Chiral symmetry restoration in hot and dense baryon-rich QCD matter

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Penetrating probes of hot high-$\mu_\mathbf{B}$ matter: theory meets experiment

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The physics case

baryon-rich QCD matter is encountered in astrophysical settings

- inner core of neutron stars

$$T \sim 0\ \mathrm{MeV}, \quad n_B/n_B^0 \gtrsim 4-6; \quad n_B^0 = 0.16\ \mathrm{fm}^{-3}$$

- core-collapse supernovae ($M_* \gtrsim 10 M_\odot$)

$$T \sim 30-40\ \mathrm{MeV}, \quad n_B/n_B^0 \lesssim 1$$

- binary-NS mergers

$$T \sim 50-70\ \mathrm{MeV}, \quad \mathrm{n_B/n_B^0} \lesssim 3-4$$

probed via:

- electroweak signals (neutrinos, photons)
- heavy-ion collisions at energies $\sqrt{s_{NN}} \sim 2-3$ GeV
- GW profiles (GW170818, GW190425)

Central questions

What is the nature of the hadron-quark transition at high density?

- chiral symmetry restoration and confinement-deconfinement transition
- chiral cross-over or true first-order transition?
- where is the location of the chiral CEP?

What are the implications?

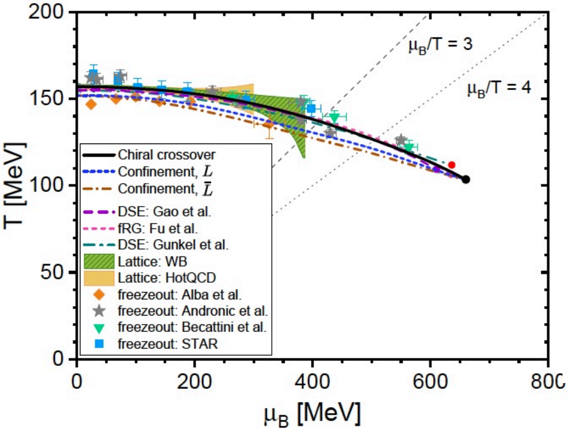
- for the EoS of nuclear and neutron matter
- the spectral properties (electroweak response)
- transport properties (electrical and thermal conductivity, ..)

How does this compare the observation?

- properties of isolated NS (M-R relation, cooling...)
- Heavy-ion collisions at GSI/FAIR energies (hadron yields, dileptons..)
- NS mergers (GW signals, neutrino emissivities and propagation, ..)

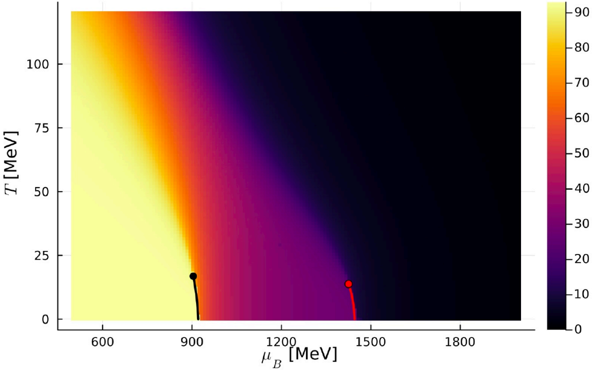
Location of the QCD CEP

pseudo-critical line



[J. Pawlowski et al., arXiv:2504.050990]

Parity-Doublet model



[J. Eser, J.-P Blaizot, Phys.Rev. C 109,045201 (2024)]

[M. Recchi, L. von Smekal, J. Wambach, in progress]

Outline

I) Theoretical setup

- parity doubling in QCD
- parity-doublet model: a primer
- RG-invariant mean-field description
- $\mu_B = 0$, $\mu_I \neq 0$: LQCD comparison

II) Results

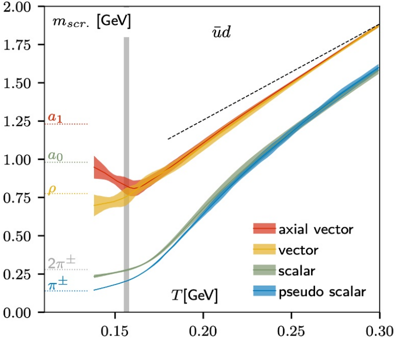
- thermodynamics for symmetric and asymmetric nuclear matter
- chiral symmetry restoration
- β-equilibrated matter and $M - R$ relation

III) Summary and outlook

Parity doubling in QCD

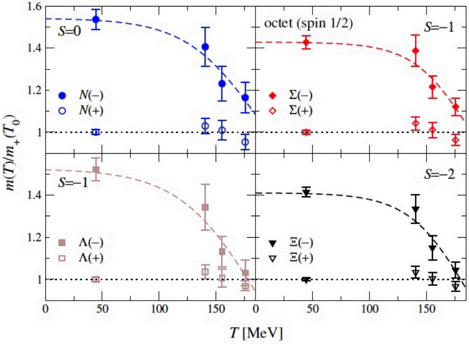
- **chiral symmetry restoration $\leftrightarrow$ spectral degeneracy of parity partners**

mesons



[A. Bazavov et al., Phys. Rev. D 100 094510 (2019)]

baryons



[G. Aarts et al., Phys. Rev. D 99, 074503 (2019)]

- **hadron masses remain finite at the chiral transition!**

The parity-doublet model (PDM)

implements parity degeneracy for massive fermions

- elementary constituents are **nucleons and mesons**
- to respect chiral symmetry the **parity partners** must be included

  mesons: $\phi = (\sigma, \pi)$ baryons: $N(939)$ and $N^*(1535)$

- axial transformation of opposite-parity fermions, N_1, N_2, under chiral $SU(2)_V \times SU(2)_A$

$$\text{naive assignment :} \quad N_1 \to e^{i\vec{\alpha}\cdot\vec{\tau}/2\gamma_5} N_1, \quad N_2 \to e^{i\vec{\alpha}\cdot\vec{\tau}/2\gamma_5} N_2$$

$$\text{mirror assignment :} \quad N_1 \to e^{i\vec{\alpha}\cdot\vec{\tau}/2\gamma_5} N_1, \quad N_2 \to e^{-i\vec{\alpha}\cdot\vec{\tau}/2\gamma_5} N_2$$

[C. E. DeTar, T. Kunihiro, Phys. Rev. D 39, 2805 (1989)]

- **mirror assignment allows for finite fermion mass M_0 in a chirally-invariant way!**

The parity-doublet model (PDM)

- **PDM chiral Lagrangian:**

$$\mathcal{L} = \left(\bar{N}_1, \bar{N}_2\right) \begin{pmatrix} (i\not{D} - g_1(\sigma + i\tau \cdot \vec{\pi}\gamma_5) & -M_0\gamma_5 \\ M_0\gamma_5 & i\not{D} - g_2(\sigma - i\vec{\tau} \cdot \vec{\pi}\gamma_5) \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + \mathcal{L}_{mes}$$

M_0 parameter of the model: $0 \leq M_0 \lesssim 850$ MeV

- include vector repulsion

$$\not{D} \equiv \not{\partial} + i\gamma^\mu (g_\omega \omega_\mu + g_\rho \vec{\rho}_\mu \cdot \vec{\tau})$$

- mesonic Lagrangian (drop kinetic terms)

$$\mathcal{L}_{mes} = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{c_6}{6}\phi^6 + \frac{c_8}{8}\phi^8 - c\sigma - \frac{m_V^2}{2}\left(\omega_\mu\omega^\mu + \vec{\rho}_\mu \cdot \vec{\rho}^\mu\right)$$

- leads to **massive baryons in the chirally restored phase**

[H. Nishihara, M. Harada, JPSCP.10.022017 (2016)], [S. Kim. S.H. Lee, Phys. Rev. D 105, 014014 (2022)]

Origin of the nucleon mass in QCD

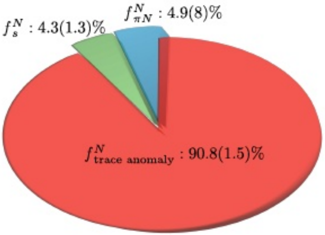
- the **hadron mass** can be obtained from the **trace of the energy-momentum tensor** of QCD

  [J. C. Collins, A. Duncan, S. D. Joglekar, Phys. Rev. D 16, 438 (1977)], [N. K. Nielsen, Nucl. Phys. B 120, 212 (1977)]

$$M_H = T^\mu_\mu = \sum_{l=u,d,s} m_l \langle \bar{q}_l q_l \rangle_H + \left\langle \frac{\tilde{\beta}(g)}{2g} G^{\mu\nu a} G^a_{\mu\nu} + \sum_{l=u,d,s} m_l \gamma_{m_l} \bar{q}_l q_l \right\rangle_H$$

- LQCD results for nucleon mass decomposition

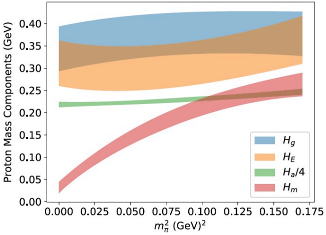
$$M_N = \langle H_m \rangle_N + \langle H_a \rangle_N$$



[K-F. Liu, Phys.Rev.D 104, 076010 (2021)]

$$M_N = \langle H_m \rangle_N + \langle H_E \rangle_N (\mu) + \langle H_g \rangle_N (\mu) + \langle H_a \rangle_N /4$$

[X.-D. Ji, Phys. Rev. Lett. 74, 1071 (1995)]



[Yi-Bo Yang et al., Phys. Rev. Lett. 121, 212001(2018)]

PDM (extended) mean-field description

- meson fields are treated classically → σ, ω_μ and $\vec{\rho}_\mu$ acquire non-vanishing expectation values

$$\sigma, \omega_\mu, \vec{\rho}_\mu \quad \rightarrow \quad \langle\sigma\rangle = \bar{\sigma}, \;\; \langle\omega_\mu\rangle = \bar{\omega}_0, \;\; \langle\vec{\rho}_\mu\rangle = \bar{\rho}_0$$

- after diagonalization → masses of physical baryon states: $N(939)$ and $N^*(1535)$

$$M_\pm(\bar{\sigma}) = \frac{1}{2}\left[\sqrt{\bar{\sigma}^2(g_1+g_2)^2 + 4M_0^2} \pm \bar{\sigma}(g_2 - g_1)\right] \qquad \bar{\sigma} = 0 \rightarrow M_\pm(0) = M_0$$

from QCD sum rules: $M_0 = 525$ MeV

[S. Kim. S.H. Lee, Phys. Rev. D 105, 014014 (2022)]

- diagonalized Lagranian

$$\mathcal{L} = \bar{N}_+\left(i\not{D} - M_+\right)N_+ + \bar{N}_-\left(i\not{D} - M_-\right)N_- + U_{mes}(\bar{\sigma}, \bar{\omega}, \bar{\rho}_0)$$

$$U_{mes}(\bar{\sigma}, \bar{\omega}, \bar{\rho}_0) = \frac{m^2}{2}\bar{\sigma}^2 + \frac{\lambda}{4}\bar{\sigma}^4 + \frac{c_6}{6}\bar{\sigma}^6 + \frac{c_8}{8}\bar{\sigma}^8 - c\bar{\sigma} - \frac{m_V^2}{2}\left(\bar{\omega}_0^2 + \bar{\rho}_0^2\right)$$

PDM thermodynamics

- Grand Canonical potenial

$$\Omega(T,\mu_B,\mu_I) = -2T \sum_{\pm,\tau=\pm} \int_p \Big(\ln\cosh\left(\frac{E_\pm - \tilde{\mu}_B - \tau\tilde{\mu}_I}{2T}\right) + \ln\cosh\left(\frac{E_\pm + \tilde{\mu}_B + \tau\tilde{\mu}_I}{2T}\right)\Big) + U_{mes}$$

with $\quad E_\pm = \sqrt{p^2 + M_\pm^2(\bar{\sigma})}, \quad \tilde{\mu}_B = \mu_B - g_\omega \bar{\omega}_0; \;\; \tilde{\mu}_I = \mu_I - g_\rho \bar{\rho}_0/2$

$$\Omega(T,\mu_B,\mu_I) = \Omega_{\rm med}(T,\mu_B,\mu_I) + U_{mes} + \Omega_{\rm vac}^F, \qquad \Omega_{\rm vac}^F = -4\sum_\pm \int_p \sqrt{p^2 + M_\pm^2(f_\pi)}$$

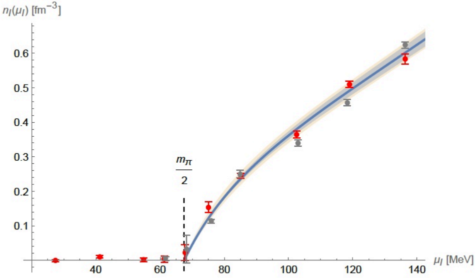
- divergent part in $\Omega_{\rm vac}^F$ can be removed in an RG-invariant way!
- setting the RG-scale $\Lambda_{RG} = M_0$ leads to

$$\Omega(T,\mu_B,\mu_I) = \Omega_{\rm med}(T,\mu_B,\mu_I) + \tilde{U}_{mes} - \frac{1}{4\pi}\sum_{\pm,\tau=\pm} M_\pm^4(f_\pi) \ln\frac{M_\pm(f_\pi)}{M_0}$$

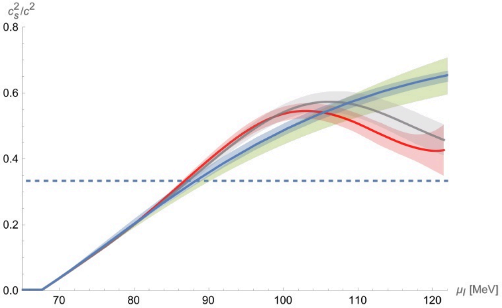
QCD matter at $\mu_B = 0, \mu_I \neq 0$: LCD comparison

- for vanishing μ_B the quark fermion determinant is real and the QCD partition function can be evaluated in LQCD
- $\rightarrow$ effective chiral theories can be tested against exact results
- quark-meson model in RG-MF vs LDCD

isospin number density



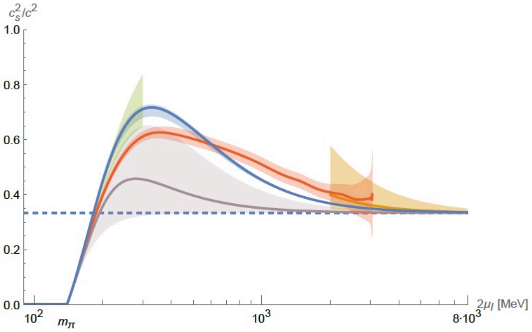
speed of sound



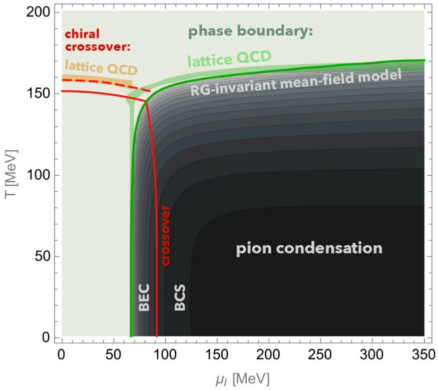
[B. Brandt et al., arXiv:2502.04025]

QCD matter at $\mu_B = 0, \mu_I \neq 0$: LQCD comparison

speed of sound



phase diagram



PDM Best-fit values

No Vacuum Contribution

m_0 [MeV]	B [MeV]	n_0 [fm^{-3}]	E_{sym} [MeV]	L [MeV]	K_∞ [MeV]	$< \sigma_{n_0} >$ [MeV]	T_c [MeV]
500	16.0	0.157	31.0	84	240	67.19	17.2
600	16.0	0.157	31.0	83	226	67.19	17.3
700	16.0	0.157	31.0	82	240	67.19	18.0
800	16.0	0.157	31.0	80	240	67.19	21.7

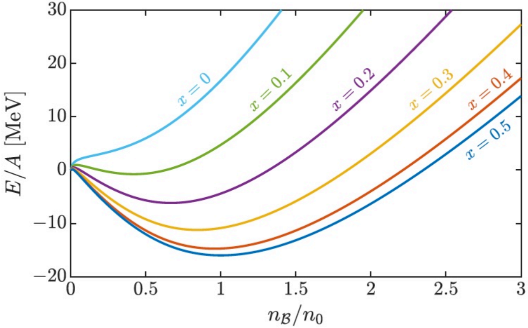
Vacuum Contribution

m_0 [MeV]	B [MeV]	n_0 [fm^{-3}]	E_{sym} [MeV]	L [MeV]	K_∞ [MeV]	$< \sigma_{n_0} >$ [MeV]	T_c [MeV]
500	15.9	0.162	30.99	82	244	72.13	14.1
600	15.8	0.161	31.01	82	244	71.21	15.5
700	16.0	0.157	30.95	81	240	69.49	16.5
800	15.8	0.164	31.01	80	259	66.73	21.5

[M. Recchi, L. von Smekal, J. Wambach, in progress)]

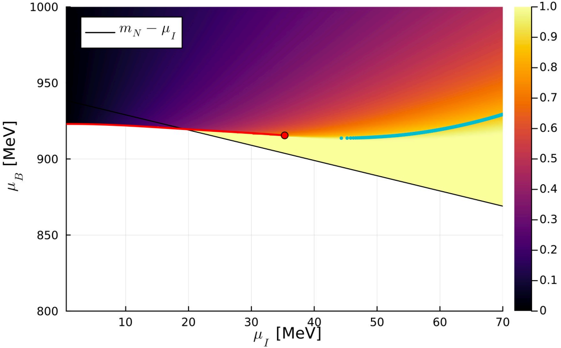
PDM results: T=0

energy/particle vs. proton fraction



[J. Eser, J.-P. Blaizot, Phys.Rev. C 110, 065205 (2024)]

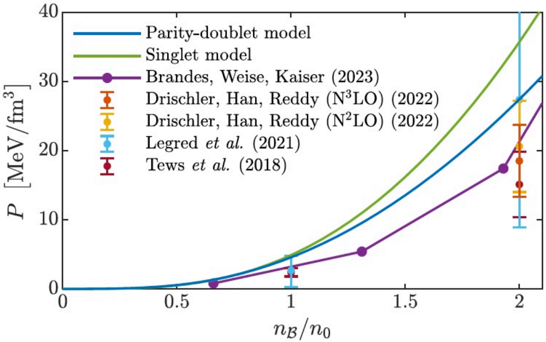
μ_B-μ_I phase diagram



[M. Recchi, L. von Smekal, J. Wambach., in progress]

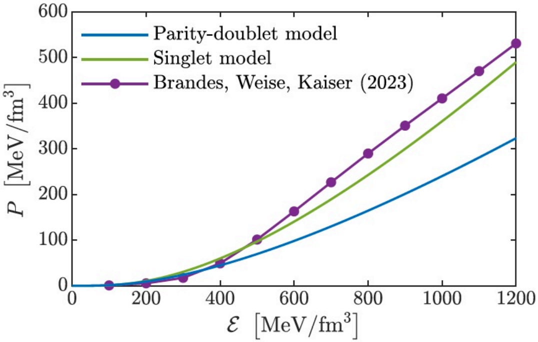
PDM results: neutron matter T=0

EoS



[J. Eser, J.-P. Blaizot, Phys.Rev. C 110, 065205 (2024)]

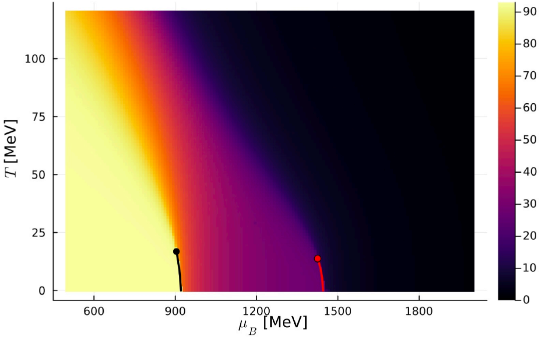
EoS



[J. Eser, J.-P. Blaizot, Phys.Rev. C 110, 065205 (2024)]

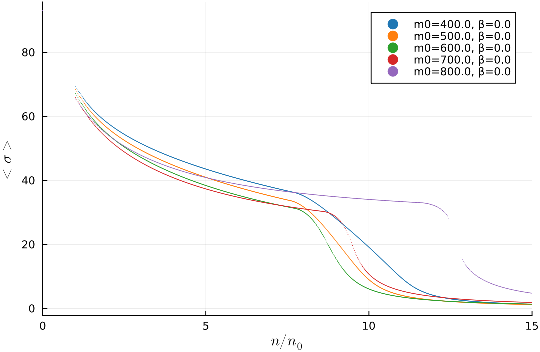
PDM chiral transition: symmetric nuclear matter

chiral condensate $(M_0 = 800\ \text{MeV})$



[M. Recchi, L. von Smekal, J. Wambach, in progress)]

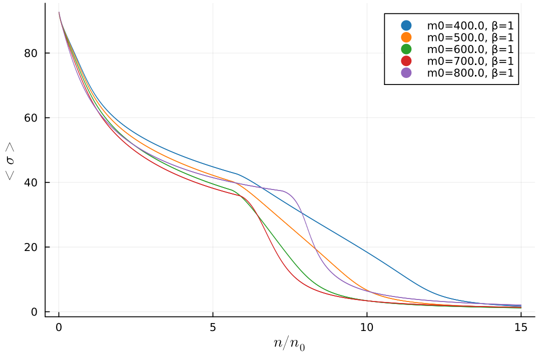
chiral condensate $(T = 0)$



[M. Recchi, L. von Smekal, J. Wambach, in progress)]

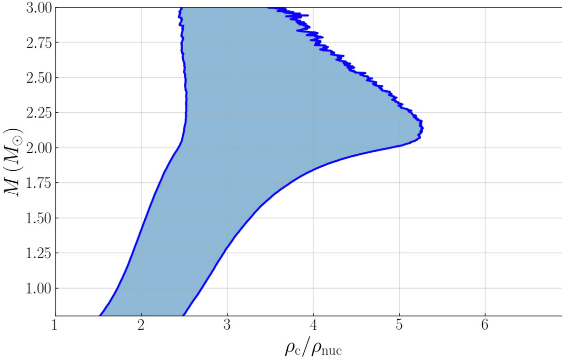
Astrophysical relevance?

chiral condensate: neutron matter



[M. Recchi, L. von Smekal, J. Wambach, in progress)]

central density: Baeysian inference



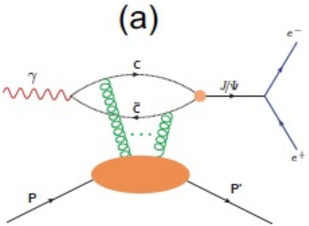
[K. Chatziioannou et al., arXiv:2407.11153]

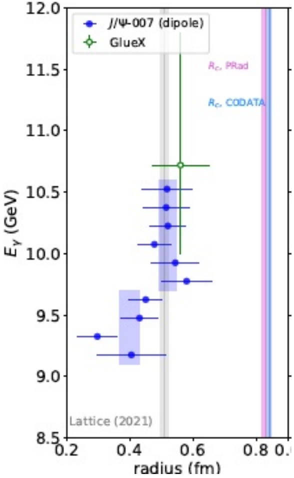
"Gravitational radius" of the proton

- **gravitational radius of the proton** can be obtained as derivative w.r.t. momentum transfer $t = q^2 = (p_1 - p_2)^2$:

$$\langle \vec{p}_1 | T^\mu_\mu | \vec{p}_2 \rangle \propto G(q^2) \to \langle R_m^2 \rangle = \frac{6}{M} \frac{dG}{dt}\Big|_{t=0}$$

- measurable in exclusive J/ψ production near threshold





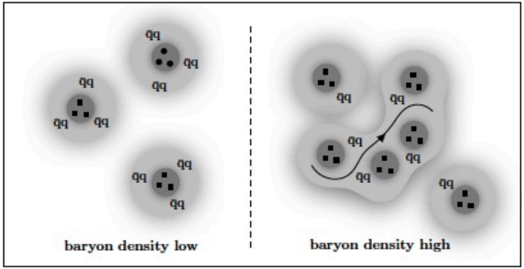
$\to$ data lead to $R_m \approx 0.55$ fm, as opposed to $R_c \approx 0.84$ fm

$$\mathbf{\Delta V \sim 0.3!}$$

[D. E. Kharzeev, Phys.Rev.D 104, 054015 (2021)], [B. Duran et al. (2022) arXiv:2207.05212]

baryons at high density (Weise)

TWO-SCALES Scenario for DENSE BARYONIC MATTER



- Baryon densities
  $n_B \sim n_0 = 0.16\,\mathrm{fm}^{-3}$

  Tails of mesonic clouds overlap :
  two-body exchange forces
  between nucleons

- $n_B \gtrsim 2 - 3\,n_0$

  Soft $\bar{q}q$ clouds delocalize :
  percolation → many-body forces
  baryonic cores still separated, but subject to increasingly strong repulsive Pauli effects

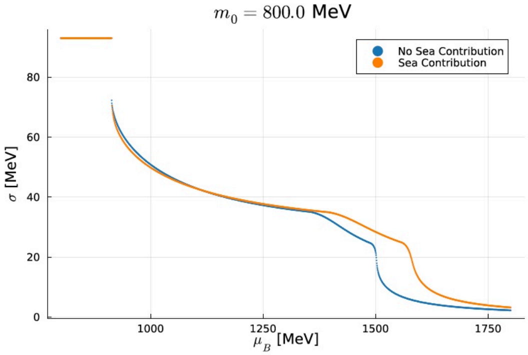
- $n_B > 5\,n_0$ (beyond central densities of neutron stars)

  Compact nucleon cores begin to touch and overlap at distances $d \lesssim 1\,\mathrm{fm}$
  (but still have to overcome the repulsive NN hard core)

K. Fukushima, T. Kojo, W.W.
Phys. Rev. D 102 (2020) 096017

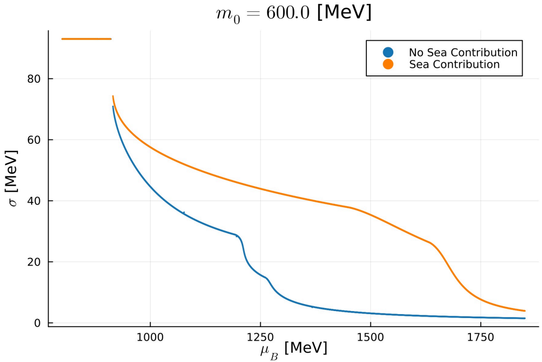
Importance of the sea contribution

chiral condensate: β-equilibrium



[M. Recchi, L. von Smekal, J. Wambach, in progress)]

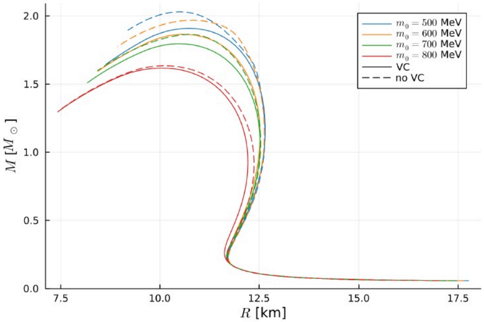
chiral condensate: β-equilibrium



[M. Recchi, L. von Smekal, J. Wambach, in progress)]

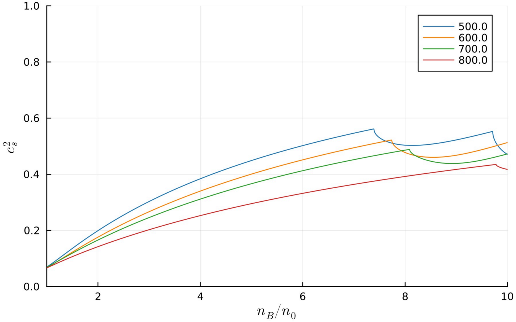
Mass-radius relation and sound speed

M-R relation



[M. Recchi, L. von Smekal, J. Wambach, in progress)]

speed of sound



[M. Recchi, L. von Smekal, J. Wambach, in progress)]

Summary and Outlook

- restoration through **parity degeneracy** of baryonic and mesonic states
- well motivated by "mass generation" in QCD and seen in LQCD
- **PDM in RG-invariant mean-field theory** (sea contributions)
- RG-MF compares well with lattice QCD results at vanishing μ_B
- the chiral transition most likely a cross-over transition (sea contributions essential!)
- PDM EoS too soft at high densities? (especially for large M_0)

Outlook

- include strangeness (hyperon puzzle?)
- compute electroweak response (photons, neutrinos)
- $g_A = -g_A^* \leq 1$ for PDM: fix via 'kinetic mixing'
- infer signals for HIC and NS mergers

Thanks for your attention