Electro-scattering on light nuclei and beyond

Next generation ab-initio nuclear theory
ECT*, Trento, Italy
July 17, 2025

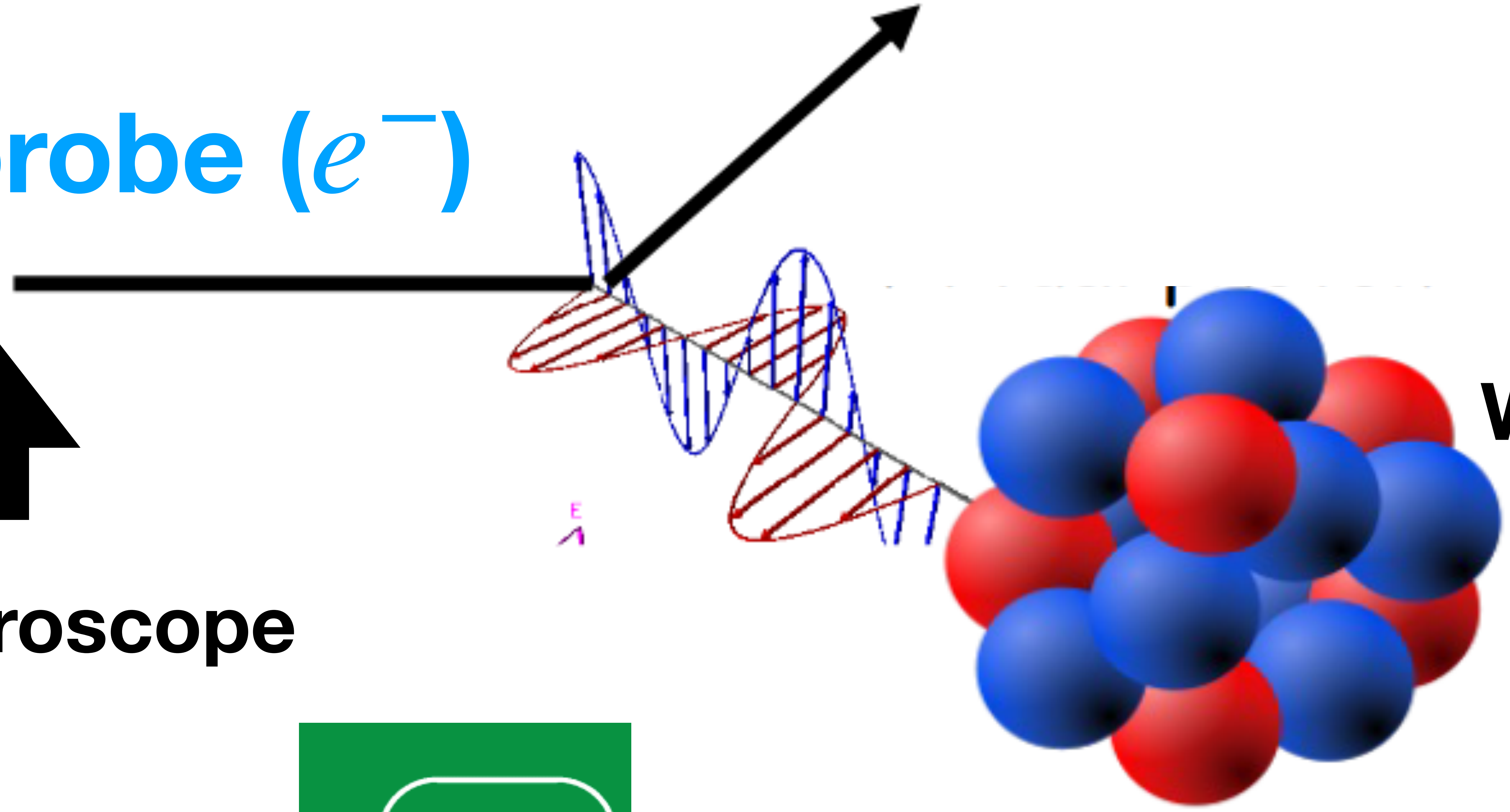
Alex Gnech (agnech@odu.edu)

OLD DOMINION UNIVERSITY

Jefferson Lab

Studying the nature of the strong interaction

External probe (e^-)



The microscope

What we want to study

Nucleus

Jefferson Lab

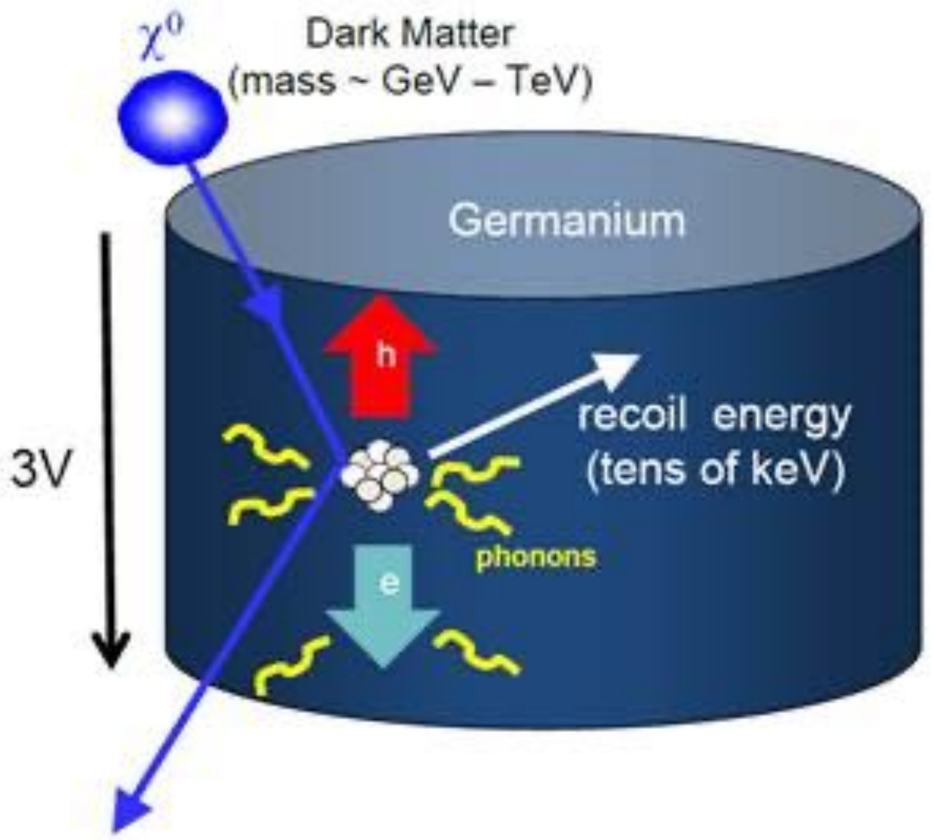
mami MAINZER MIKROTRON

EIC

Fundamental physics with nuclei

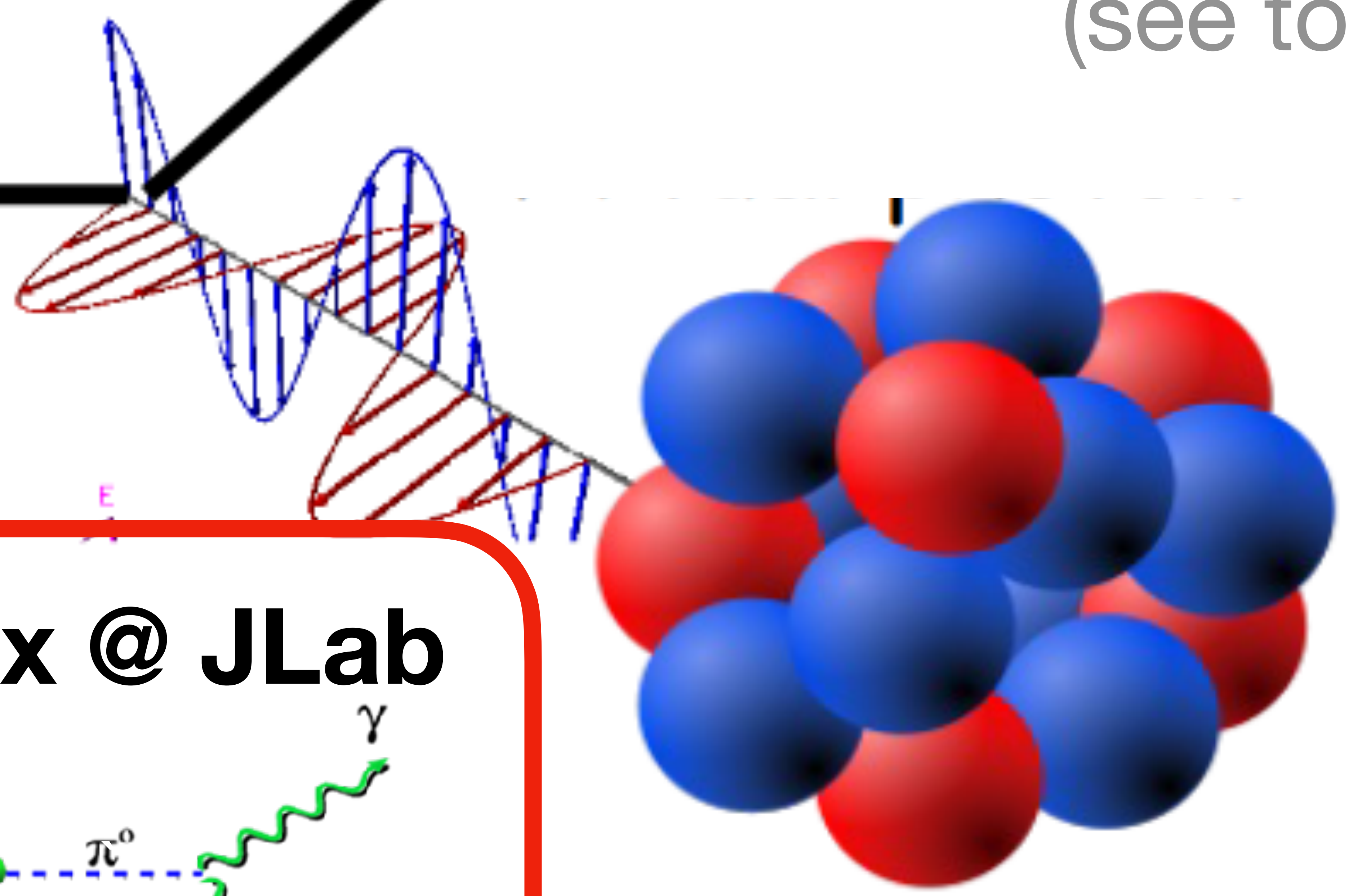
What we want to study

Dark matter



(see tomorrow session)

External probes

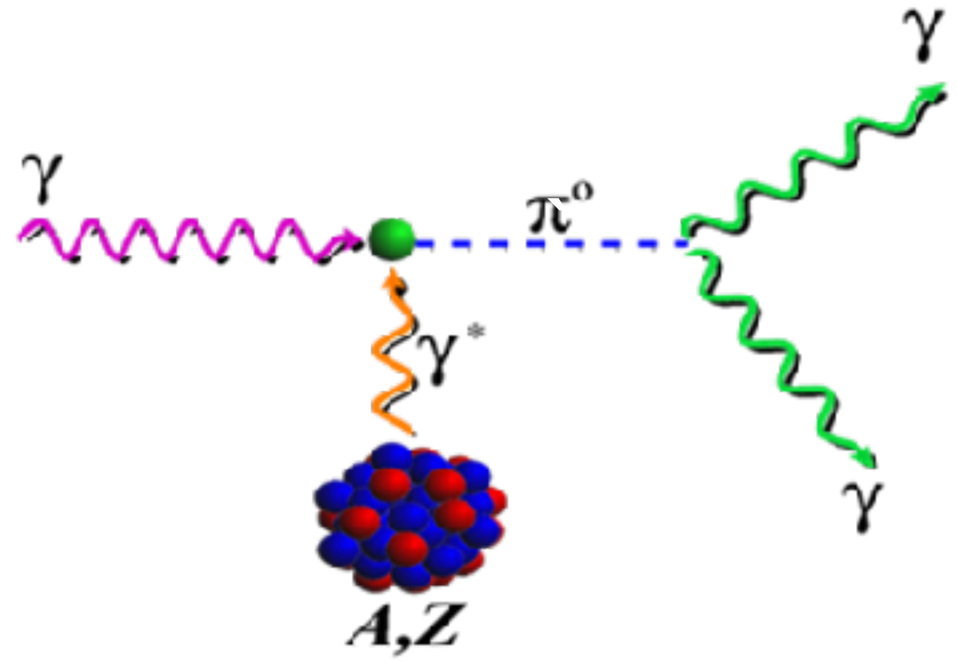


Neutrino Physics

DUNE

T2K

PrimEx @ JLab



Nucleus

The microscope

Scattering of electrons on nuclei (elastic)

$$\frac{d\sigma}{d\Omega} = 4\pi\sigma_M f_{\text{rec}}^{-1} \left[\frac{Q^4}{q^4} F_L^2(q) + \left(\frac{Q^2}{2q^2} + \tan^2 \theta_e/2 \right) F_T^2(q) \right]$$

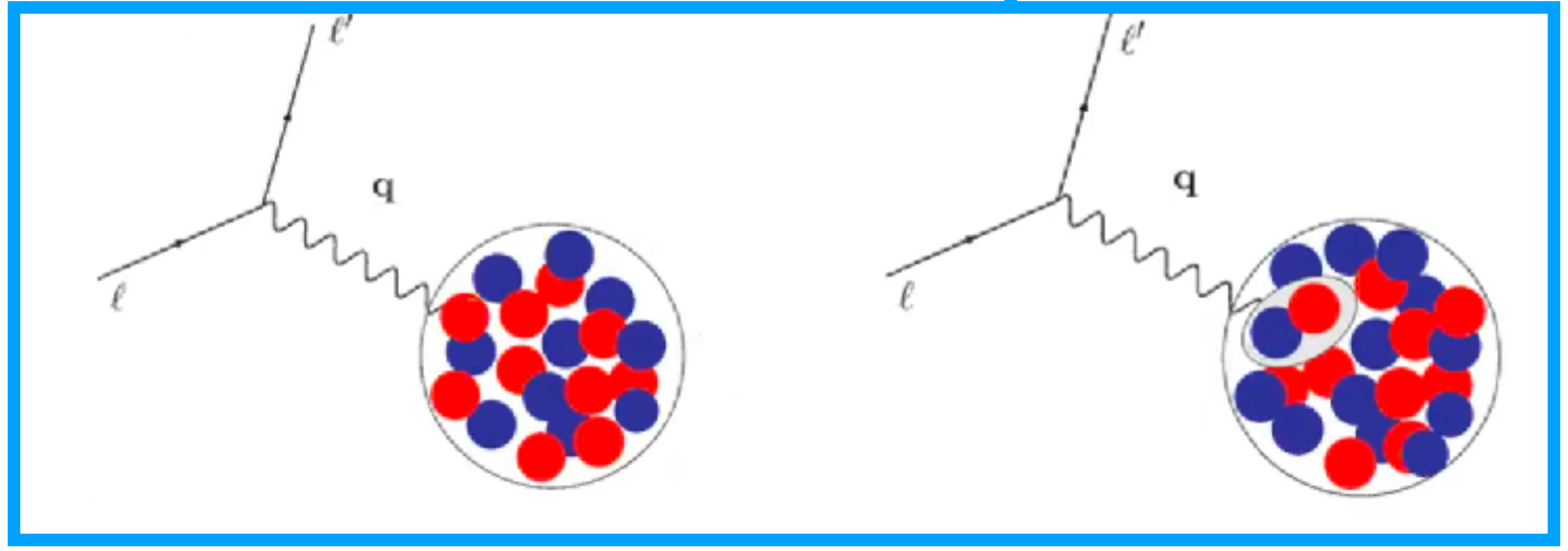
$$F_L(q) \propto \langle \psi | \rho(q) | \psi \rangle$$

$$F_T(q) \propto \langle \psi | j_y(q) | \psi \rangle$$

$$H | \psi \rangle = E | \psi \rangle$$

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

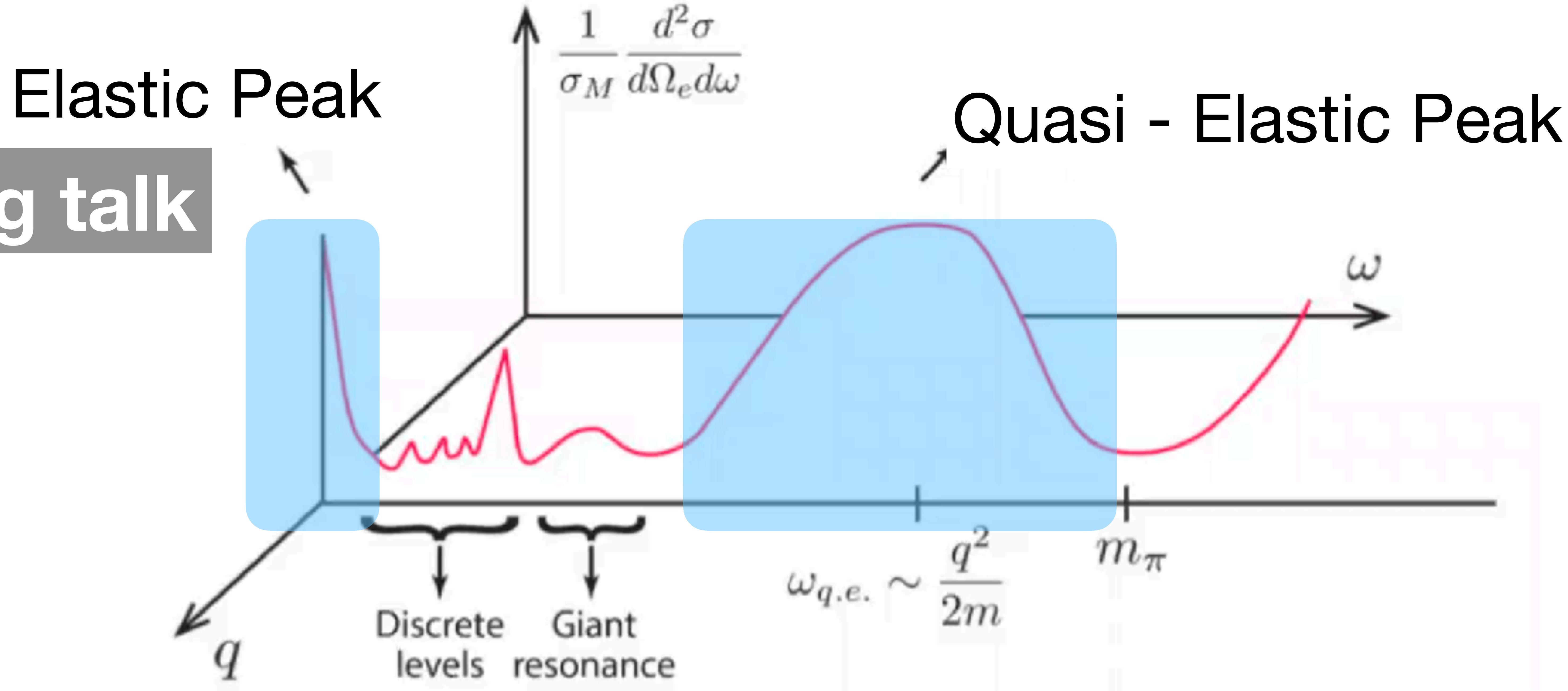
+ Quantum Monte Carlo method (VMC)

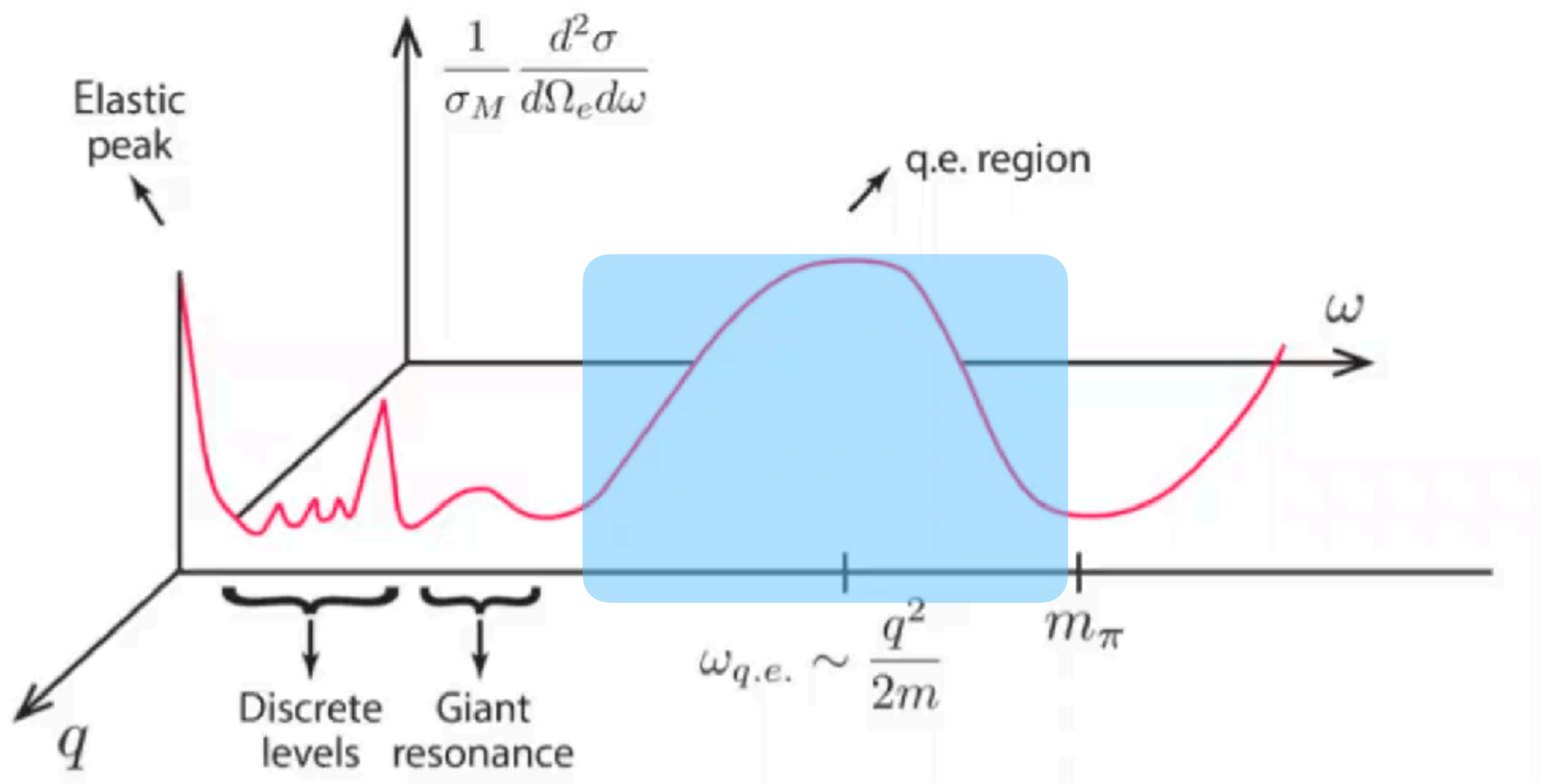


Contents

- **Response function of light nuclei**
- **Primakoff production of pseudoscalar mesons**

see G.B. King talk





Electromagnetic response function of light nuclei: relativistic corrections

[Phenomenological interactions and currents AV18 + UIX]

L. Andreoli, R. Weiss, G. Chambers-Wall, J. Carlson,
G. B. King, S. Gandolfi, A.G., S. Pastore, M. Piarulli, R. B. Wiringa

[In preparation]

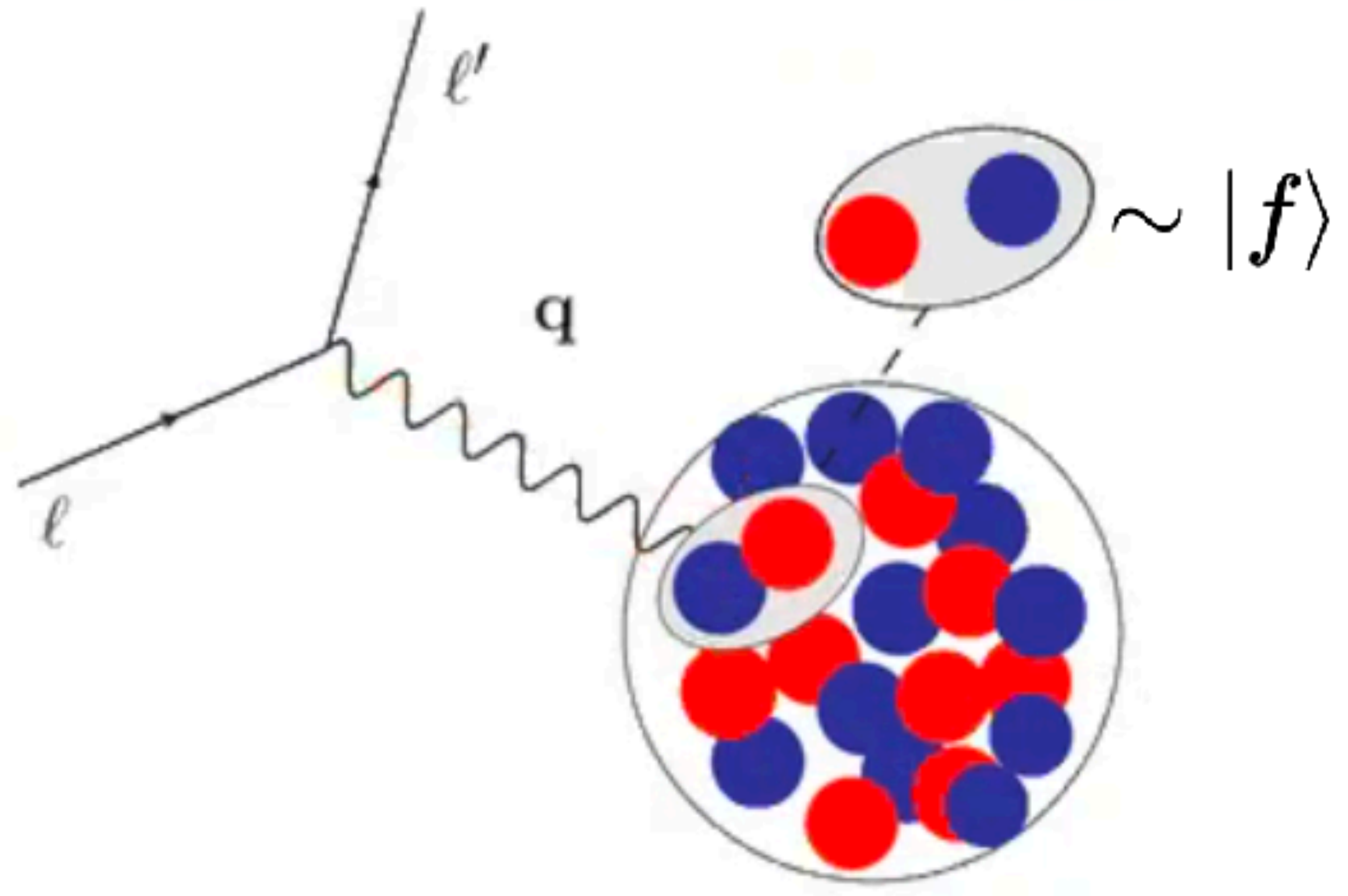
S. Pastore et al., PRC 101, 044612 (2020)

Short time approximation

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M [v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)]$$

We can't really compute it

$$R_\alpha(q,\omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$



$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

The sum over all final states is replaced by a two nucleon propagator:

inclusion of full two-body dynamics

see S. Bacca and F. Marino talk for a different approach

Figures and formulas courtesy of L. Andreoli

Response densities

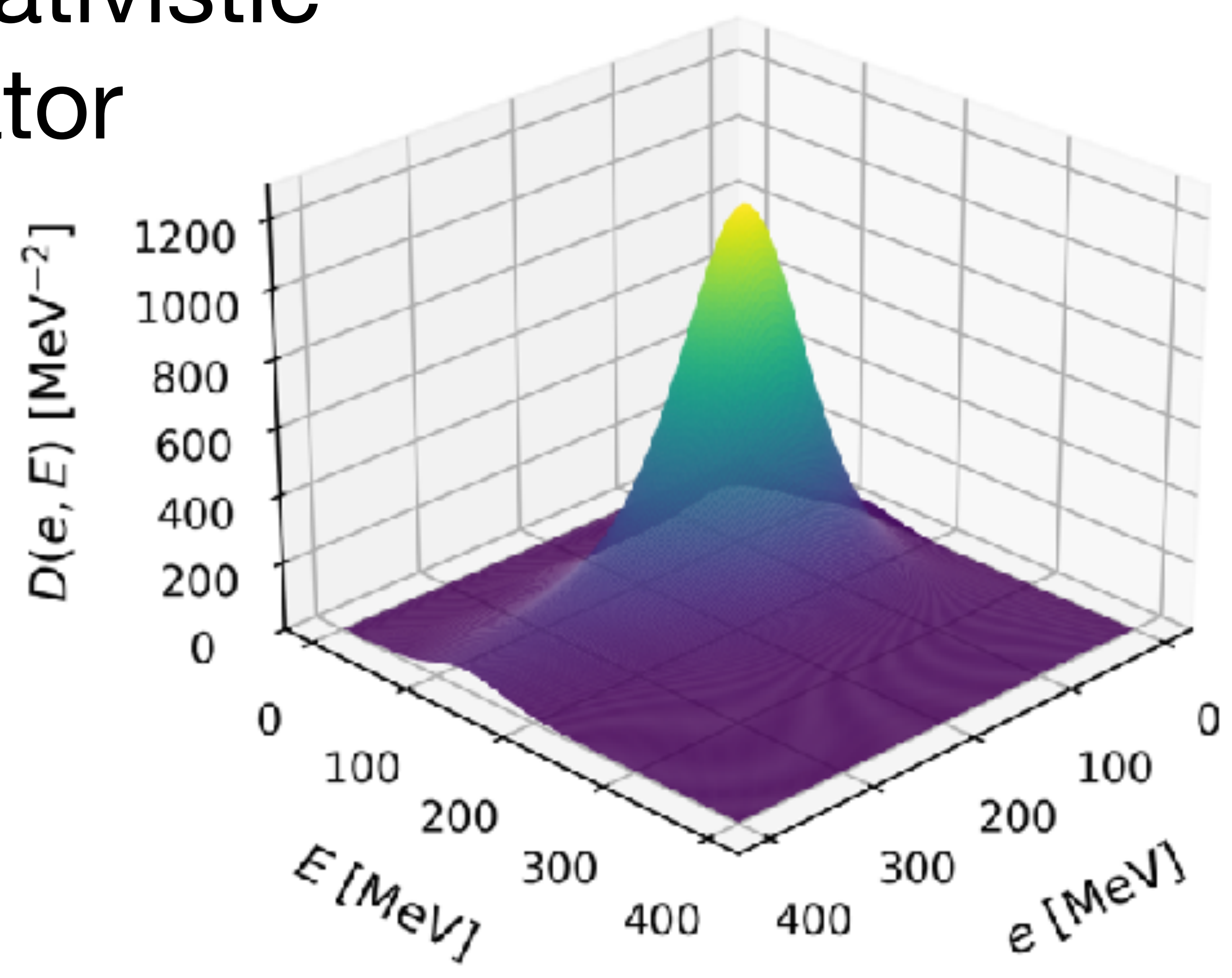
$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

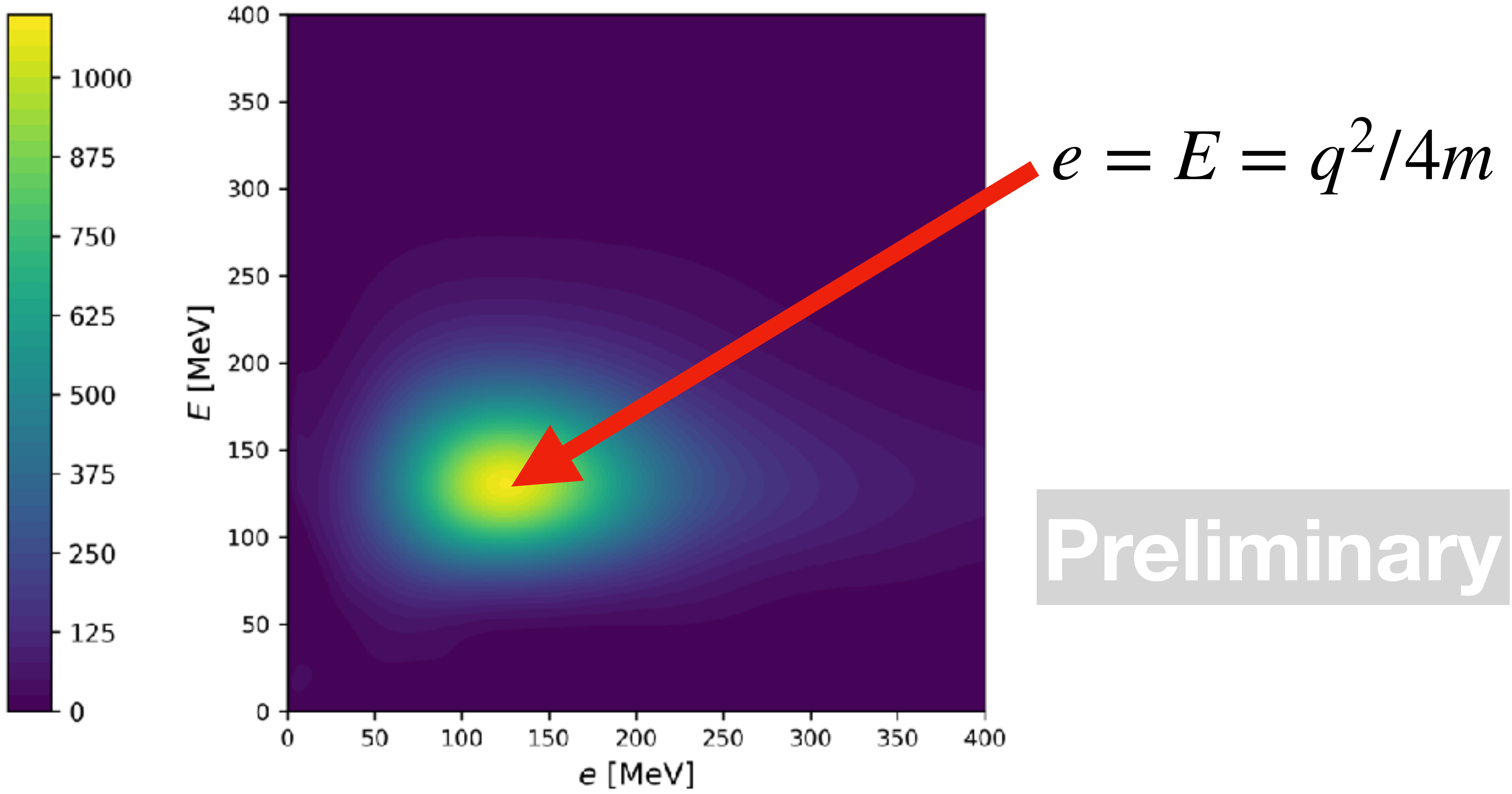
STA

$$R_\alpha(\mathbf{q},\omega) = \int de dE_{cm} D(e, E_{cm}) \delta(\omega - E_{cm} - e)$$

q=700 MeV/c
Transverse response
Full non—relativistic
propagator

^{4}He



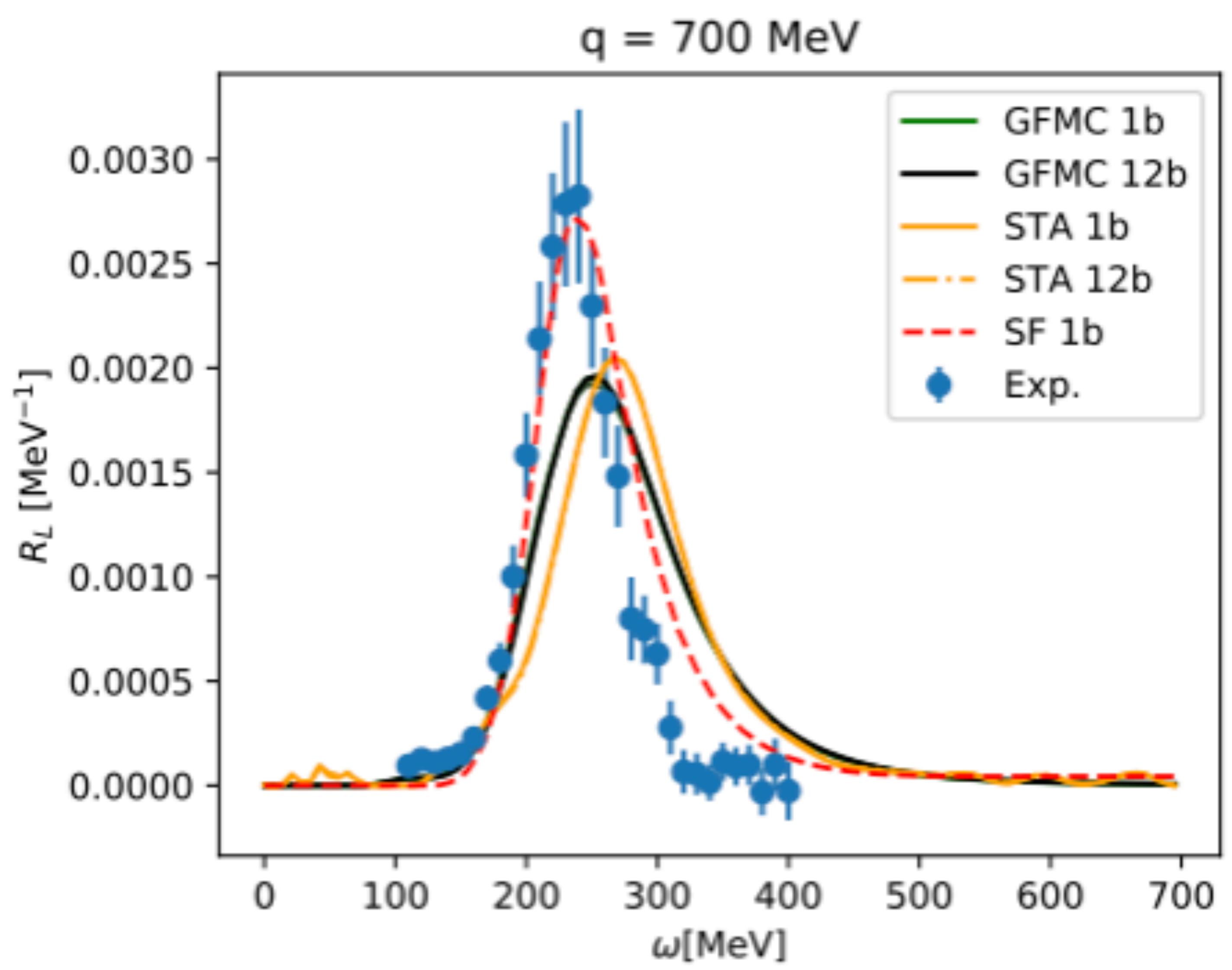


L. Andreoli et al., PRC 105, 014002 (2022)

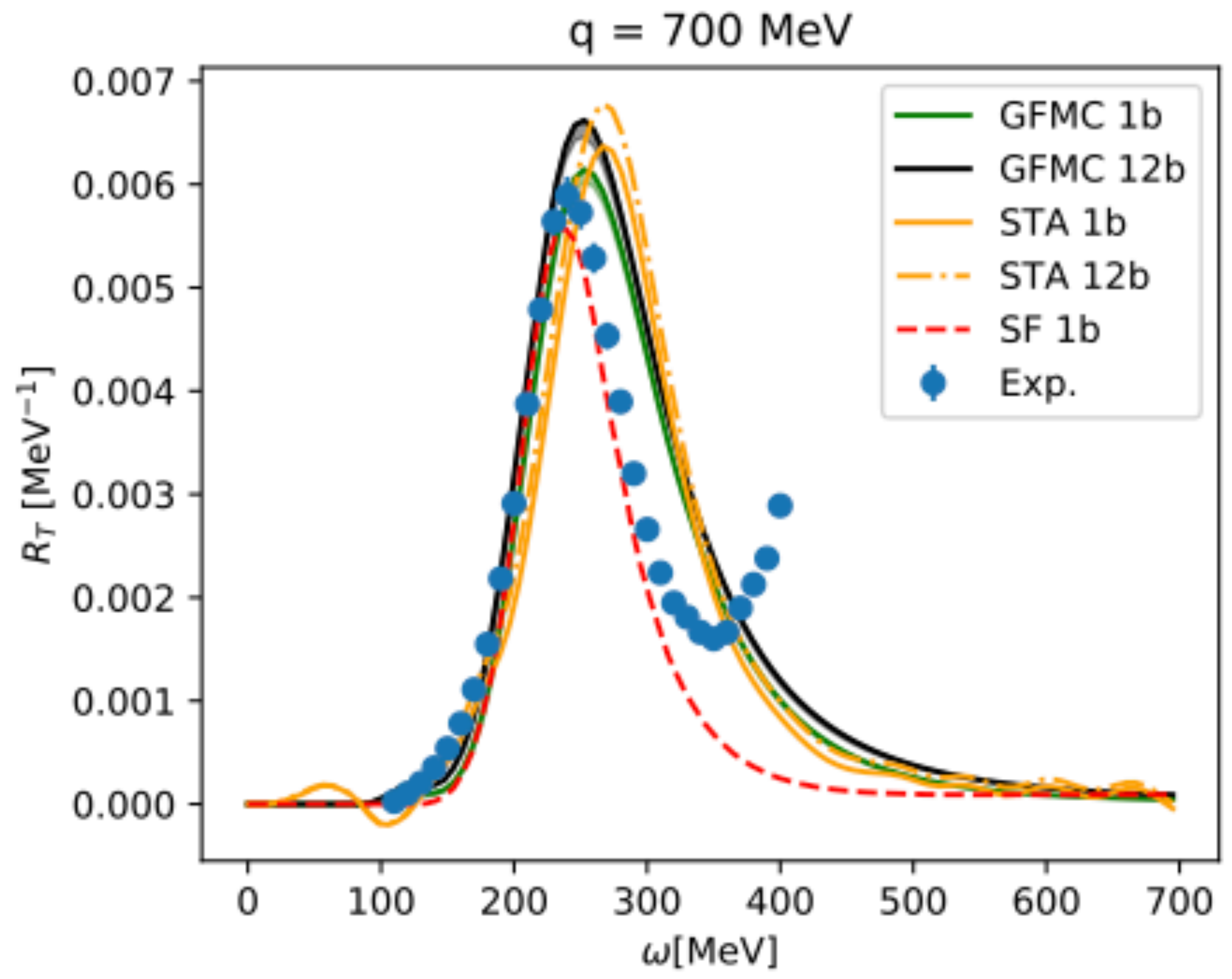
Previous results

- The STA contain the full 2-body dynamics.
- The spectral function can use the full relativistic dynamic exactly.

Correct the STA including (at least partially) the relativistic dynamic and kinematic



Helium-3



Response densities relativistic kinematic

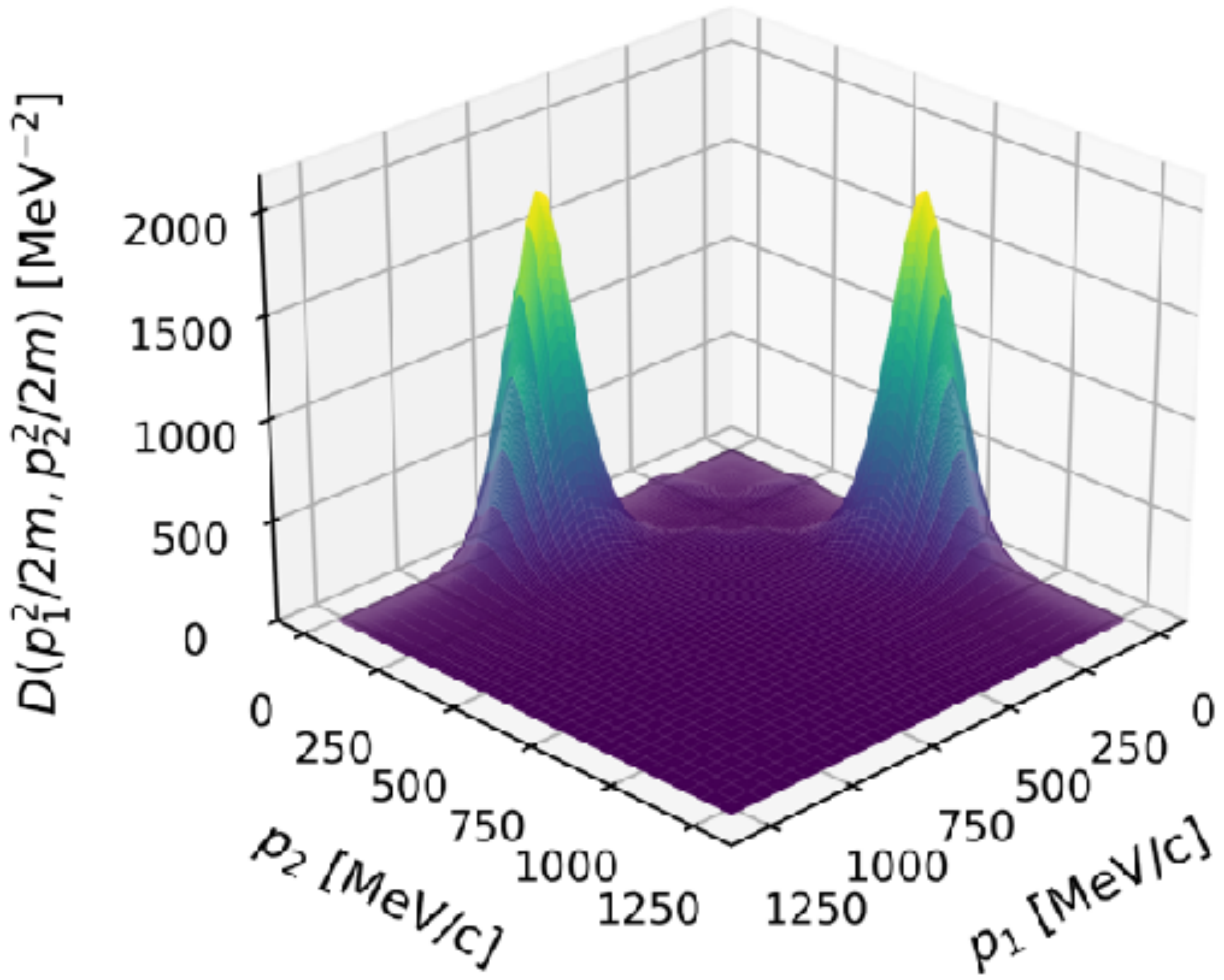
$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

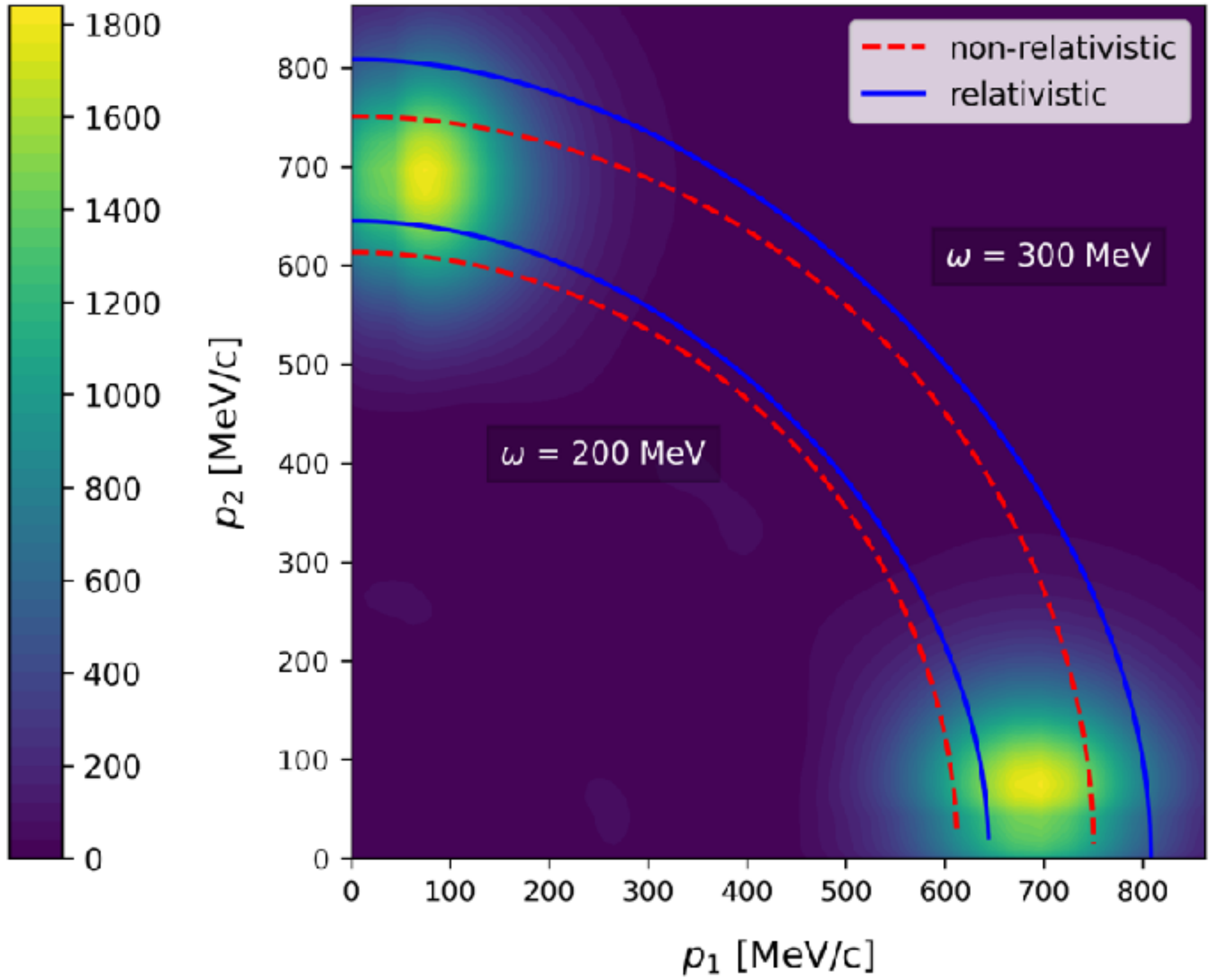
STA

$$R(\mathbf{q},\omega) = \int_0^\infty dp_1 \int_0^\infty dp_2 D(p_1,p_2) \delta\left(\omega - \sqrt{p_1^2+m^2} - \sqrt{p_2^2+m^2} + 2m\right)$$

q=700 MeV/c
Transverse response
No FSI (no full 2-body propagator)

^{4}He





Preliminary

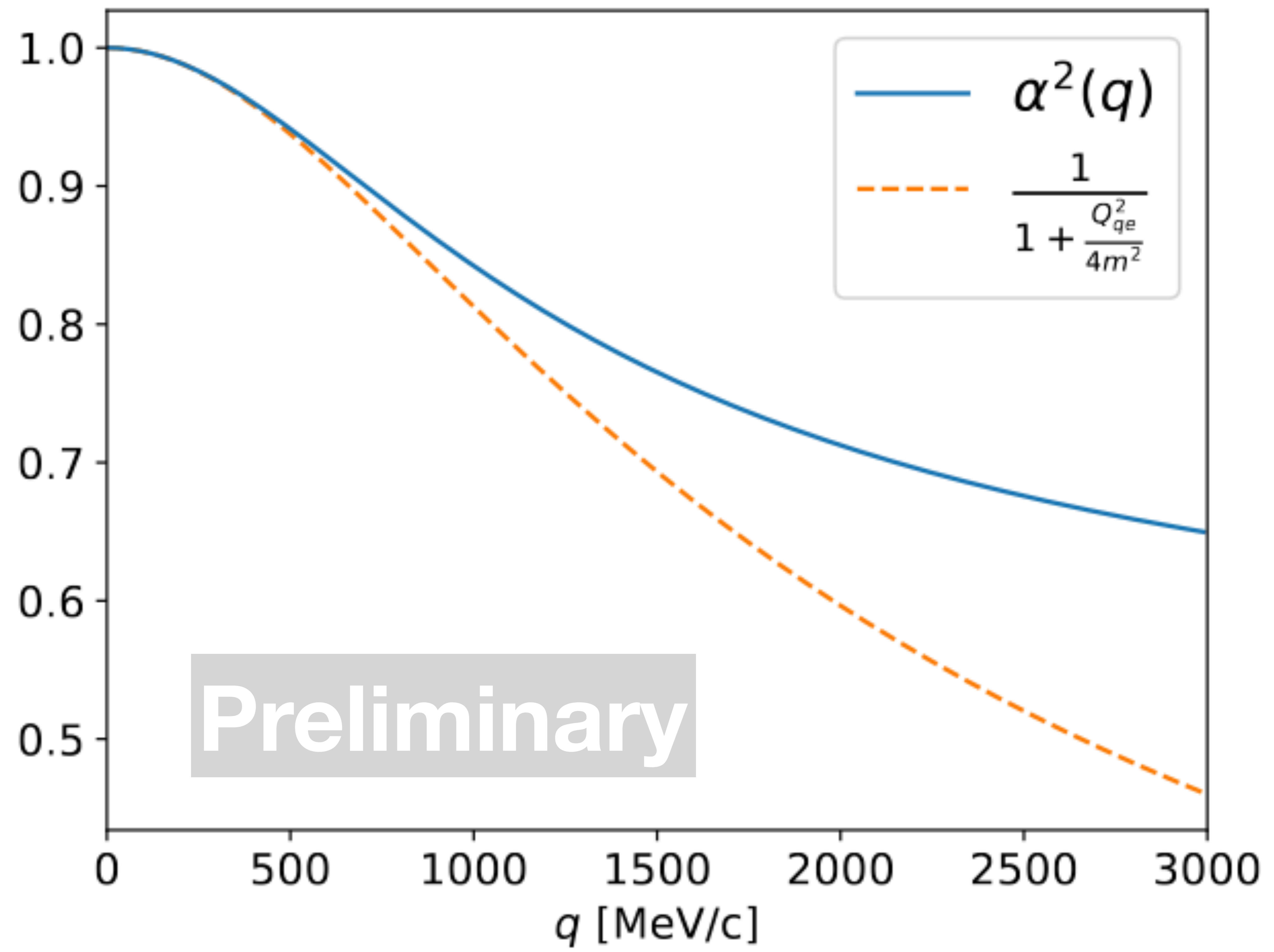
Relativistic Currents

New relativistic expansion of the currents

$$j^0_{nr} = \frac{G_E(Q^2_{qe})}{\sqrt{1 + Q^2_{qe}/4m^2}} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$

$$\boldsymbol{p}' = \boldsymbol{p} + \boldsymbol{q}$$

$$j^0_{p_0} = \alpha(q) G_E(Q^2_{qe}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$



L. Andreoli, R. Weiss et al., in preparation (2025)

Relativistic currents + kinematic

Higher momentum transfer

- New expansion of the currents

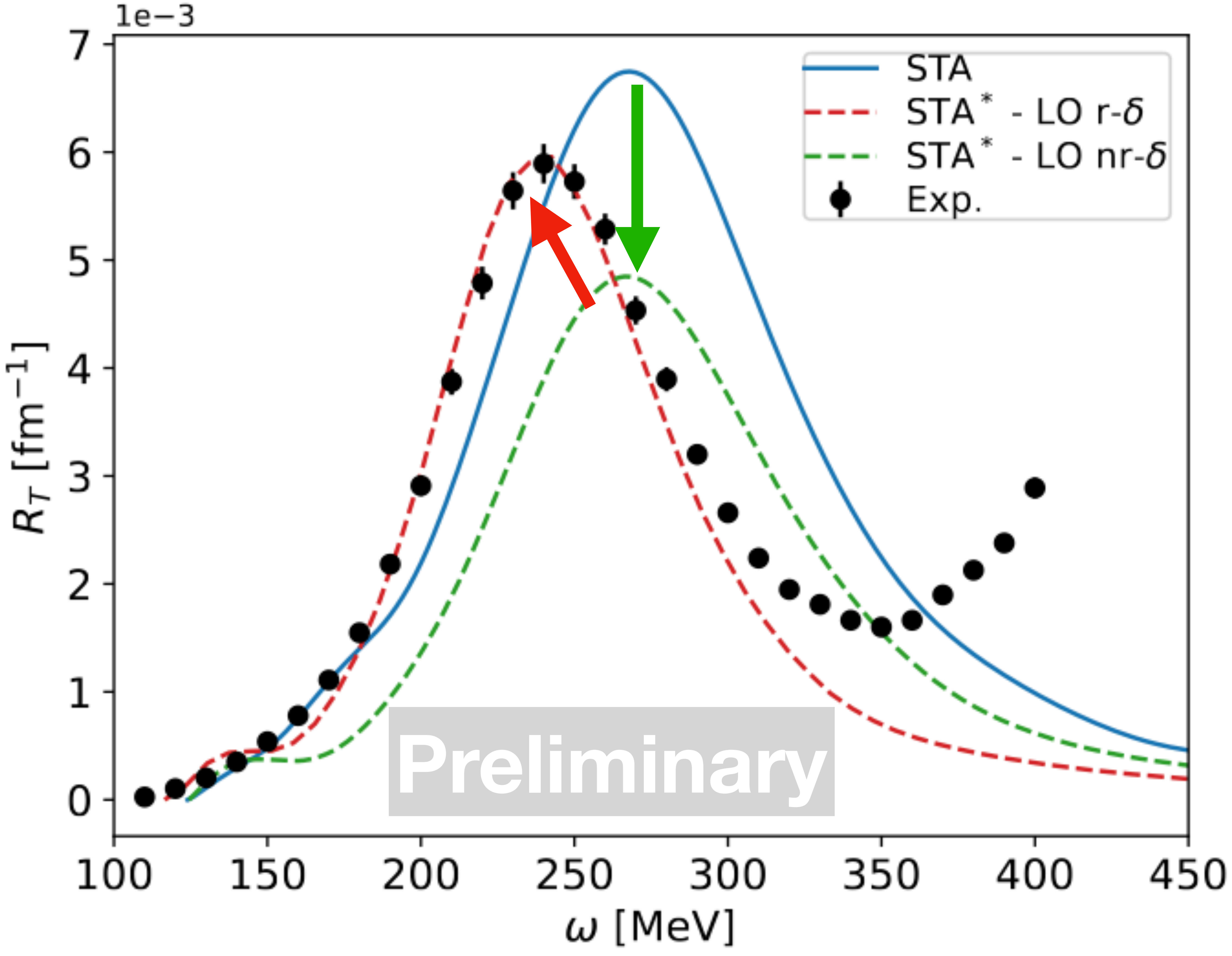
$$\boldsymbol{p}' = \boldsymbol{p} + \boldsymbol{q}$$

$$j^0_{p^0} = \alpha(q) G_E(Q^2_{qe}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$

$$\boldsymbol{j}^{\perp}_{p^0} = \frac{2m\tau_{qe}}{q^2} G_M(Q^2_{qe}) \alpha(q) i(\boldsymbol{\sigma} \times \boldsymbol{q}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$

- Relativistic treatment of the energy conservation

Helium-3 q=700 MeV/c

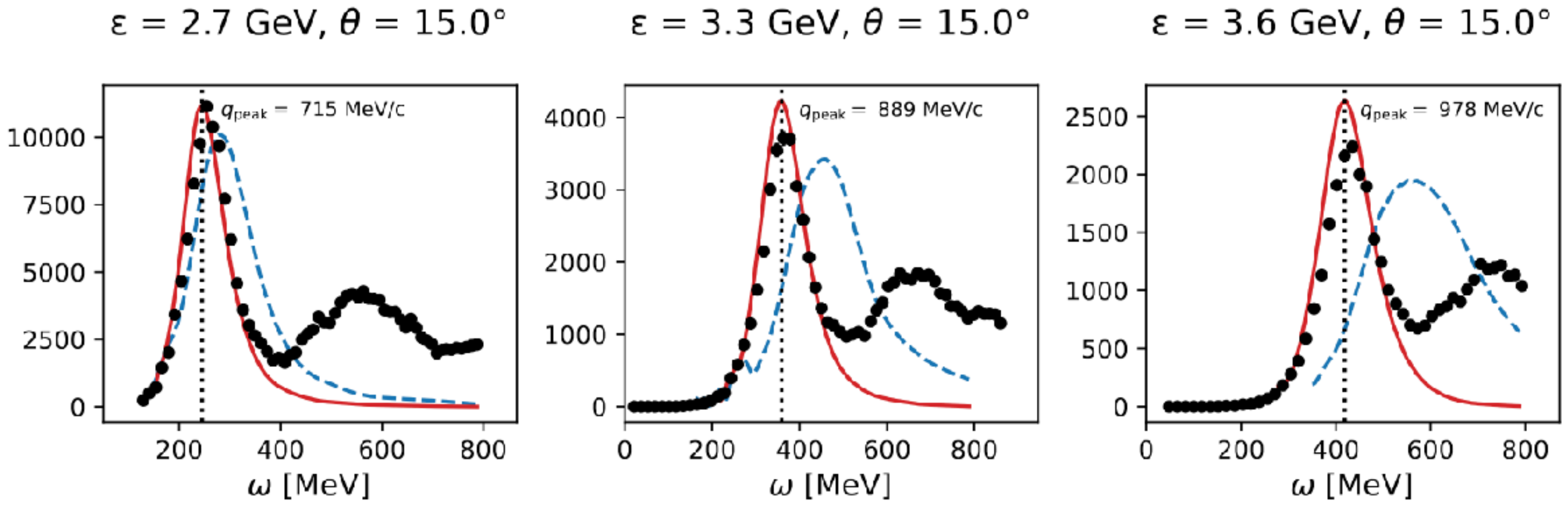


L. Andreoli, R. Weiss et al., in preparation (2025)

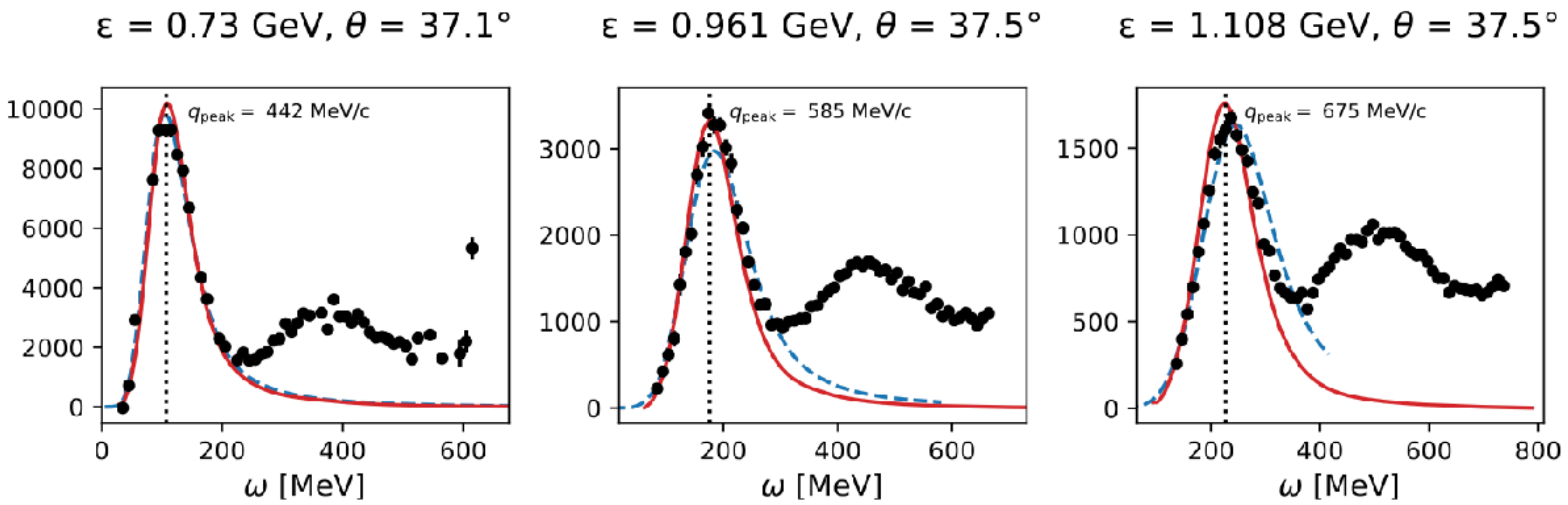
New cross sections

AV18+UIX with phenomenological 1-body current

Helium-3



Preliminary



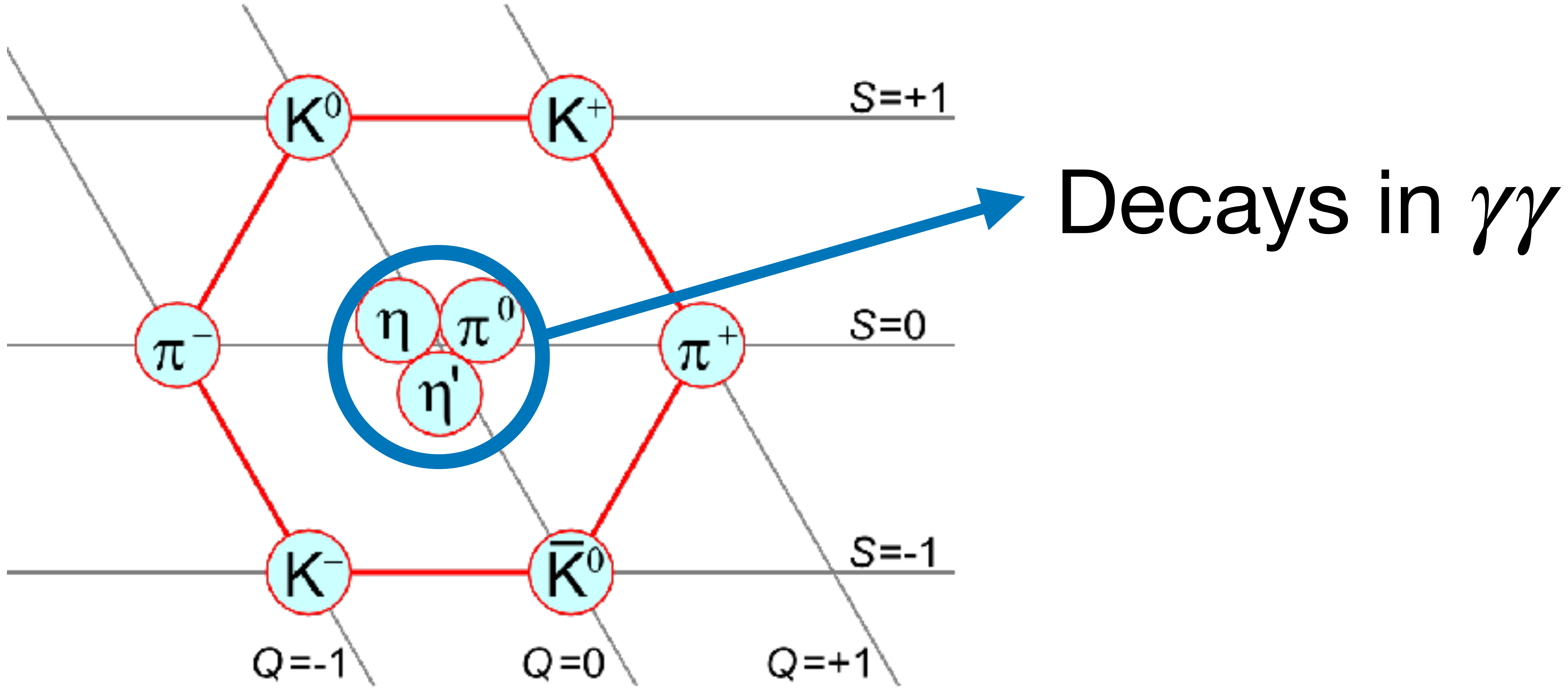
Helium-4

Primakoff production of pseudoscalar mesons

L. Andreoli, A.G., J. Carlson, G. Chambers-Wall, G. B. King
S. Pastore, M. Piarulli, R. B. Wiringa, R. Weiss

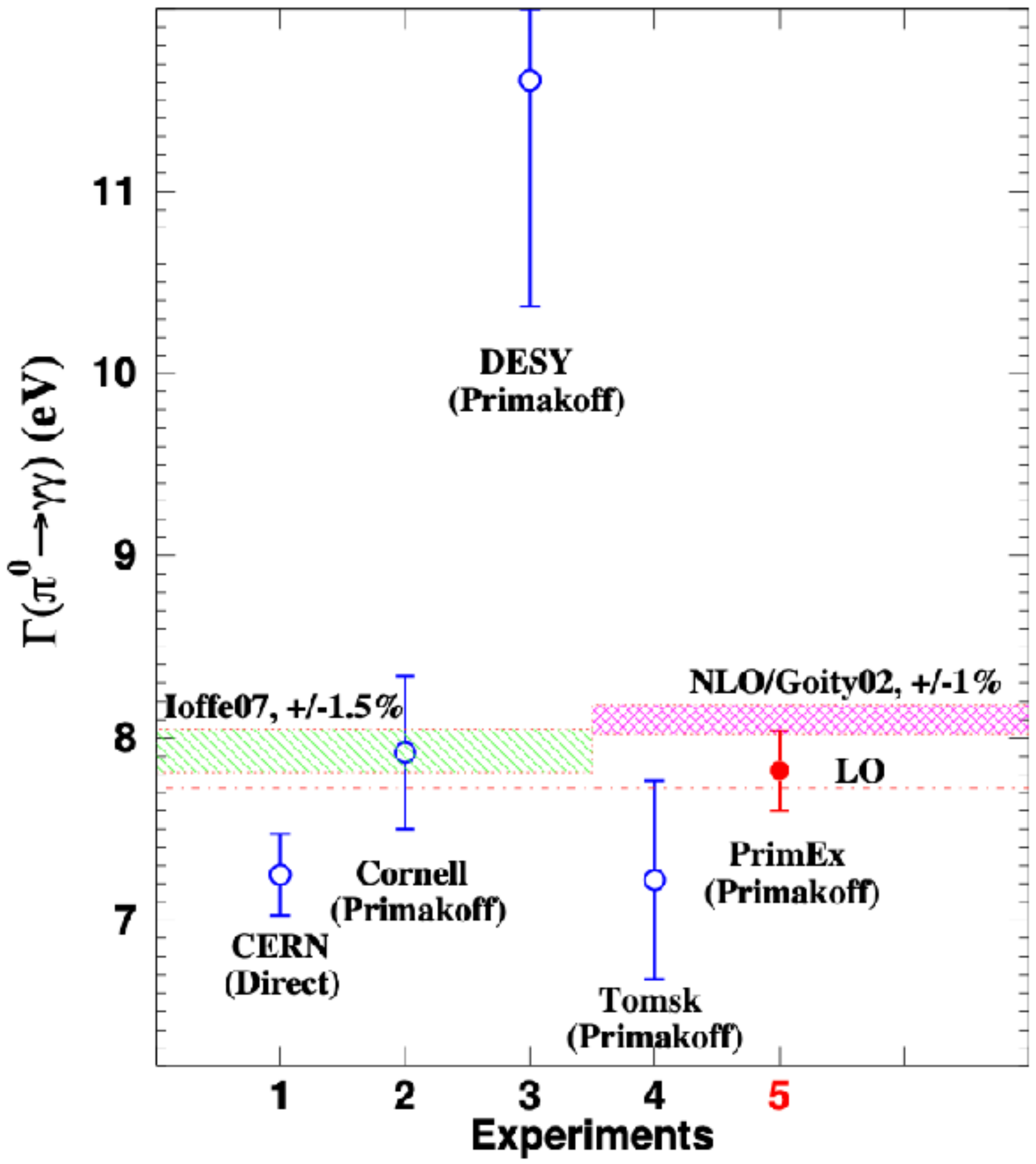
Thanks to
M. Albrecht (JLab)
I. Jaegle (JLab)
D. Smith (JLab)
V. Flechas (FIU)

Primakoff production of pseudoscalar mesons



- Precision tests of chiral symmetry and anomalies
- Determination of light quark mass ratio
- $\eta - \eta'$ mixing angle

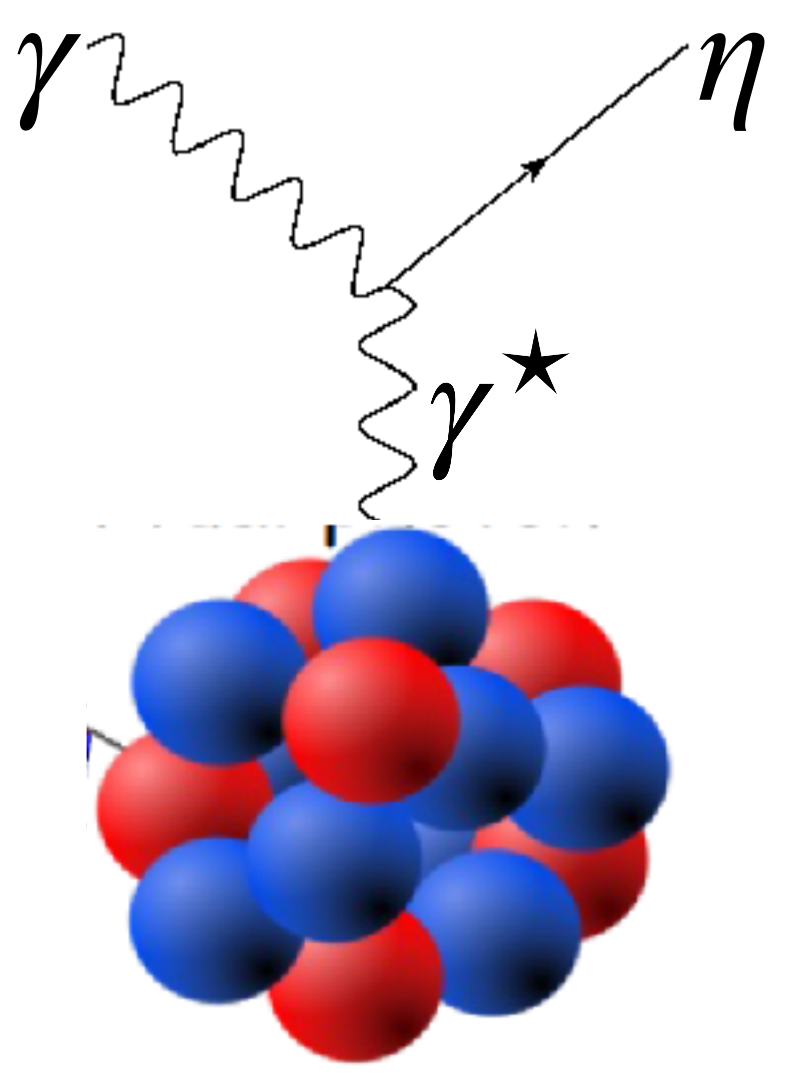
PrimEx experiment @JLab

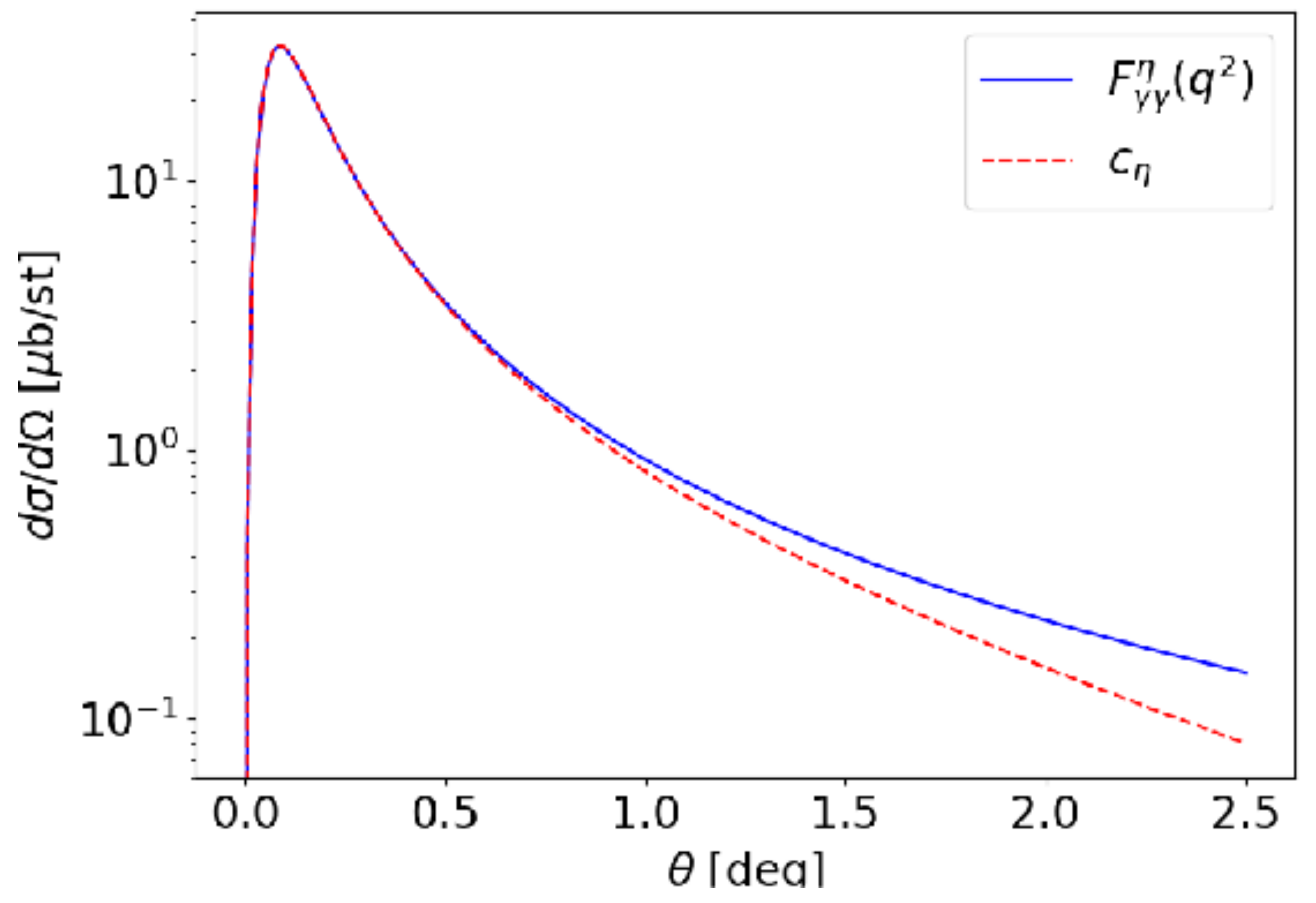


PrimeEx Coll., Phys.Rev.Lett. 106, 162303 (2011)

Primakoff production of η

- Production of neutral pseudoscalar mesons from interaction photon - nuclear Coulomb field
- Modeling based on Glauber theory + FSI + Shadowing (parametrization of the nuclear effects) [1]
- Remove spurious contribution from nuclear structure represents the main challenge





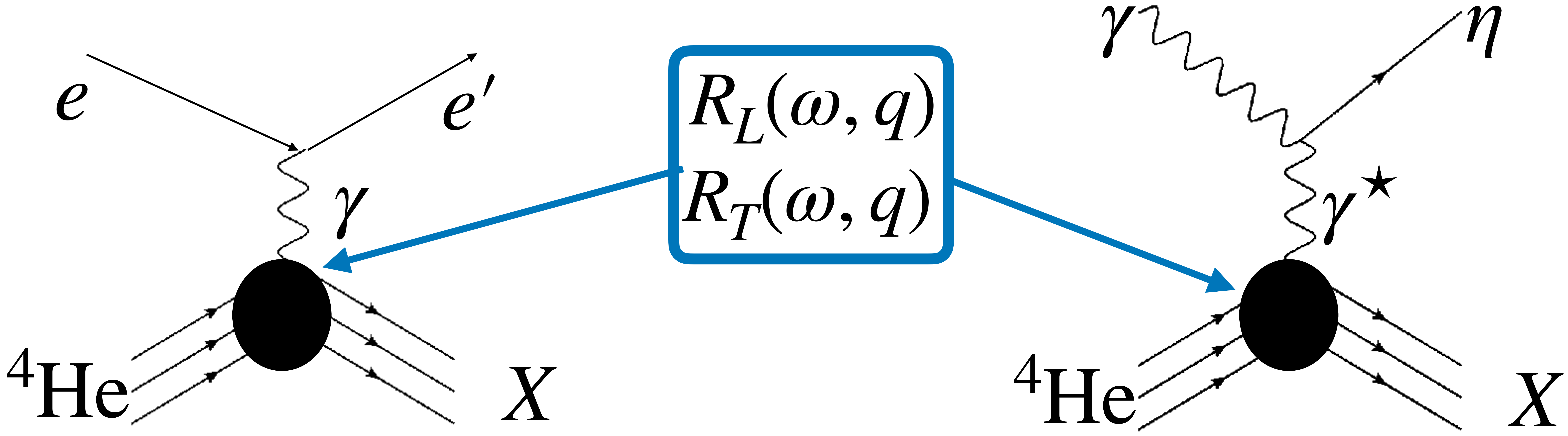
$$\frac{d\sigma}{d\Omega} = \Gamma_{\eta\to\gamma\gamma}\frac{8Z^2\alpha}{m_\eta^3}\frac{|\vec{k}'|^3 k}{q^4}\sin^2\theta F_A^2(q^2)$$

$$\Gamma_{\eta\to\gamma\gamma} = \frac{\alpha^2}{64\pi^3 f_\pi^2} m_\eta^3 c_\eta^2 .$$

[1] S. Gevorkyan et al., Phys. Rev. C **80**, 055201 (2009)

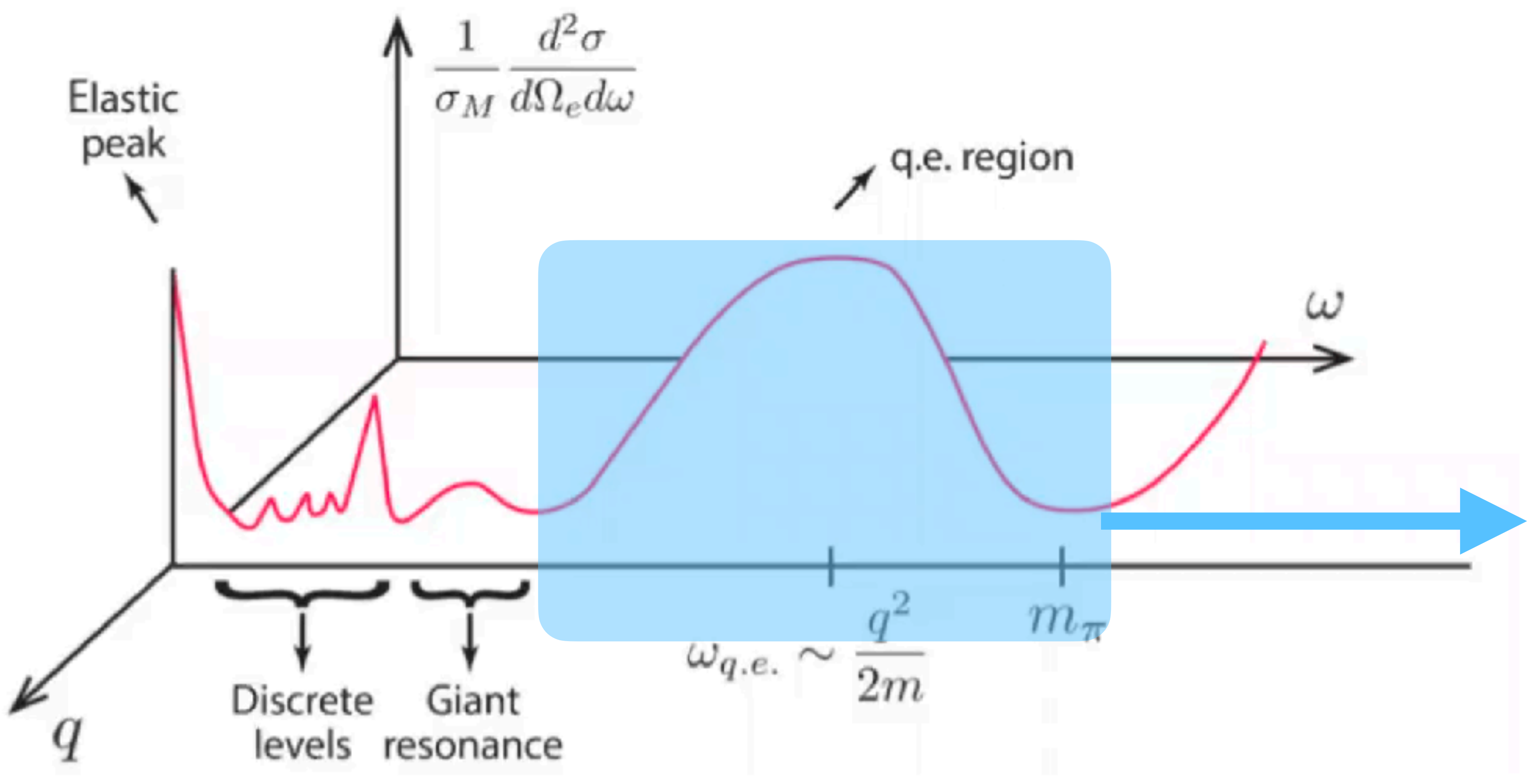
Primakoff production and electro scattering

The nuclear vertex in the Primakoff production is identical to the one of electro-scattering

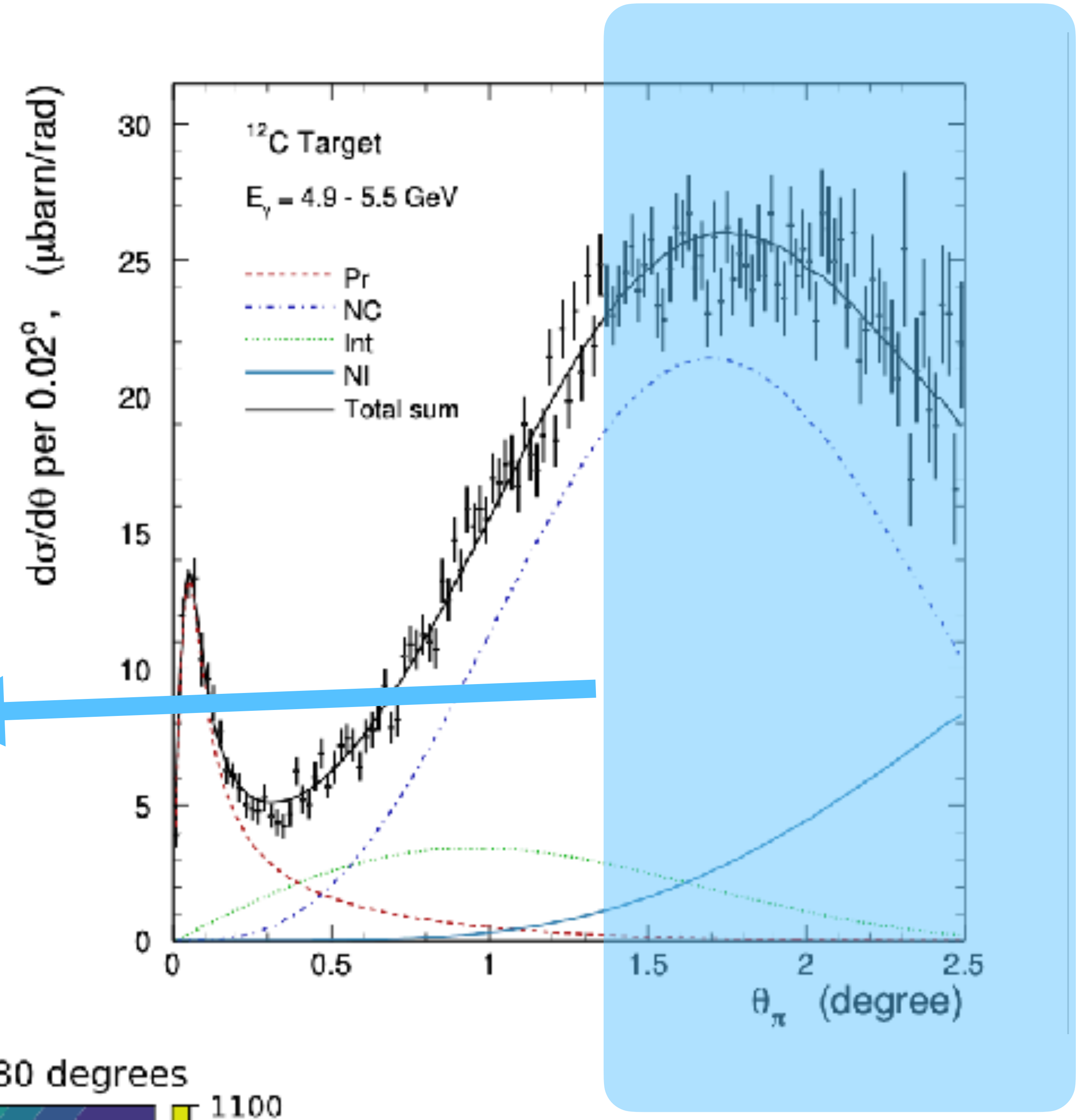


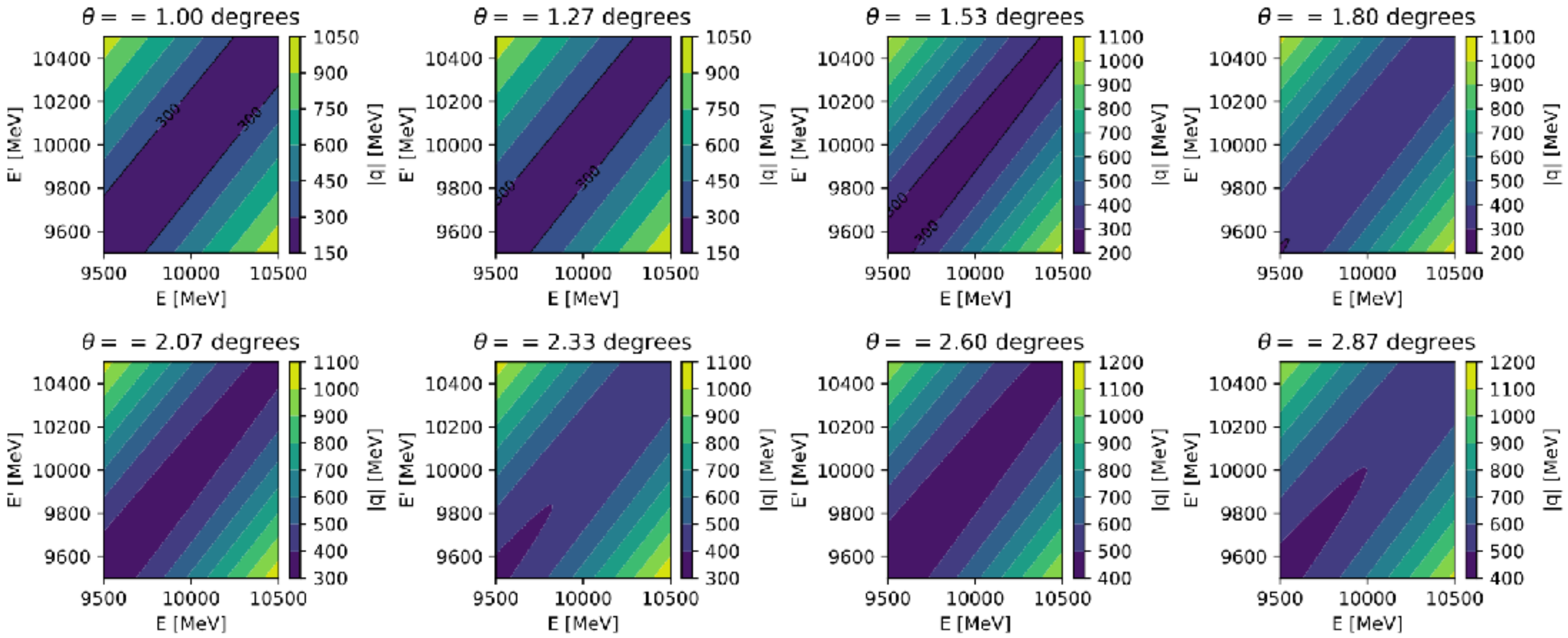
Kinematic and STA

Phys.Rev.Lett. 106, 162303 (2011)



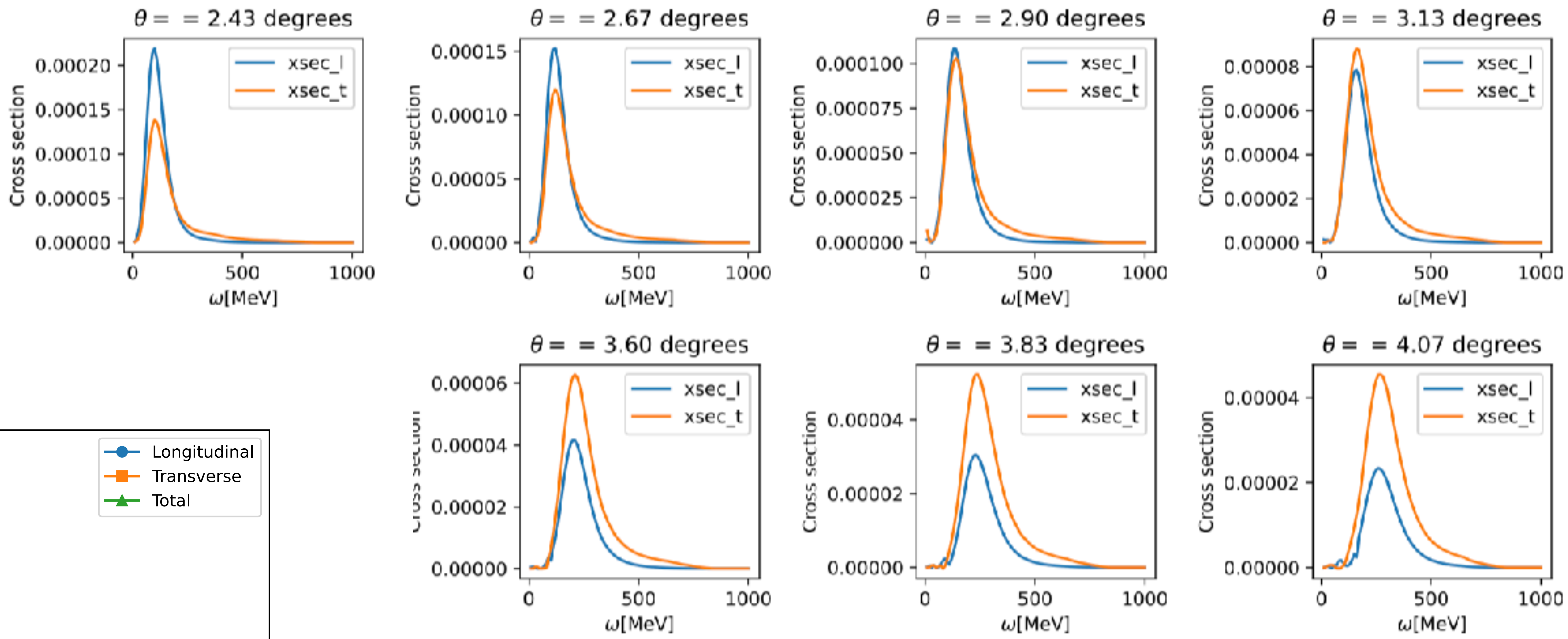
STA + relativistic correction kinematic validity region

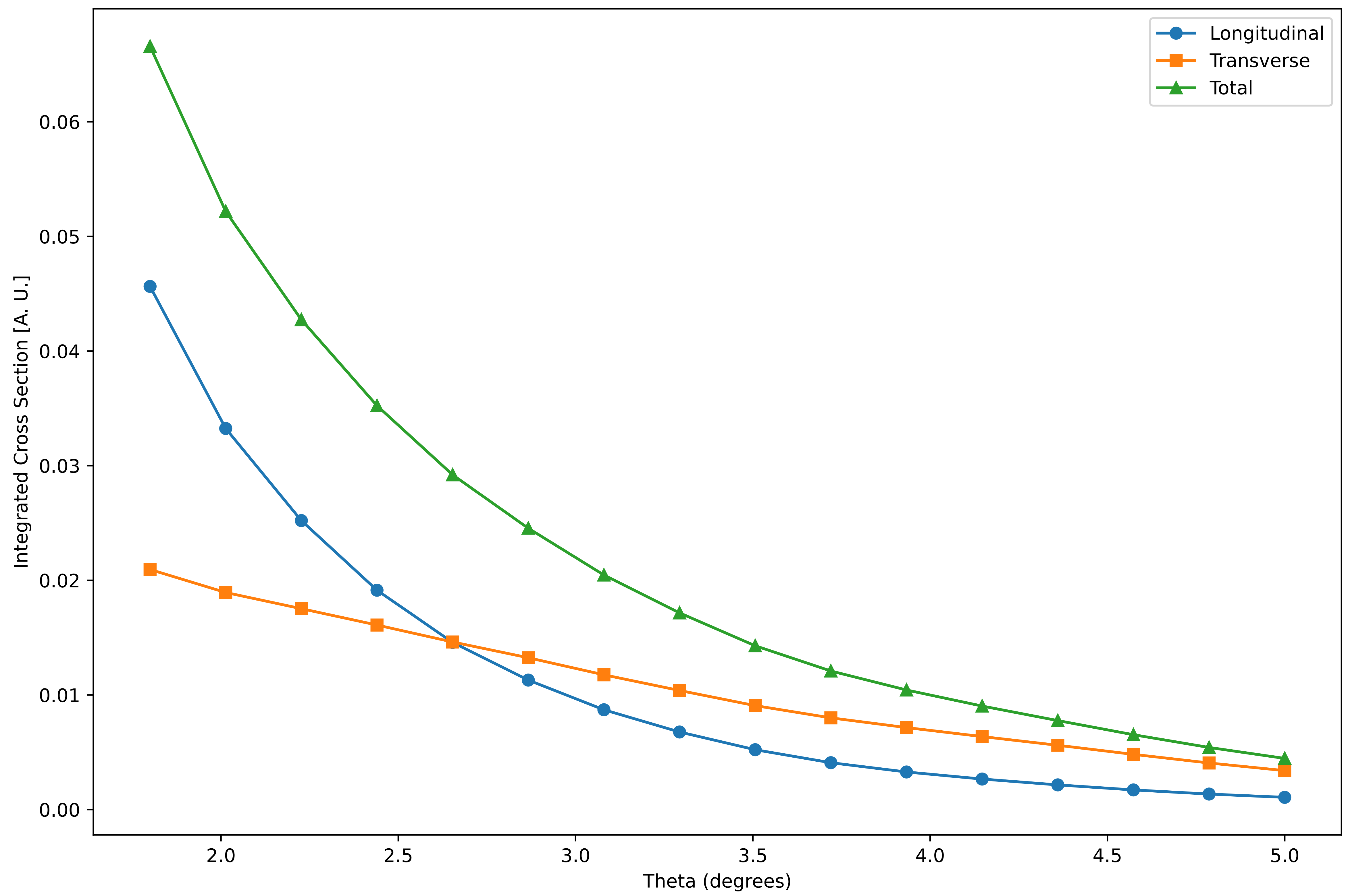




Cross sections

Super Preliminary





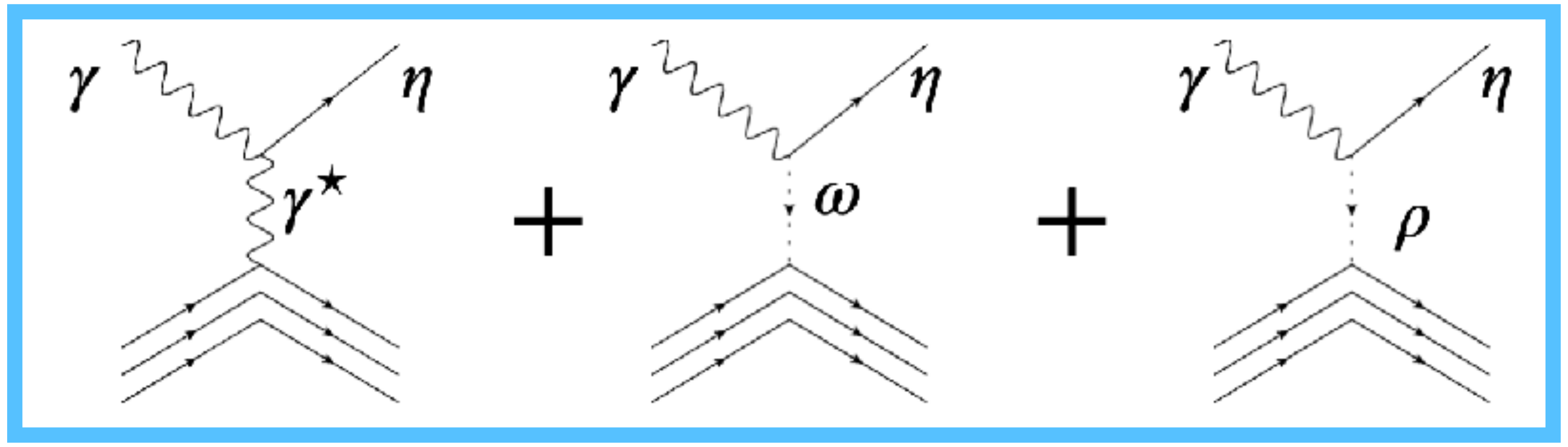
The cross sections are integrated over the energy of the incoming photon and the outgoing meson

$$\frac{1}{\Delta\nu}\int dE' d\nu \frac{d\sigma}{dE' d\Omega}$$

Not so easy…

Vector meson dominance and final/initial state interactions

- Vector meson dominance ⟶ New form factors (from Regge theory)



$$\rightarrow j_0 = \sum_{i=1,A} \frac{A^S(q^2,s) + A^V(q^2,s)\tau_z^i}{2} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$

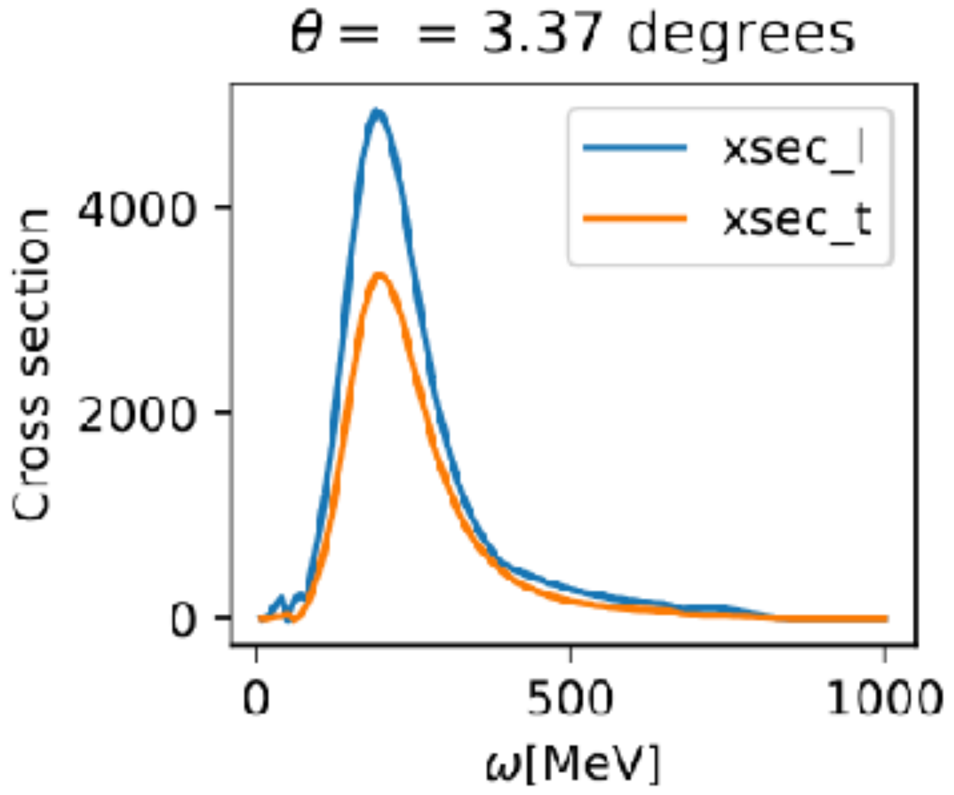
- Final state interactions and Shadowing

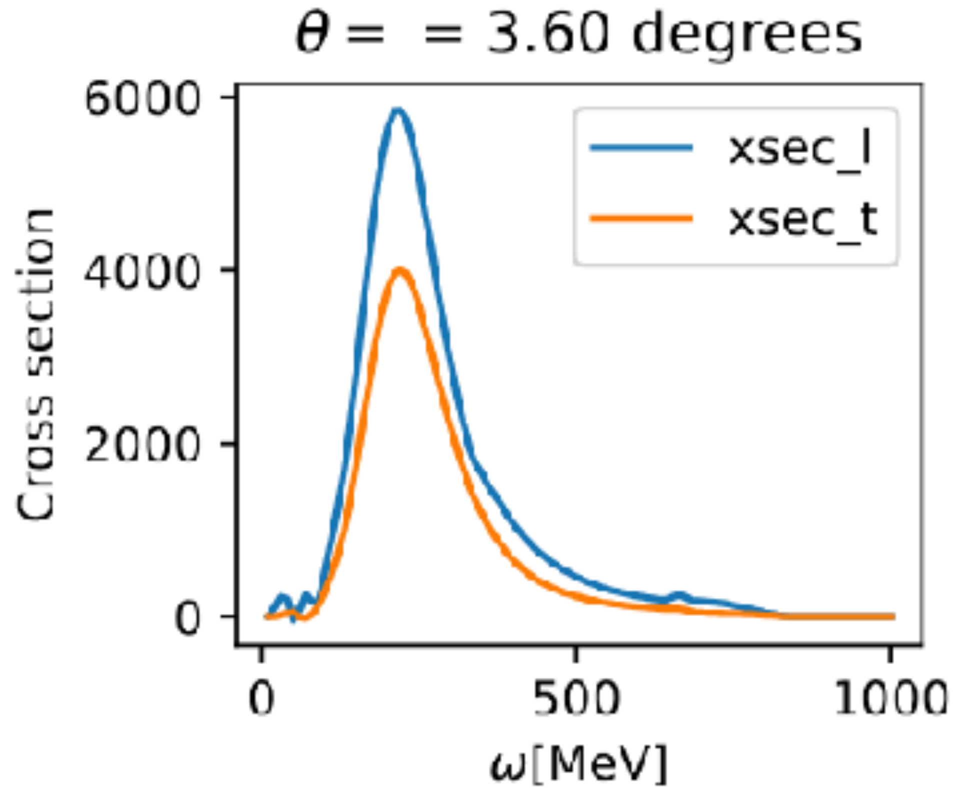
Modification of the plane wave of the outgoing meson and incoming photon

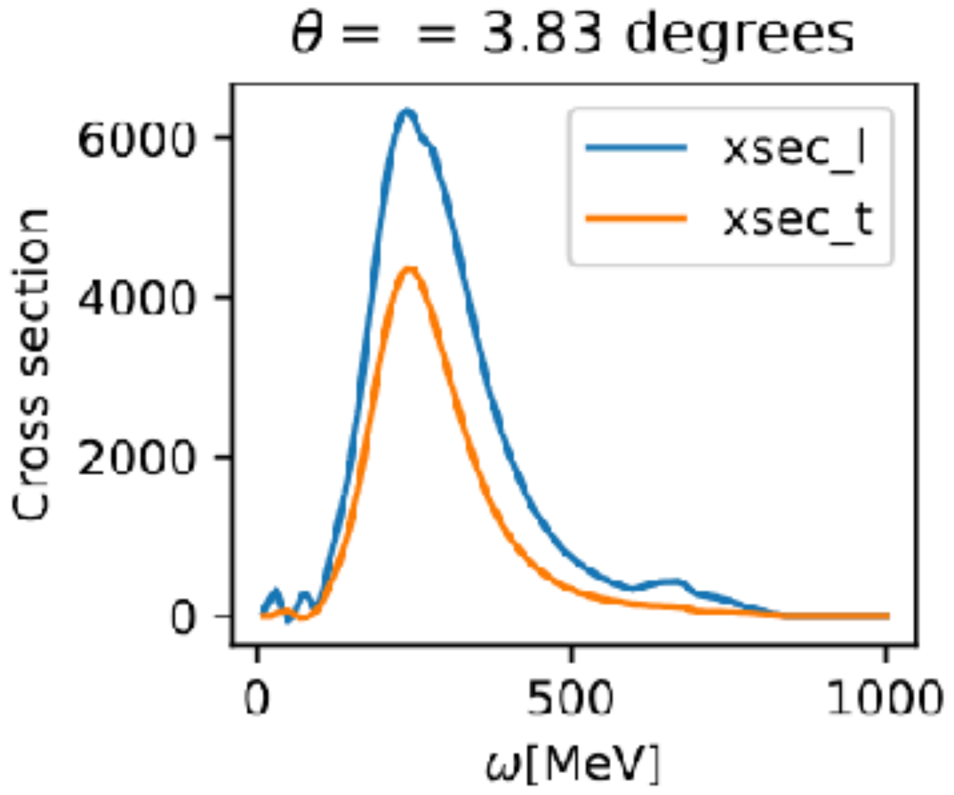
$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i} \rightarrow e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i}\chi_{\boldsymbol{k}}(\boldsymbol{r_i})$$

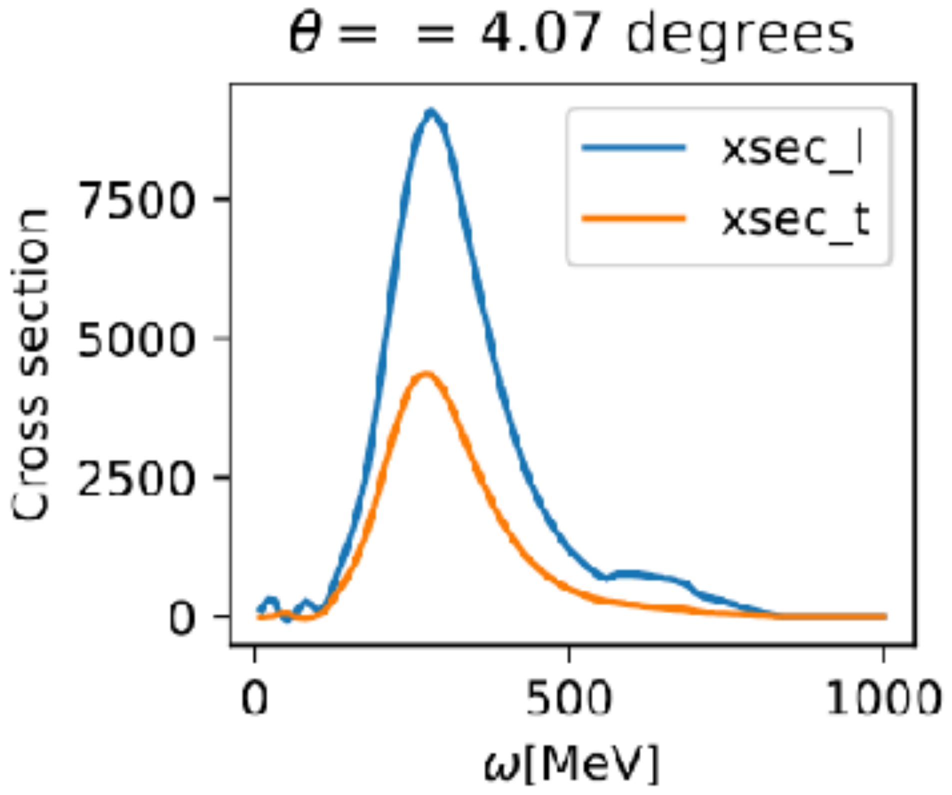
Vector Meson Dominance

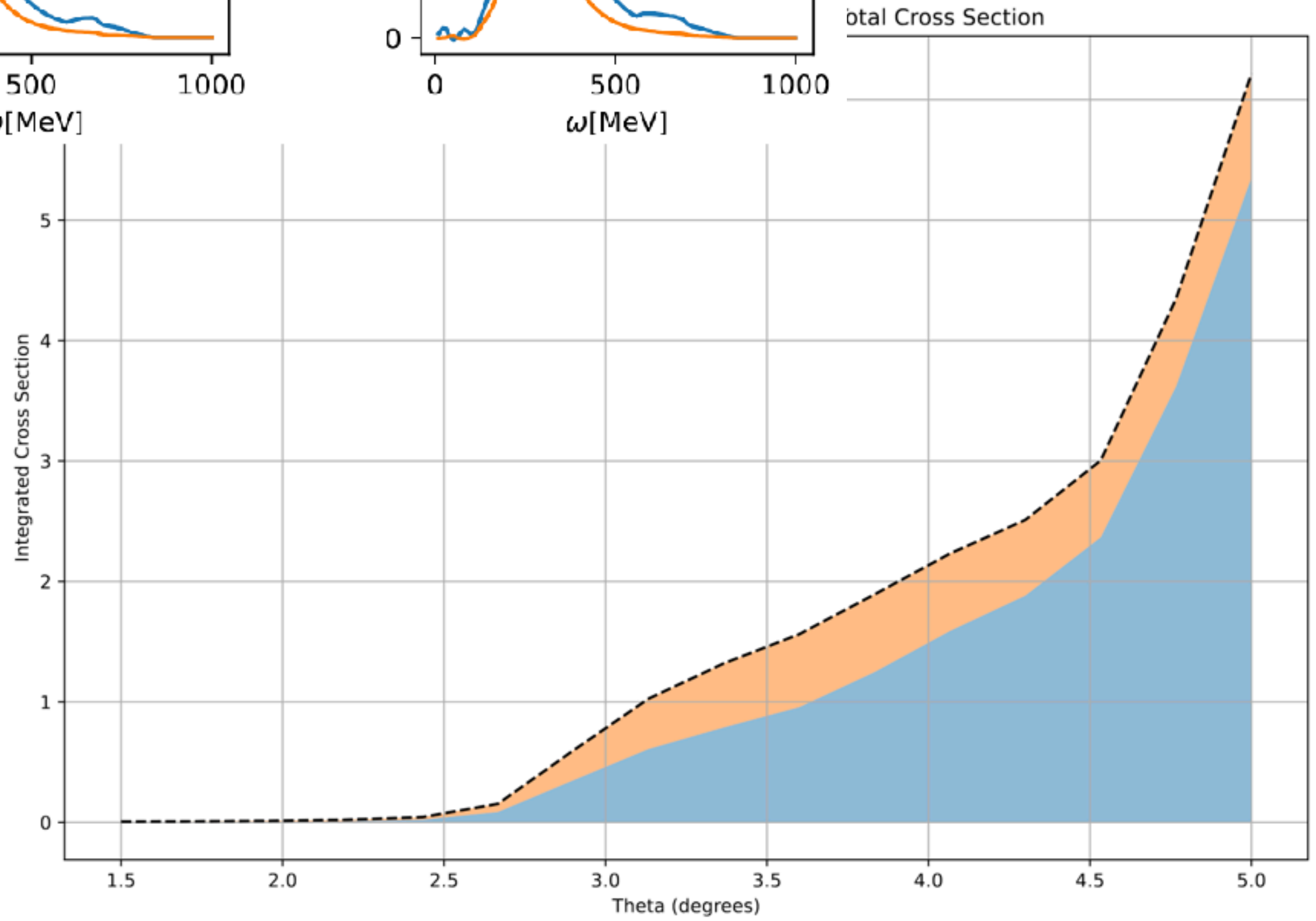
Contribution of ρ and ω exchange











Summary

- **Short-time approximation responses are in good agreement with the data**
    - New relativistic currents and kinematical effects extend the range of calculations at higher values of q.
    - Accounts for two-body physics both currents and correlations and it is promising to be extended beyond $A \geq 12$.
- **STA responses can be used to compute primakoff production of** π^0, η, η'
    - Full inclusion of the nuclear dynamics.
    - Inclusion of VMD, FSI and Shadowing is in progress.

Challenges and perspective

- Error estimation
- Tagging exclusive processes
- Mesonic degrees of freedom? (Lucas Madeira talk)
- Inclusion of full relativistic approach for matching other non perturbative QCD approaches (light front quantization)

Collaborators

L. Andreoli (JLab & ODU)
J. Carlson (LANL)
G. Chambers-Wall (WashU)
S. Gandolfi (LANL)
G. B. King (WashU)
S. Pastore (WashU)
M. Piarulli (WashU)
R. Schiavilla (Retired)
R. B. Wiringa (ANL)
R. Weiss (WashU)

Acknowledgments

NTNP
DOE Topical Collaboration

U.S. DEPARTMENT OF ENERGY

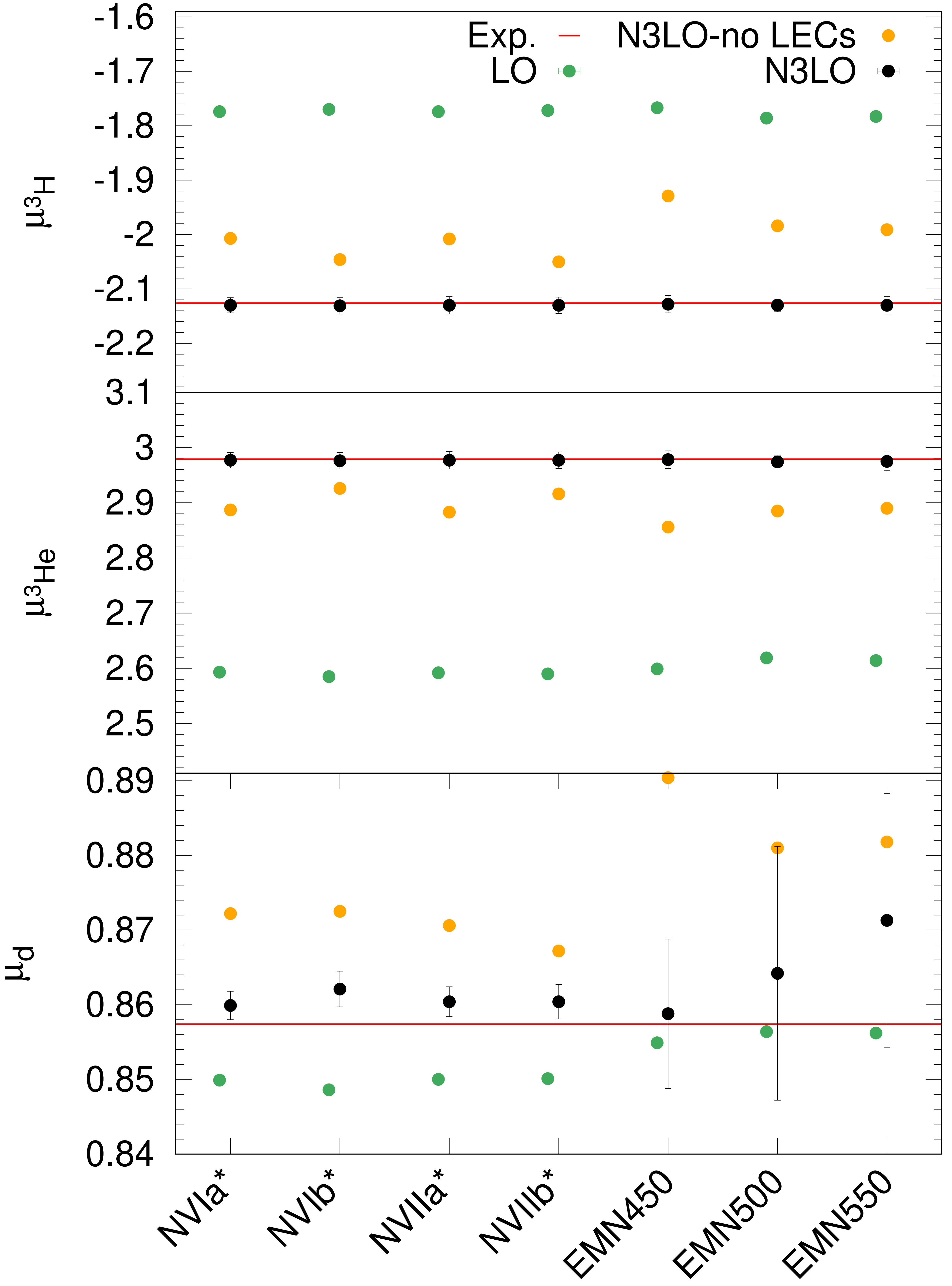
NERSC National Energy Research Scientific Computing Center

Sparse

Results of the fit

Pot.	χ^2/ndf	χ^2/ndf (no Rand)
NVIa*	9.9	2.0
NVIb*	10.2	2.3
NVIIa*	11.6	2.5
NVIIb*	11.6	2.6
EMN450	11.3	2.8
EMN500	14.7	4.7
EMN550	17.7	7.9

- ndf~40
- Removing Rand *et al.* data, χ^2improves

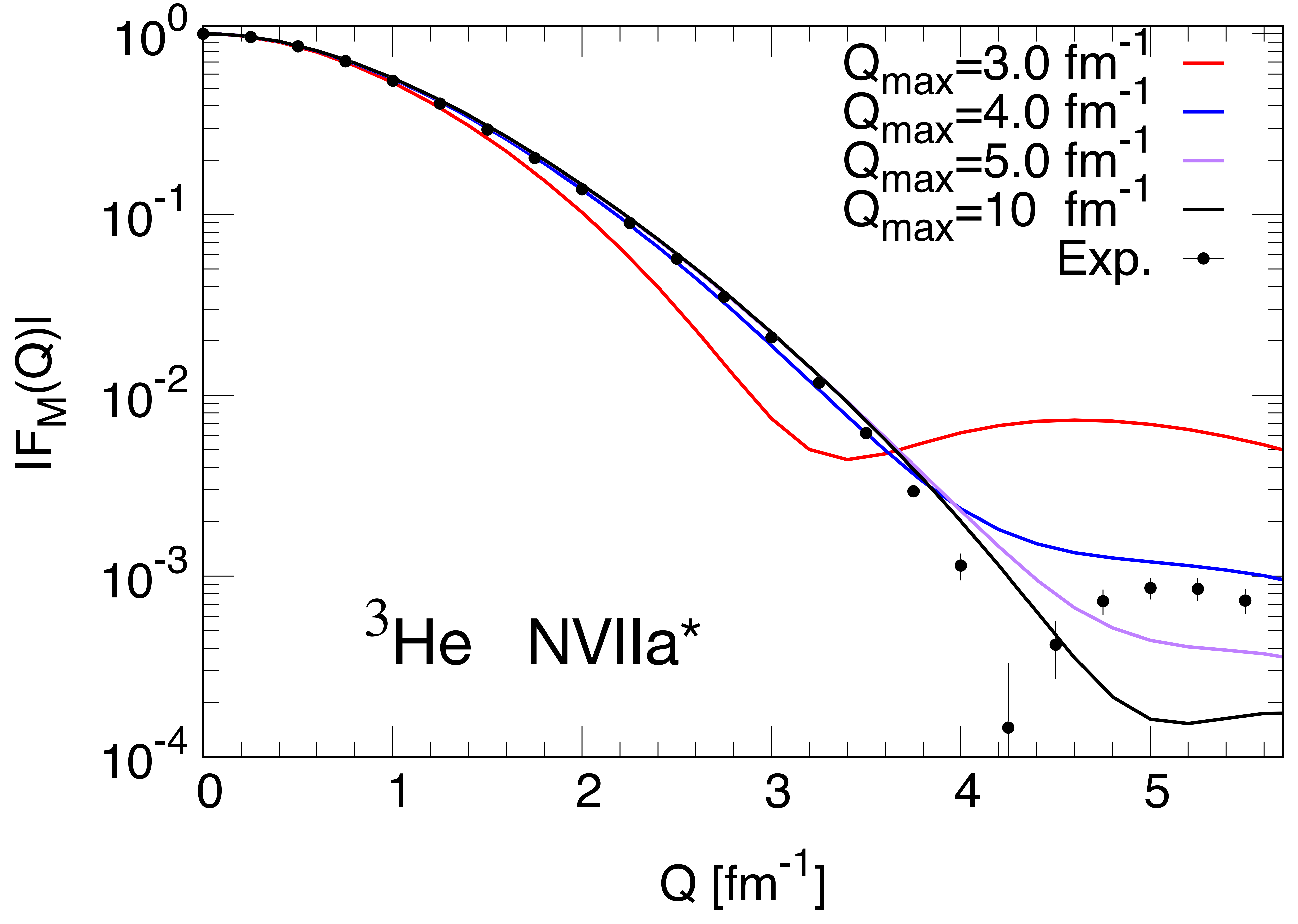


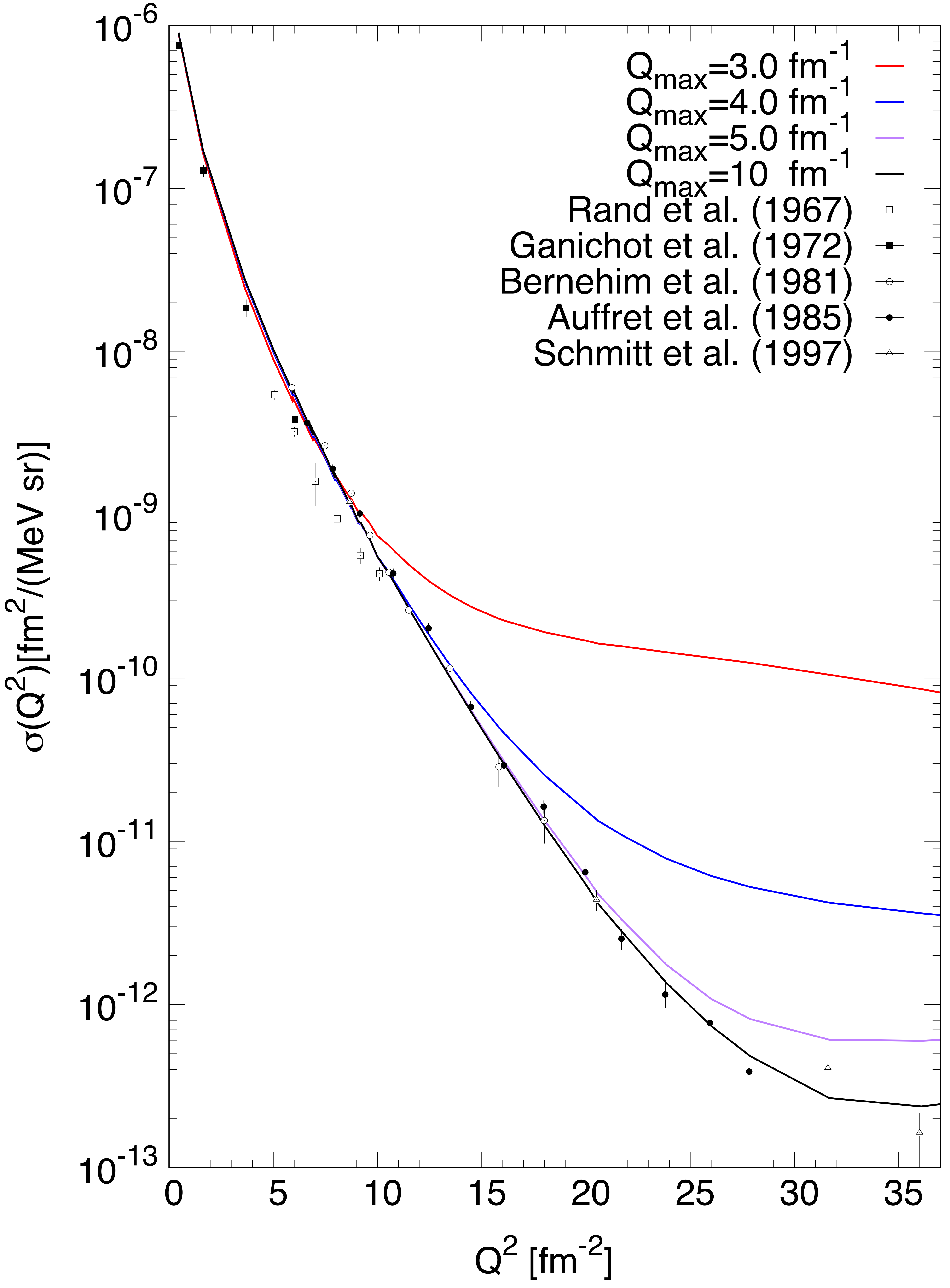
Summary I

- **First ab-initio calculation of magnetic and charge form factors of nuclei $7 \leq A \leq 10$.**
    - Good overall agreement with the experimental data.
    - Two-body currents account up to 40-50% of the total contribution to the magnetic form factors. Almost negligible for the charge
    - First observation of M_1/M_3 inversion in mirror p-shell nuclei (not observed experimentally yet).
    - Extraction of magnetic and charge radii.

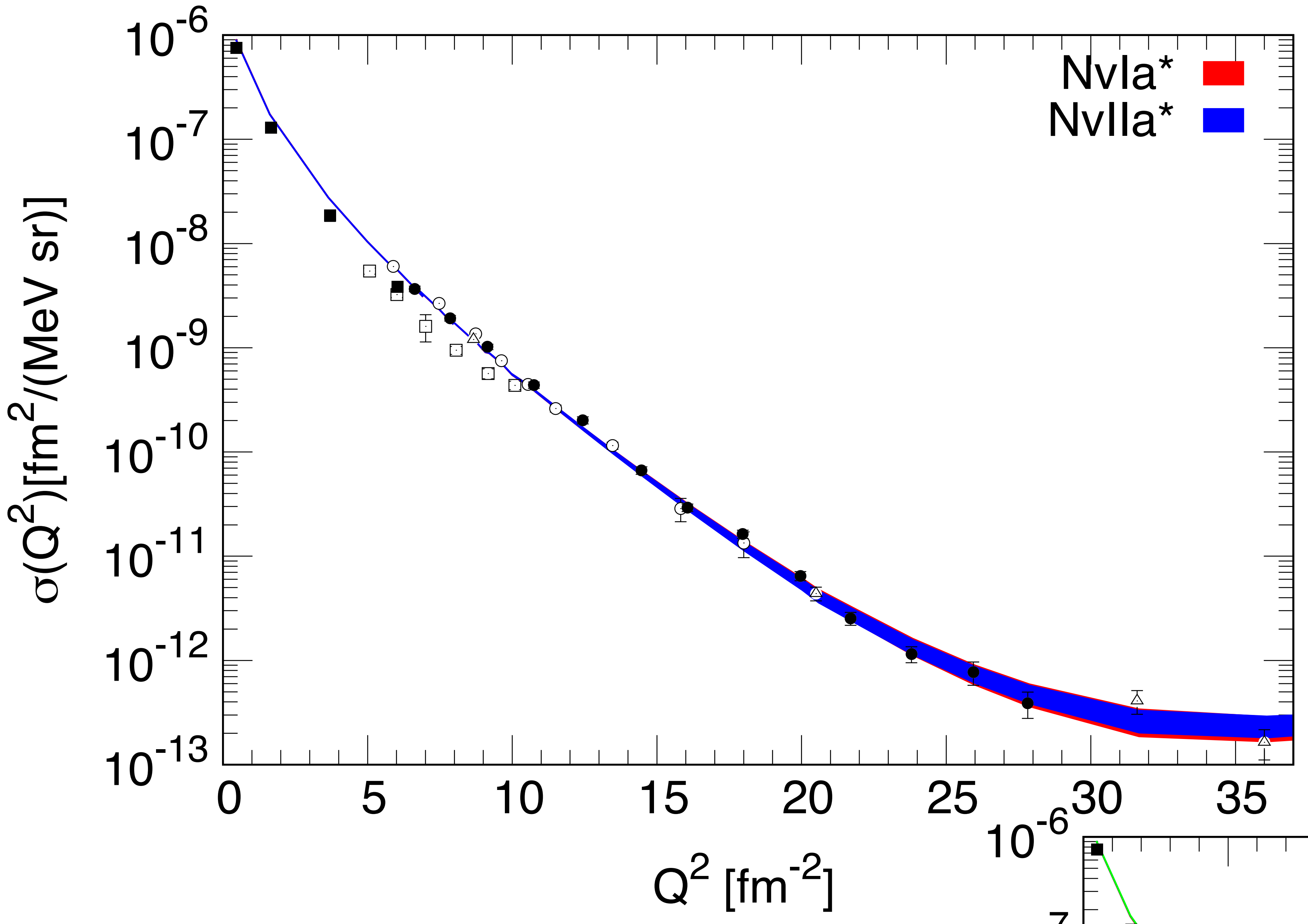
More precise data on more nuclei would permit to constrain better our models

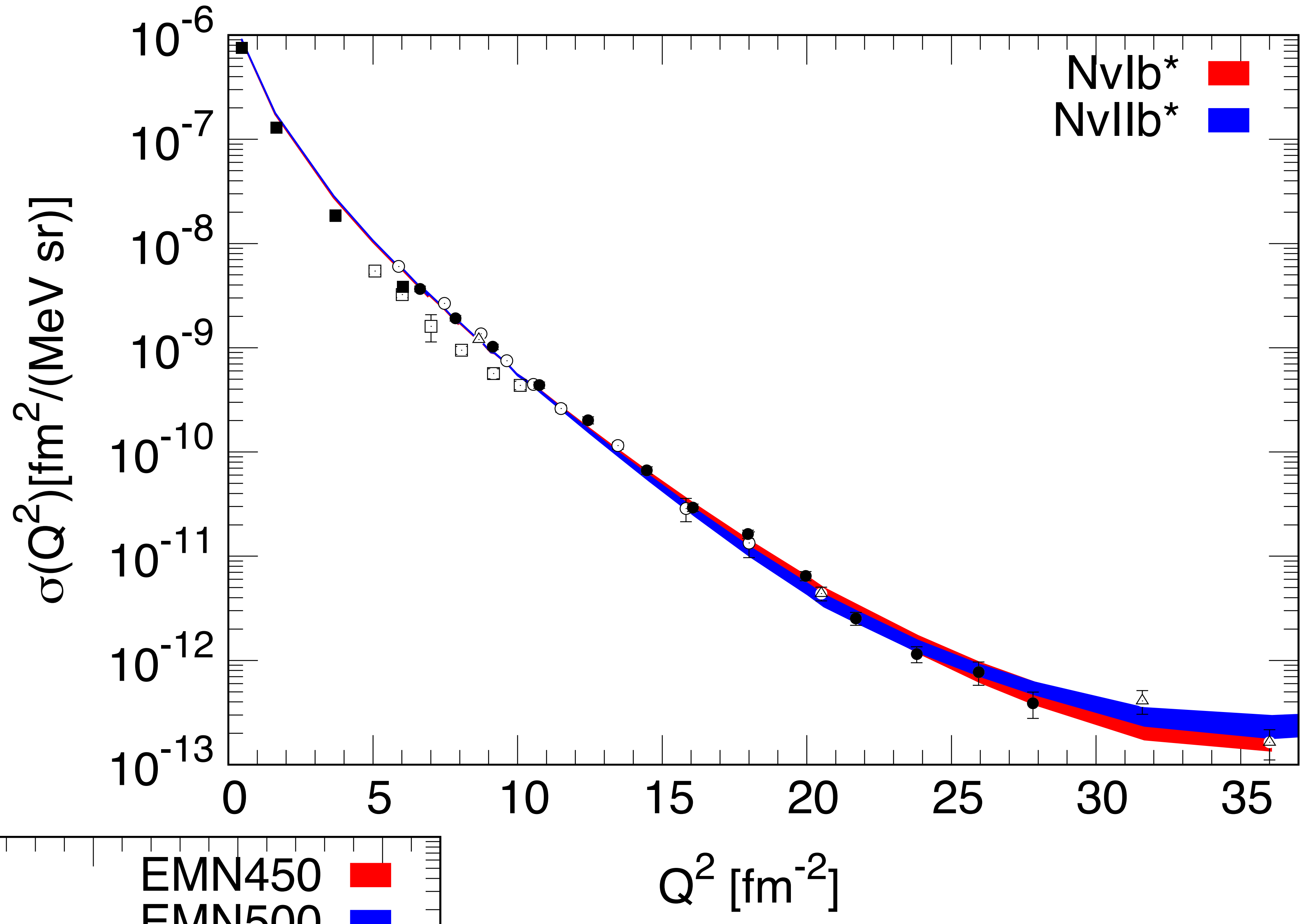
Dependence on Q^2_{max}

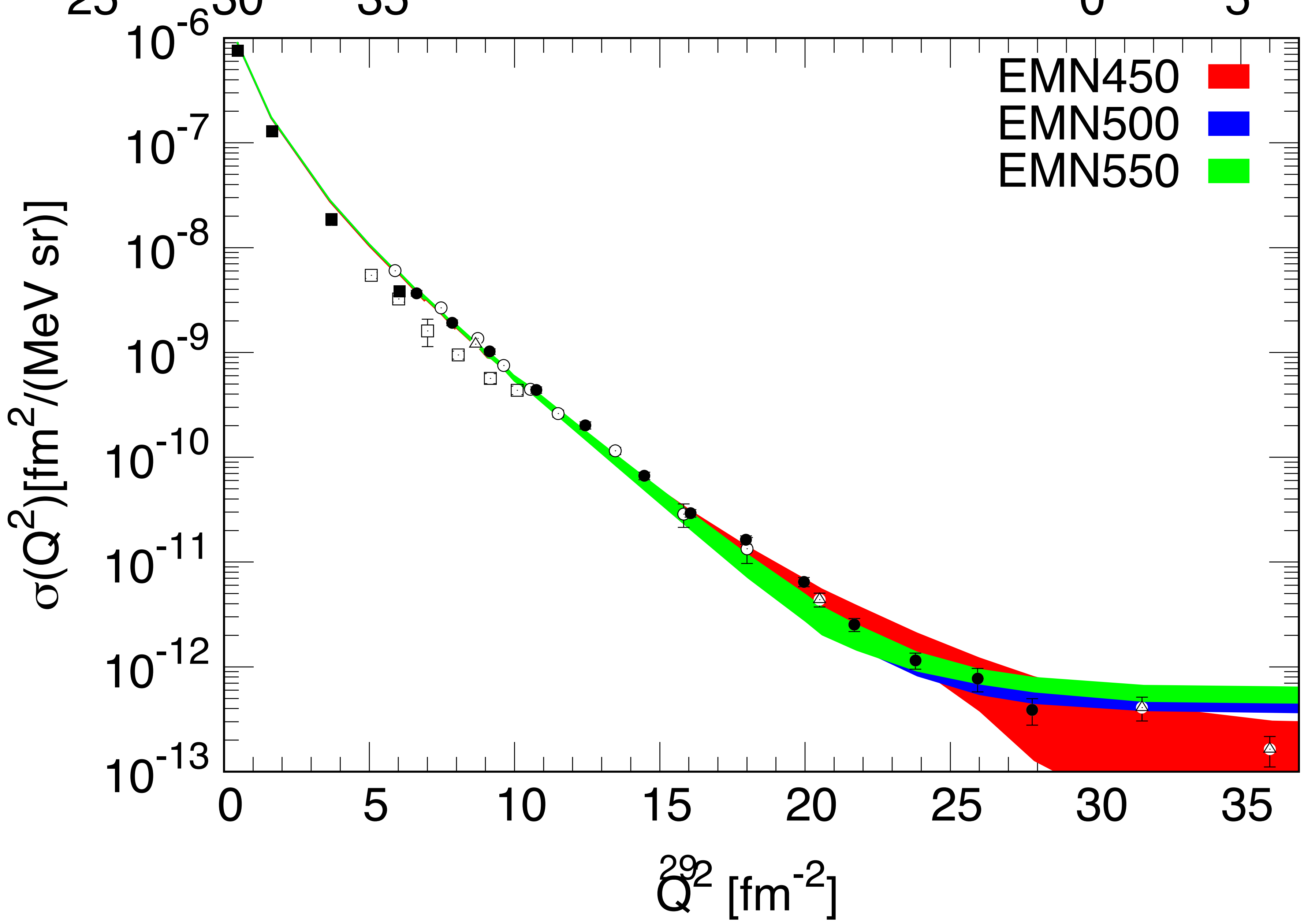




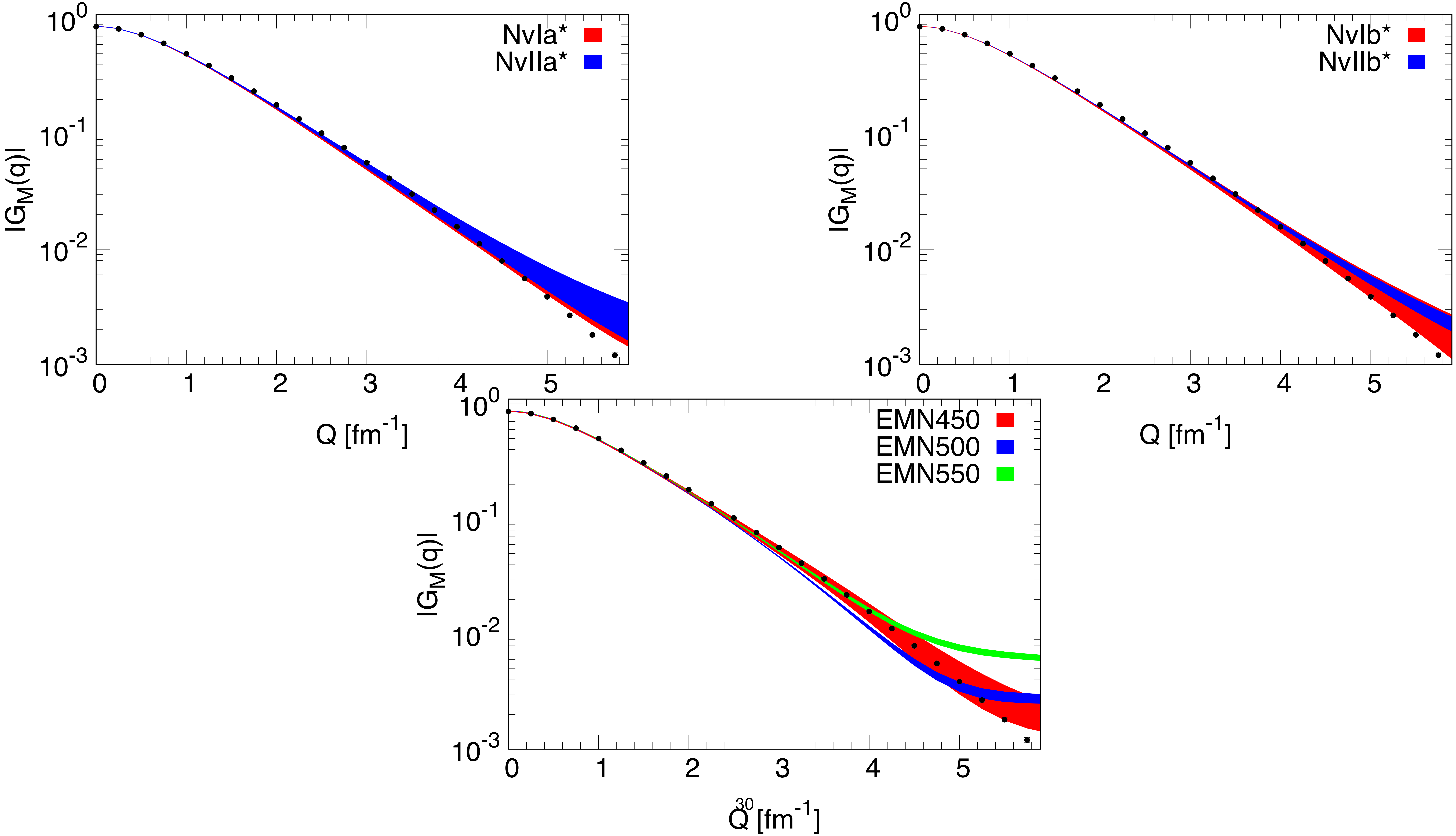
d-threshold


NvIa*
NvIIa*
σ(Q²)[fm²/(MeV sr)]
Q² [fm⁻²]

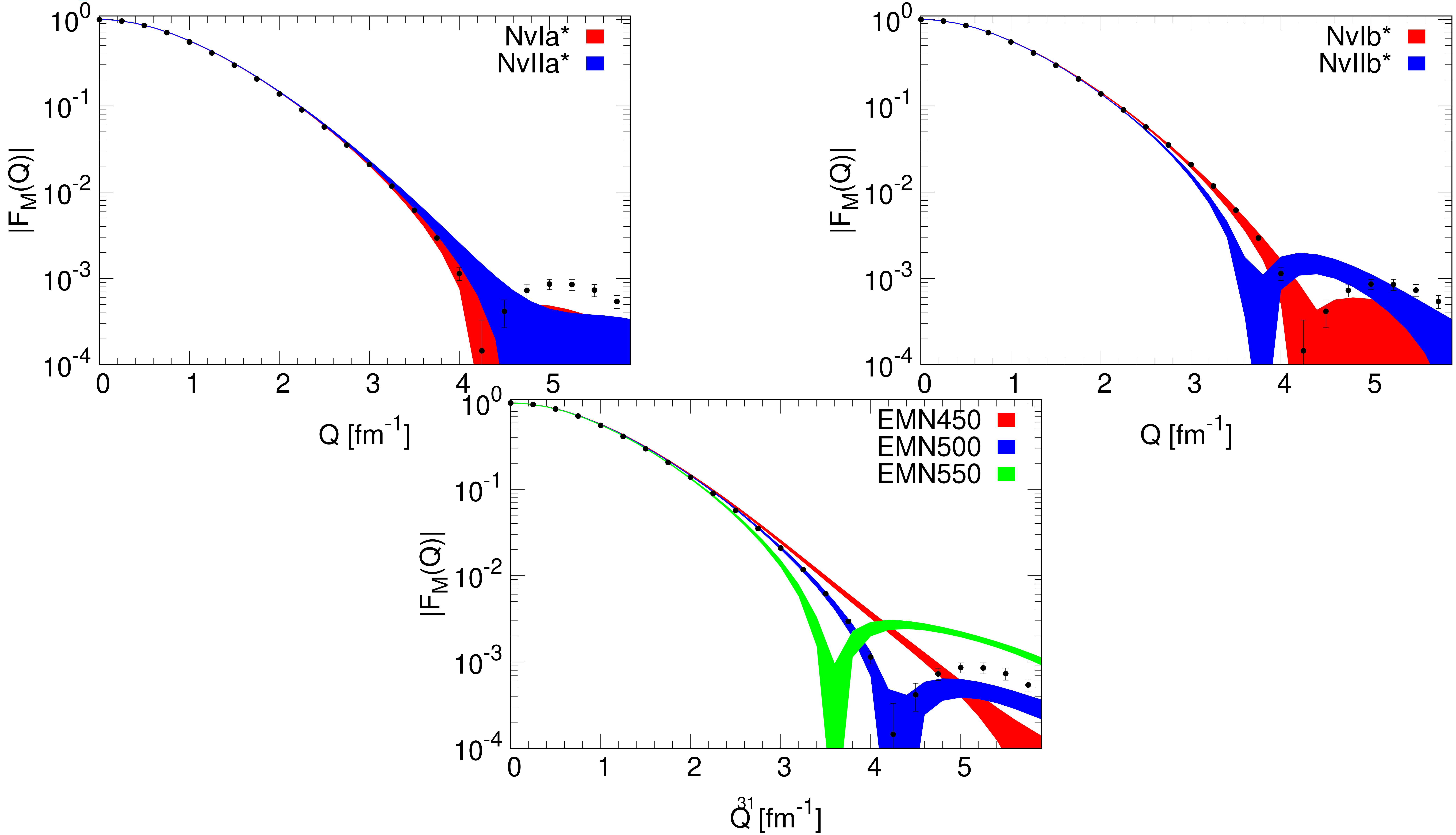

NvIb*
NvIIb*
σ(Q²)[fm²/(MeV sr)]
Q² [fm⁻²]


EMN450
EMN500
EMN550
σ(Q²)[fm²/(MeV sr)]
Q² [fm⁻²]

Magnetic form factors of ^{2}H



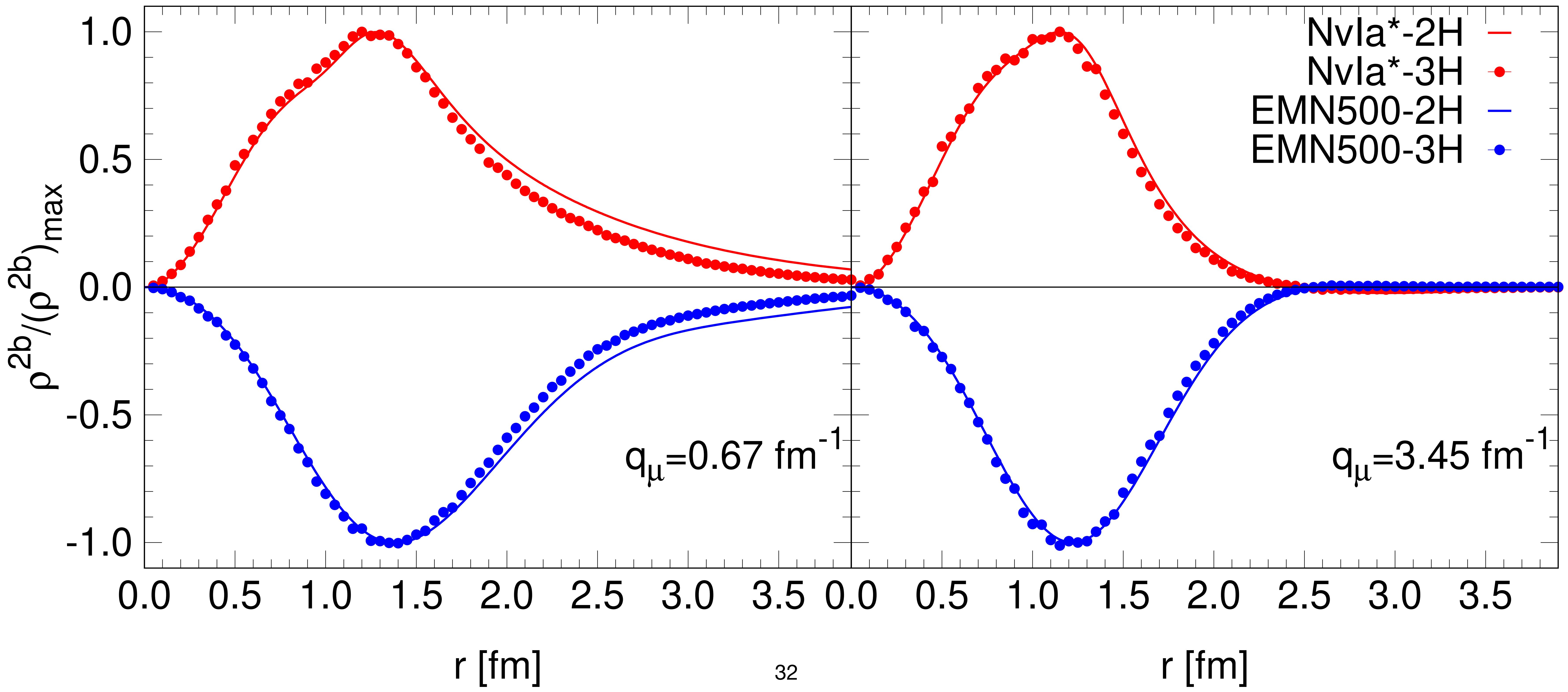
Magnetic form factors of ^{3}He



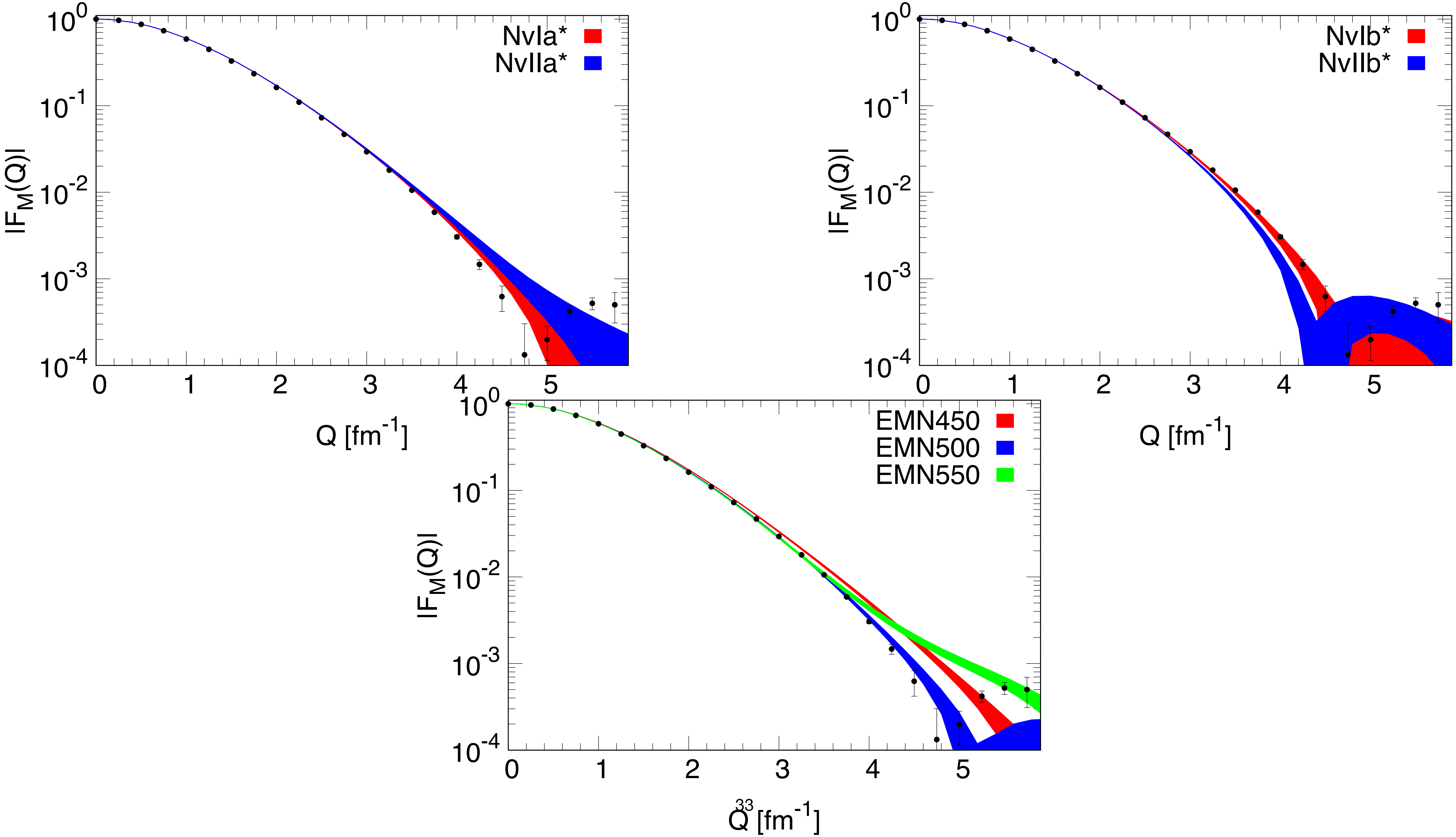
Why does it work?

Universal behavior of isovector transitions

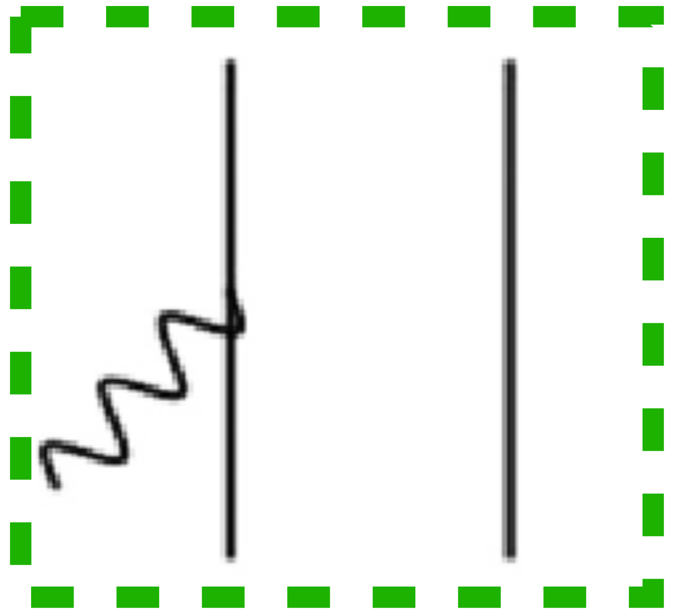
Correlated np pairs ⟷ Universal 2-body wave functions ⟷ Universal 2-body transition densities

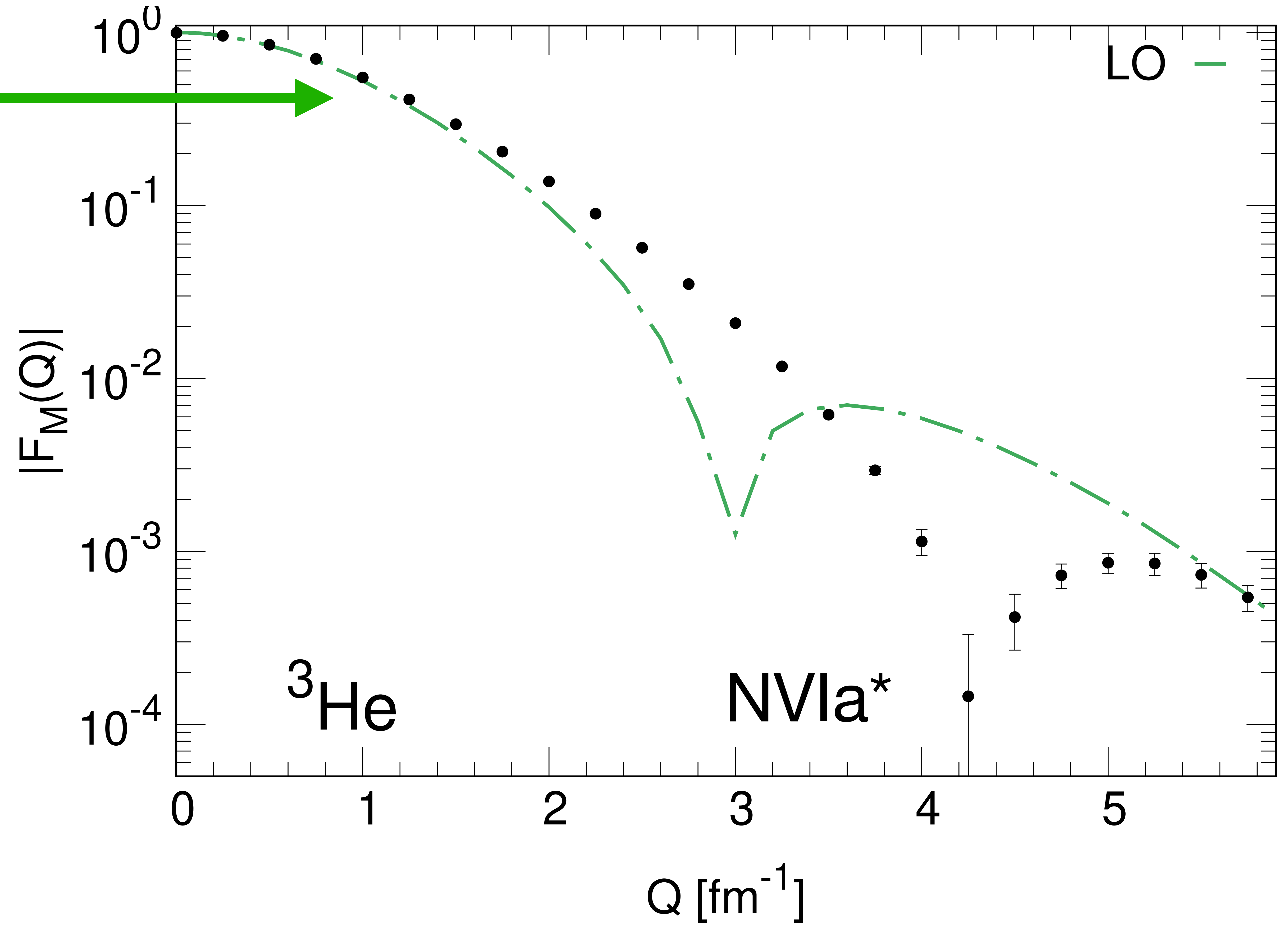


Magnetic form factors of ^{3}H

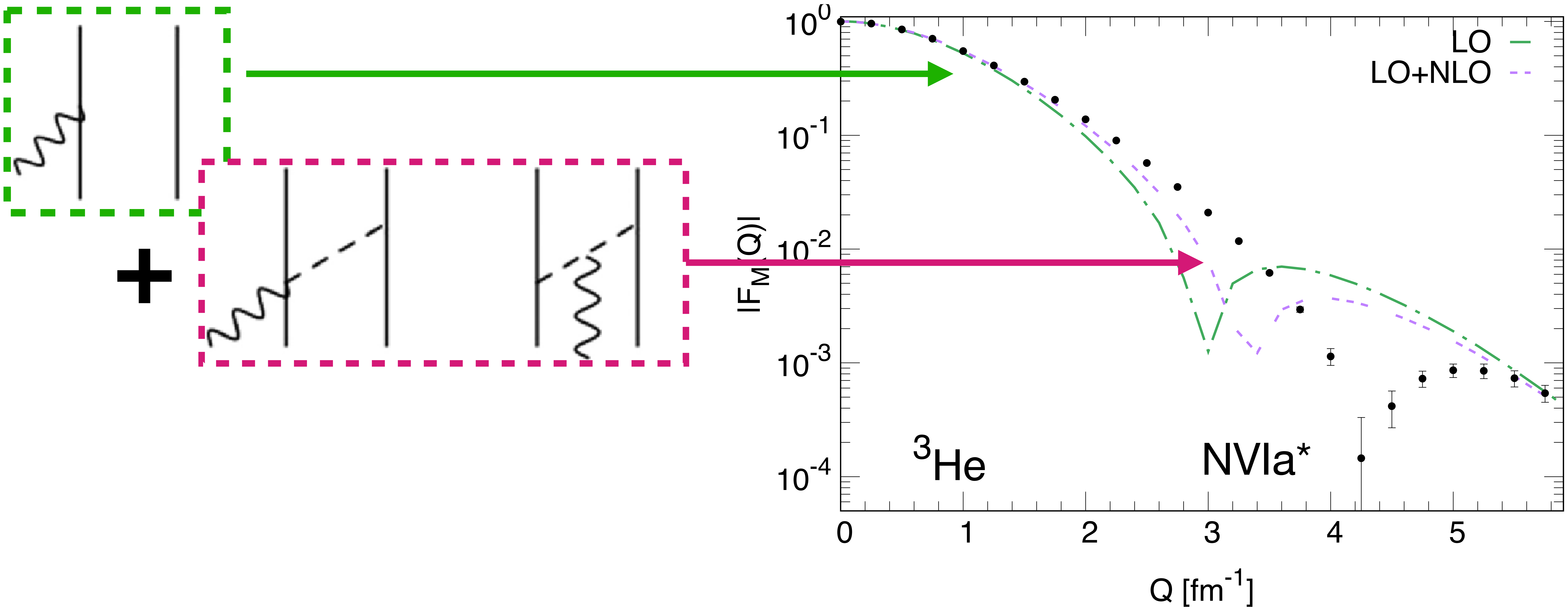


Prediction of A=3 Magnetic Form Factors

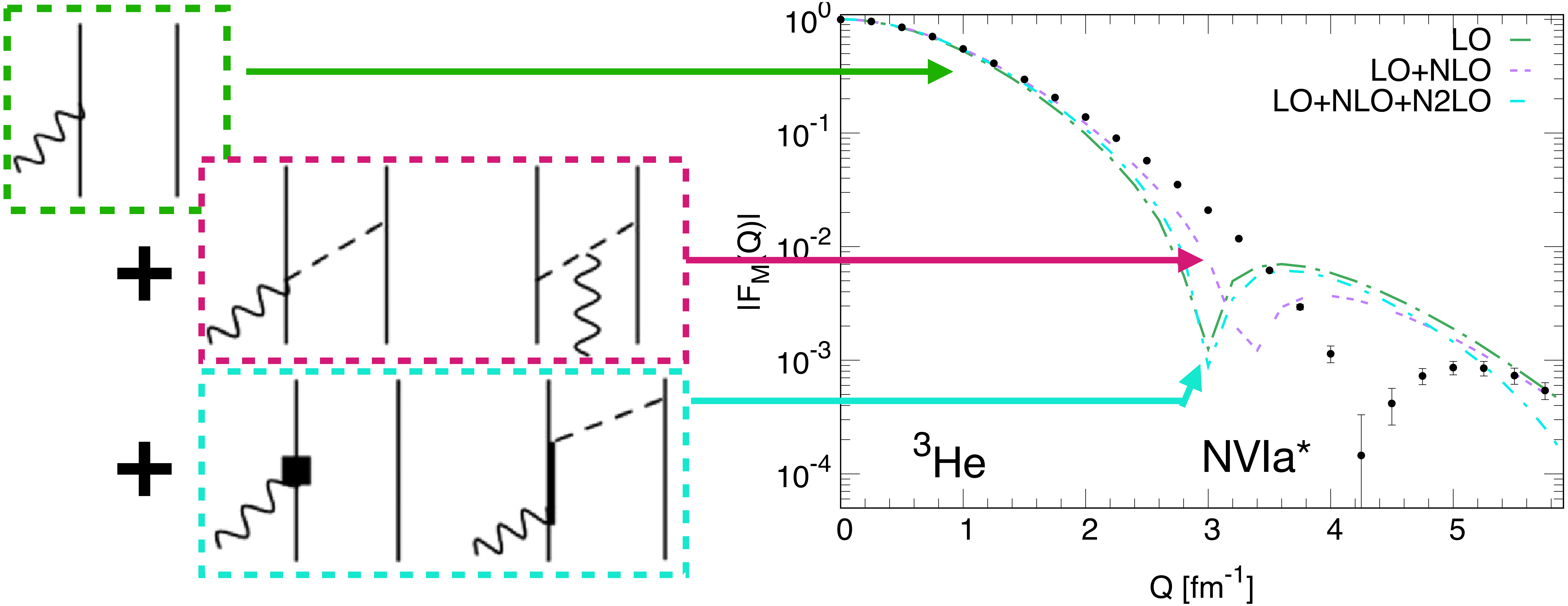




Prediction of A=3 Magnetic Form Factor



Prediction of A=3 Magnetic Form Factor



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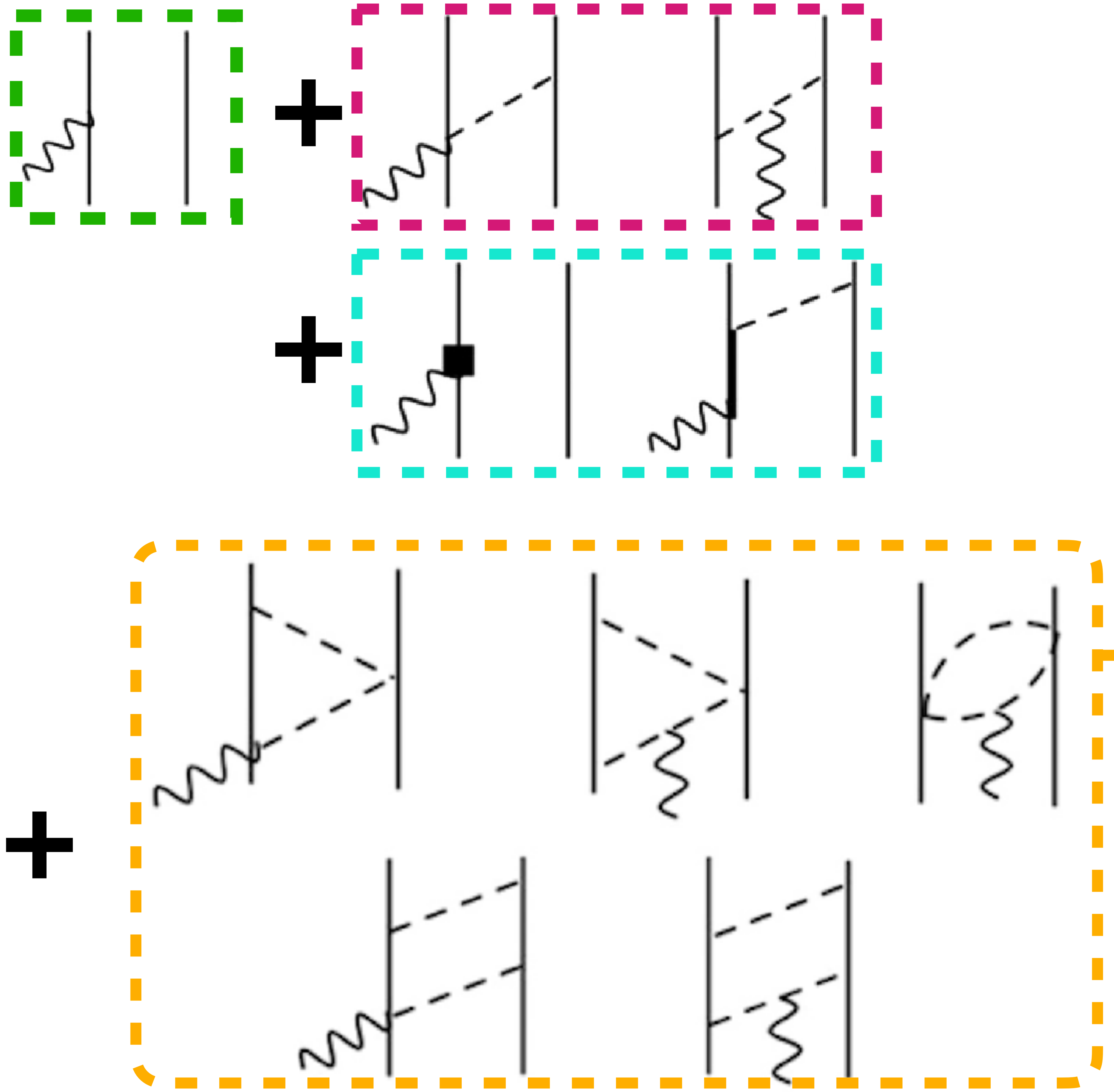

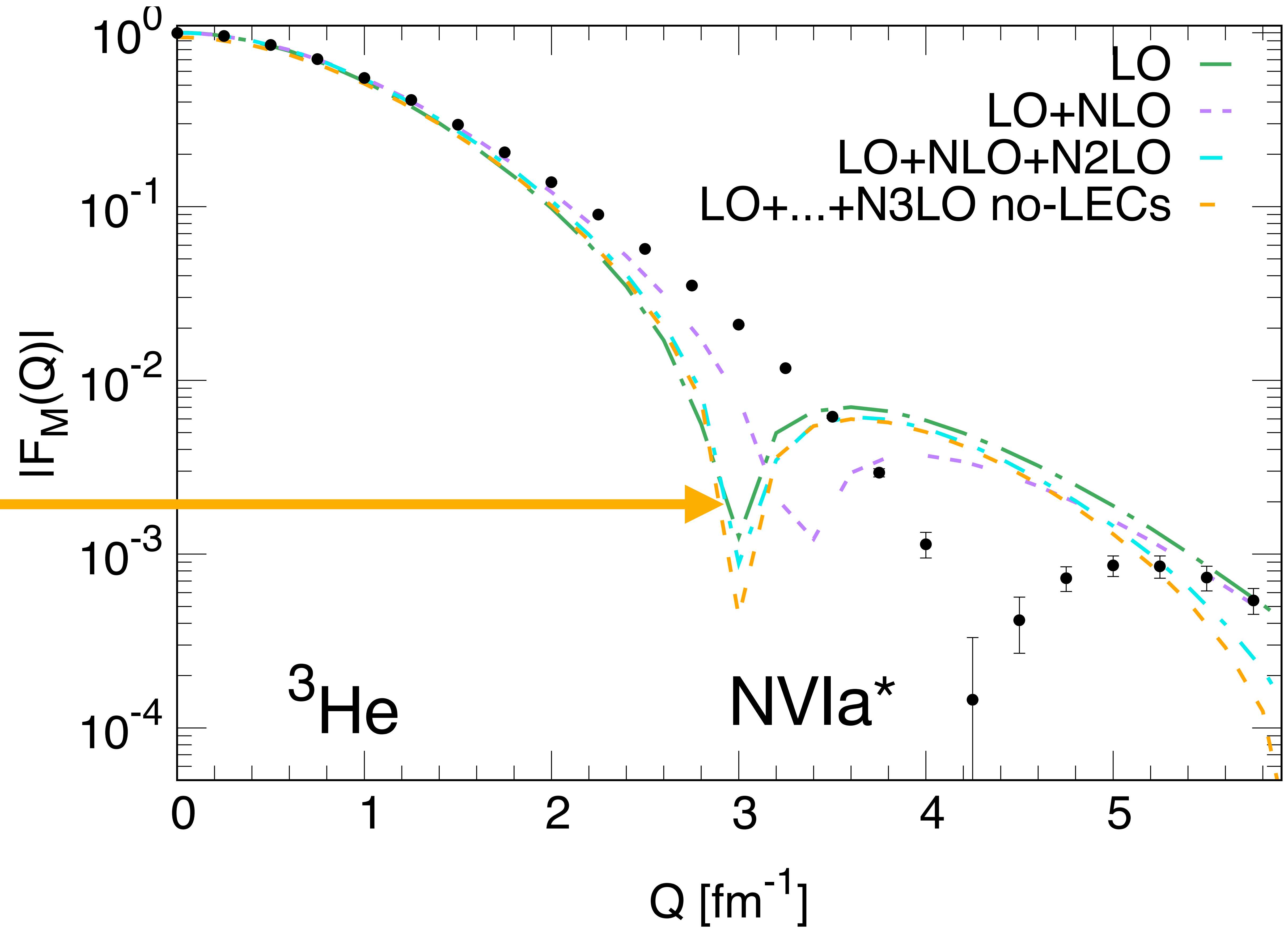

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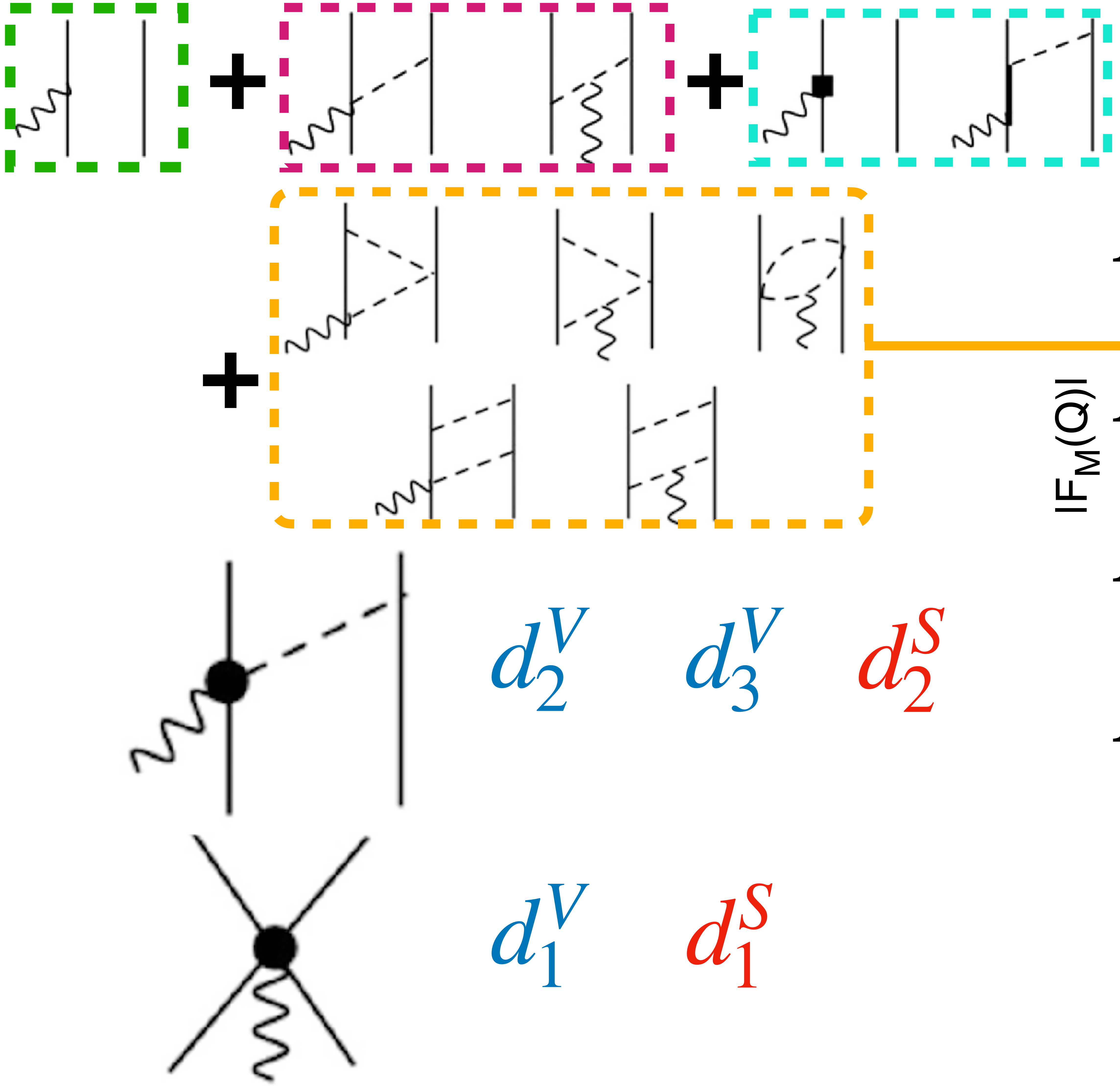
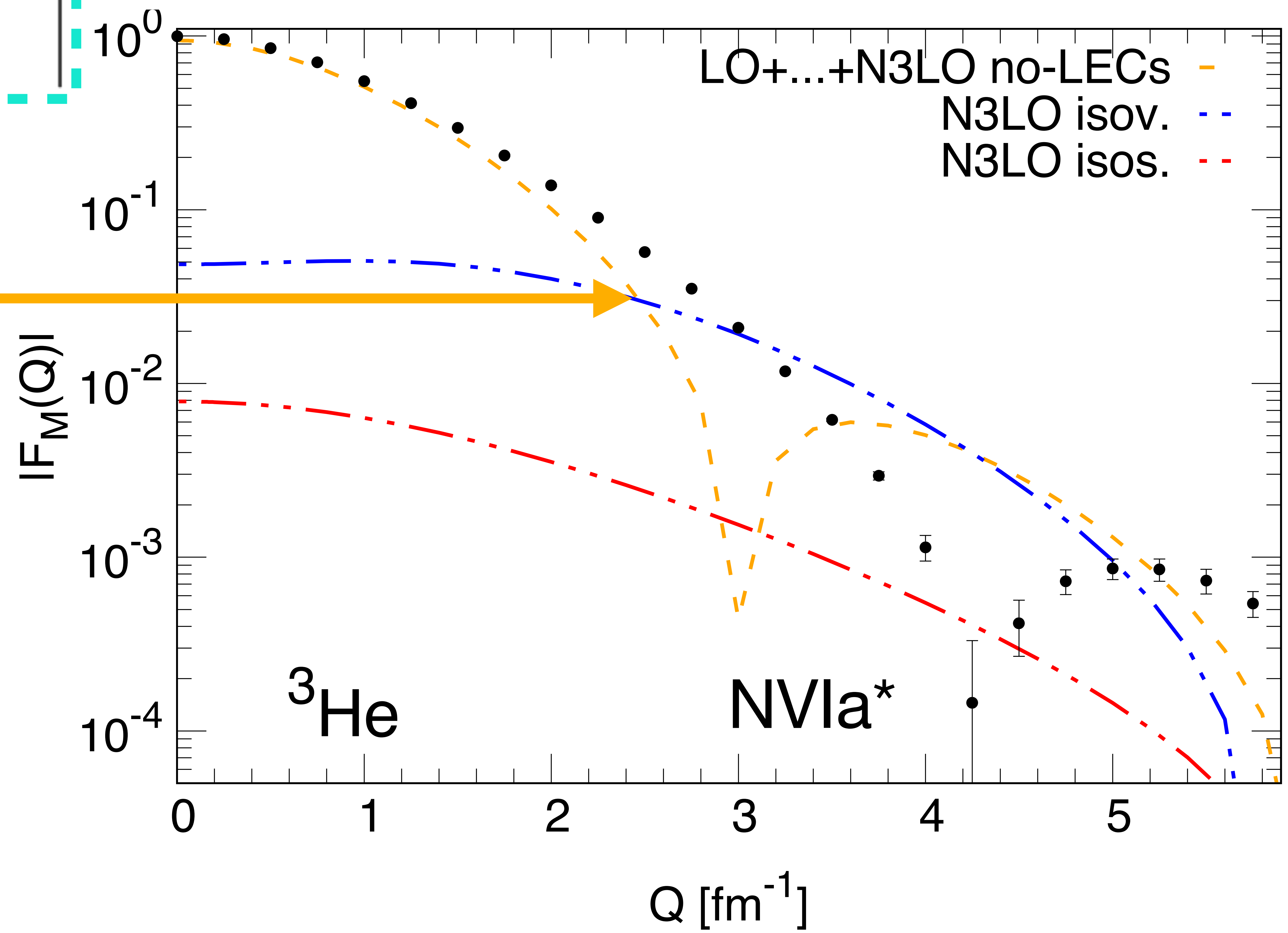




Reliability of the predictions

Is χEFT able to describe large Q?

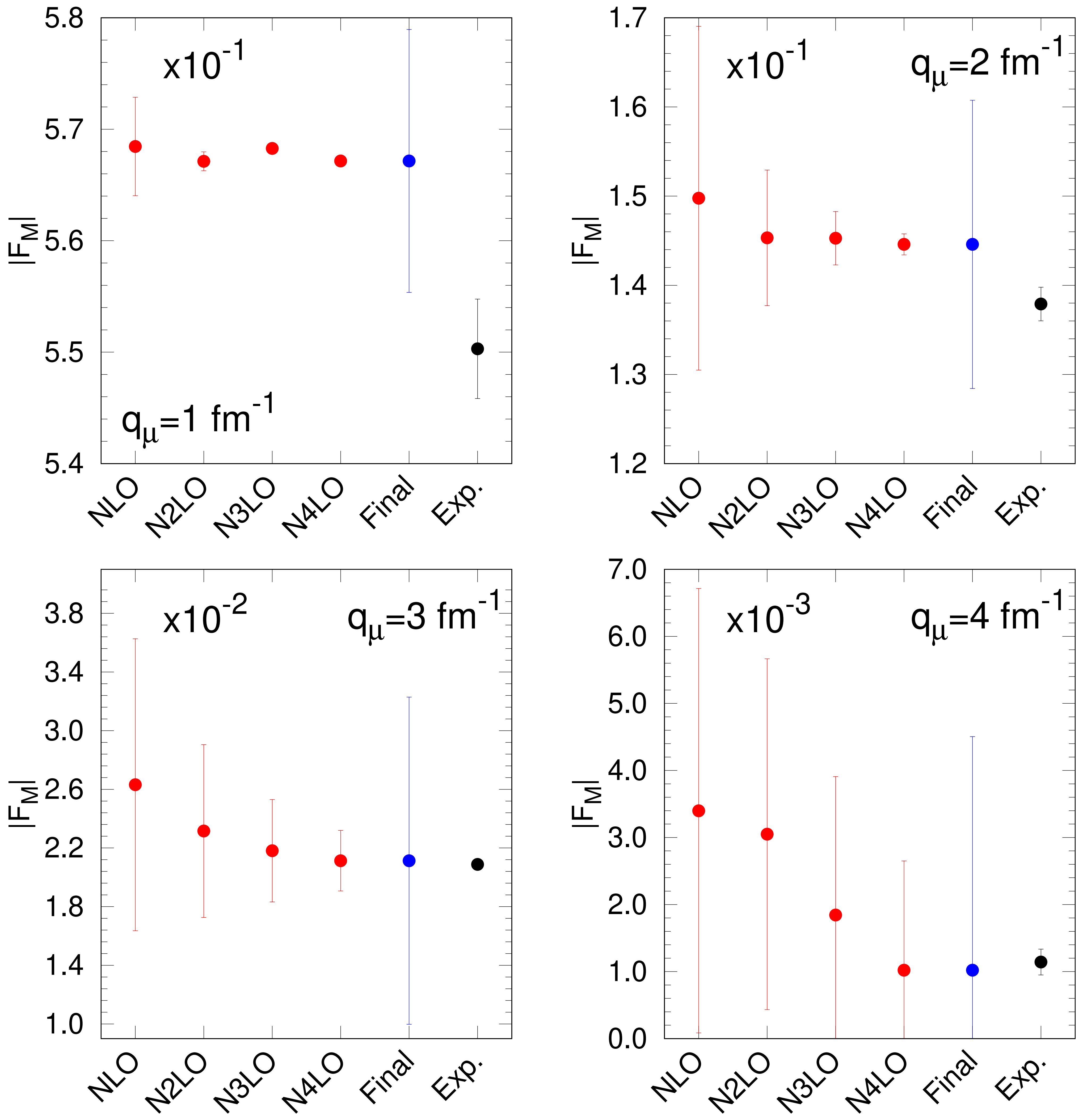
- Truncation errors (as [EPJA 51, 53 (2015)])

$$\alpha = \max\left\{\frac{Q}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right\} \quad \Lambda_b = 1 \text{ GeV}$$

- Nuclear interaction + currents

- Systematic explodes after

$$Q^2 > 0.5 \text{ GeV}^2$$



^{3}He EMN500

Naive truncation error estimate

Is χEFT able to describe large Q^2?

- Truncation errors (as [EPJA 51, 53 (2015)])

$$\alpha = \max\left\{\frac{Q}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right\} \quad \Lambda_b = 1 \text{ GeV}$$

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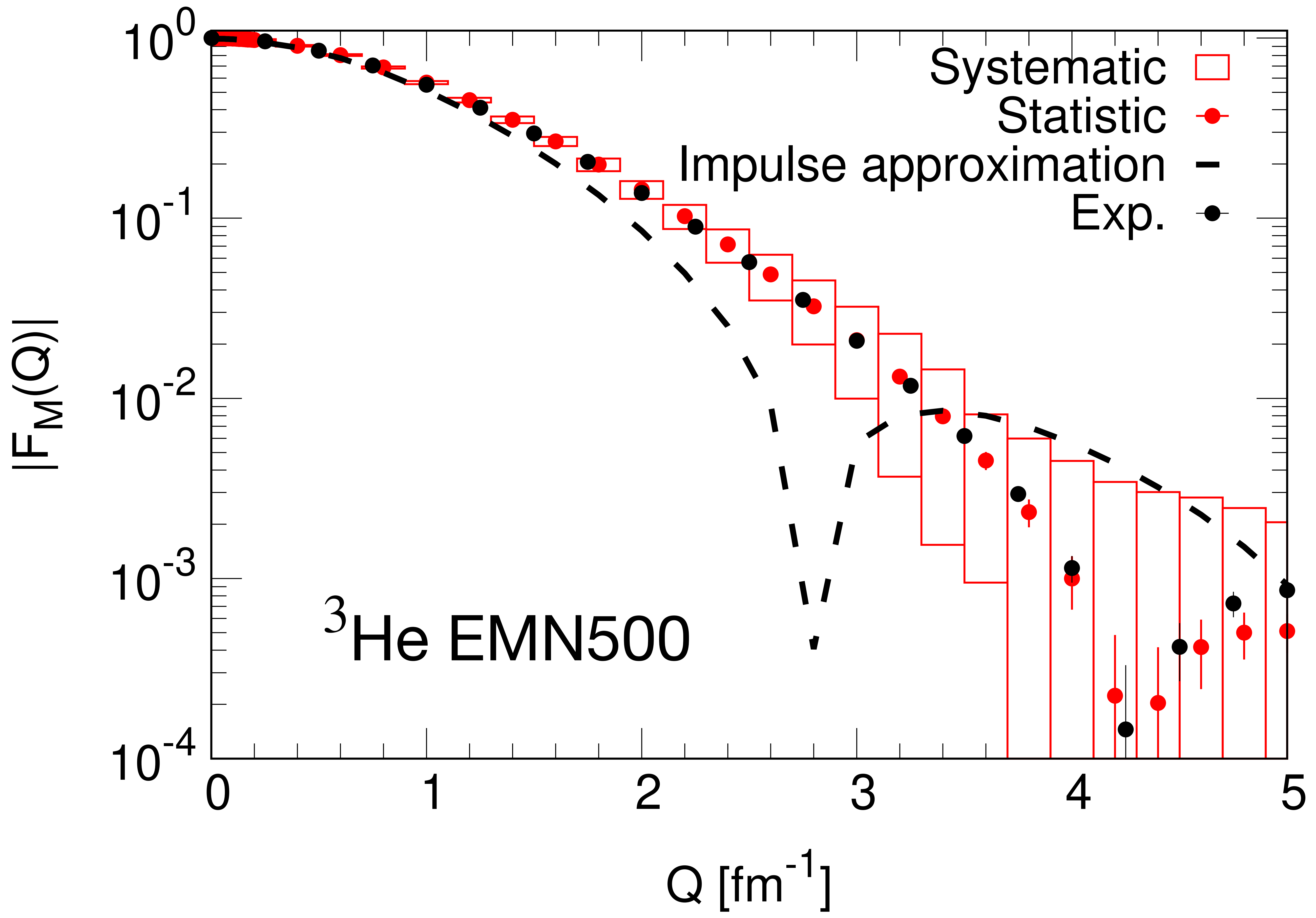
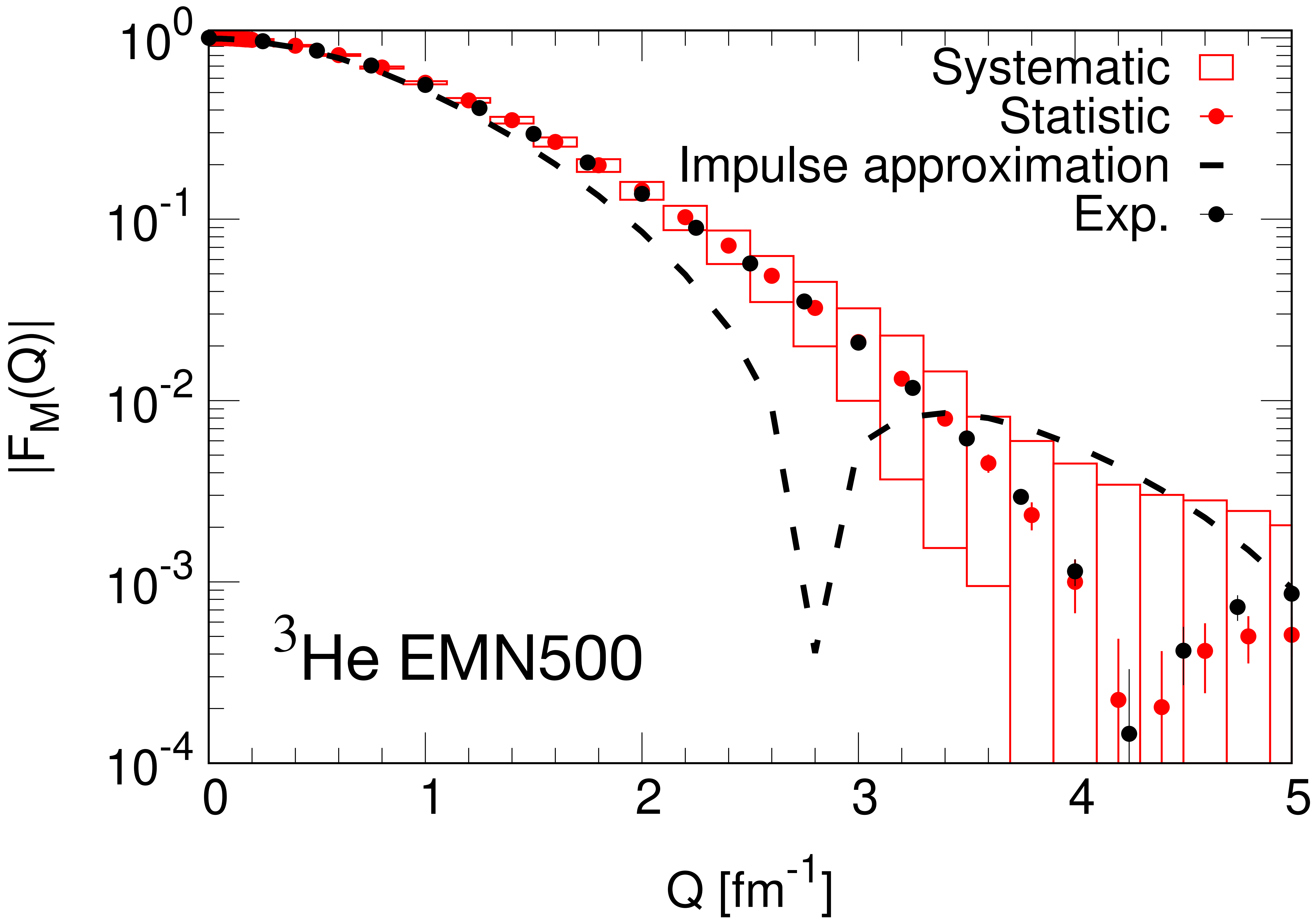
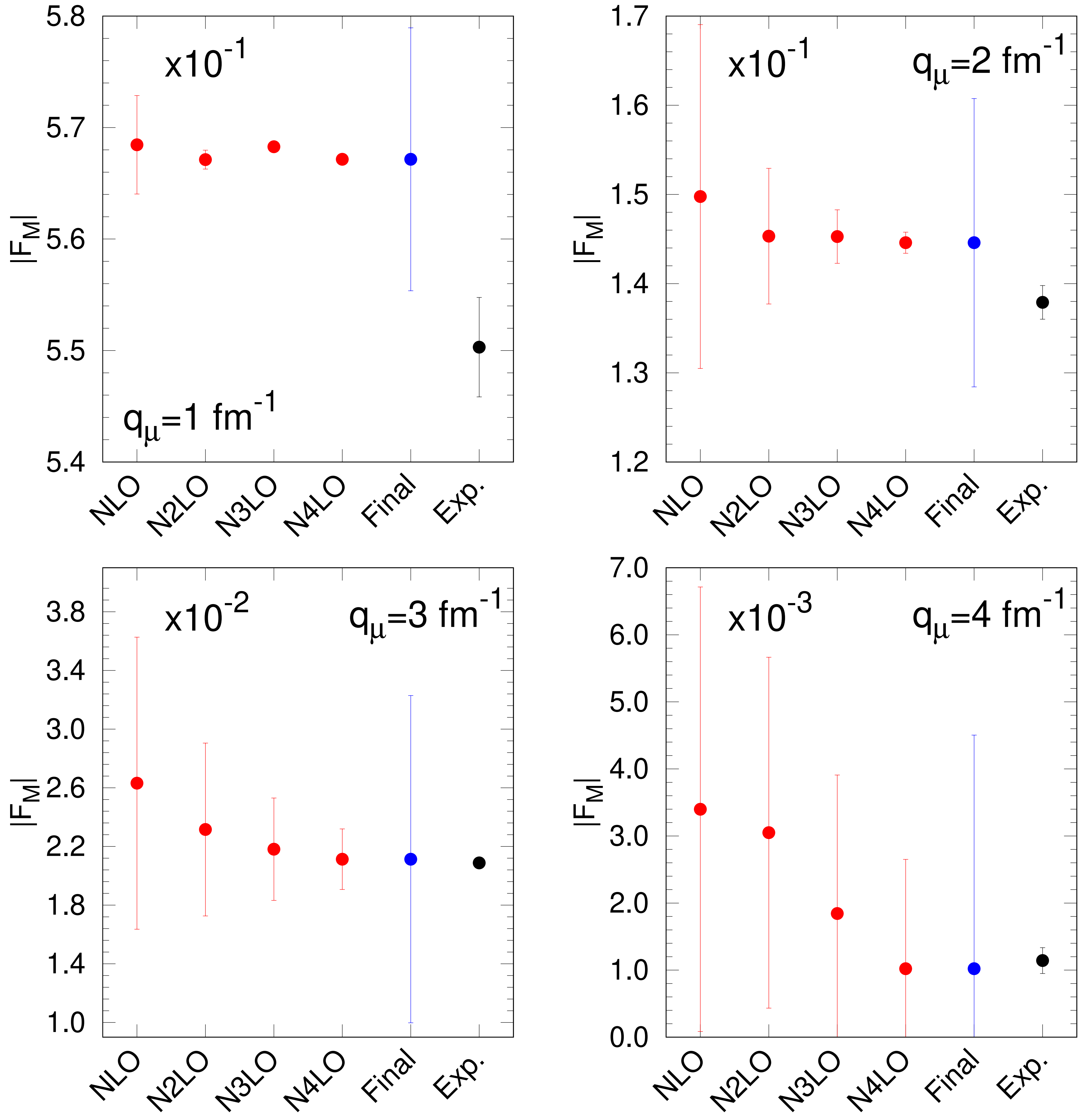


Error estimate

- Truncation errors (as [EPJA 51, 53 (2015)])

$$\alpha = \max\left\{\frac{q}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right\} \qquad \Lambda_b = 1 \text{ GeV}$$





Mirror nuclei structure

The reason

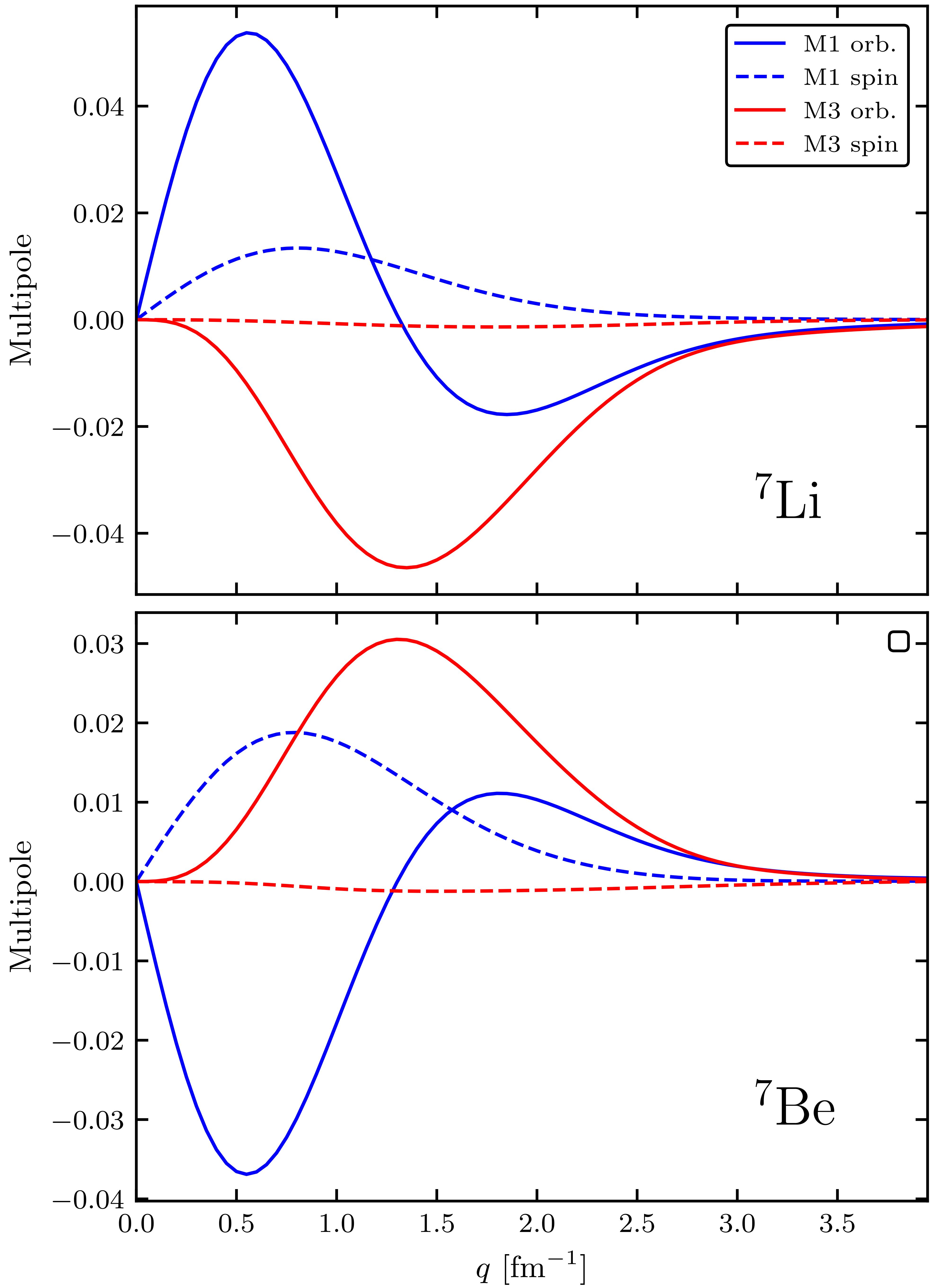
$$\mathbf{j}^{\mathrm{LO}}(\mathbf{q}) = \frac{\epsilon_i(q_\mu^2)}{2\,m}\,\left[\mathbf{p}_i\,,\,\mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}_i}\right]_+ + i\,\frac{\mu_i(q_\mu^2)}{2\,m}\,\mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}_i}\,\sigma_i\times\mathbf{q}\,,$$

Convection current

Magnetic current

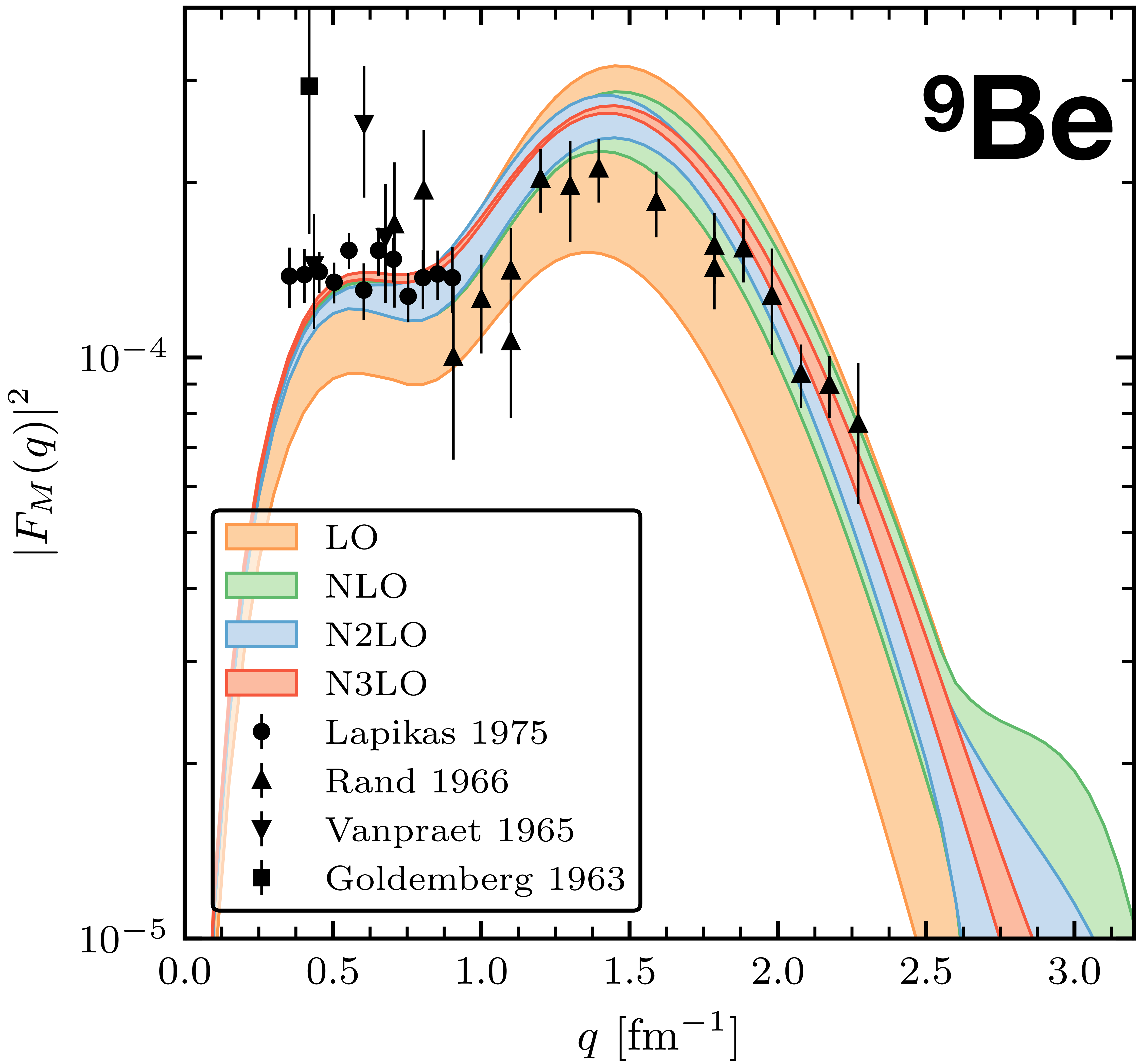
No contribution to M_3

Change sign if there is an unpaired neutron/proton



Order by order expansion

- Error analysis based on: [EPJA 51, 53 (2015)]
- Expansion parameter: $Q = (A-1)/A \times q/\Lambda_b,\ \Lambda_b = 700\,\mathrm{MeV}$
- Chiral expansion seems to have in general a good behavior.
- N3LO corrections for some nuclei are of the same size of N2LO.



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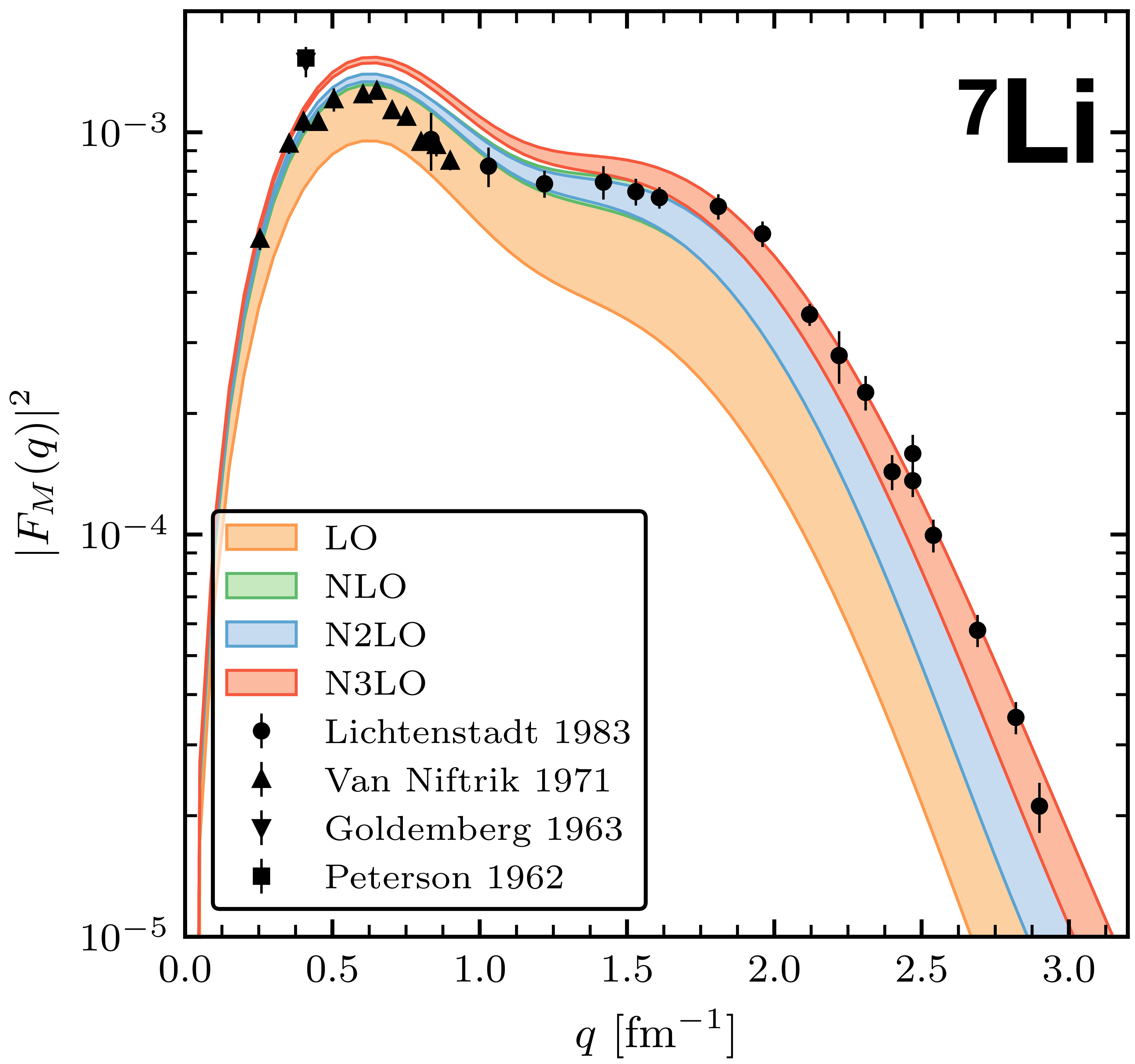
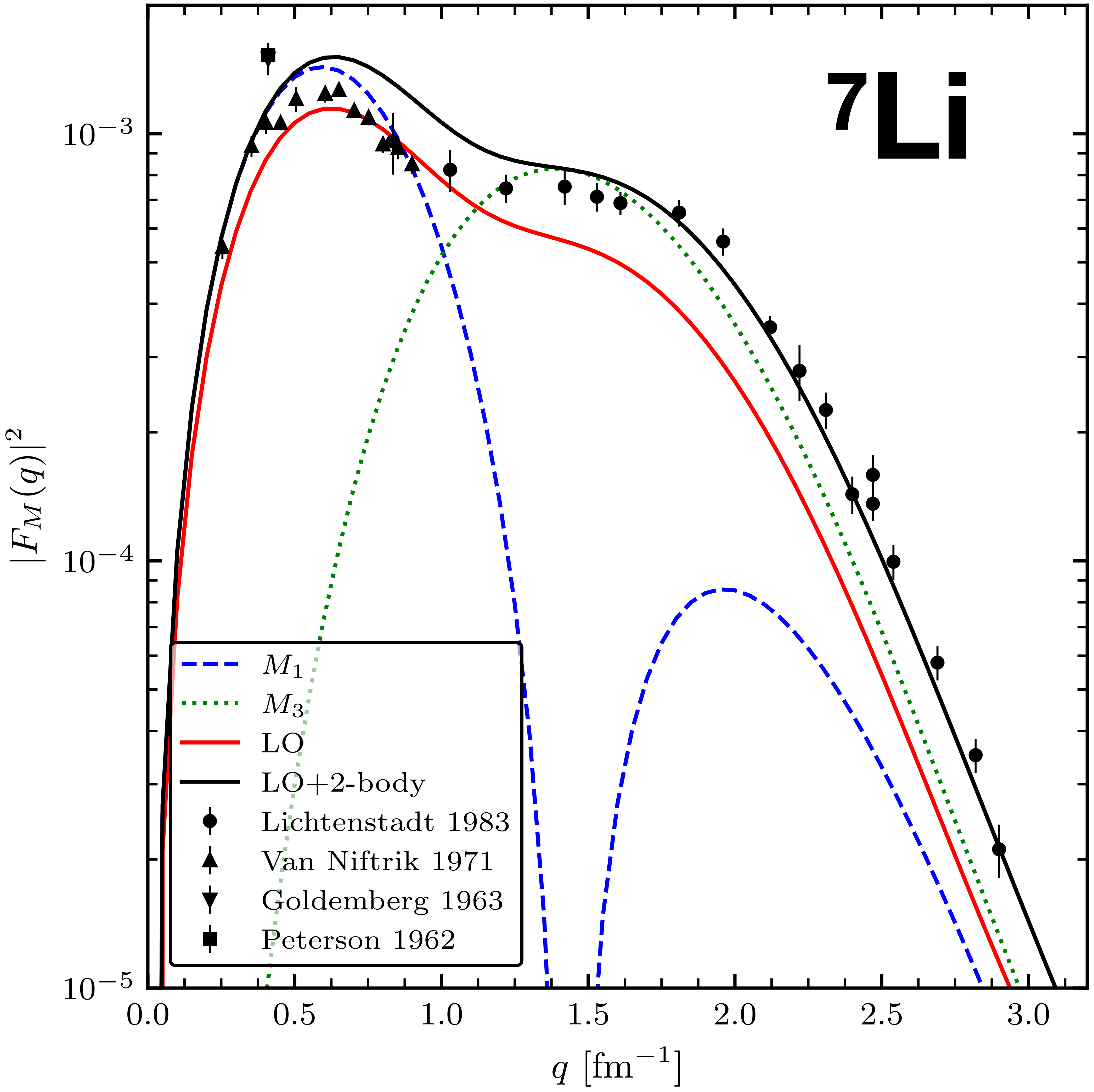
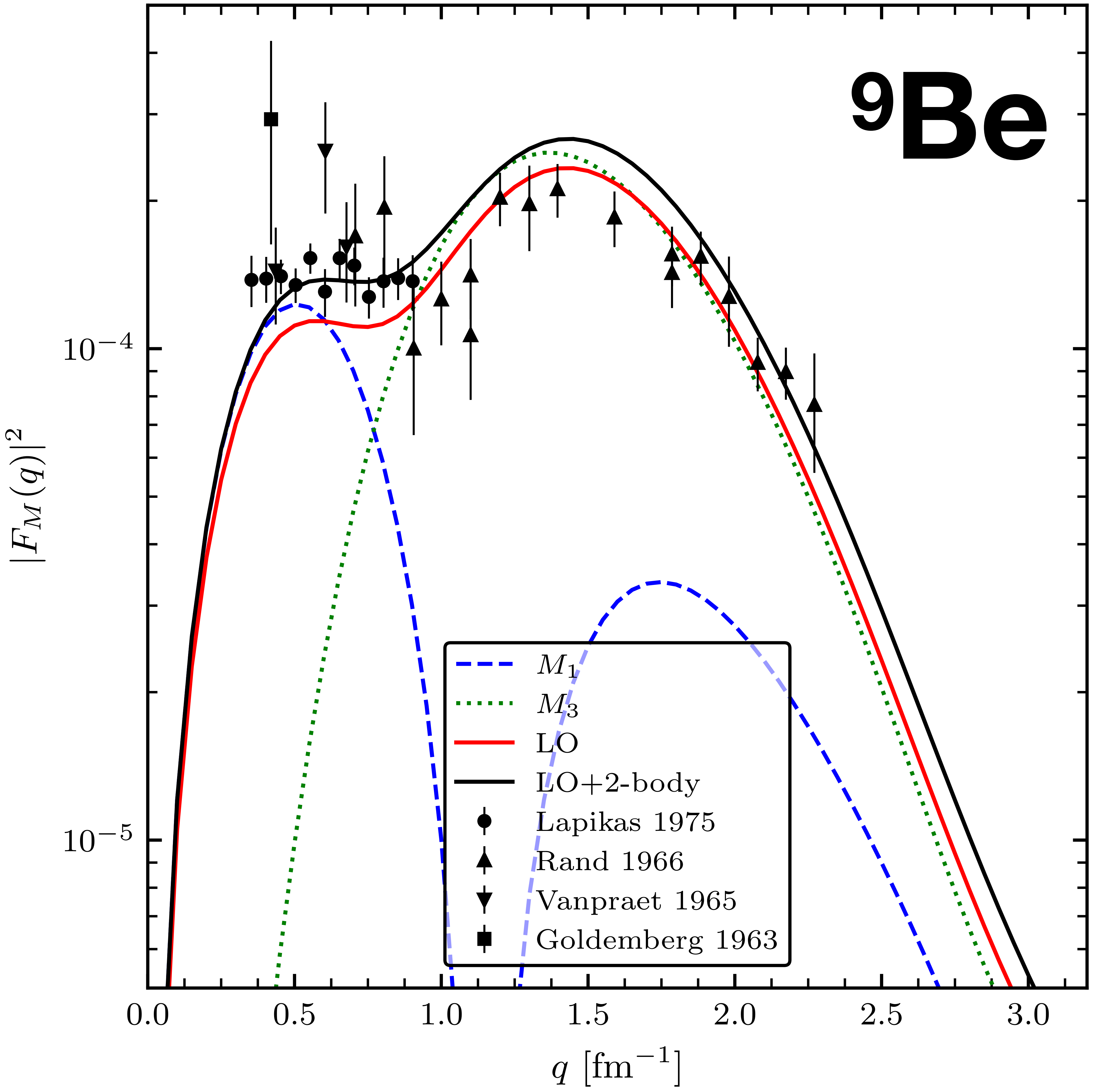


Magnetic form factor predictions

Lithium-7 and Berilium-9 (isovector dominated)





Magnetic form factor predictions

Lithium-6 and Boron-10 (isoscalar transition)