Spectral functions from lattice QCD, from light hadrons to heavy quarks

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(2) University of Liverpool, U.K.
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(5) INFN Firenze, Italy
(6) Trinity College, Dublin, Ireland
(7) National University of Ireland Maynooth, Ireland

Overview

FASTSUM approach

- *Anisotropic*

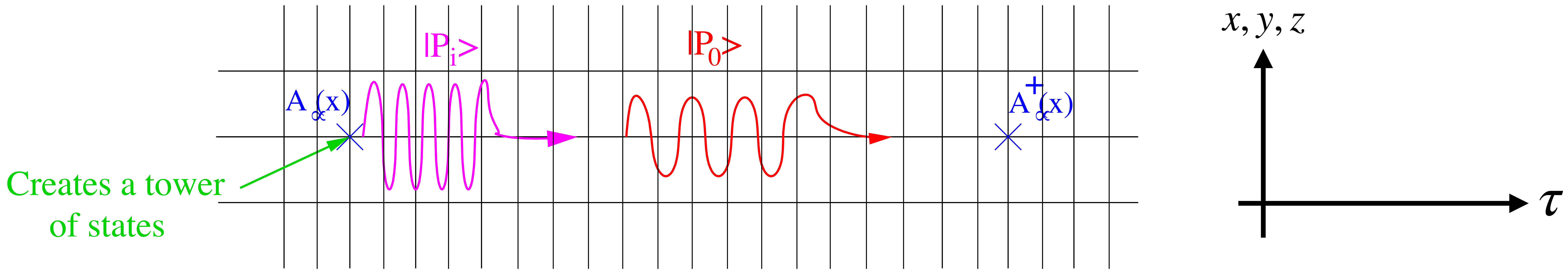
$T_C(\mu)$ curve from mesonic spectrum

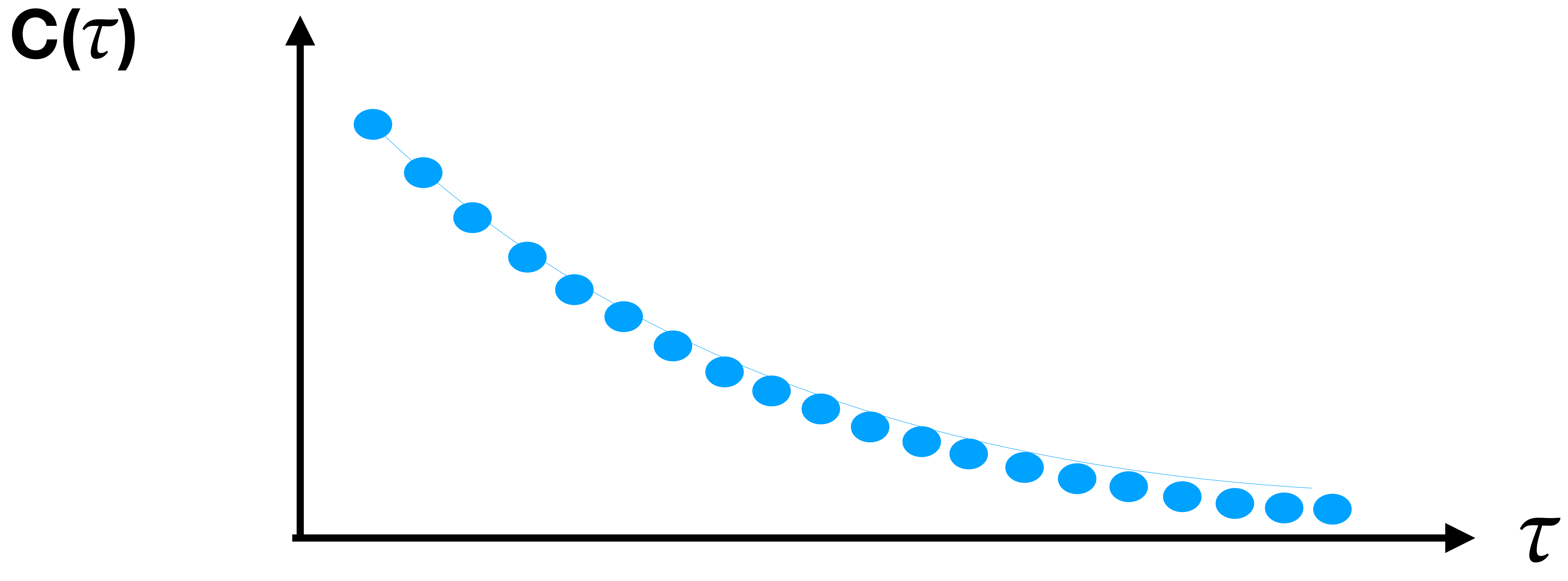
Charm Baryons and Parity Doubling

Spectral Functions for NRQCD mesons

FASTSUM Approach:

Anisotropic Lattice





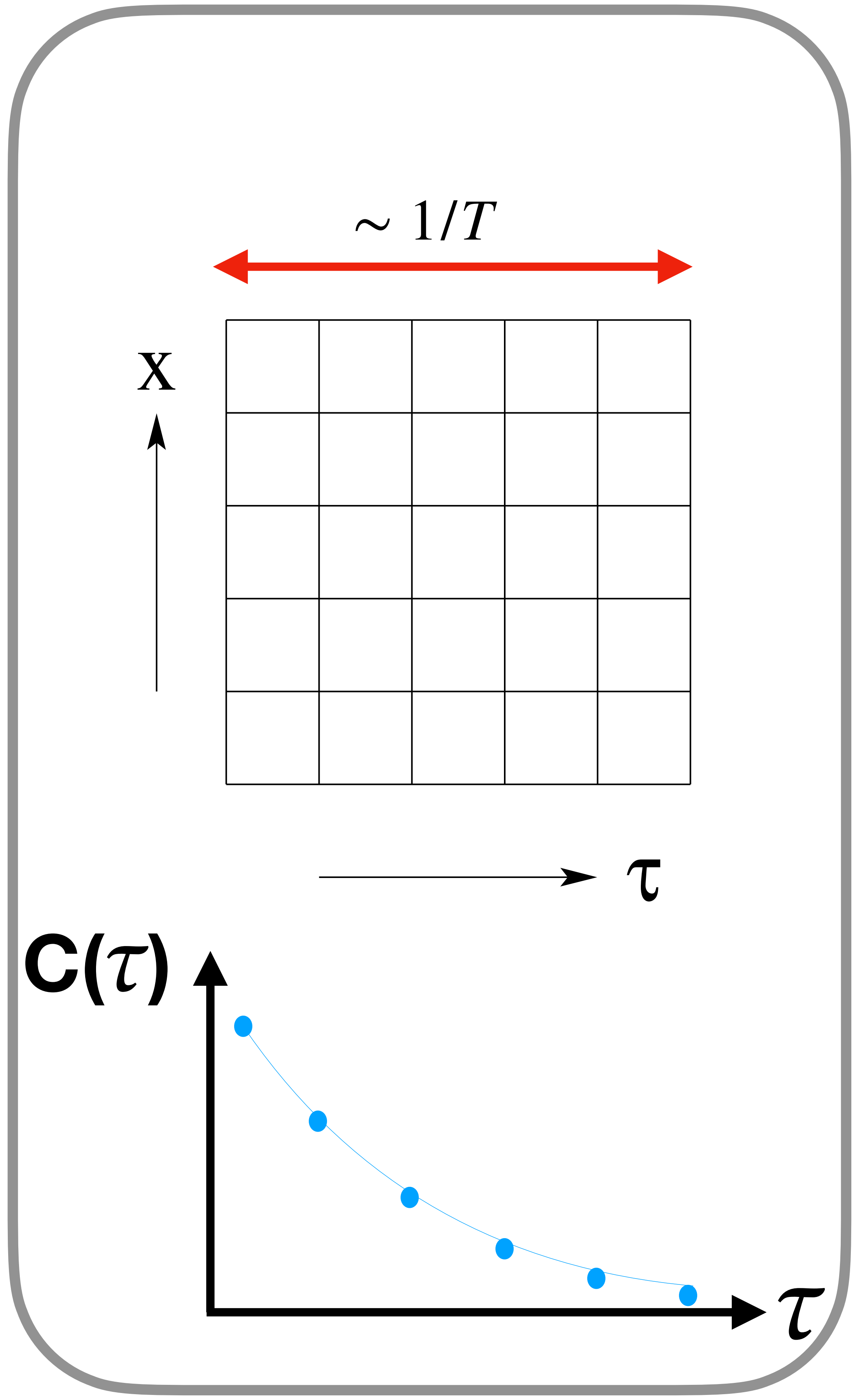
Spectral Quantities:
- Bottomonium
- Charmed mesons
- Heavy Baryons
- Light Hadrons

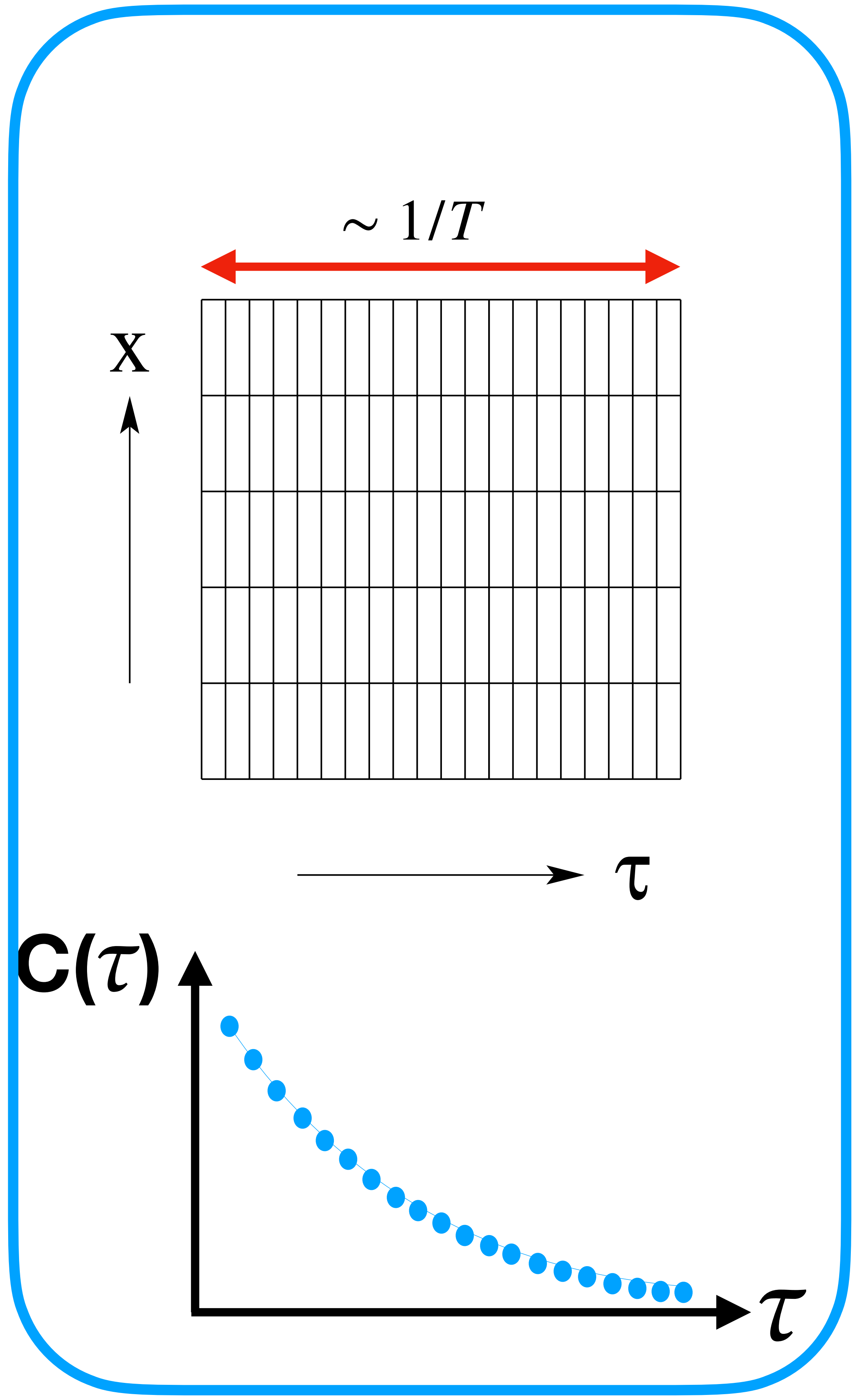
Interquark potential

Conductivity

FASTSUM Approach:

Anisotropic Lattice



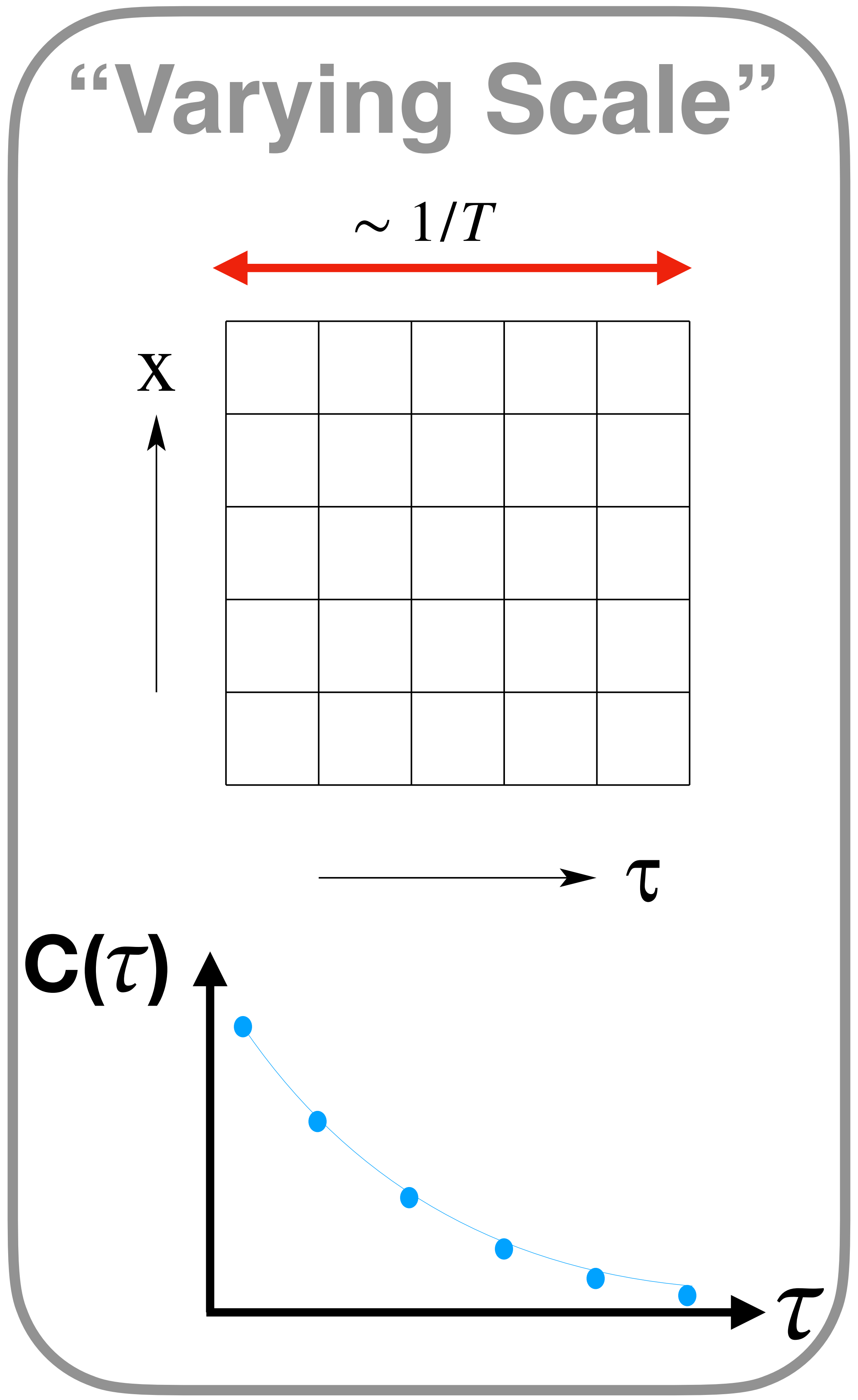


$$\sum_i \langle i | e^{-H L_\tau} | i \rangle = \sum_i \langle i | e^{-H/T} | i \rangle$$

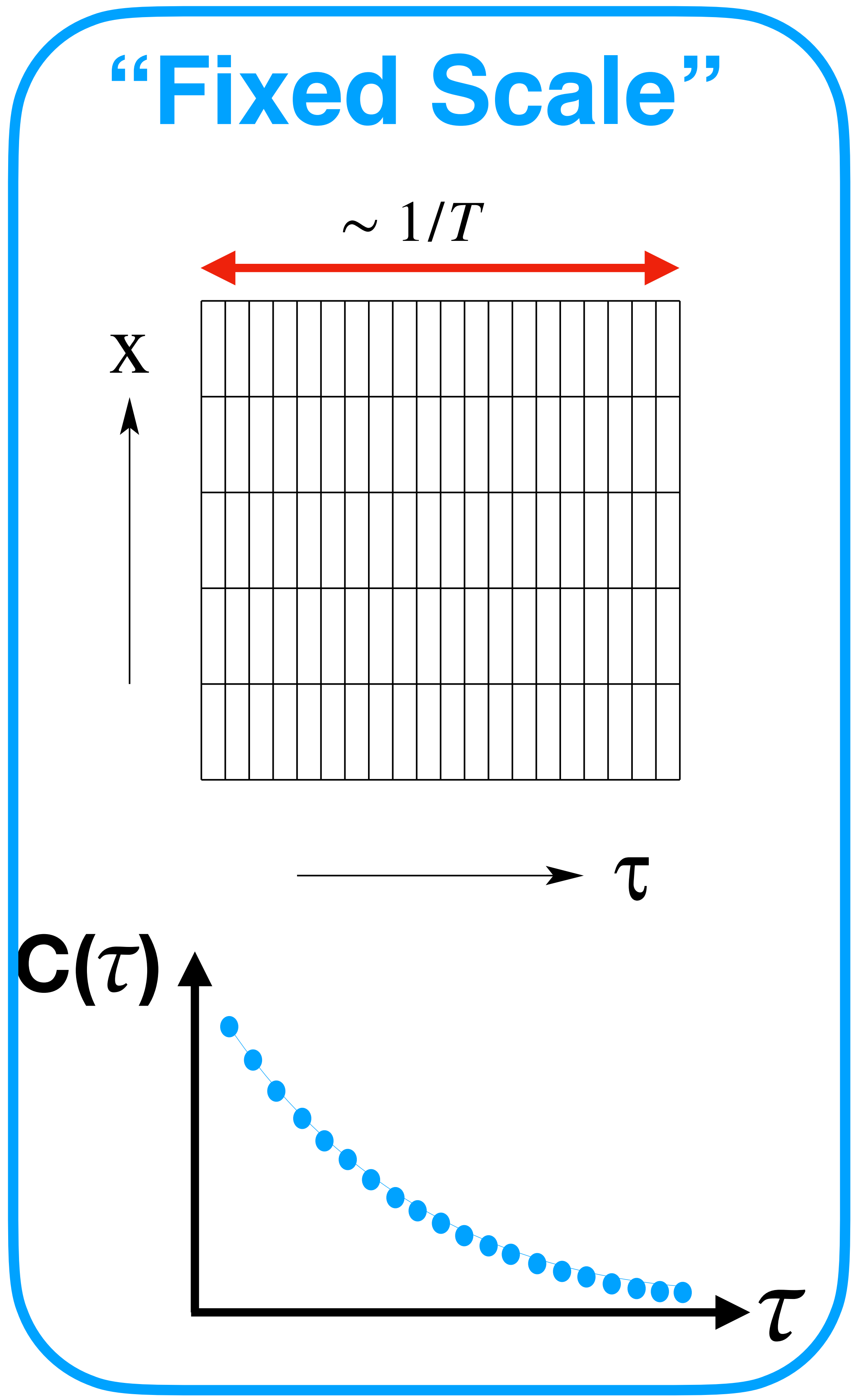
$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

FASTSUM Approach:

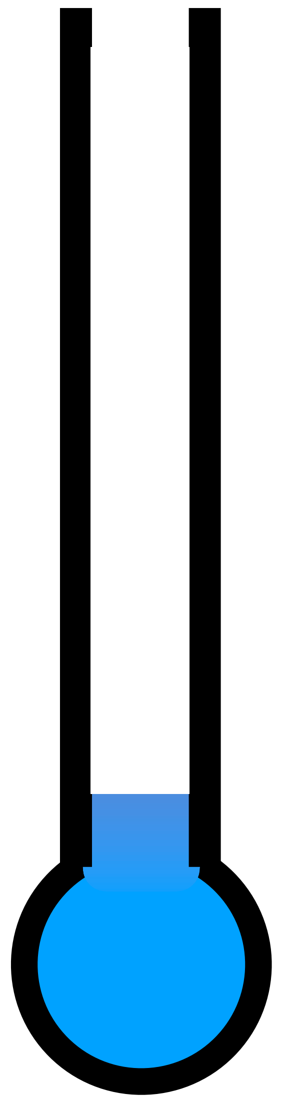
Anisotropic Lattice



$a_\tau \rightarrow 0$



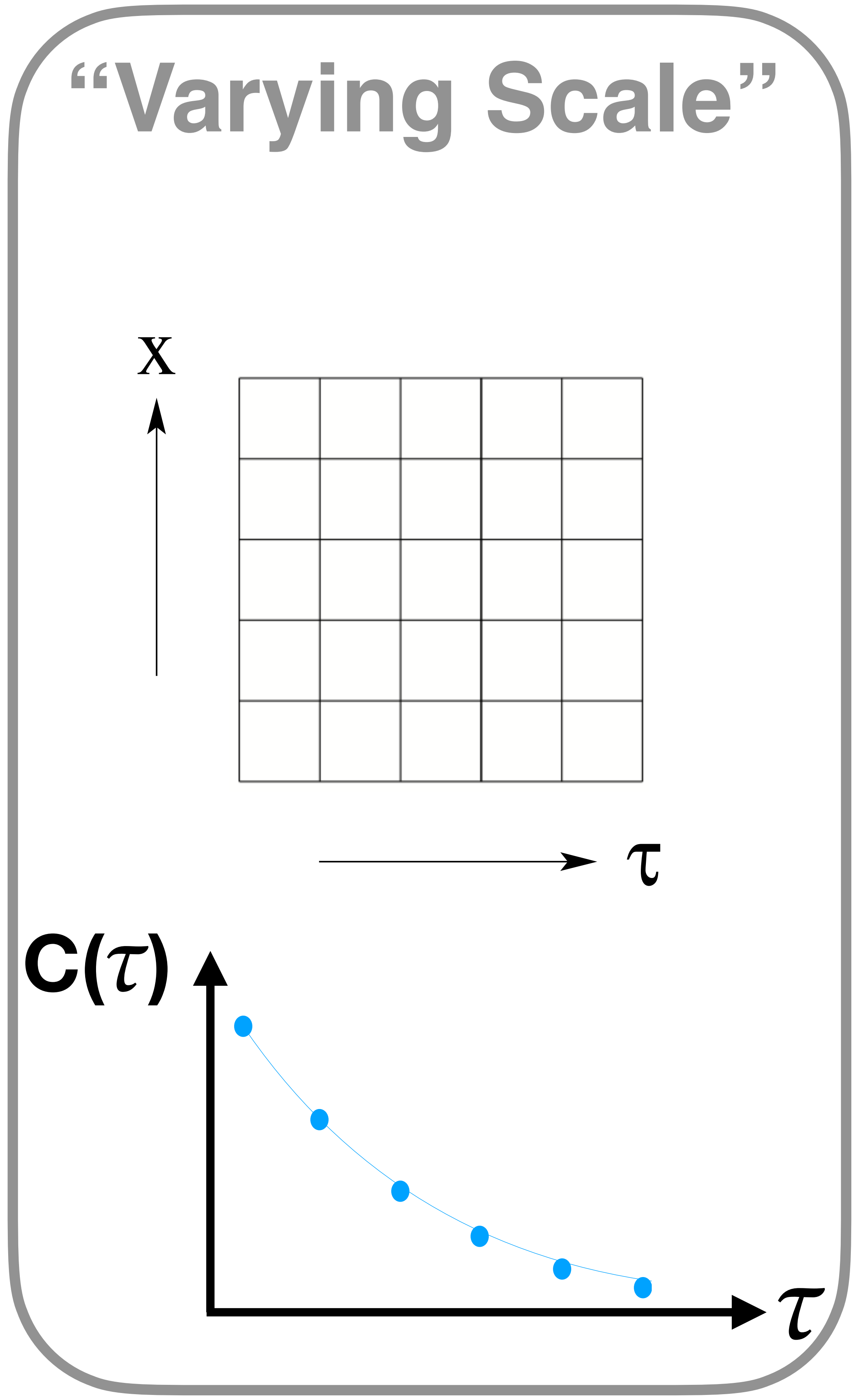
$N_\tau \rightarrow 0$



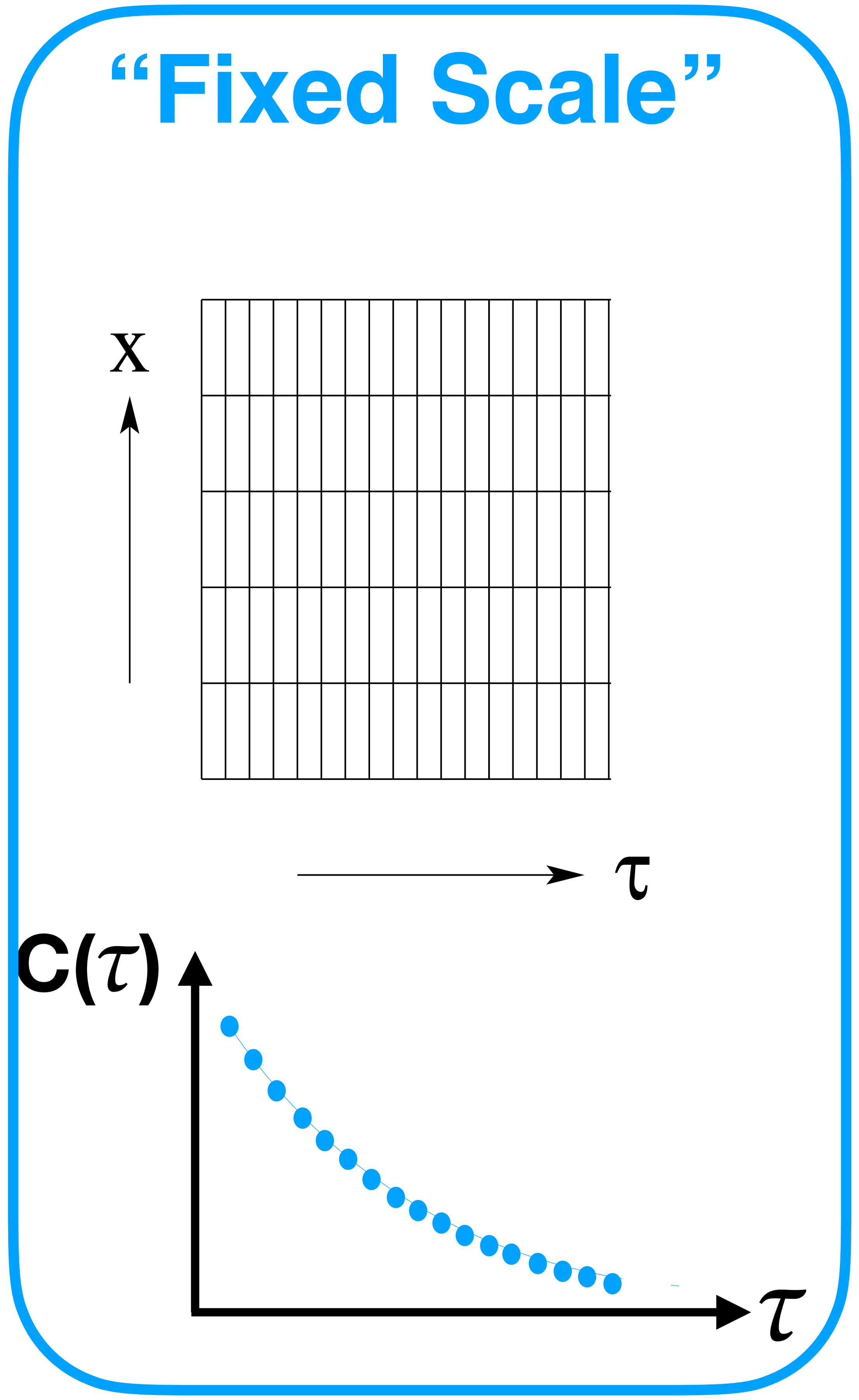
Going hotter…

$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

FASTSUM Approach:
Anisotropic Lattice



$$a_\tau \to 0$$



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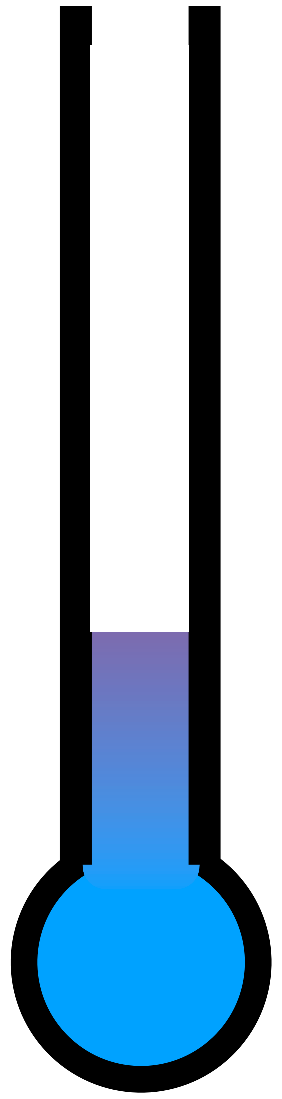

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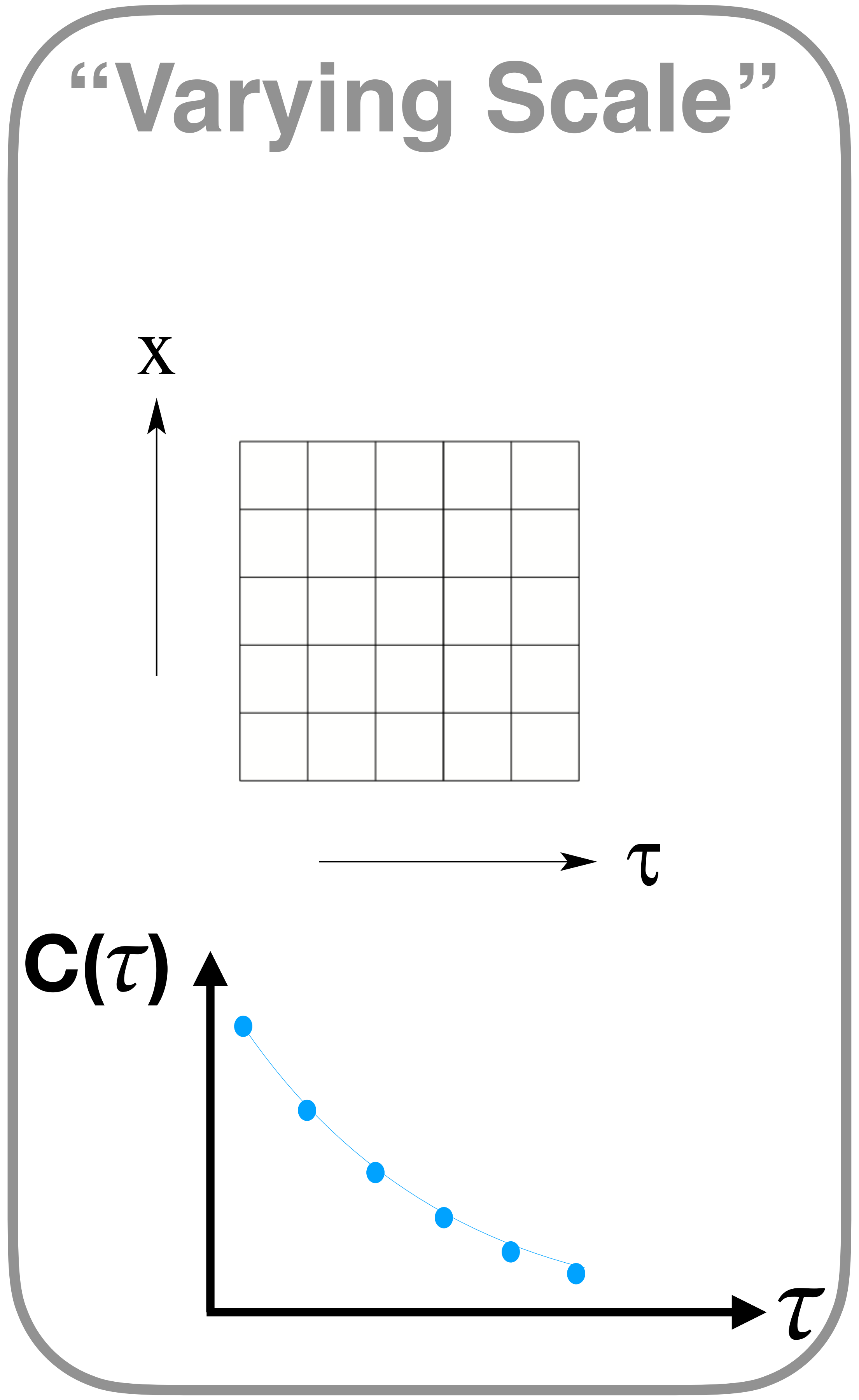

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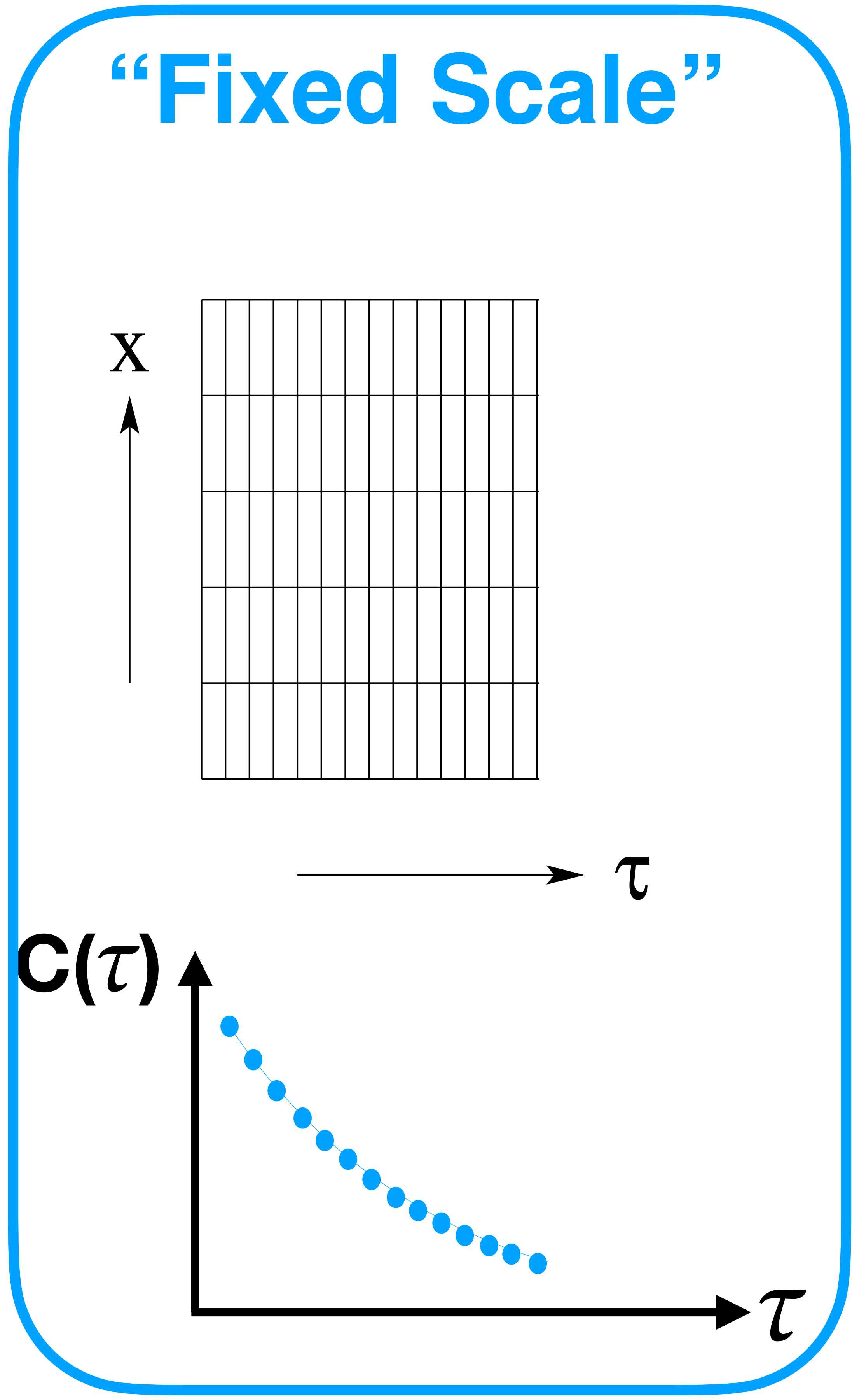

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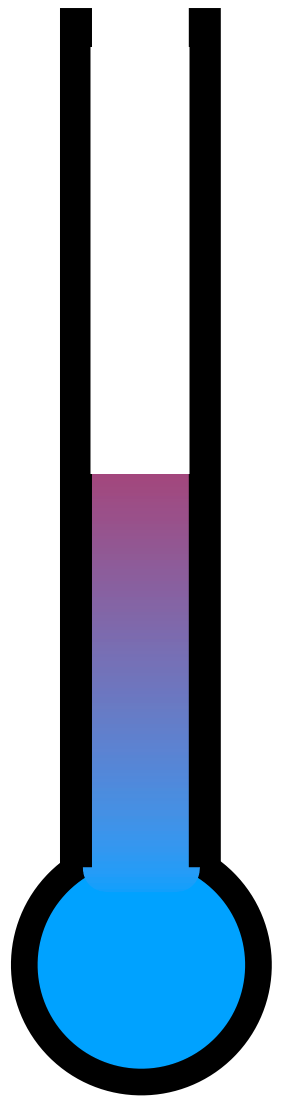

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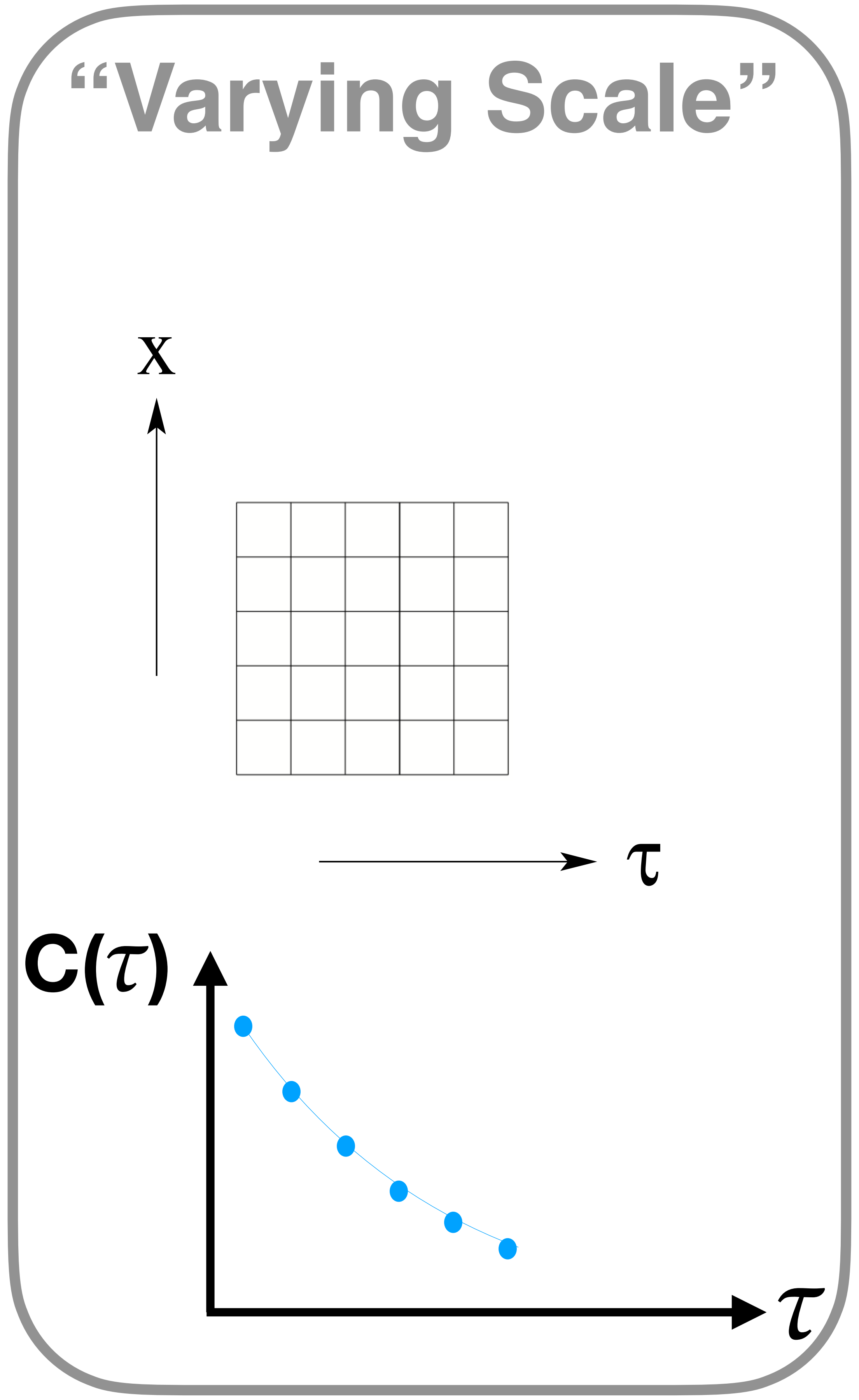

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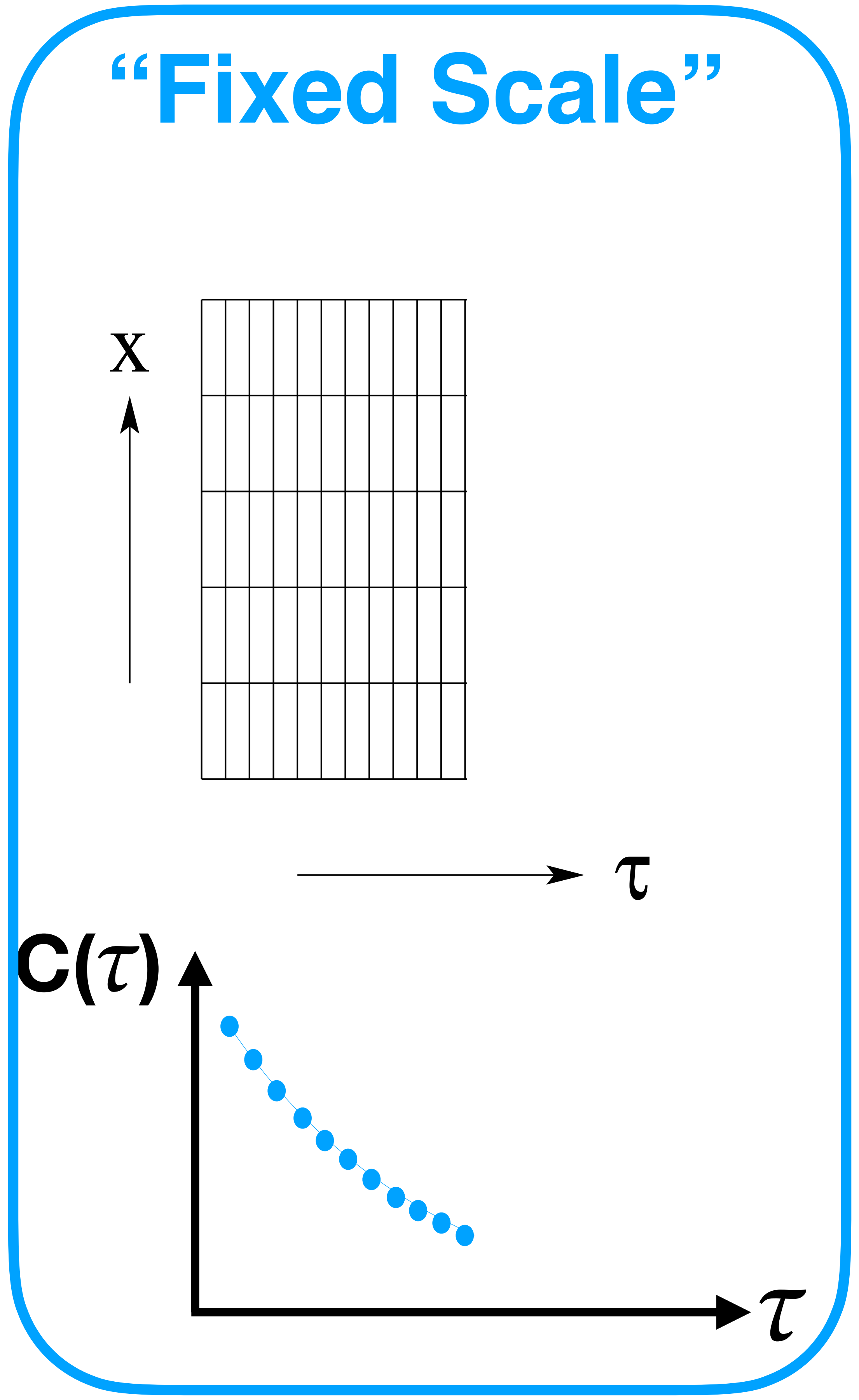

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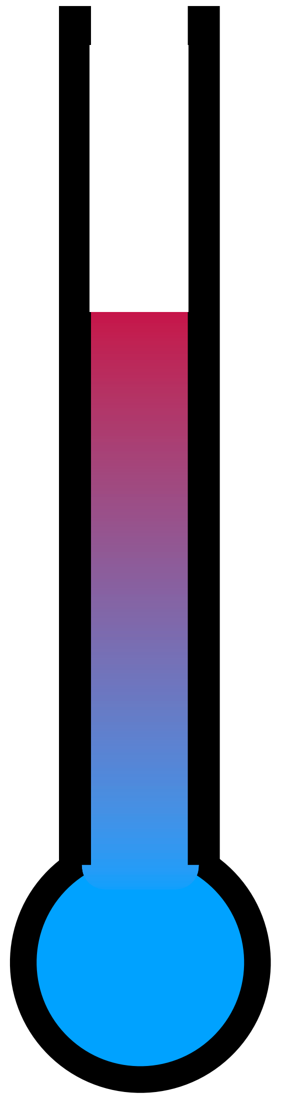

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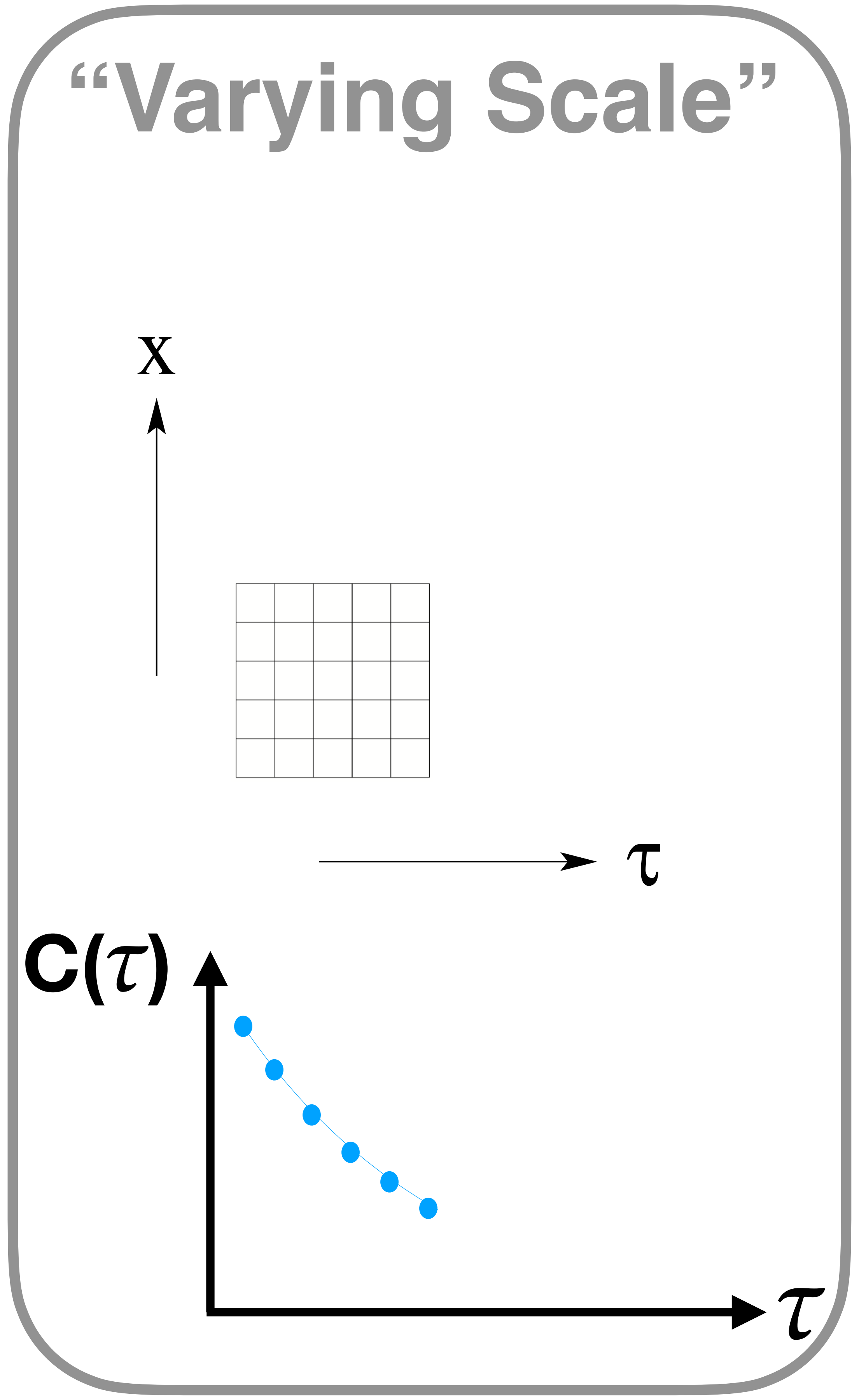

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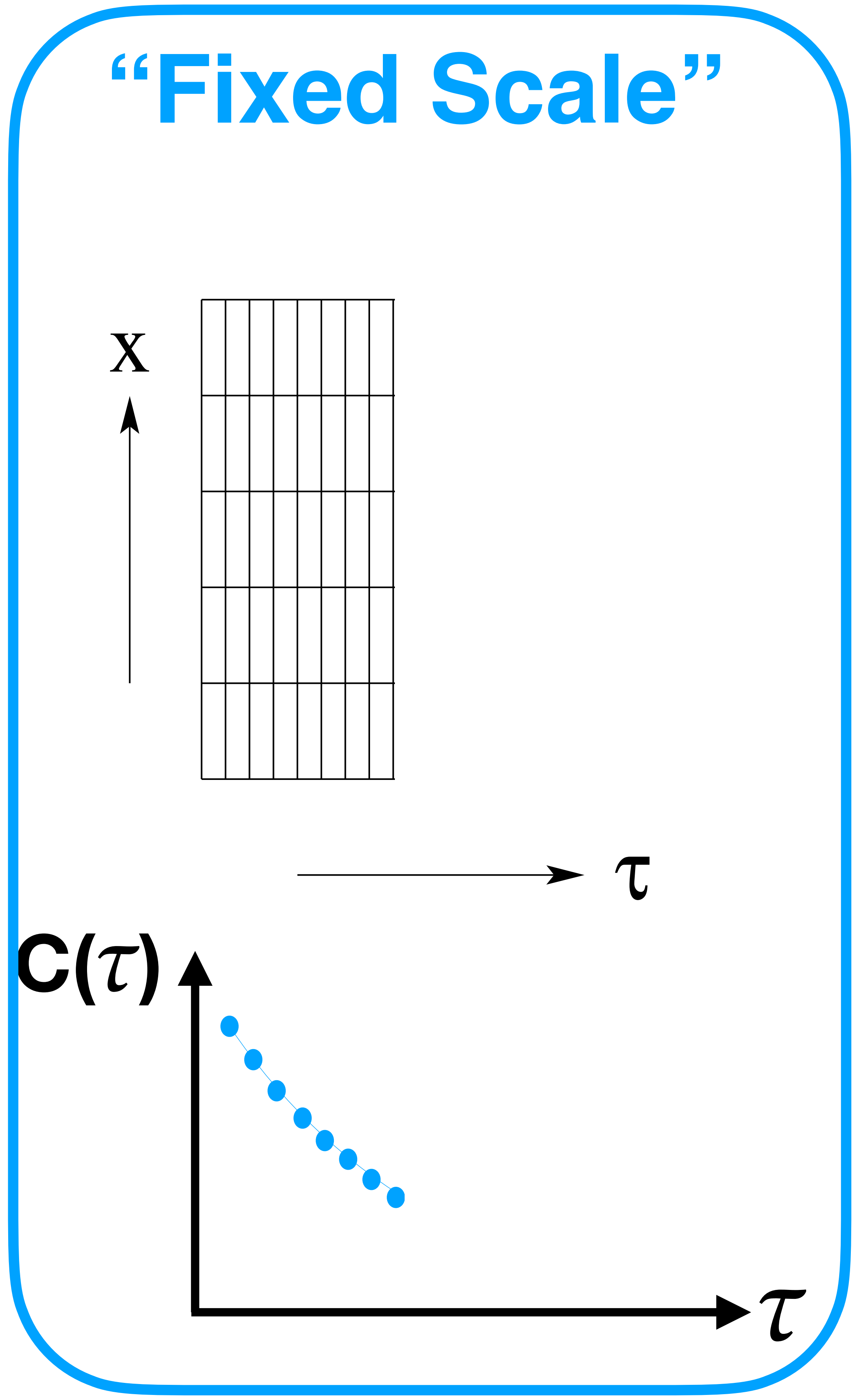

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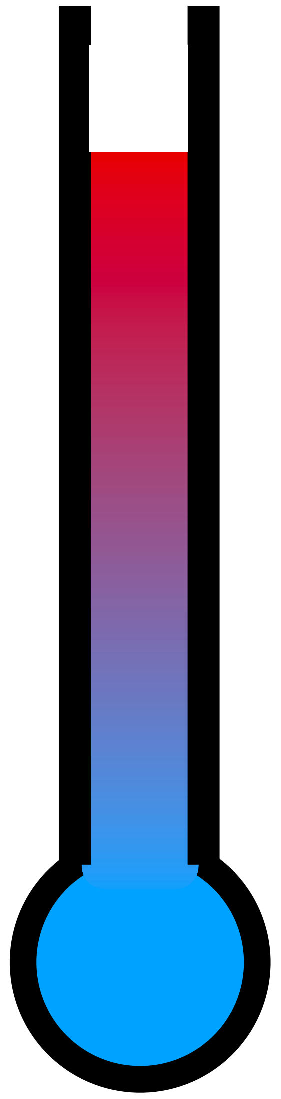

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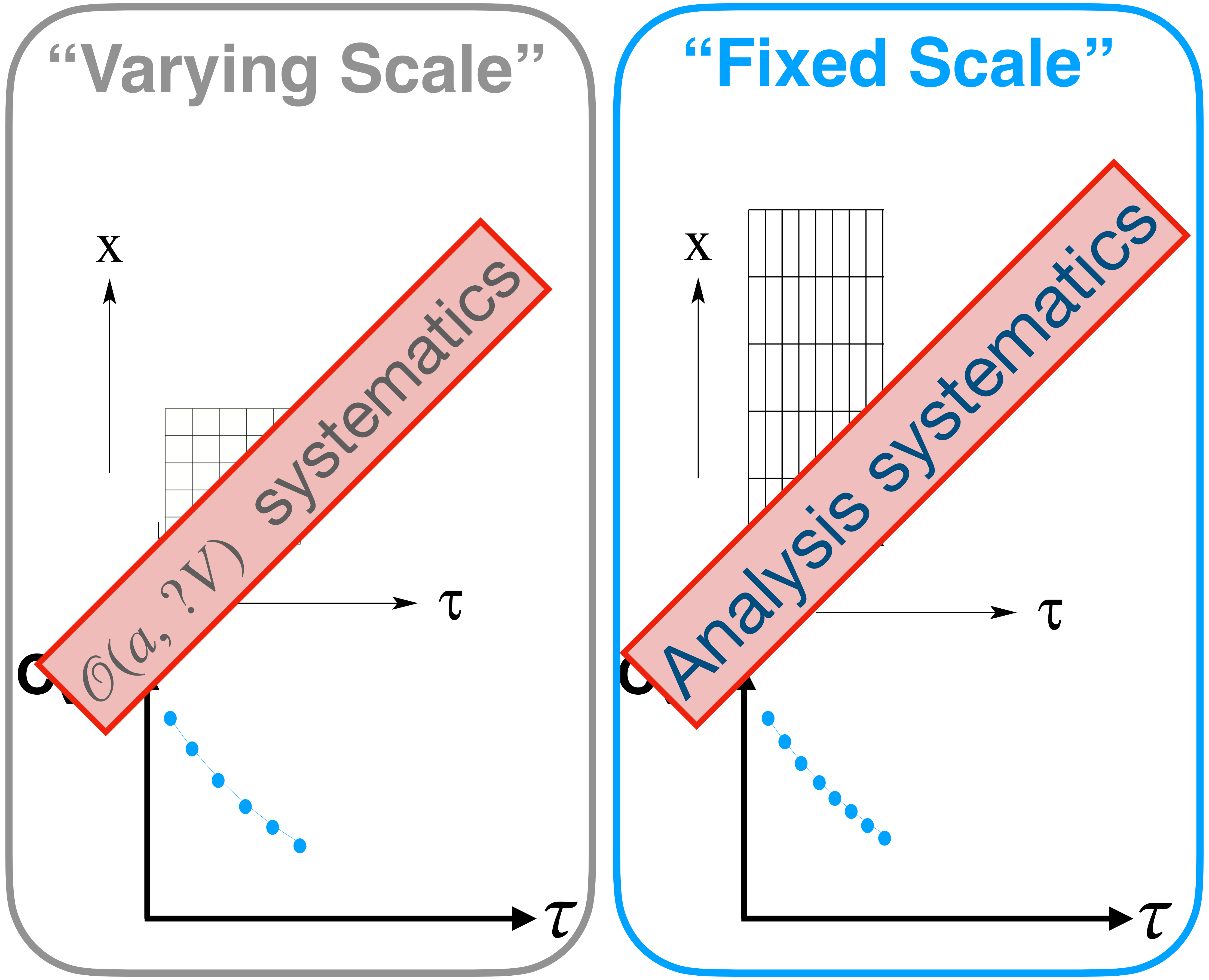

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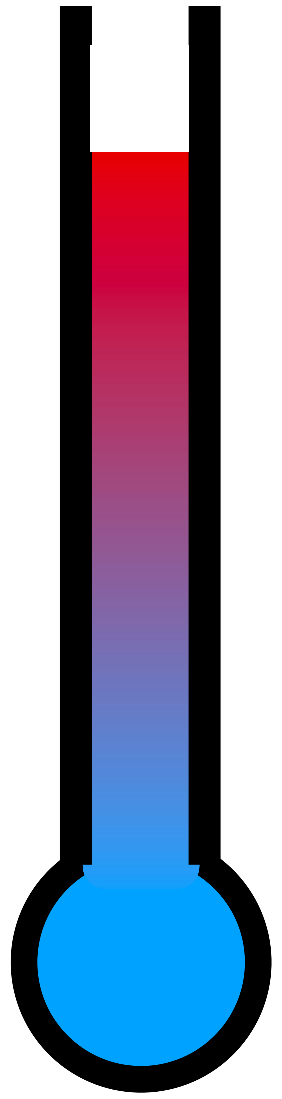

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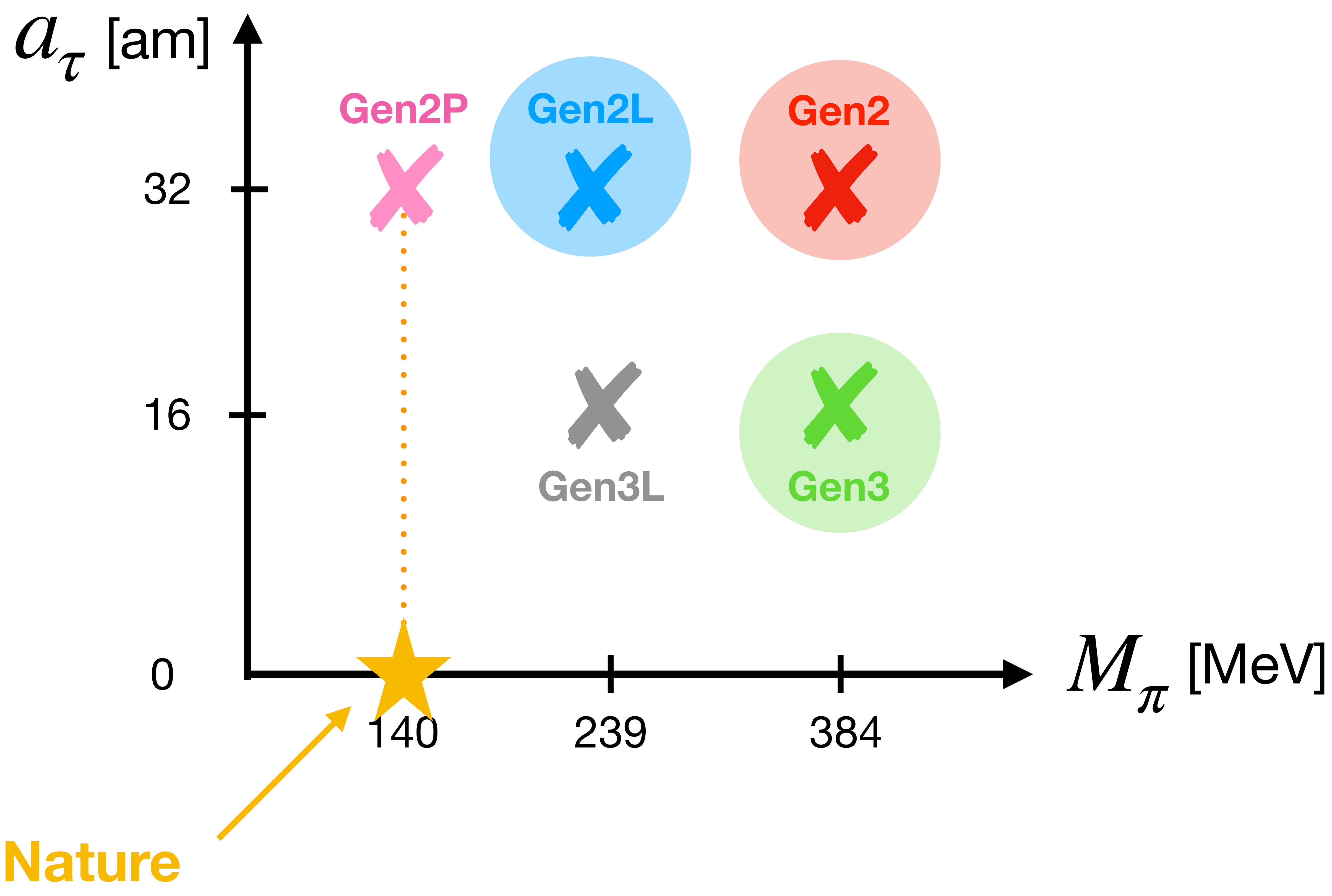
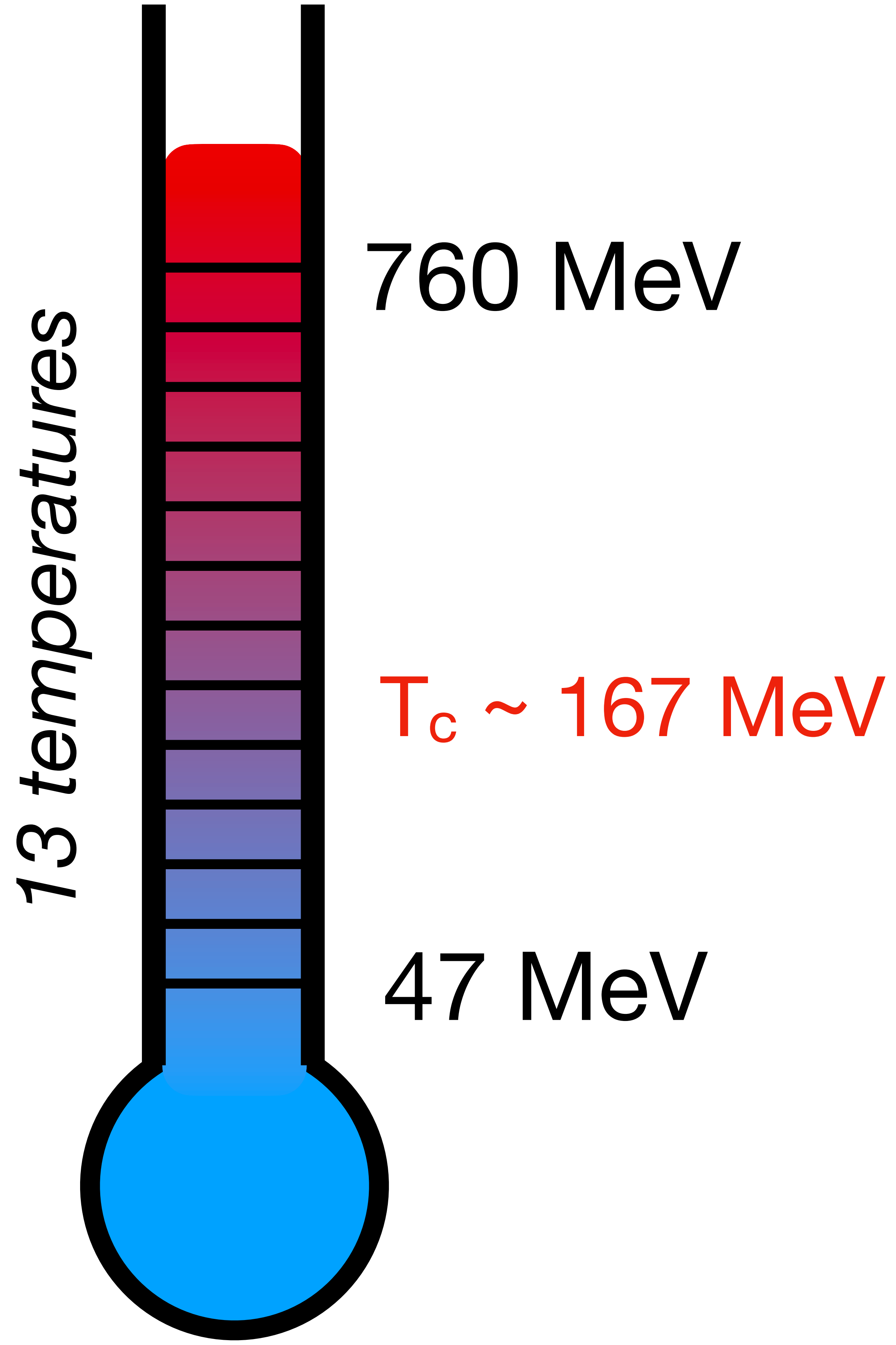
FASTSUM Approach:
Lattice Parameters





Generation 2L
(2+1) flavour
a_s ~ 0.112 fm

Gauge Action:
Anisotropic,
Symanzik-improved

Fermion Action:
Wilson-clover,
tree-level tadpole,
stout-smeared links

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Tc(μ) curve from mesonic spectrum

Antonio Smecca

FASTSUM arXiv: 2412.20922

Curvature:

$$\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B=0)}\right)^2 + \mathcal{O}(\mu_B^4)$$

Taylor expansion of mesonic correlation f'ns in μ

$$G(\tau;\mu_q) = G(\tau)\big|_{\mu_q=0} + \frac{\mu_q}{T} T \frac{\partial G(\tau)}{\partial \mu_q}\bigg|_{\mu_q=0} + \frac{1}{2}\frac{\mu_q^2}{T^2} T^2 \frac{\partial^2 G(\tau)}{\partial \mu_q^2}\bigg|_{\mu_q=0} + \mathcal{O}\left(\frac{\mu_q^3}{T^3}\right),$$

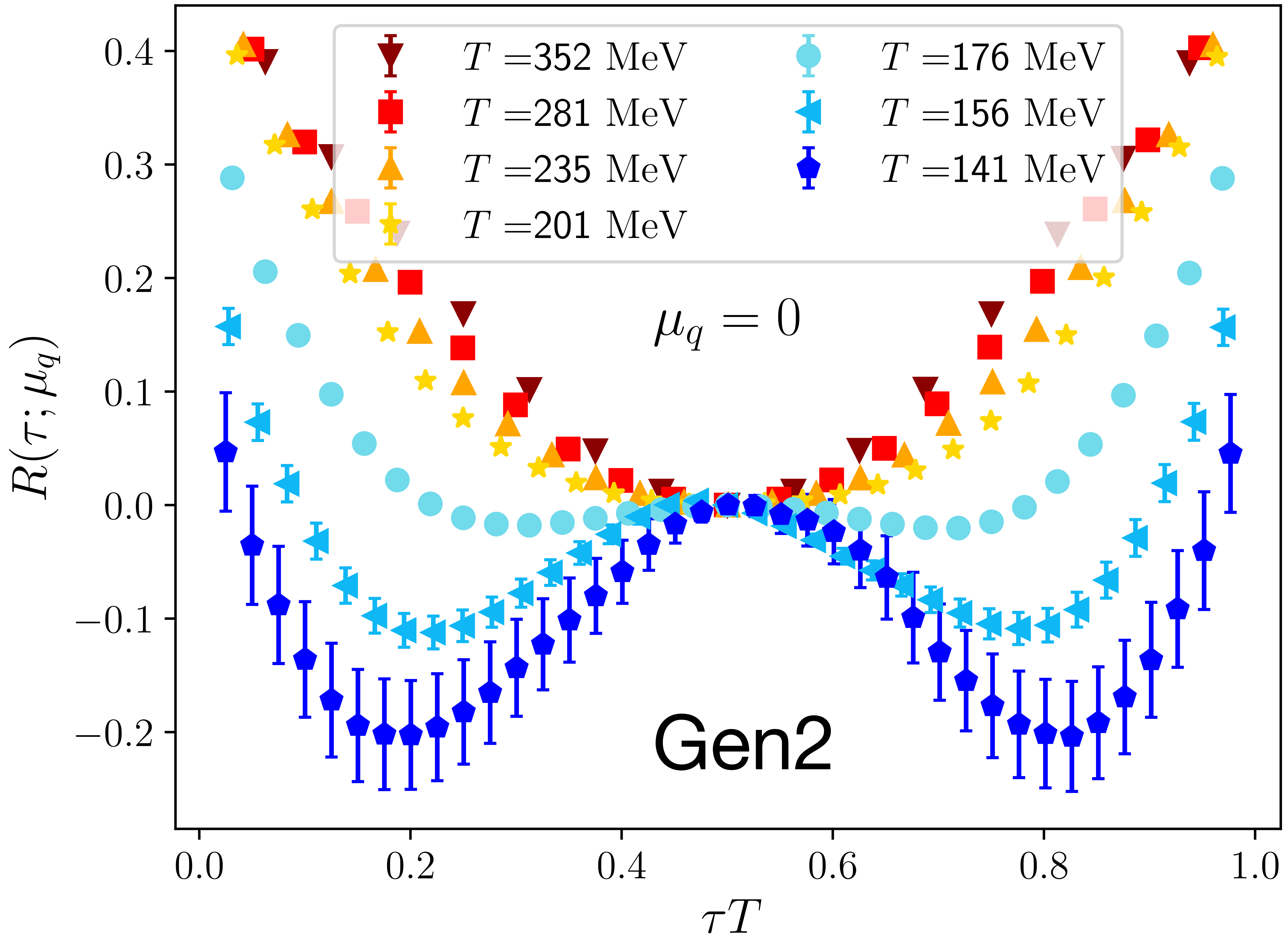
Vector & Axial Vector: $$G_{V/A}(\tau;\mu_q) = G_{V/A}(\tau) + \frac{1}{2}\mu_q^2 G''_{V/A}(\tau)$$ (No odd terms)

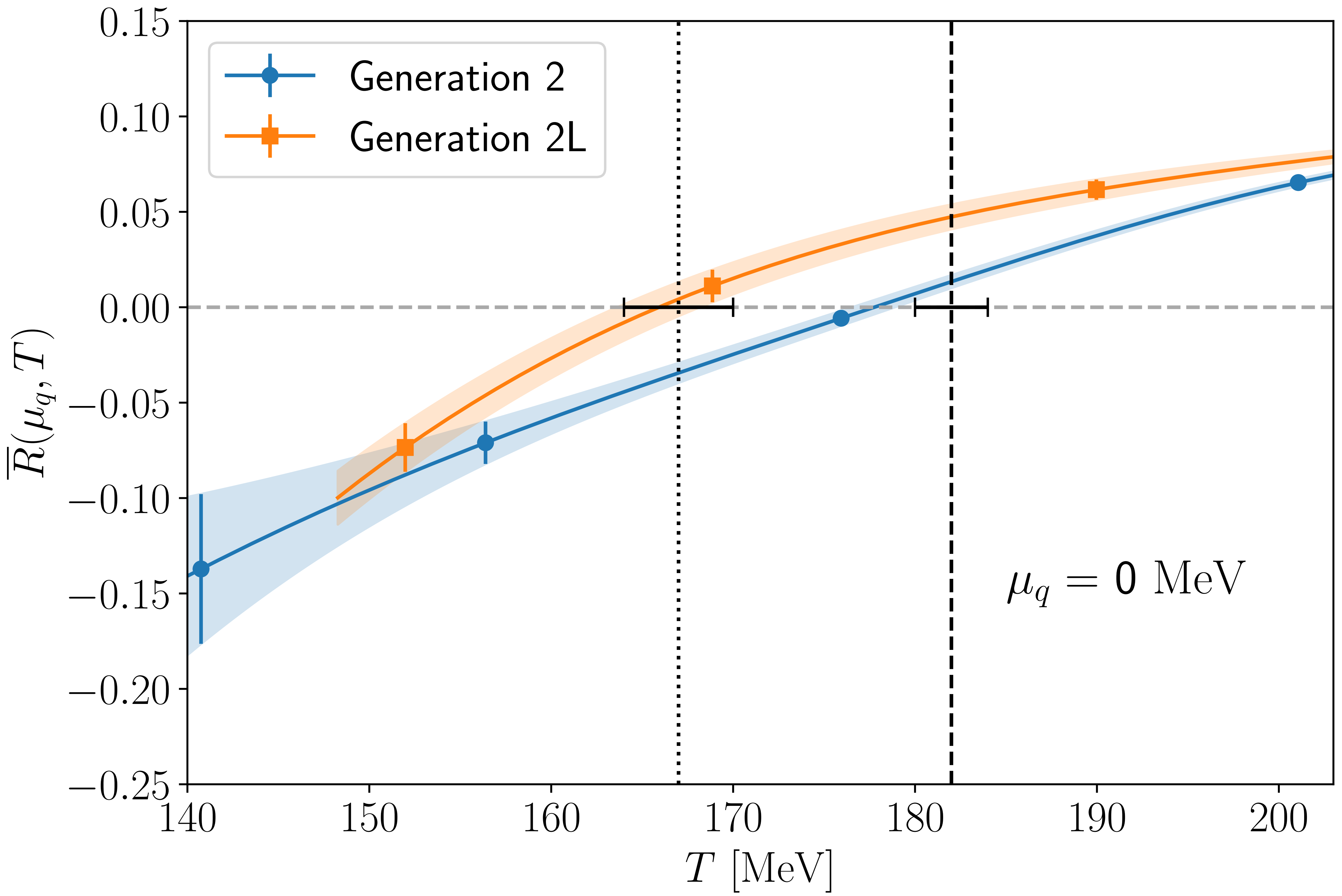
Tc(μ) curve from mesonic spectrum

R-ratio: $$R(\tau;\mu_q) = \frac{\tilde{G}_V(\tau;\mu_q) - \tilde{G}_A(\tau;\mu_q)}{\tilde{G}_V(\tau;\mu_q) + \tilde{G}_A(\tau;\mu_q)}$$

$\overline{R}(\mu_q, T)$ = time average of $R(\mu_q, T)$

$\tilde{G}(\tau) = \frac{G(\tau)}{G(N_\tau/2)}$ (Normalised at midpoint) $\longrightarrow$ $R(N_\tau/2) \equiv 0$





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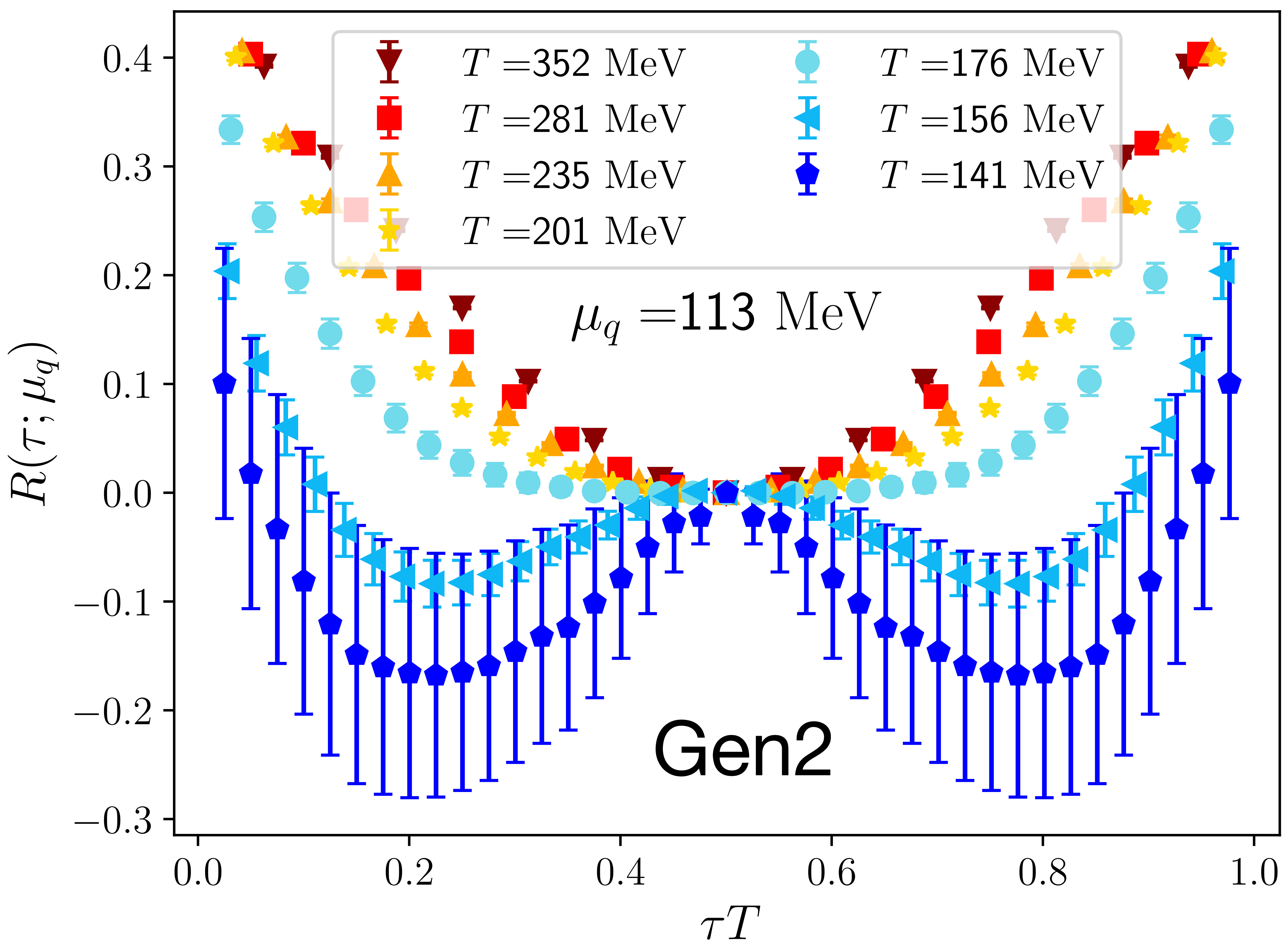

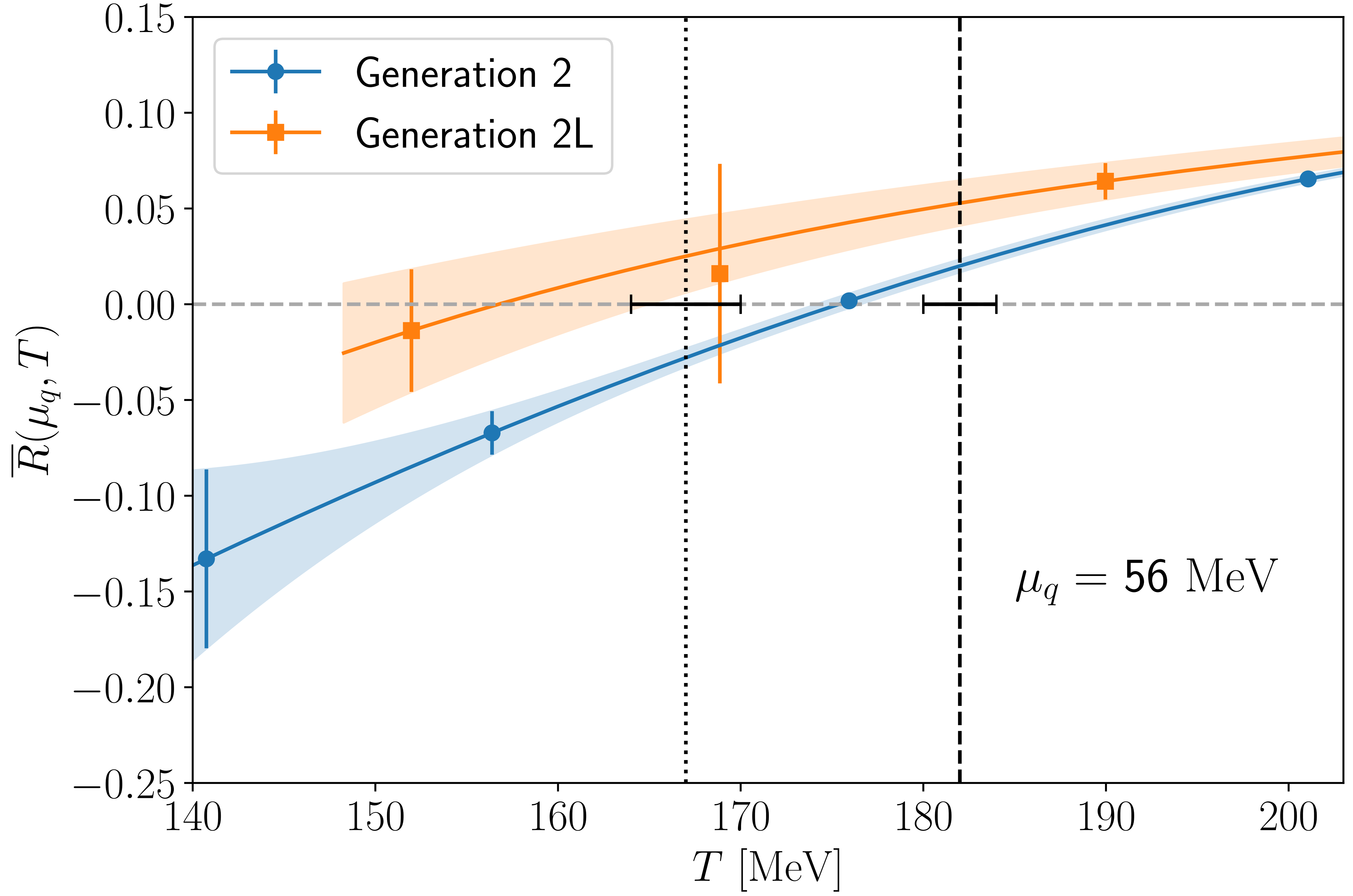

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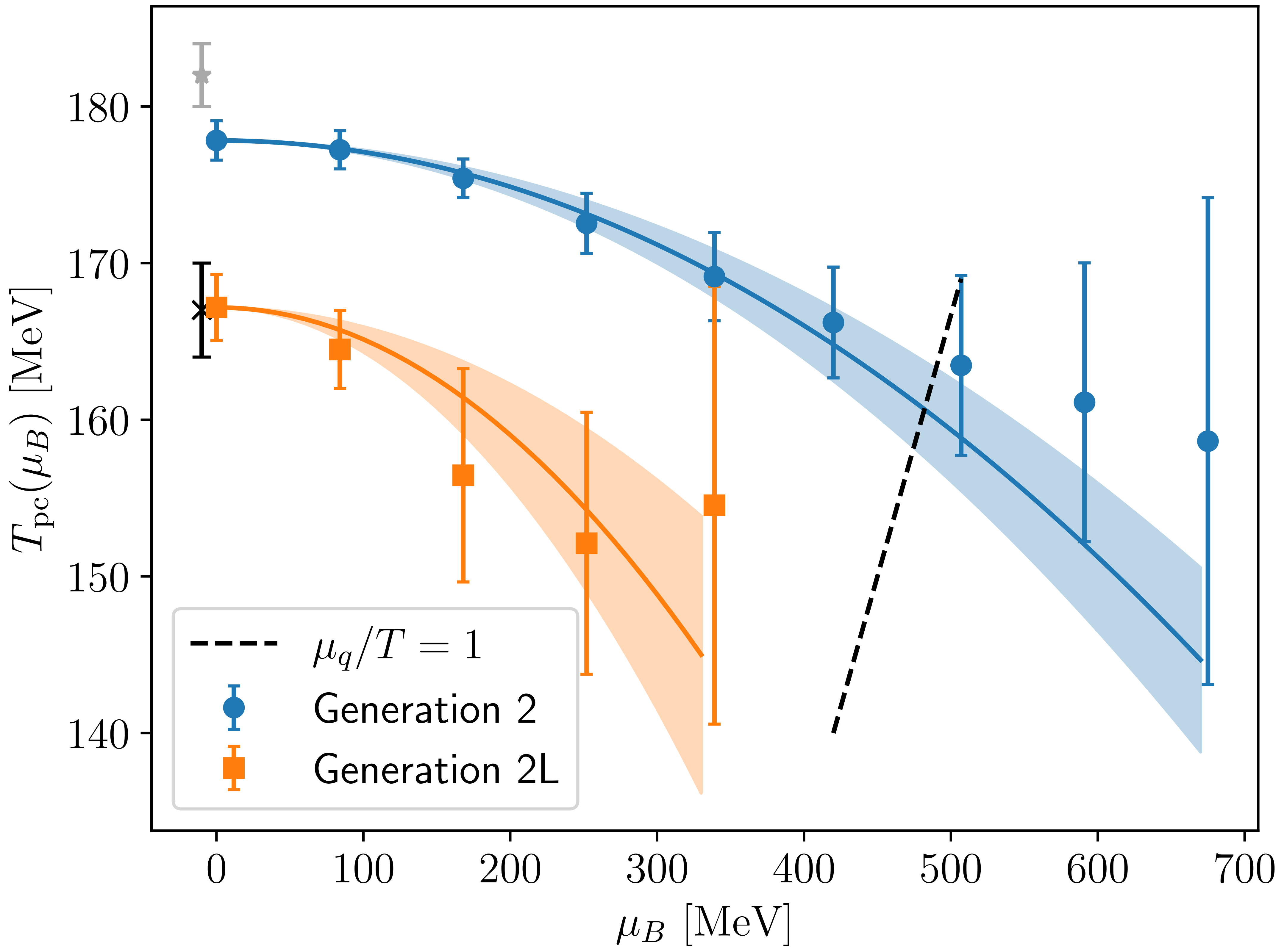

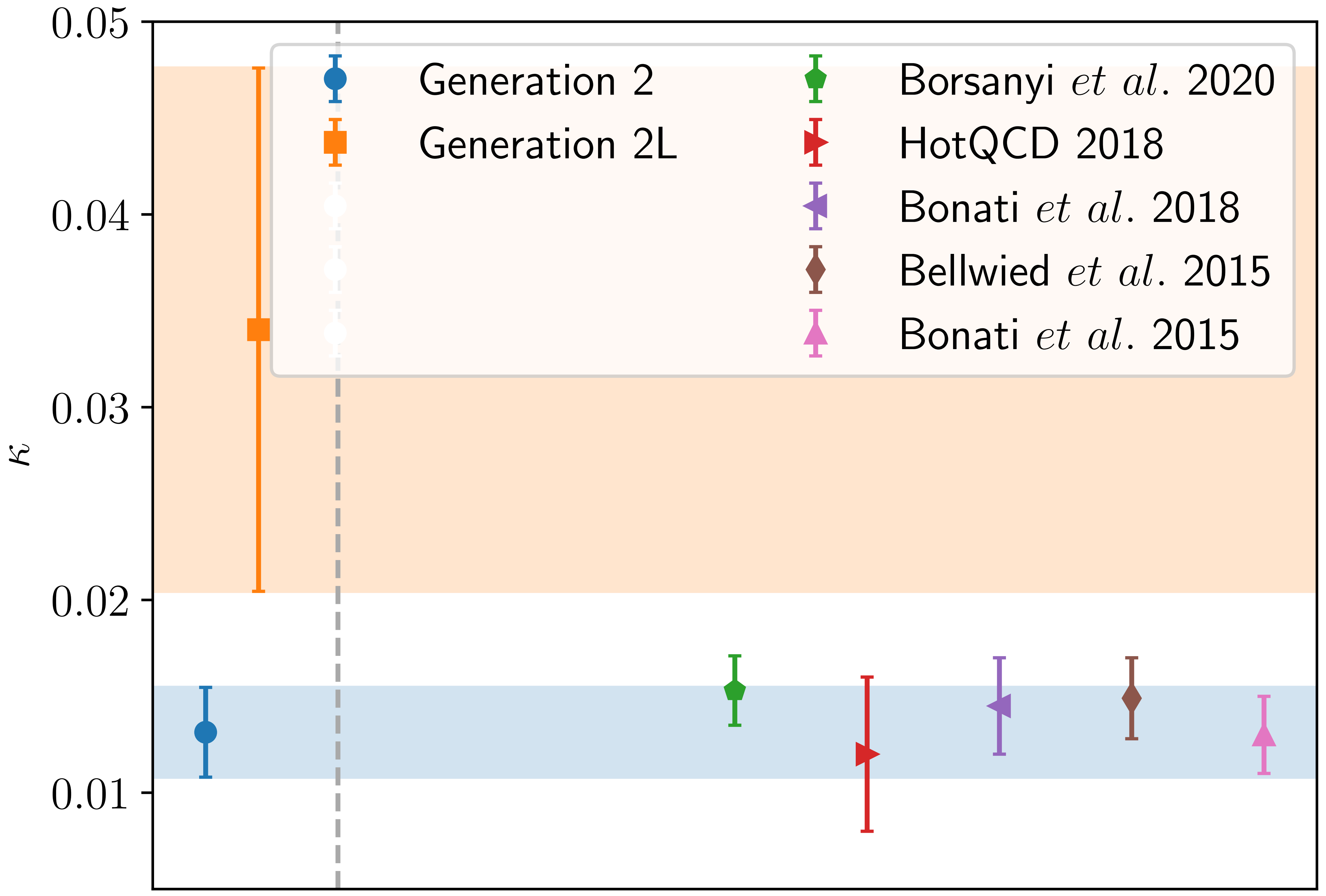

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Parity in the Baryonic Spectrum

Ryan Bignell

PRD 92 (2015) 014503 [arXiv:1502.03603]
JHEP 06 (2017) 034 [arXiv:1703.09246]
Phys.Rev. D99 (2019) no.7, 074503 [arXiv:1812.07393]
Eur.Phys.J.A 60 (2024) 3, 59 [arXiv: 2308.12207]

No parity doubling in (T=0) Nature:

+ve parity: $m_+ = m_N = 0.939$ GeV
-ve parity: $m_- = m_{N*} = 1.535$ GeV

Question: What happens as T increases?

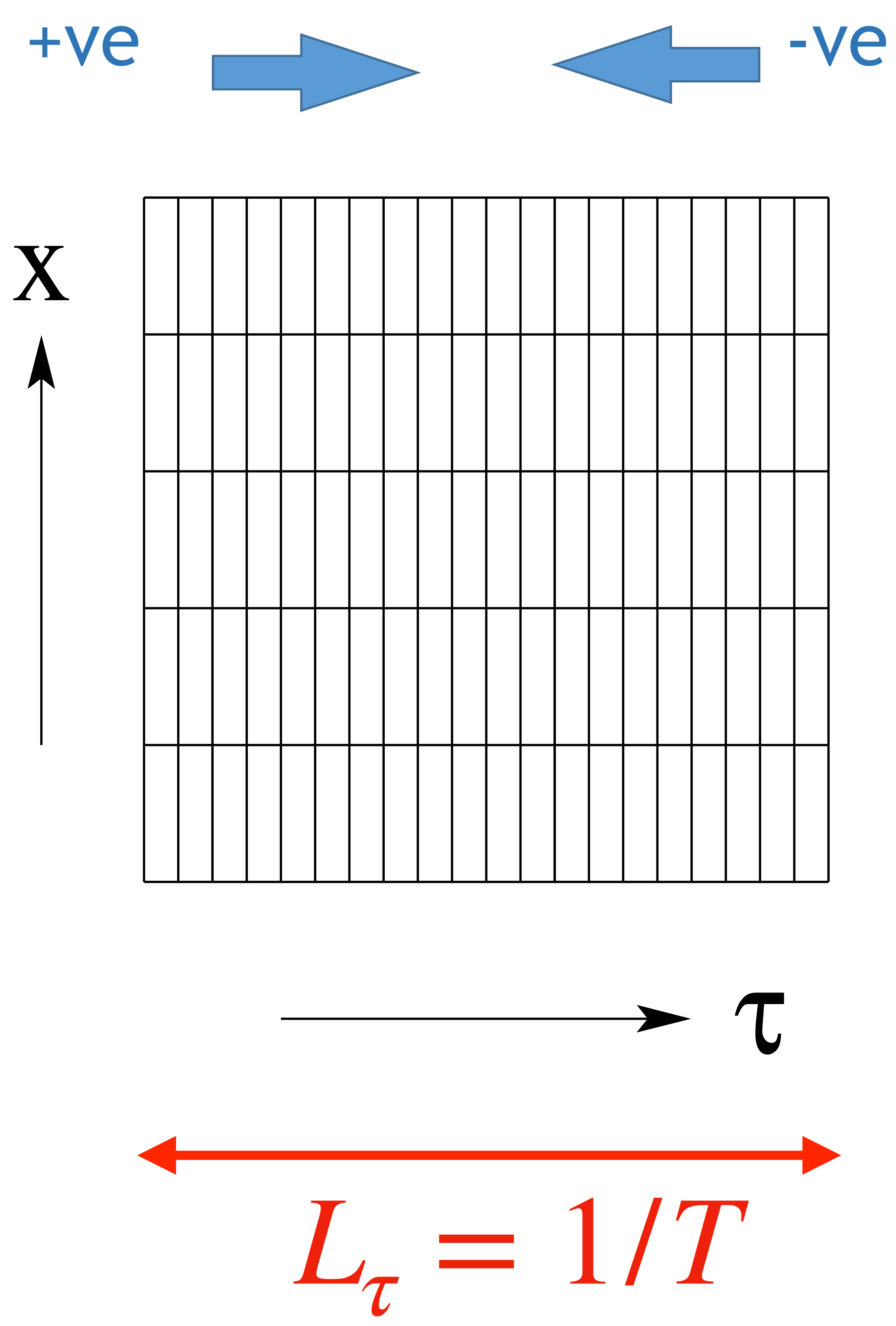
Lattice: Parity operation: $P\mathcal{O}(\tau,\vec{x})P^{-1} = \gamma_4\mathcal{O}(\tau,-\vec{x})$

- Use this to construct correlation f'ns

Charge conjugation (zero density): $G_\pm(\tau) = -G_\mp(1/T-\tau)$

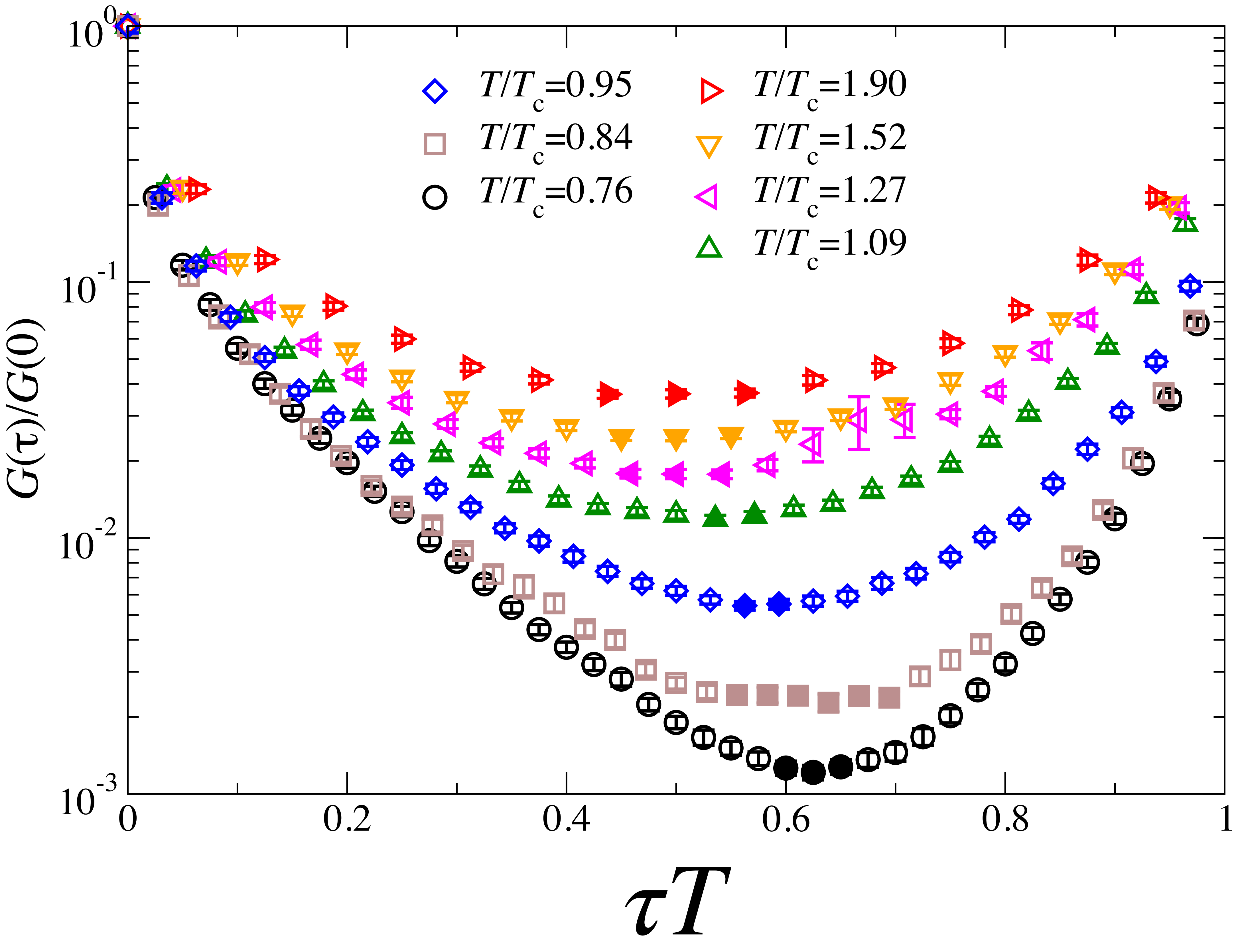
Chiral symmetry: $G_+(\tau) = -G_-(\tau)$

$$\Big\} \; G_+(\tau) = G_+(1/T-\tau)$$

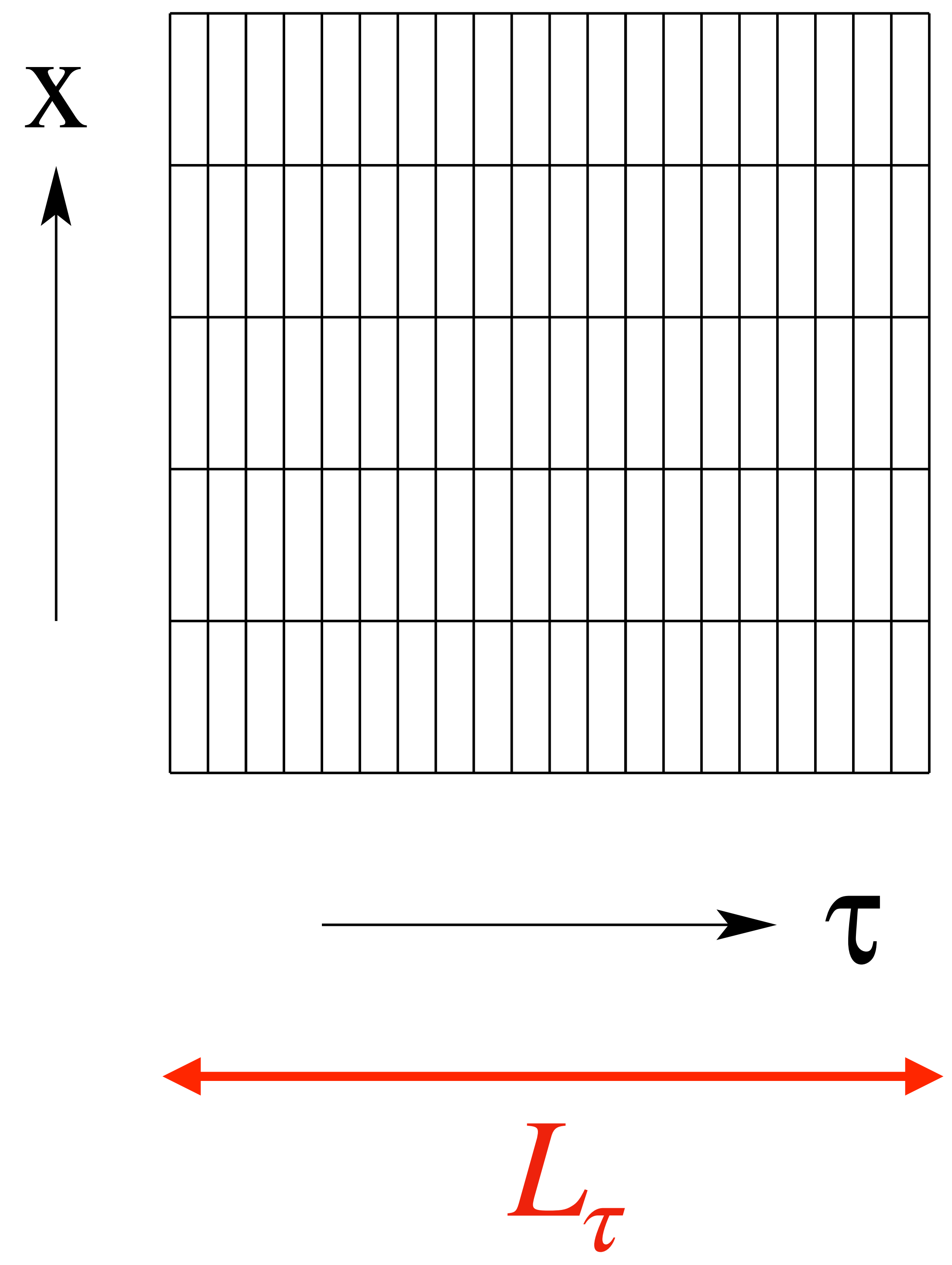


Lattice Nucleon Correlator: G_+

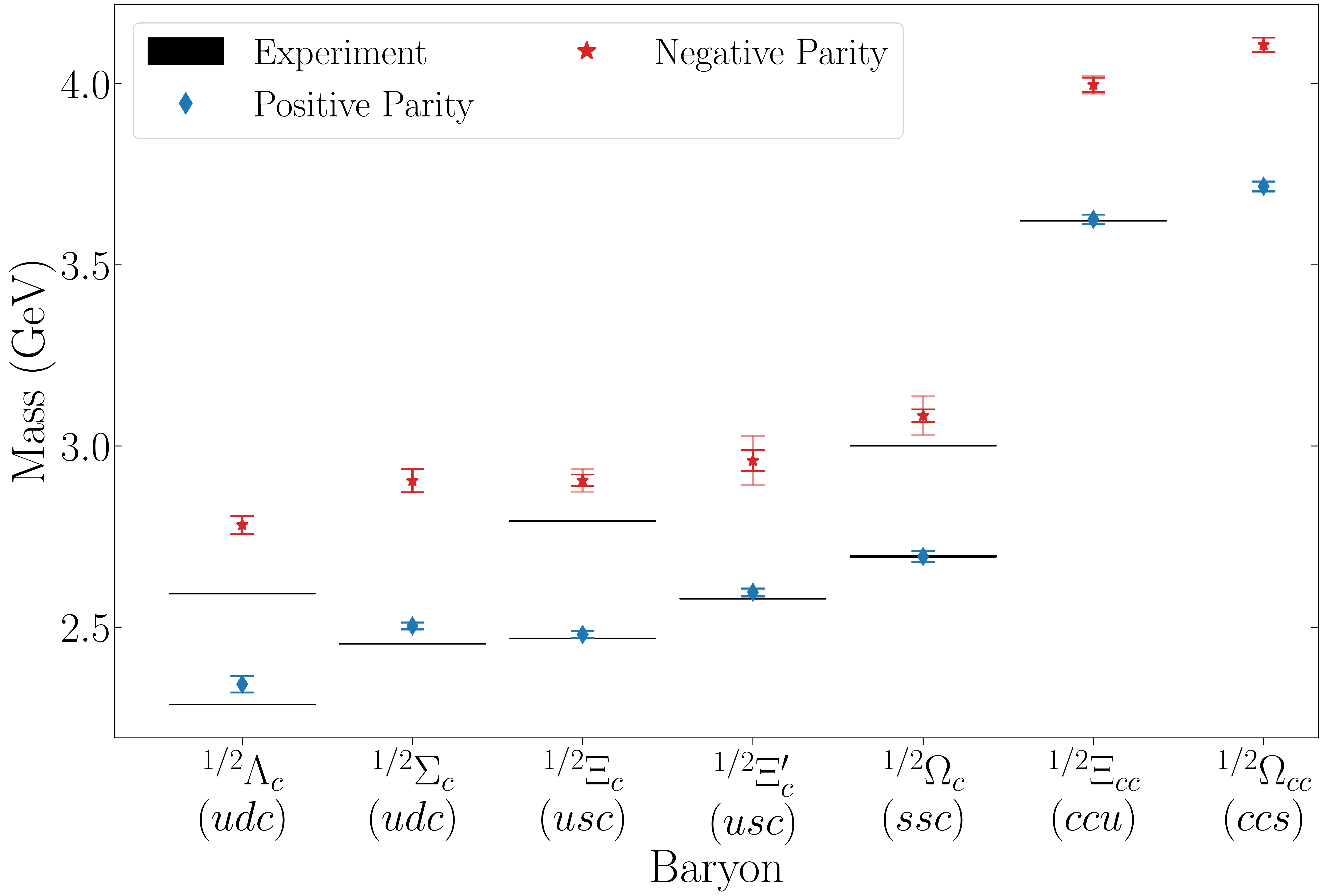
+ve parity → ← -ve parity



$$= \frac{\tau}{a_\tau N_\tau} \equiv \frac{\tau}{L_\tau}$$



T=0 Spectrum Results



Results — "Reconstructed" Correlators

$$G(\tau; T) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K_F(\tau, \omega; T) \rho(\omega, T)$$ where the *fermionic* kernel is: $$K_F(\tau, \omega; T) = \frac{e^{-\omega/T}}{1 + e^{-\omega/T}}$$

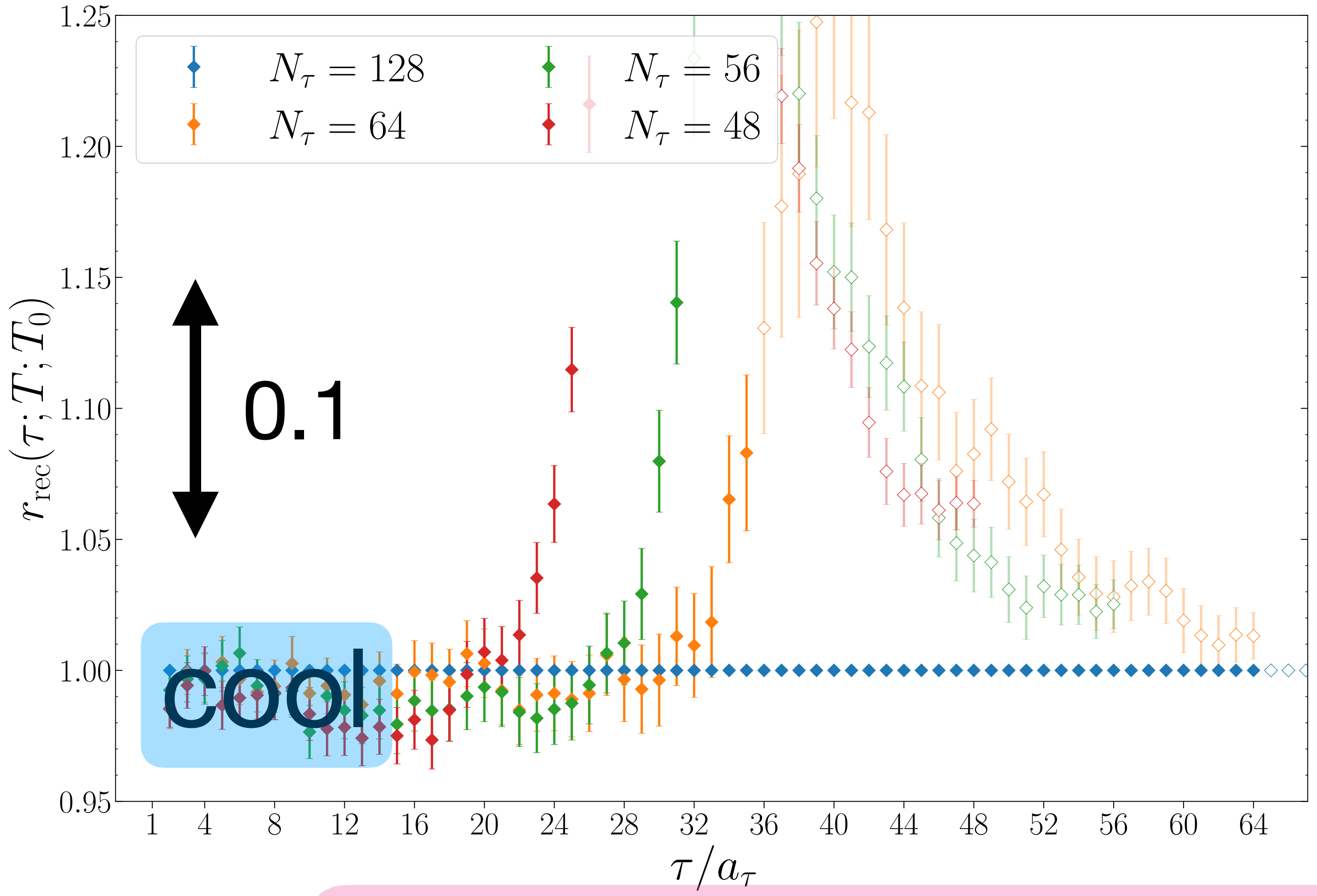
Following: H. T. Ding et al, Phys. Rev. D 86 (2012) 014509, [arXiv:1204.4945]

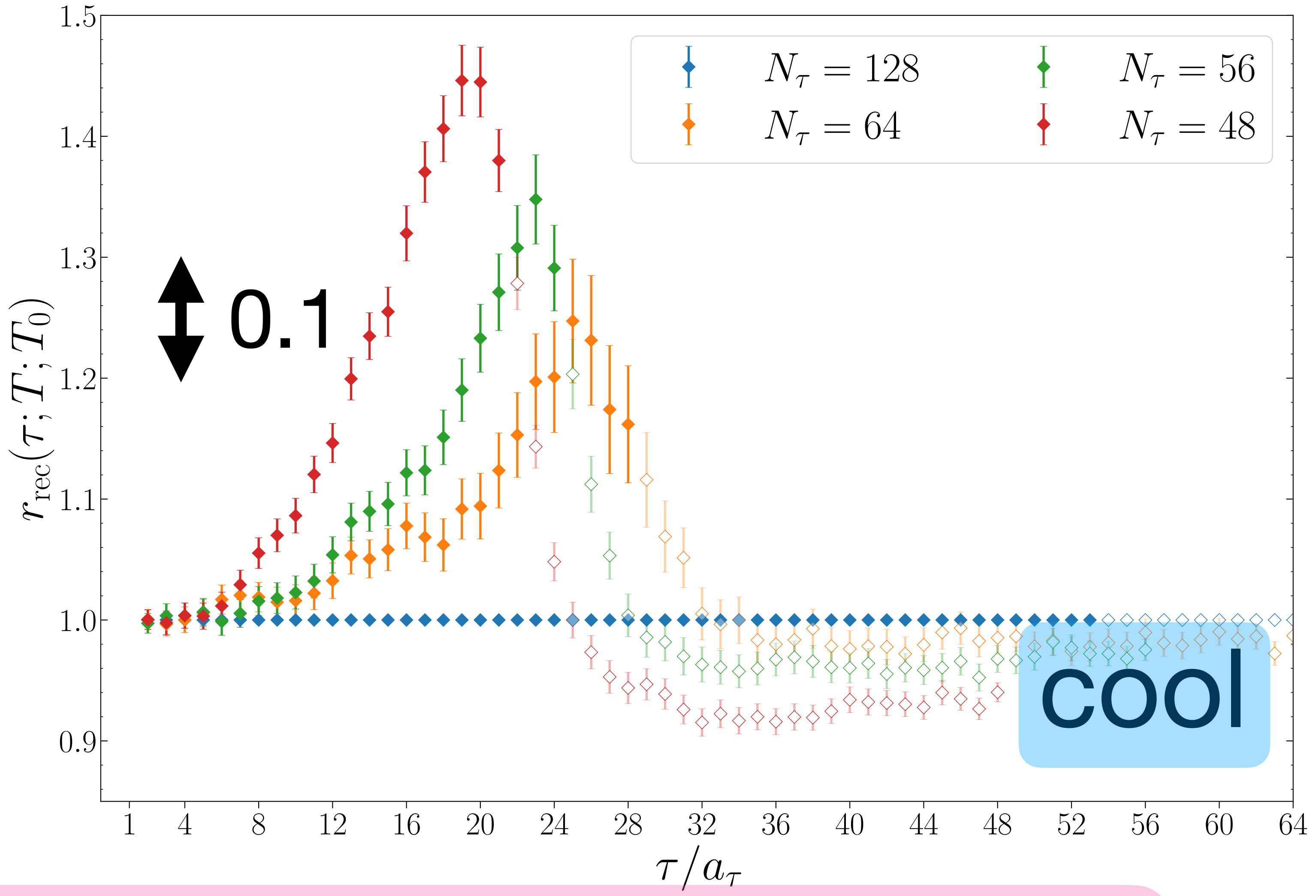
we write $$1 + e^{-\omega m N_\tau} = (1 + e^{-\omega N_\tau}) \sum_{n=0}^{m-1} (-1)^n e^{-n\omega N_\tau}$$ where $N_0 = m\, N_\tau$ and m is odd

$$K_F(\tau, \omega; 1/N_\tau) = \frac{e^{-\omega\tau}}{1 + e^{-\omega N_\tau}} = \sum_{n=0}^{m-1} (-1)^n \frac{e^{-\omega(\tau + nN_\tau)}}{1 + e^{-\omega m N_\tau}} = \sum_{n=0}^{m-1} (-1)^n K_F(\tau + nN_\tau, \omega; 1/(mN_\tau))$$

Suppose $\rho(\omega)$ was indept of T : $$G_{\text{rec}}(\tau;\ 1/N_\tau;\ 1/N_0) = \sum_{n=0}^{m-1} (-1)^n G(\tau + nN_\tau;\ 1/N_0)$$

Results - "Reconstructed" ratio: G_{rec}/G $\Sigma_c\,(udc)$

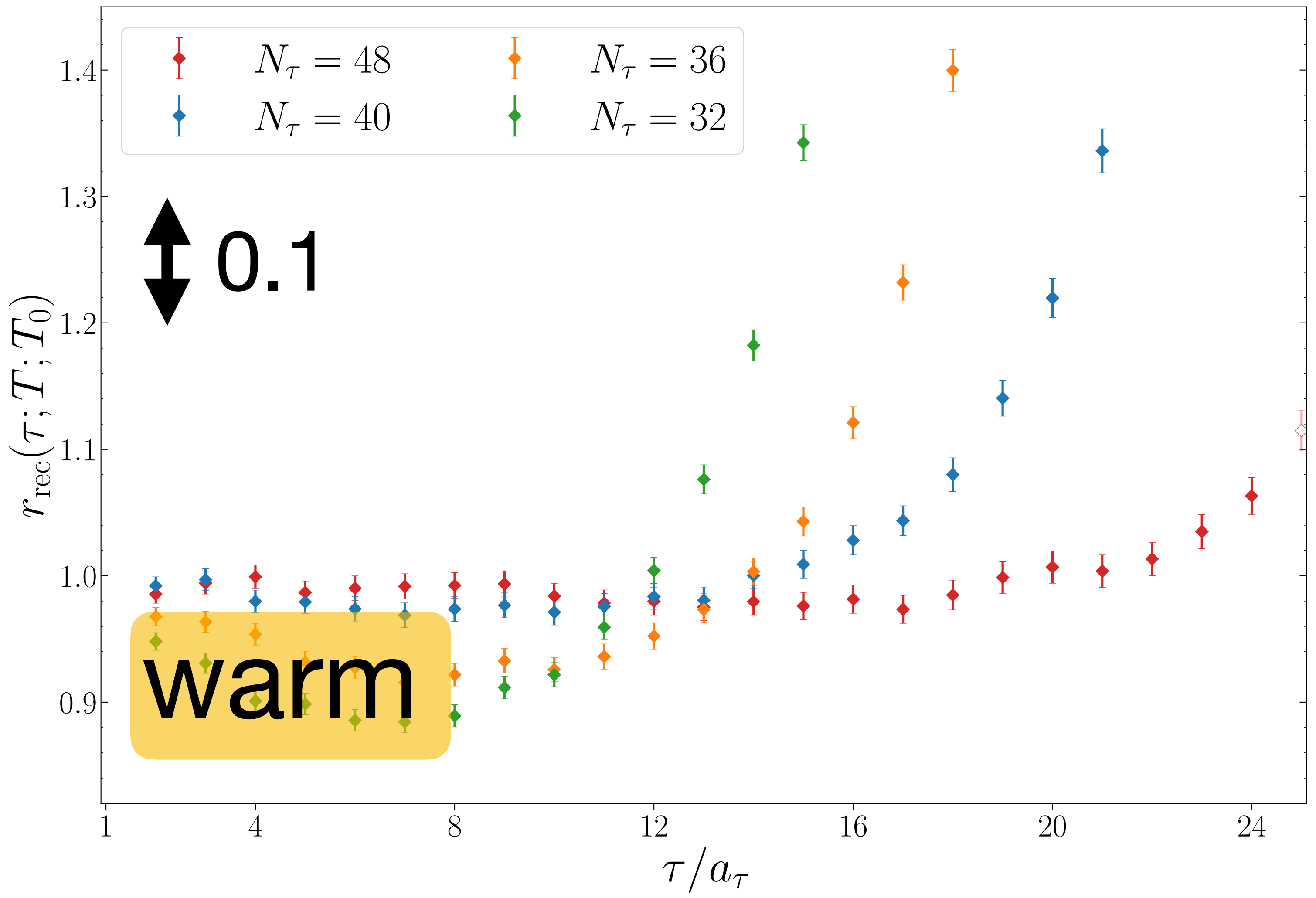


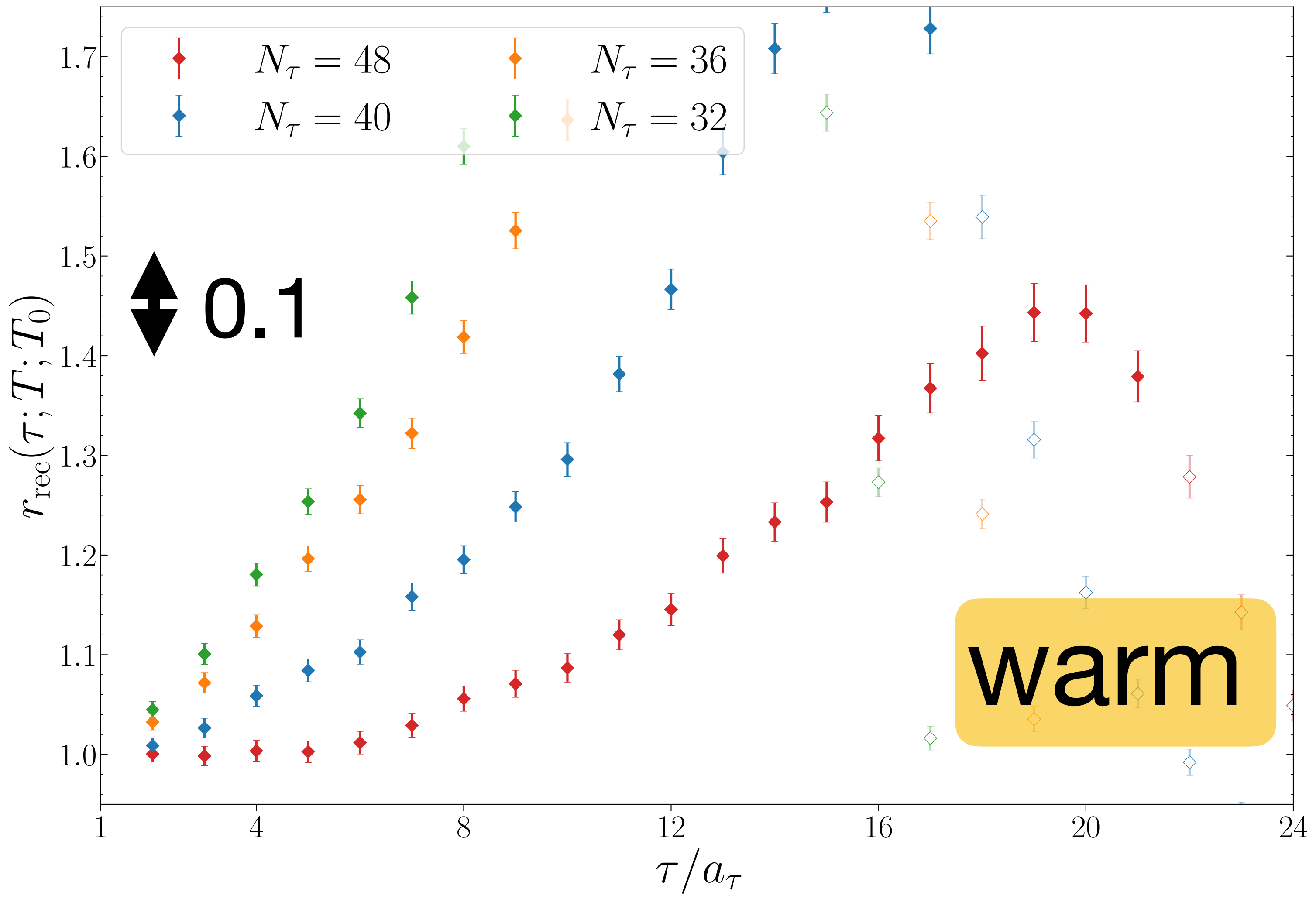


+ve parity

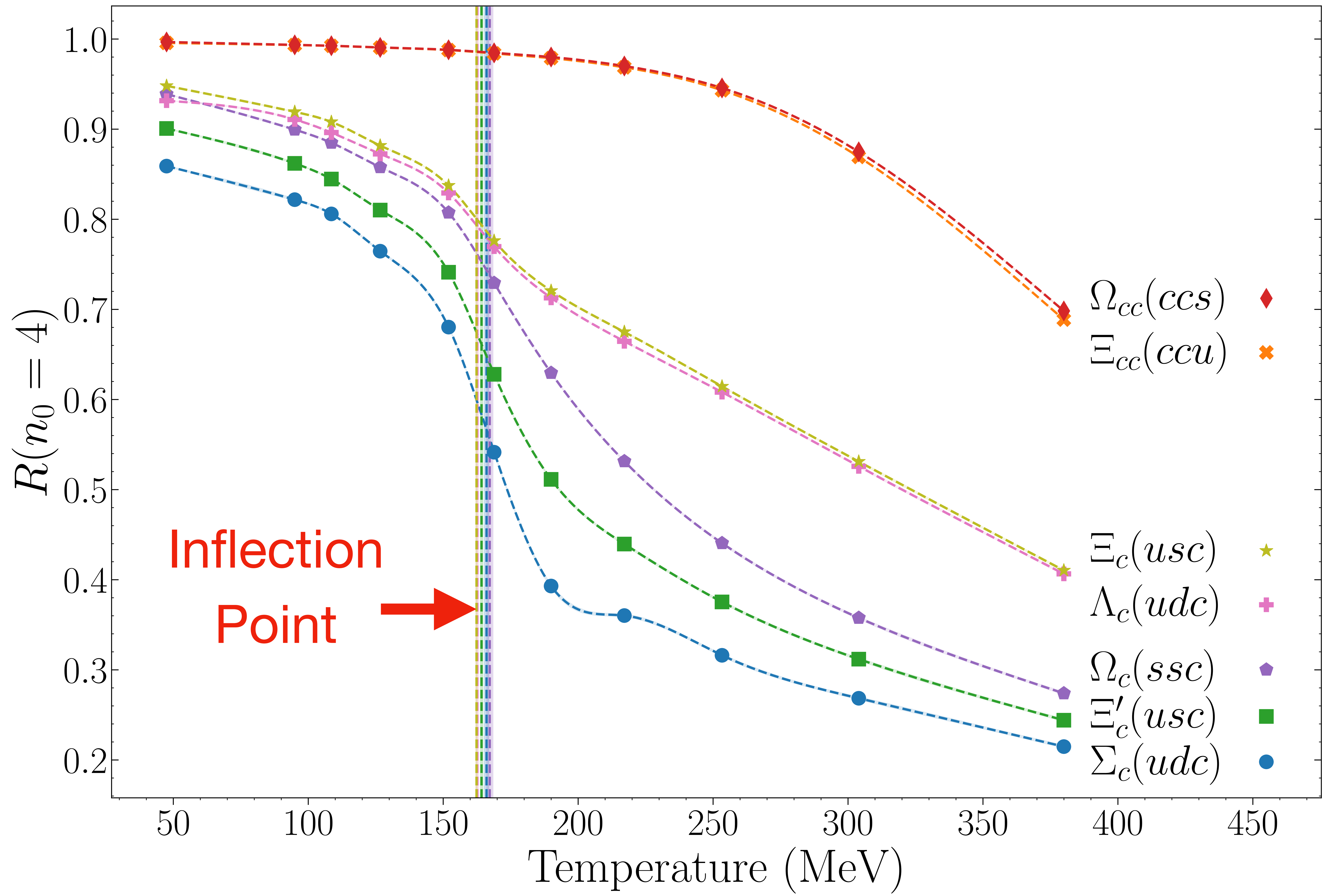
+ve parity sector less thermally sensitive than -ve parity

-ve parity





Parity doubling in the correlators



$$R(\tau) = \frac{G_+(\tau) - G_+(1/T - \tau)}{G_+(\tau) + G_+(1/T - \tau)}$$

Parity doubling:

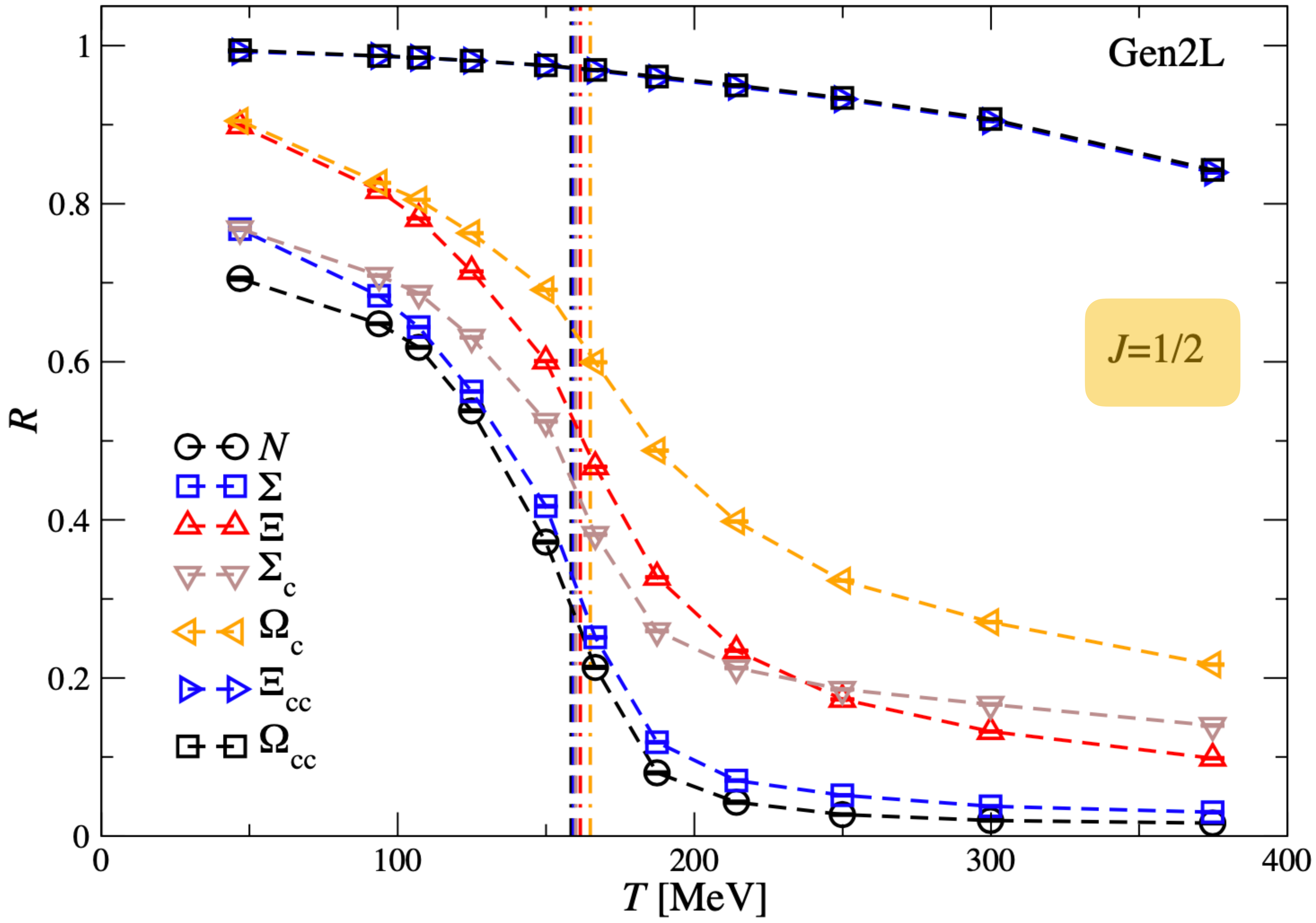
$G_+ = G_- \rightarrow R(\tau) \sim 0$

Parity max broken:

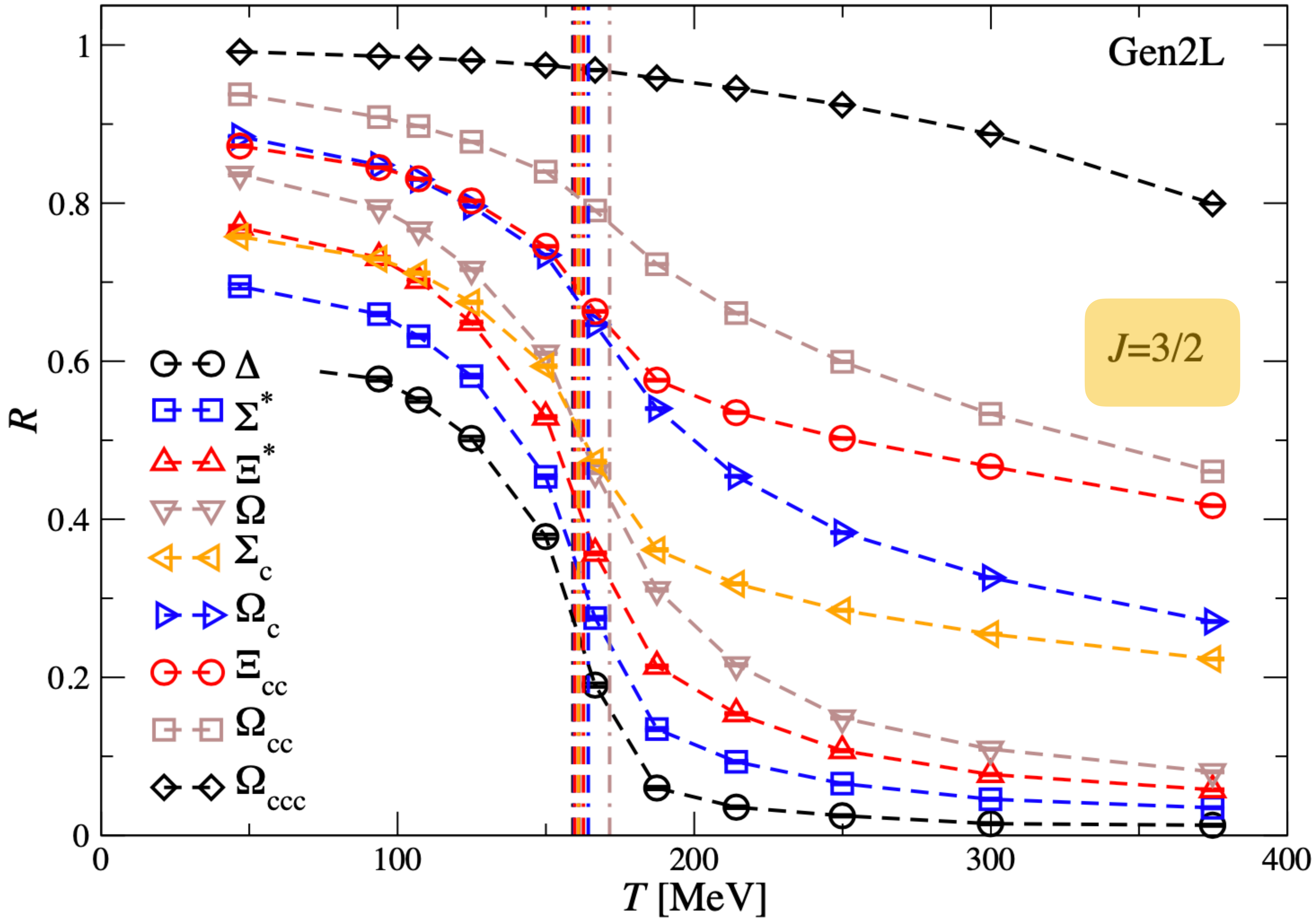
$G_+ \gg G_- \rightarrow R(\tau) \sim 1$

$$R = \frac{\sum_\tau R(\tau)/\sigma^2(\tau)}{\sum_\tau 1/\sigma^2(\tau)}$$

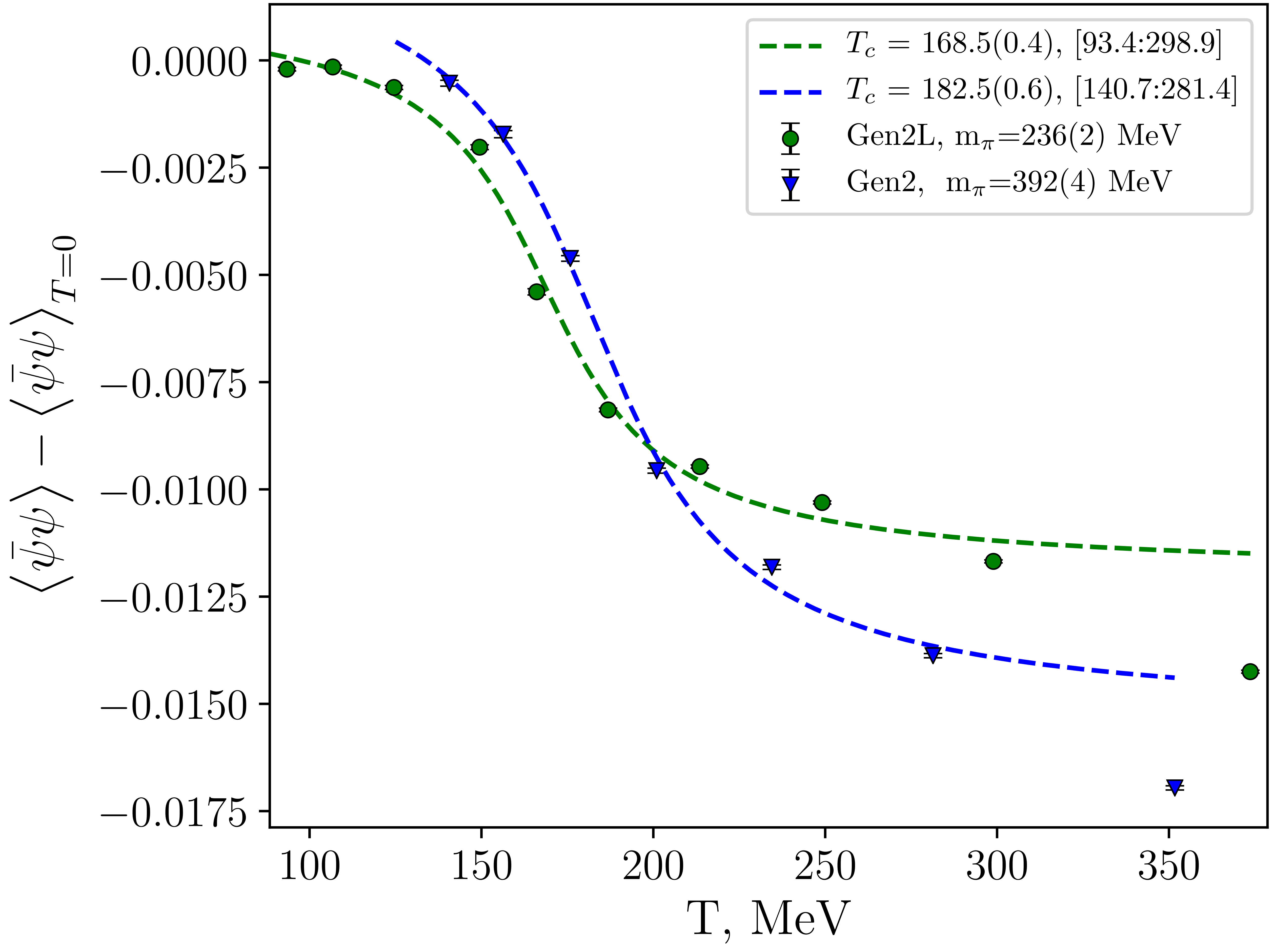
Generation 2L results -
Comparison with J=1/2 light hadrons



Generation 2L results -
Comparison with J=3/2 light hadrons



Generation 2 & 2L results -
Comparison with chiral condensate



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Many Approaches to Extract Spectral Information

Infamous *Inverse Problem*

1. Exponential (Conventional δ f'ns)
2. Gaussian Ground State (+ δ f'n excited)

Tom Spriggs, Ryan Bignell

} Maximum Likelihood

3. Moments of Correlation F'ns

Rachel Horohan D'arcy

} Direct Method - "no" fit

4. Backus Gilbert
5. HLT
6. HMR

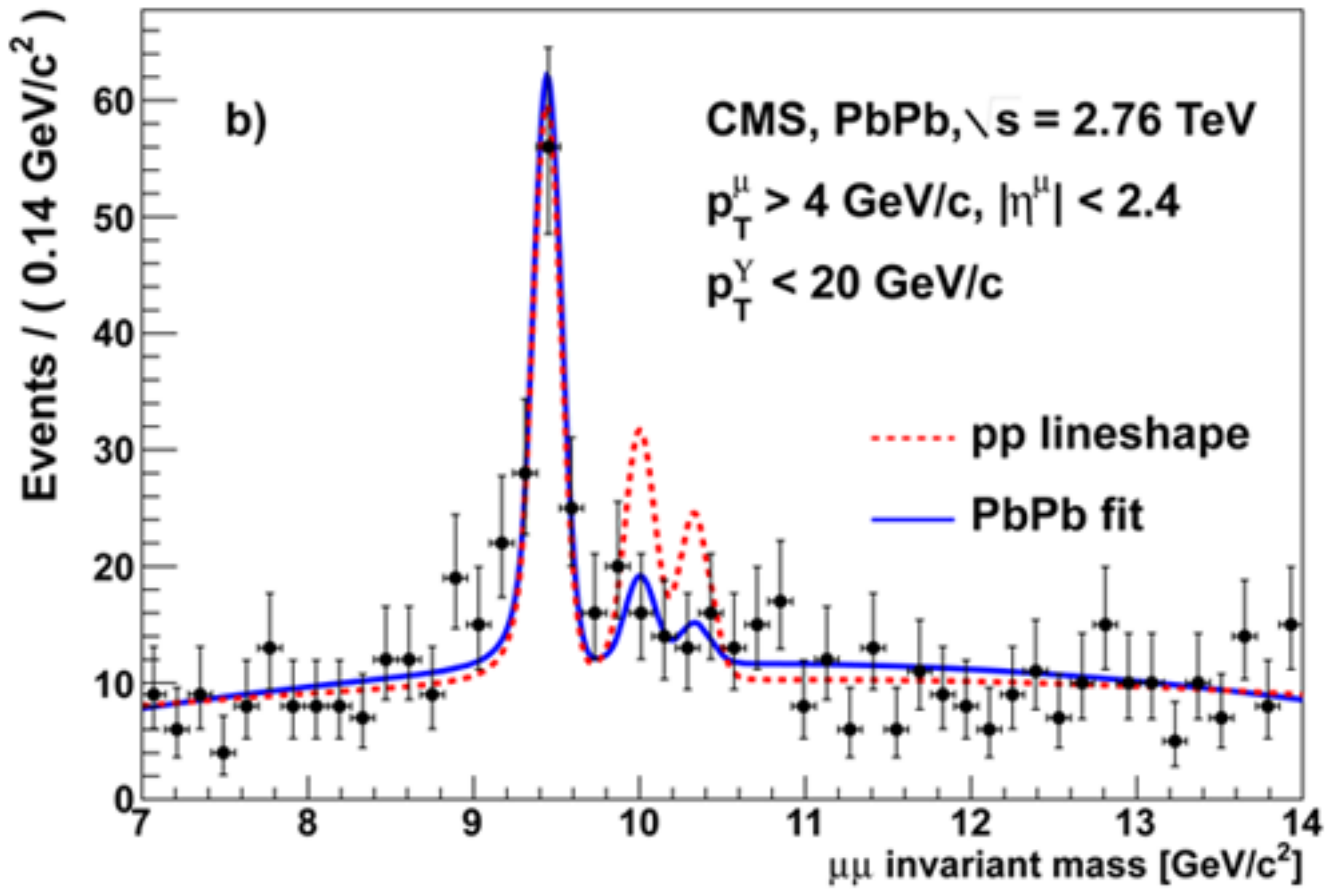
Ben Page

Antonio Smecca

} Linear Methods

7. Maximum Entropy Method
8. BR Method

} Bayesian Approaches



CMS

Many Approaches to Extract Spectral Information

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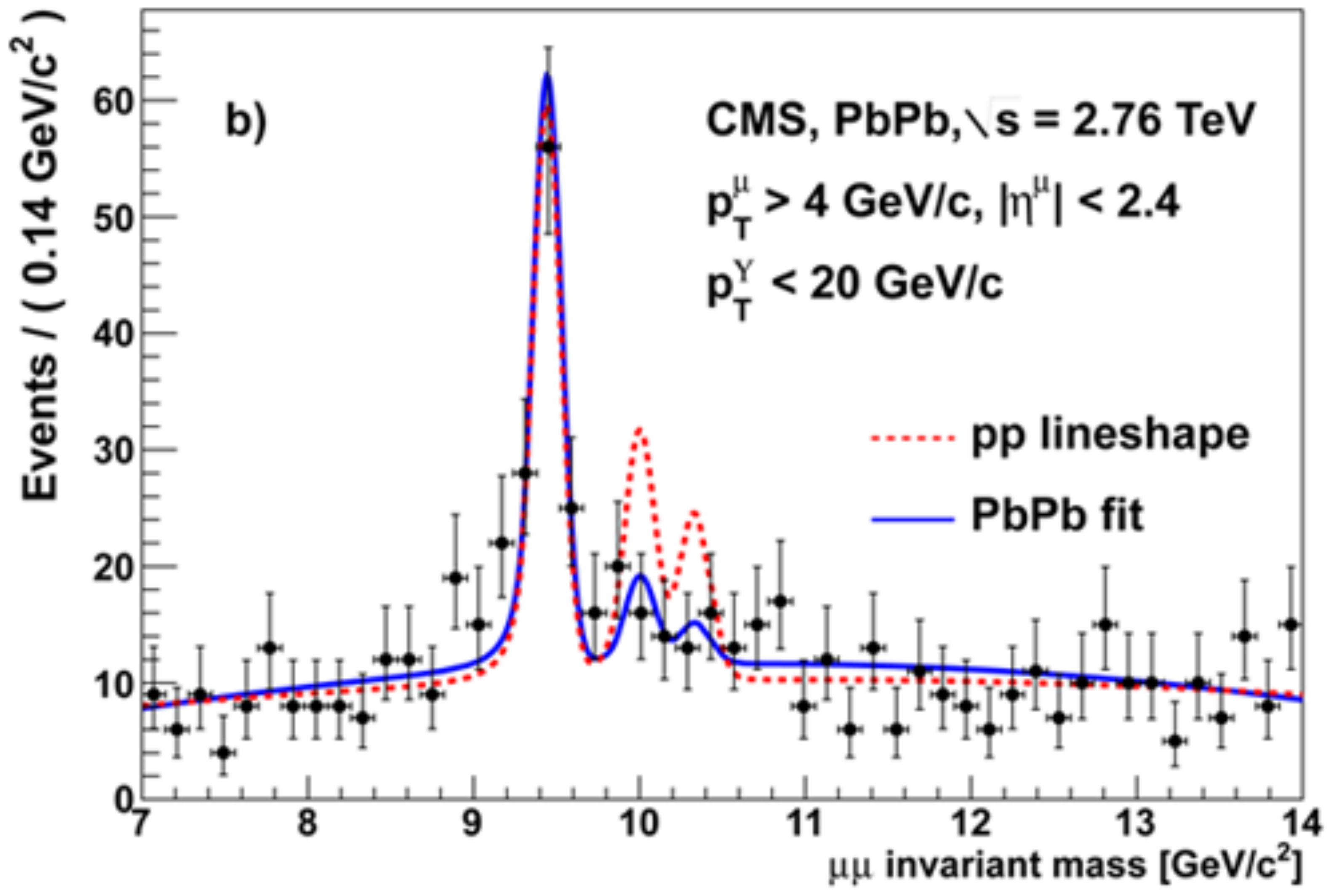


CMS

Studying Thermal Effects via Spectral Functions

Correlation Function's Spectral Representation:

$$G(\tau; T) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega; T)\, \rho(\omega; T)$$

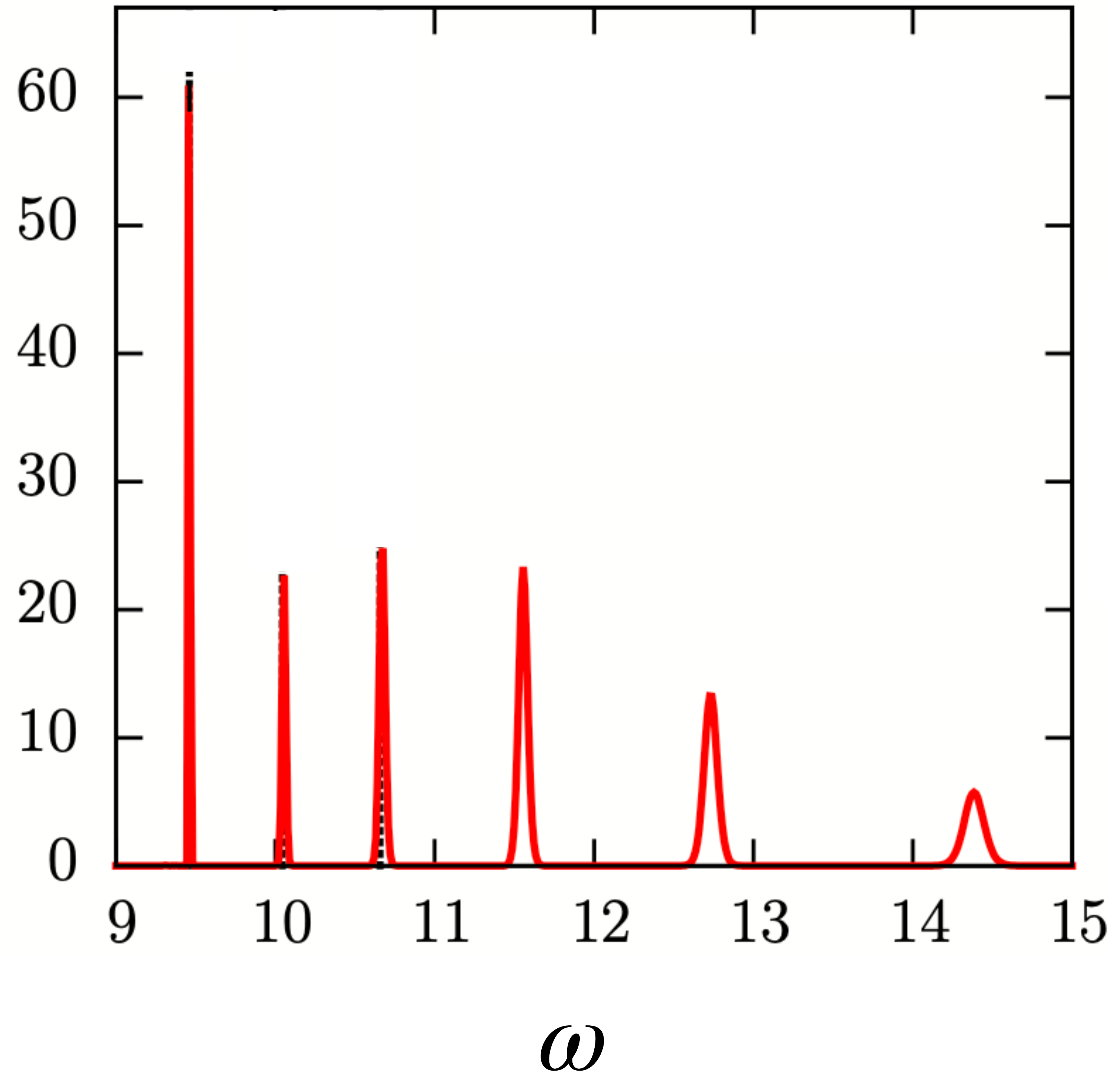
Two sources of Thermal Effects:

Kernel (Geometry / Periodicity)

Spectral F'n (Physics)

Kernel: NRQCD: $K(\tau, \omega; T) = \exp(-\omega\tau)$

Spectral F'n: $\rho(\omega; T)$



Linear Methods: Backus Gilbert & HLT

G. Backus and F. Gilbert, Geophysical Journal International 16 (1968) 169

M. Hansen, A. Lupo and N. Tantalo, Phys. Rev. D 99 (2019) 094508, [arXiv:1903.06476]

smearing f'n

Define a *smeared spectral f'n:*

$$\hat{\rho}(\omega_0) = \int_{\omega_{min}}^{\omega_{max}} \Delta^{lin}(\omega, \omega_0)\, \rho(\omega)\, d\omega$$

kernel

Express smearing f'n in terms of kernel:

$$\Delta^{lin}(\omega, \omega_0) = \sum_{\tau} g_\tau(\omega_0)\, e^{-\omega\tau}$$

coefts

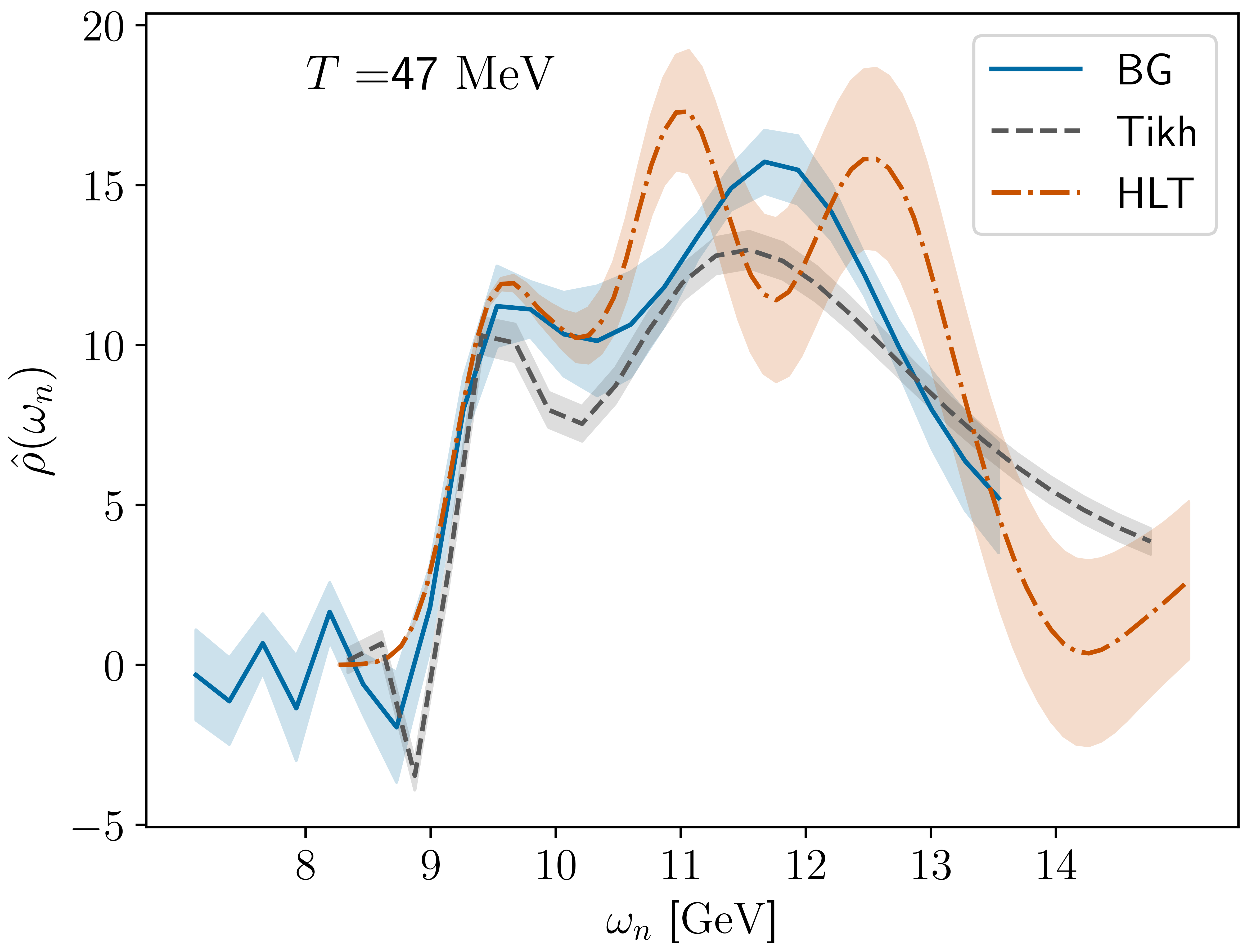
Obtain smeared spectral f'n in terms of coeffts

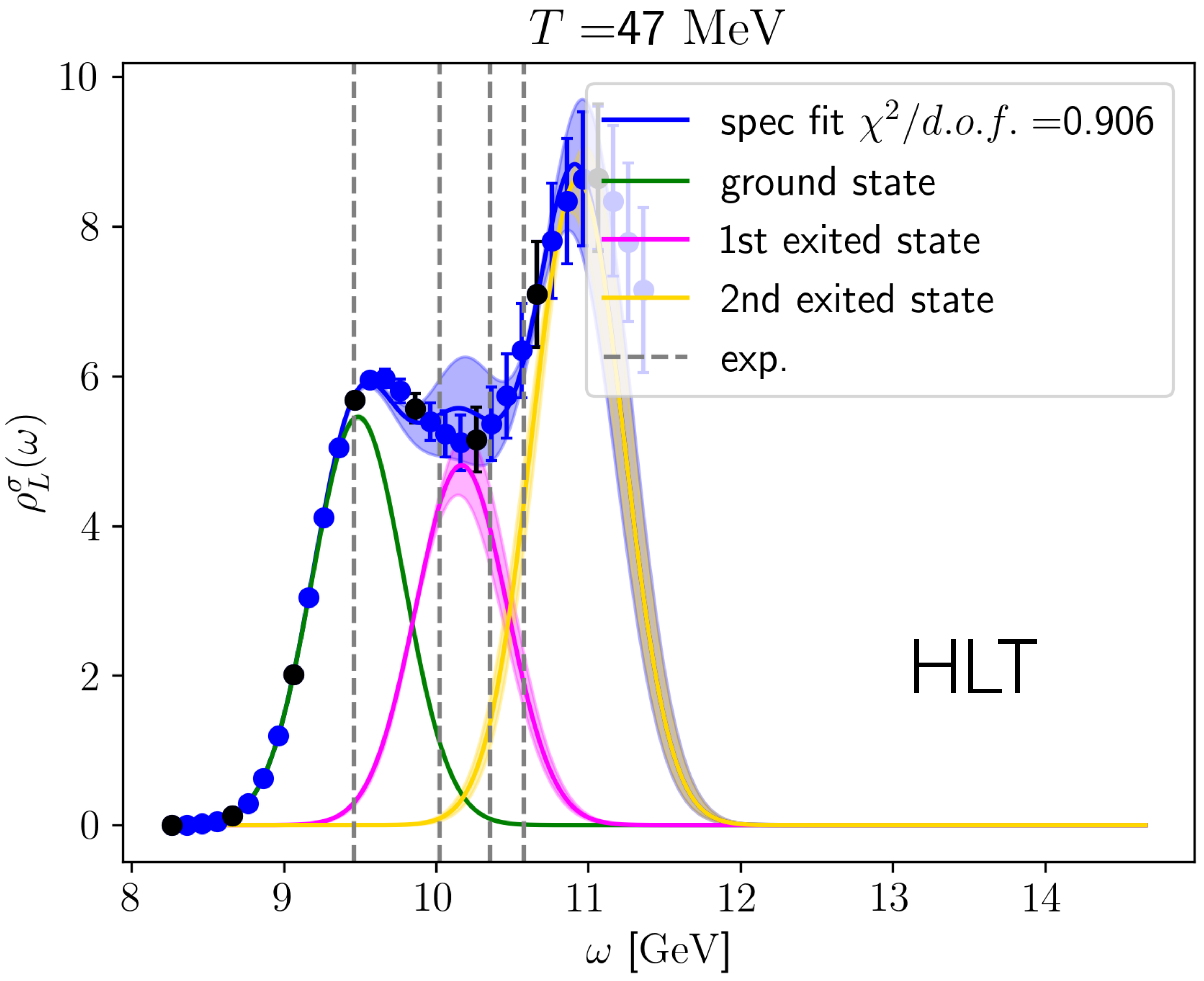
$$\hat{\rho}(\omega_0) = \sum_{\tau} g_\tau(\omega_0)\, G(\tau)$$

Aim is to get

$$\Delta^{lin}(\omega, \omega_0) \to \delta(\omega, \omega_0)$$

Linear Methods Υ

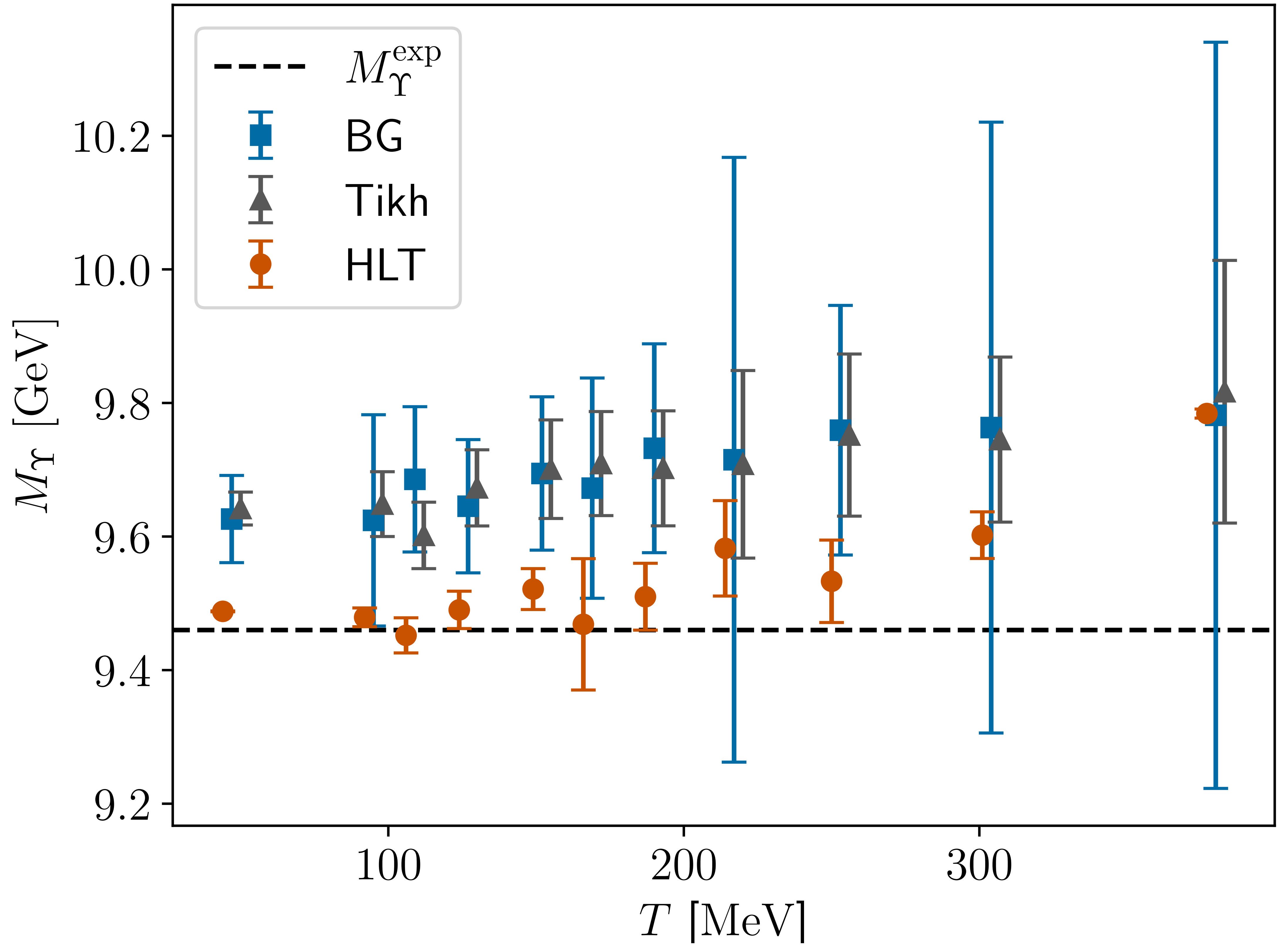




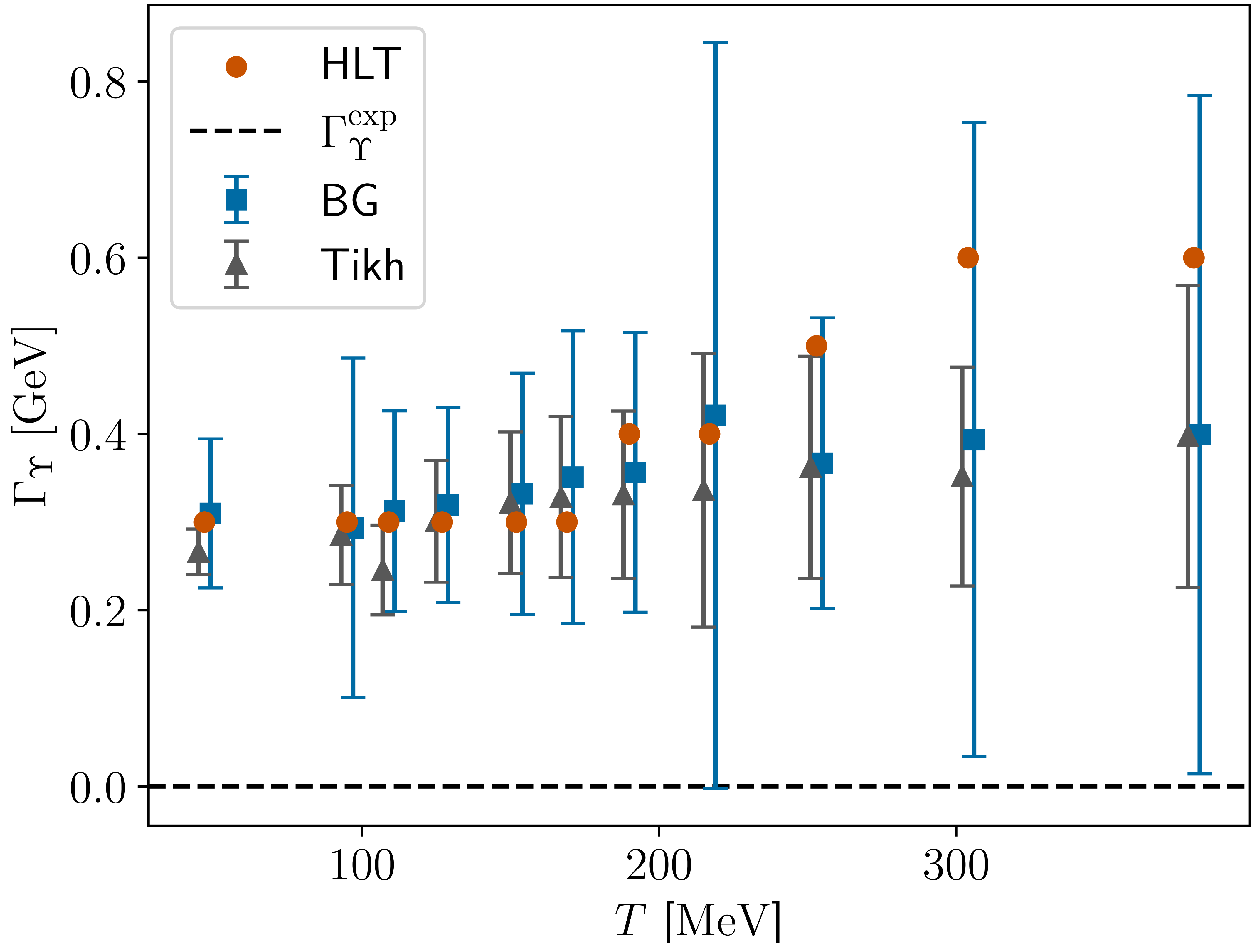
Preliminary

Linear Methods Υ

Mass



Width



Bayesian Methods

Fitting a f'n F to Data D

Need to maximise $P(F|D)$

Bayes Theorem:

$$P(F|D)P(D) = P(D|F)P(F)$$

i.e. $$P(F|D) = \frac{P(D|F)P(F)}{P(D)}$$

But $P(D|F) \sim e^{-\chi^2}$ $\longrightarrow$ minimising χ^2 $\neq$ maximising $P(F|D)$

$\longrightarrow$ *Maximum Likelihood Method* wrong??

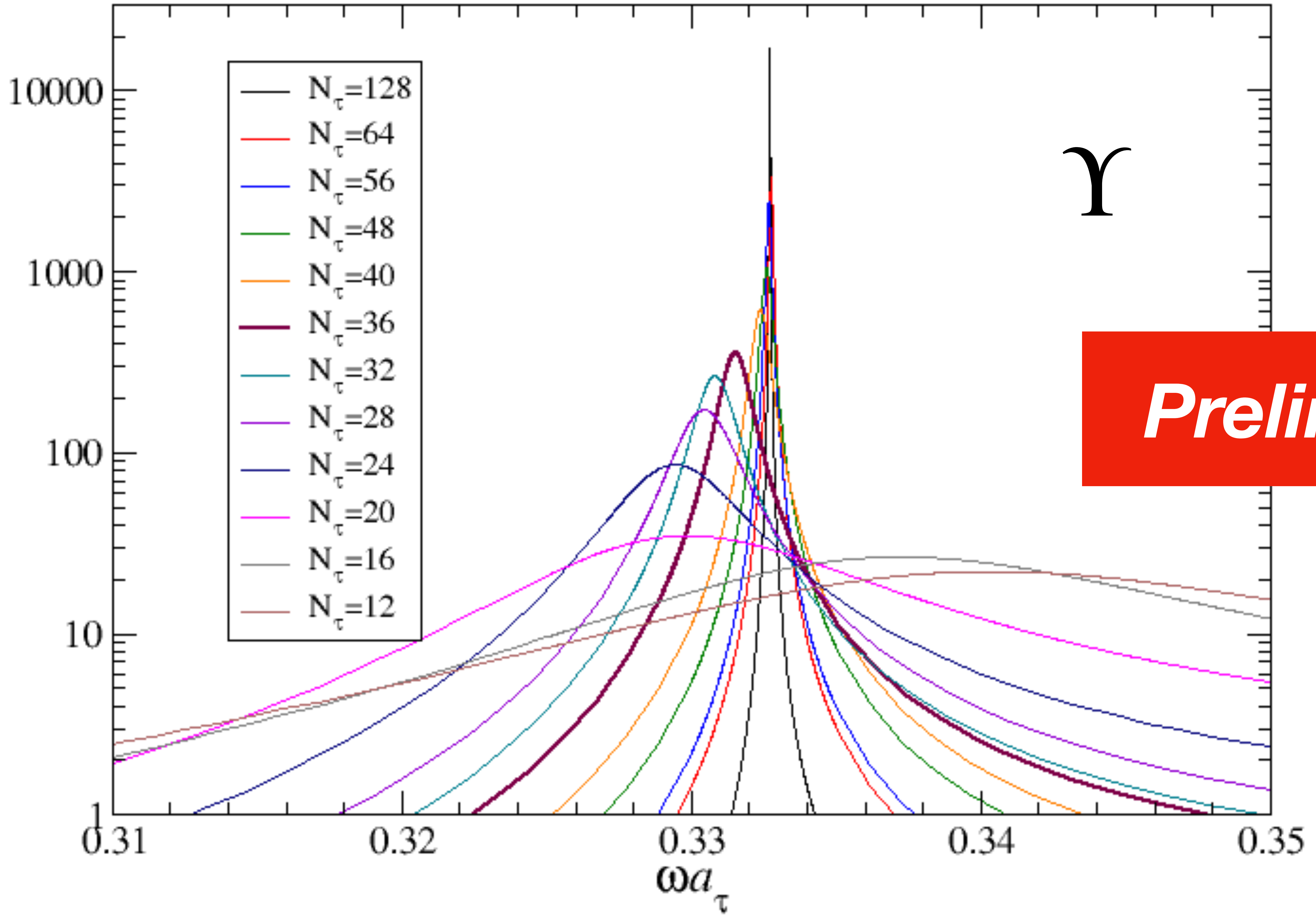
P(F) contains *Prior Information* encoded by "Entropy"

Battle between Data *P(D|F)* and Prior *P(F)*

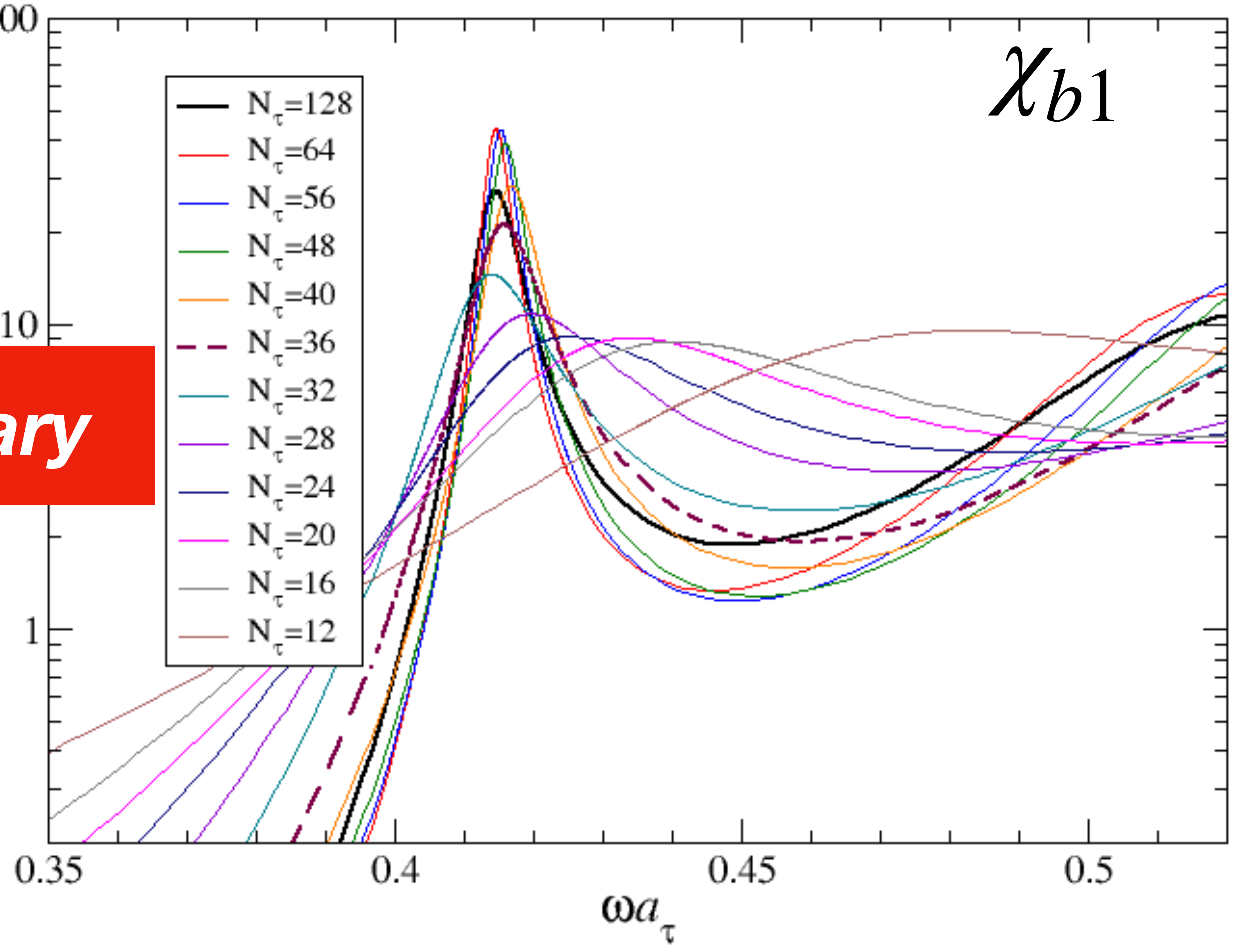
BR (Bayesian Reconstruction)

Y. Burnier and A. Rothkopf, Phys. Rev. Lett. 111 (2013) 182003, [arXiv:1307.6106],
A. Rothkopf, Front. Phys. 10 (2022) 1028995, [arXiv:2208.13590]

BR Entropy: $$S_{BR}[\rho] = \int d\omega \left(1 - \frac{\rho\omega}{m(\omega)} + \ln \frac{\rho(\omega)}{m(\omega}\right)$$



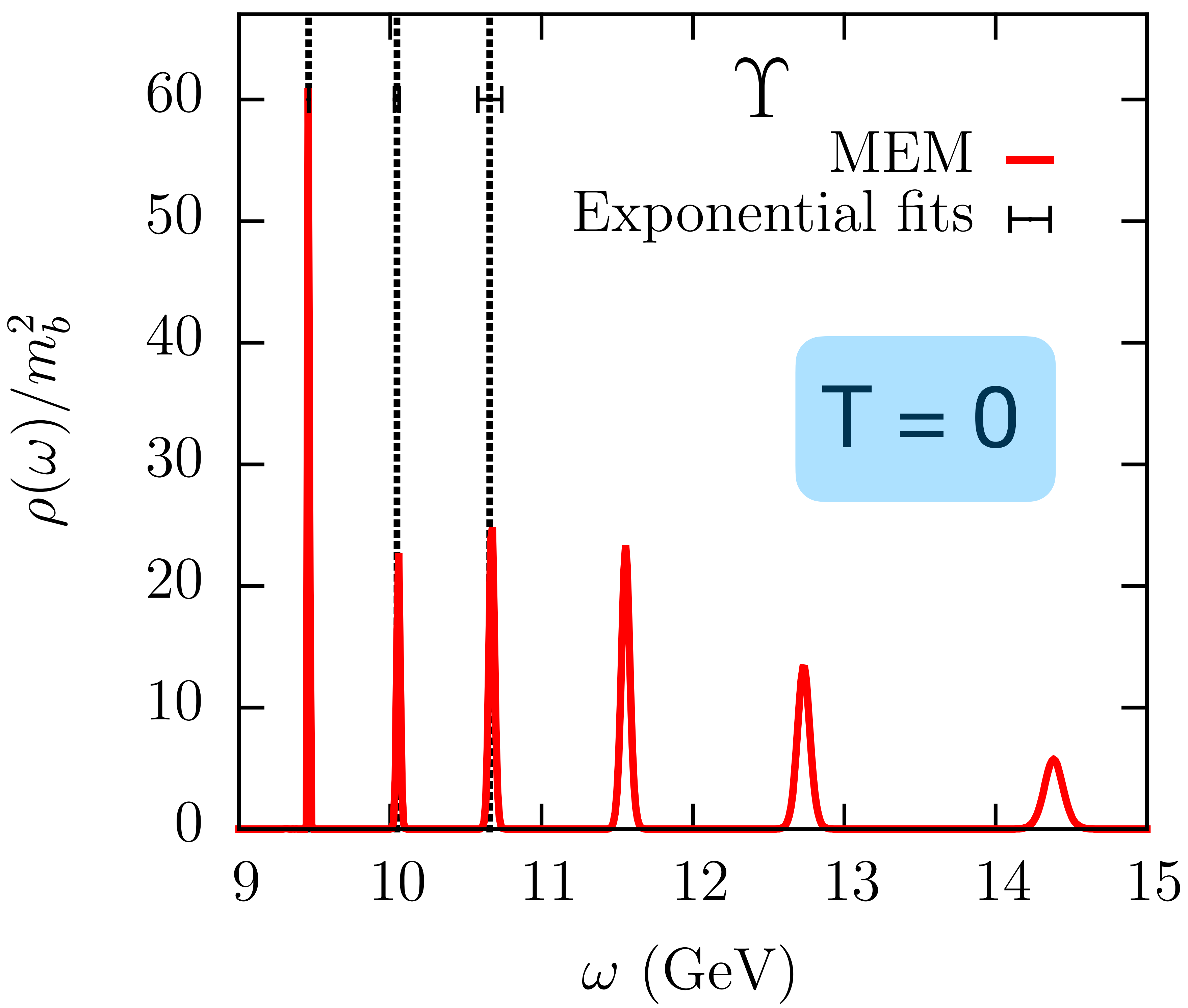
Preliminary

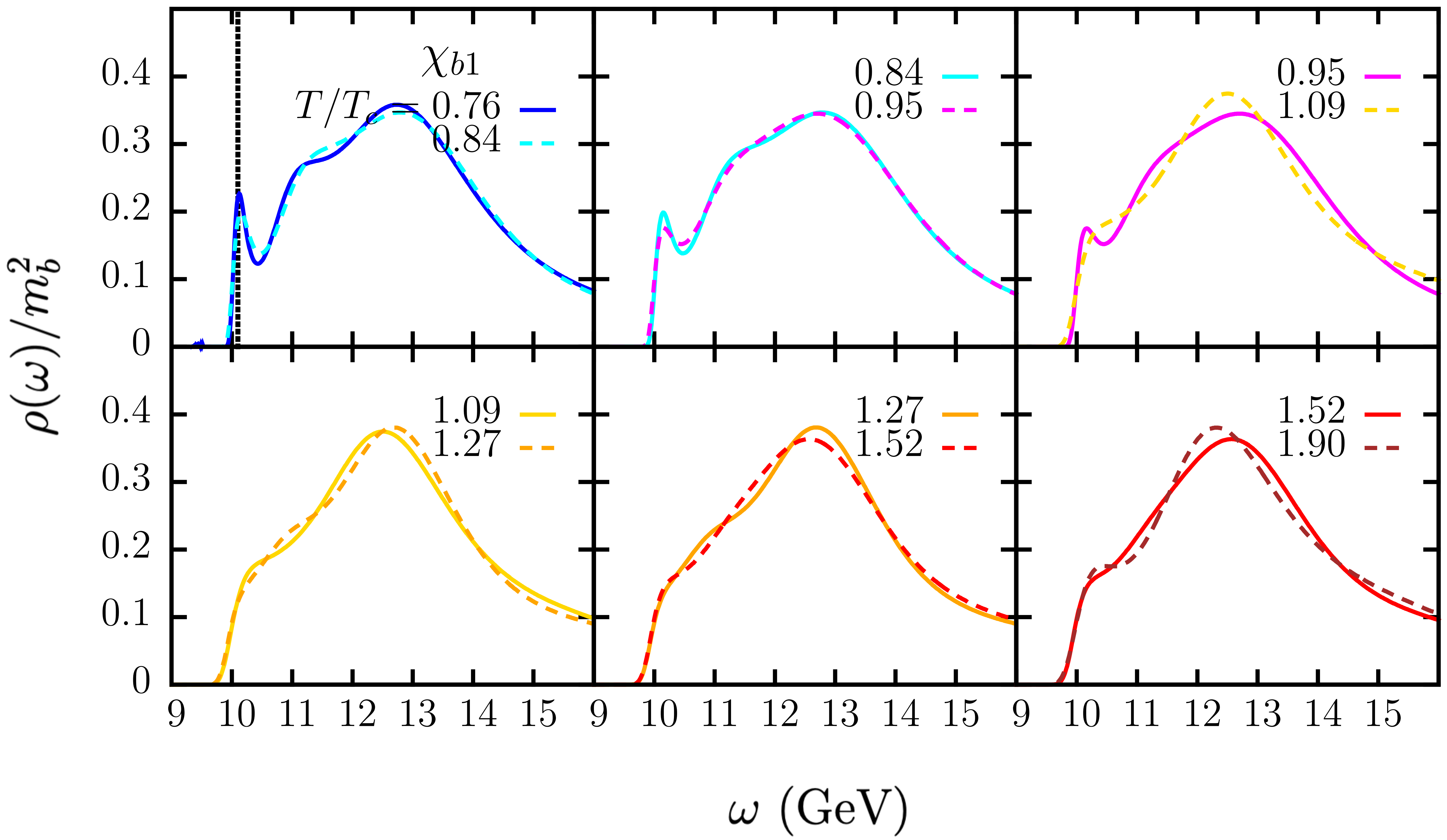


MEM

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459

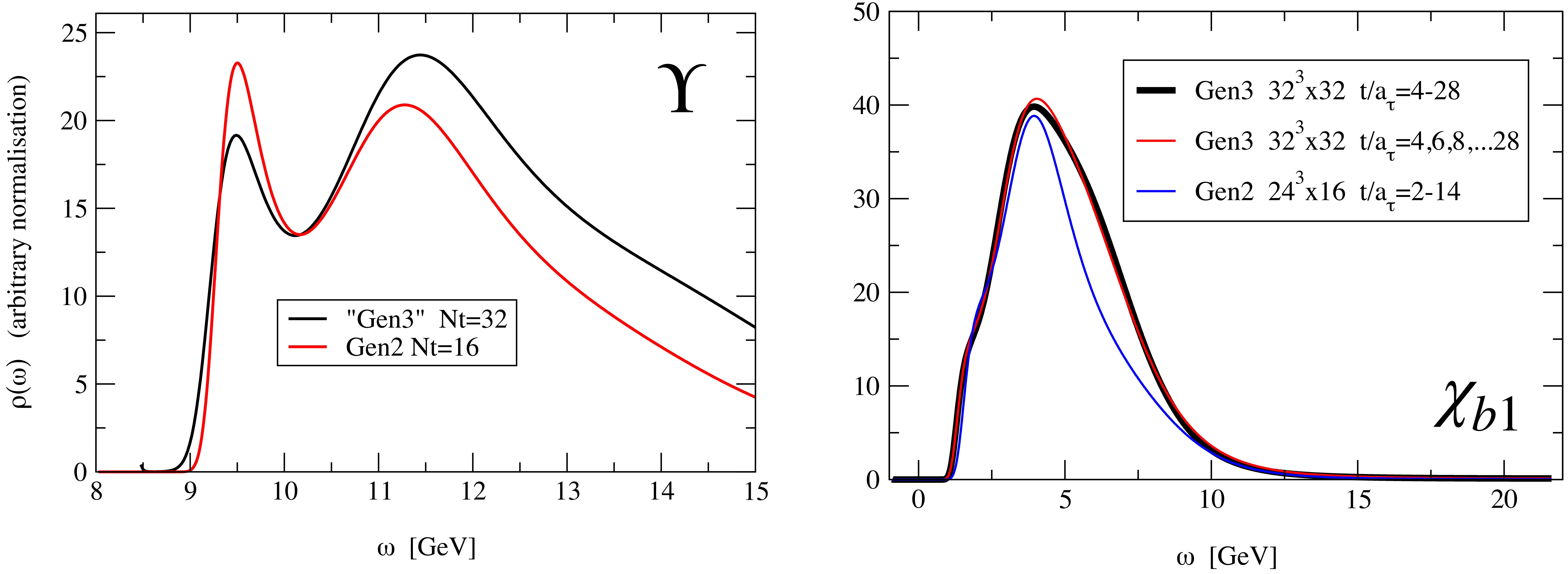
MEM Entropy: $$S_{MEM}[\rho] = \int \frac{d\omega}{2\pi} \left(\rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)} \right)$$





(MEM) Comparison of Gen2 and "Gen3"

$a_\tau(\text{Gen2}) = 32\text{am} \approx 2\, a_\tau(\text{Gen3})$



Summary

FASTSUM approach

- *Anisotropic*

$T_c(\mu)$ curve from mesonic spectrum

- *Curvature agrees with thermodynamic approaches*

Charm Baryons and Parity Doubling

- *+ve parity less T dependent than -ve*
- *Signs of approx parity doubling*

Spectral Functions for NRQCD mesons

- *Comprehensive study in progress!*

Back-Up Slides

Generation 2L

a_τ [am]	a_τ^{-1} [GeV]	$\xi = a_s/a_\tau$	a_s [fm]	m_π [MeV]	$T_{\rm pc}^{\bar{\psi}\psi}$ [MeV]
32.46(7)	6.079(13)	3.453(6)	0.1121(3)	239(1)	167(2)(1)

Generation 2L, $32^3 \times N_\tau$										
N_τ	128	64	56	48	40	36	32	28	24	20
T [MeV]	47	95	109	127	152	169	190	217	253	304
$N_{\rm cfg}$	1024	1041	1042	1123	1102	1119	1090	1031	1016	1030

T_c ~ 167 MeV

a^{-1} = 6.079(13) GeV from HadSpec calculation of **Ω** baryon,

D. J. Wilson, et al., Phys. Rev. Lett. 123 (2019)