Ab initio reactions up to medium-mass nuclei with the symmetry-adapted resonating group method

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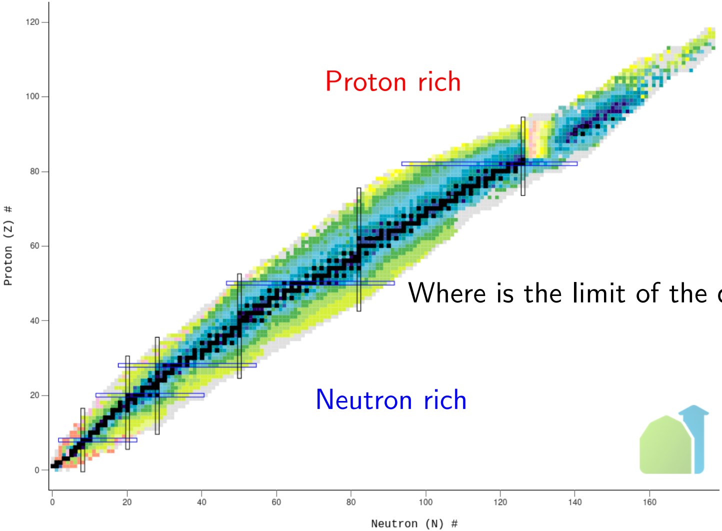
Outline

Exotic nuclei

Why are exotic nuclei important?

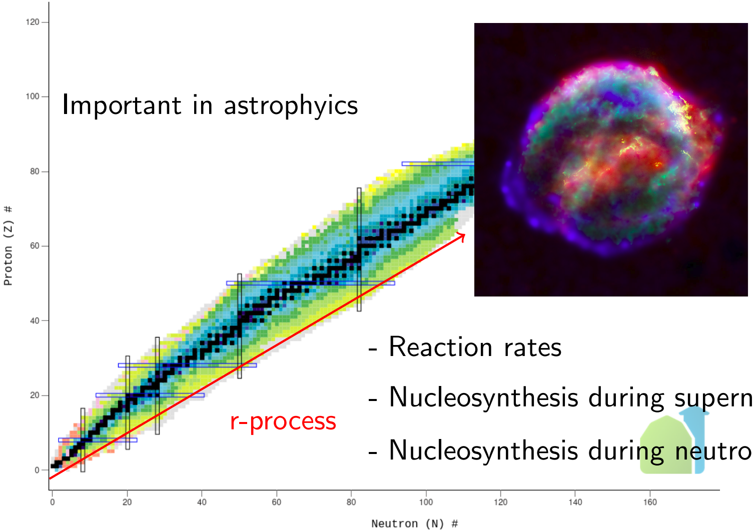


Where is the limit of the driplines?

Exotic nuclei

Why are exotic nuclei important?

Important in astrophyics



- Reaction rates
- Nucleosynthesis during supernovae
- Nucleosynthesis during neutron star mergers

Exotic nuclei

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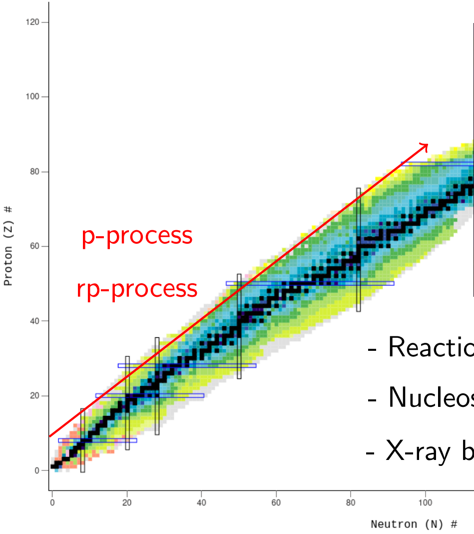
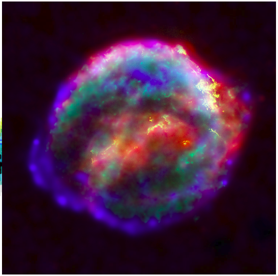


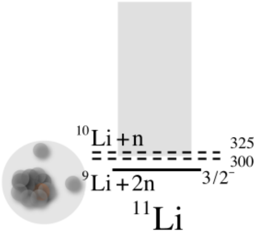


- Reaction rates
- Nucleosynthesis during supernovae
- X-ray bursts

Exotic nuclei

Exotic nuclei exhibit properties like:

- Short lifetimes
- Near threshold clustering
- Halo nuclei



These brings the following challenges:

Coupling to the continuum $\longrightarrow$ Structure calculation cannot ignore the coupling to the continuum.

How do we address the low energy regime where individual resonances are important?

Towards a unified nuclear structure and nuclear reaction approaches!

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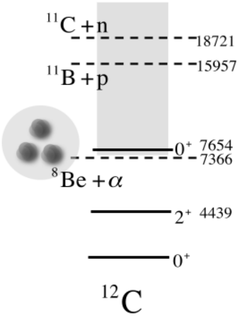

These brings the following challenges:

Coupling to the continuum $\longrightarrow$ Structure calculation cannot ignore the coupling to the continuum.

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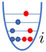
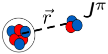
Towards a unified nuclear structure and nuclear reaction approaches!

We consider the ansatz:

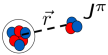
$\sum_i c_i$ 

$$|(c,r)^{J^\pi}\rangle = \mathcal{A}_c \left\{ |\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle \right\}^{J^\pi} = \mathcal{A}_c |$$  $$\rangle.$$

Antisymmetrized!

The wave functions are expanded in this basis

Relative motion between T and P.

$$|\Psi^{J^\pi}\rangle = \sum_c \int_0^\infty \left(\frac{g_c^{J^\pi}(r)}{r} \right) \mathcal{A}_c |$$  $$\rangle r^2 dr.$$

Which lead to the coupled channel equation (Hills-Wheeler)

$$\sum_c \int_0^\infty \left[H_{cc'}^{J^\pi}(r,r') - E\, N_{cc'}^{J^\pi}(r,r') \right] \frac{g_c^{J^\pi}(r)}{r} r^2 dr = 0\,.$$

Channels are not orthogonal!

Resonating group method (RGM)

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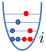
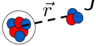
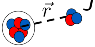
$\sum_i c_i$  $_i$

$$|(c,r)^{J^\pi}\rangle = \mathcal{A}_c \left\{ |\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle \right\}^{J^\pi} = \mathcal{A}_c |$$  $$^{J^\pi}\rangle.$$

Antisymmetrized!

The wave functions are expanded in this basis

Asymptotics yield scattering information.

$$|\Psi^{J^\pi}\rangle = \sum_c \int_0^\infty \left(\frac{g_c^{J^\pi}(r)}{r} \right) \mathcal{A}_c |$$  $$^{J^\pi}\rangle r^2 dr.$$

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Channels are not orthogonal!

Resonating group method (RGM)

First, we go from a generalized eigenvalue problem to the standard case by:

$$H|\Psi\rangle = EN|\Psi\rangle \rightarrow \bar{H}|\bar{\Psi}\rangle = E|\bar{\Psi}\rangle\,,$$

where we have defined and orthogonalized channel basis

$$\bar{H} = \hat{N}^{-1/2}\hat{H}\hat{N}^{-1/2} \qquad |\bar{\Psi}\rangle = \hat{N}^{1/2}|\Psi\rangle\,.$$

This allows us to get the interaction between channels as a non-local potential:

$$\left(T_c(r) + E_{I_1^{\pi_1}} + E_{I_2^{\pi_2}}\right)\frac{u_c^{J^\pi}(r)}{r} + \sum_{c'}\int dr'\, r'^2\, W_{cc'}(r,r')\frac{u_{c'}^{J^\pi}(r')}{r'} = 0,$$

which we can compute in matrix form as:

$$W = N^{-1/2}T_{\mathrm{rel}}N^{1/2} - T_{\mathrm{rel}} + N^{-1/2}V_{\mathrm{rel}}\hat{N}^{-1/2}.$$

We compute scattering observables with a microscopic R-matrix calculation.

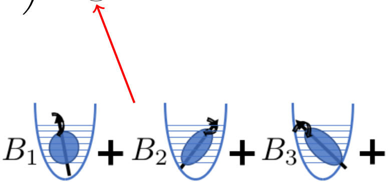
Resonating group method (RGM)

Let's use the wavefunctions from a microscopic model!

$$|\Psi\rangle = \sum_c \int_0^\infty \left(\frac{g_c(r)}{r}\right) \mathcal{A}|\,\bullet \overset{\vec{r}}{- - -} \bullet\,\rangle r^2 dr.$$

Symmetry-adapted No-core Shell Model

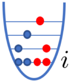
- All nucleons active.
- Chiral EFT interaction
- RGM version only nucleon projectiles (for now).



Symmetry-adapted no-core shell model

SA-NCSM is based on standard configuration interaction (CI) approach

- Harmonic oscillator basis.
- Model space set by $N_{\max}\hbar\omega$
- Hamiltonian from first principles $V_{NN} = V_{\chi_{\mathrm{EFT}}}$.

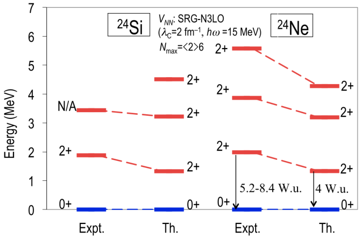
CI $\rightarrow |$  $_i\rangle$

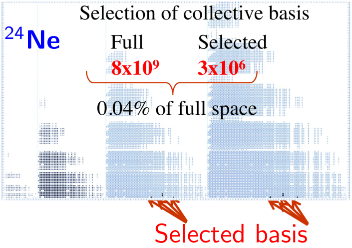
SA-NCSM $\rightarrow |$  $\rangle$

The SA-NCSM leverages near-exact symmetries

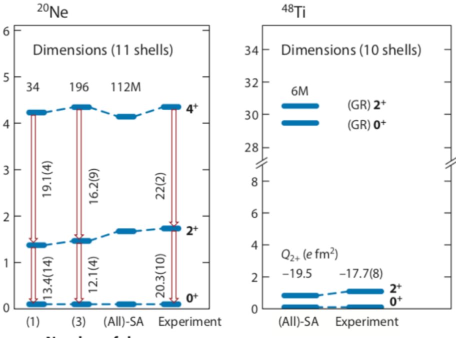
- Reorganization of the Slater determinant basis into SU(3) coupled states.
- This allows the construction of model spaces containing only the most physically relevant configurations.
- This truncation still has exact CM factorization.
- In a complete model space equivalent to CI.

Symmetry-adapted no-core shell model





K. D. Launey et al., Annu. Rev. Nucl. Part. Sci. **71**, 253–277 (2021)



- Few basis states contribute and they capture relevant correlations.
- Practically exact calculations with selections.

Two important questions to address

- What do I mean by SU(3) coupled states?

- How do we actually do the selection of the basis?

The SU(3)×SU(2) basis

SU(2) coupling:

$$|j_1 j_2 JM\rangle = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | JM\rangle \, |j_1 m_1 j_2 m_2\rangle$$

Clebsch–Gordan (SU(2) coupling) coefficients

SU(3) coupling:

$$|(\lambda_3\mu_3)\alpha_3\rangle_\rho = \sum_{\alpha_1\alpha_2} \langle(\lambda_1\mu_1)\alpha_1;(\lambda_2\mu_2)\alpha_2|(\lambda_3\mu_3)\alpha_3\rangle \, |(\lambda_1\mu_1)\alpha_1\rangle \, |(\lambda_2\mu_2)\alpha_2\rangle$$

SU(3) coupling coefficients

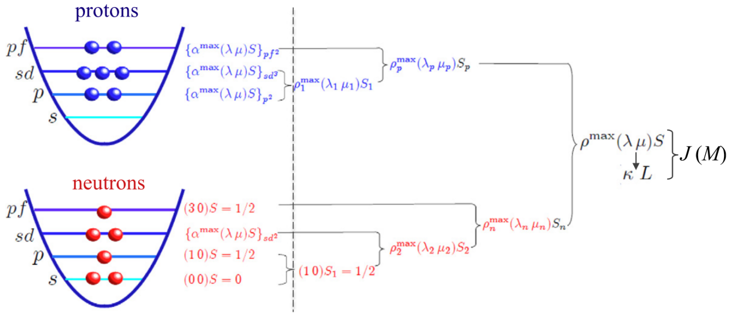
The $(\lambda\mu)$ projection convention used is:

$$\alpha \to \kappa LM$$

Later we can couple in SU(2):

$$\{LS\} \to \{j\}$$

The SU(3)×SU(2) basis



K. D. Launey et al., Prog. Part. Nuc. Phys. **89**, 101–136 (2016)

In the HO basis, we typically use:

$$N = 2n + \ell,$$

but we can also use Cartesian:

$$N = n_x + n_y + n_z$$

We can obtain (λ, μ) by:

$$\lambda = n_z - n_x \qquad \mu = n_x - n_y$$

Physical interpretation of an SU(3) irrep

We have two quantum numbers (λ, μ)

$(\lambda, 0) \sim n_z - n_x \rightarrow$ prolate shape

$(0, \mu) \sim n_x - n_y \rightarrow$ oblate shape

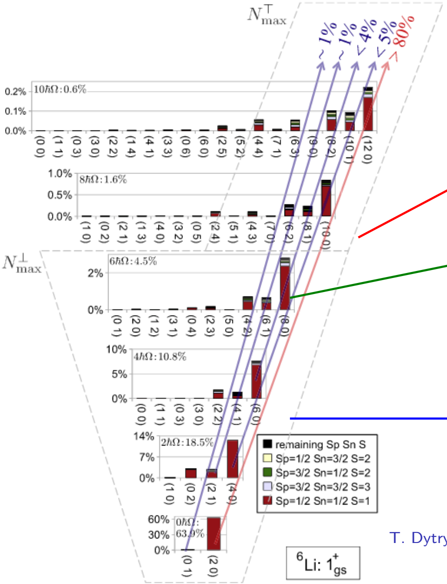
Specific superposition of(λ, μ)
creates triaxial shapes

We can relate them to β and γ too:

$$\beta^2 \propto \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + u) \qquad \gamma = \arctan\left(\frac{\sqrt{3}\mu}{2\lambda + \mu}\right)$$

J. P. Draayer et al., Phys. Rev. Lett. **62**, 20–23 (1989)

Model space selection



Selected SU(3) irreps

- Relevant contibutions
- Controlled approximation

$2\hbar\omega$ raising operator

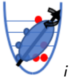
$\lambda + \mu = \lambda_0 + \mu_0 + N$

$N = 0, 2, 4, \ldots$

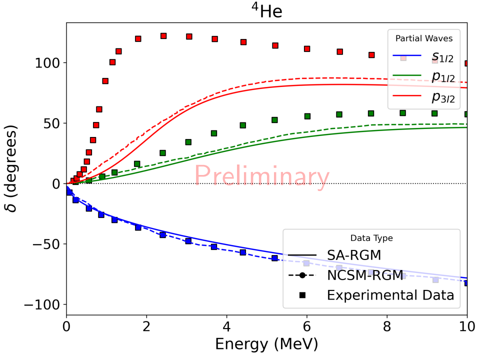
Full $N_{\max}$ (No selection)

T. Dytrych et al., Phys. Rev. Lett. **111**, 252501 (2013) nucleon (for now)

$$|(c,r)\rangle = \mathcal{A}\left\{|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle\right\}_{M_A}^{J_A}$$

$= \sum_i c_i |$  $\rangle_i$ from SA-NCSM

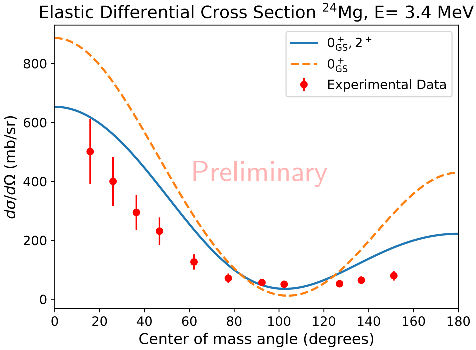
- Interaction: NNLOopt
- $N_{\max} = 14$
- Two target states: Ground state 0^+ and excited 2^-



^{24}Mg + n

We can compute cross-sections for intermediate nuclei

- Interaction: NNLOopt
- $N_{\mathrm{max}} =< 2 > 6$
- Two target states: Ground state 0^+, first excited 2^+.
- Up to $\ell = 8$ partial waves.



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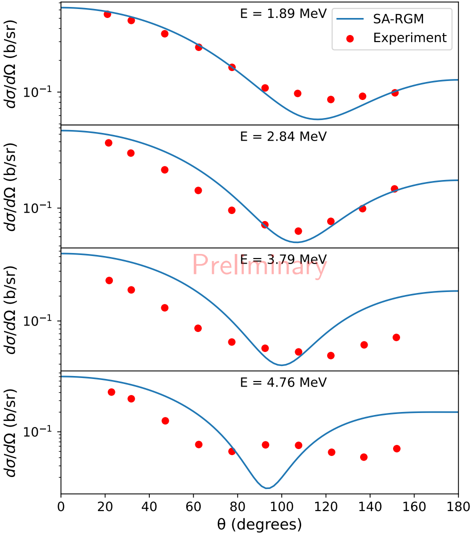
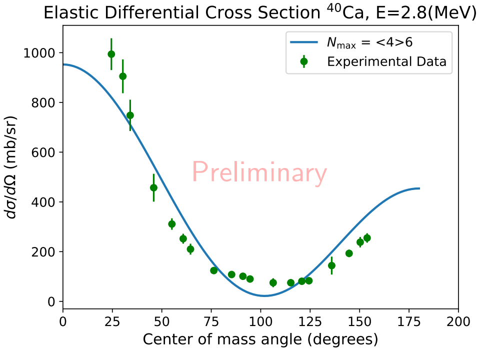


$^{40}\mathrm{Ca} + n$

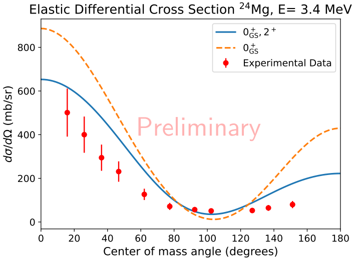
Let's push the model even more!

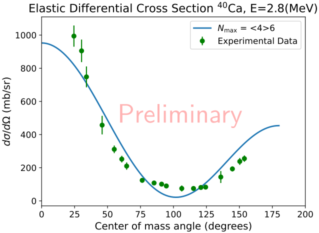
- Interaction: NNLOopt
- $N_{\max} =< 4 > 6$
- One target state: Ground state 0^{+}
- Up to $\ell = 8$ partial waves.



Conclusions and outlook

- The resonating group method provides a powerful tool that can be applied to different nuclear models to unify nuclear structure and reactions.
- The SA-RGM is a powerful method that allows to do *ab initio* reaction calculations of intermediate nuclei at bigger $N_{\max}$ spaces.
- Next step: radiative capture.





Acknowledgments

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- Noah Thompson (LSU, graduate student)

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