The QCD phase structure at finite temperature and density

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

ECT* Trento, July 21st 2025

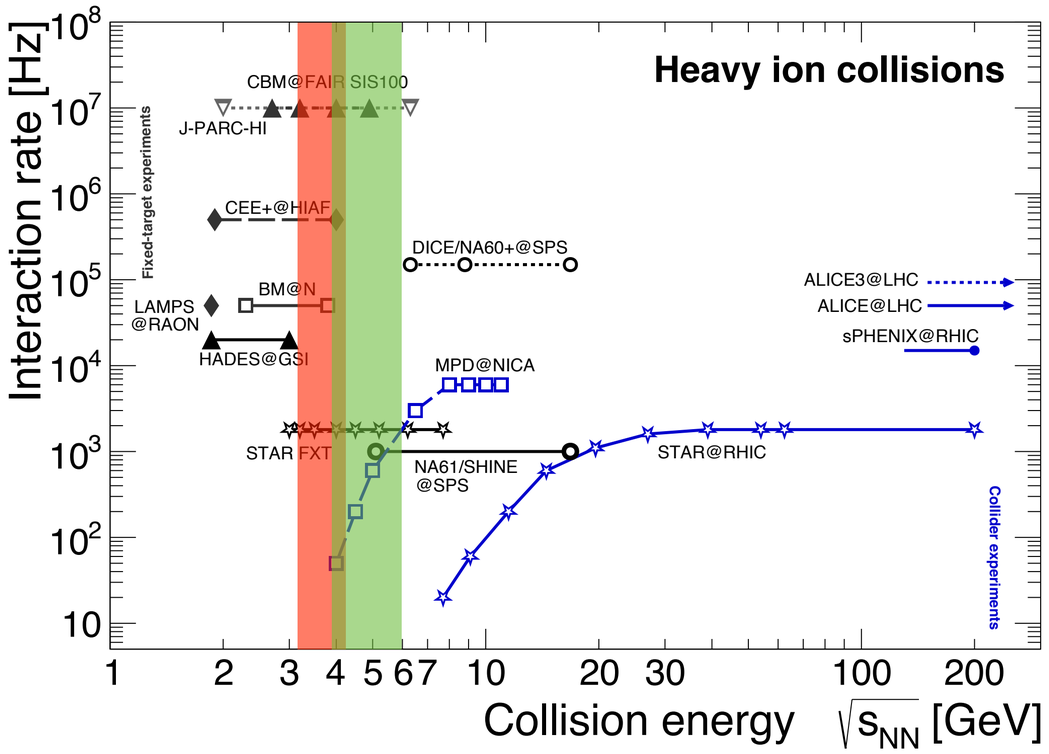
fqcd-collaboration.github.io

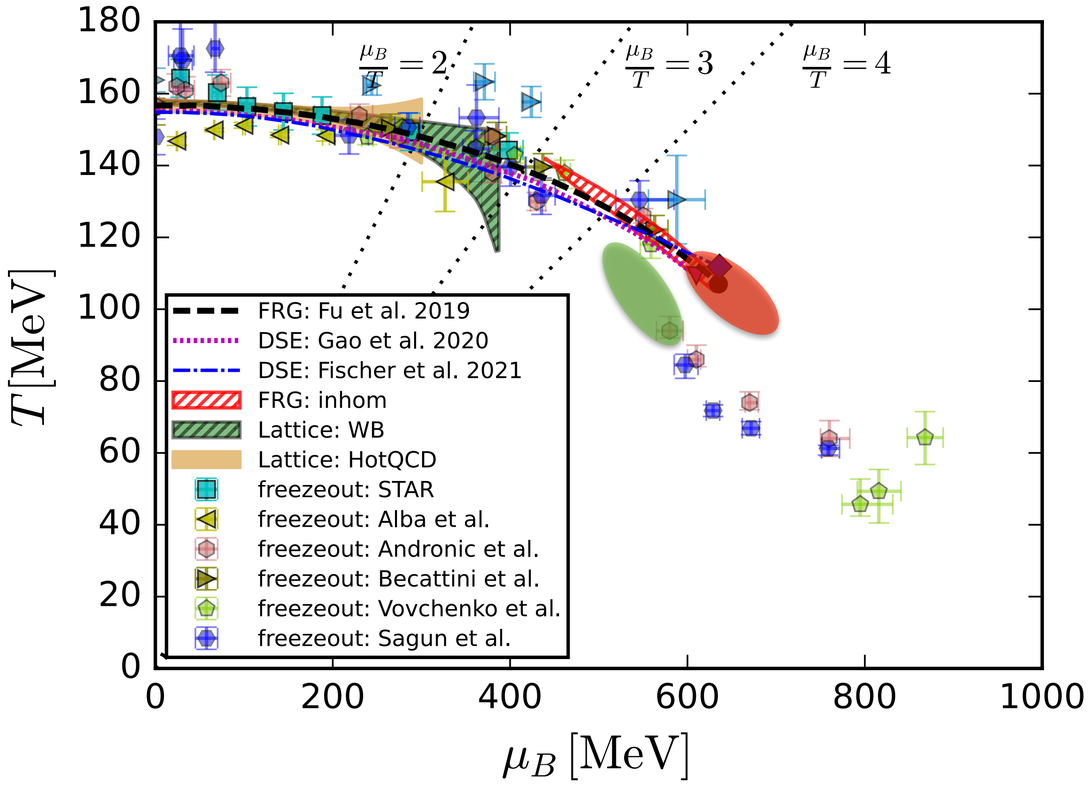
Experimental & Theoretical Landscape

Experimental landscape

Theoretical landscape

Onset of new physics (CEP)





Peak of kurtosis at the freeze-out line

Galatyuk, A982 (2019) update 2025; CBM, EPJA 53 3 (2017) 60

Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

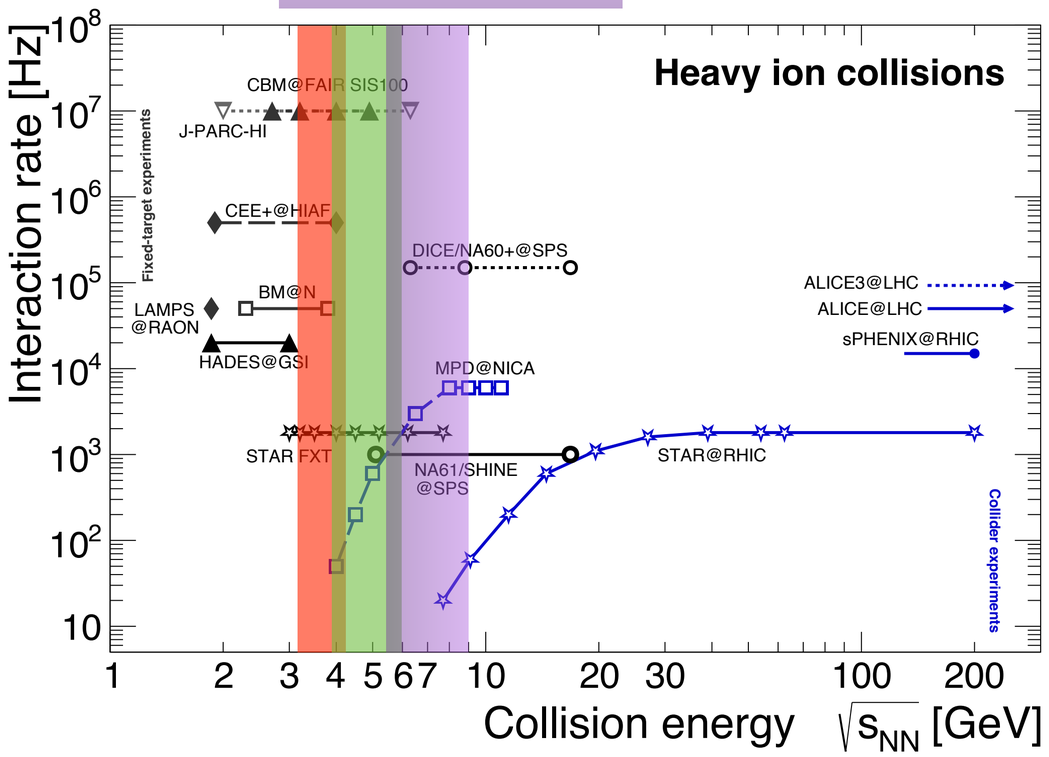
Experimental & Theoretical Landscape

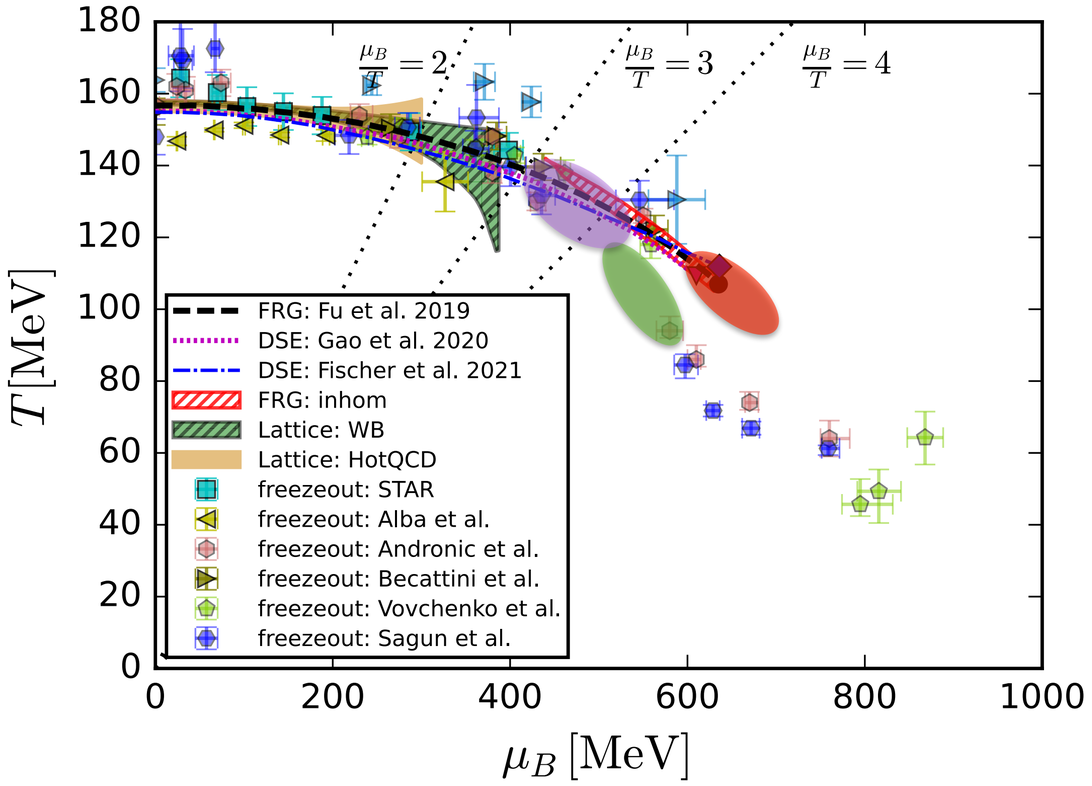
Experimental landscape

Theoretical landscape

Onset of new physics (CEP)

Transition regime





Peak of kurtosis at the freeze-out line

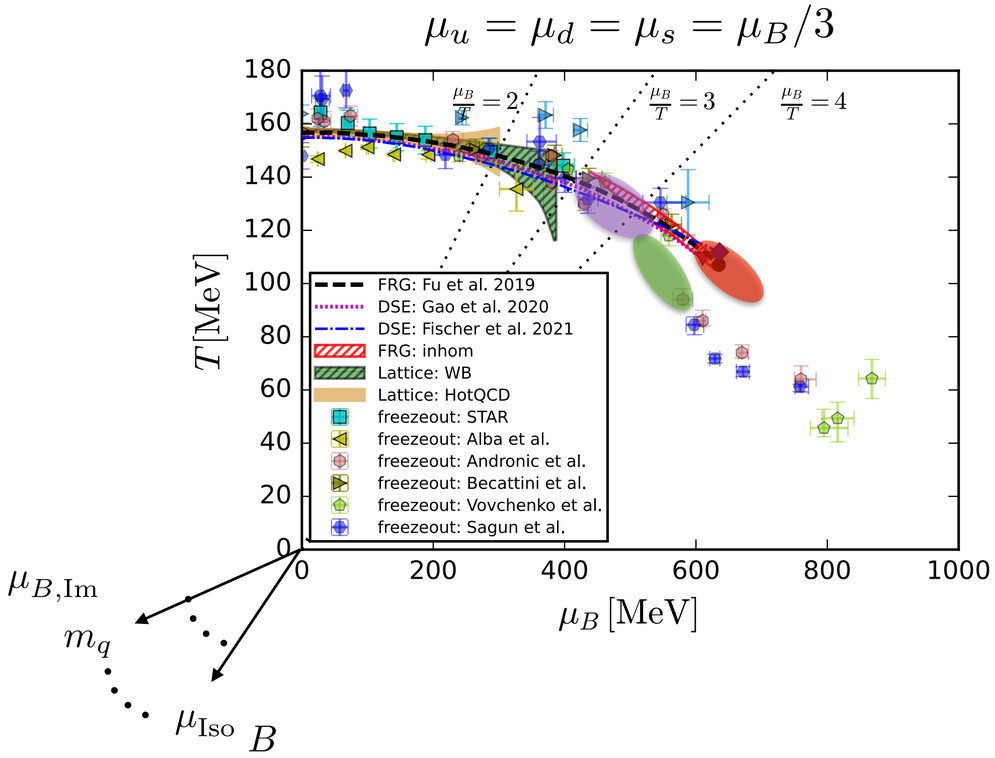
Galatyuk, A982 (2019) update 2025; CBM, EPJA 53 3 (2017) 60

Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

Phase structure of QCD and the CEP



Collect all possible information/structure for physics understanding & extrapolations

Functional QCD: CEP estimate

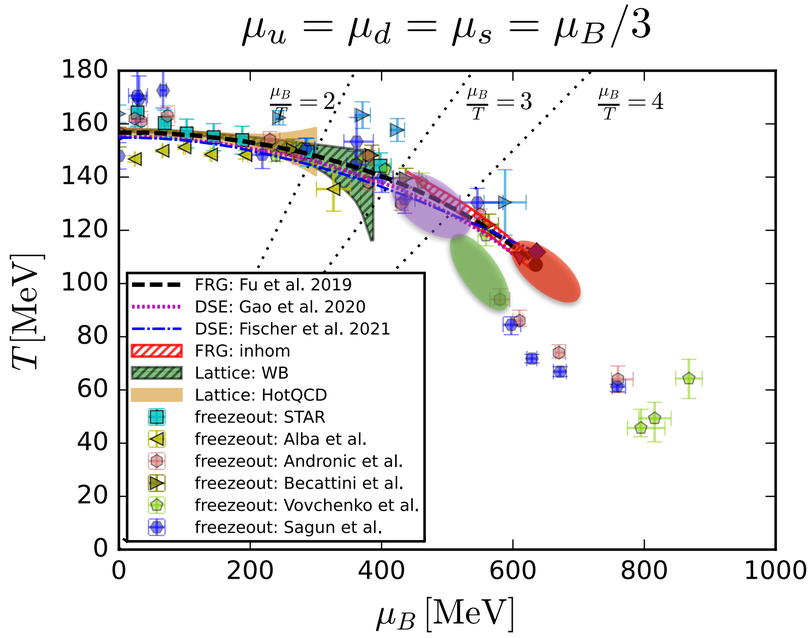
fRG: **Fu, JMP, Rennecke, PRD 101 (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**

Gunkel, Fischer, PRD 104 (2021) 054022

$$(\mu_B\,,\,T)_{\mathrm{CEP}} \sim (600-650\,,\,105-115)\ \mathrm{MeV}$$

Phase structure of QCD and the CEP



Functional QCD: CEP estimate

fRG: **Fu, JMP, Rennecke, PRD 101 (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**

Gunkel, Fischer, PRD 104 (2021) 054022

$$(\mu_B\,,\,T)_{\mathrm{CEP}} \sim (600-650\,,\,105-115)\ \mathrm{MeV}$$

Estimates & predictions

Requires computations in 1st principles QCD at

$$(\mu_B\,,\,T) \;\sim\; (\mu_B\,,\,T)_{\mathrm{CEP}}$$

Extrapolations for Pheno

Requires a discussion of the explicit & implicit assumptions

Lattice extrapolations

low energy effective theories: QM, NJL, PQM, PNJL, …, Holography

Functional approaches are the only first principles approaches *to date*

that allow for *direct* computations at finite density with

$$\frac{\mu_B}{T} \gtrsim 3$$

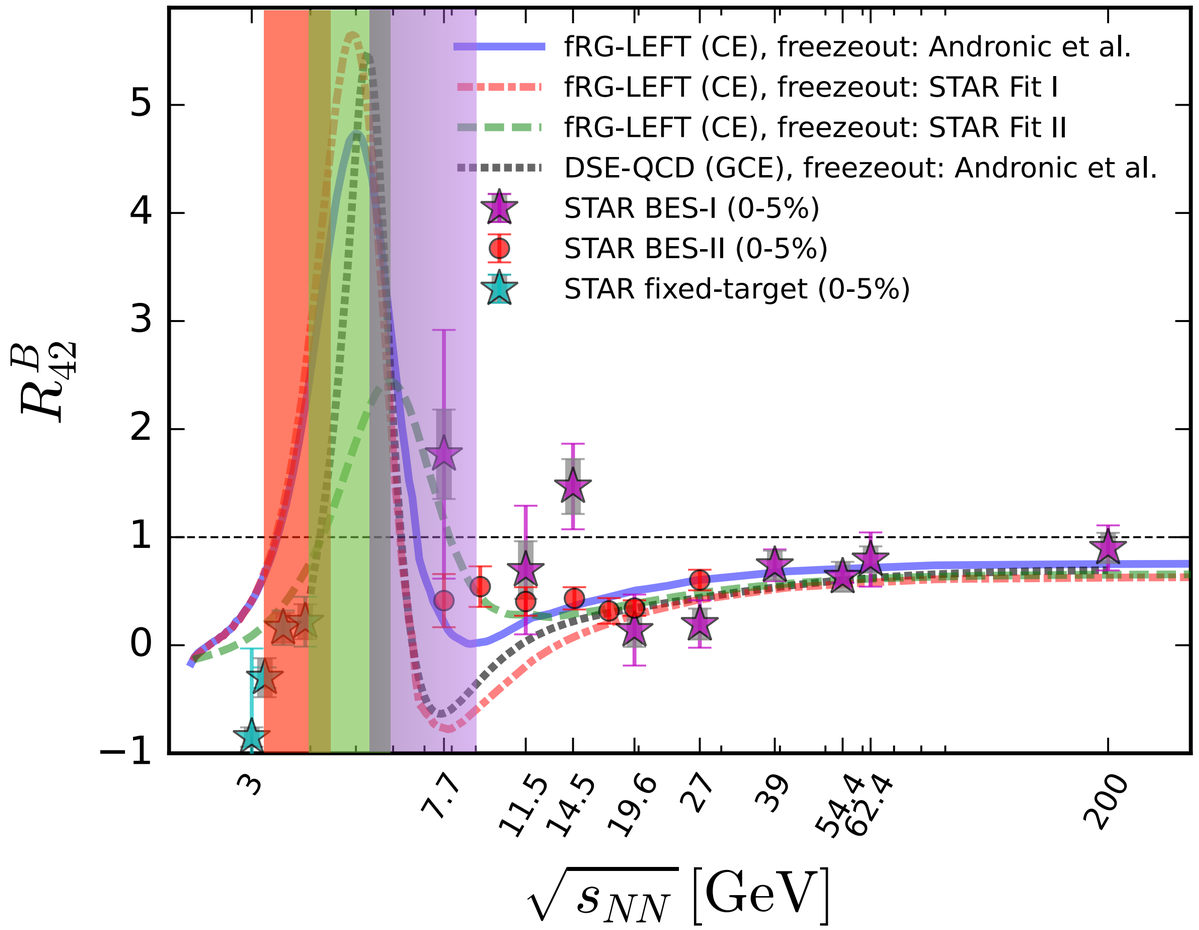
Supplementary Material:

Phase structure from functional QCD: how to

Phase structure from functional QCD: Predictions & estimates

Ripples of the CEP

net-baryon fluctuations in QCD vs net-proton fluctuations at STAR



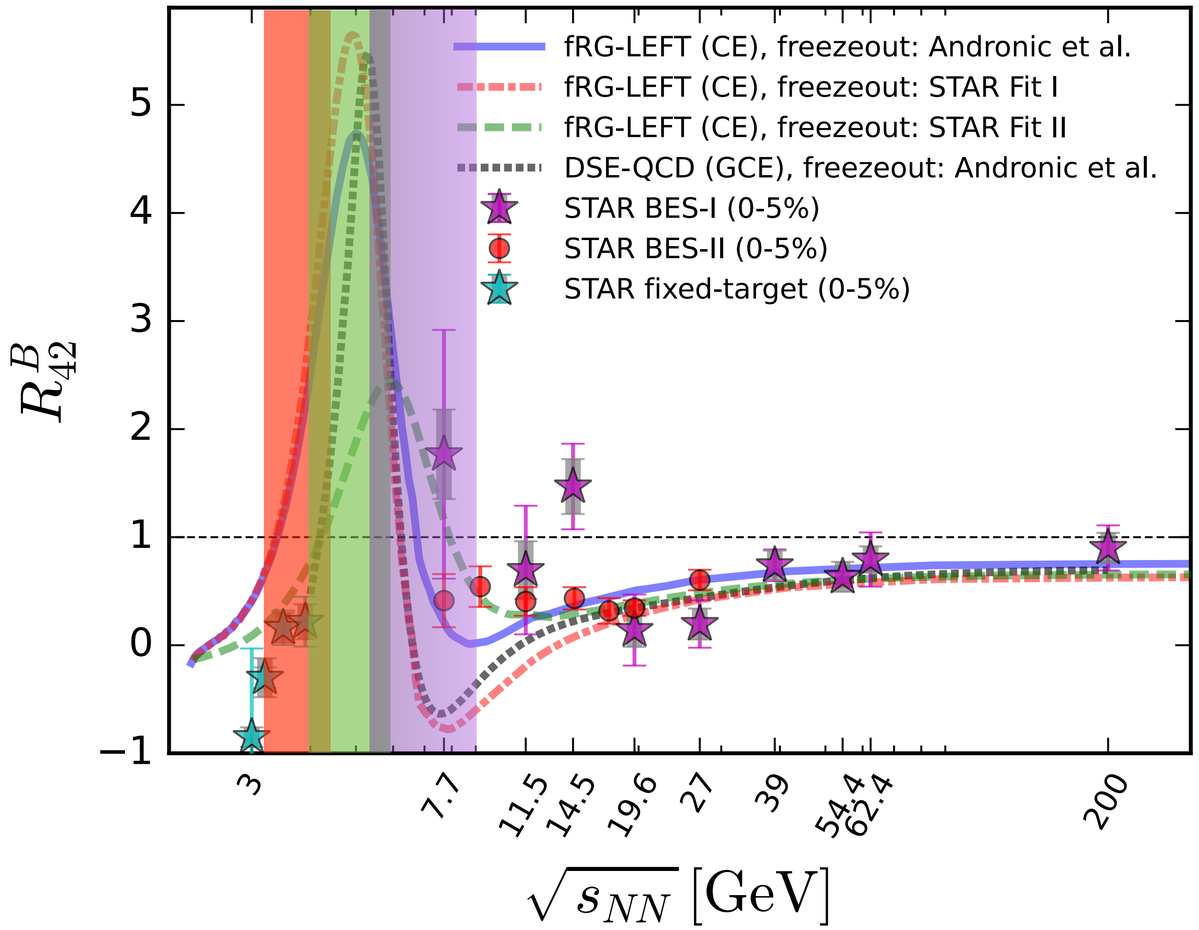
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Lu, Gao, Liu, JMP, arXiv:2504.05099

Ripples of the CEP

net-baryon fluctuations in QCD vs net-proton fluctuations at STAR



Results:

1st principles functional QCD computations

& low energy effective theories/extrapolations

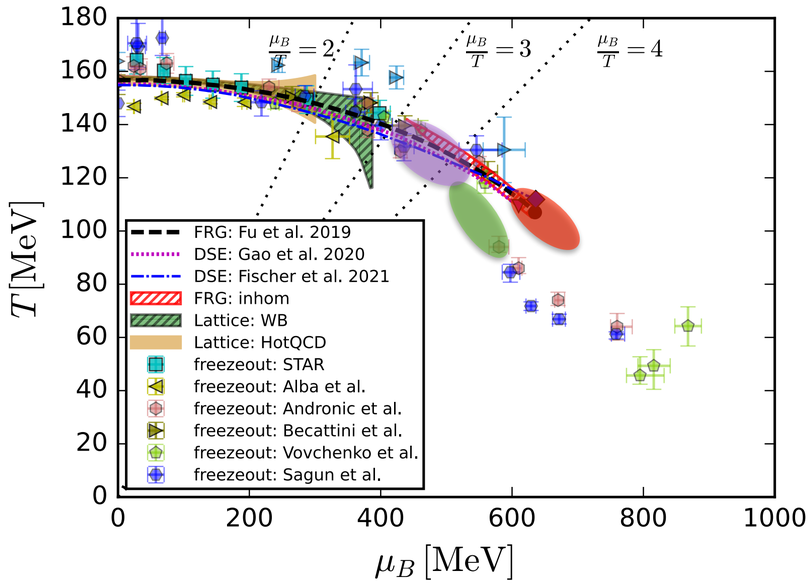
explicit & implicit assumptions

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

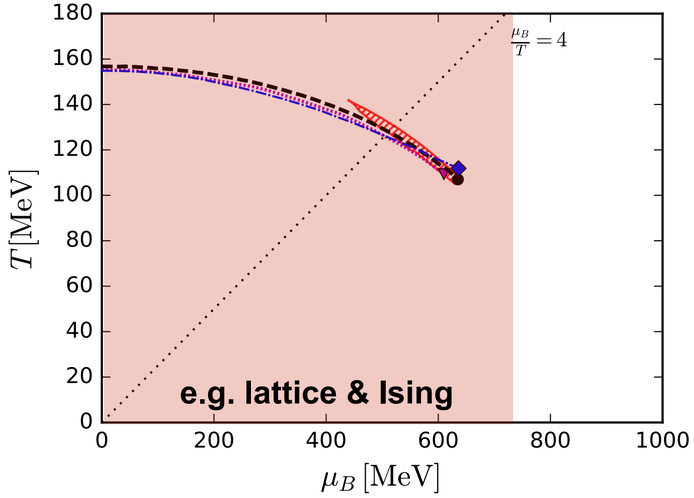
Lu, Gao, Liu, JMP, arXiv:2504.05099

Experimental signatures of the critical end point



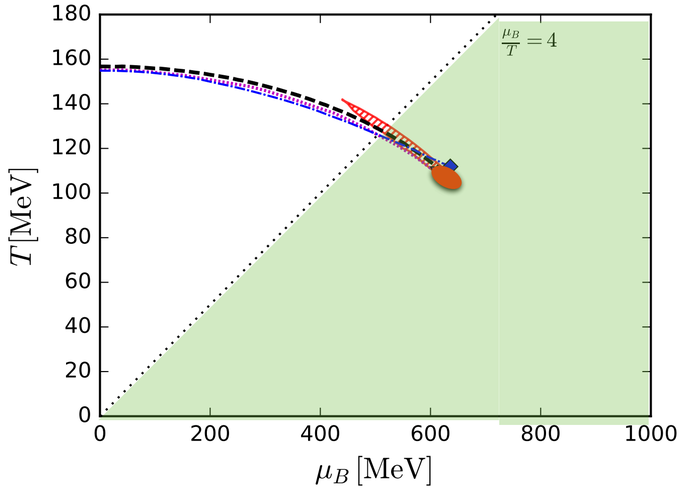
Scenario I

Scaling



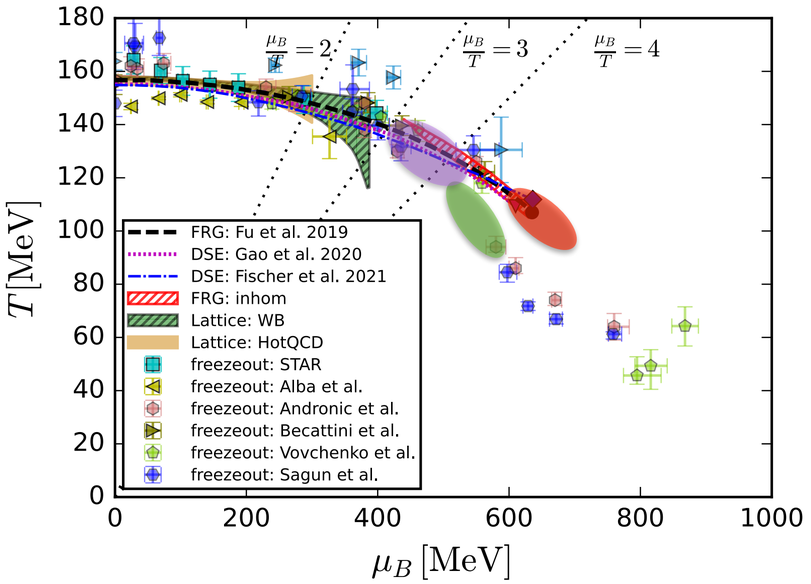
Scenario II

Minimal scaling and new phases

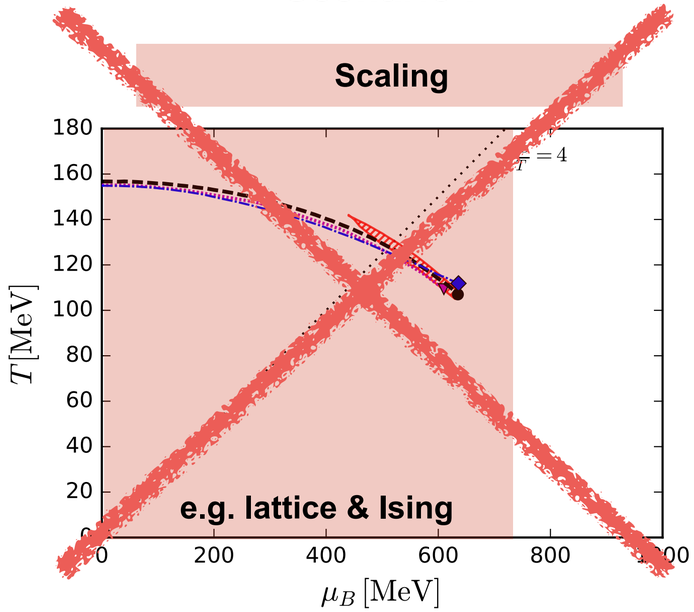


Moat, …

Experimental signatures of the critical end point

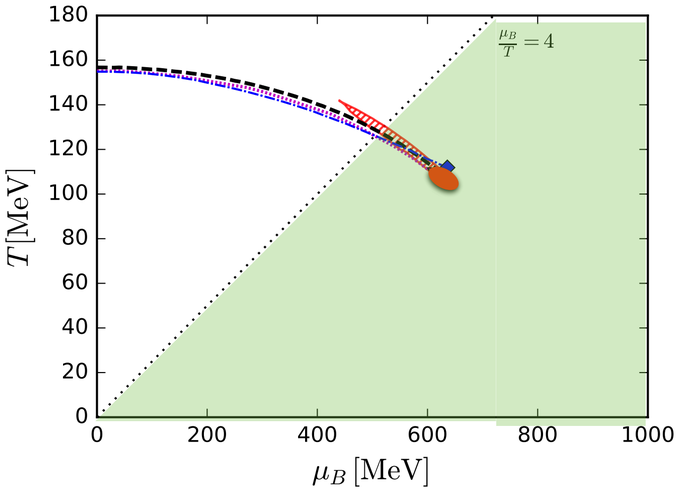


Scenario I



Scenario II

Minimal scaling and new phases



Moat, …

Supplementary Material:

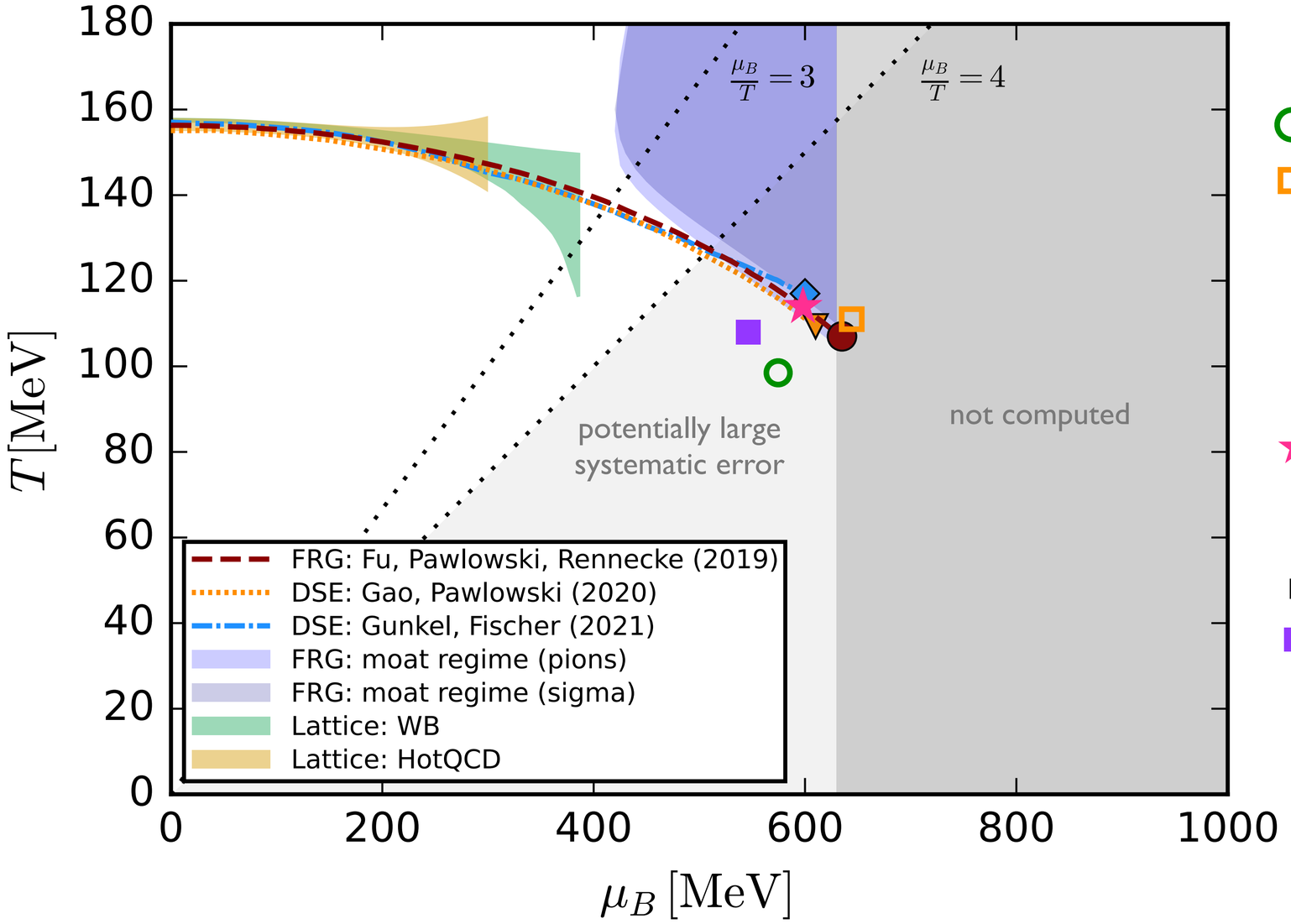
Predictions, estimates & extrapolations and how to use them

Chiral dynamics & soft modes

QCD PHASE DIAGRAM & THE CEP

Modified from Fabian Rennecke's plenary talk @ QM 2025

FRG & DSE results corroborated by subsequent "extrapolations" of lattice data



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

using Yang-Lee edge singularities:

- conformal Padé [Basar, 2312.06952]
- multi-point Padé [Clarke et al., 2405.10196]
  $N_\tau = 6, 8$ results + continuum estimate
  [Schmidt, 2504.00629]

using thermodynamics:

- constant entropy density [Shah et al. 2410.16206]

model-based extrapolation (tiny selection):

- holography [Hippert et al., 2309.00579]
  (agrees with [Cai et al., 2201.02004])

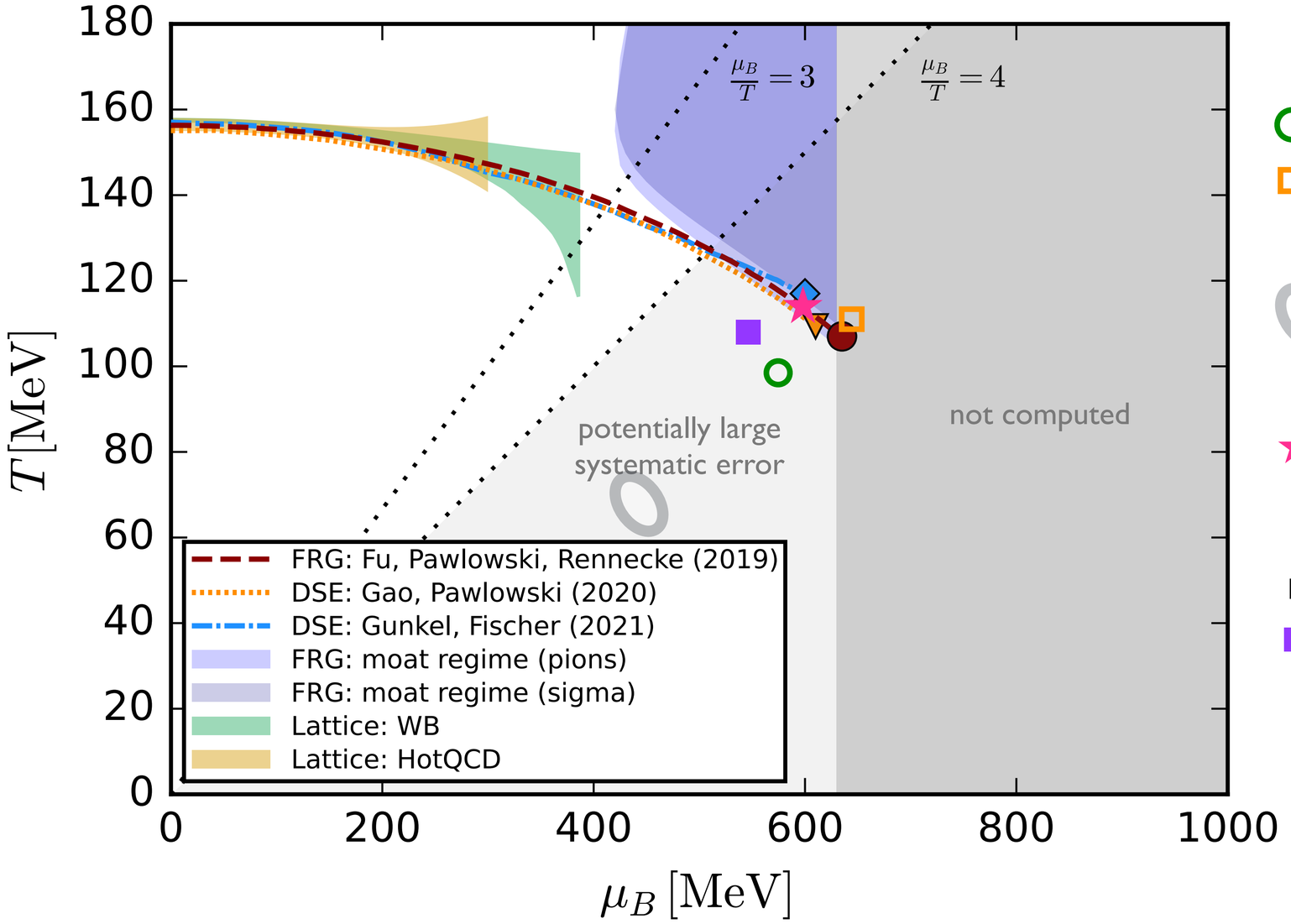
CEP location well constrained by now. And it's **in FAIR range!**

$\sqrt{s_{NN}} \approx 3.6 - 4.1\,\mathrm{GeV}$

QCD PHASE DIAGRAM & THE CEP

Modified from Fabian Rennecke's plenary talk @ QM 2025

FRG & DSE results corroborated by subsequent "extrapolations" of lattice data



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

using Yang-Lee edge singularities:

- conformal Padé [Basar, 2312.06952]
- multi-point Padé [Clarke et al., 2405.10196]
  $N_\tau = 6, 8$ results + continuum estimate
  [Schmidt, 2504.00629]
- **very recent lattice extrapolation (16^3 x 8)**
  [Adam et al, 2507.13254]

using thermodynamics:

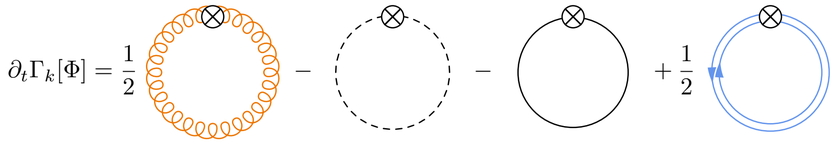
- constant entropy density [Shah et al. 2410.16206]

model-based extrapolation (tiny selection):

- holography [Hippert et al., 2309.00579]
  (agrees with [Cai et al., 2201.02004])

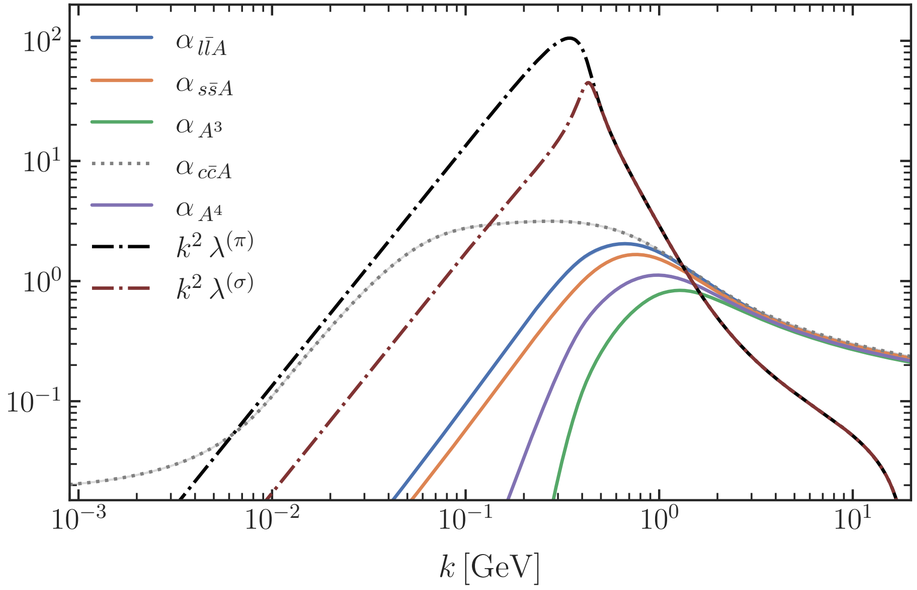
CEP location well constrained by now. And it's **in FAIR range!**

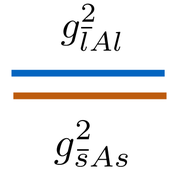
$\sqrt{s_{NN}} \approx 3.6 - 4.1\,\mathrm{GeV}$

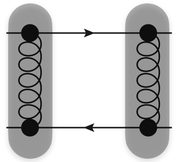


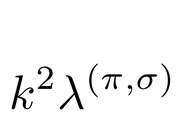
fQCD

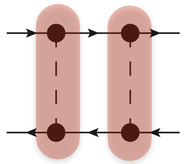
Sequential decoupling of gluon, quark, sigma, pion fluctuations





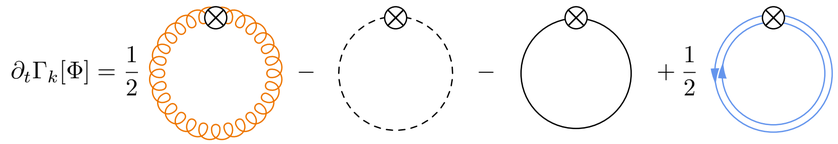






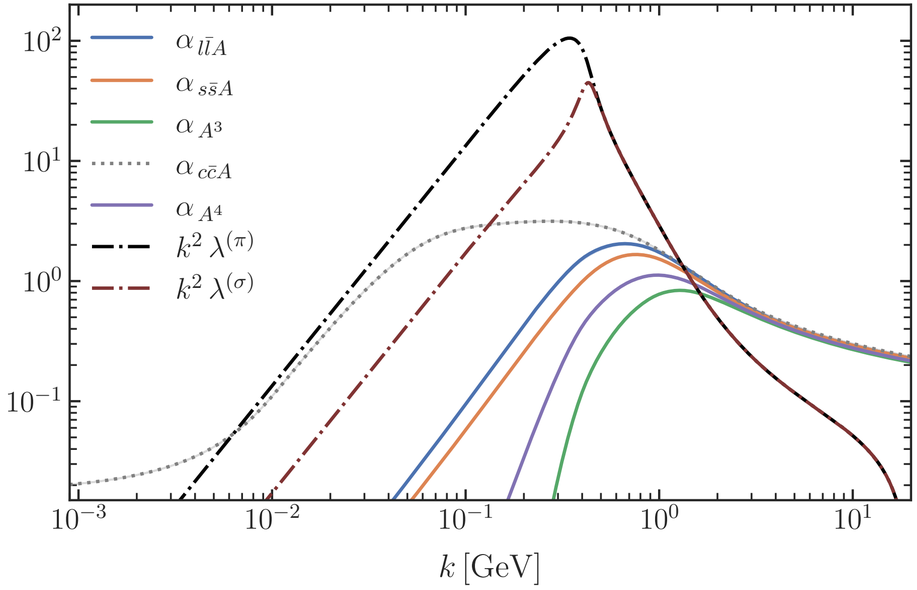
Vacuum: **Ihssen, JMP, Sattler, Wink, arXiv:2408.08413**

Phase structure: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**

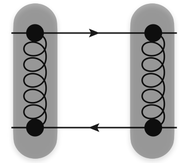


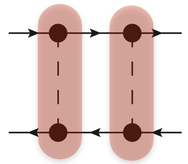
fQCD

Sequential decoupling of gluon, quark, sigma, pion fluctuations



$k^2\lambda^{(\pi,\sigma)}$

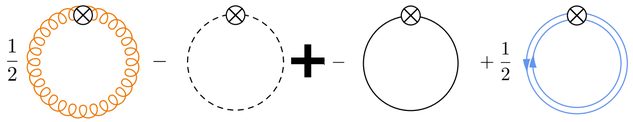




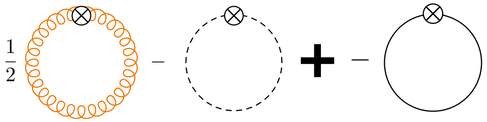
Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

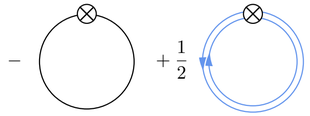
PQM-model



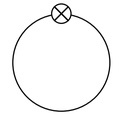
PNJL-model



QM-model



NJL-model

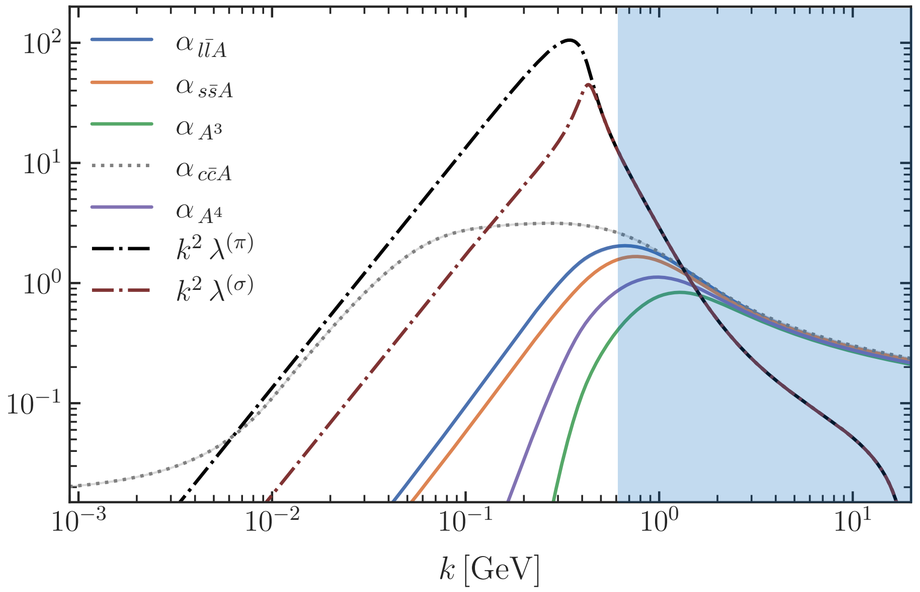


QCD-assisted low energy effective theories

fQCD

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} - - - + \frac{1}{2}$$

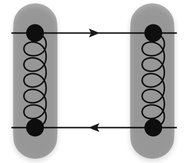
Sequential decoupling of gluon, quark, sigma, pion fluctuations

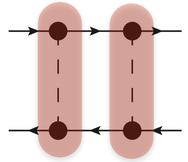


$g^2_{\bar{l}Al}$

$g^2_{\bar{s}As}$

$k^2\lambda^{(\pi,\sigma)}$





Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

PQM-model

$$\frac{1}{2} - + - + \frac{1}{2}$$

PNJL-model

$$\frac{1}{2} - + -$$

QM-model

$$- + \frac{1}{2}$$

NJL-model

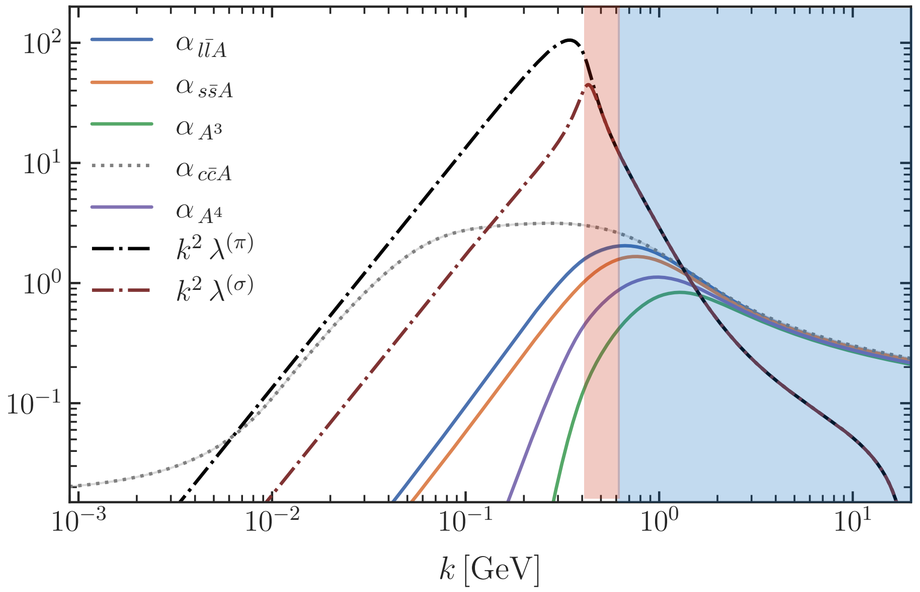
$$-$$

QCD-assisted low energy effective theories

fQCD

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} - - + \frac{1}{2}$$

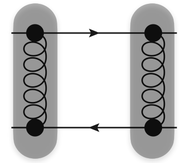
Sequential decoupling of gluon, quark, sigma, pion fluctuations

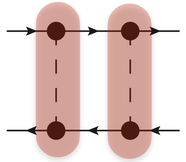


$g^2_{\bar{l}Al}$

$g^2_{\bar{s}As}$

$k^2\lambda^{(\pi,\sigma)}$





Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

PQM-model

$$\frac{1}{2} - + - + \frac{1}{2}$$

PNJL-model

$$\frac{1}{2} - + -$$

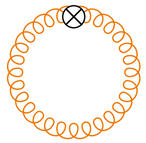
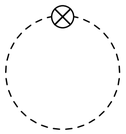
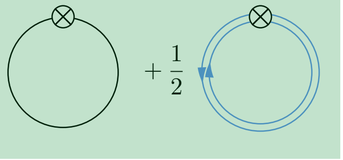
QM-model

$$- + \frac{1}{2}$$

NJL-model

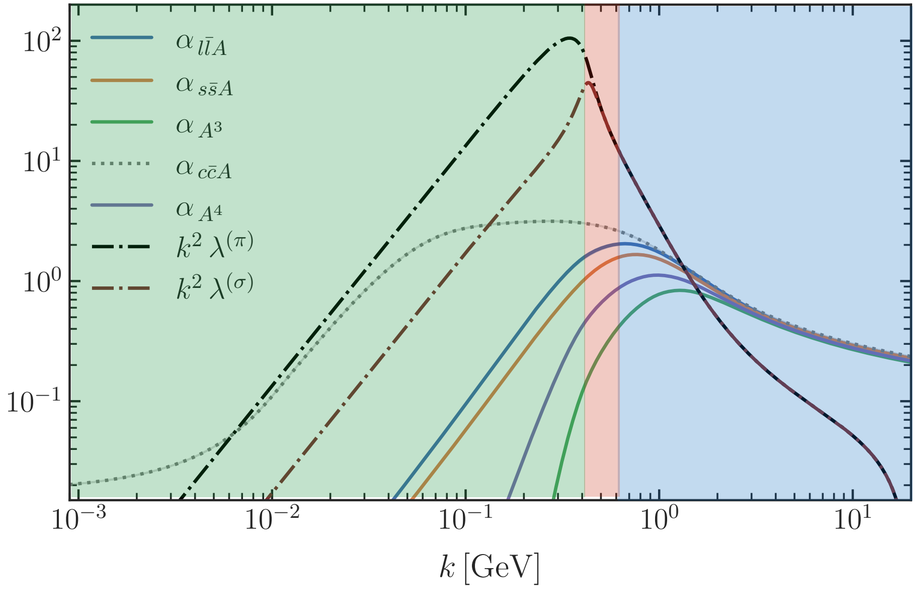
$$-$$

QCD-assisted low energy effective theories

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$$  $$-$$  $$-$$  $$+\frac{1}{2}$$

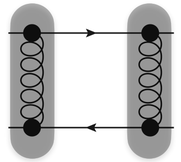
fQCD

Sequential decoupling of gluon, quark, sigma, pion fluctuations

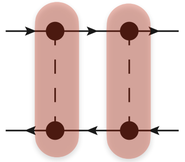


$g^2_{\bar{l}Al}$

$g^2_{\bar{s}As}$



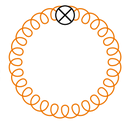
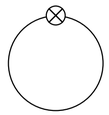
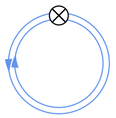
$k^2\lambda^{(\pi,\sigma)}$



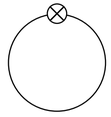
Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

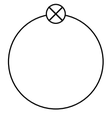
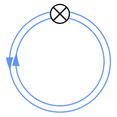
PQM-model

$\frac{1}{2}$  $-$  $+$ $-$  $+\frac{1}{2}$ 

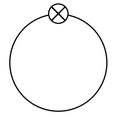
PNJL-model

$\frac{1}{2}$  $-$  $+$ $-$ 

QM-model

$-$  $+\frac{1}{2}$ 

NJL-model

$-$ 

QCD-assisted low energy effective theories

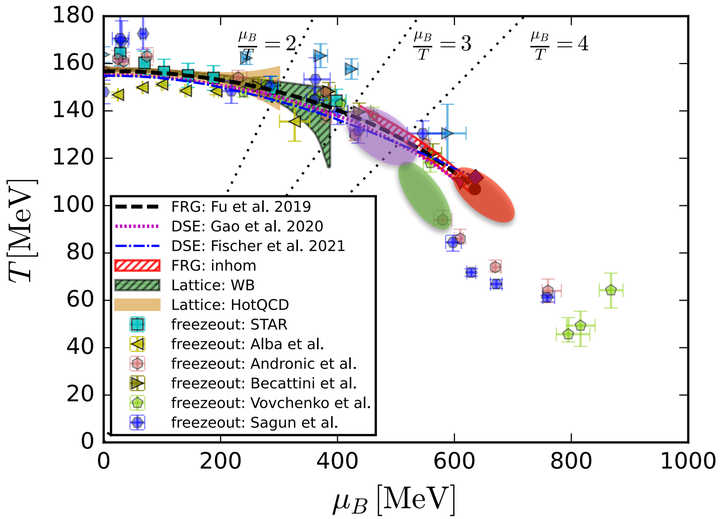
QCD-assisted low energy effective theory

Direct functional QCD input

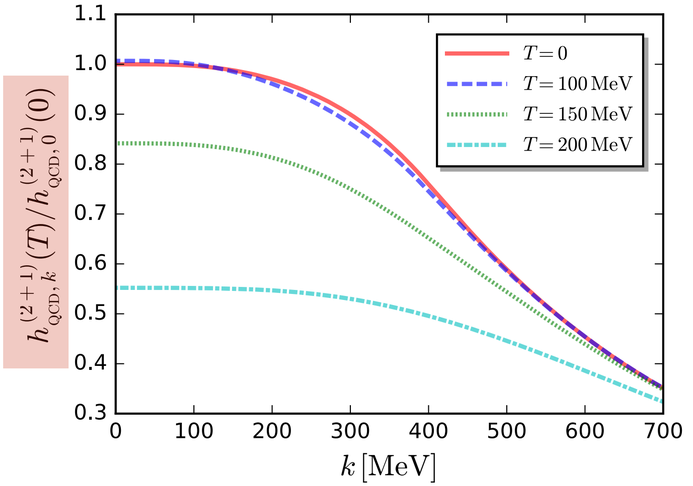
Fu, JMP, Rennecke, PRD 101 (2020) 054032

+

chiral phase structure



quark-antiquark — meson scattering



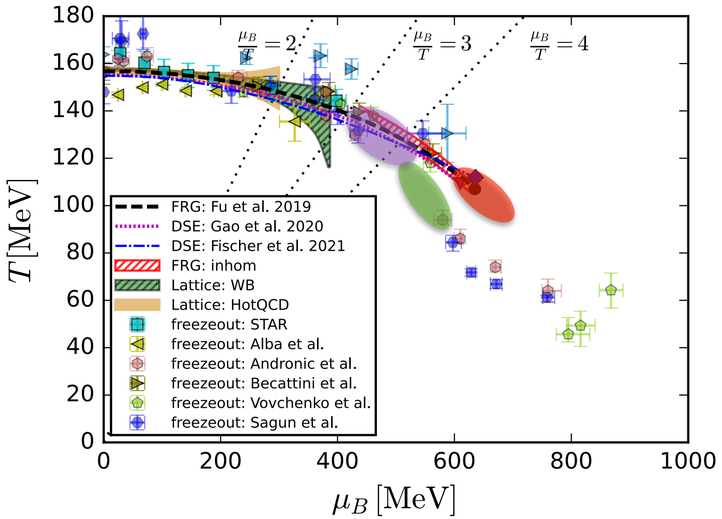
Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

QCD-assisted low energy effective theory

Direct functional QCD input

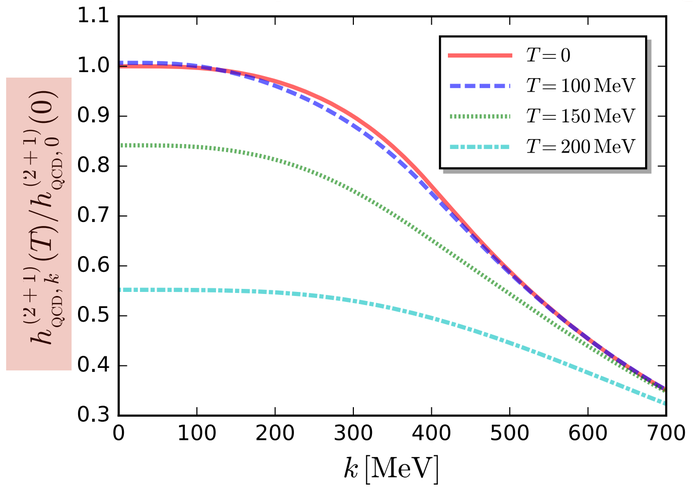
Fu, JMP, Rennecke, PRD 101 (2020) 054032

chiral phase structure



+

quark-antiquark — meson scattering



low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

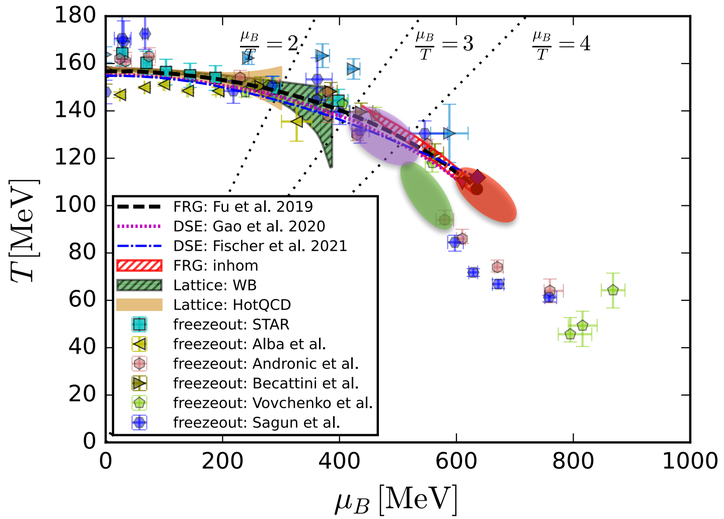
Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

QCD-assisted low energy effective theory

Direct functional QCD input

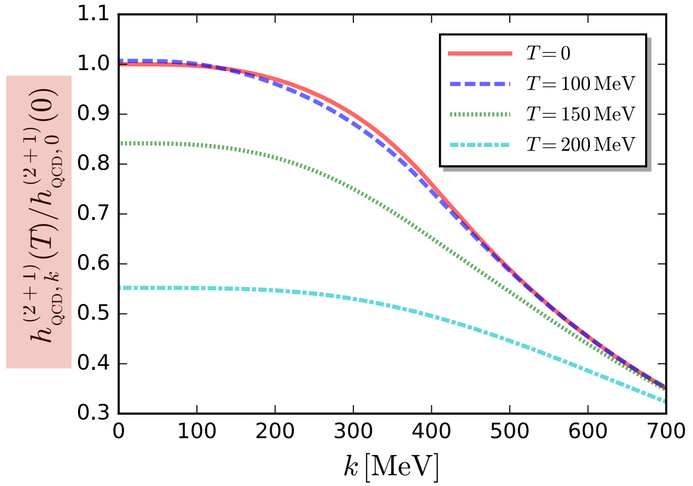
Fu, JMP, Rennecke, PRD 101 (2020) 054032

chiral phase structure



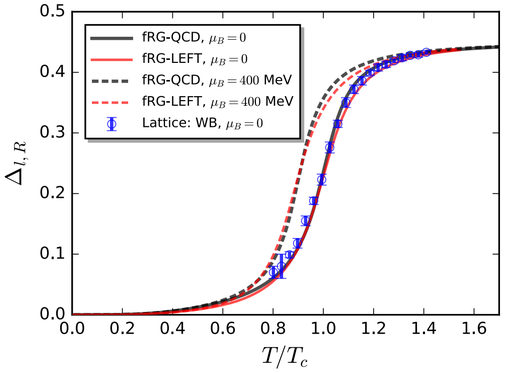
+

quark-antiquark — meson scattering



low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

renormalised chiral condensate

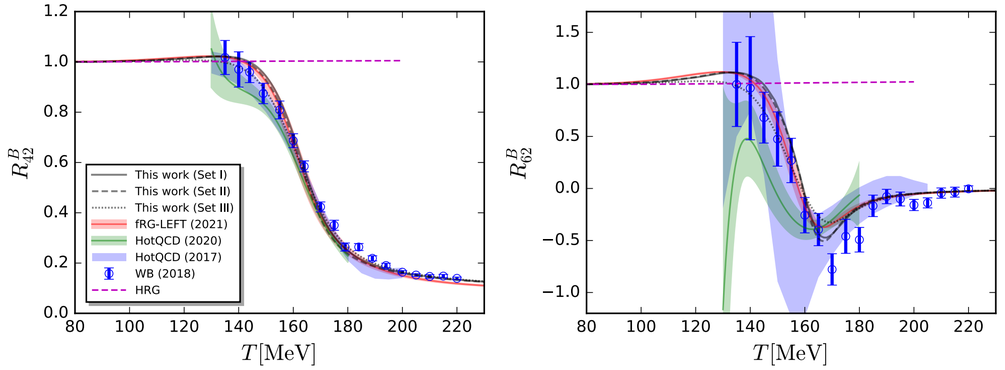


Benchmarks with lattice and fQCD

at

vanishing density and fQCD at finite density

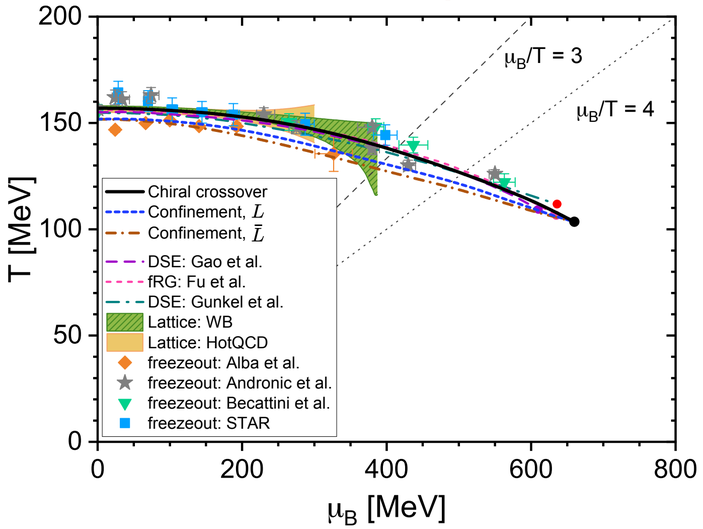
baryon number fluctuations



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Functional QCD

chiral & ‘confinement’ phase structure



L: ‘cousin’ of the Polyakov loop

Braun, Gies, JMP, PLB 684 (2010) 262
Fister, JMP, PRD 88 (2013) 045010
Herbst, Lücker, JMP, arXiv:1510.03830

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} - - - \frac{1}{6} +$$

DSE

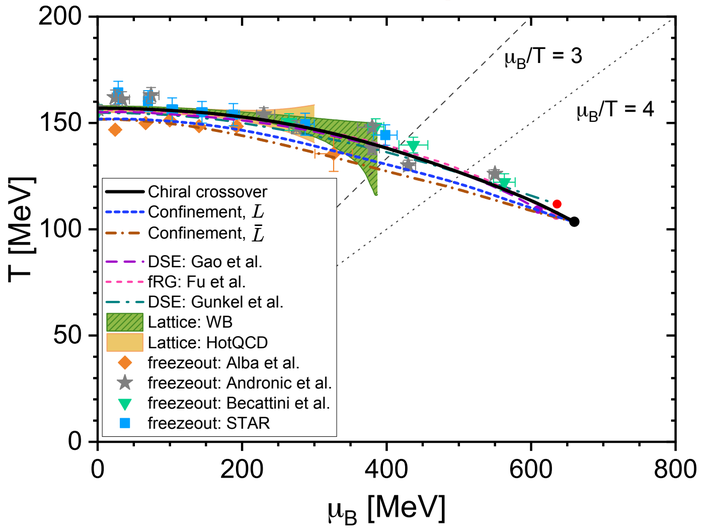
Explorative DSE studies EoS & fluctuations of the net-baryon charge

Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011
Isserstedt, Fischer, Steinert, PRD 103 (2021) 054012

Lu, Gao, Liu, JMP, arXiv: 2504.05099

Functional QCD

chiral & ‘confinement’ phase structure



On-shell

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \left(\frac{1}{2} \; - \; - \; - \frac{1}{6} \; + \; \right) \Bigg|_{L,\bar{L}} = 0$$

DSE

L: ‘cousin’ of the Polyakov loop

Braun, Gies, JMP, PLB 684 (2010) 262
Fister, JMP, PRD 88 (2013) 045010
Herbst, Lücker, JMP, arXiv:1510.03830

Explorative DSE studies EoS & fluctuations of the net-baryon charge

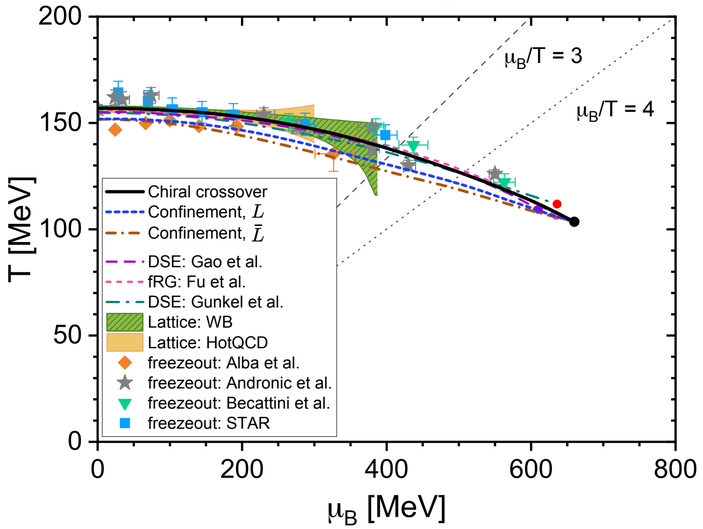
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011
Isserstedt, Fischer, Steinert, PRD 103 (2021) 054012

Off-shell $L = \bar{L} = 1$

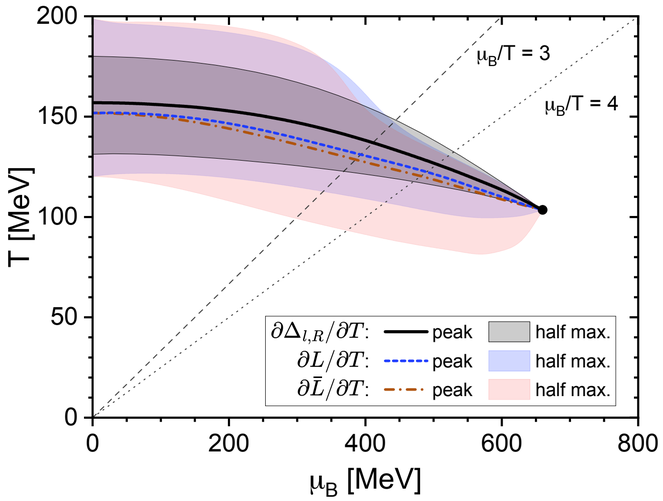
Lu, Gao, Liu, JMP, arXiv: 2504.05099

Functional QCD

chiral & ‘confinement’ phase structure

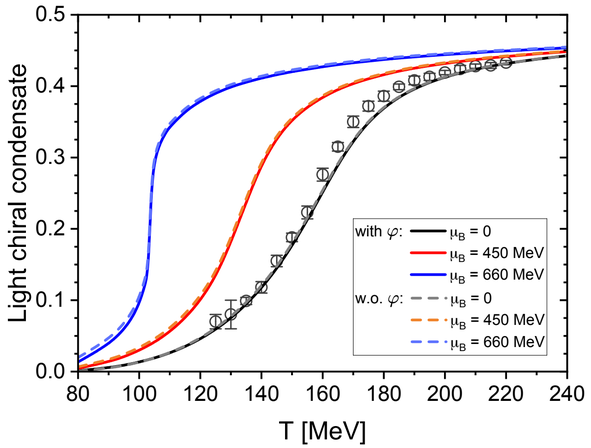


Crossovers with width



DSE

renormalised chiral condensate

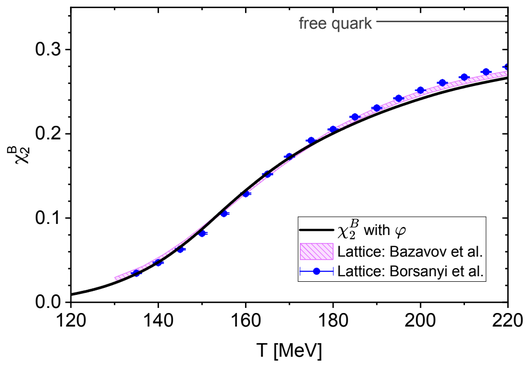


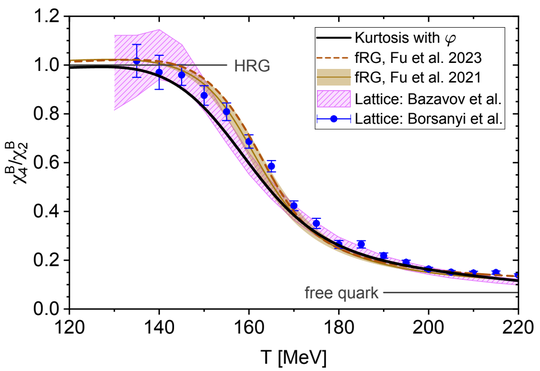
Benchmarks with lattice and fQCD

at

vanishing density and fQCD at finite density

baryon number fluctuations





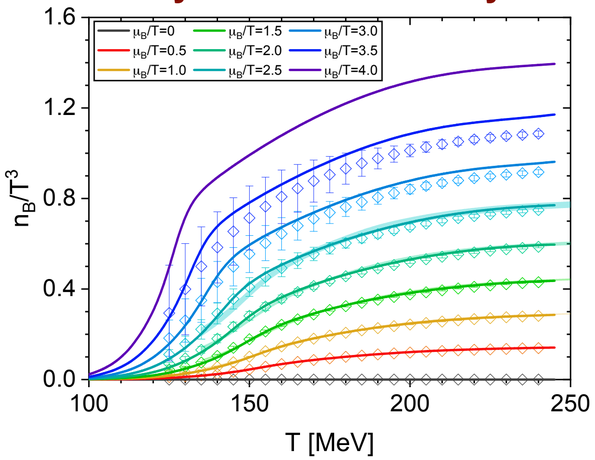
Lu, Gao, Liu, JMP, arXiv: 2504.05099

Functional QCD

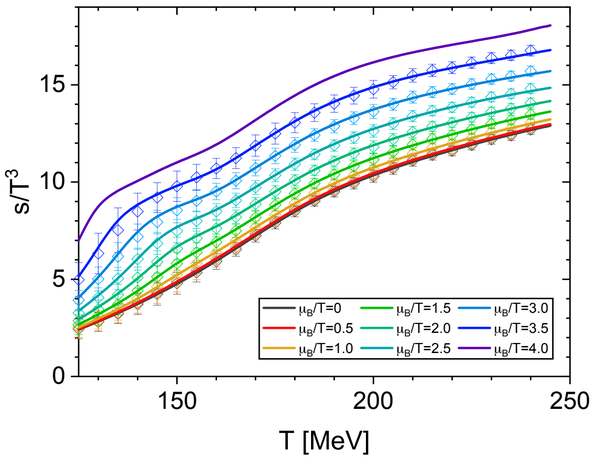
Observables

Thermodynamics

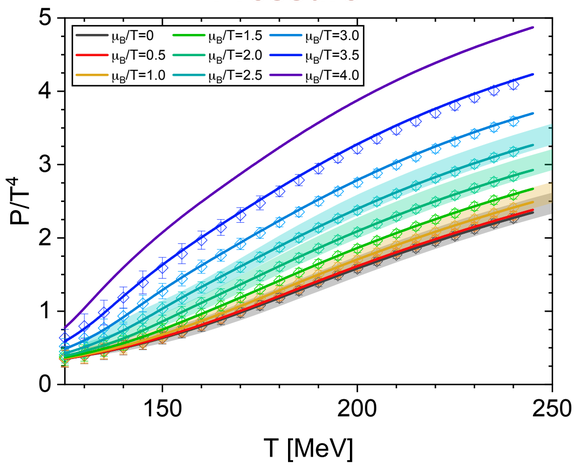
baryon number density



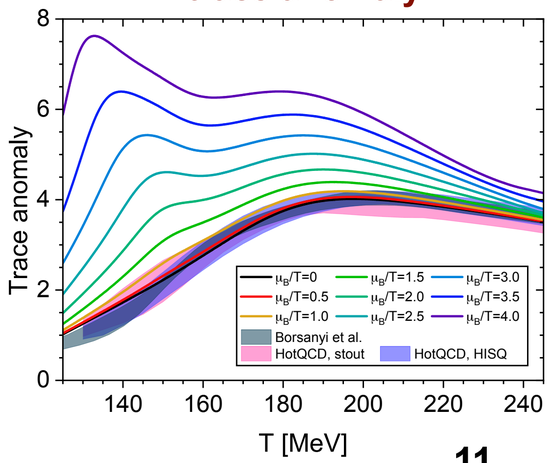
entropy



Pressure



trace anomaly



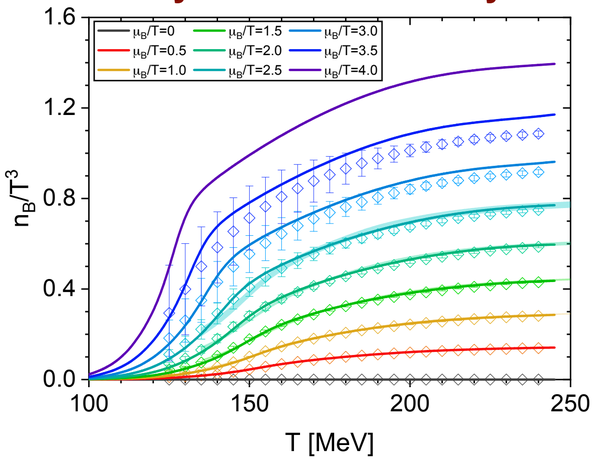
Lu, Gao, Liu, JMP, arXiv: 2504.05099

Functional QCD

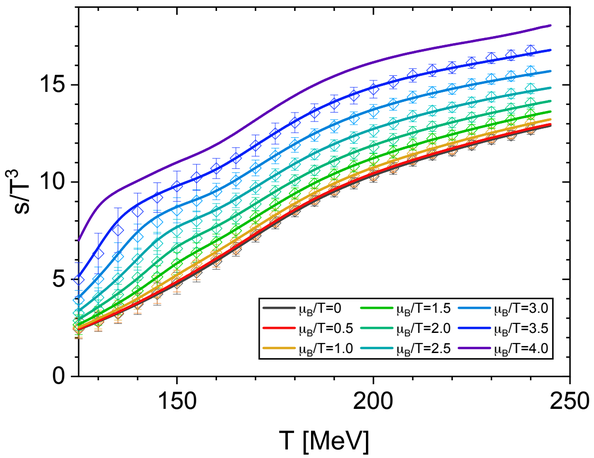
Observables

Thermodynamics

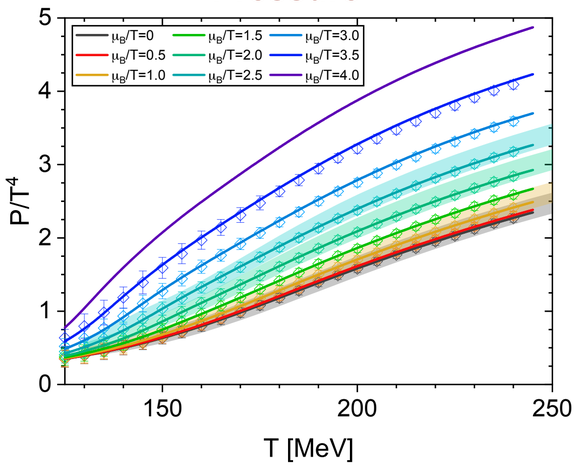
baryon number density



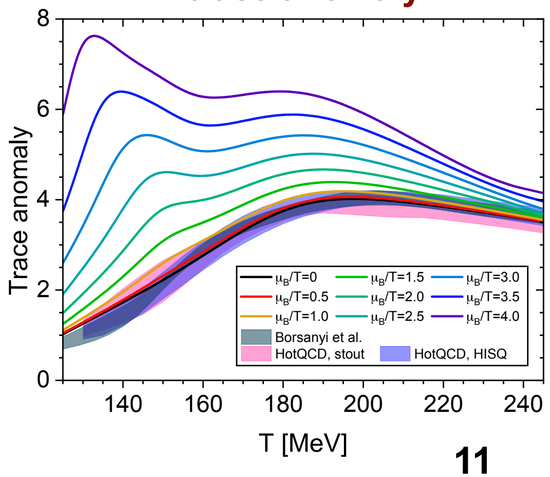
entropy



Pressure

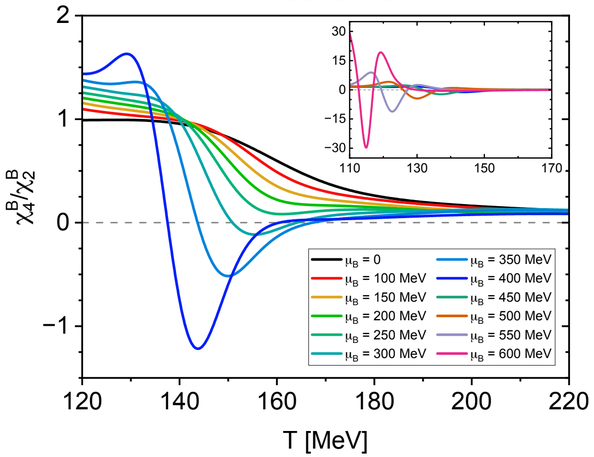


trace anomaly

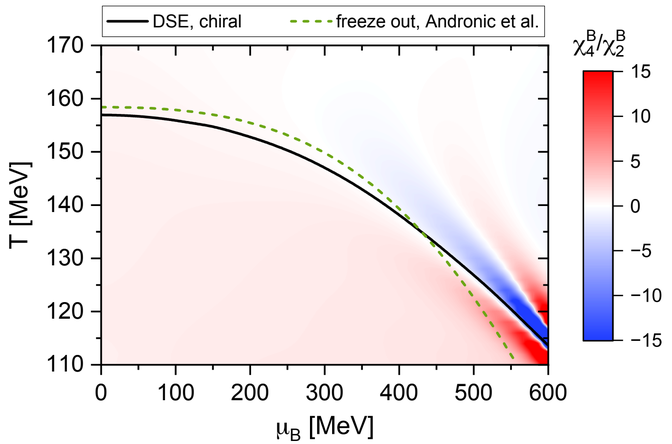


Fluctuations of net-baryon charge

kurtosis

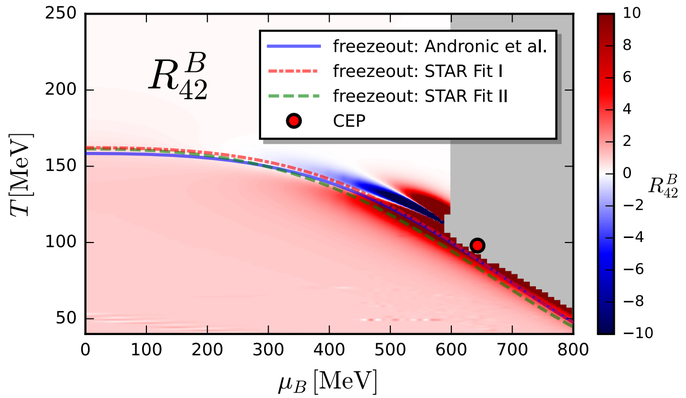


heat map kurtosis



Lu, Gao, Liu, JMP, arXiv: 2504.05099

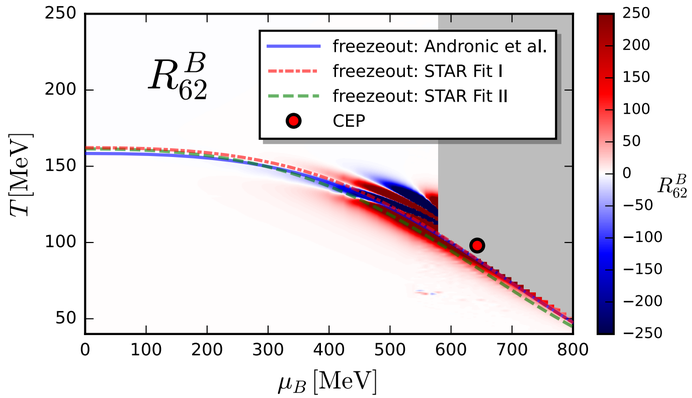
Ripples of the critical point



baryon number fluctuations in the phase structure

$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (98, 643)\,\mathrm{MeV}$$

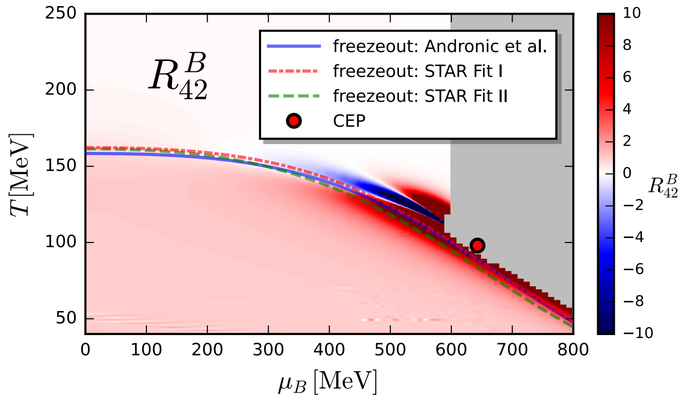
QCD-assisted LEFT



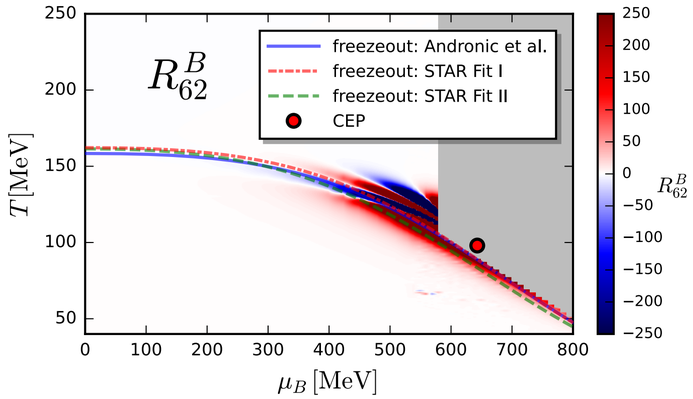
Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Ripples of the critical point

baryon number fluctuations in the phase structure

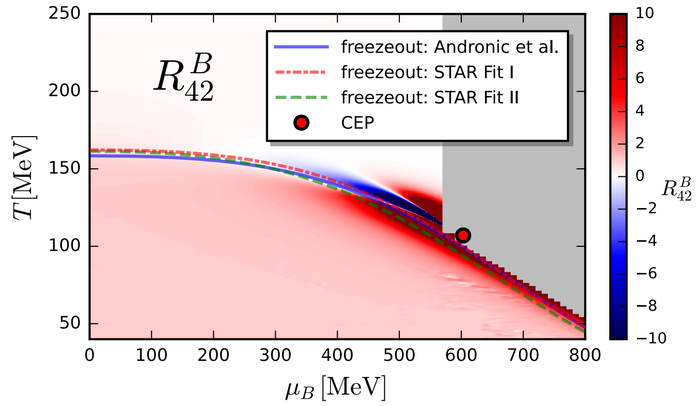


$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (98, 643)\,\mathrm{MeV}$$

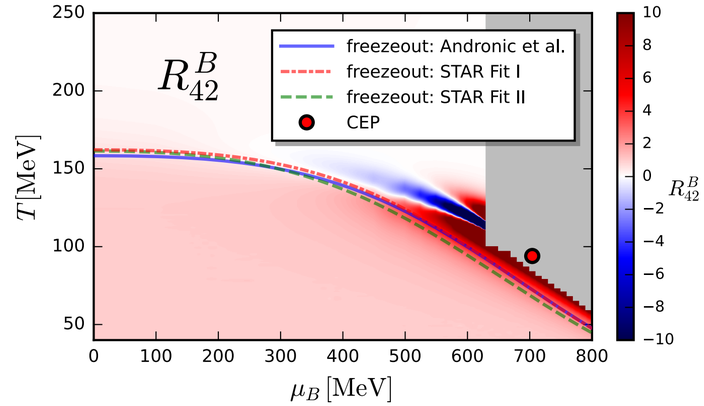


QCD-assisted LEFT

Variations of the CEP in the allowed estimate regime



$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (108, 604)\,\mathrm{MeV}$$

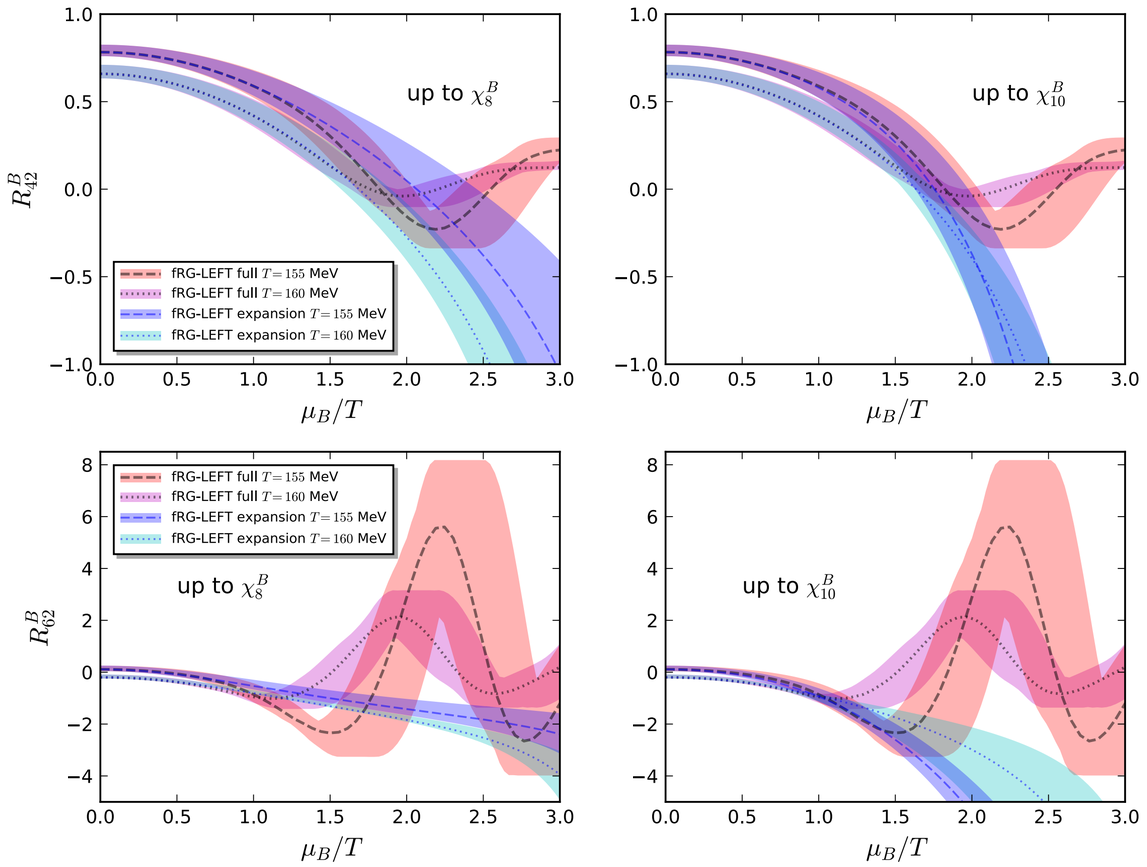


$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (94, 704)\,\mathrm{MeV}$$

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Fluctuations of conserved charges

Validity regime of extrapolations

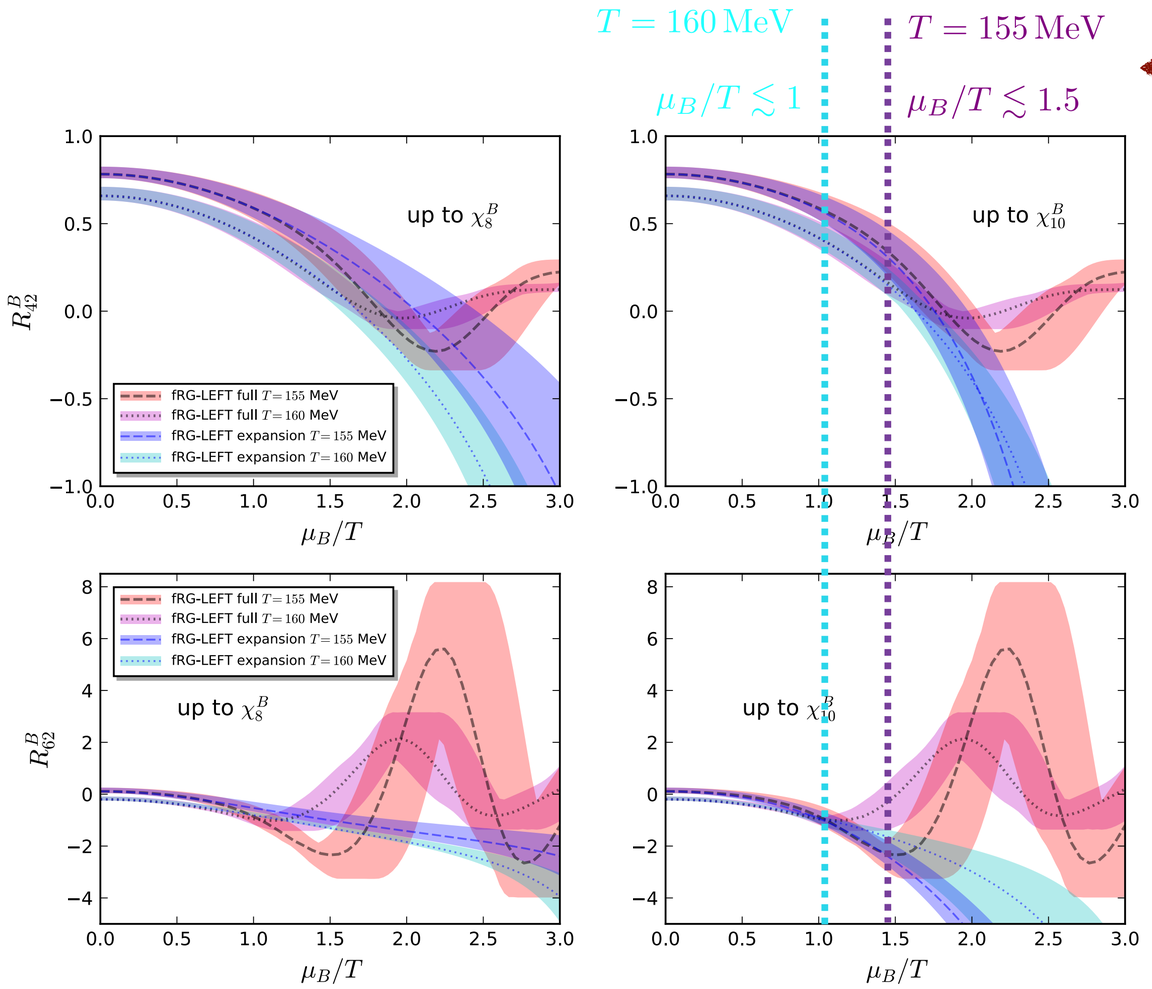


Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

Fluctuations of conserved charges

Validity regime of extrapolations



Validity of Taylor expansion

Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

Ripples of the critical point

Canonical corrections via subensemble acceptance

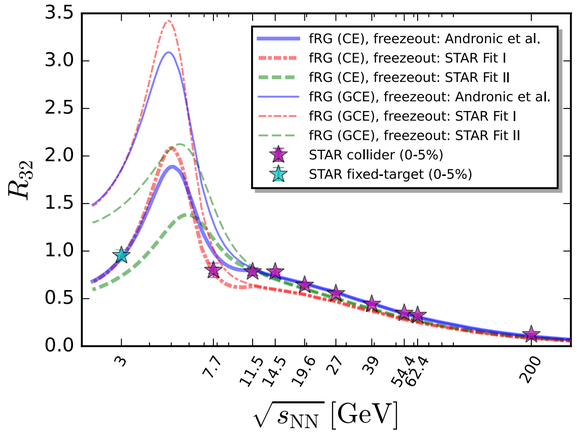
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume system volume

$$V_1 = \alpha V$$

$$\bar{R}^B_{32} = (1-2\alpha) R^B_{32}$$



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Ripples of the critical point

Canonical corrections via subensemble acceptance

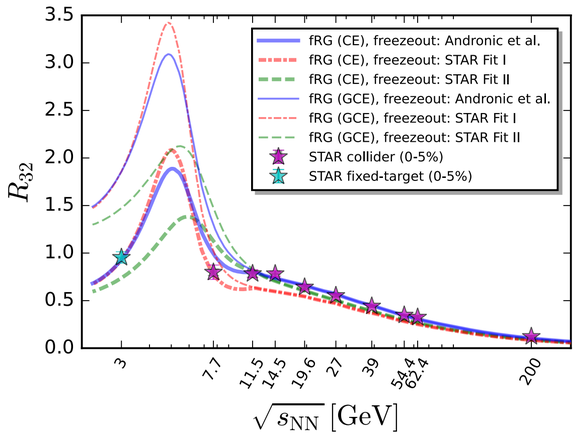
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume system volume

$$V_1 = \alpha V$$

$$\bar{R}^B_{32} = (1 - 2\alpha) R^B_{32}$$



qualitative adjustment

$$\alpha(\bar{s}) = a\left(1 - \sqrt{\bar{s}}\right)\theta\left(1 - \bar{s}\right)$$

$$a = 0.33 \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{\rm NN}}}{11.9\,\text{GeV}}$$

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Ripples of the critical point

Canonical corrections via subensemble acceptance

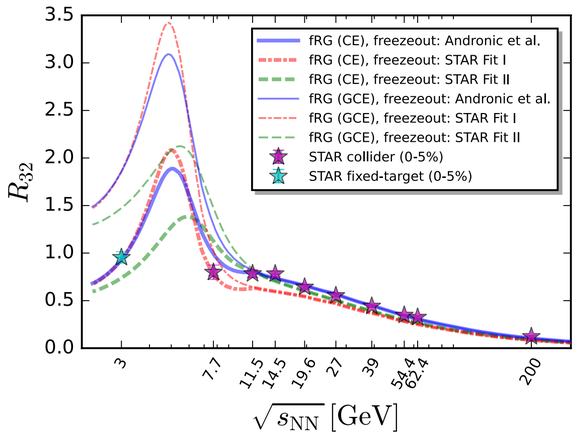
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume system volume

$$V_1 = \alpha\, V$$

$$\bar{R}^B_{32} = (1-2\alpha) R^B_{32}$$

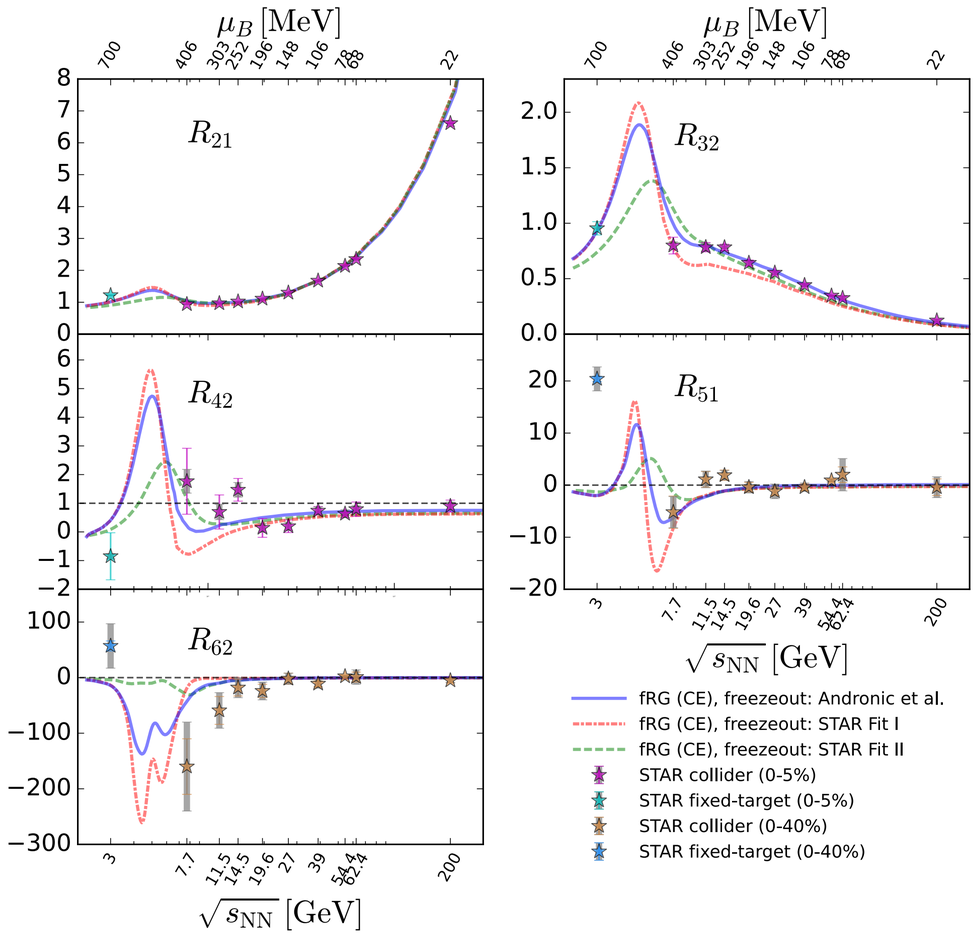


qualitative adjustment

$$\alpha(\bar{s}) = a\left(1-\sqrt{\bar{s}}\right)\theta\left(1-\bar{s}\right)$$

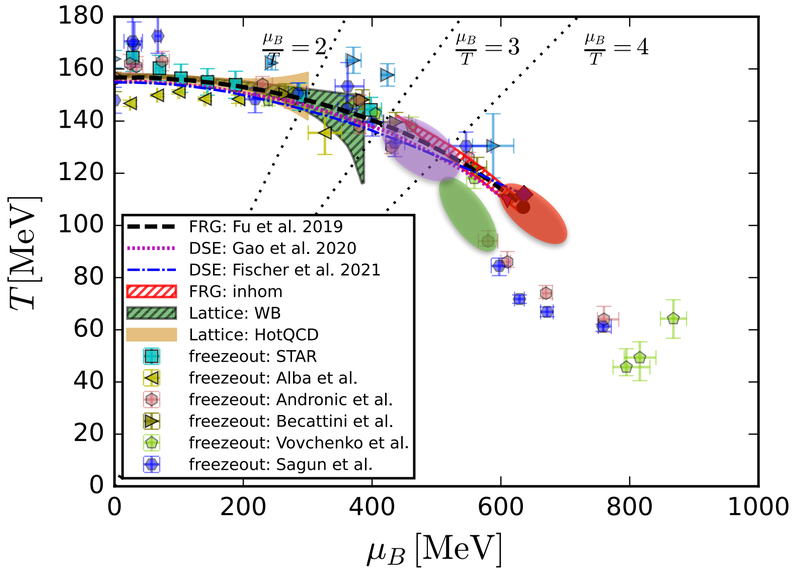
$$a = 0.33 \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{\rm NN}}}{11.9\,\text{GeV}}$$

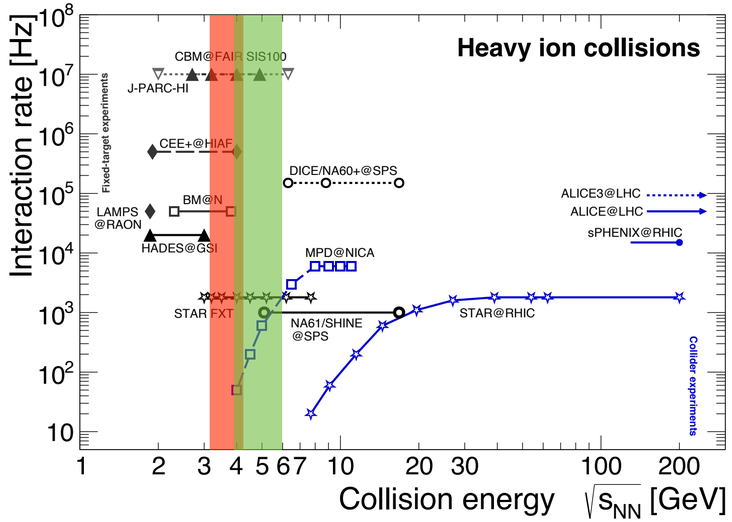
baryon & proton number fluctuations



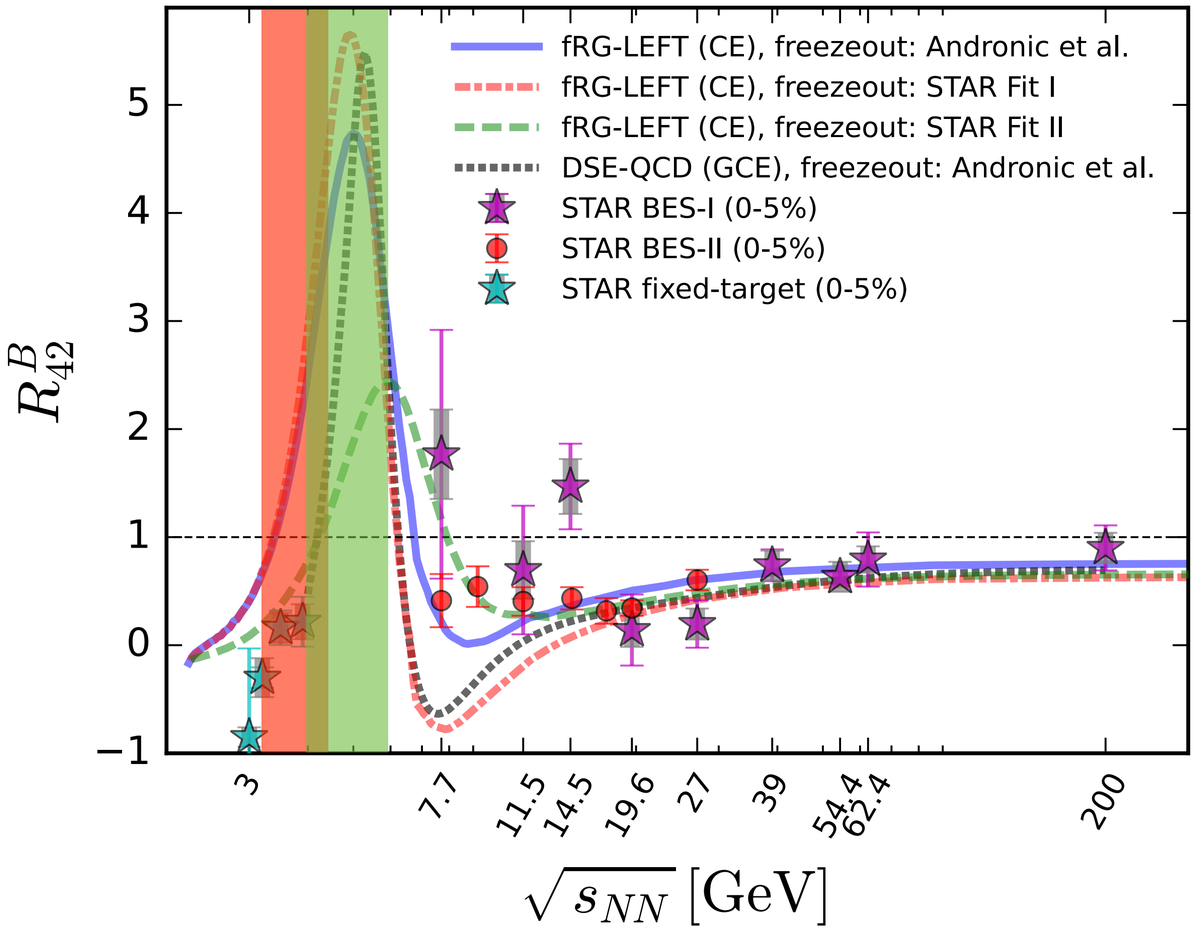
Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Ripples of the critical point





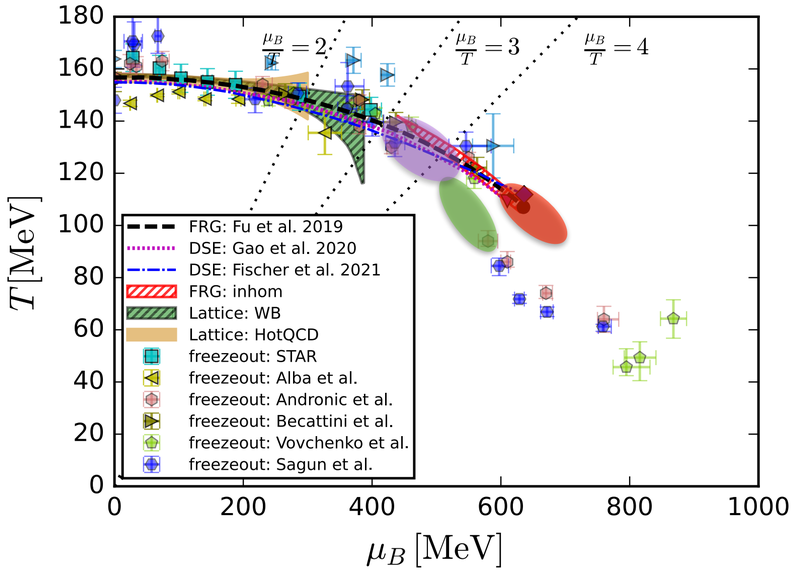
net-baryon fluctuations in QCD vs net-proton fluctuations at STAR

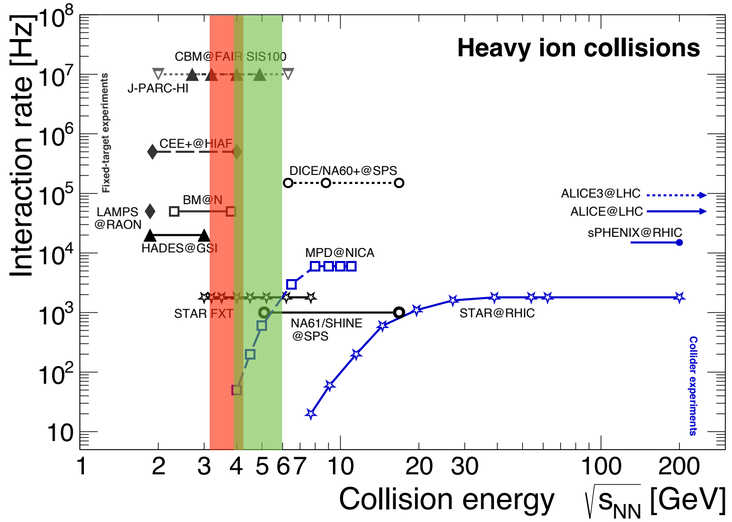


Lu, Gao, Liu, JMP, arXiv: 2504.05099

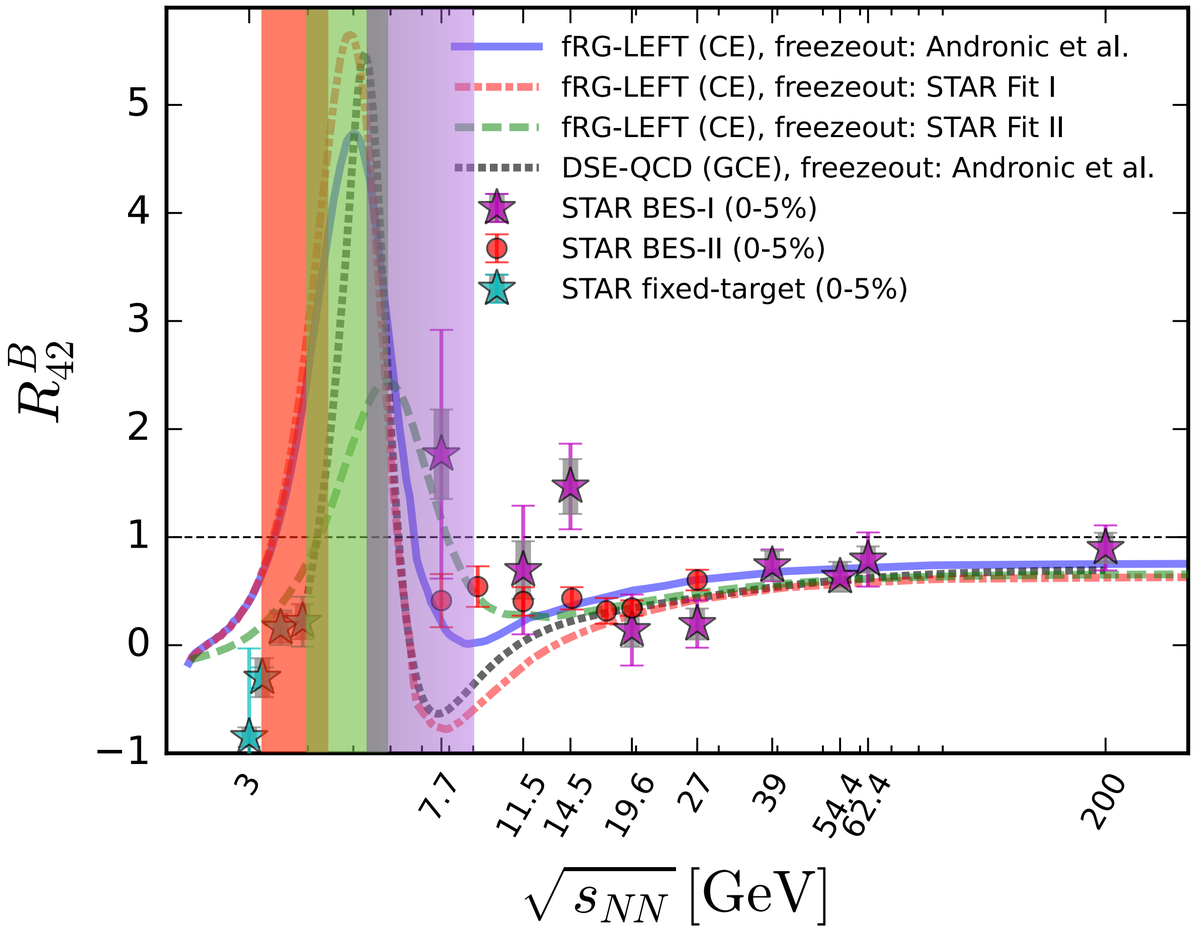
Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Ripples of the critical point





net-baryon fluctuations in QCD vs net-proton fluctuations at STAR

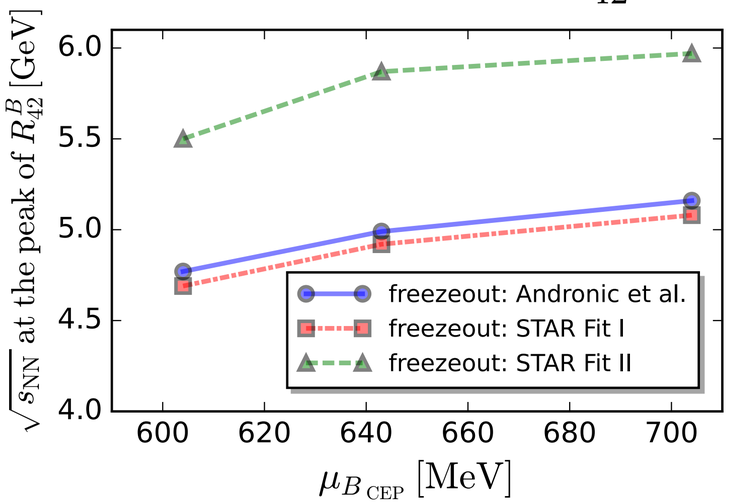


Lu, Gao, Liu, JMP, arXiv: 2504.05099

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

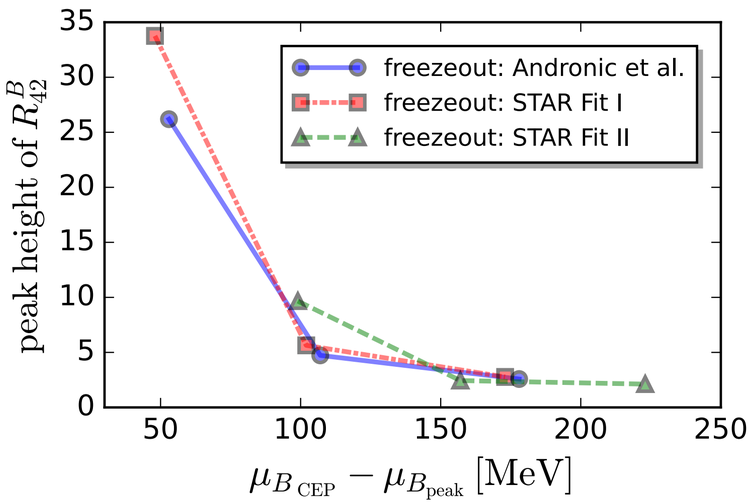
Ripples of the critical point

Position of the peak of R^B_{42}



Reconstructing the CEP

Height of the peak of R^B_{42}

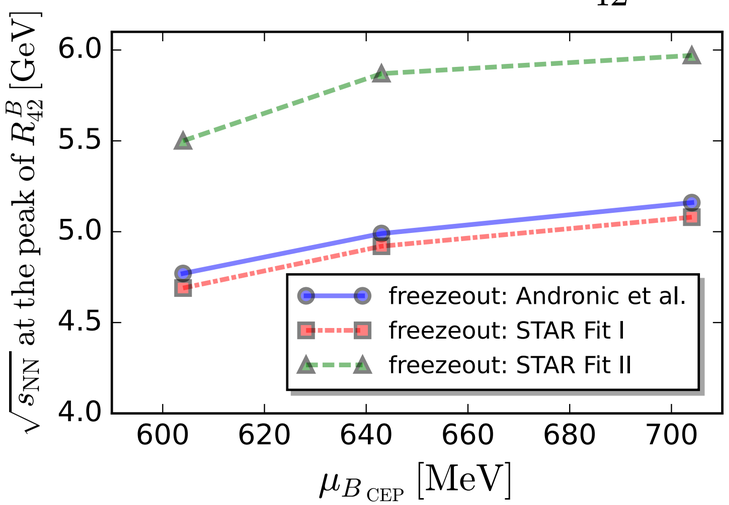


Demand: Precise freeze-out line at high density

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

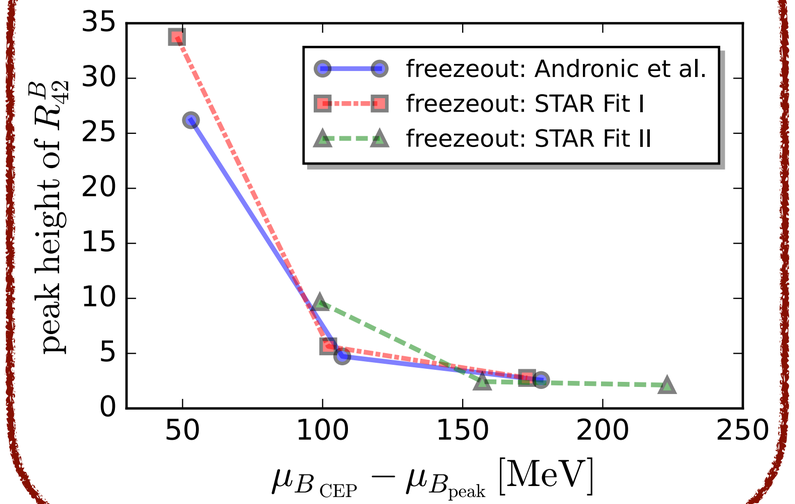
Ripples of the critical point

Position of the peak of R^B_{42}



Reconstructing the CEP

Height of the peak of R^B_{42}



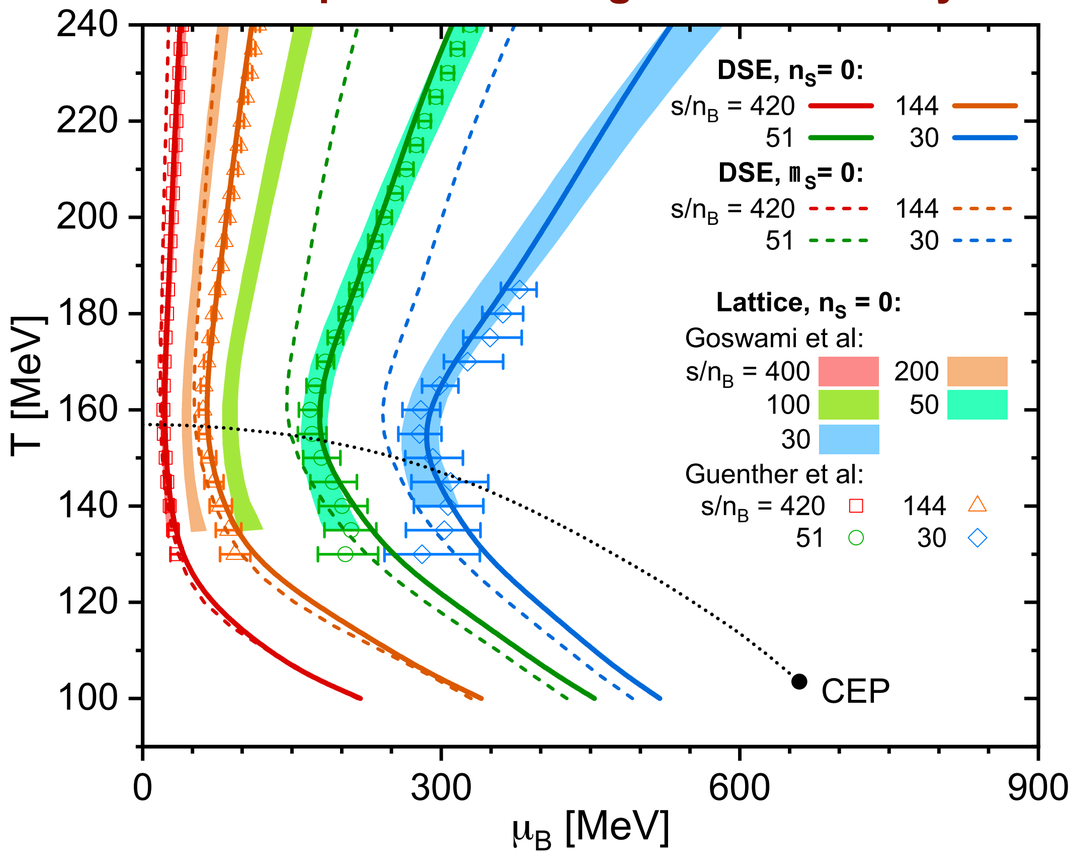
Unfolding the high density regime with new phases & physics

Great opportunity for a combined high precision analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

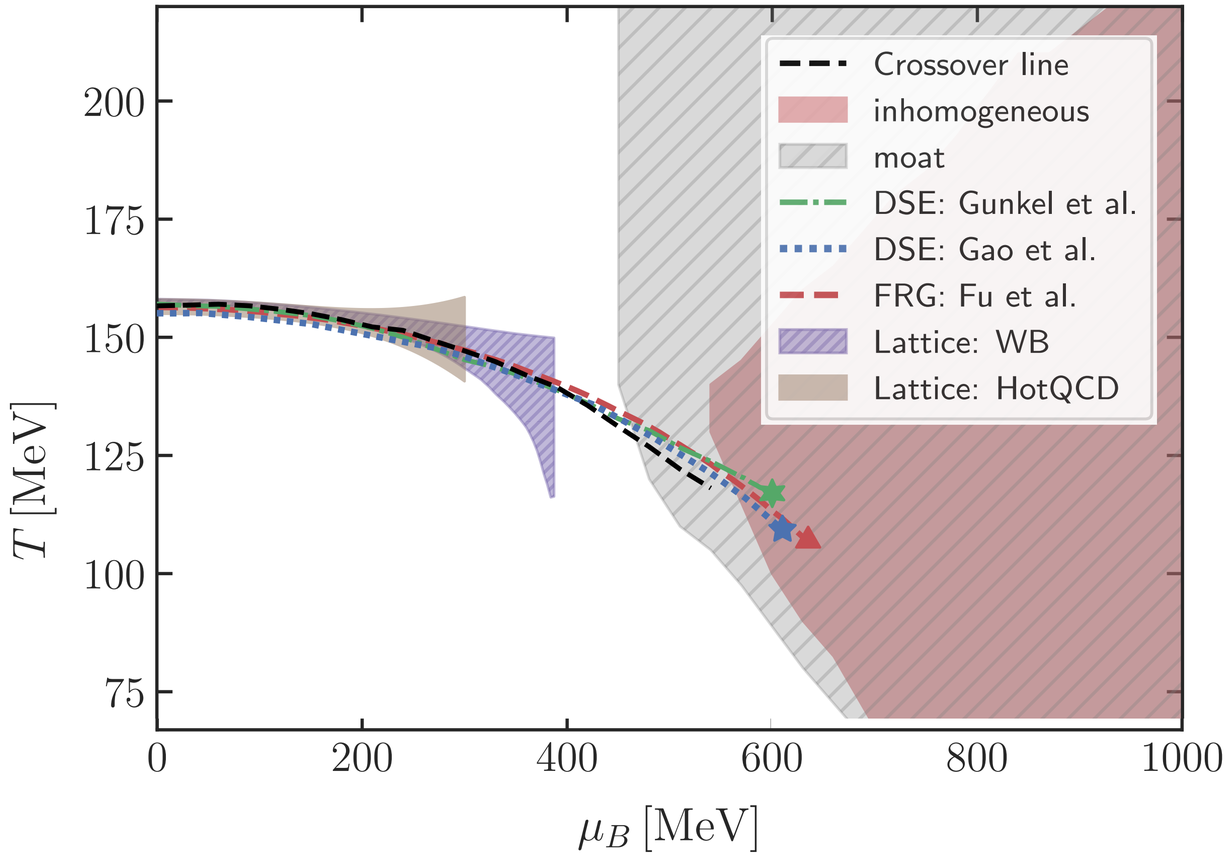
Ripples of the critical point: Sneak preview

isentropes with strangeness neutrality



Lu, Gao, Liu, Pawlowski, in preparation

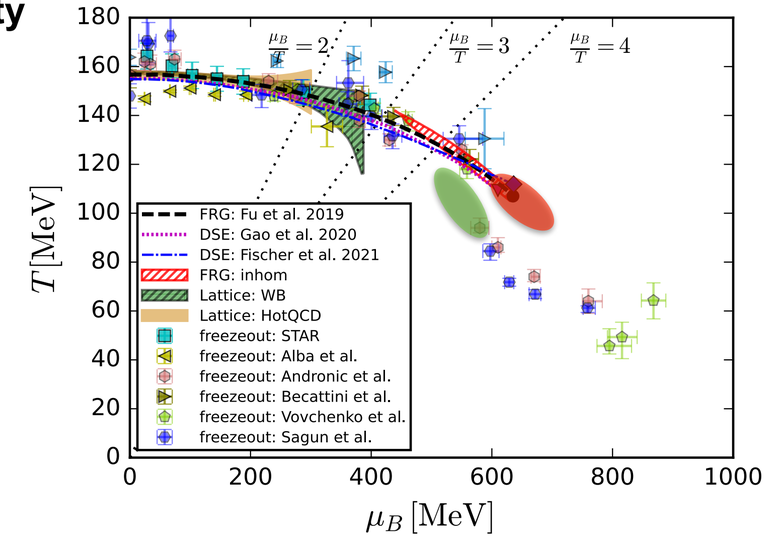
resolution of the moat regime in QCD



JMP, Rennecke, Sattler, in preparation

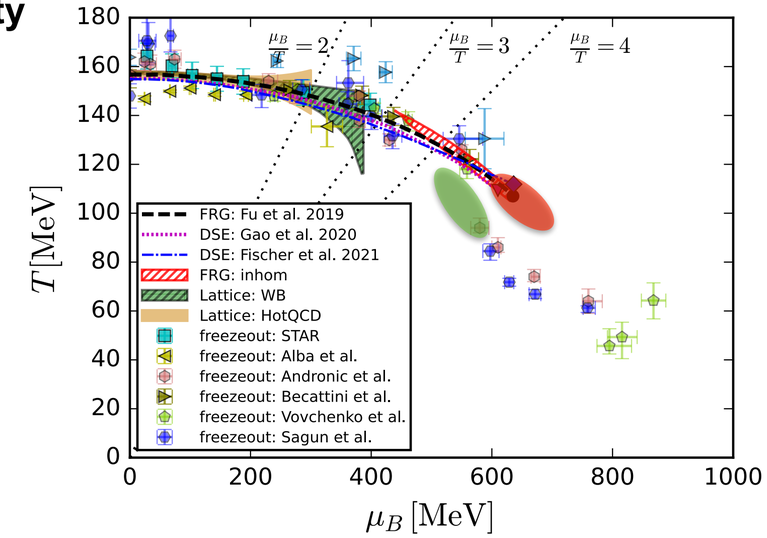
Summary

- **Functional QCD provides direct 1st principle results for the phase structure at finite density**
  - **Predictions:** $\frac{\mu_B}{T} \lesssim 4$
  - **Estimates:** $4 \lesssim \frac{\mu_B}{T} \lesssim 8$

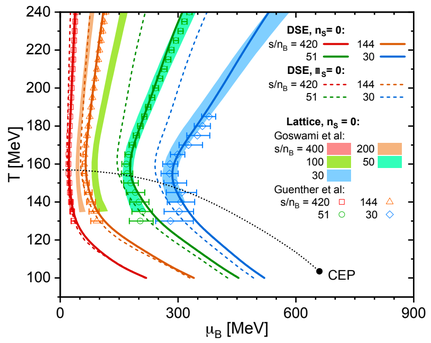


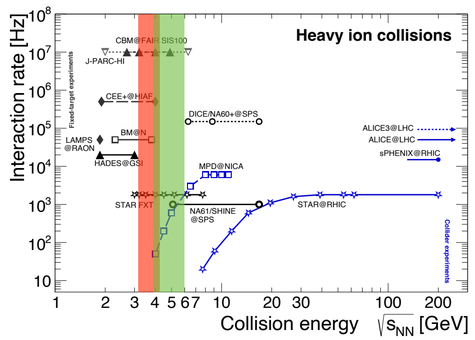
Summary

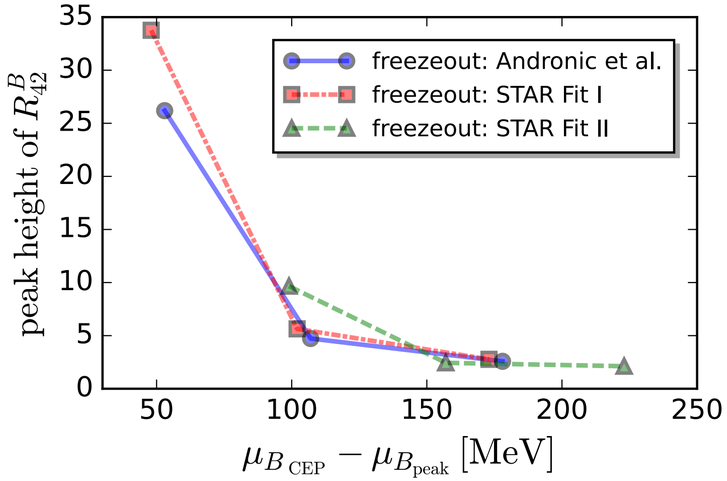
- **Functional QCD provides direct 1st principle results for the phase structure at finite density**
  - **Predictions:** $\frac{\mu_B}{T} \lesssim 4$
  - **Estimates:** $4 \lesssim \frac{\mu_B}{T} \lesssim 8$



- **Ripples of new phases aka of the critical end point**
  - **Explanation for the convergence of CEP locations of extrapolation approaches**
  - **Results & observables: EoS, fluctuations of conserved charges, tiny critical regime & onset of new phases; moat regime, ….**
  - **Peak height of kurtosis carries the location of the CEP**



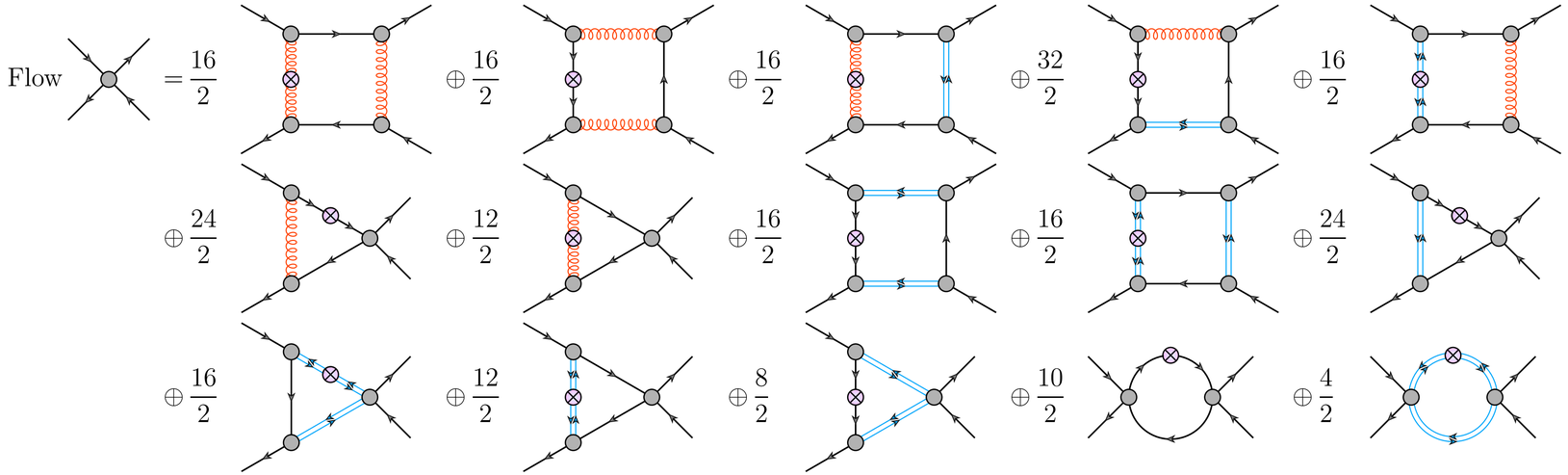




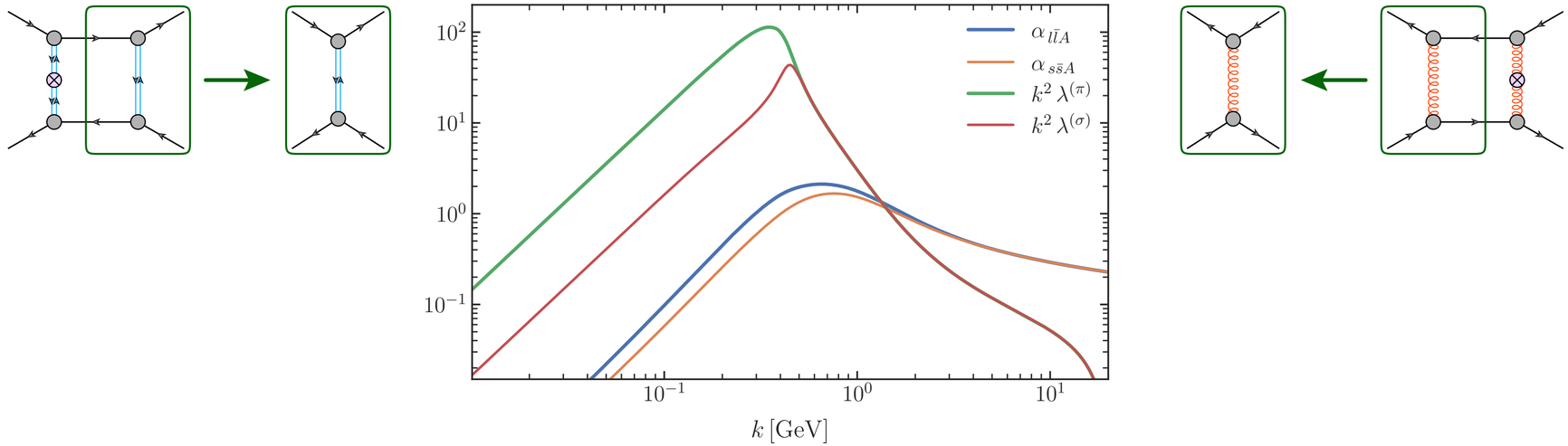
Supplementary material

Phase structure from functional QCD: how to

How to: systematic error estimates & the LEGO® principle

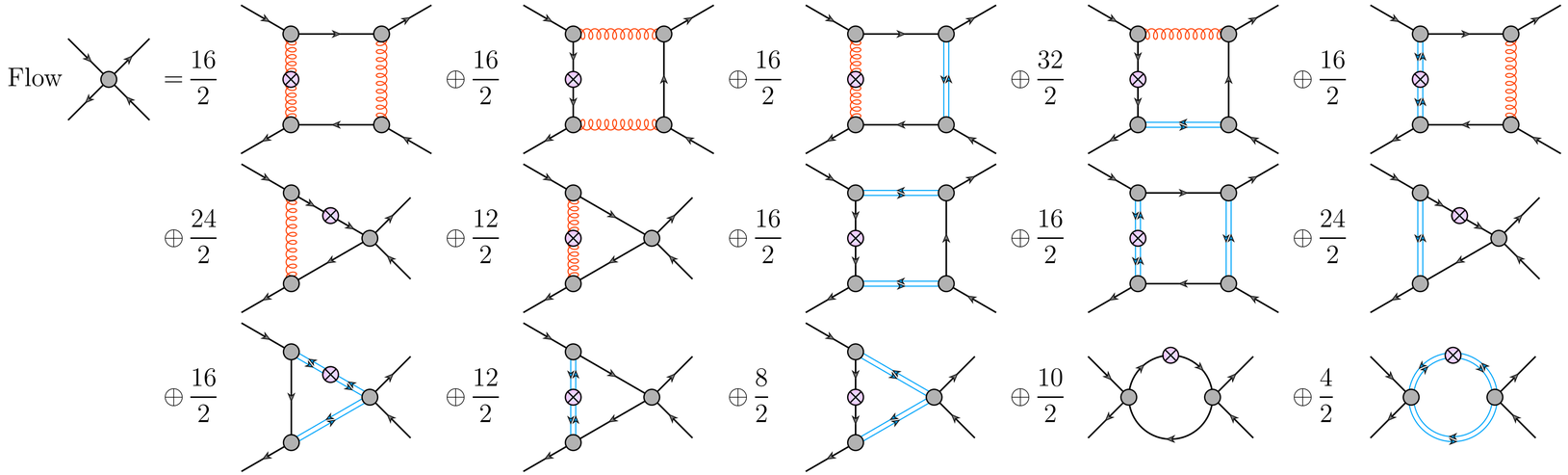


Example: 4-quark scattering vertex

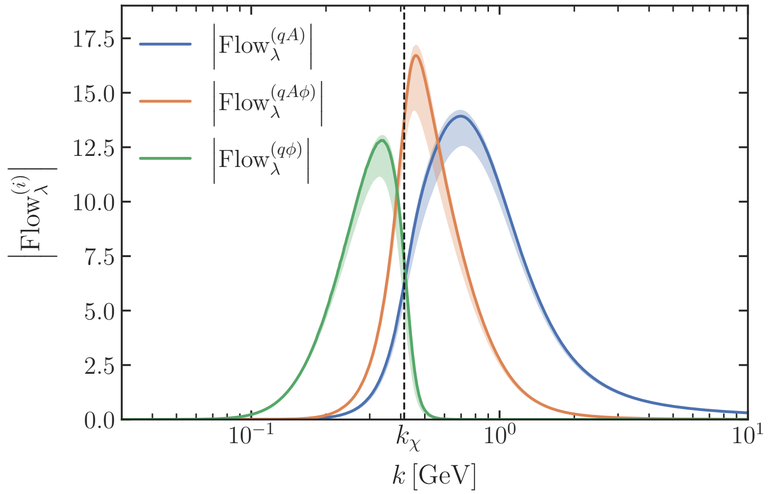


Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

How to: systematic error estimates & the LEGO® principle

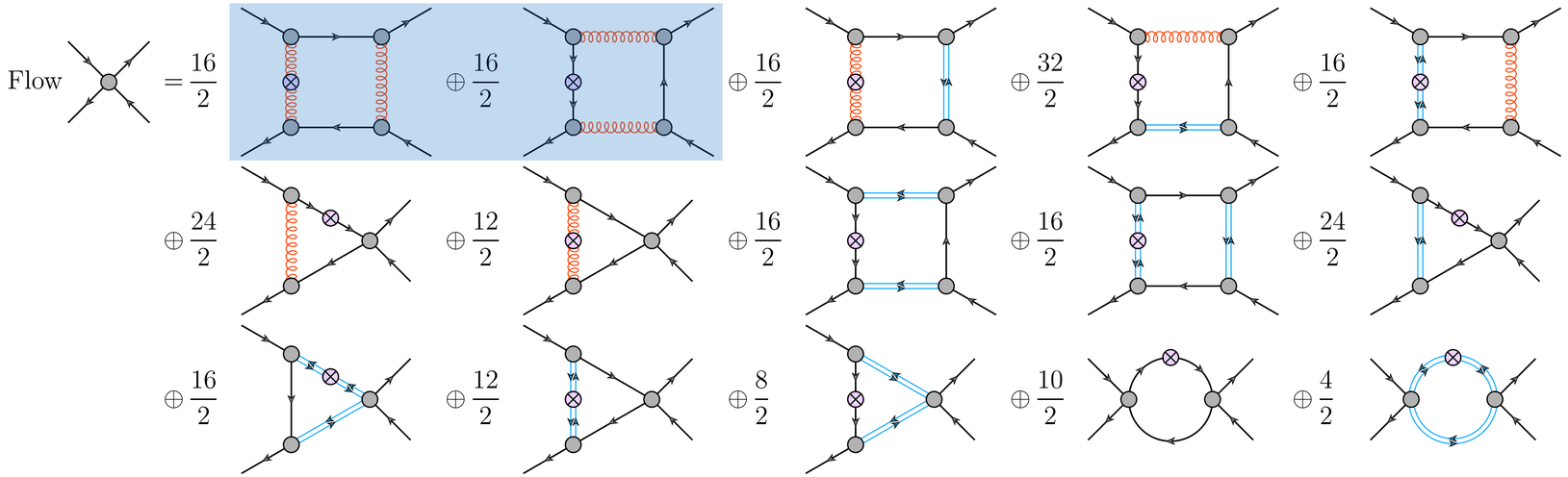


Example: 4-quark scattering vertex

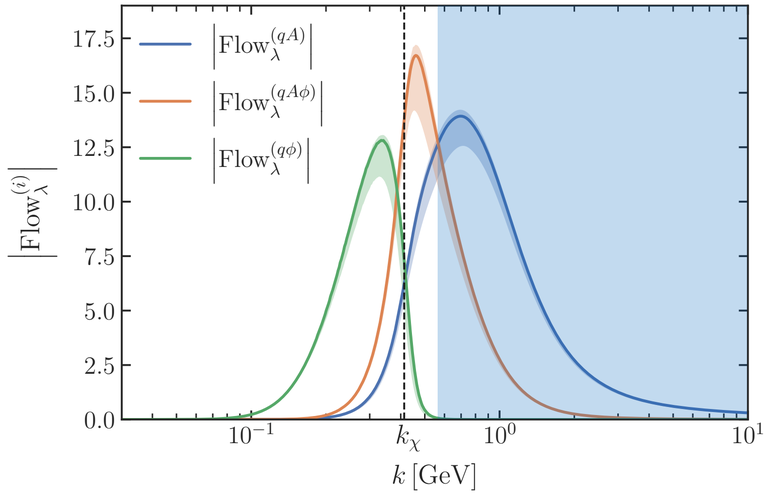


Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

How to: systematic error estimates & the LEGO® principle

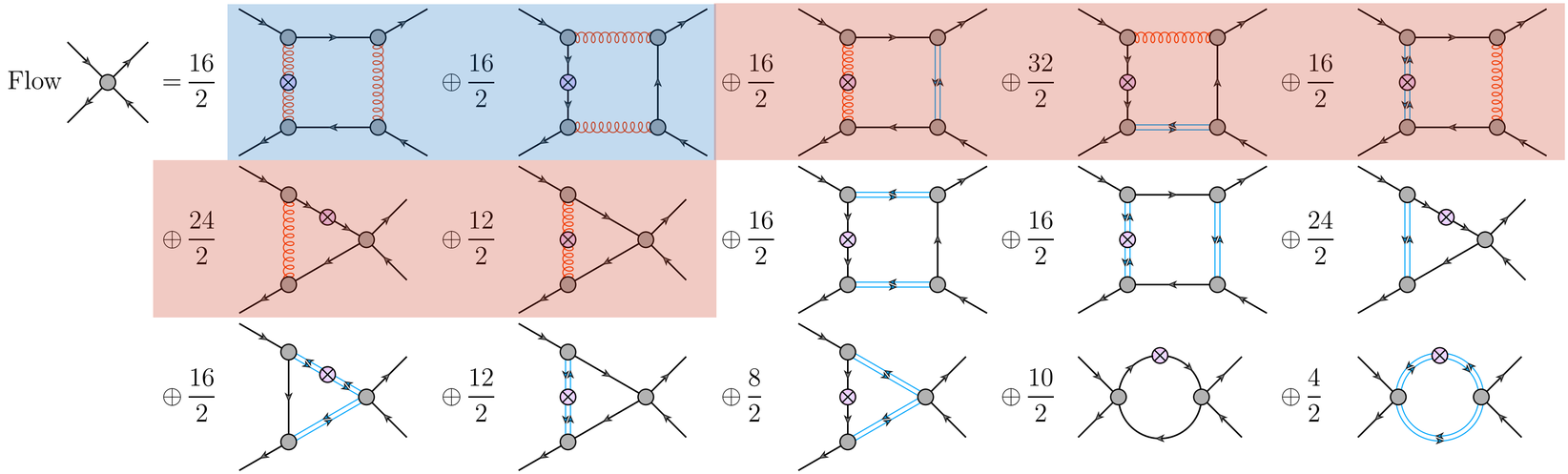


Example: 4-quark scattering vertex

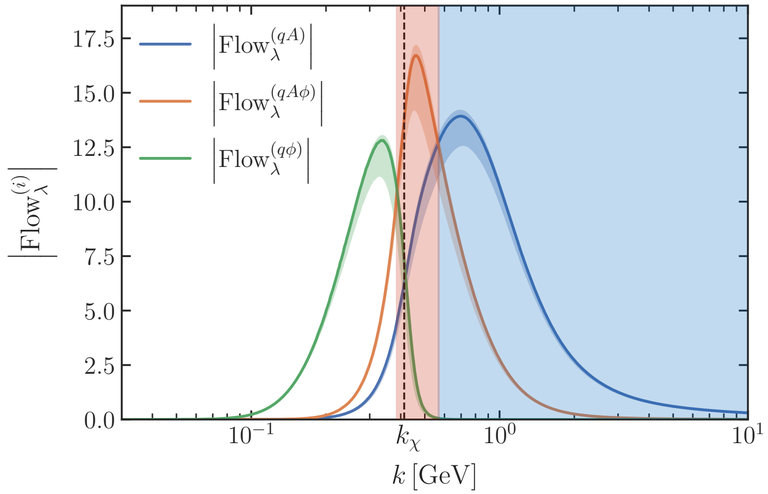


Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

How to: systematic error estimates & the LEGO® principle

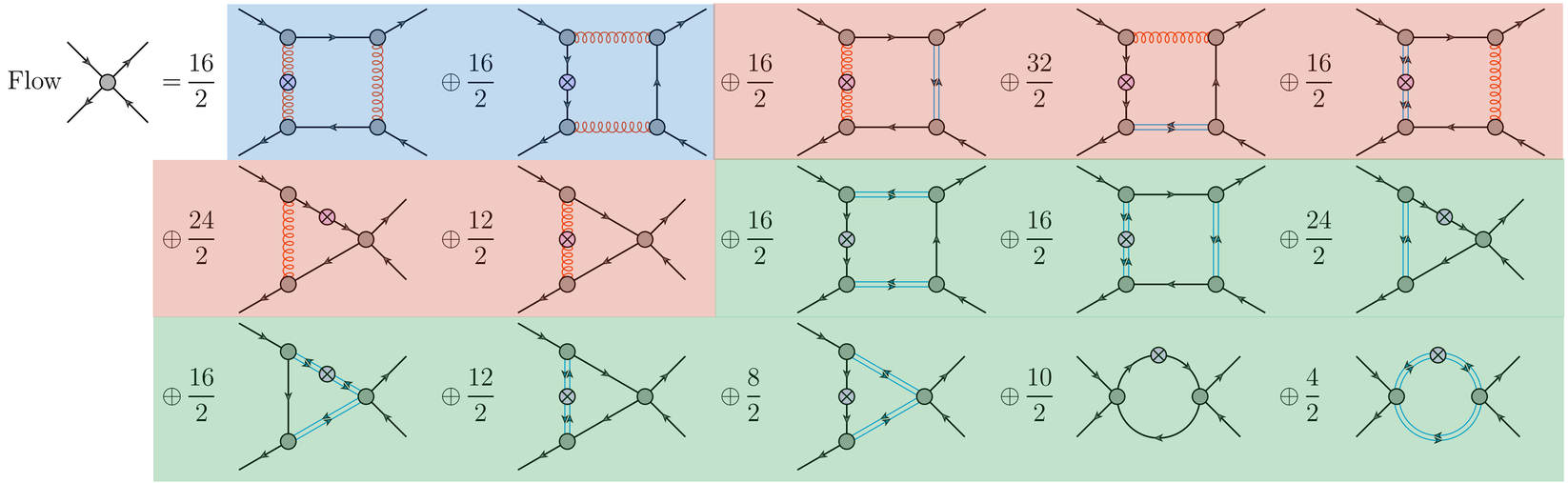


Example: 4-quark scattering vertex

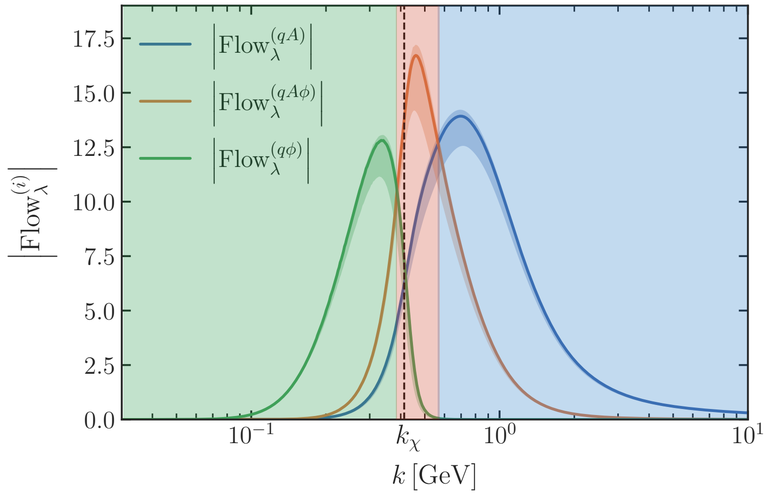


Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

How to: systematic error estimates & the LEGO® principle

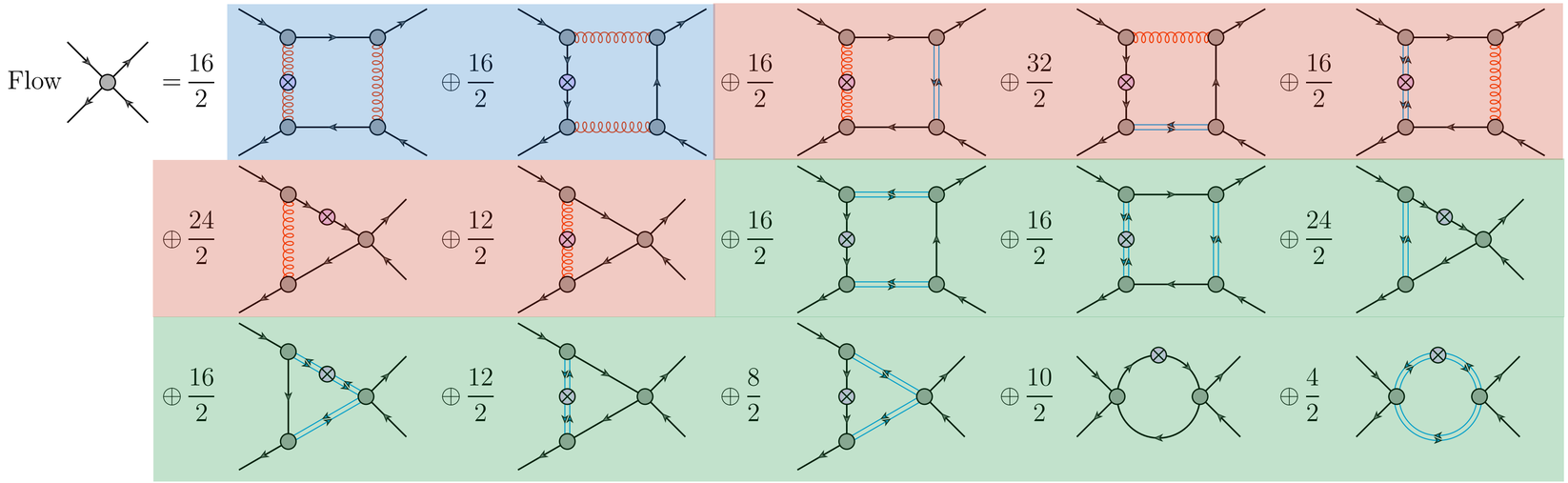


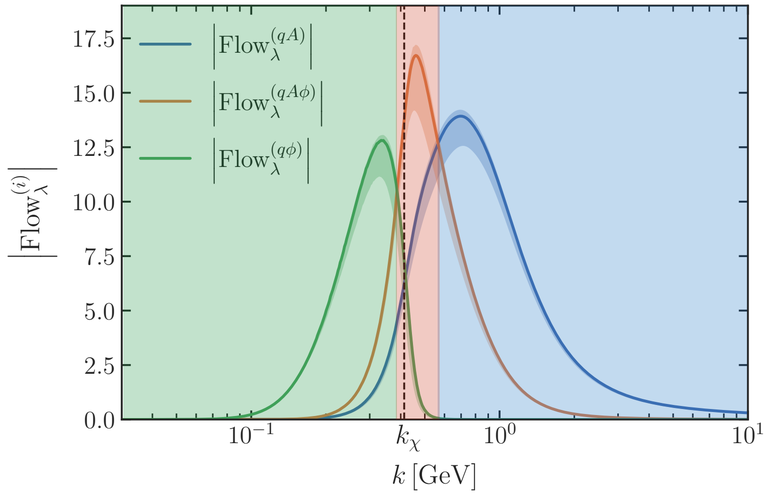
Example: 4-quark scattering vertex



Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

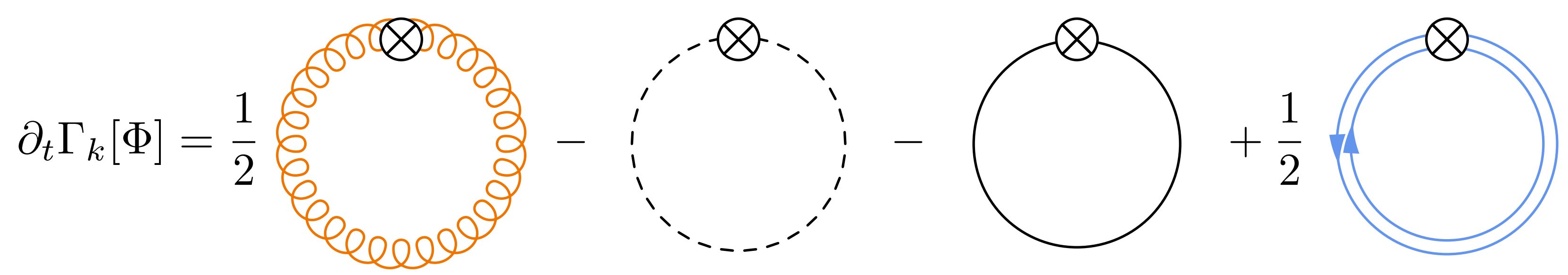
How to: systematic error estimates & the LEGO® principle



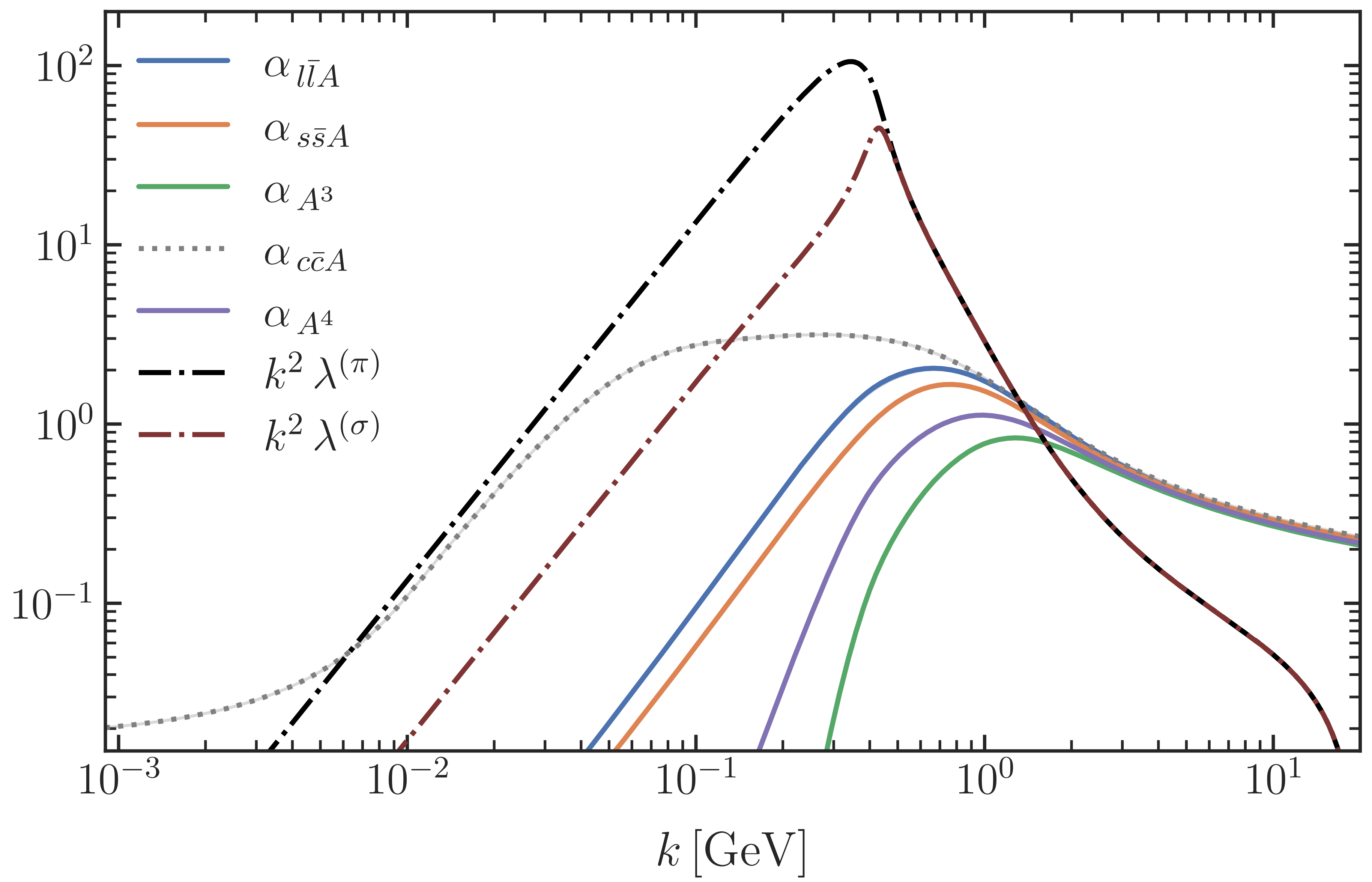


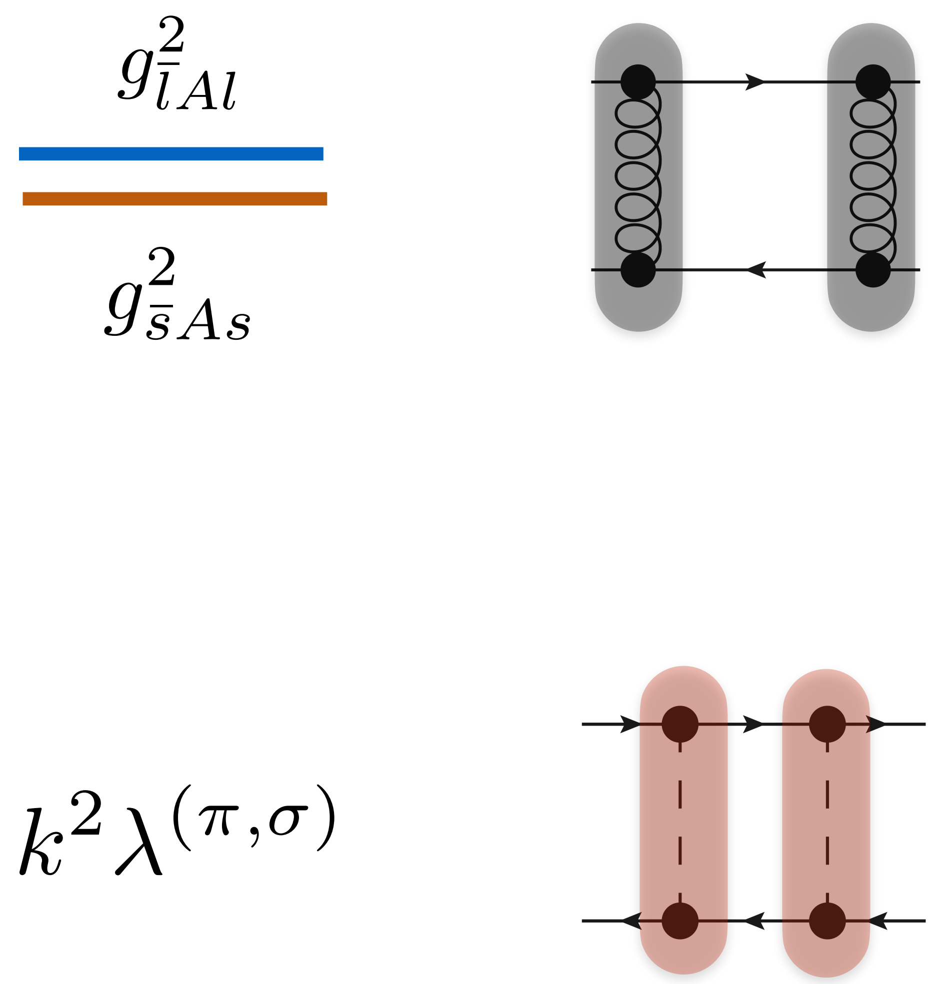
Access and combined use of error estimates from functional QCD & LEFTs

Ihssen, JMP, Sattler, Wink, arXiv:2408.08413



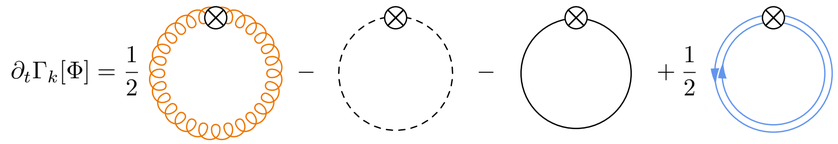
Sequential decoupling of gluon, quark, sigma, pion fluctuations





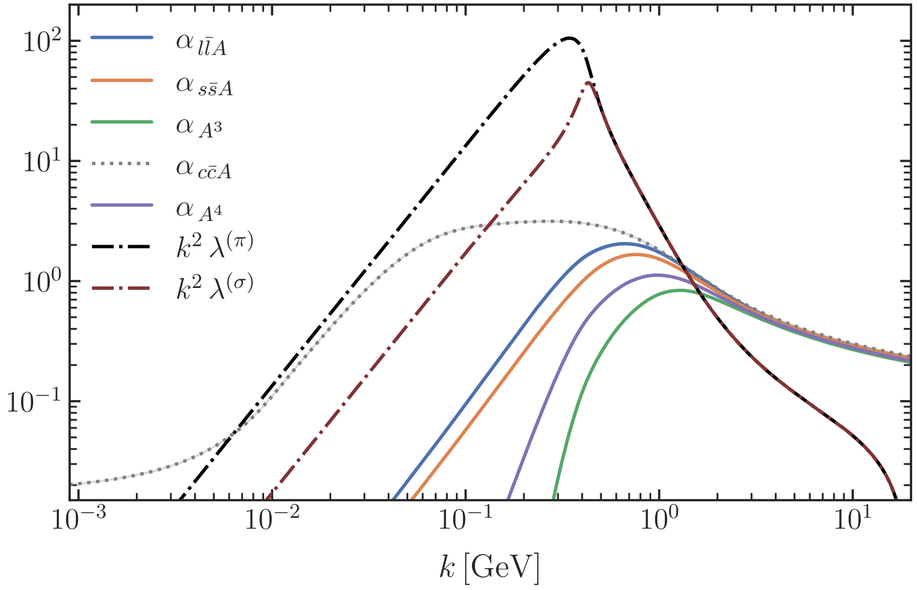
Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

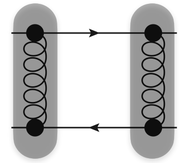


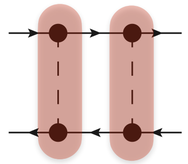
fQCD

Sequential decoupling of gluon, quark, sigma, pion fluctuations



$k^2\lambda^{(\pi,\sigma)}$

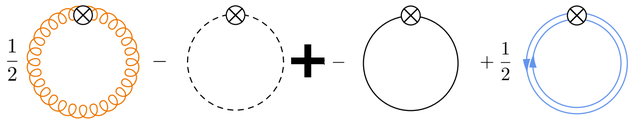




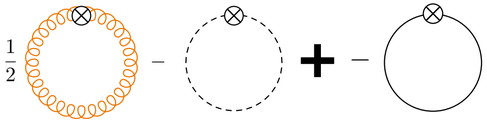
Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

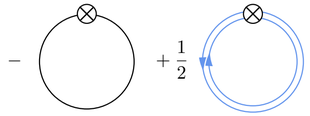
PQM-model



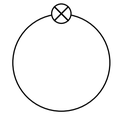
PNJL-model



QM-model

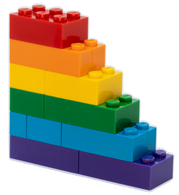


NJL-model

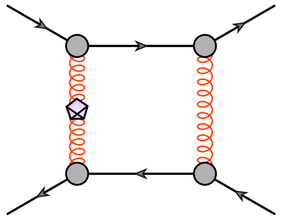


QCD-assisted low energy effective theories

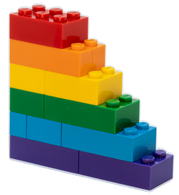
How to: systematic error estimates & the LEGO® principle



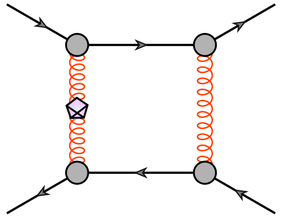
Example: Disect quark-gluon box



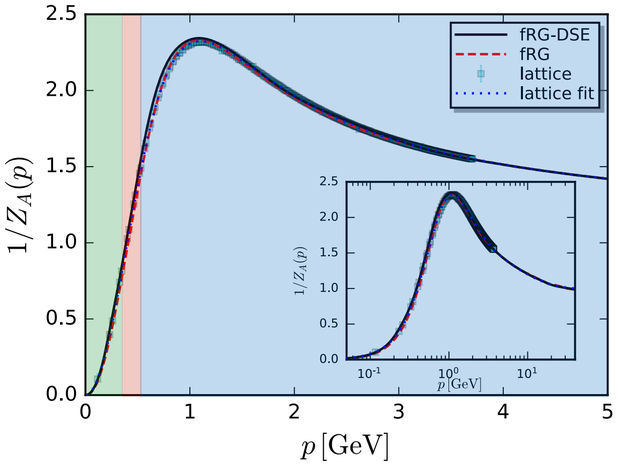
How to: systematic error estimates & the LEGO® principle

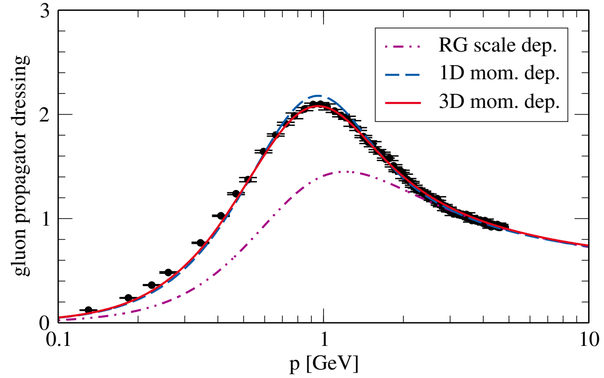


Example: Disect quark-gluon box

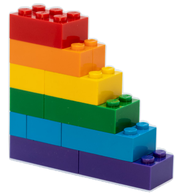


gluon two-point correlator

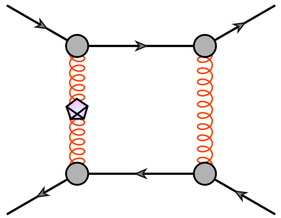




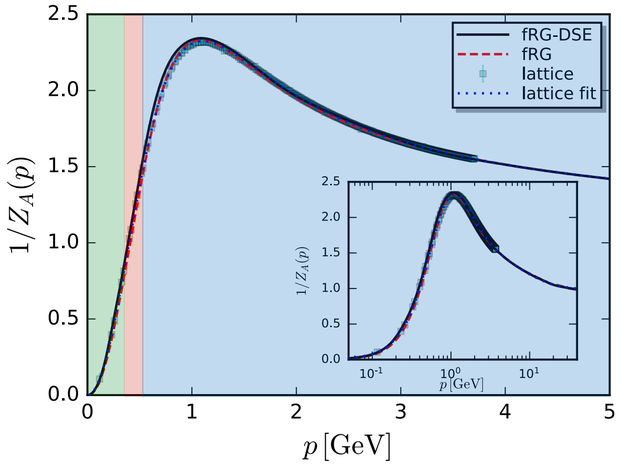
How to: systematic error estimates & the LEGO® principle

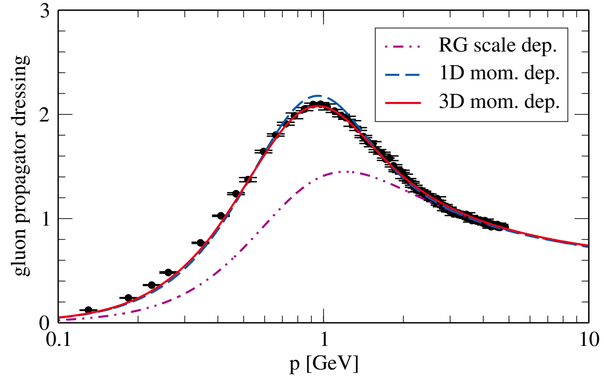


Example: Disect quark-gluon box

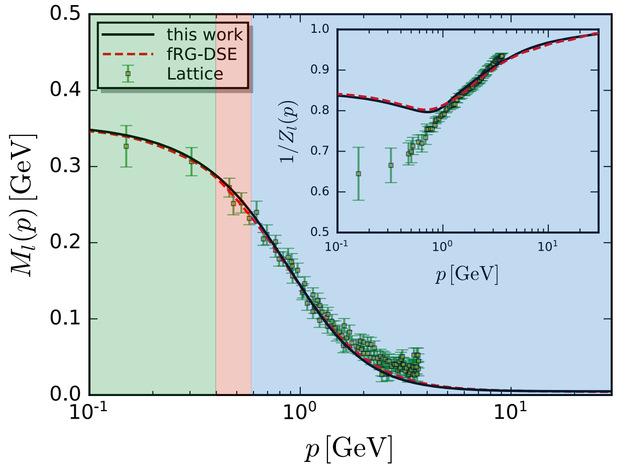


gluon two-point correlator

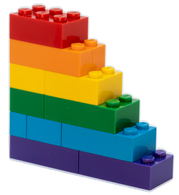




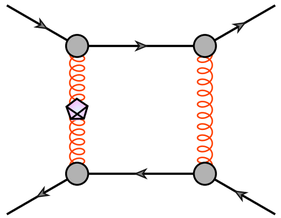
quark two-point correlator



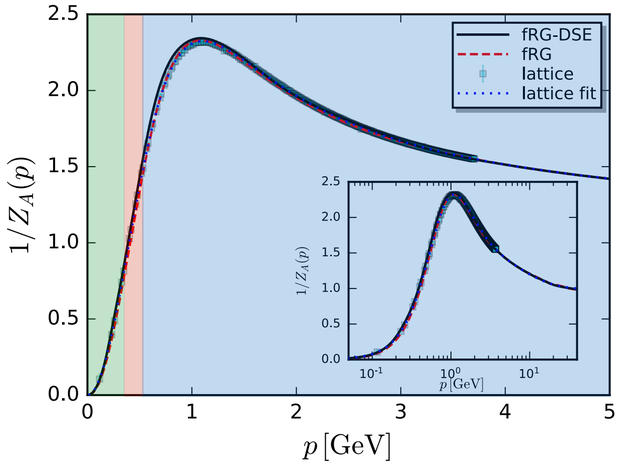
How to: systematic error estimates & the LEGO® principle

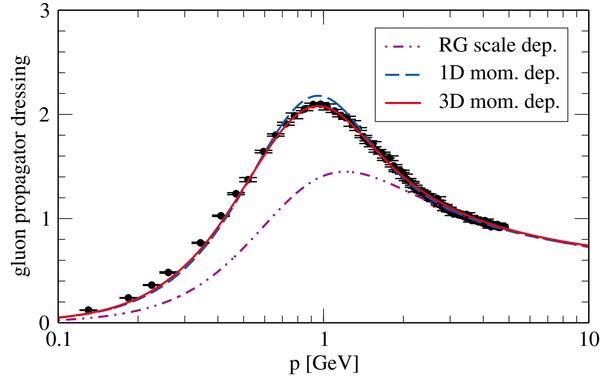


Example: Disect quark-gluon box

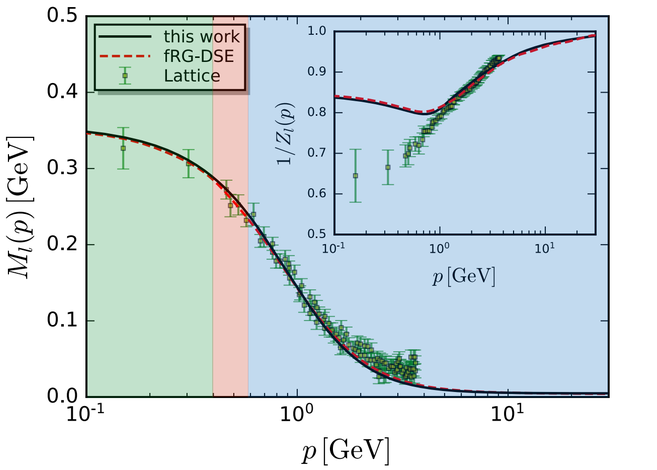


gluon two-point correlator

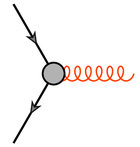




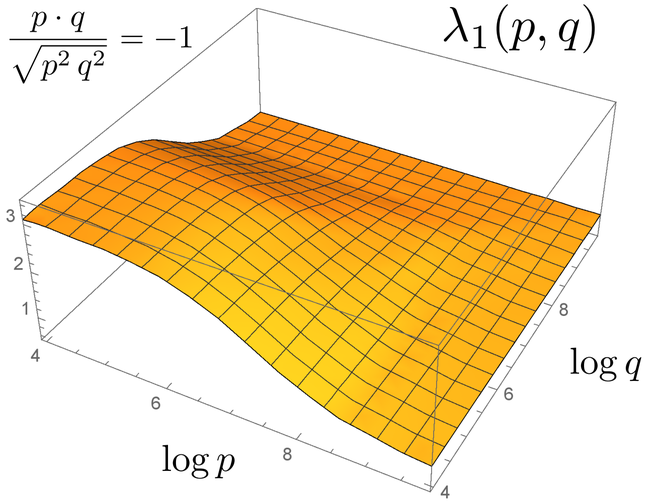
quark two-point correlator



quark-gluon scattering



$$\frac{p\cdot q}{\sqrt{p^2\,q^2}} = -1$$



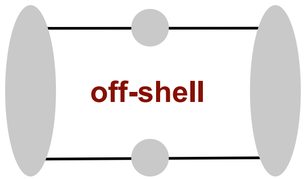
$$\lambda_1(p,q), \cdots, \lambda_8(p,q)$$

Relevant: $\lambda_1\,,\ \lambda_4\,,\ \lambda_7$

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$

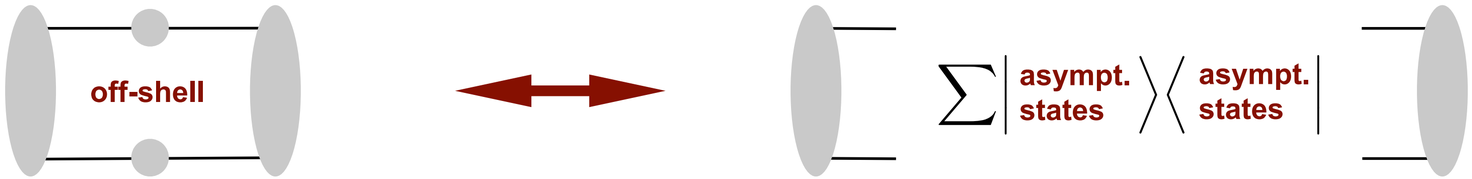


pressure, trace anomaly,
fluctuations, volume flucs., ...

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$



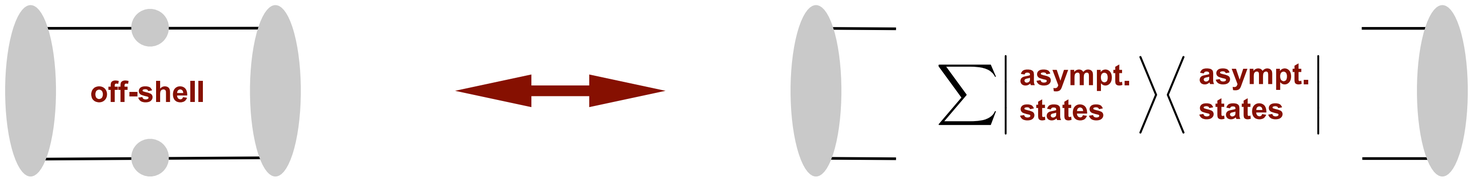
pressure, trace anomaly, fluctuations, volume flucs., ...

e.g. hadron resonances

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$



pressure, trace anomaly, fluctuations, volume flucs., ...

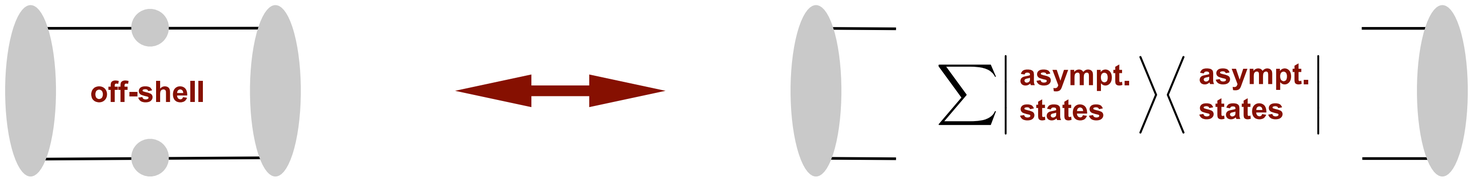
e.g. hadron resonances

- **gauge fixing = parameterisation**

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$



pressure, trace anomaly, fluctuations, volume flucs., ...

e.g. hadron resonances

- **gauge fixing = parameterisation**

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$

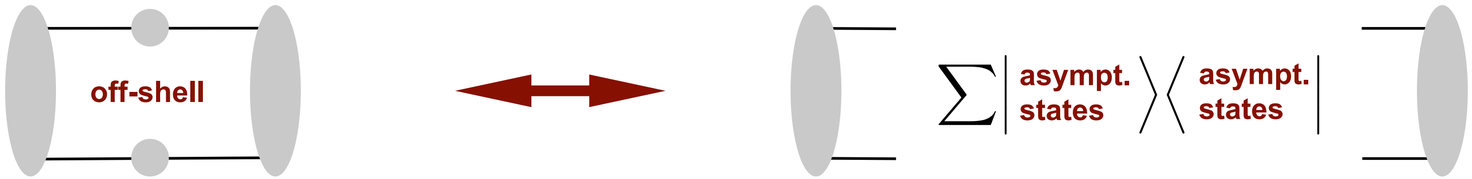
Consequences

I: simple correlations

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$



pressure, trace anomaly, fluctuations, volume flucs., ...

e.g. hadron resonances

- **gauge fixing = parameterisation**

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$

Consequences

I: simple correlations

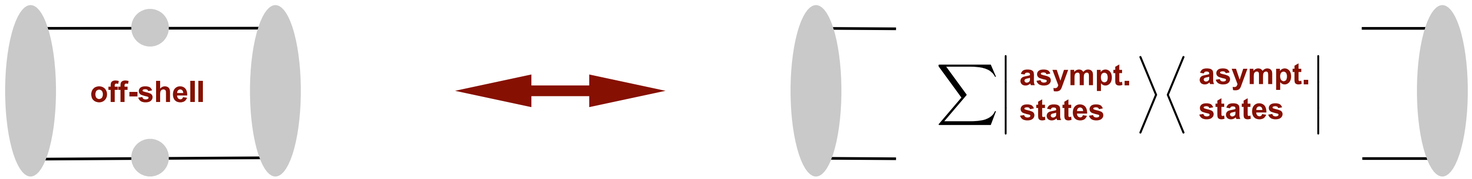
II: Difficult access to some observables

'No free lunch theorem'

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$



pressure, trace anomaly, fluctuations, volume flucs., ...

e.g. hadron resonances

- **gauge fixing = parameterisation**

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$

Consequences

I: simple correlations

II: Difficult access to some observables

'No free lunch theorem'

- **'Your mean field is not my mean field'**

$$\left.\frac{\delta S_{\rm cl}[\phi]}{\delta\phi}\right|_{\phi=\bar{\phi}} = 0 \qquad \left.\frac{\delta\Gamma[\phi]}{\delta\phi}\right|_{\phi=\bar{\phi}_{\rm quant}} = 0$$

full quantum equation of motion

How to: direct computations and the minimal point of view

Those are my interpretations,
and if you don't like them....
well, I have others

- **Self-consistent truncations to functional relations define analytic functions in μ_B , eg:**

$$\partial_t \Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B) = \mathrm{Loop}\left[\Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B),\ \Big\langle q(x)A_\mu(y)\bar{q}(z)\Big\rangle(\mu_B), \cdots;\ \mu_B\right]$$

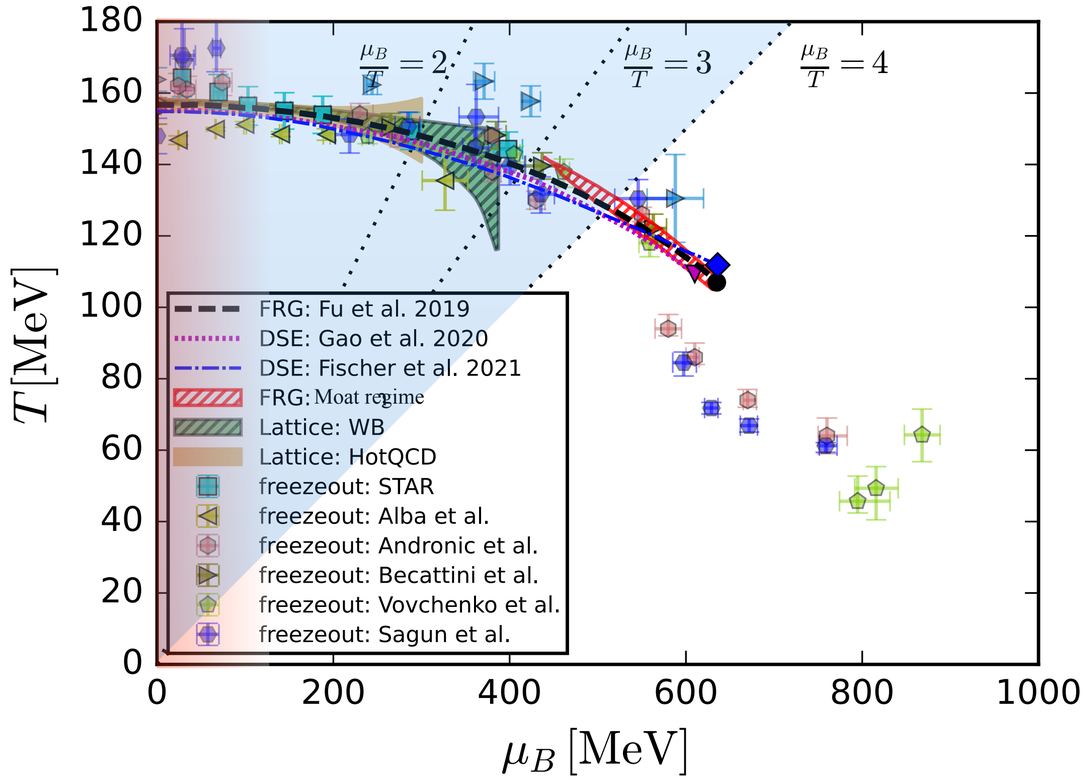
How to: direct computations and the minimal point of view

Those are my interpretations, and if you don't like them.... well, I have others

- **Self-consistent truncations to functional relations define analytic functions in μ_B, eg:**

$$\partial_t \Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B) = \mathrm{Loop}\left[\Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B),\ \Big\langle q(x)A_\mu(y)\bar{q}(z)\Big\rangle(\mu_B), \cdots;\ \mu_B\right]$$

- **Consequences for functional QCD predictions at finite density**



By now the best truncations to functional QCD pass lattice benchmark tests at vanishing and small chemical potential

Regime of quantitative reliability of current best truncation

Unique: QCD-based analytic continuations that satisfy the lattice benchmarks at small chemical potential.

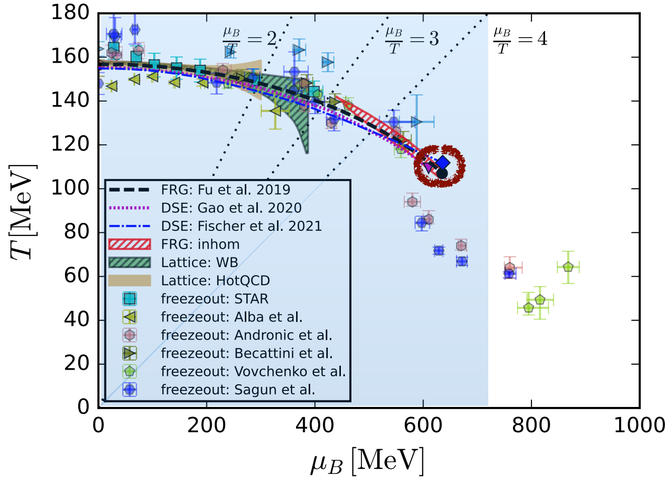
Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD + LEFTs)

Phase structure from functional QCD: Predictions & estimates

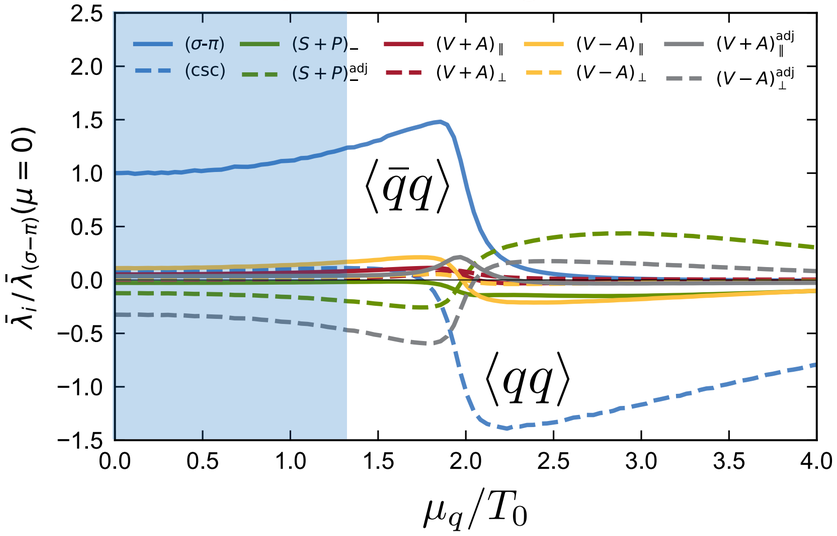
Predictions & estimates

Dominance of scalar-pseudoscalar fluctuations

Pions & sigma mode

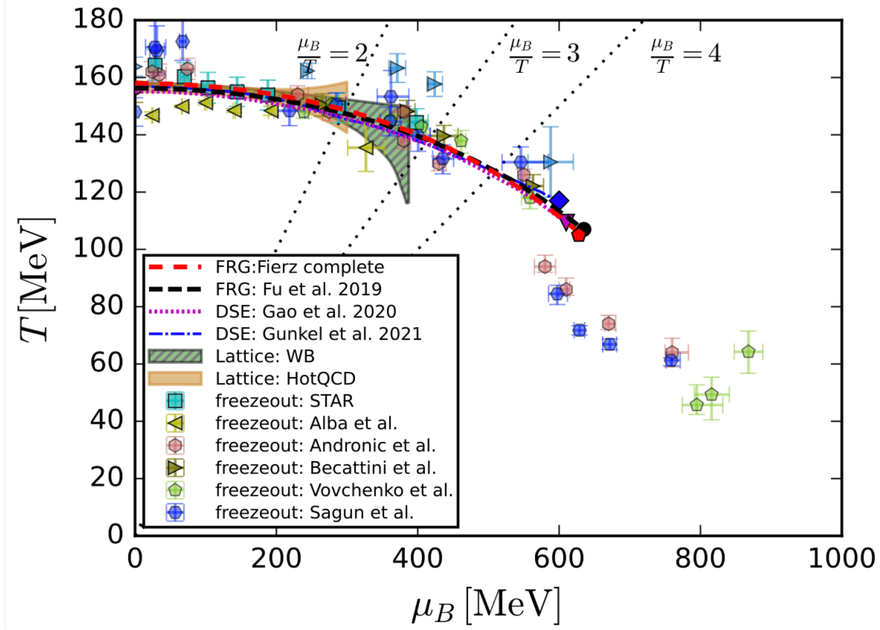


Four-quark scattering channels



Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Fierz-complete phase structure



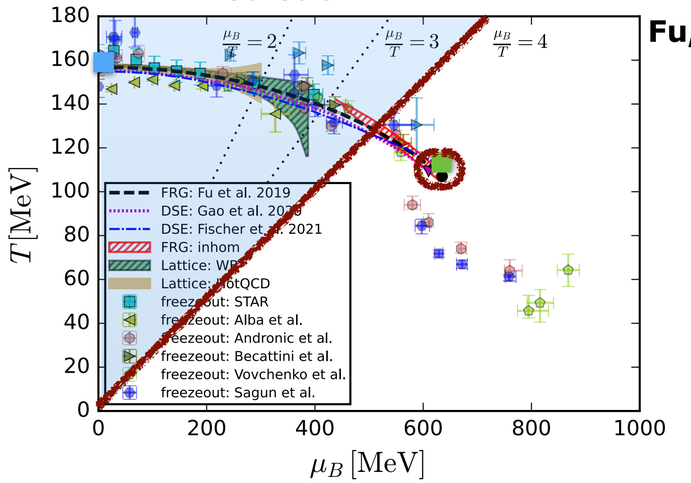
Wang et al, in preparation

Predictions & estimates

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

Prediction

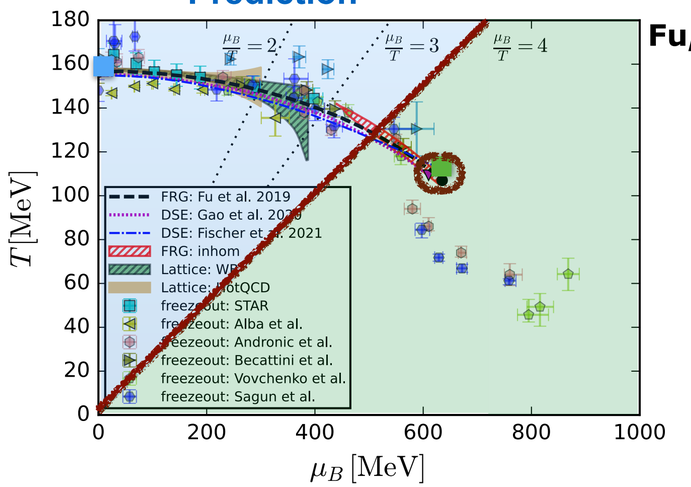


Fu, JMP, Rennecke, PRD 101 (2020) 054032

Regime of quantitative reliability of current best truncation

Predictions & estimates

Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032

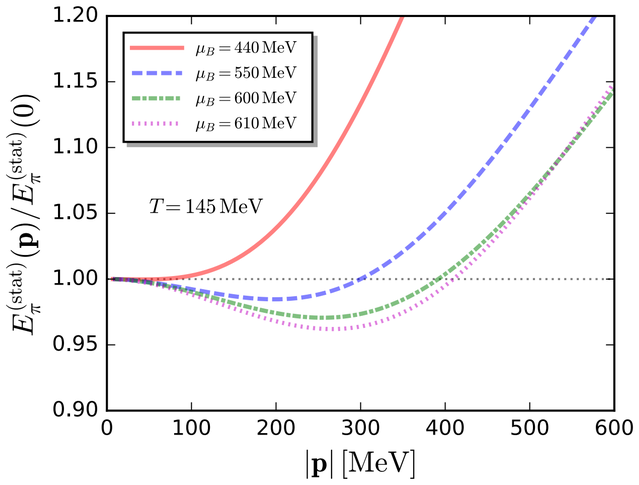
Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

Regime of quantitative reliability of current best truncation

Estimate

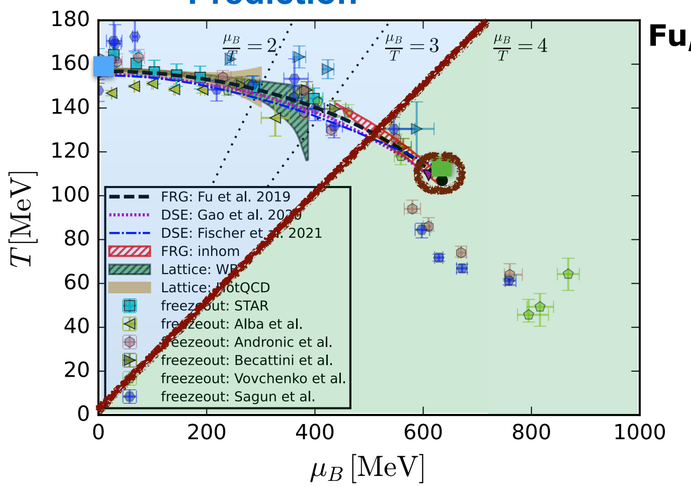
Moat regime is not captured quantitatively



Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032

Moat regime

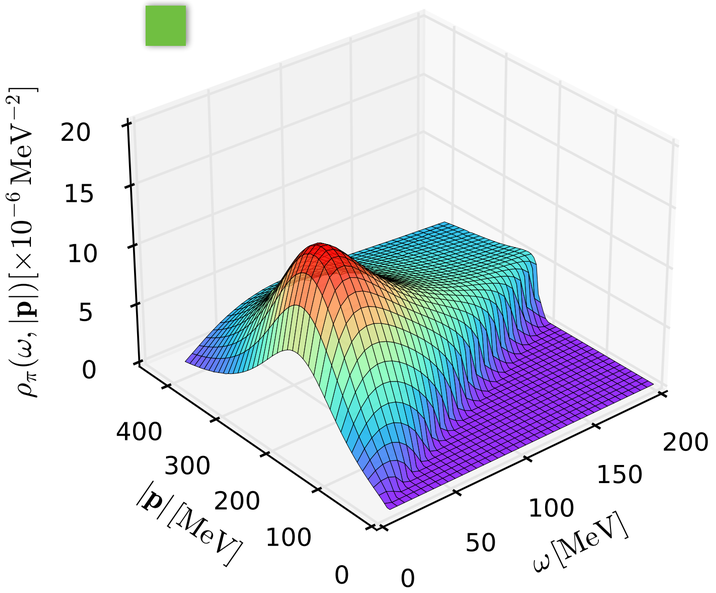
Pisarski, Rennecke, PRL 127 (2021) 152302

Regime of quantitative reliability of current best truncation

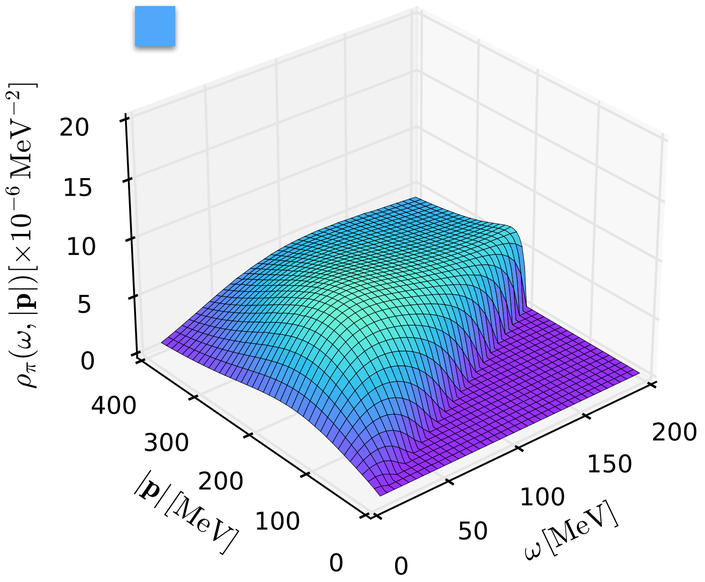
Estimate

Moat regime is not captured quantitatively

T=114 MeV & μ_B =630 MeV



T=160 MeV & μ_B =0 MeV

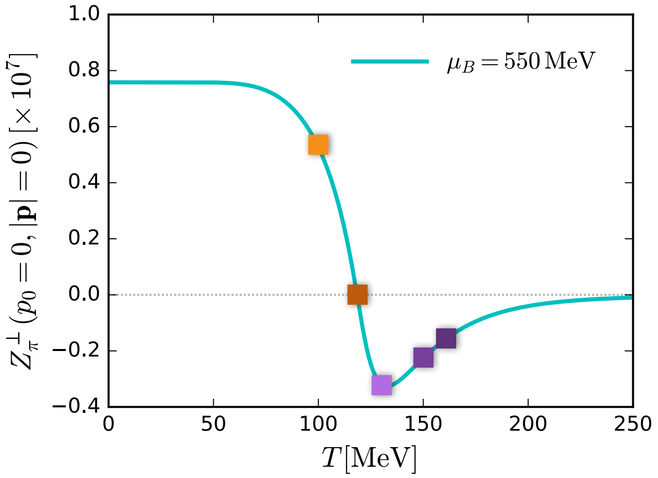


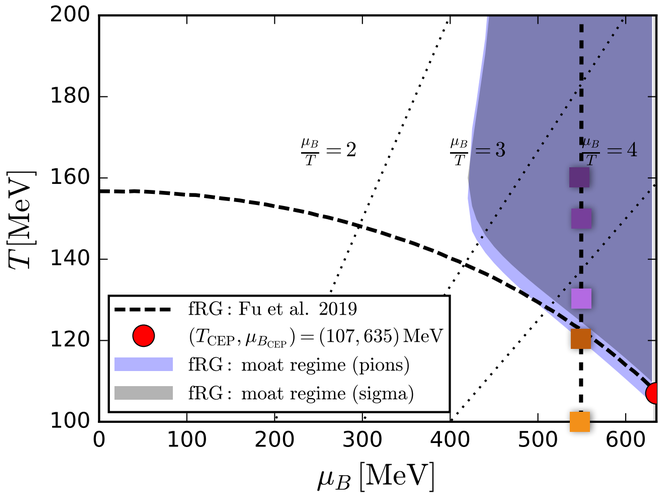
Pion spectral functions

Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

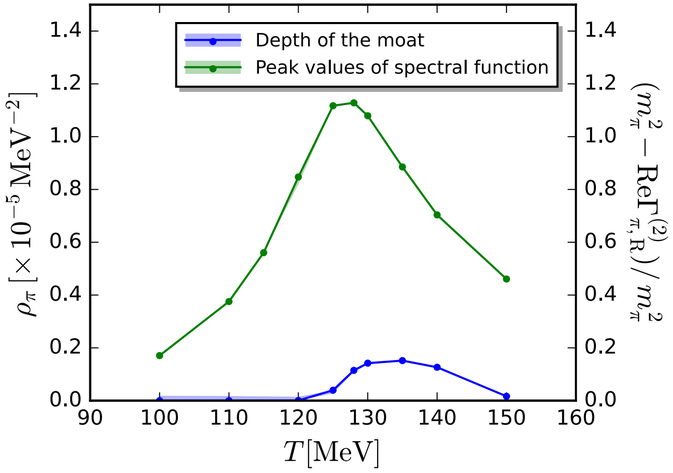
Predictions & estimates

Spatial wave function of the pion at p=0





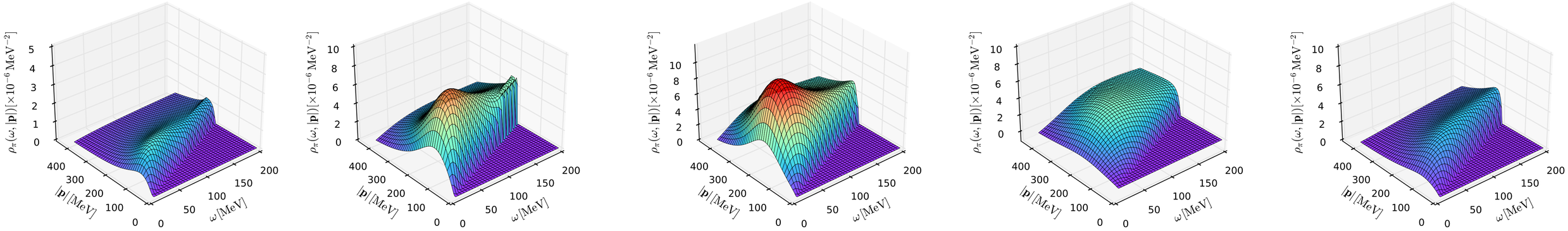
Depth of the moat & spectral peak of the pion



Dissecting the moat & the moaton

μ_B = 650 MeV

T=100 MeV T=120 MeV T=130 MeV T=150 MeV T=160 MeV

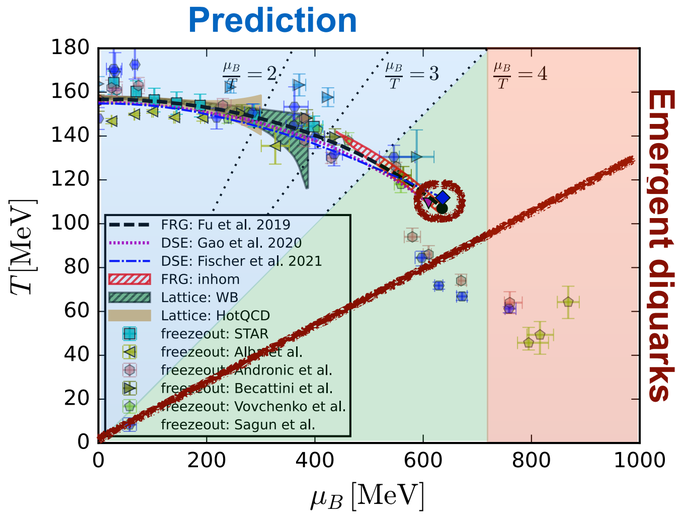


Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

Emergent diquarks

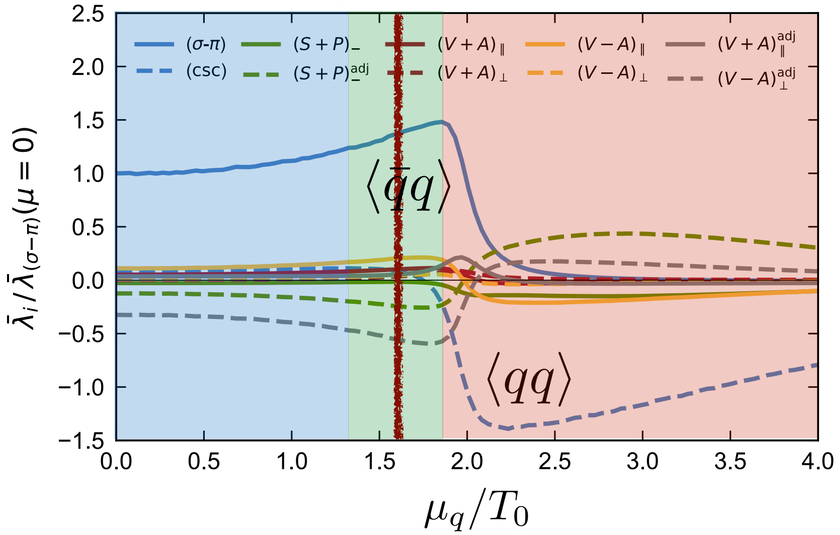
Prediction



Estimate

Regime of quantitative reliability of current best truncation

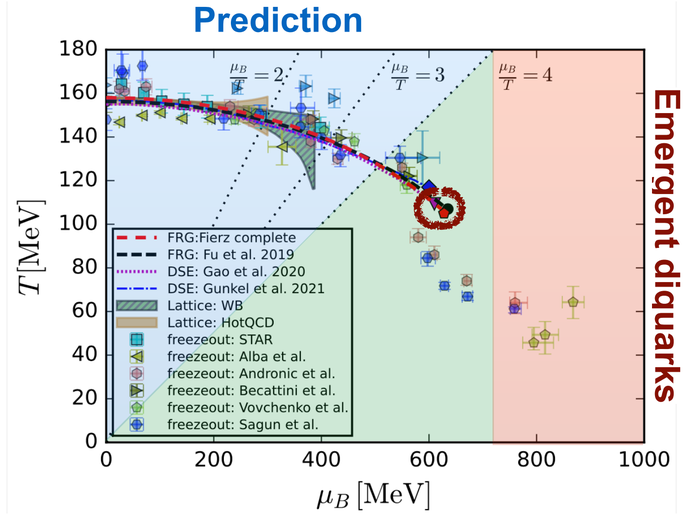
Emergent diquarks (fRG)



Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Fierz-complete phase structure

Prediction



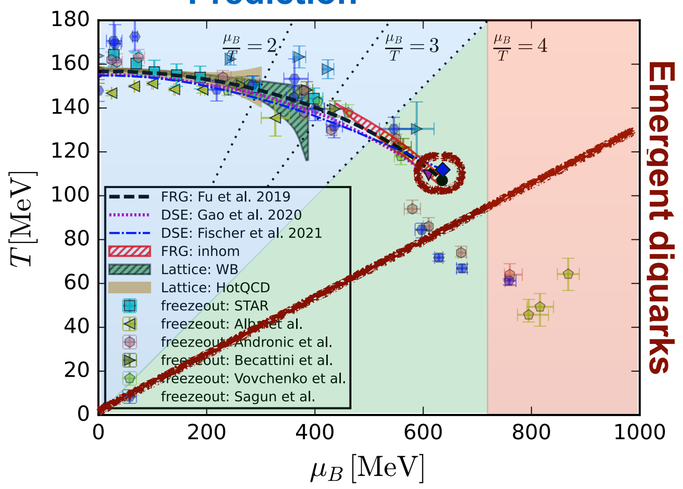
Estimate

Wang et al, in preparation

Predictions & estimates

Emergent diquarks

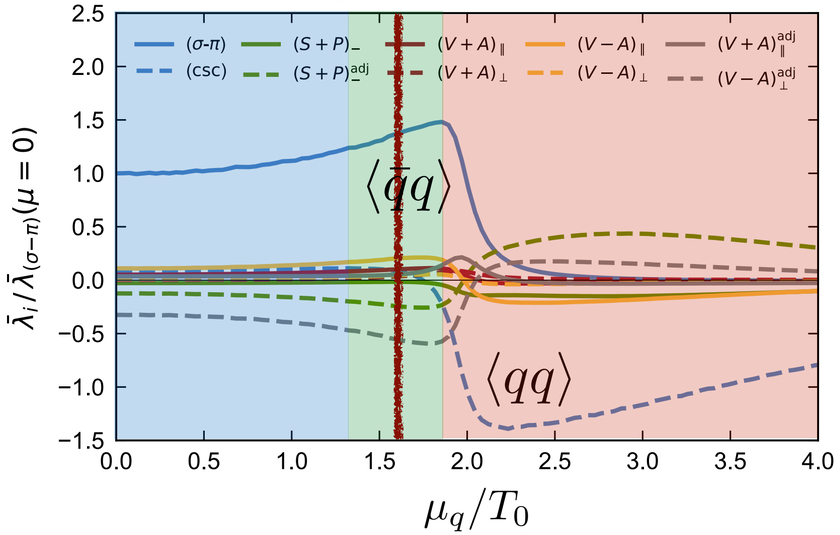
Prediction



Estimate

Regime of quantitative reliability of current best truncation

Emergent diquarks (fRG)

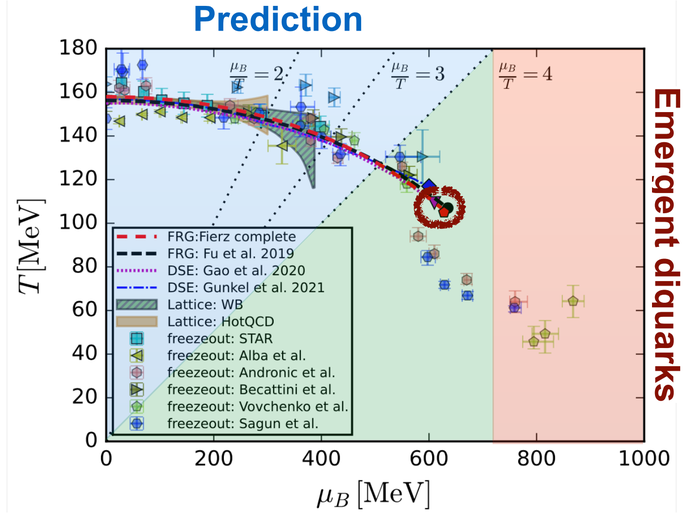


Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Emergent diquarks are not captured by extrapolations

Fierz-complete phase structure

Prediction



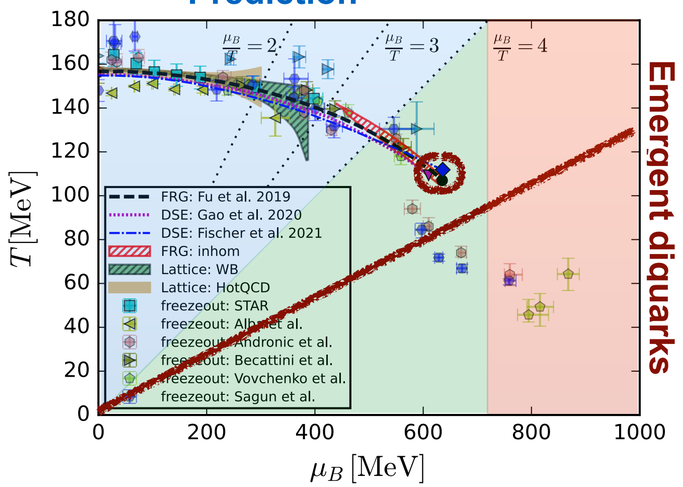
Estimate

Wang et al, in preparation

Predictions & estimates

Emergent diquarks

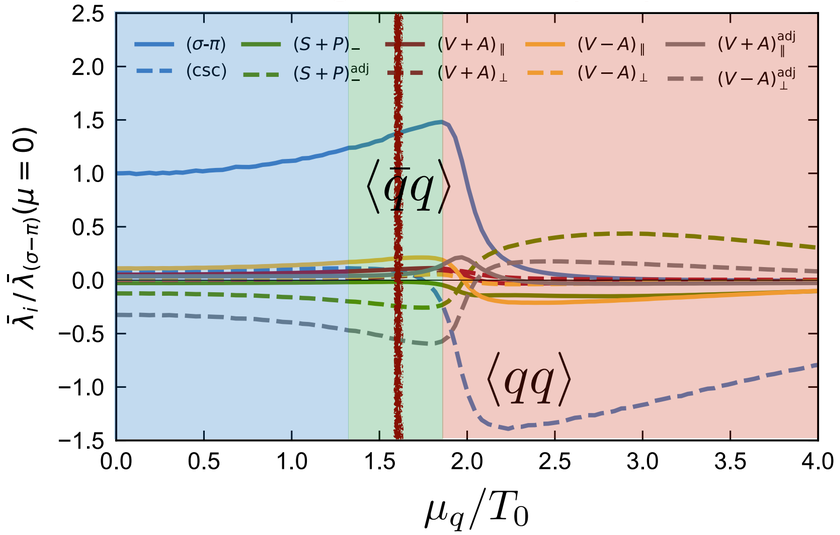
Prediction



Estimate

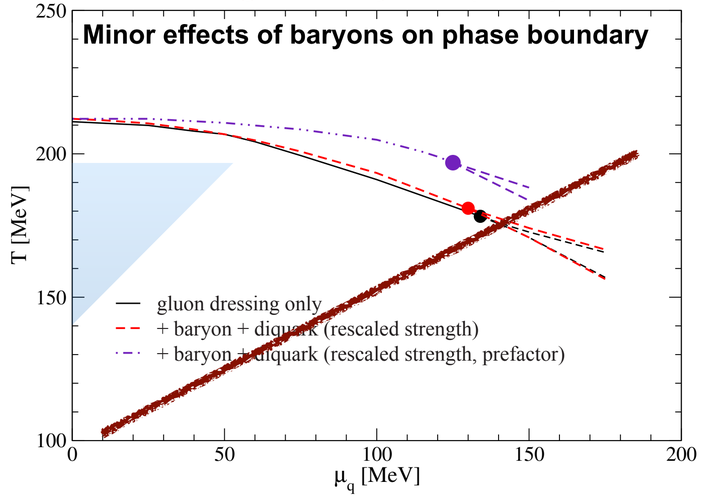
Regime of quantitative reliability of current best truncation

Emergent diquarks (fRG)



Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Emergent baryons (DSE)

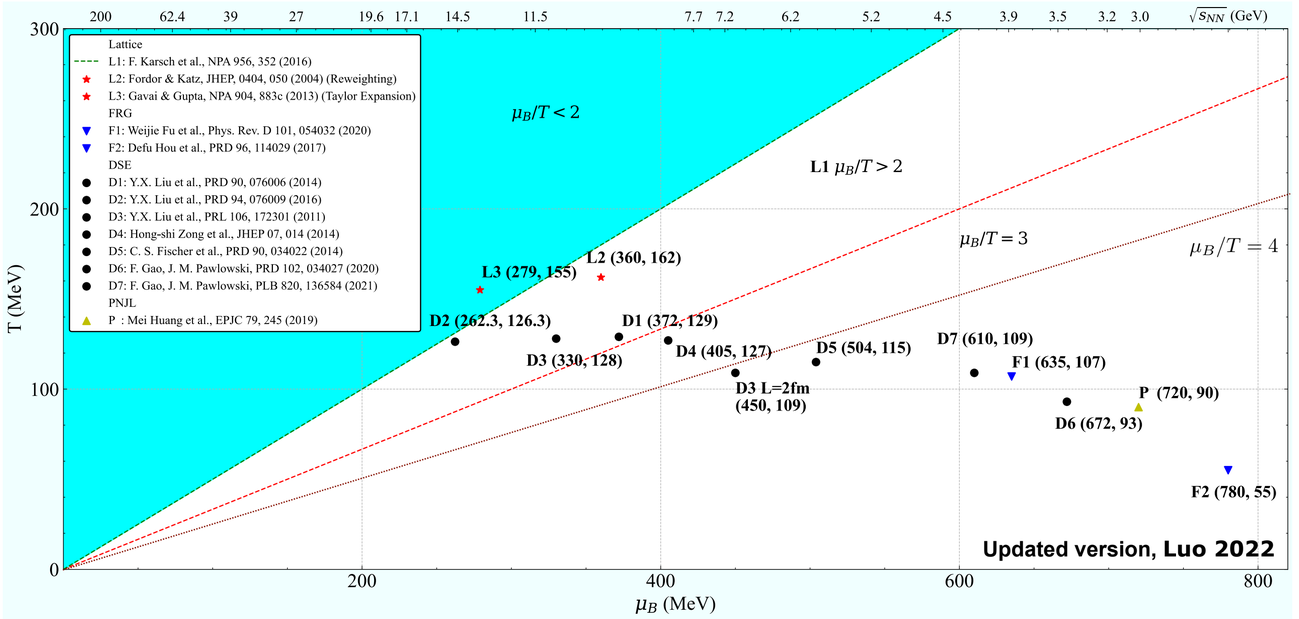


Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

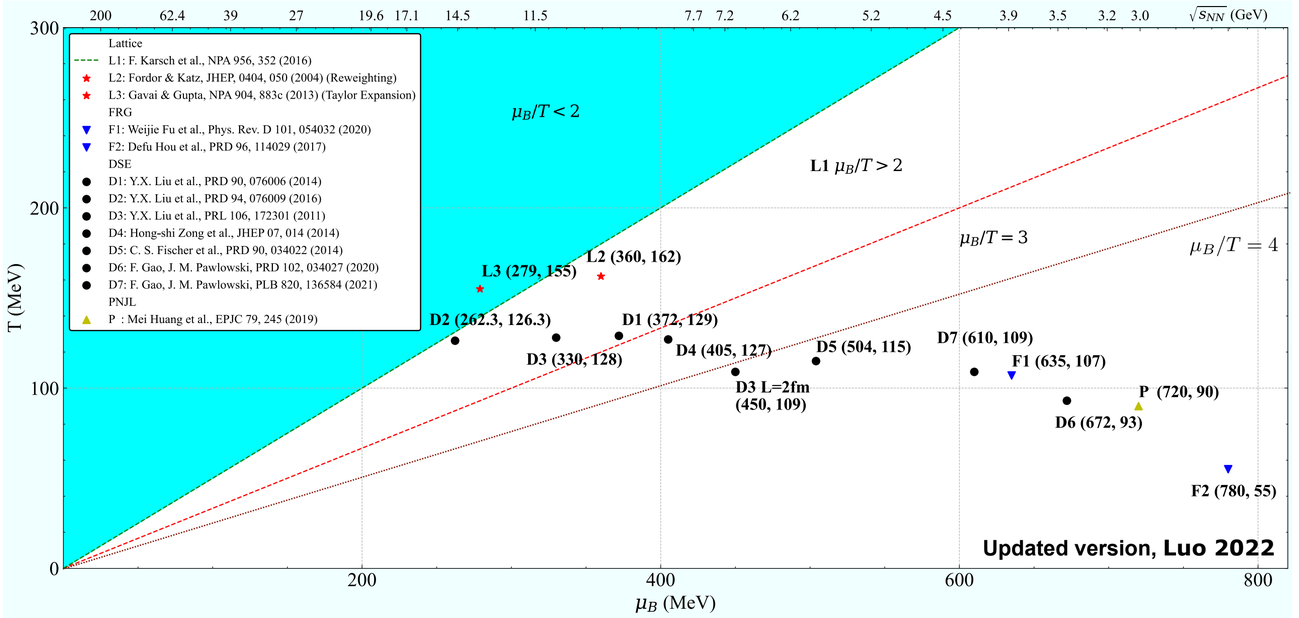
Lack of predictive power for CEP-predictions is no quality measure!

CEP is standing for 'regime with new physics'

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Common folklore since ~2004 ⟷ Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

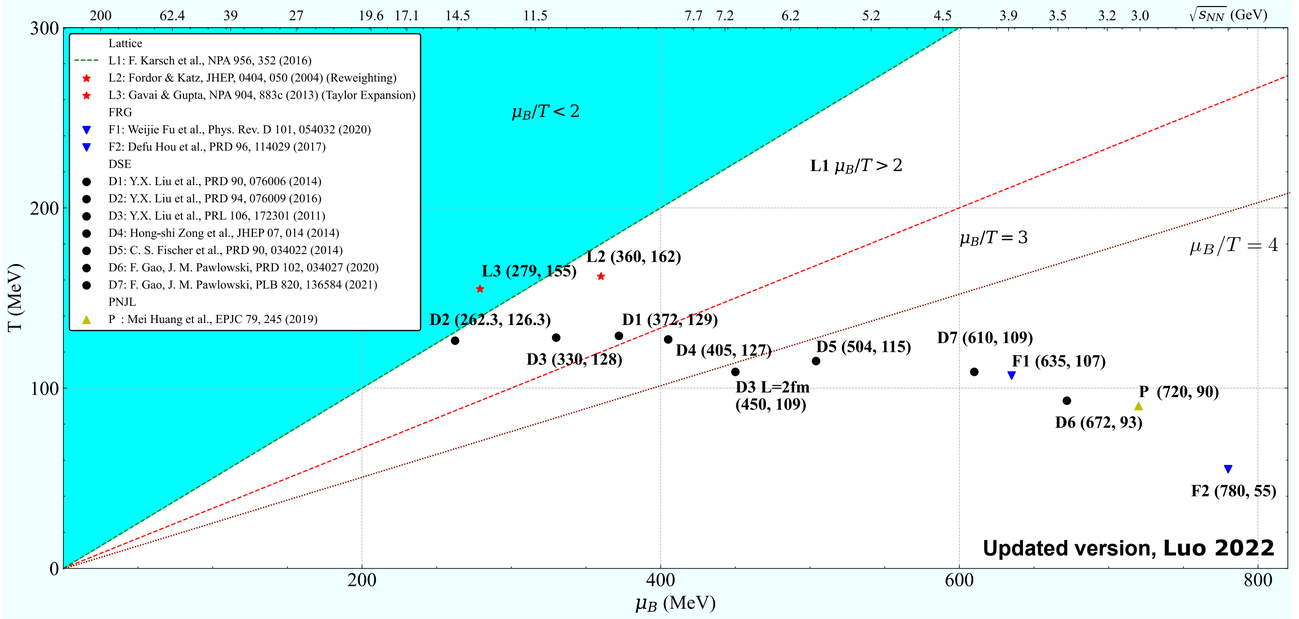
Lack of predictive power for CEP-predictions is no quality measure!

CEP is standing for 'regime with new physics'

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Remove CEP-predictions

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

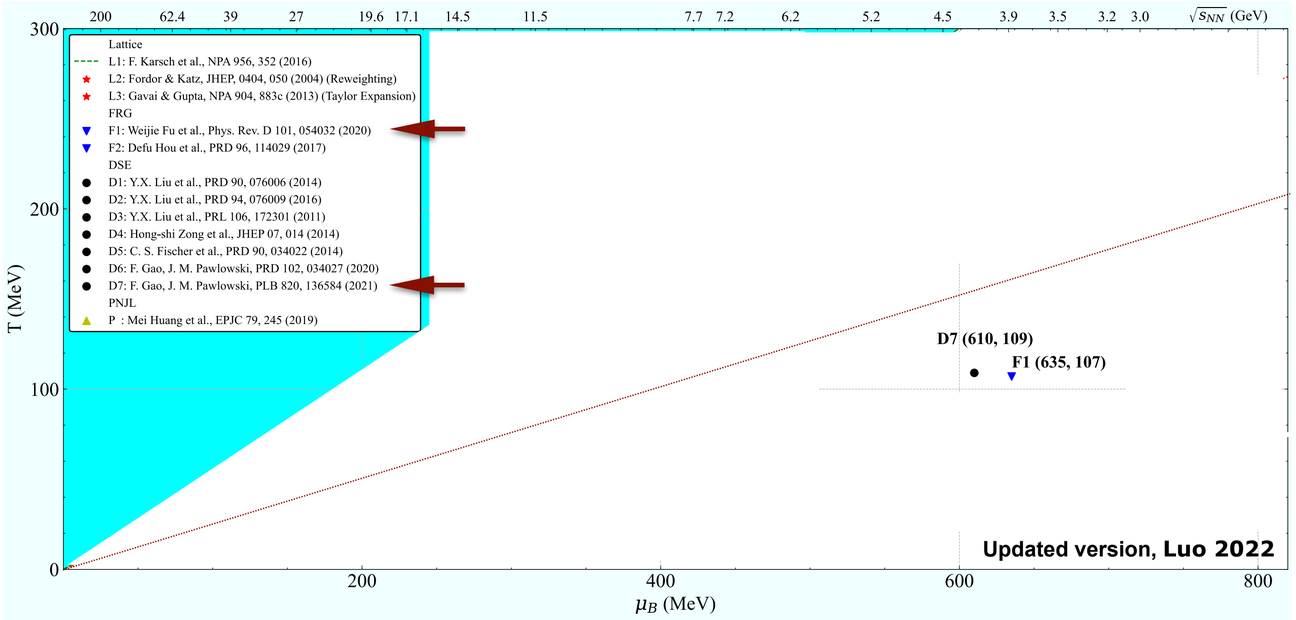
(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)

Functional QCD



Large uncertainties for the estimation of CP location.

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) ‘old’ CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

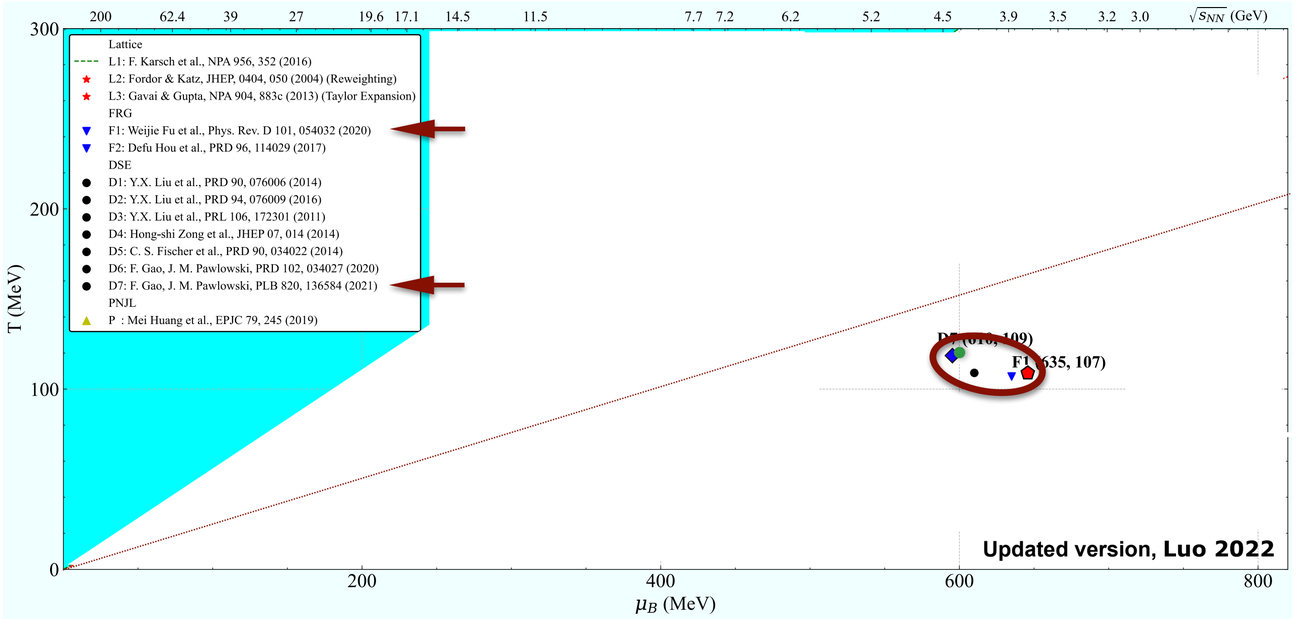
Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2023)

Functional QCD

- **Gao, Lu, JMP, Schneider, in prep (DSE)**
- **Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)**
- **Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)**



Large uncertainties for the estimation of CP location.

Remove CEP-predictions

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

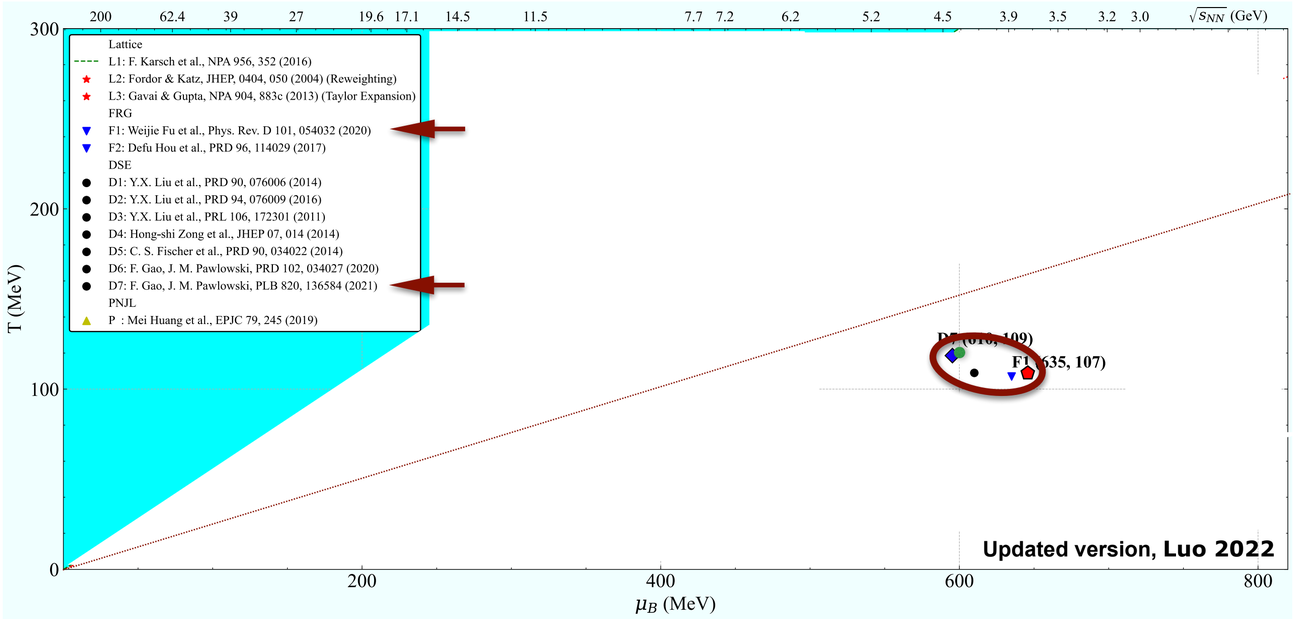
Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2023)

Functional QCD

- **Gao, Lu, JMP, Schneider, in prep (DSE)**
- **Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)**
- **Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)**



Still small uncertainties for the estimation of CP location **/Onset of new phases**

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2023)

Functional QCD

- Gao, Lu, JMP, Schneider, in prep (DSE)
- Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)
- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)

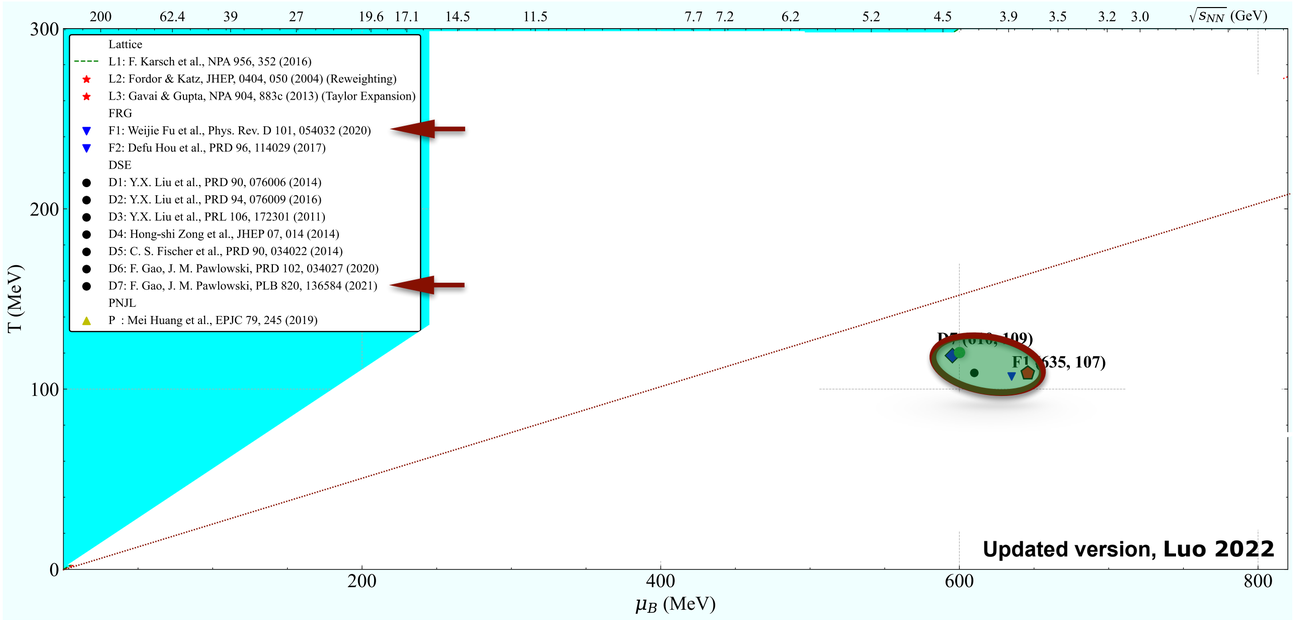
Extrapolations

Lattice extrapolations: Basar, PRC 110 (2024) 015203
Bielefeld-Parma, arXiv:2405.10196

⋮

Holographic models: Hippert, Grefa, Manning, Noronha, Noronha-Hostler, Portillo Vazquez, Ratti, Rougemont, Trujillo, arXiv: 2309.00579

⋮



Still small uncertainties for the estimation of CP location **/Onset of new phases**

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

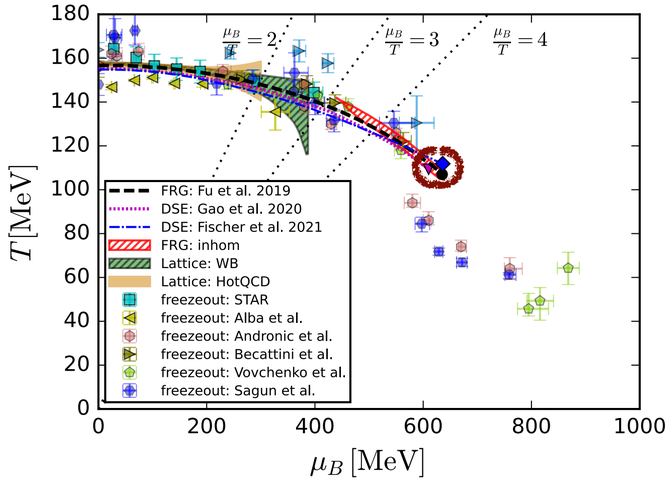
(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

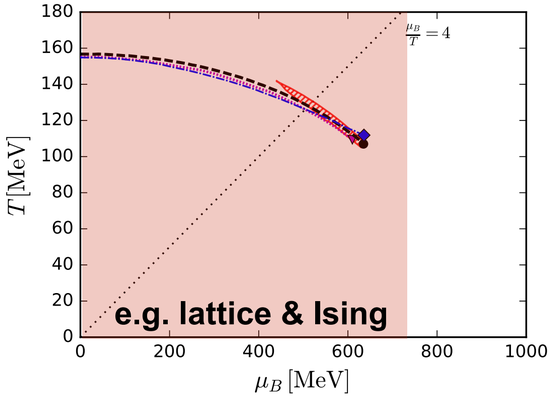
Predictions, estimates & extrapolations and how to use them

Predictions, estimates & extrapolations and how to use them



Scenario I

Scaling

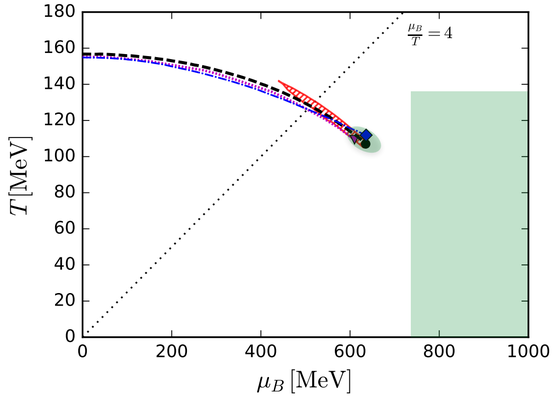


Extrapolations for Pheno

✓

Scenario II

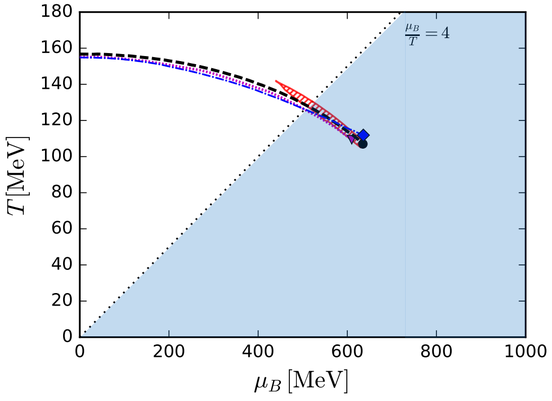
Minimal scaling and new phases



✓

Scenario III

Scaling and/or new phases



✗

Predictions, estimates & extrapolations and how to use them

Out by the LEGO ®principle

Fu, JMP, Rennecke, PRD 101 (2020) 054032

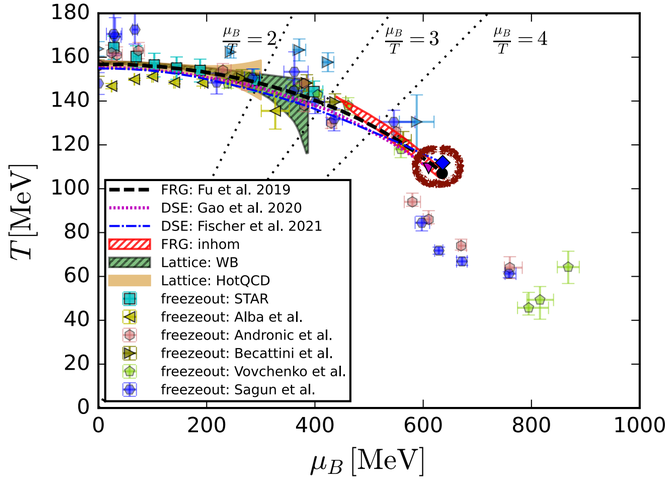
+

Size of scaling regime in LEFTs

Schaefer, Wambach, PRD 75 (2007) 085015

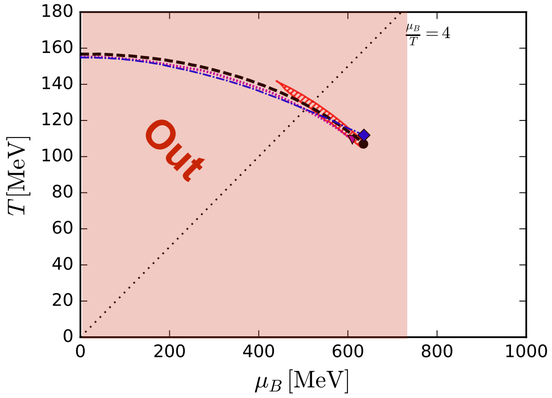
Braun, Klein, Piasecki, EPJC 71 (2011) 1576

⋮



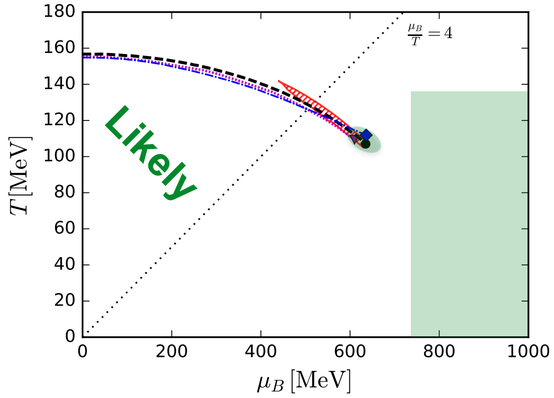
Scenario I

Scaling



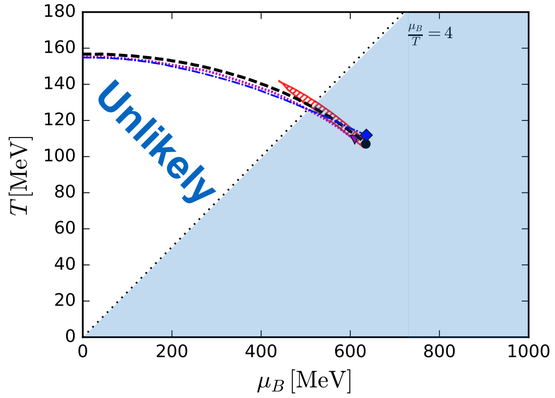
Scenario II

Minimal scaling and new phases



Scenario III

Scaling and/or new phases



Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

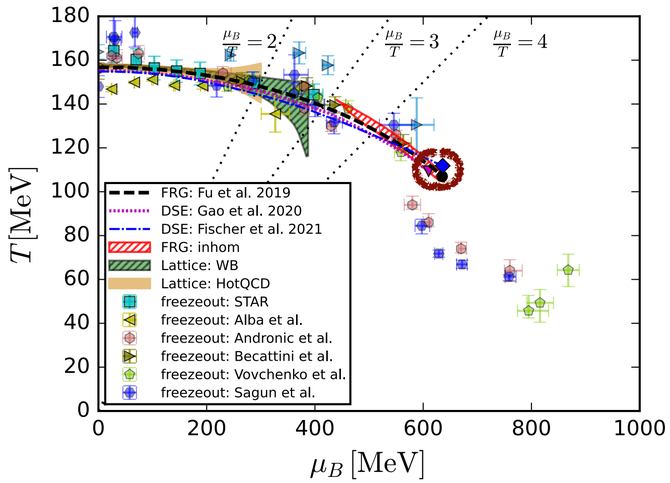
Gao, JMP, PRD 105 (2022) 094020

Soft modes in hot QCD matter: Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, arXiv:2310.19853

+ many results in dynamical low energy effective theories

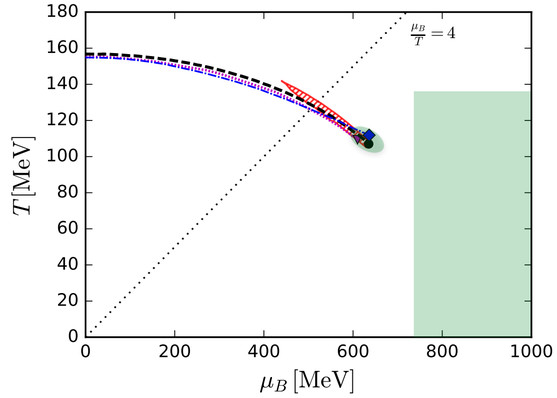
LEGO® principle

Predictions, estimates & extrapolations and how to use them



Scenario II

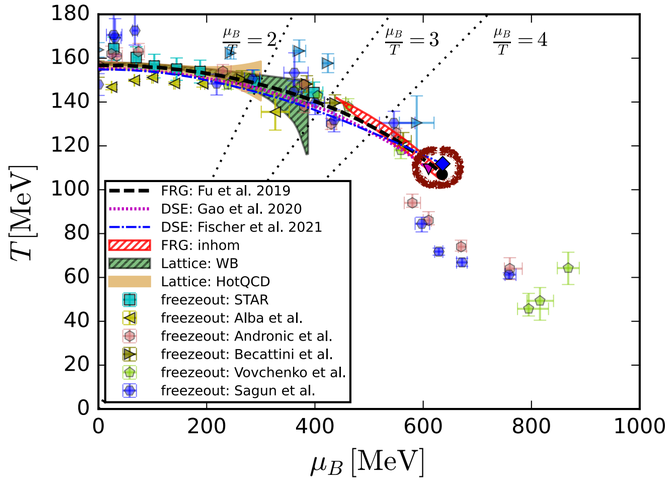
Minimal scaling and new phases



Extrapolations for Pheno

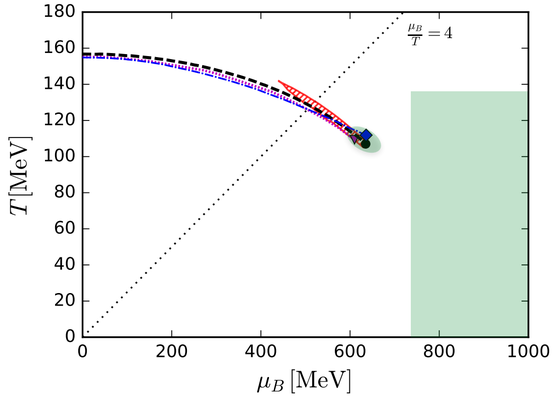
✓

Predictions, estimates & extrapolations and how to use them



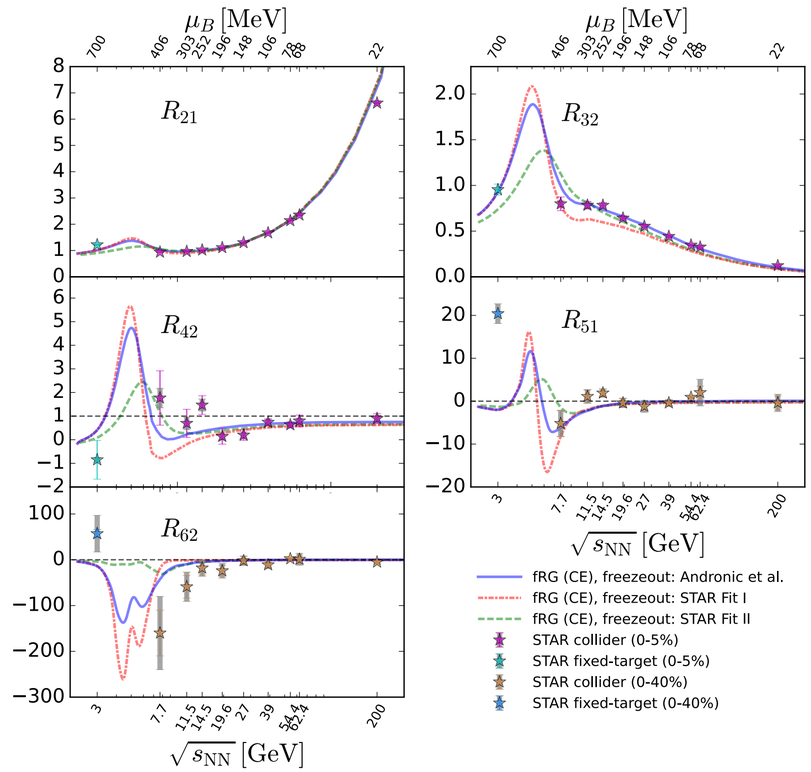
Scenario II

Minimal scaling and new phases



Ripples of the critical end point

baryon & proton number fluctuations



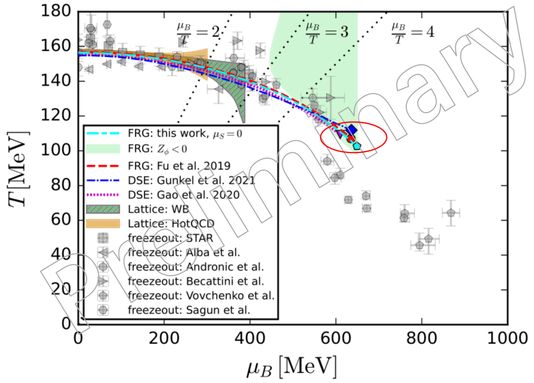
Extrapolations for Pheno

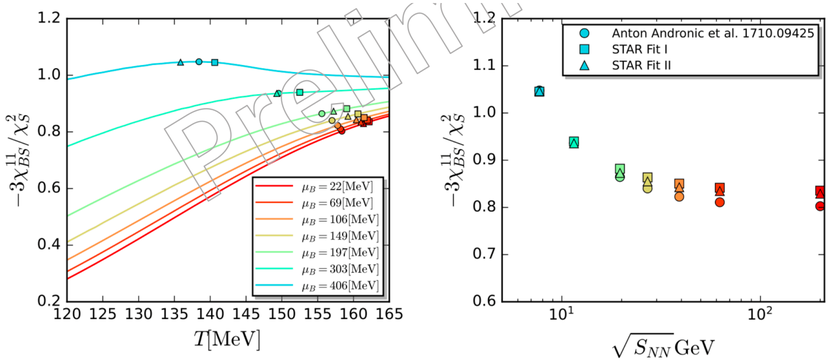
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

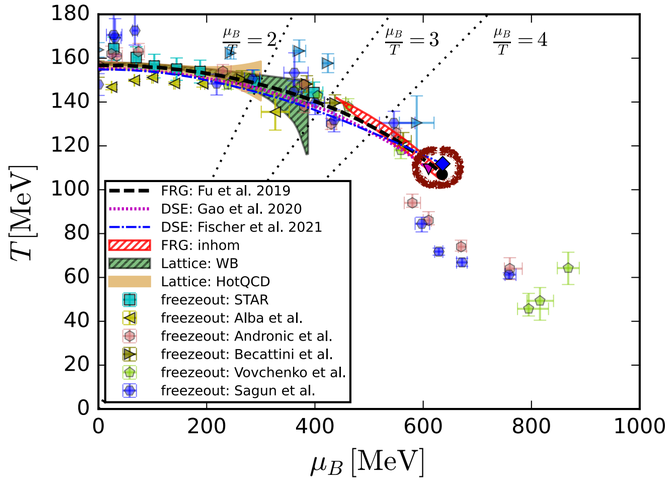
Predictions, estimates & extrapolations and how to use them

Strangeness neutrality

baryon & proton number fluctuations

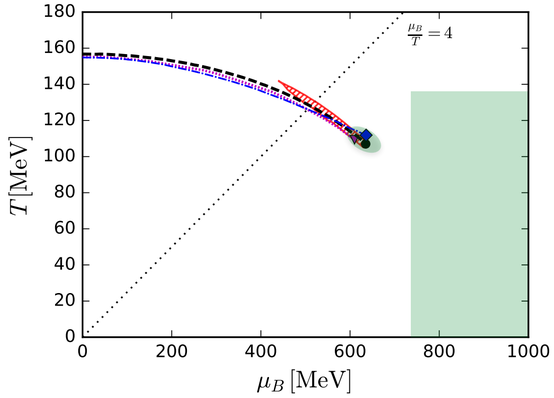






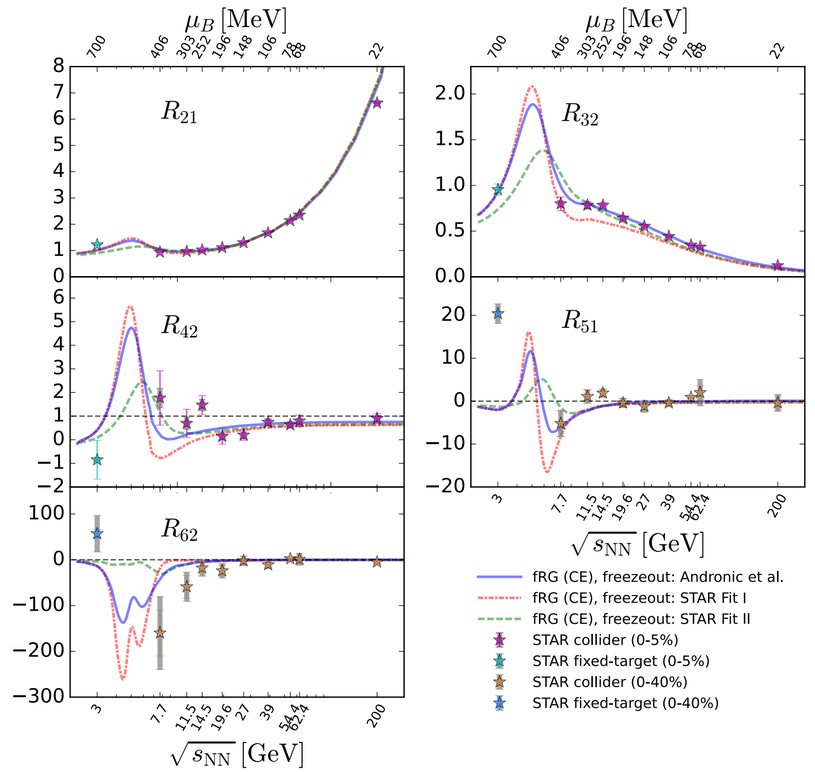
Scenario II

Minimal scaling and new phases



Ripples of the critical end point

baryon & proton number fluctuations

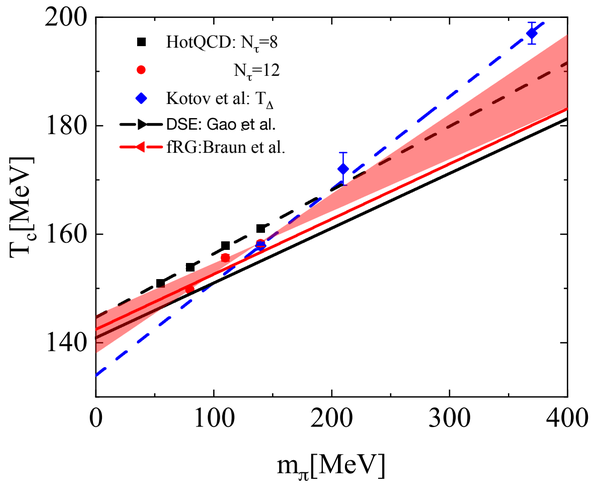


Fu, Huang, JMP, Rennecke, Wen, Yin, in preparation

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

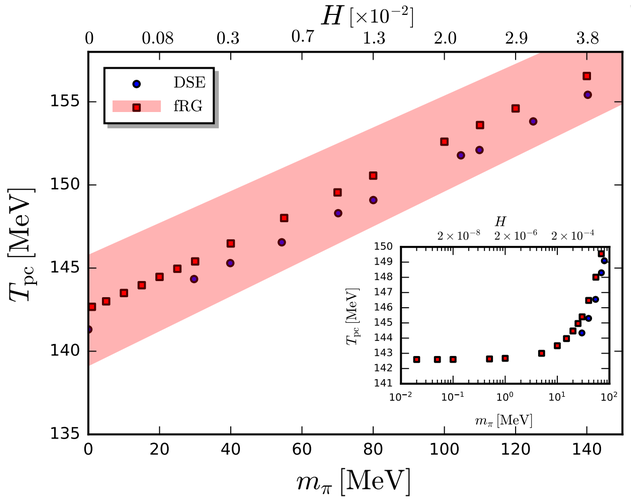
Chiral dynamics & soft modes

To be (critical) or not (to be)

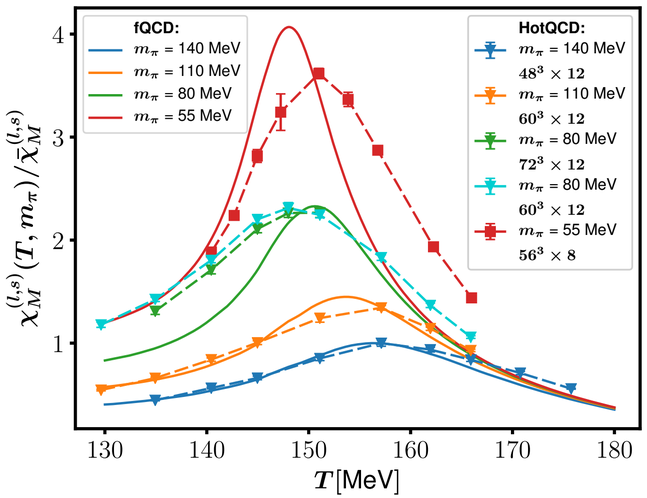


Chiral transition temperature

$$H = \frac{m_l}{m_s}$$



Magnetic susceptibility

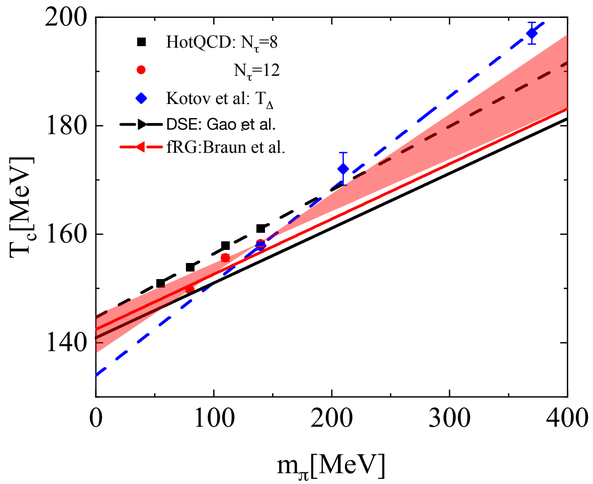


Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

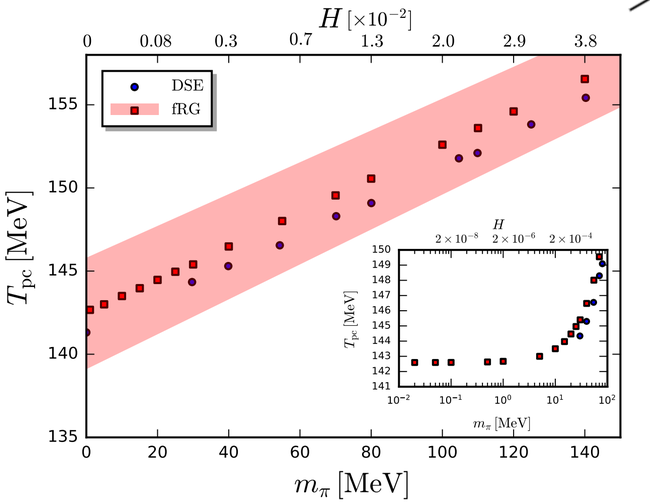
Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

To be (critical) or not (to be)



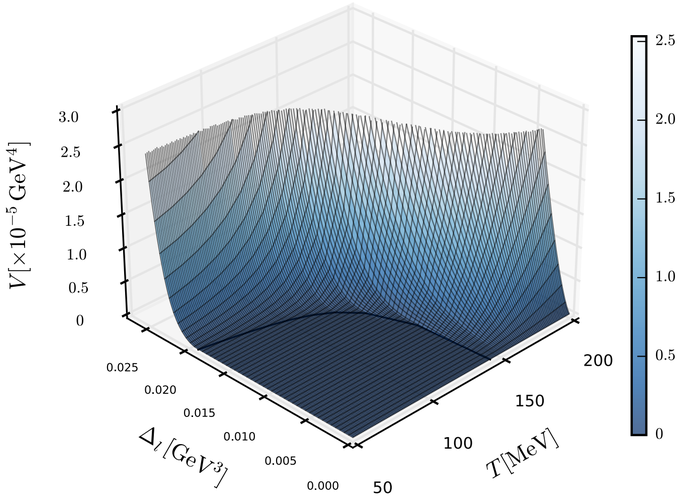
Chiral transition temperature

$$H = \frac{m_l}{m_s}$$



Order parameter potential & scaling

$$V_\chi \approx \Delta_l^n \quad \Longleftrightarrow \quad \Delta_l(H) \propto H^{\frac{1}{n-1}}$$



$$V_\chi^{(\mathrm{fRG})} \approx V_\chi^{(\mathrm{DSE})}$$

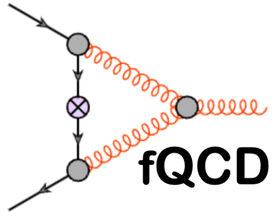
(Critical) exponent: $\frac{1}{\delta} = \frac{1}{n-1}$

Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

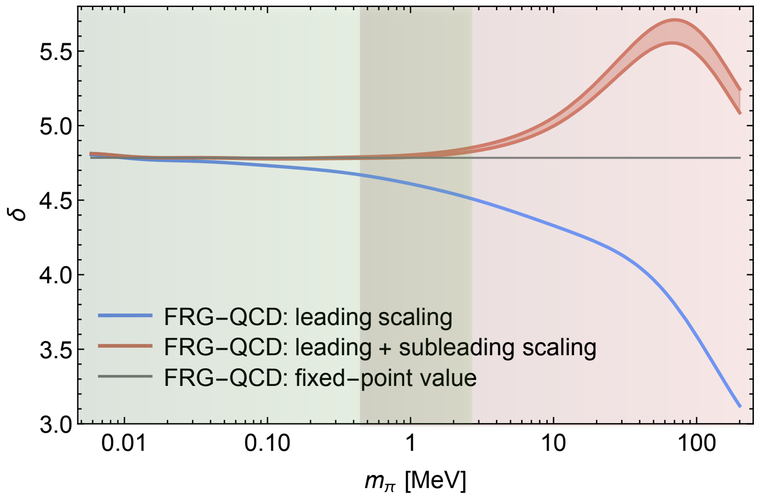
Gao, JMP, PRD 105 (2022) 094020

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes



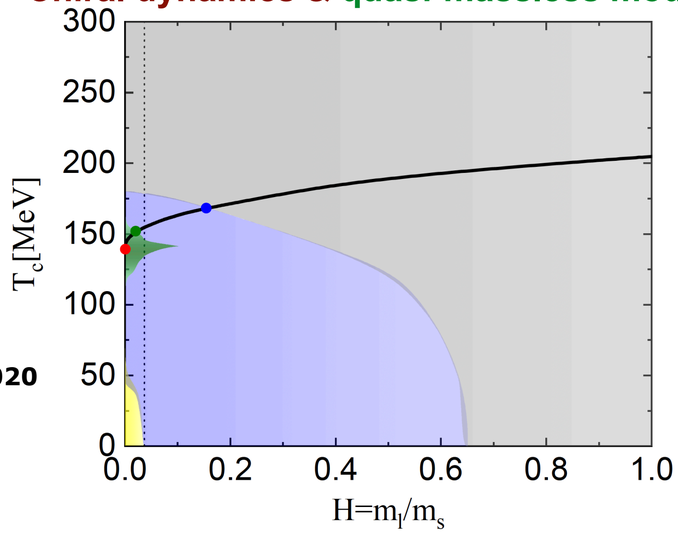
Scaling coefficient as function of the pion mass



Critical O(4) scaling

Trivial $\Delta_l^{1+\delta}$ scaling

Chiral dynamics & quasi-massless modes



Critical scaling

Gao, JMP, PRD 105 (2022) 094020

Far away from the critical regime for $m_\pi \gtrsim 1\,\mathrm{MeV}$

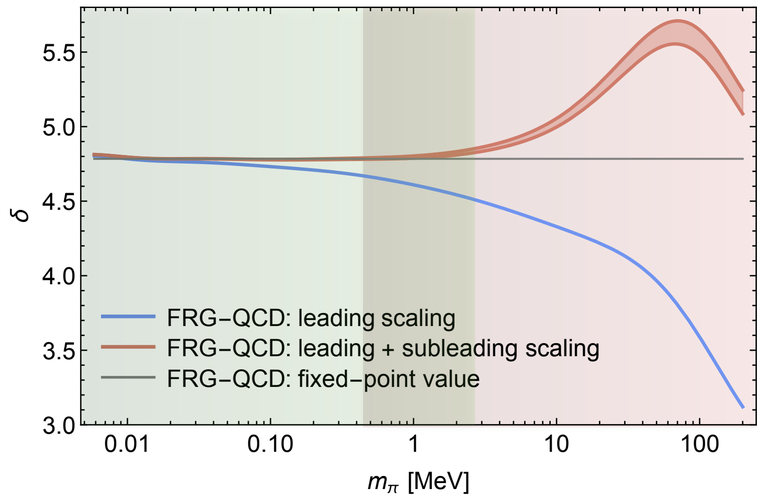
$$\Delta_l(T,H) \approx \Delta_{l,\chi}(0)\left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H\right)$$

$$V_\chi(\Delta_l) \propto \qquad \Delta_l^6 \qquad \Delta_l^4 \qquad \Delta_l^2$$

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



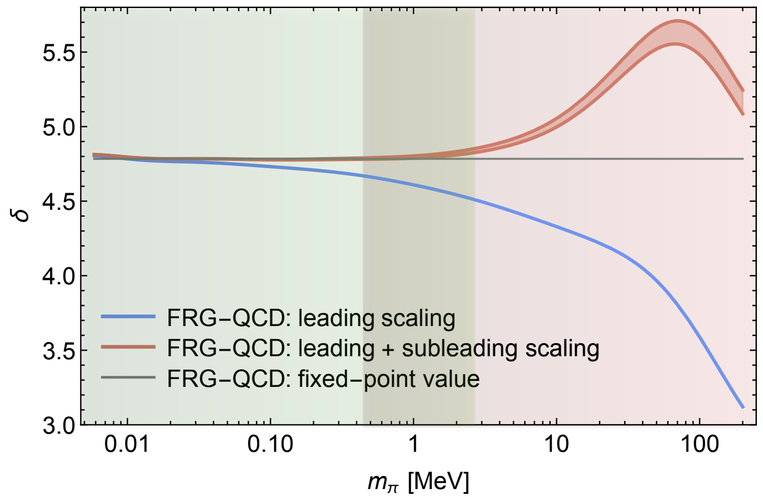
Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



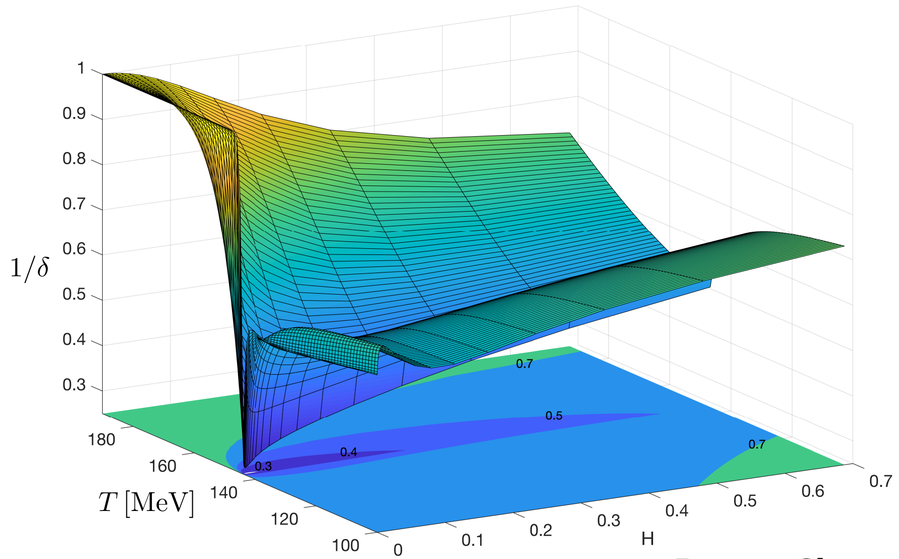
Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$

Small chiral scaling regime

↓

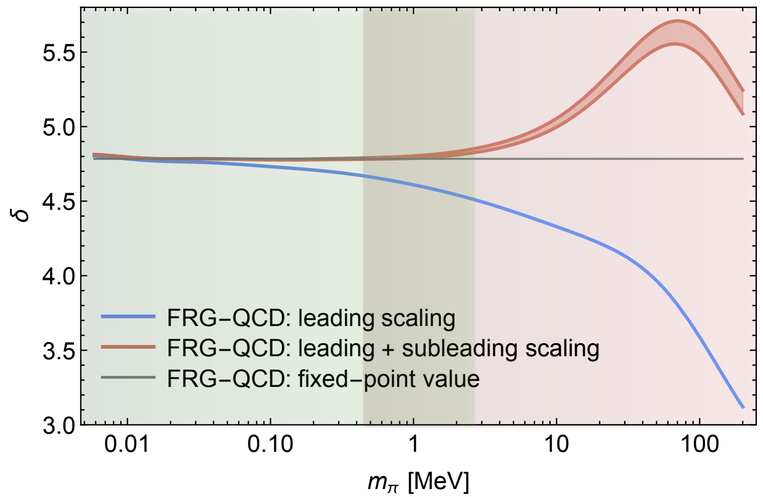
Small critical regime around pot. CEP



Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



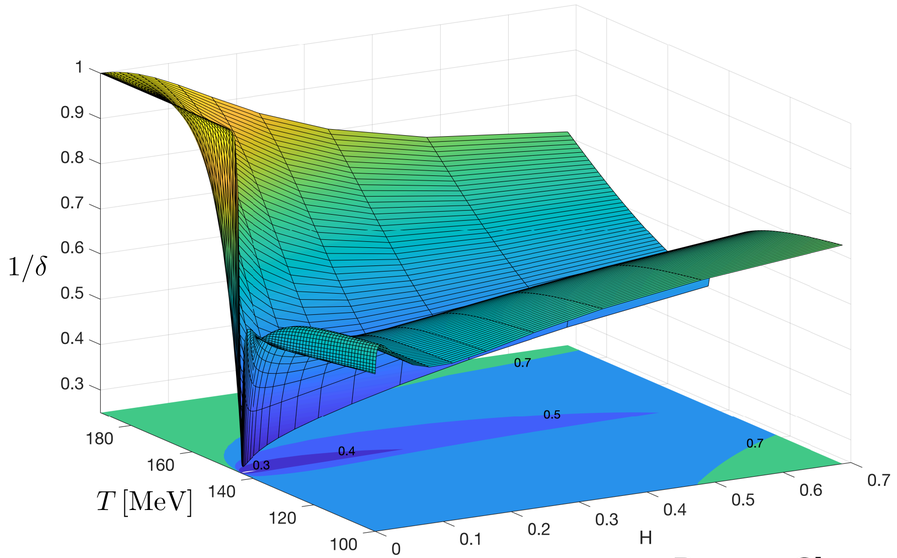
Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$

Small chiral scaling regime

↓

Small critical regime around pot. CEP



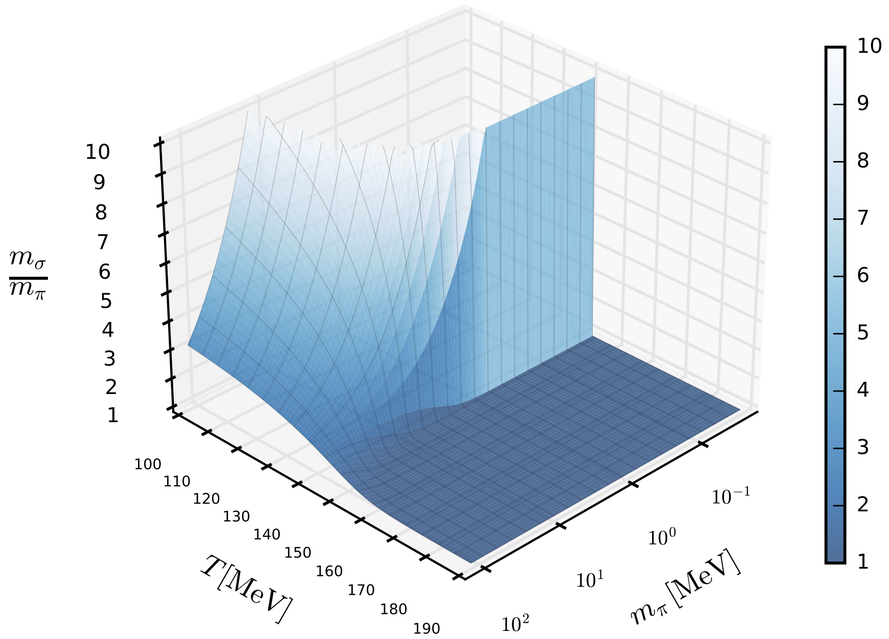
!!Great News!!

Location of CEP/New phase accessible via combination of precision measurements & computations

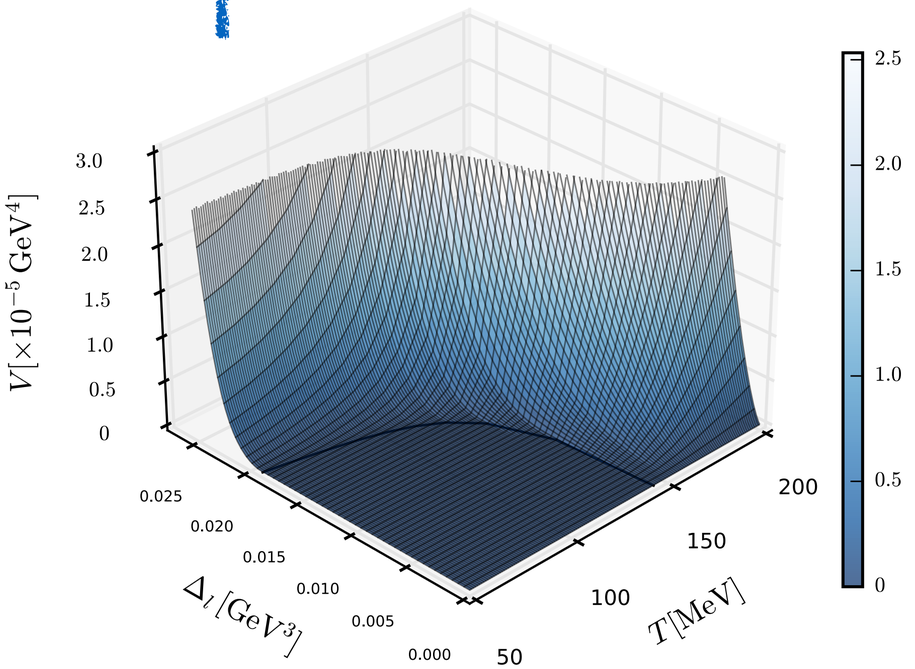
Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Full order parameter potential

Measure: correlation length



Use for chiral dynamics in heavy ion collisions



Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853