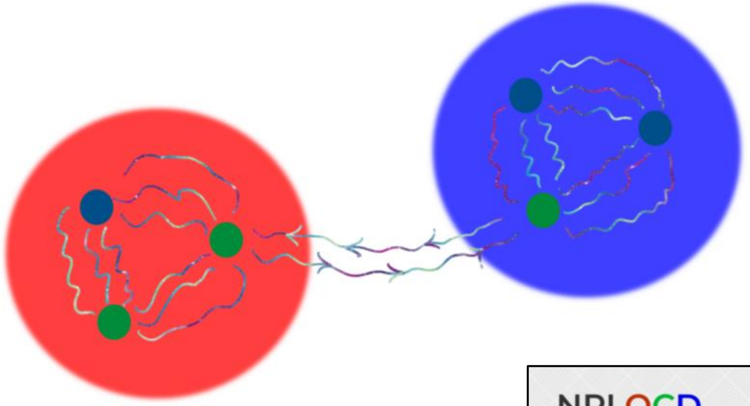
# An *ab initio* framework for the modelling of electroweak processes in light nuclei


Michael Gennari

Collaborators

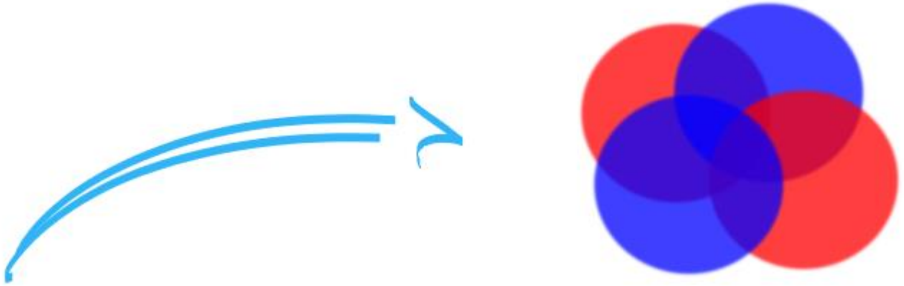
D. A. Najera, L. Jokiniemi, M. Drissi, C.-Y. Seng, M. Gorchtein, P. Navratil

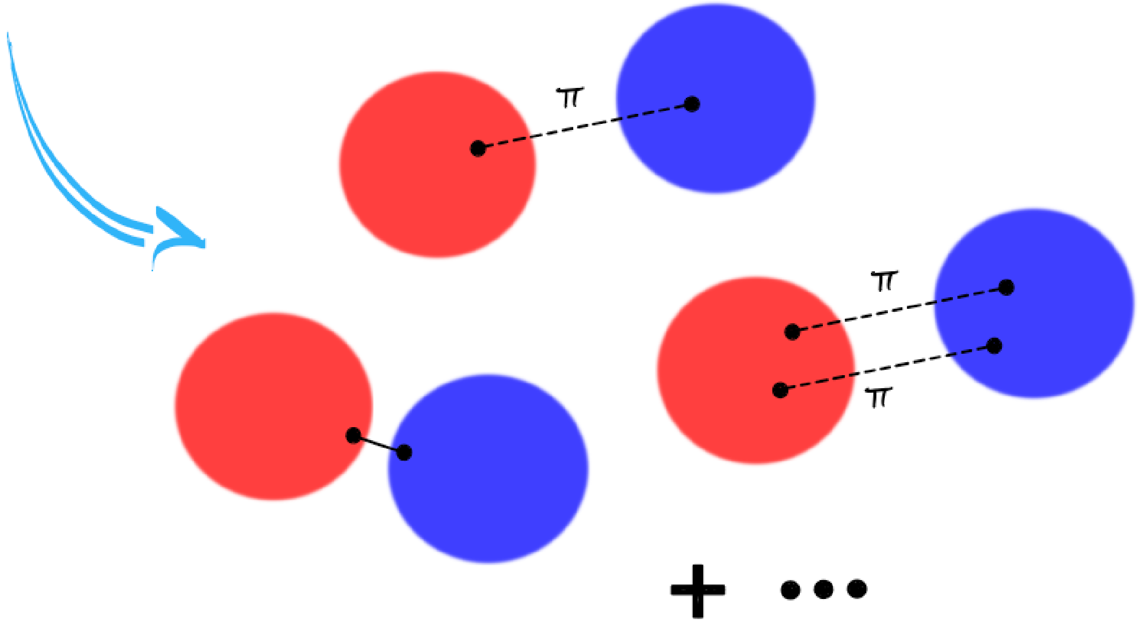



NPLQCD
Nuclear Physics with Lattice QCD

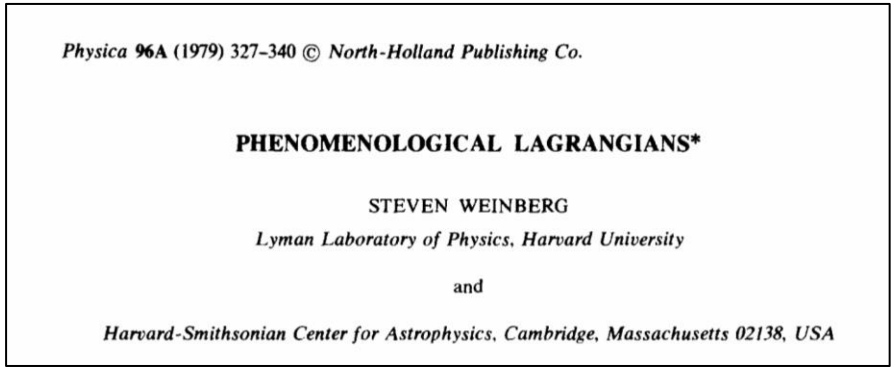
$$SU(3)_{\mathrm{C}}$$

$$V = \sum_{i<j}^{A} V_{NN}^{(ij)} + \sum_{i<j<k}^{A} V_{NNN}^{(ijk)} + \cdots$$






$$SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \sim SO(4)$$


*Physica* **96A** (1979) 327–340 © *North-Holland Publishing Co.*

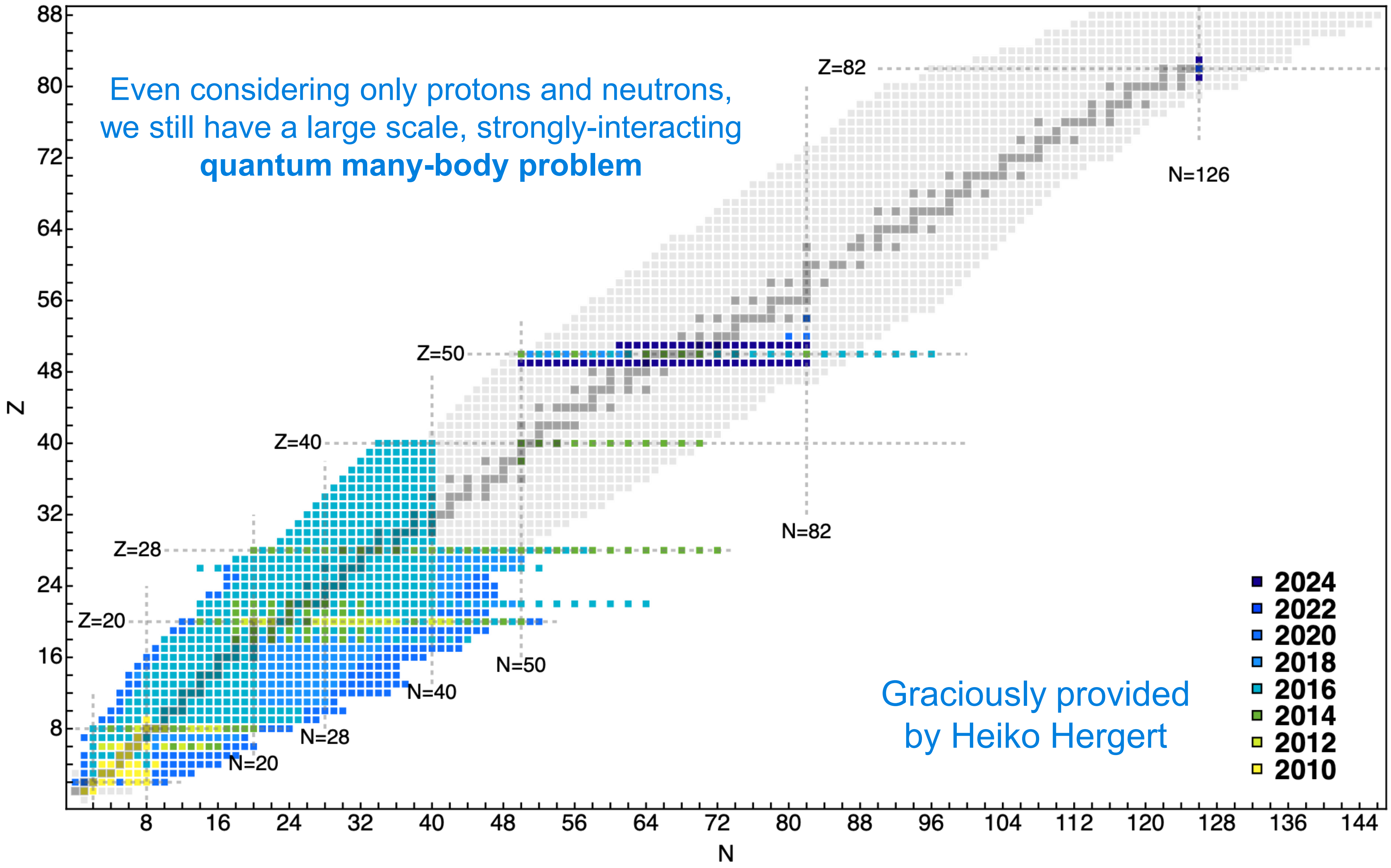
**PHENOMENOLOGICAL LAGRANGIANS***

STEVEN WEINBERG

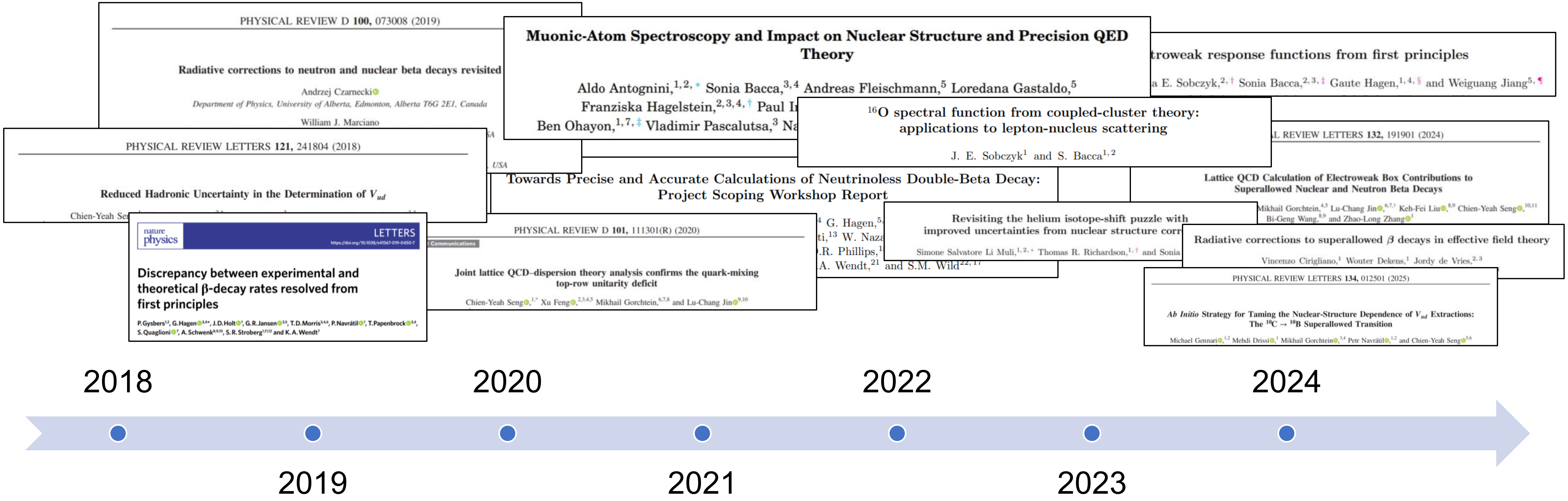
*Lyman Laboratory of Physics, Harvard University*

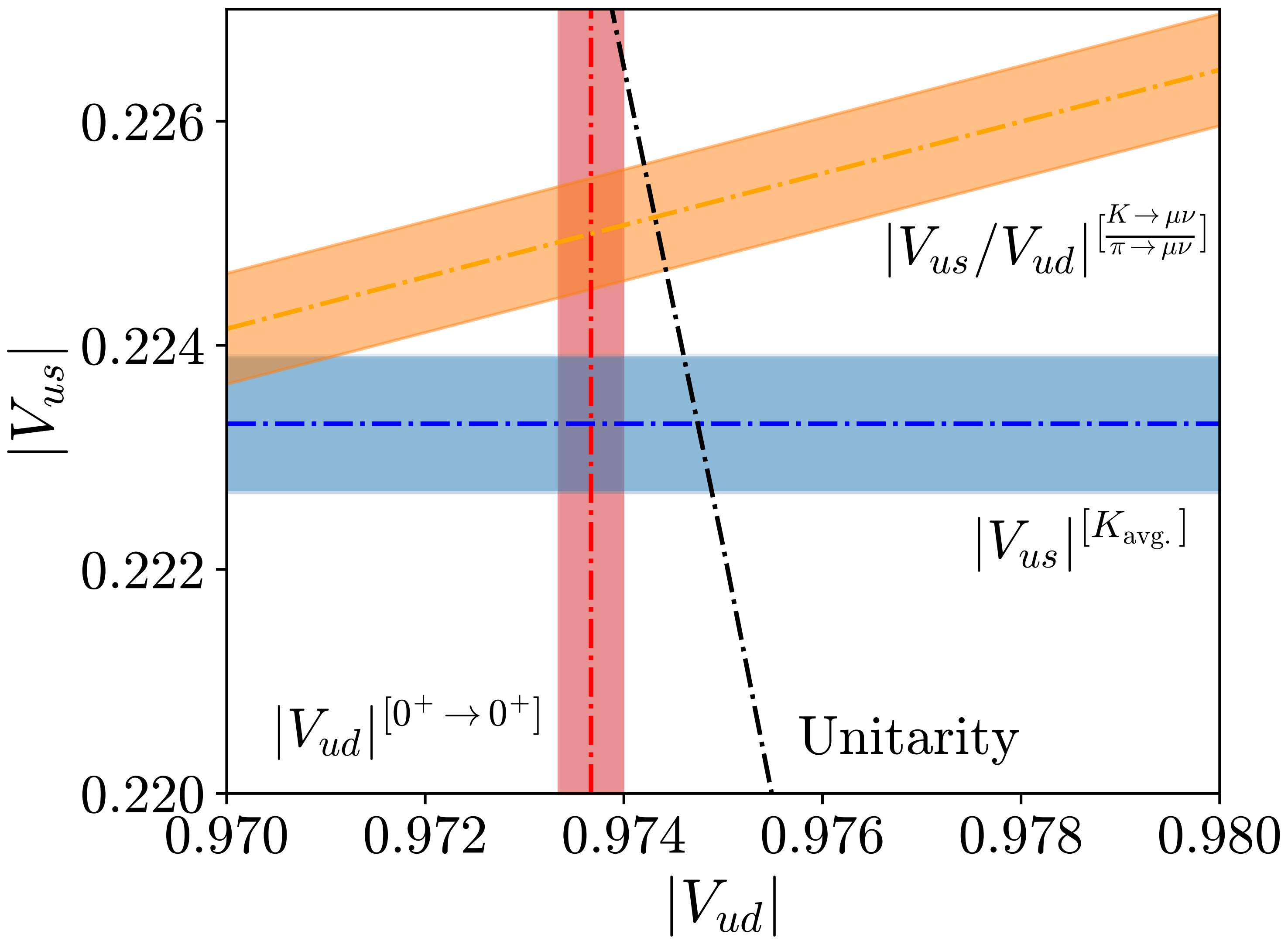
and

*Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA*

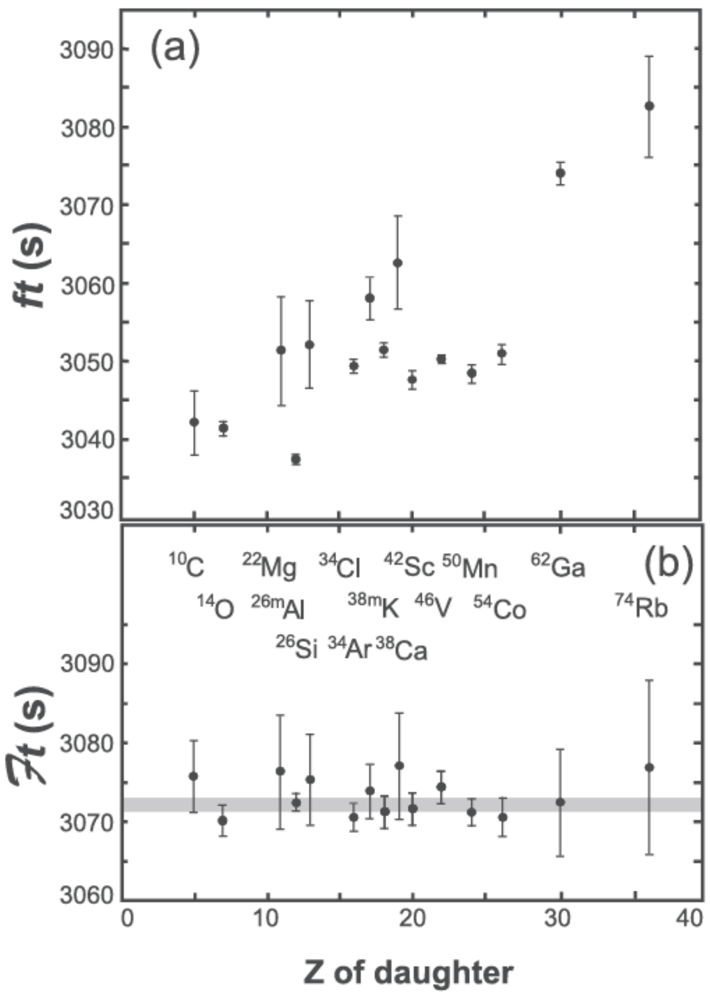

Even considering only protons and neutrons,
we still have a large scale, strongly-interacting
quantum many-body problem
Graciously provided
by Heiko Hergert
Z=82
Z=50
Z=40
Z=28
Z=20
N=126
N=82
N=50
N=40
N=28
N=20
2024
2022
2020
2018
2016
2014
2012
2010
Z
N

# We are entering an era in which the precision modelling of strongly-interacting many-body systems is becoming possible





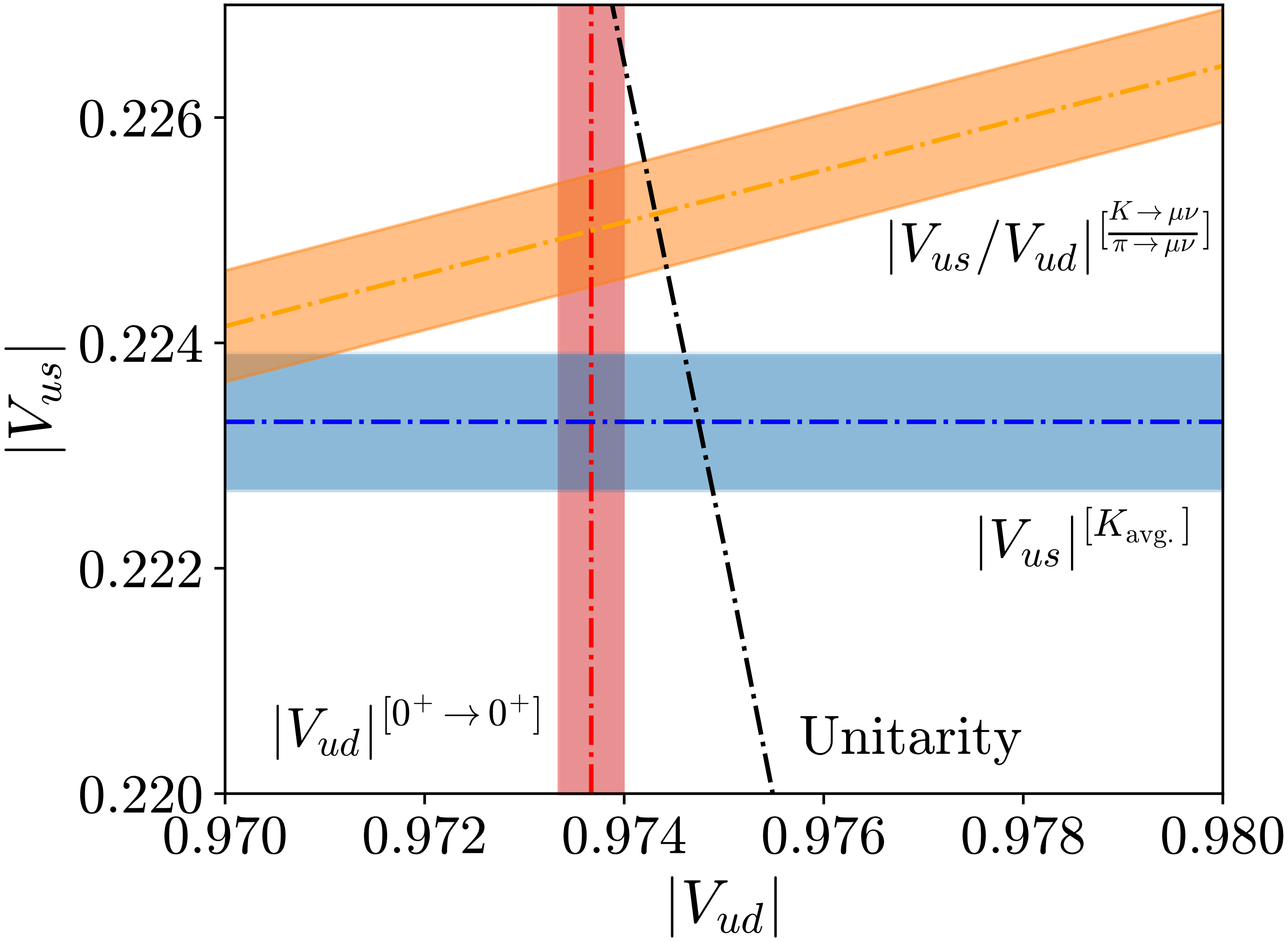

Hardy et al. PRC **102**, 045501 (2020)

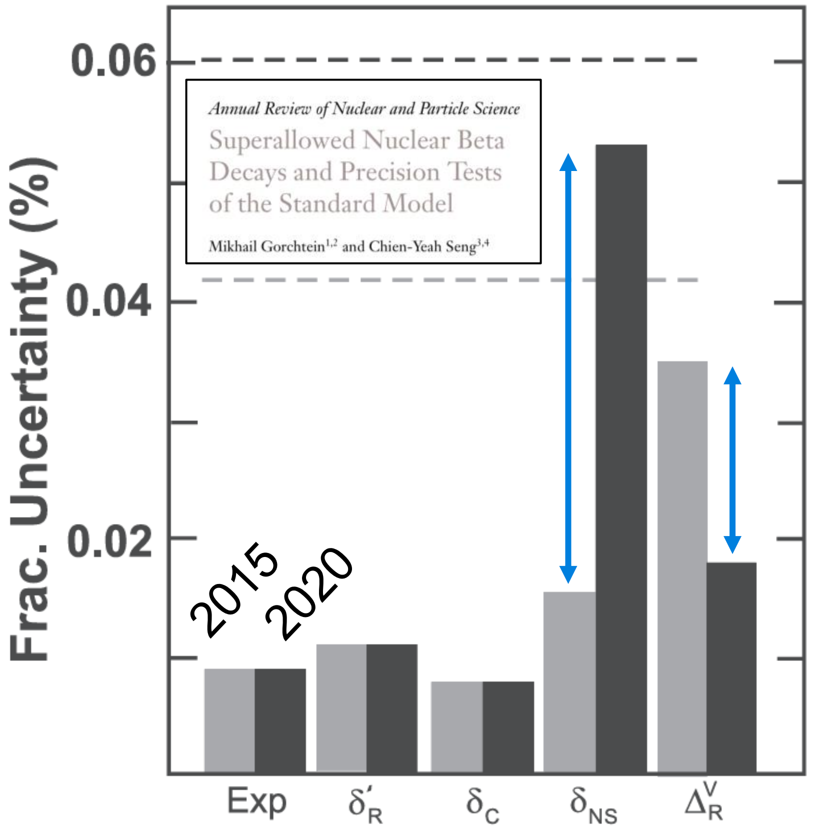



Requires **nuclear theory**

Particle Data Group. PRD **110**, 030001 (2024) and references therein

Hardy et al. PRC **102**, 045501 (2020)

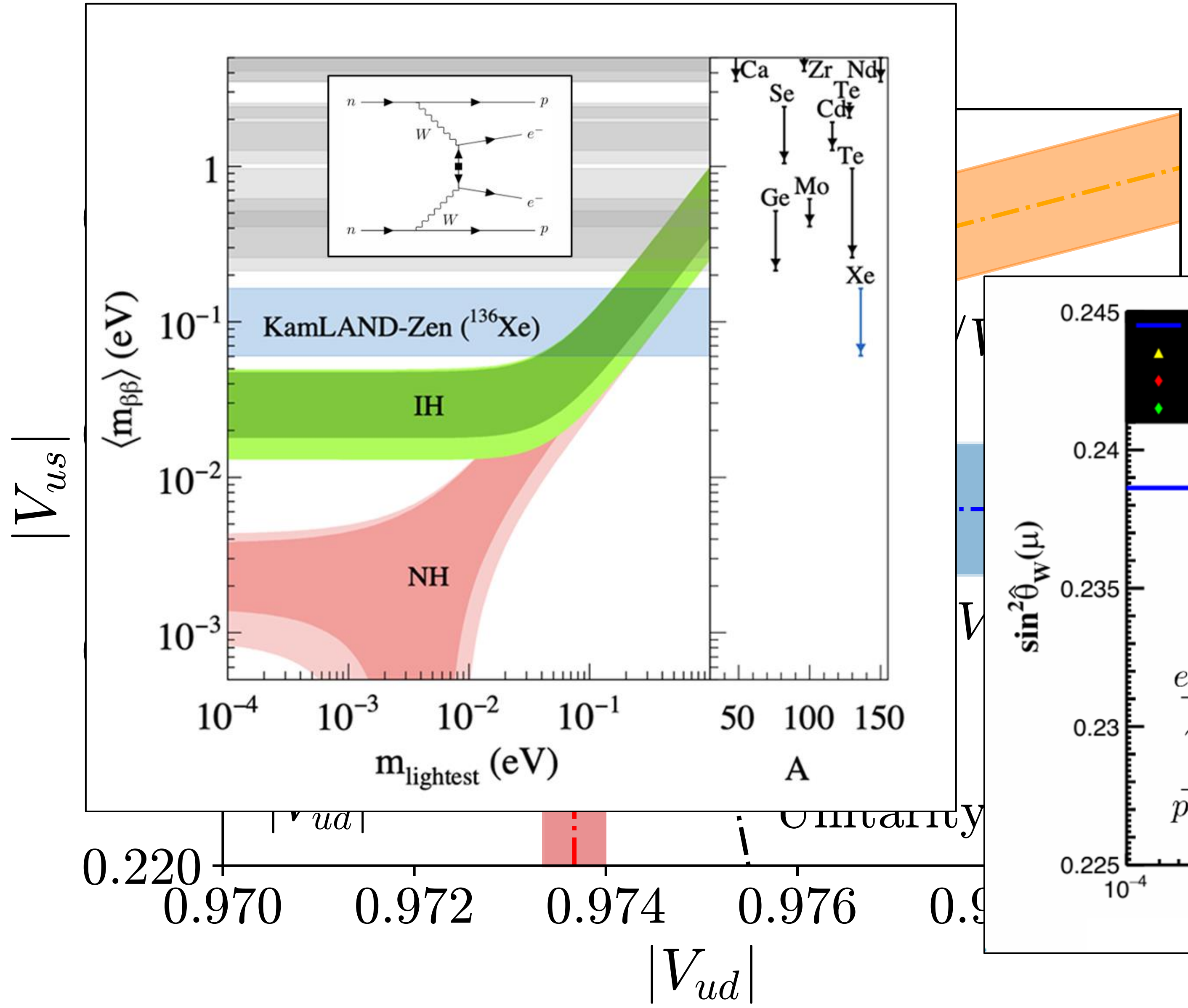



Seng et al. PRL **121**, 241804 (2018)

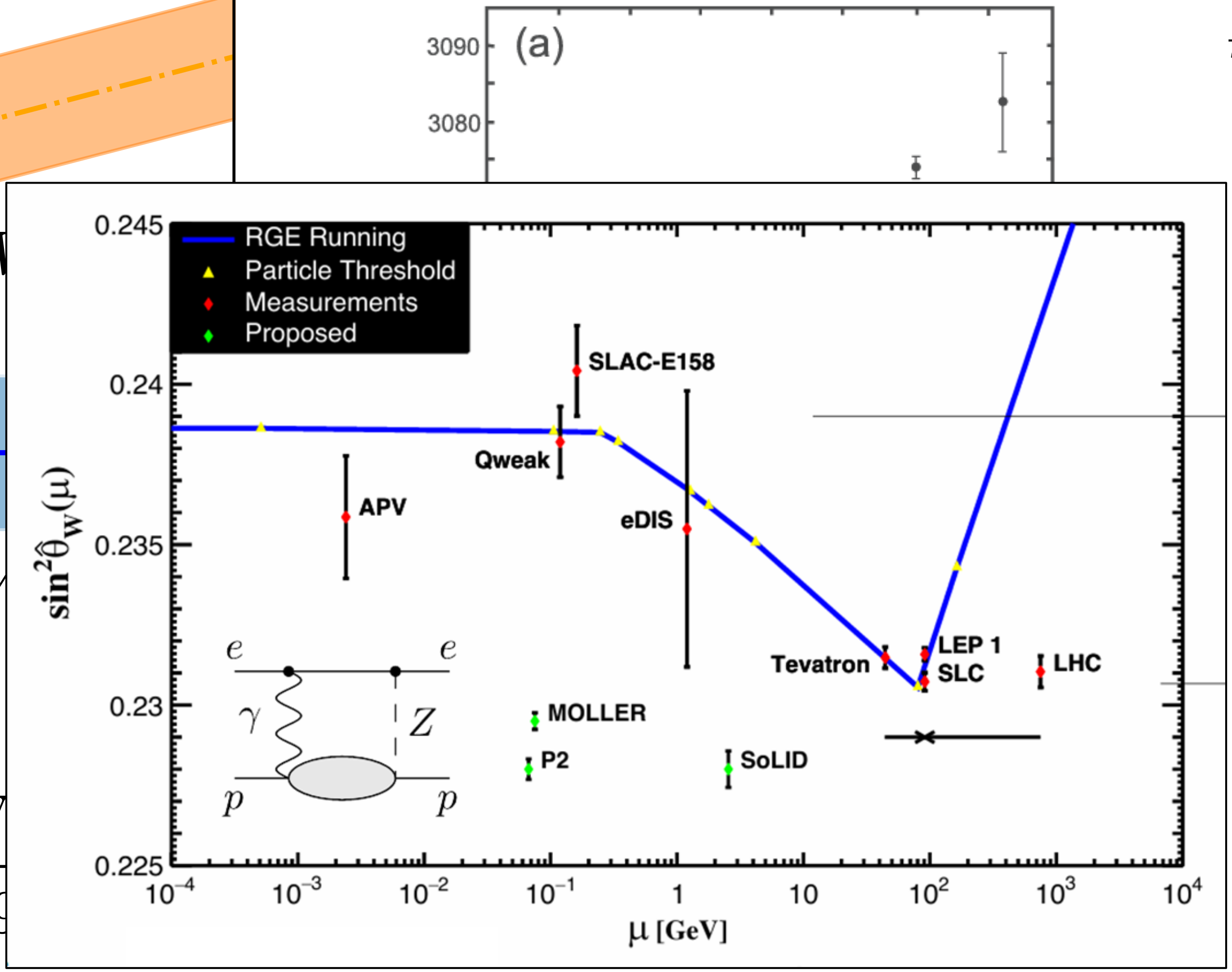
Requires **nuclear theory**

Particle Data Group. PRD **110**, 030001 (2024) and references therein




Hardy et al. PRC **102**, 045501 (2020)



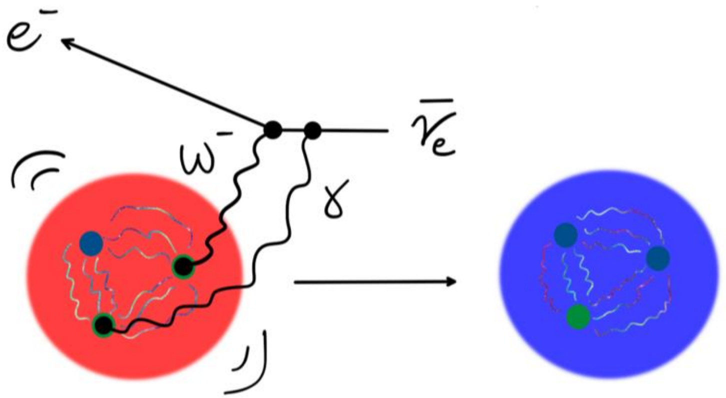



Requires **nuclear theory**

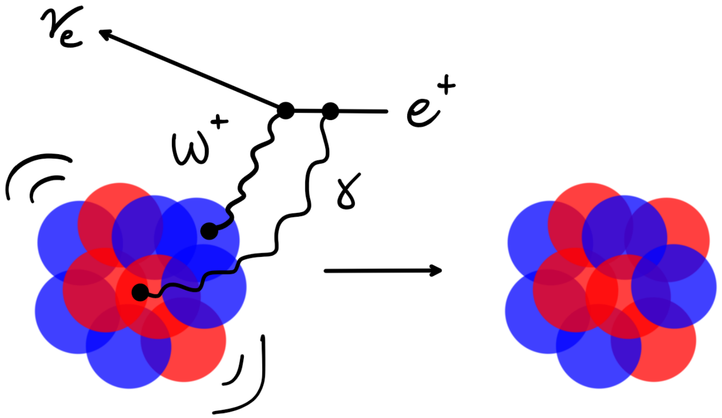
Particle Data Group. PRD **110**, 030001 (2024) and references therein

# Nuclear beta decay in the Standard Model






$$|V_{ud}|^2 = \frac{K}{\mathcal{F}t\,(1+\Delta_R^V)}$$



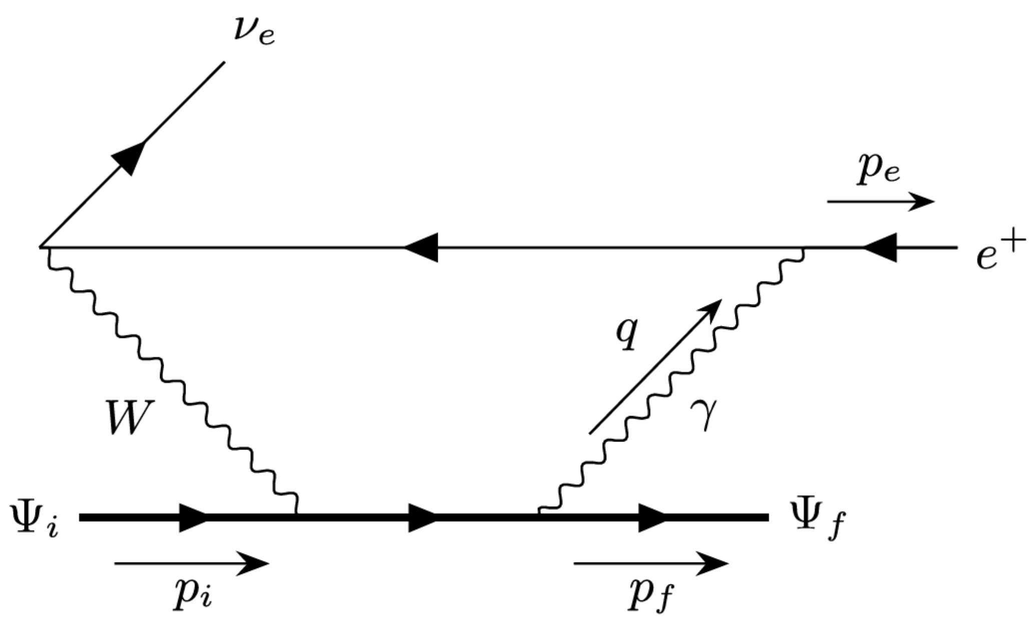

$$\mathcal{F}t = ft(1+\delta_R')(1-\delta_C+\delta_{NS})$$

Accounts for isospin symmetry breaking and electroweak radiative corrections

Particle Data Group. PRD **110**, 030001 (2024)

# One-loop radiative correction






Convenient [ not critical ] approximations

- Forward scattering limit
- Neglect recoil of final nucleus

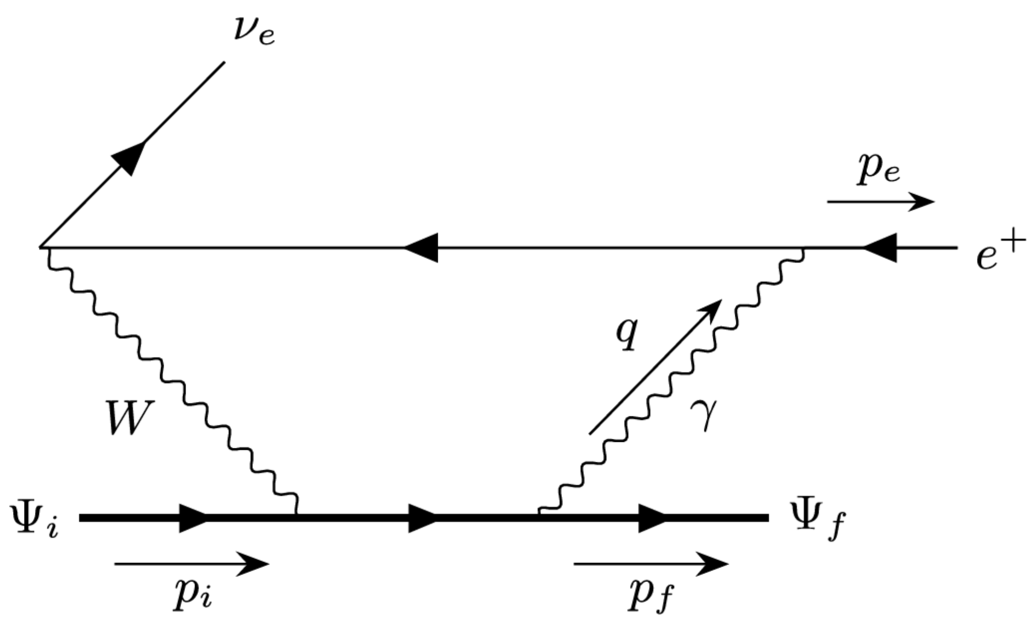
$$\delta\mathcal{M}_{\mathrm{one-loop}} = \Box_{\gamma W}(E_e)\,\mathcal{M}_{\mathrm{tree}}$$

$$T^{\mu\nu}(p_f, p_i; q) = \langle \Phi_f\,;\, p_f | \left\{ \frac{1}{2} \int d^4x\; e^{iq\cdot x}\, \mathrm{T}\left[ J^{\mu}_{\mathrm{em}}(x)\, J^{\nu}_{W}(0)^{\dagger} \right] \right\} | \Phi_i\,;\, p_i \rangle$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

# One-loop radiative correction






## Convenient [ not critical ] approximations

- Forward scattering limit
- Neglect recoil of final nucleus

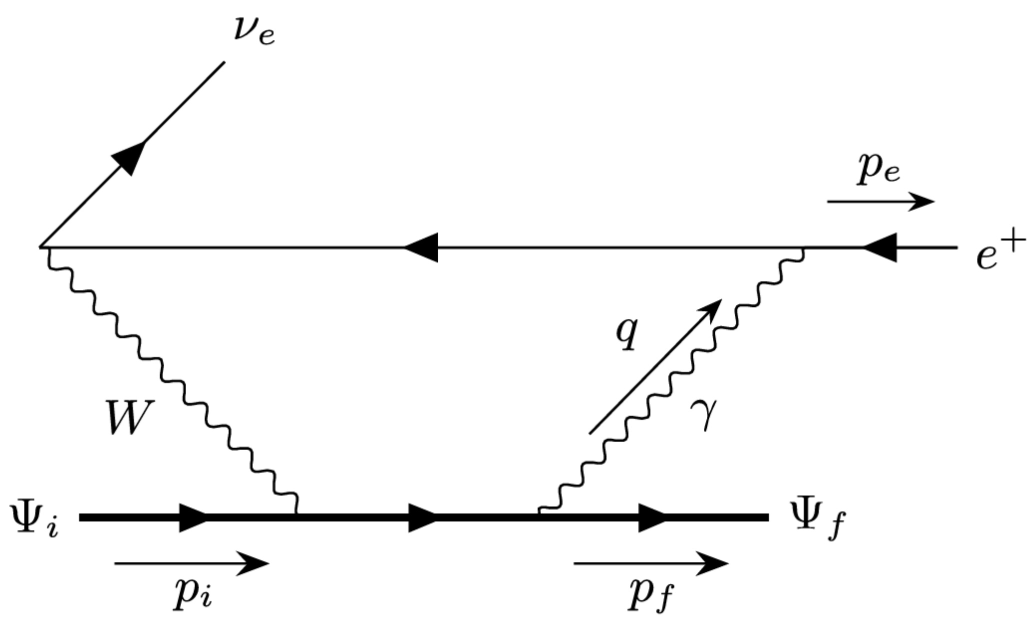
$$\delta\mathcal{M}_{\mathrm{one-loop}} = \Box_{\gamma W}(E_e)\,\mathcal{M}_{\mathrm{tree}}$$

$$\Box^b_{\gamma W}(E_e) = \frac{e^2}{M}\int \frac{d^4q}{(2\pi)^4}\,R_W(q)\,R_e(q)\,R_\gamma(q)\left(M\frac{p_e\cdot q}{p\cdot p_e} - \frac{q^2}{\nu}\right)\frac{T_3(\nu,|\vec{q}|)}{f_+}$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

# One-loop radiative correction






## Convenient [ not critical ] approximations

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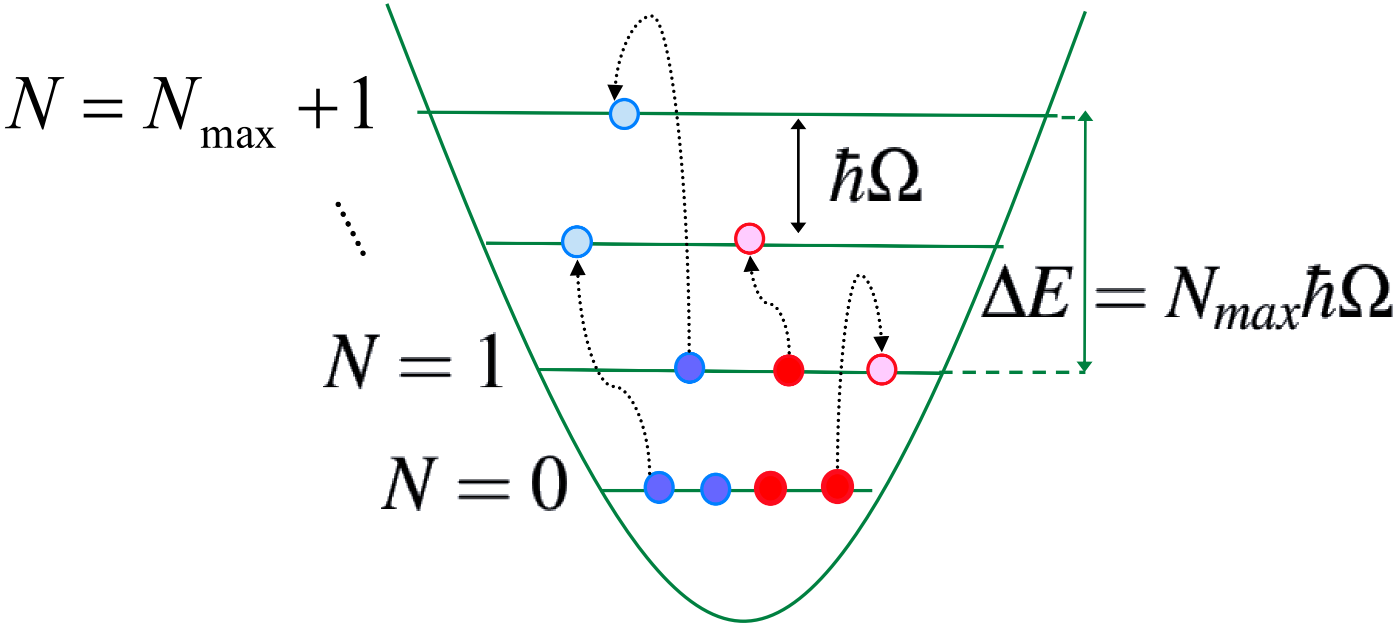
Seng & Gorchtein. PRC **107**, 035503 (2023)

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces




$$N = \sum_i 2n_i + l_i \leq N_{\text{LPC}} + N_{\text{max}}$$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

$$\mathcal{F}(\mathcal{H}) = \oplus_A \left[\mathcal{H}^{\otimes A}\right] = \mathbb{1} \oplus \mathcal{H} \oplus \left[\mathcal{H} \otimes \mathcal{H}\right] \oplus \cdots$$

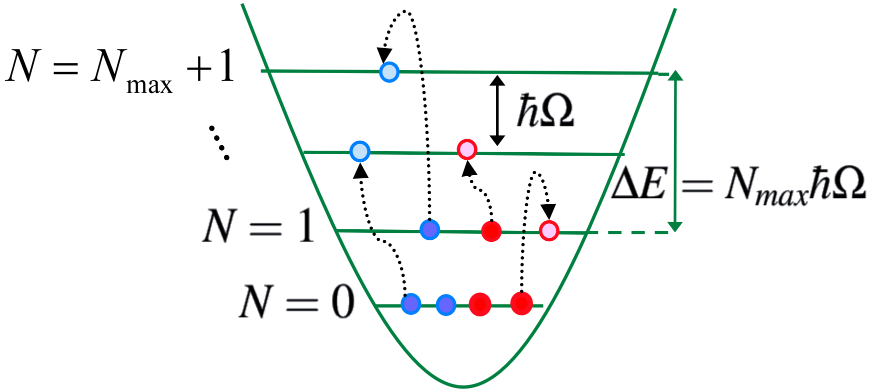
Anti-symmetrized products of many-body harmonic oscillator states

$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_{N=0}^{N_{max}} \sum_{\alpha} c_{N\alpha}^{J^\pi T} \left|\Phi_{N\alpha}^{J^\pi T}\right\rangle$$

Barrett et al. PPNP **69** (2013), pp.181-131

# We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

$$H = \frac{1}{A}\sum_{i<j}^{A}\frac{\left(\vec{p}_i - \vec{p}_j\right)^2}{2m_N} + \sum_{i<j}^{A} V_{NN}^{(ij)} + \sum_{i<j<k}^{A} V_{NNN}^{(ijk)} + \cdots$$




Barrett et al. PPNP **69** (2013), pp.181-131

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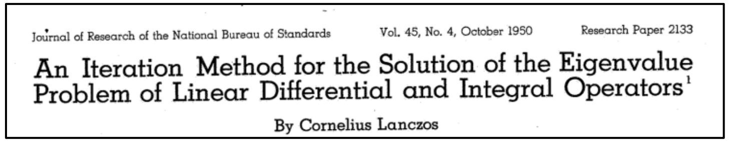
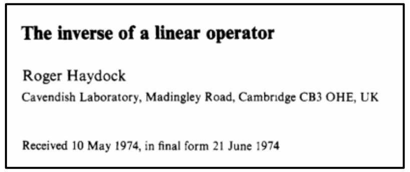


Journal of Research of the National Bureau of Standards Vol. 45, No. 4, October 1950 Research Paper 2133

An Iteration Method for the Solution of the Eigenvalue Problem of Linear Differential and Integral Operators[1]

By Cornelius Lanczos



**The inverse of a linear operator**

Roger Haydock

Cavendish Laboratory, Madingley Road, Cambridge CB3 OHE, UK

Received 10 May 1974, in final form 21 June 1974

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

## Lanczos Algorithm

$$
\begin{aligned}
H|\eta_0\rangle &= \alpha_0|\eta_0\rangle + \beta_0|\eta_1\rangle \\
H|\eta_1\rangle &= \beta_0|\eta_0\rangle + \alpha_1|\eta_1\rangle + \beta_1|\eta_2\rangle \\
H|\eta_2\rangle &= \qquad\qquad \beta_1|\eta_1\rangle + \alpha_2|\eta_2\rangle + \beta_2|\eta_3\rangle \\
H|\eta_3\rangle &= \qquad\qquad\qquad\qquad \beta_2|\eta_2\rangle + \alpha_3|\eta_3\rangle + \beta_3|\eta_4\rangle \\
&\vdots
\end{aligned}
$$

$$E = P^{-1} H_{\text{Lanczos}} P$$

Lanczos. JRNBS **45** no. 4 (1950)

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

## Lanczos Strengths Method

$$\mathcal{A}_{fi} = \langle \Phi_f | O_2 \, (z - H)^{-1} \, O_1 | \Phi_i \rangle = \langle \Phi_f | O_2 | \Phi_{\mathrm{R}} \rangle$$

$$(z - H) | \Phi_{\mathrm{R}} \rangle = O | \Phi_i \rangle$$

Method for extracting many-body resolvent amplitudes

Haydock. JPA **7**, 2120 (1974)    Bessis & Villani. JMP **16**, 462 (1975)    Dagotto. RMP **66**, 763 (1994)

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces



## Lanczos Strengths Method

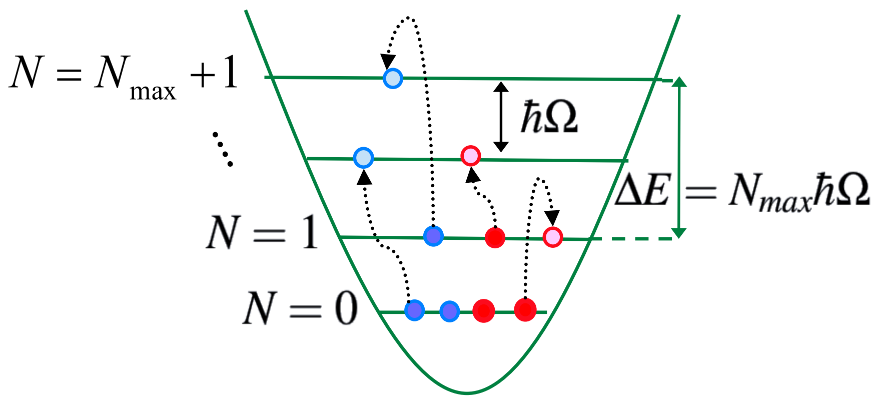
$$\langle \Psi_n | O_1 | \Psi_i \rangle = \left| \langle \Psi_i | O_1^\dagger O_1 | \Psi_i \rangle \right| \langle \eta_n | P^\dagger | \eta_0 \rangle$$

$$\langle \Psi_f | O_2 | \Psi_n \rangle = \left| \langle \Psi_f | O_2^\dagger O_2 | \Psi_f \rangle \right| \sum_m \langle \eta_m | P | \eta_n \rangle \langle \zeta_0 | \eta_m \rangle$$

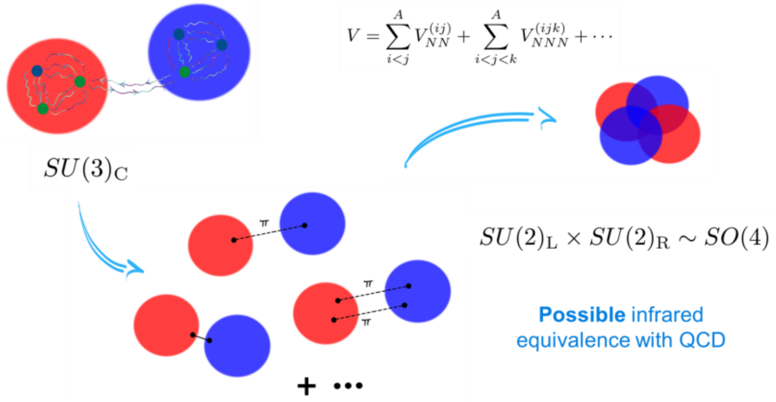
Haydock. JPA **7**, 2120 (1974) Bessis & Villani. JMP **16**, 462 (1975) Dagotto. RMP **66**, 763 (1994)

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

## No-core Shell Model



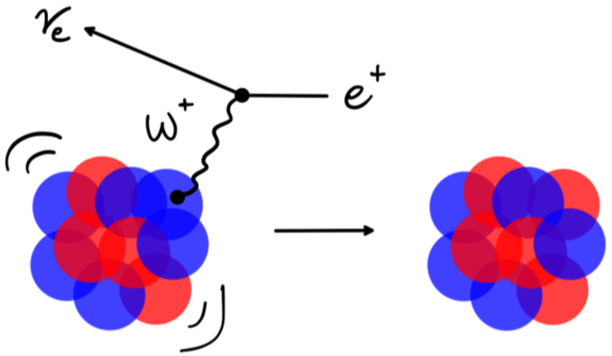

## Chiral Effective Field Theory




## Lanczos Strengths Method

$$(z - H)\big|\Phi_{\mathrm{R}}\big\rangle = O\big|\Phi_i\big\rangle$$

# We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces



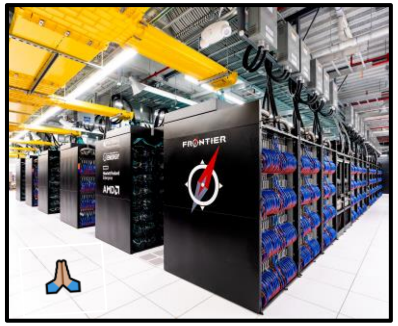

## Super-allowed beta decay

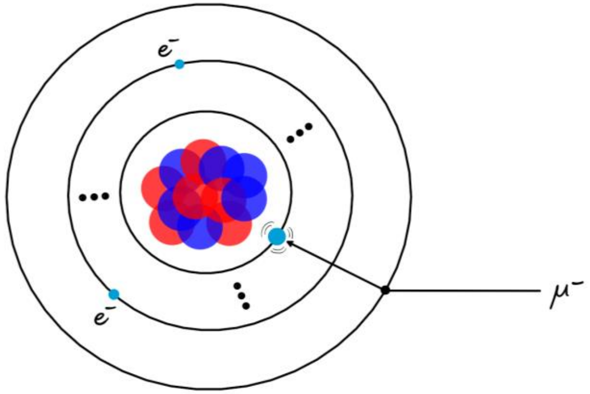
Gennari et al.
PRL **134**, 012501 (2025)

PHYSICAL REVIEW LETTERS 134, 012501 (2025)

*Ab Initio* Strategy for Taming the Nuclear-Structure Dependence of $V_{ud}$ Extractions: The $^{10}C \to {}^{10}B$ Superallowed Transition

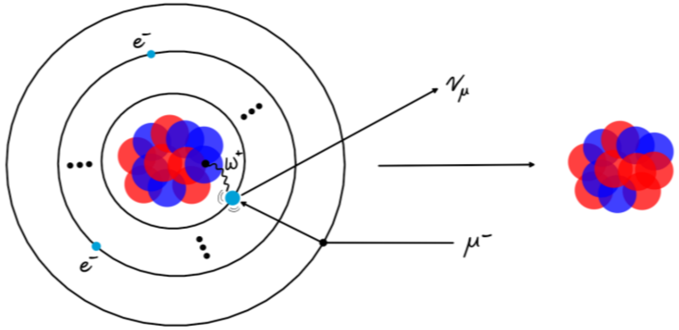
Michael Gennari,[1,2] Mehdi Drissi,[1] Mikhail Gorchtein,[3,4] Petr Navrátil,[1,2] and Chien-Yeah Seng[5,6]




## Muonic atoms

Drissi et al. In prep.




## Muon capture

Najera et al. In progress.

# We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

**No resolution** for Compton amplitude above pion threshold, thus $\delta_{\mathrm{NS}}$ matched with the free nucleon Born contribution **only**

$$\delta_{\mathrm{NS}} = 2\left\{ \left(\Box^{b,\mathrm{nuc}}_{\gamma W}\right)_{\mathrm{a.i.}} - \left(\Box^{b,n}_{\gamma W}\right)_{\mathrm{el}} + \delta\left(\Box^{b,n}_{\gamma W}\right)_{\mathrm{sh}} \right\}$$

Gennari et al. PRL **134**, 012501


PHYSICAL REVIEW LETTERS 134, 012501 (2025)

*Ab Initio* Strategy for Taming the Nuclear-Structure Dependence of $V_{ud}$ Extractions: The $^{10}C \rightarrow {}^{10}B$ Superallowed Transition

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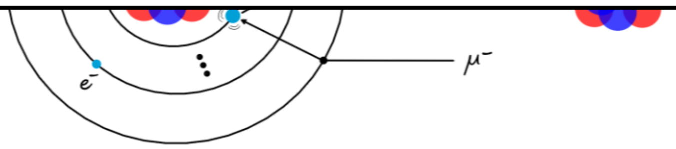


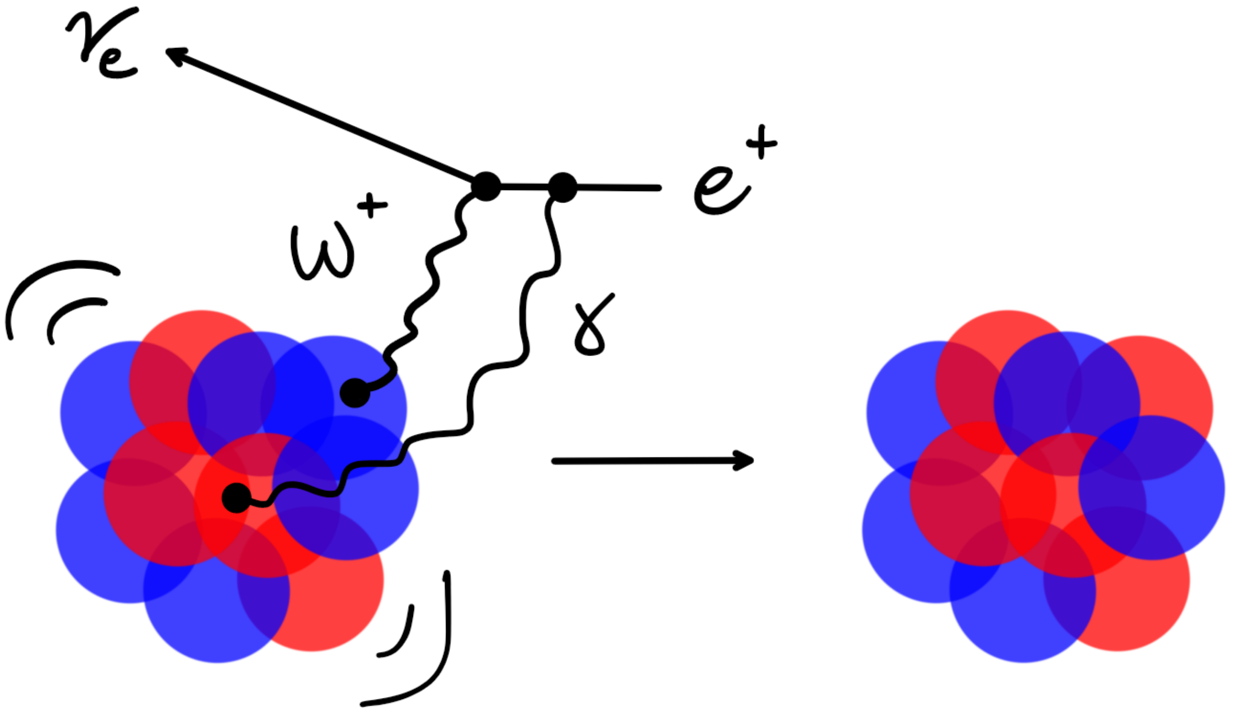

Muon capture

Najera et al. In progress.

# Amplitude and pole structure



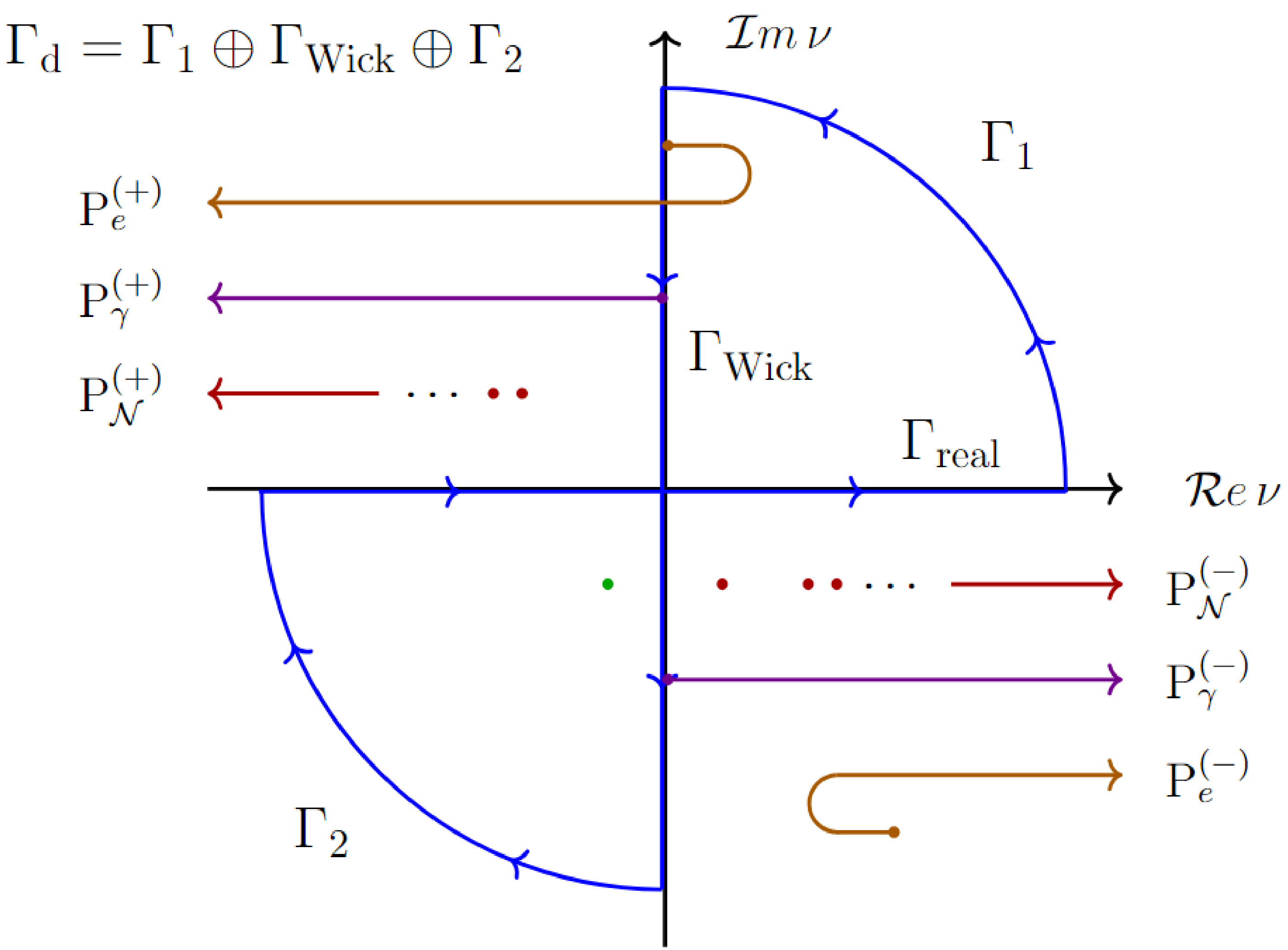
$$\mathcal{A}_{fi} = \langle \Phi_f | O_2 \, (z - H)^{-1} \, O_1 | \Phi_i \rangle$$



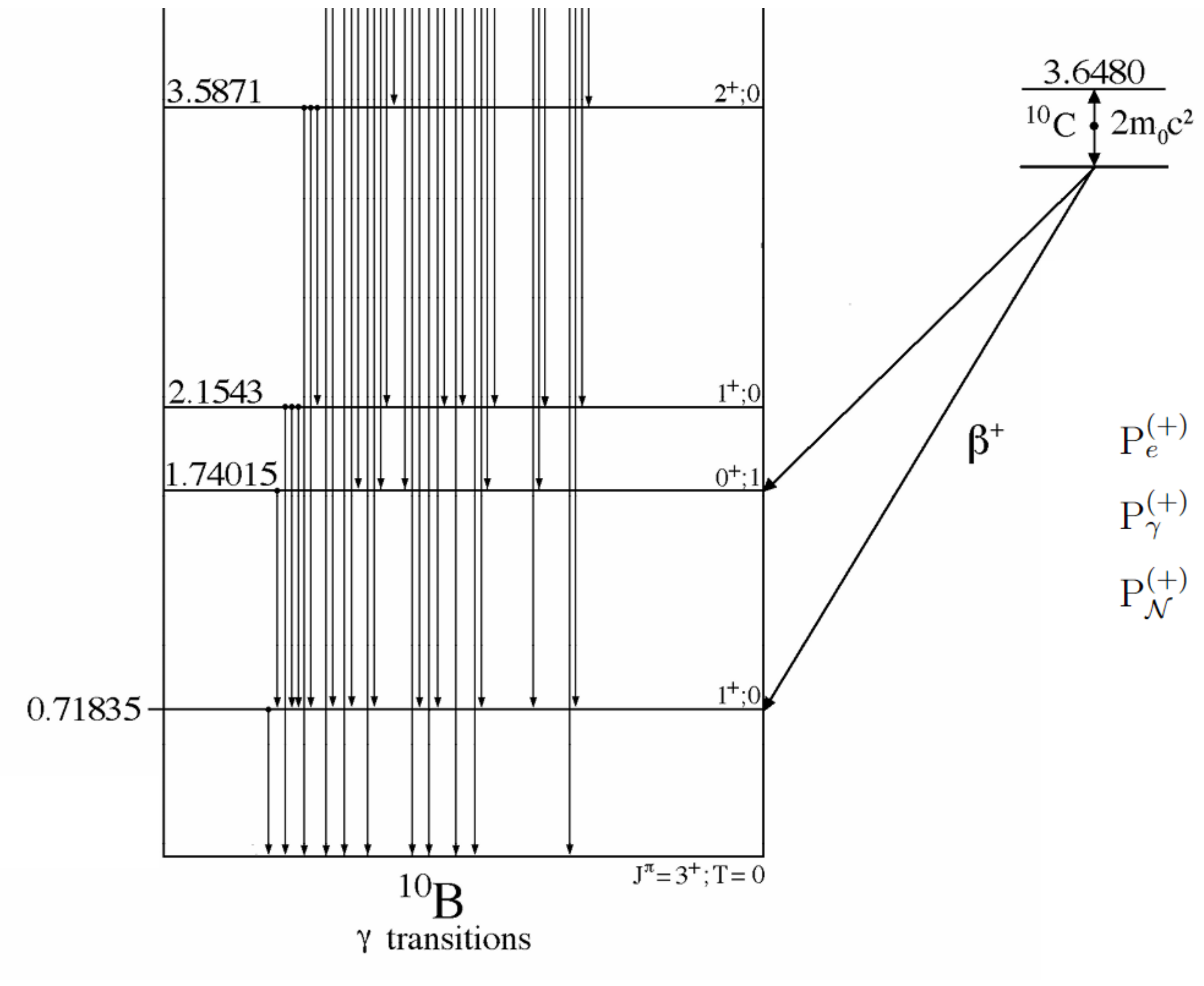

Gennari et al. PRL **134**, 012501

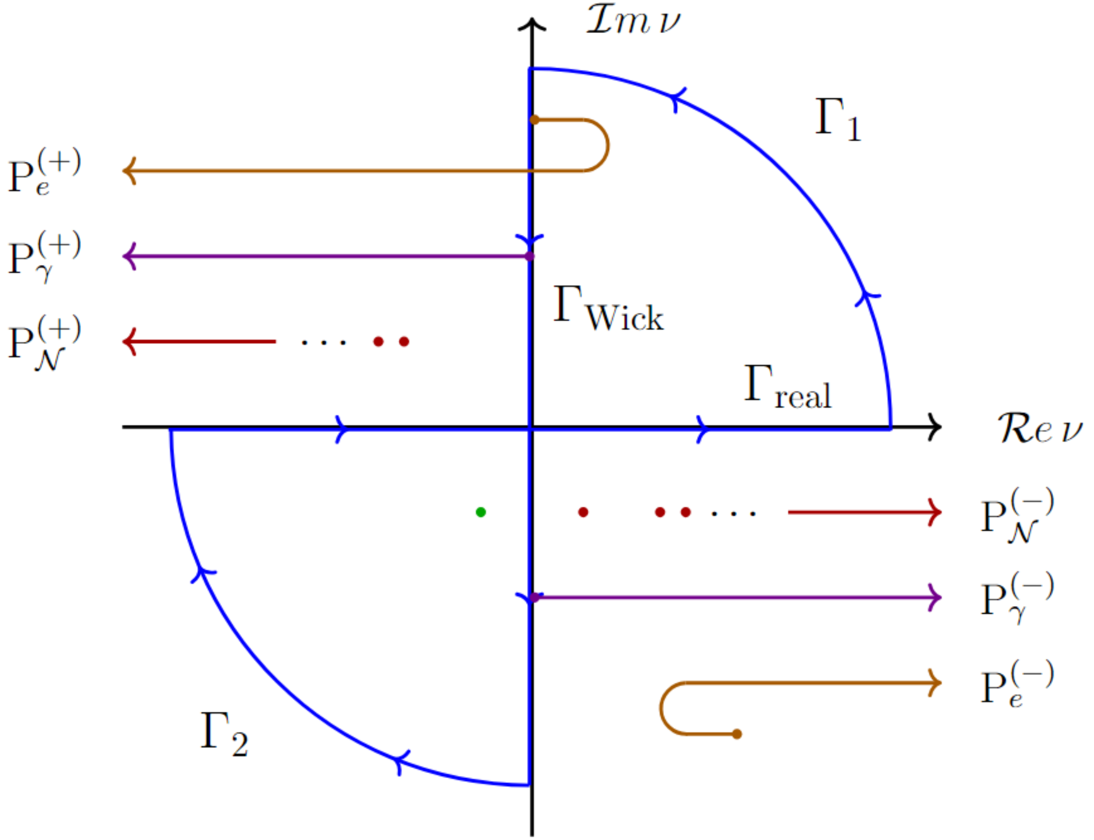
# Amplitude and pole structure



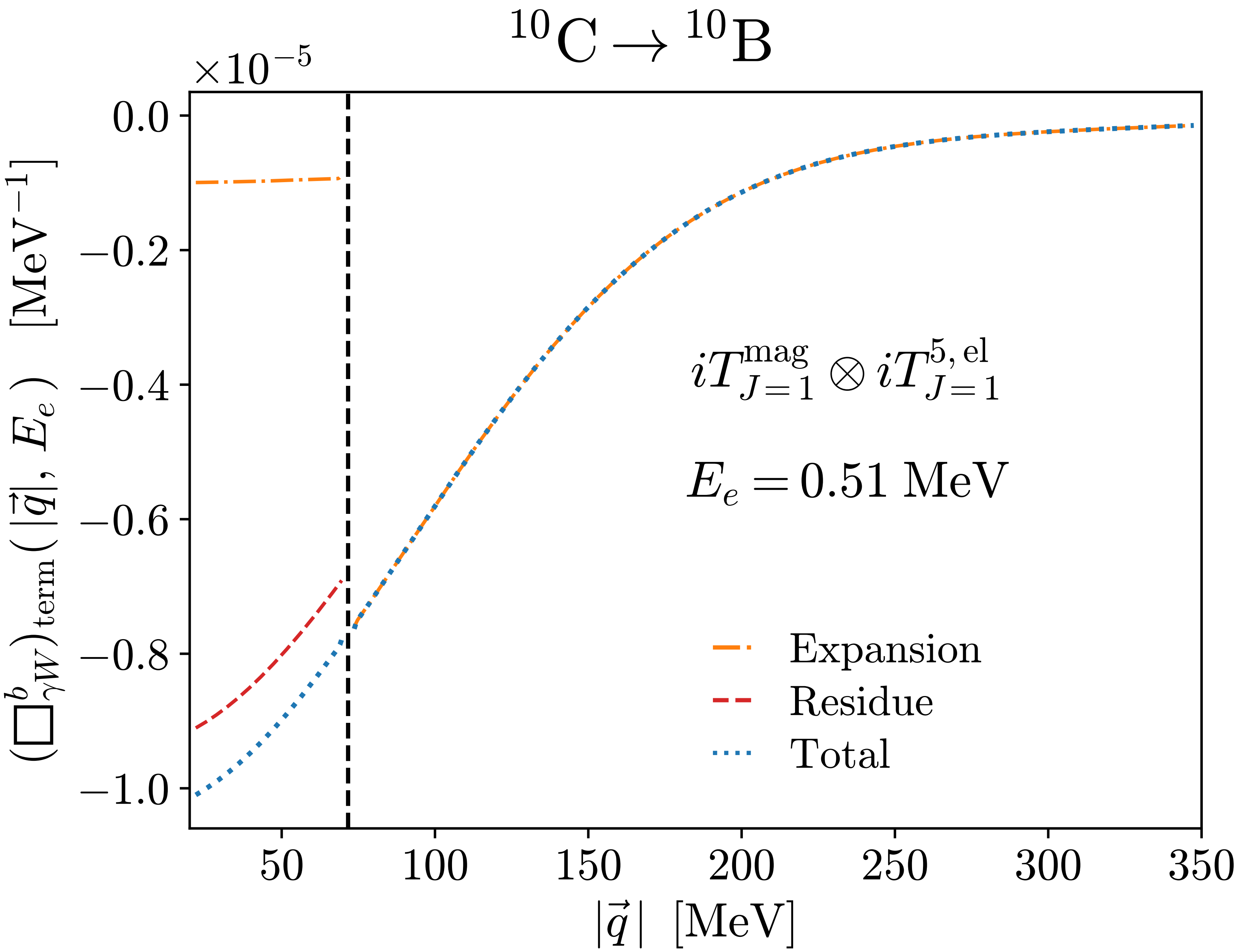



Gennari et al. PRL **134**, 012501



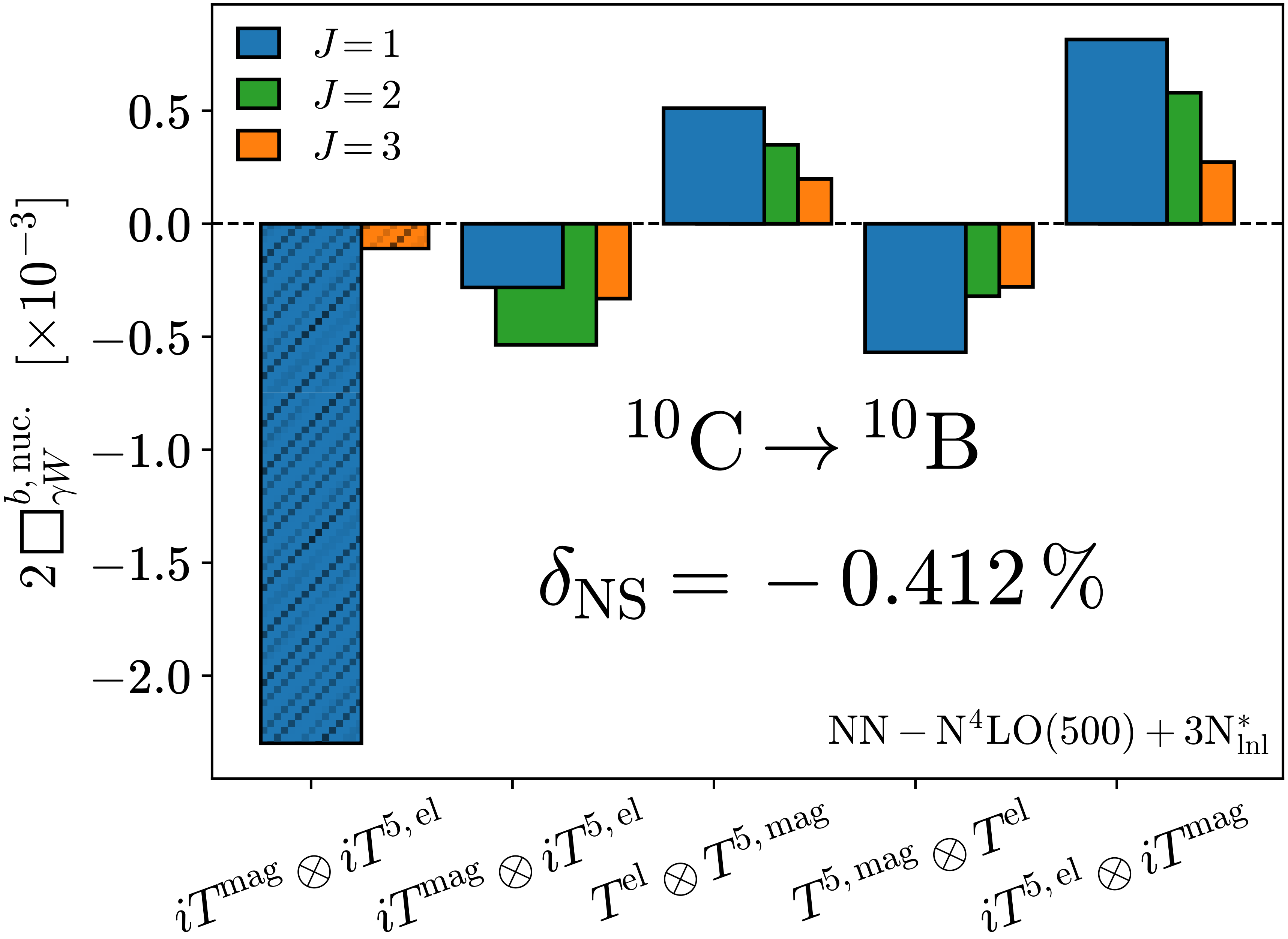




Gennari et al. PRL **134**, 012501



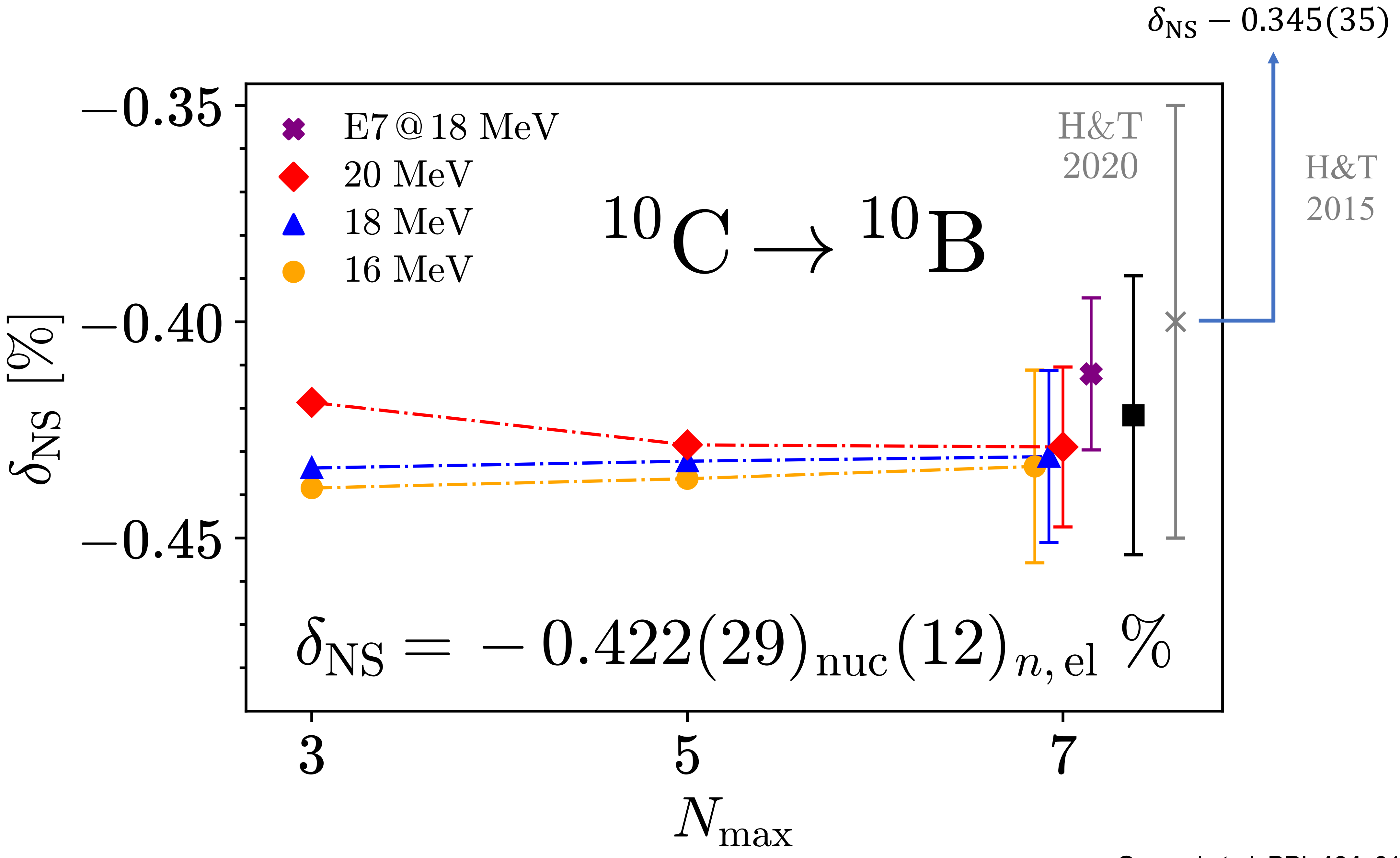



Gennari et al. PRL **134**, 012501



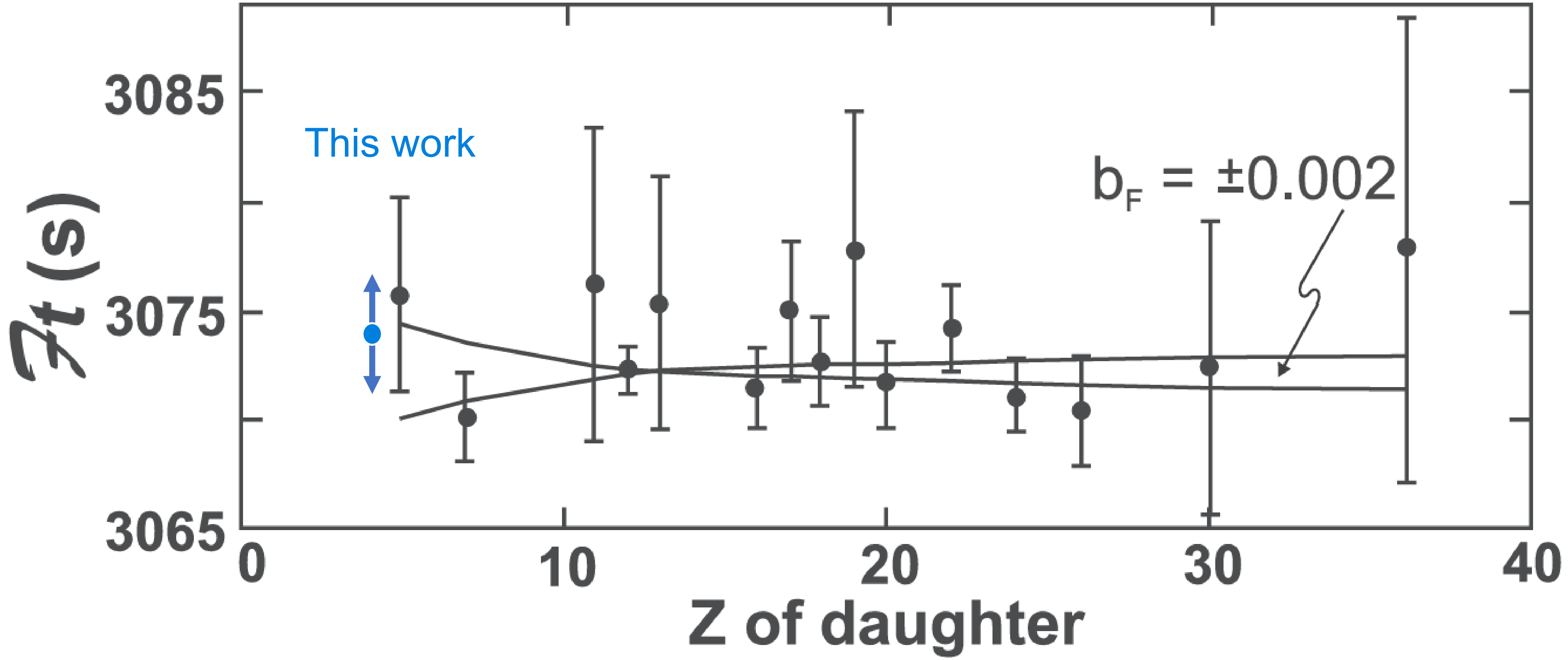


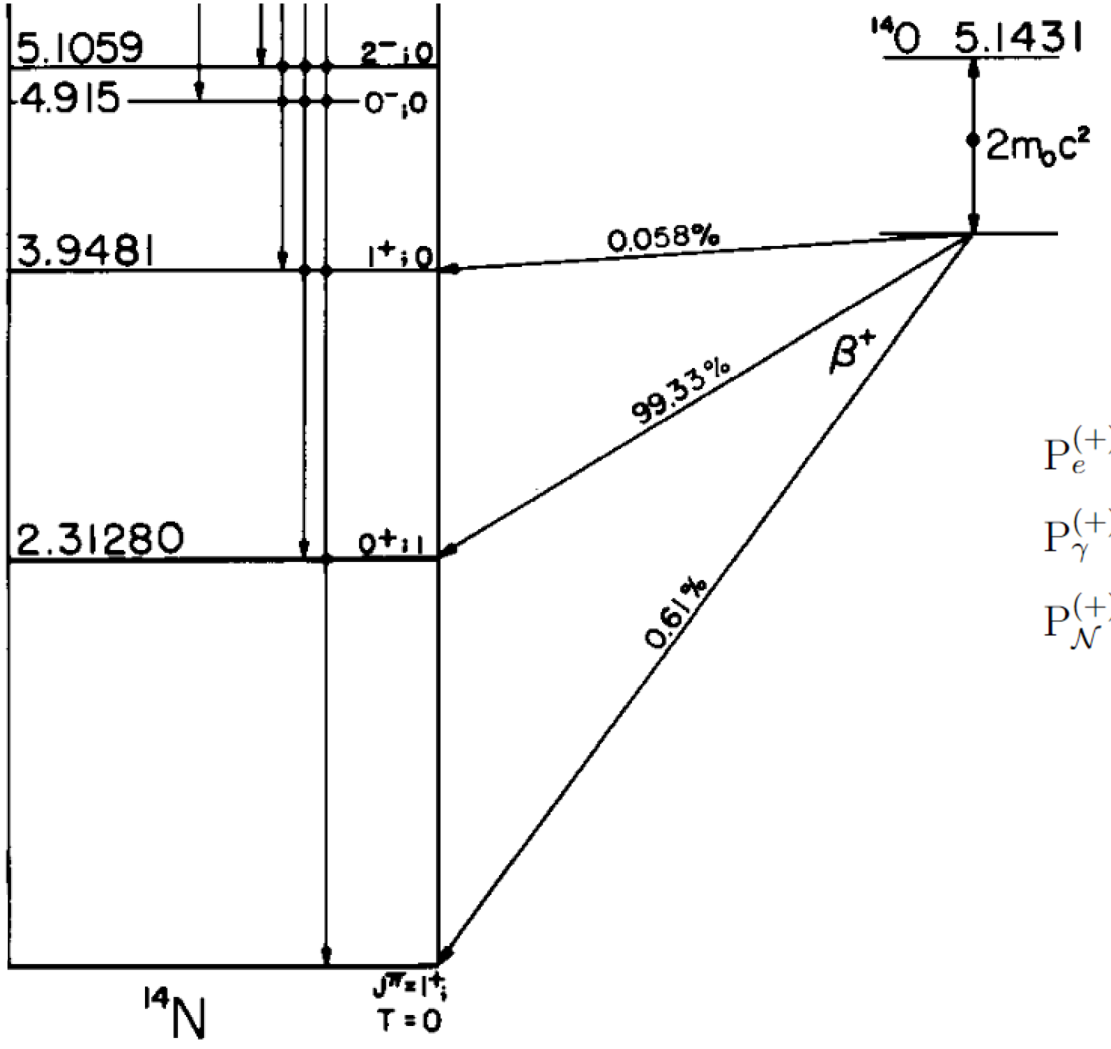

Gennari et al. PRL **134**, 012501

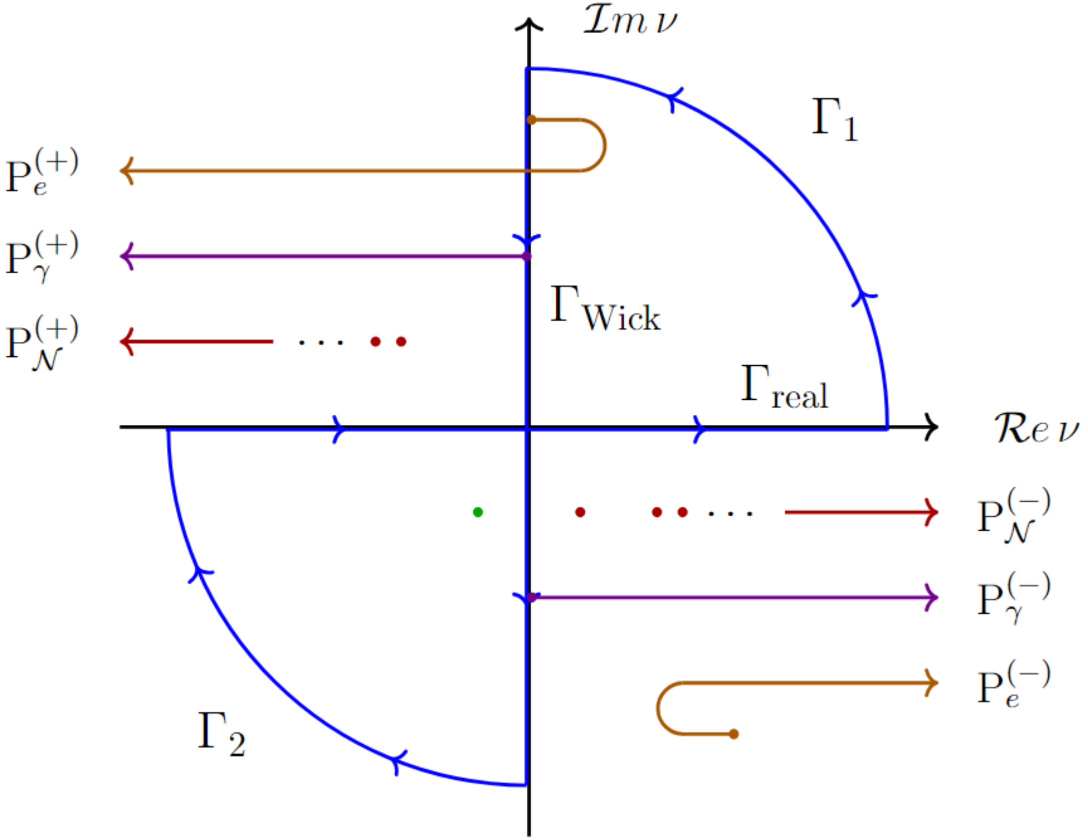



Gennari et al. PRL **134**, 012501

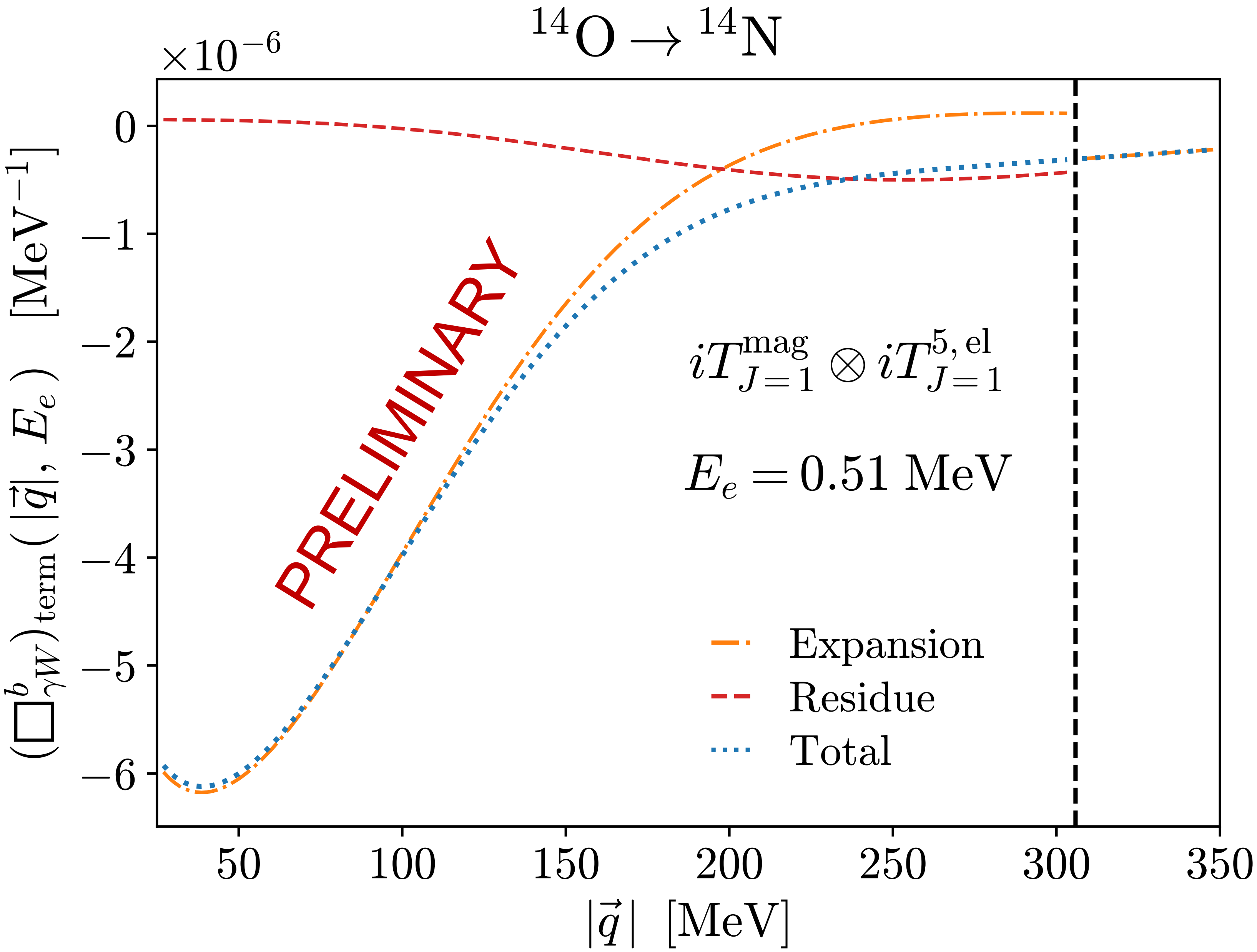
H&T 2020

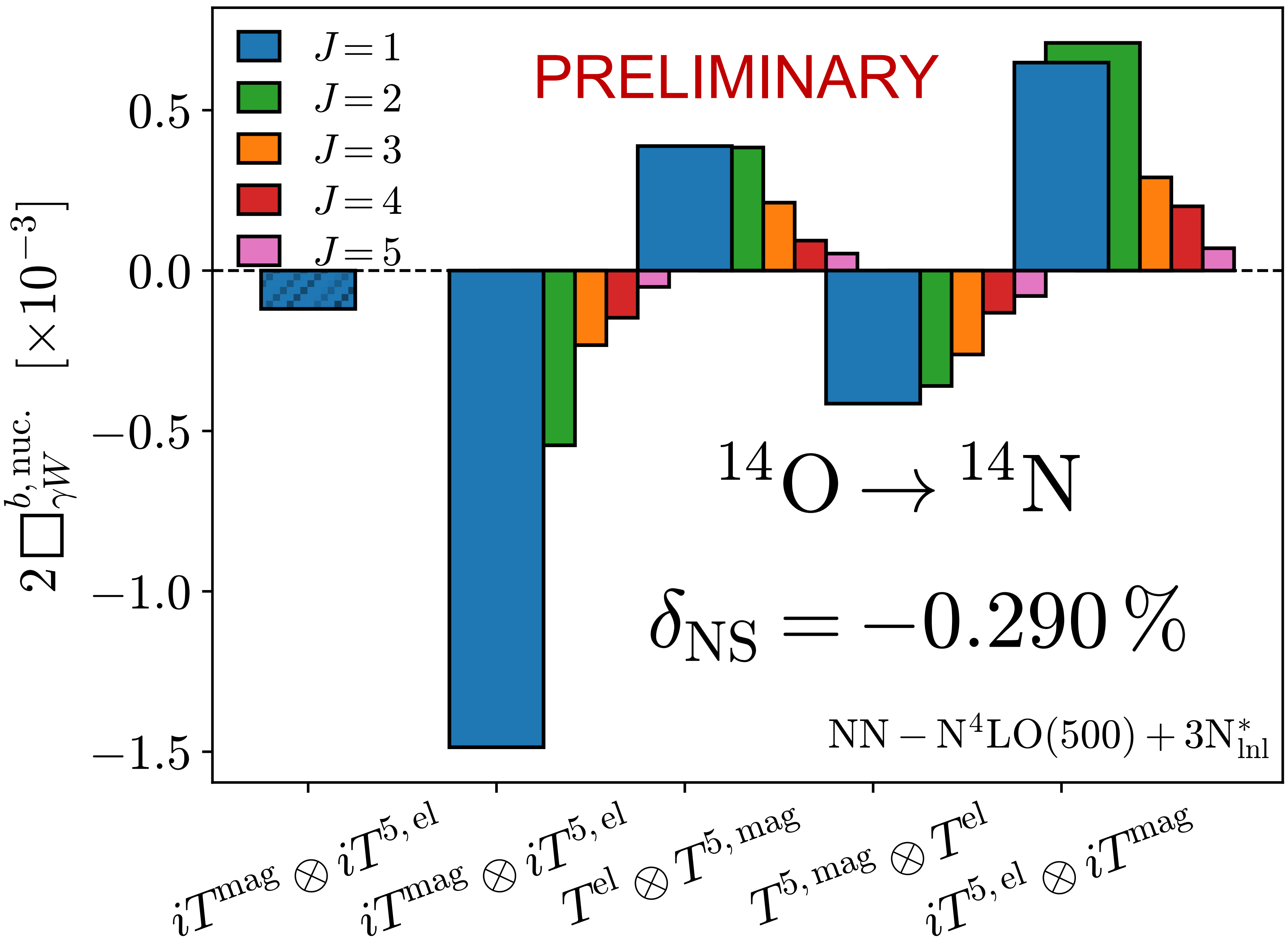

This work
$b_F = \pm 0.002$
$\mathcal{F}t$ (s)
3085
3075
3065
0
10
20
30
40
Z of daughter

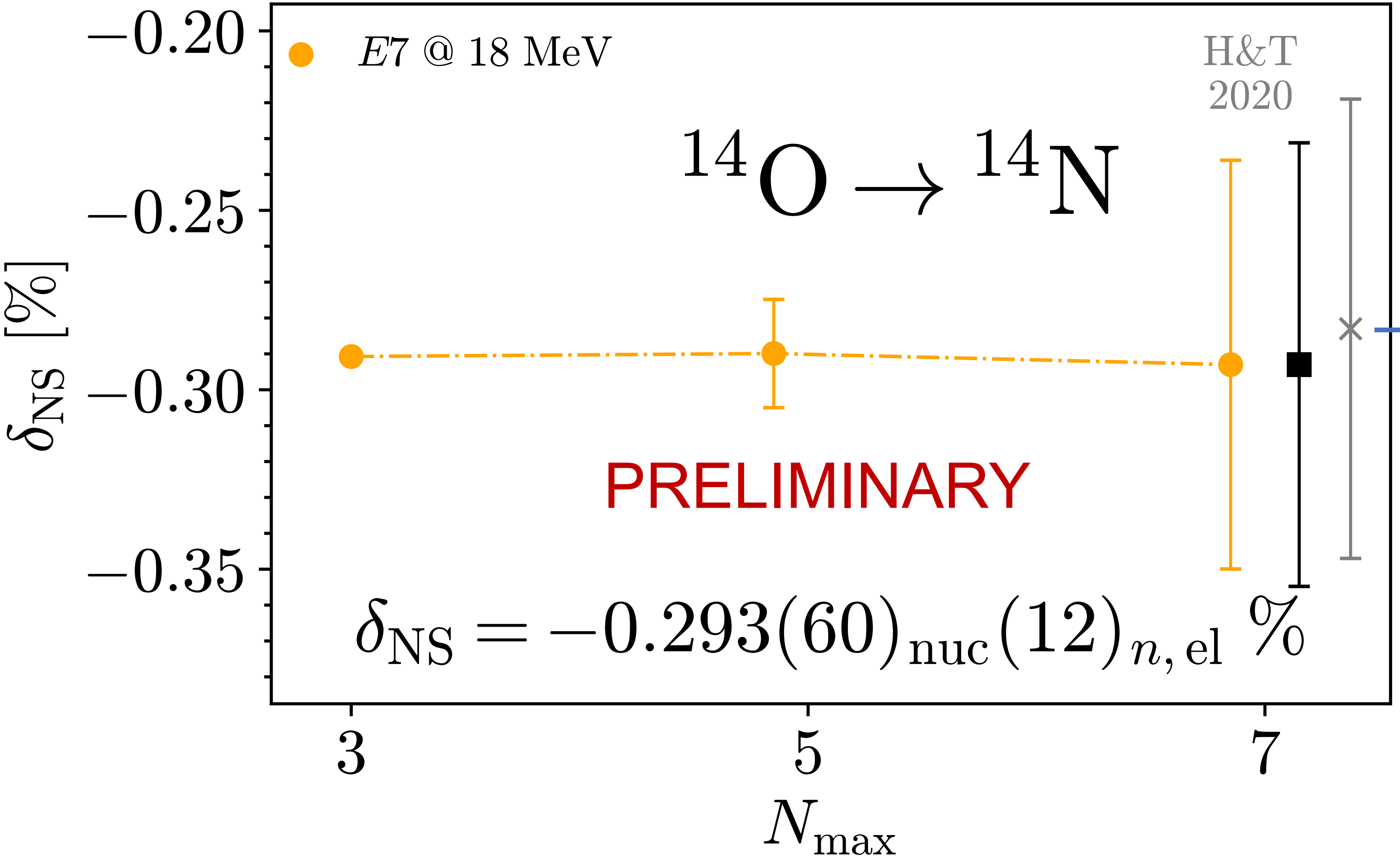



Gennari et al. PRL **134**, 012501


$^{14}O \to {}^{14}N$
$\times 10^{-6}$
$(\Box^b_{\gamma W})_{\rm term}(|\vec{q}|,\ E_e)$ [MeV$^{-1}$]
0
−1
−2
−3
−4
−5
−6
50
100
150
200
250
300
350
$|\vec{q}|$ [MeV]
PRELIMINARY
$iT^{\rm mag}_{J=1} \otimes iT^{5,\,\rm el}_{J=1}$
$E_e = 0.51$ MeV
Expansion
Residue
Total

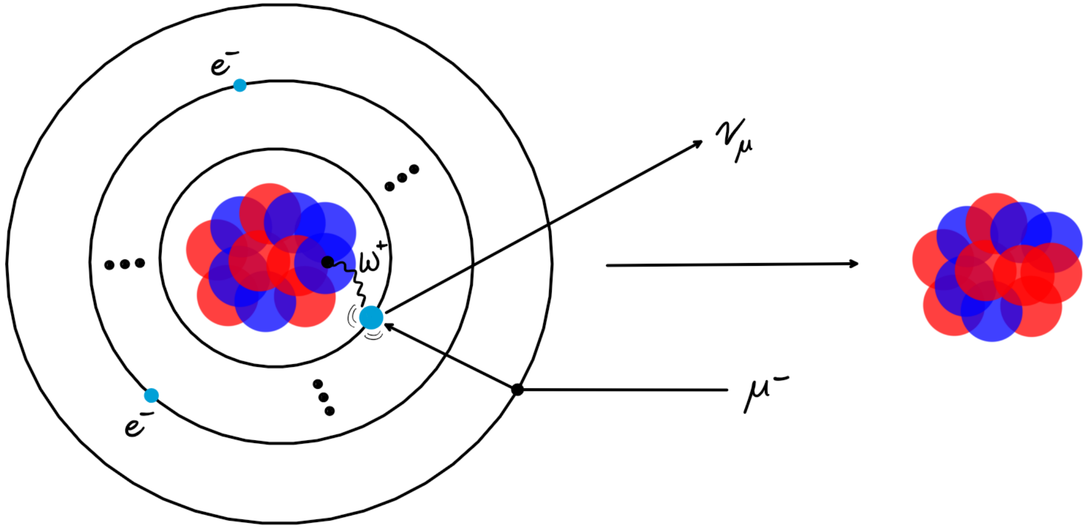

$J=1$
$J=2$
$J=3$
$J=4$
$J=5$
PRELIMINARY
$^{14}\mathrm{O} \to {}^{14}\mathrm{N}$
$\delta_{\mathrm{NS}} = -0.290\,\%$
$\mathrm{NN-N^4LO(500)+3N^*_{lnl}}$
$2\square_{\gamma W}^{b,\,\mathrm{nuc.}}\ [\times 10^{-3}]$
0.5
0.0
−0.5
−1.0
−1.5
$iT^{\mathrm{mag}} \otimes iT^{5,\,\mathrm{el}}$
$iT^{\mathrm{mag}} \otimes iT^{5,\,\mathrm{el}}$
$T^{\mathrm{el}} \otimes T^{5,\,\mathrm{mag}}$
$T^{5,\,\mathrm{mag}} \otimes T^{\mathrm{el}}$
$iT^{5,\,\mathrm{el}} \otimes iT^{\mathrm{mag}}$

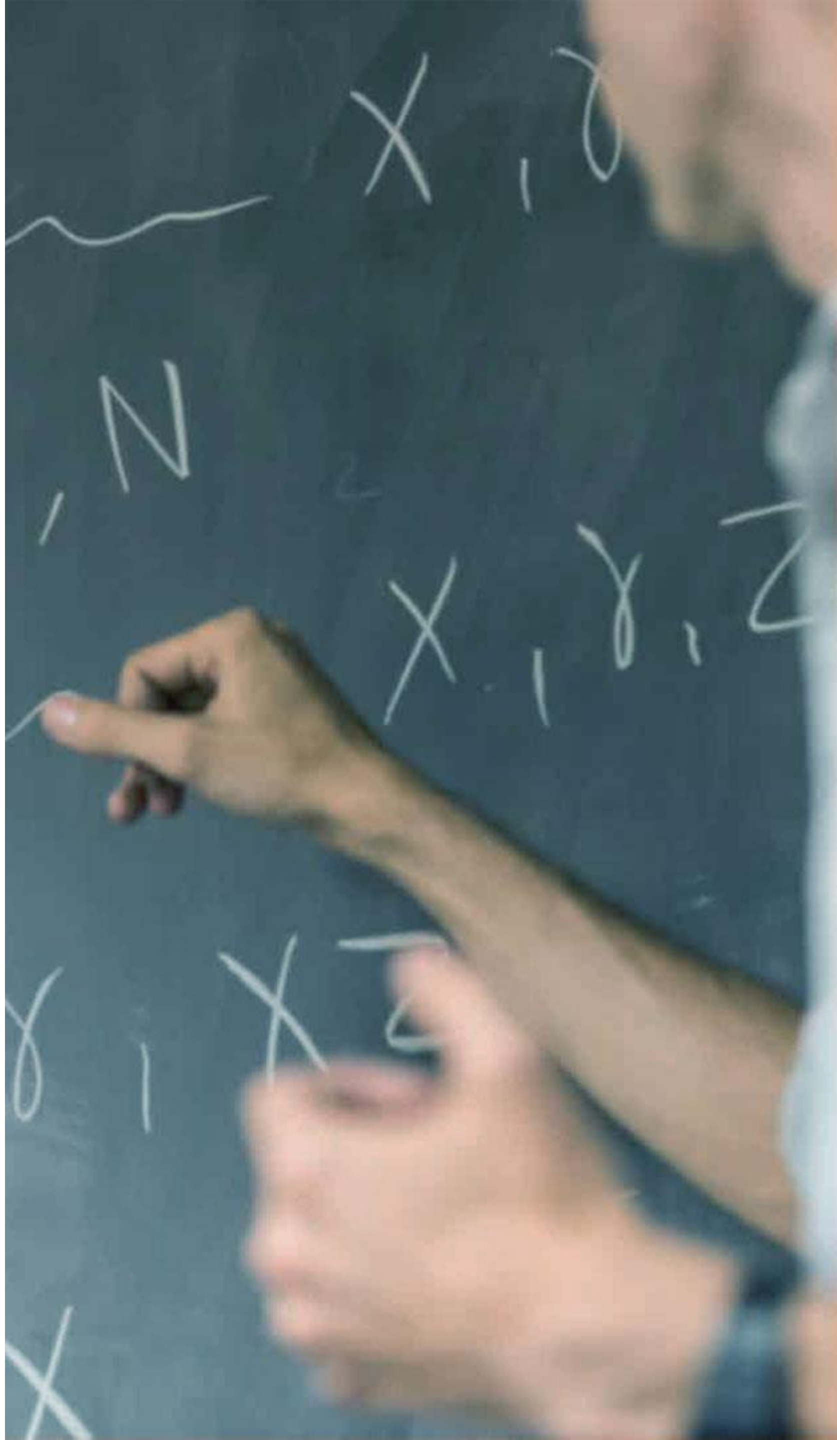



$\delta_{NS} = -0.245(50)$

H&T
2015

# Ordinary muon capture



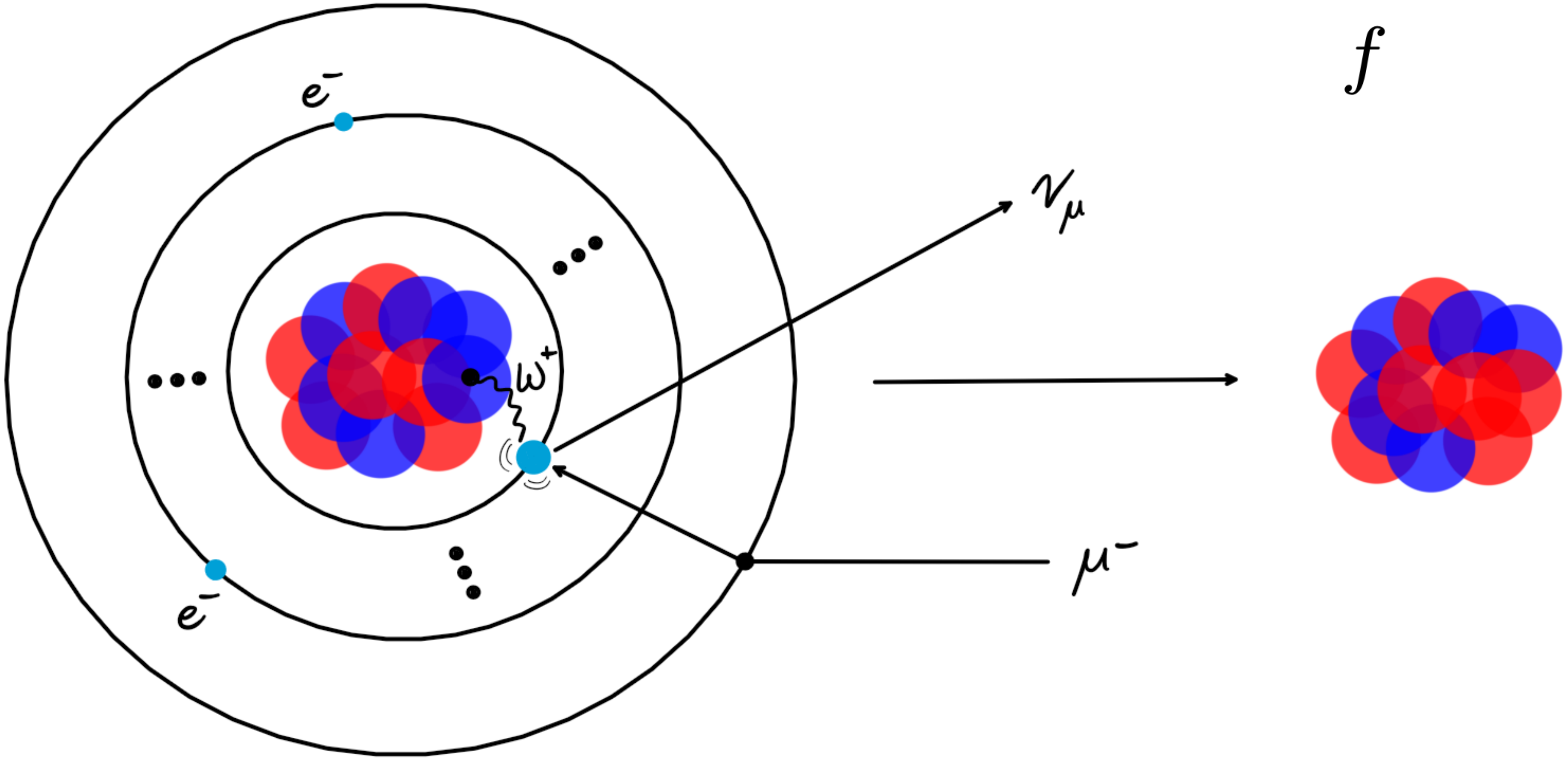



Najera et al. In progress.

# Ordinary muon capture



$$\omega_{\text{total}} = \sum_f \omega_{fi}$$



# Ordinary muon capture

Why study muon capture?

Neutrinoless double beta decay of course!

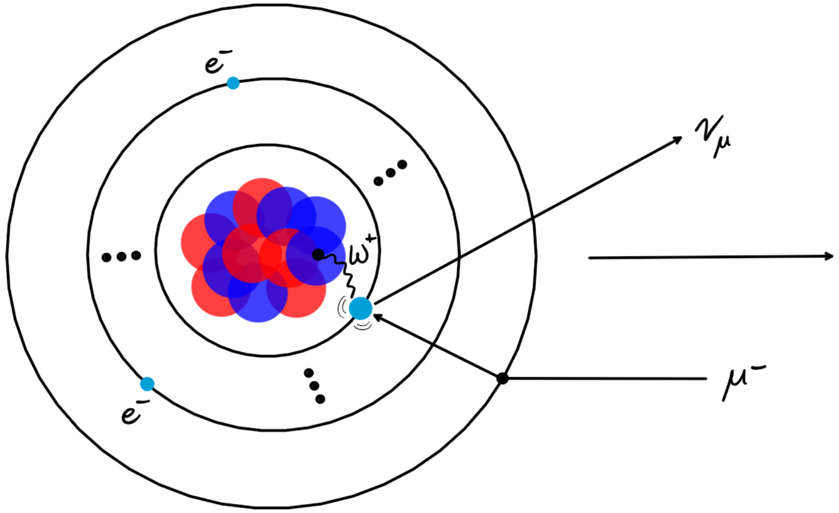
1. Identical currents and coupling constants
2. Similar phase-space (different from electron-induced beta decay)

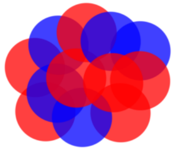
A **measurable process** which probes physics relevant to **neutrinoless double beta decay**

# Ordinary muon capture



$$\omega_{fi} \propto \langle \phi_{1s} \rangle_\rho \sum_{J=0}^{\infty} \left| \langle \Phi_f \,;\, p_f | O_L(|\vec{q}|) | \Phi_i \,;\, p_i \rangle \right|^2 + \left| \langle \Phi_f \,;\, p_f | O_T(|\vec{q}|) | \Phi_i \,;\, p_i \rangle \right|^2$$



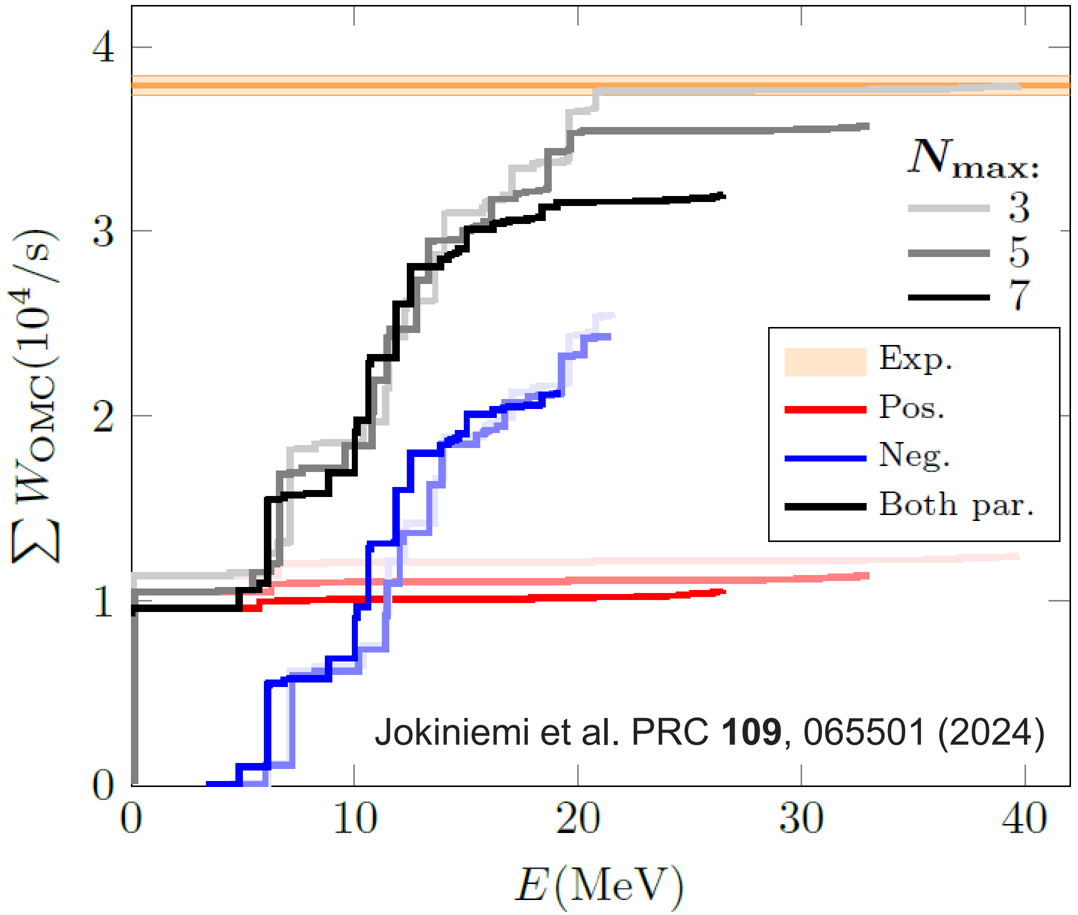
## Devil is in the details [ with the LSM ]

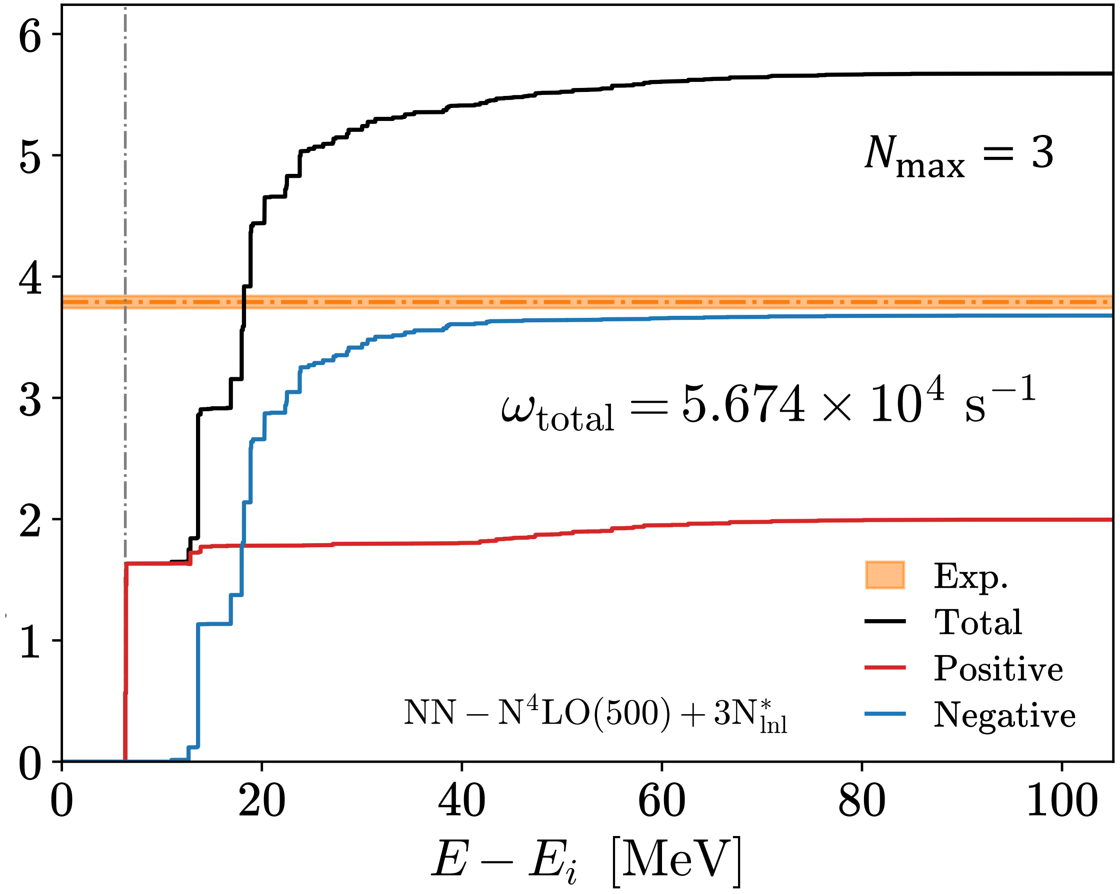
- Cast as energy-weighted sum of resolvent amplitudes
- Numerically evaluate off-shell and later cast as on-shell

Walecka. NPA **258** (1976), pp. 397-416

# Comparison to literature for $^{12}C \to {}^{12}B$ capture





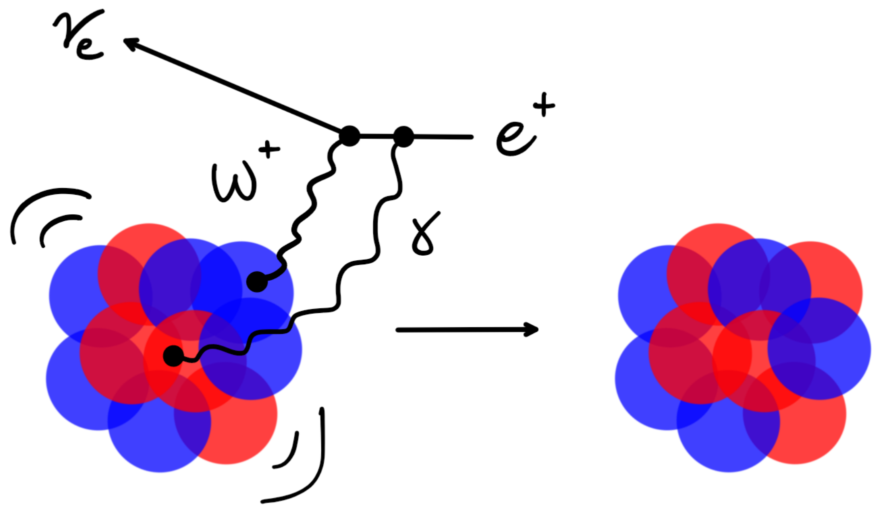



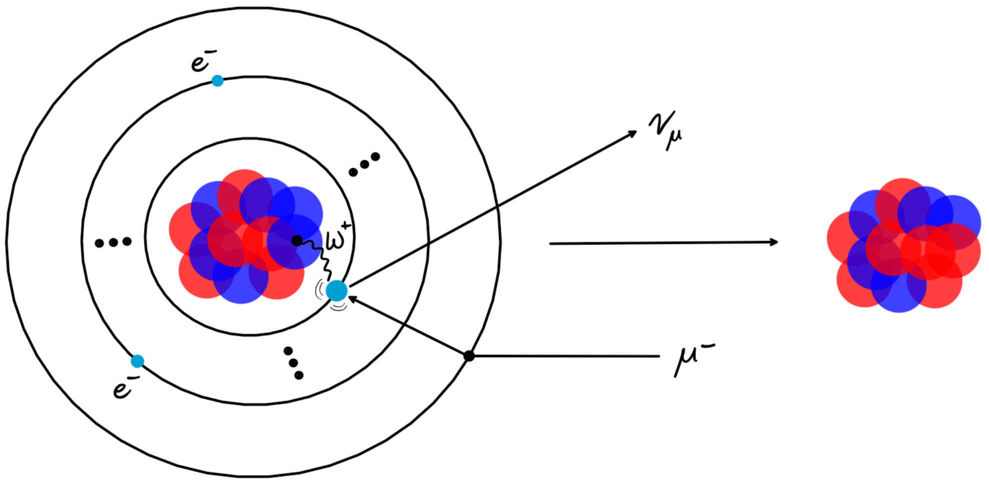

DANGEROUSLY PRELIMINARY

# The future of electroweak theory in light nuclei



- **Improved precision for electroweak radiative corrections in nuclei** with the Lanczos Strengths Method coupled to the ab initio NCSM
- Systematic improvements available, e.g., consistent currents in chiral EFT
- Limited by matching of high-energy QCD to low-energy nuclear theory



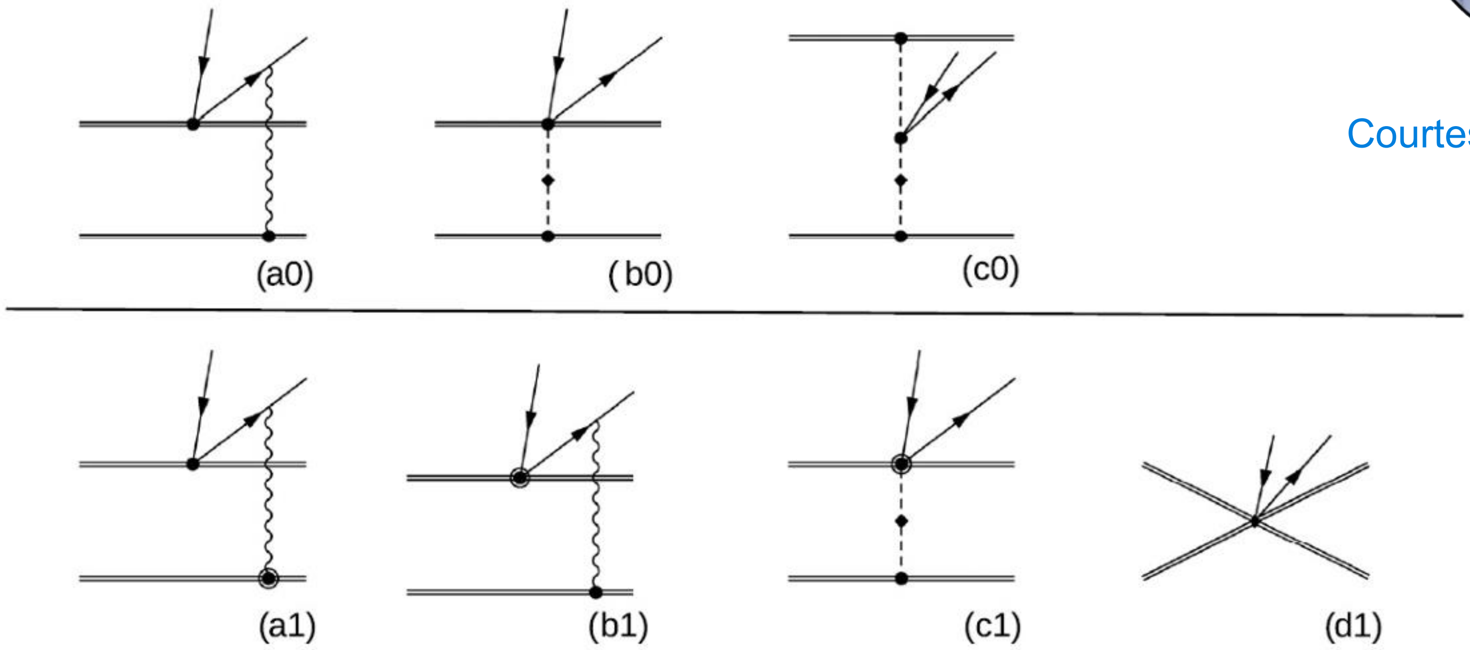



# What is feasible in the near future?

# Constraints on LECs of pNRQED



Courtesy of C.-Y. Seng




$$\delta_{\rm NS} = \frac{2}{M_F} \langle \mathcal{V}_0^{\rm mag}(\vec{r}) + \mathcal{V}_0^{\rm rec,1}(\vec{r}) + \mathcal{V}_0^{\rm CT}(\vec{r}) \rangle_{fi}$$

$$\mathcal{A}_{fi} = \langle \Phi_f | O_2 \, (z - H)^{-1} \, O_1 | \Phi_i \rangle$$

Large cost reduction

PHYSICAL REVIEW C **72**, 065501 (2005)

**Piecewise moments method: Generalized Lanczos technique for nuclear response surfaces**

Wick C. Haxton,[1] Kenneth M. Nollett,[2] and Kathryn M. Zurek[1]
[1]*Institute for Nuclear Theory and Department of Physics, University of Washington, Seattle, Washington 98195, USA*
[2]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

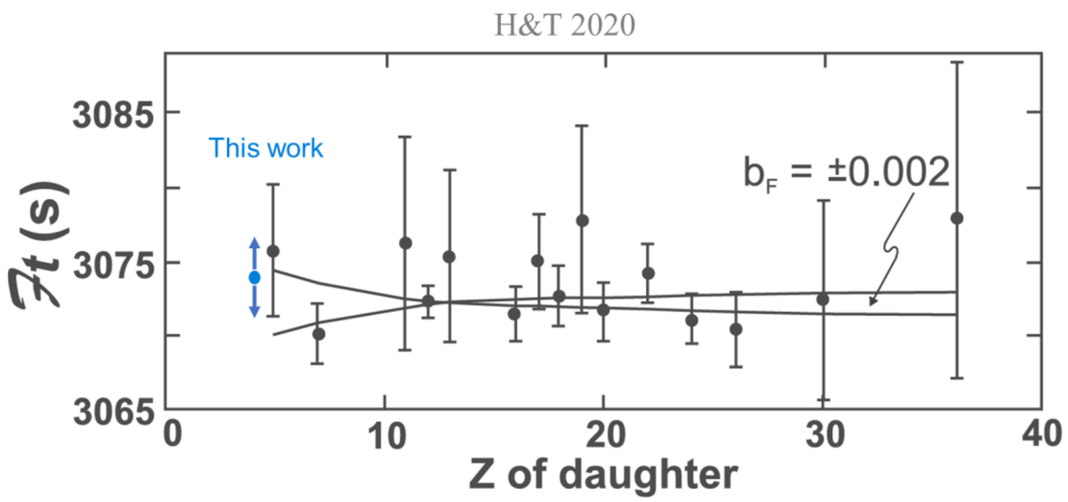
(Received 22 August 2005; published 29 December 2005)

UQ bounds on numerics

Do We Fully Understand the Symmetric Lanczos Algorithm Yet? *

Beresford N. Parlett †

# Were I to dream




What about $^4$He?

**A non-Hermitian quantum mechanics approach for extracting and emulating continuum physics based on bound-state-like calculations: technical details**

Xilin Zhang[1,*]

[1] *Facility for Rare Isotope Beams, Michigan State University, Michigan 48824, USA*

(Dated: November 12, 2024)

Constraints on scalar and pseudo-scalar dark matter!

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# Thank you
# Merci

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Discovery,
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# One-loop radiative correction



$$\mathcal{M} = \mathcal{M}_{\rm tree} + \delta\mathcal{M}_{\rm one-loop} + \cdots$$

$$\delta\mathcal{M}_{\rm one-loop} = -i\sqrt{2}\,G_F e^2\, L_\lambda(k_f, k_i) \int \frac{d^4q}{(2\pi)^4} R_W(q)\, R_e(q)\, R_\gamma(q) \times \left[\epsilon^{\mu\nu\alpha\lambda} q_\alpha T_{\mu\nu}(p_f, p_i; q)\right]$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

# Wick rotation and electron energy expansion



$$\Box^b_{\gamma W}(E_e) = \underline{(\Box^b_{\gamma W})_{\mathrm{Wick}}(E_e) + (\Box^b_{\gamma W})_{\mathrm{Res},e}(E_e)} + \underline{(\Box^b_{\gamma W})_{\mathrm{Res},T_3}(E_e)}$$

Wick rotated box diagram and electron residue contribution are regular as $E_e \to 0$

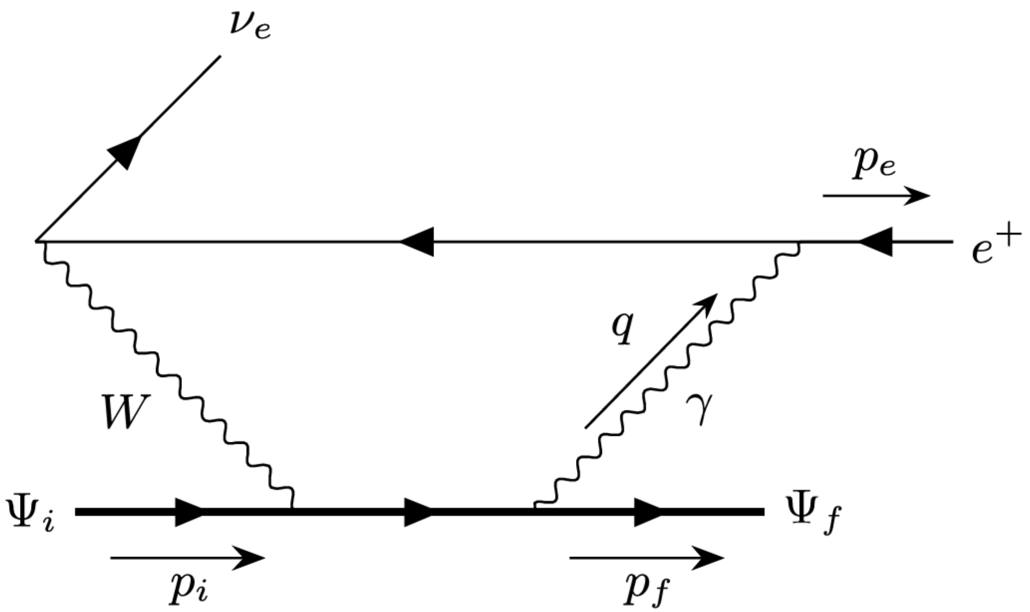
Nuclear residue contribution is **singular**

$$\Box^b_{\gamma W}(E_e) = \boxminus_0 + E_e \boxminus_1 + (\Box^b_{\gamma W})_{\mathrm{Res},T_3}(E_e) + \mathcal{O}(E_e^2)$$

Gennari et al. PRL **134**, 012501

# Deriving the non-relativistic Compton amplitude






$$J(\vec{r}) = \int \frac{d^3 r}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}} J(\vec{q})$$

$$J^\mu(t,\vec{x}) = e^{-iHt} J^\mu(0,\vec{x})\, e^{iHt}$$

$$T^{\mu\nu}(p_f, p_i; q) = -\frac{i}{2}\langle \Phi_f\,;\,p_f |\, J^\mu_{\mathrm{em}}(-\vec{q})(z_f - H)^{-1} J^{\dagger\,\nu}_{\mathrm{W}}(\vec{q}) | \Phi_i\,;\,p_i\rangle$$
$$- \frac{i}{2}\langle \Phi_f\,;\,p_f |\, J^{\dagger\,\nu}_{\mathrm{W}}(-\vec{q})(z_i - H)^{-1} J^\mu_{\mathrm{em}}(\vec{q}) | \Phi_i\,;\,p_i\rangle$$

# Deriving the non-relativistic Compton amplitude



$$M_{JM}(q) := \int d^3r \; \mathcal{M}_{JM}(q,\vec{r})\,\rho(\vec{r})$$

$$L_{JM}(q) := \int d^3r \; \frac{i}{q}\left(\vec{\nabla}\mathcal{M}_{JM}(q,\vec{r})\right)\cdot\vec{J}(\vec{r})$$

$$T^{\mathrm{el}}_{JM}(q) := \int d^3r \; \frac{1}{q}\left(\vec{\nabla}\times\vec{\mathcal{M}}^{M}_{JJ}(q,\vec{r})\right)\cdot\vec{J}(\vec{r})$$

$$T^{\mathrm{mag}}_{JM}(q) := \int d^3r \; \vec{\mathcal{M}}^{M}_{JJ}(q,\vec{r})\cdot\vec{J}(\vec{r})$$

Donnelly et al. ADNDT **23**, pp. 103-176 (1979)