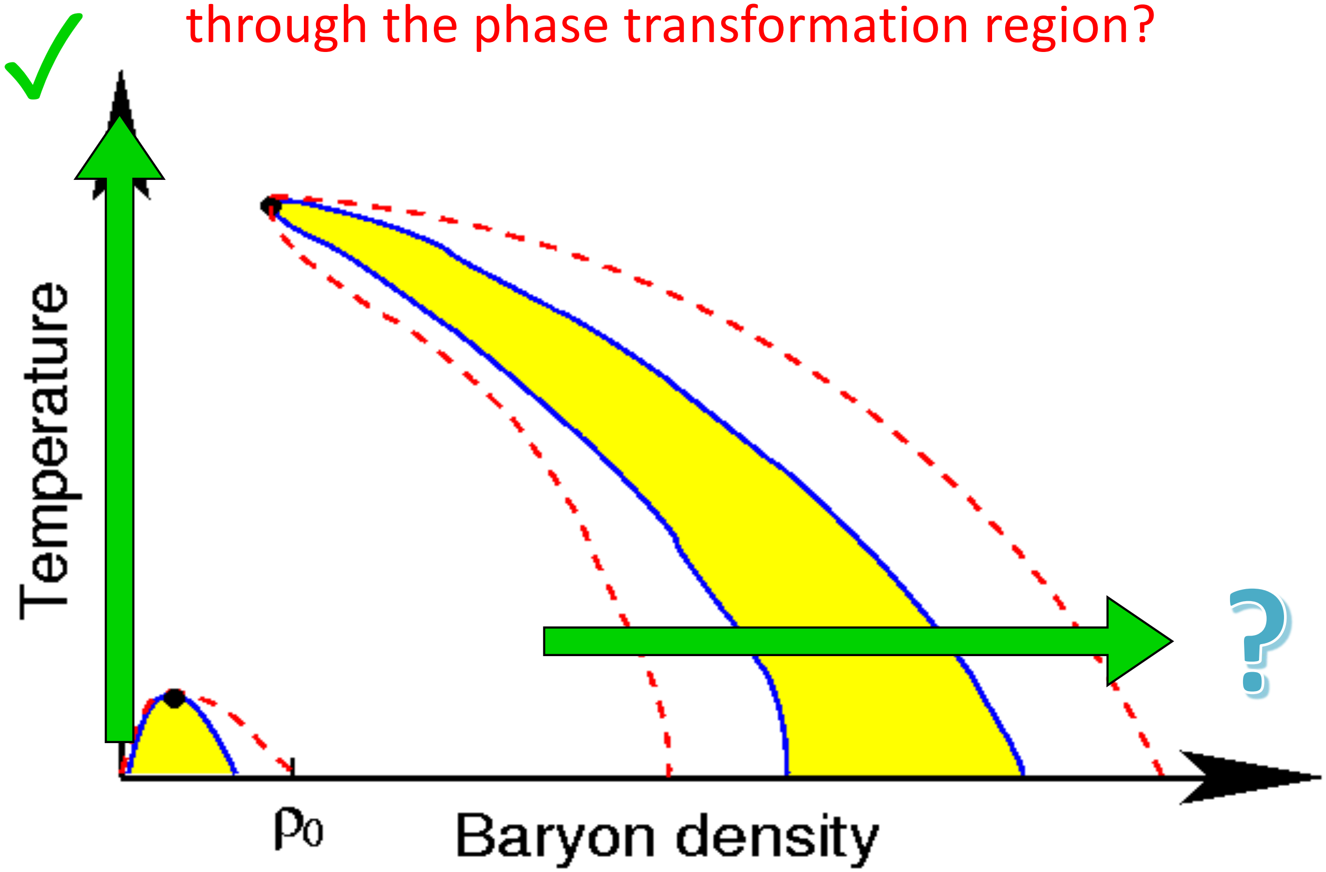
If so, what are the observable effects?

Hadronic observables: nothing spectacular

J Steinheimer, J Randrup, V Koch,
Phys Rev C **89**, 034901 (2014)

Penetrating probes: dileptons?

How does the microscopic structure of matter evolve through the phase transformation region?

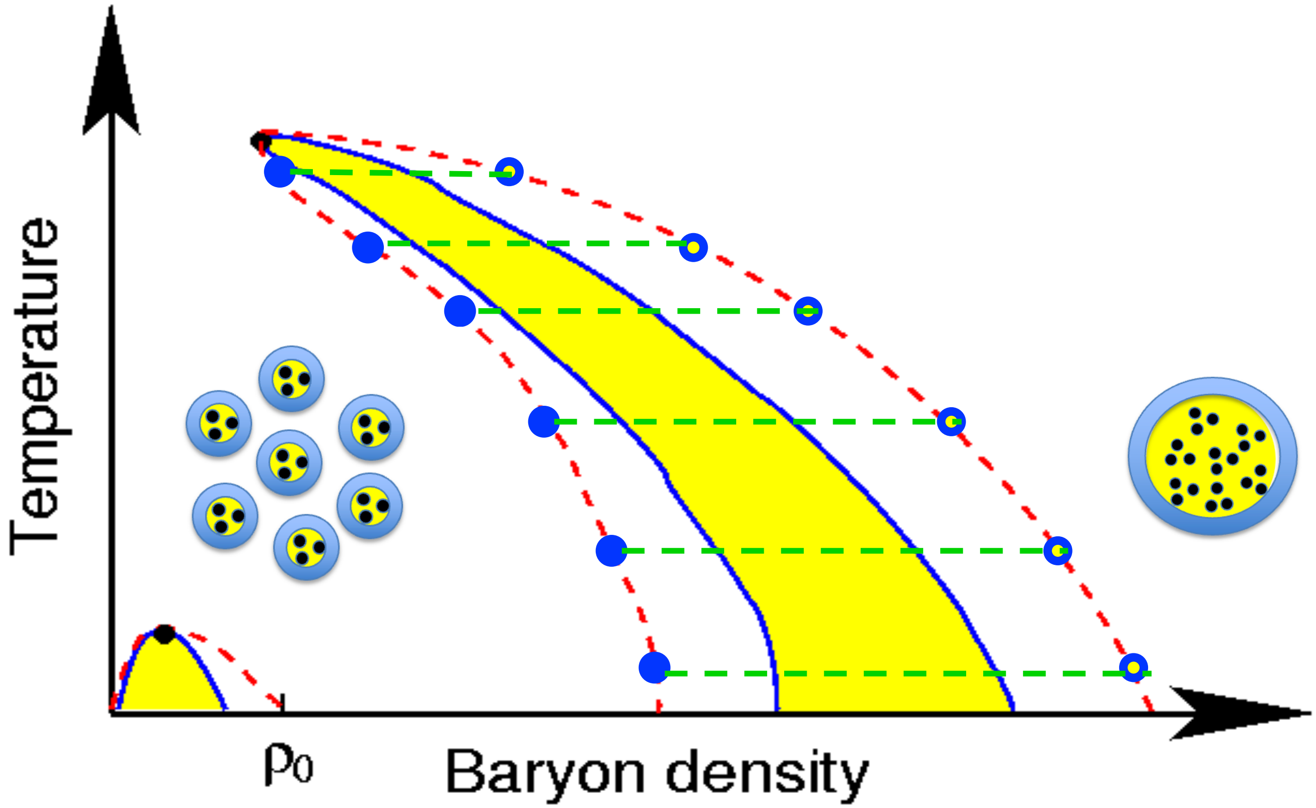


Lattice QCD at $\mu = 0$ may elucidate how the system evolves from a hadron gas to a quark-gluon fluid as the temperature is increased though the phase transformation region

How does the dilepton production evolve?

"HG" at density ρ_H coexists with "QGP" at density ρ_Q
(*Coexistence*: the two systems have the *same* p, T, μ)

(How) do dileptons emitted from such coexisting systems differ?



TASK

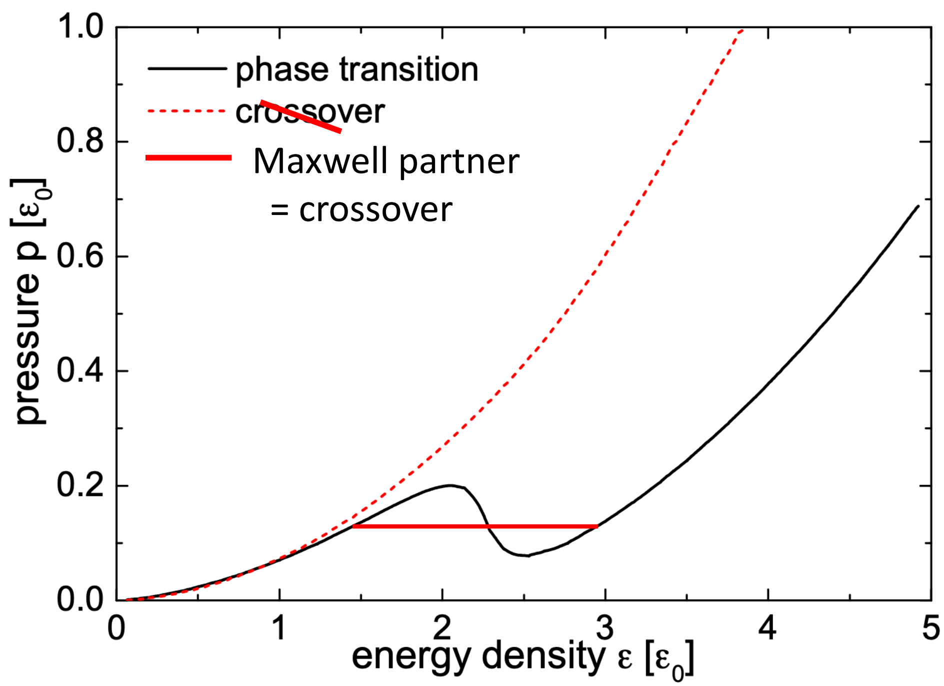
A systematic comparison of dilepton emission
from *coexisting* confined and deconfined matter

Probing the phase-coexistence region with leptons

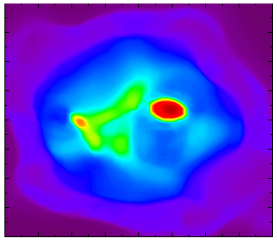
Recent dynamical studies

Dilepton signature of a first-order phase transition,
F. Seck, T. Galaytuk, A. Mukheerjee, R. Rapp, J. Steinheimer, J. Stroth,
Phys Rev C **106**, 014904 (2022)

Enhanced dilepton emission from a phase transition in dense matter,
O. Savchuck, A. Motornenko, J. Steinheimer, V. Vovchenko, M. Bleicher,
M. Gorenstein, T. Galatyuk, J Phys G: Nucl Part Phys **50**, 125104 (2023)

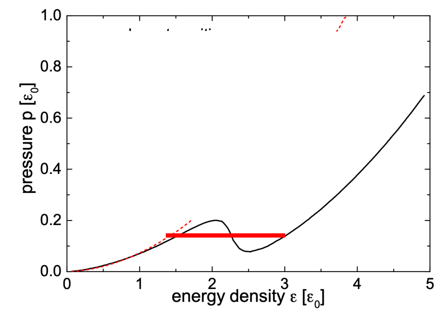


Concluding remarks

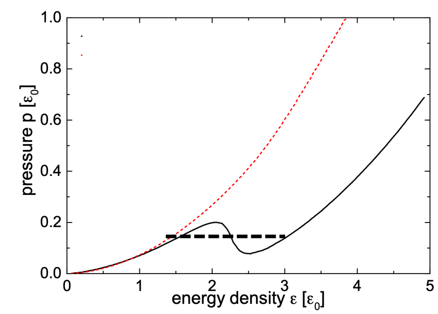


A first-order phase transition might produce
transient blobs of quark matter
(whereas a smooth phase change would not)

It may be *im*possible to determine experimentally whether the QCD phase diagram has a *first-order* phase transition and an associated *critical point*



But it may be feasible to observe the *softening* associated with a deconfinement phase change, due to the enhancement resulting from the increased time spent in the transformation zone



PENETRATING PROBES OF HOT HIGH$-\mu_B$ MATTER:
THEORY MEETS EXPERIMENT

ECT* Workshop, 21-25 July 2025

Probing the phase-coexistence region with leptons?

Jørgen Randrup

Lawrence Berkeley National Laboratory

Thank You!