ECT* Workshop:
Next Generation Ab Initio Nuclear Theory, July 2025

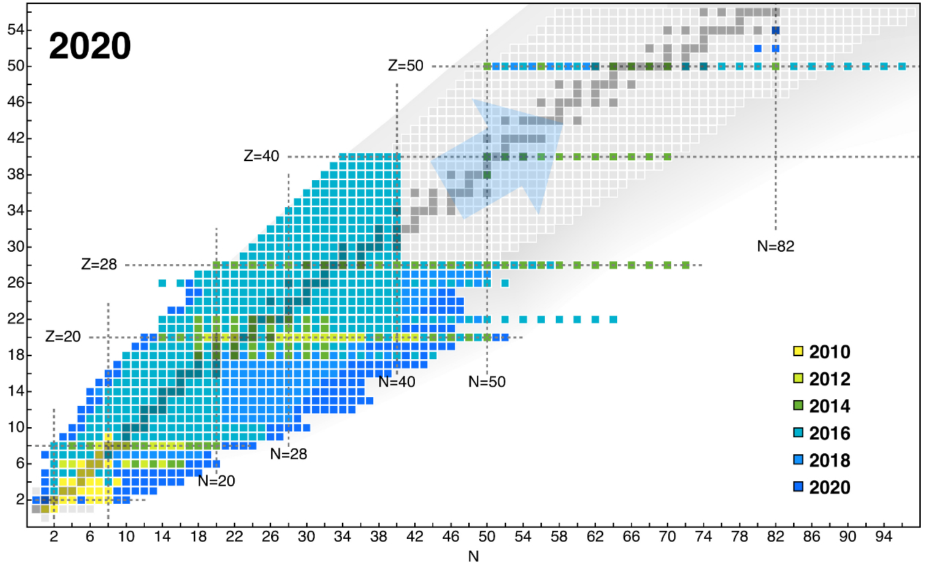
Spin-isospin symmetries of nuclear beta decays

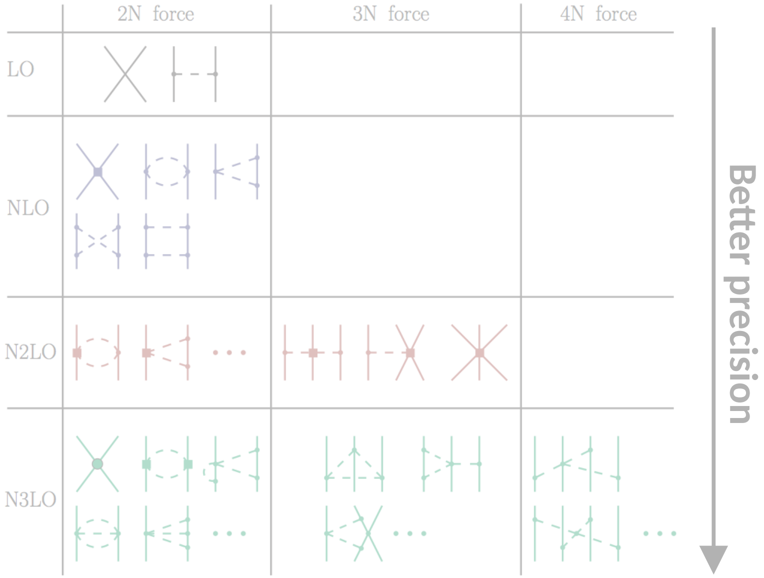
Simone Salvatore Li Muli

Chalmers University of Technology

Present and past ab-initio nuclear theory

H. Hergert Front. Phys. 8:379. (2020)

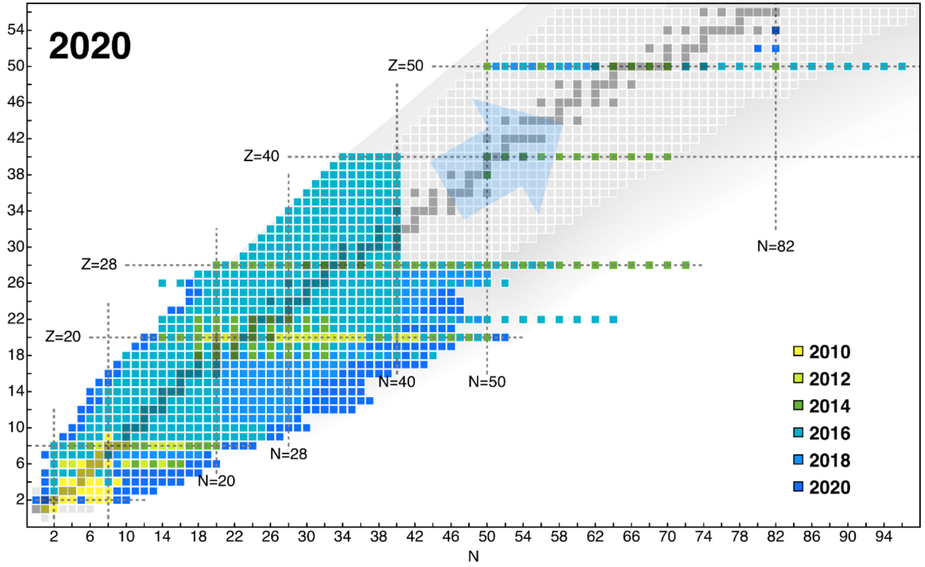


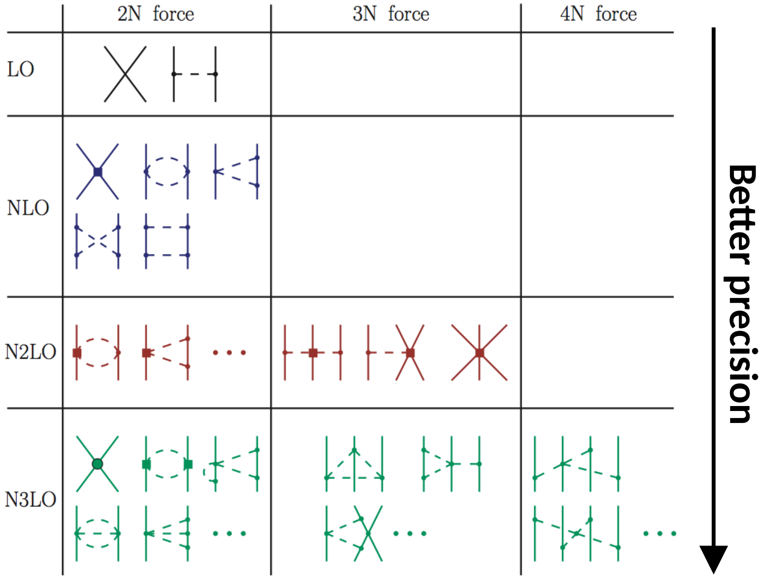


- Degrees of freedom
- Symmetries
- Power counting

Present and past ab-initio nuclear theory

H. Hergert Front. Phys. 8:379. (2020)





- **Degrees of freedom**
- **Symmetries**
- **Power counting**

Next generation ab-initio nuclear theory

- **Do atomic nuclei posses symmetries, that are not of QCD?**

- **How to use them to increase the predictive power of ab-initio nuclear theory?**

Wigner SU(4) symmetry

$$\mathrm{H}\,|\Psi_A\rangle = \mathrm{E}\,|\Psi_A\rangle$$

Wigner SU(4) Symmetry

The nuclear Hamiltonian contains forces that can be written such that they depend on space coordinates alone.
E. Wigner, Phys. Rev. 51, 106–119 (1937)

Interest revived in the 90'

NN forces at LO in the large number of colors of QCD are SU(4) symmetric for S-waves.
D. B. Kaplan and M. J. Savage, Physics Letters B 365 (1996) 244-251

Proposed as a medium- and long-distance symmetry of the NN force. Broken at short-distances.
A. Calle Cordon and E. Ruiz Arriola, Phys. Rev. C 78, 054002 (2008)

Argued that symmetry becomes evident in NN forces at a particular resolution scale.
D. Lee et al, Phys. Rev. Lett 127, 062501 (2021)

Single-particle states

$$\begin{bmatrix} \uparrow \\ \downarrow \end{bmatrix}_S \otimes \begin{bmatrix} p \\ n \end{bmatrix}_T = \begin{bmatrix} \uparrow p \\ \uparrow n \\ \downarrow p \\ \downarrow n \end{bmatrix}_{ST}$$

For SU(4) invariant Hamiltonians, nuclear states populate **irreducible representations** of the SU(4) group.

Irreducible representations of SU(2) & SU(3)

SU(2) of isospin

3 Generators: $\mathrm{T}_a = \frac{1}{2}\sum_i \tau_a^{(i)},$

One can be diagonalized

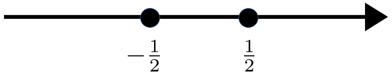
Gell-Mann SU(3)

8 Generators: $\mathrm{F}_a = \frac{1}{2}\sum_i \lambda_a^{(i)},$

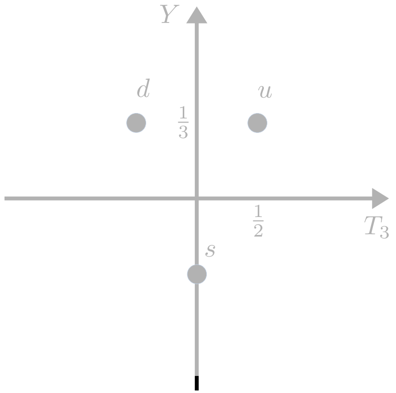
Two can be diagonalized

Single-particle states

T=1/2

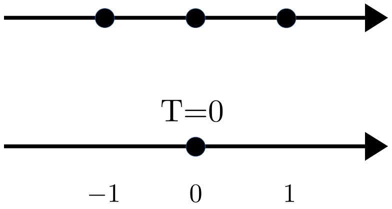


$\left(\frac{1}{2}, \frac{1}{3}\right)$



Two-particle states

T=1



Irreducible representations of SU(2) & SU(3)

SU(2) of isospin

3 Generators: $\mathrm{T}_a = \frac{1}{2}\sum_i \tau_a^{(i)},$

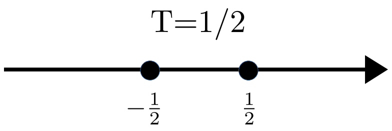
One can be diagonalized

Gell-Mann SU(3)

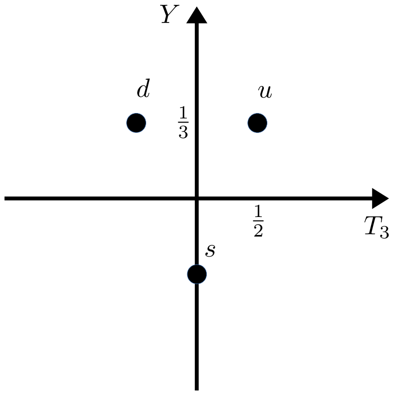
8 Generators: $\mathrm{F}_a = \frac{1}{2}\sum_i \lambda_a^{(i)},$

Two can be diagonalized

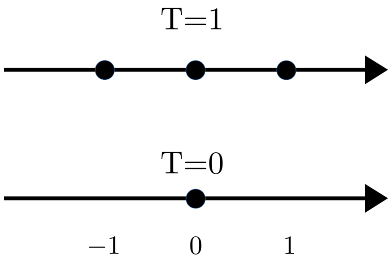
Single-particle states



$\left(\frac{1}{2}, \frac{1}{3}\right)$



Two-particle states



Irreducible representations of SU(4)

There are 15 group generators:

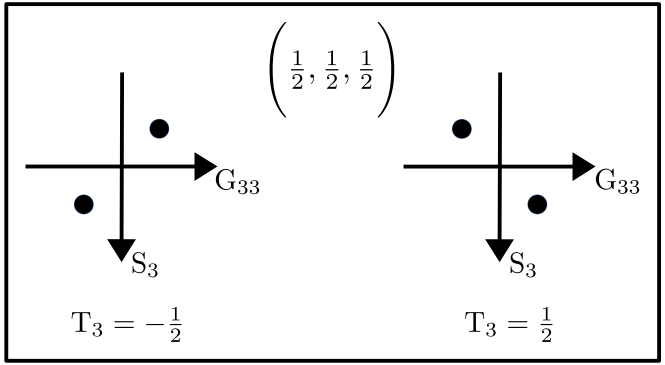
$$\mathrm{T}_a = \frac{1}{2}\sum_i \tau_a^{(i)}, \qquad \mathrm{S}_b = \frac{1}{2}\sum_i \sigma_b^{(i)}, \qquad \mathrm{G}_{ab} = \frac{1}{2}\sum_i \tau_a^{(i)}\sigma_b^{(i)}$$

It is possible to diagonalize simultaneously T_3, S_3, G_{33} and the quadratic Casimir operator

$$\mathrm{C}_2 = T^a T_a + S^b S_b + G^{ab} G_{ab}$$

Single-particle states (Fundamental irreducible representation):

$$\begin{bmatrix} \uparrow \\ \downarrow \end{bmatrix}_S \otimes \begin{bmatrix} p \\ n \end{bmatrix}_T = \begin{bmatrix} \uparrow p \\ \uparrow n \\ \downarrow p \\ \downarrow n \end{bmatrix}_{ST}$$

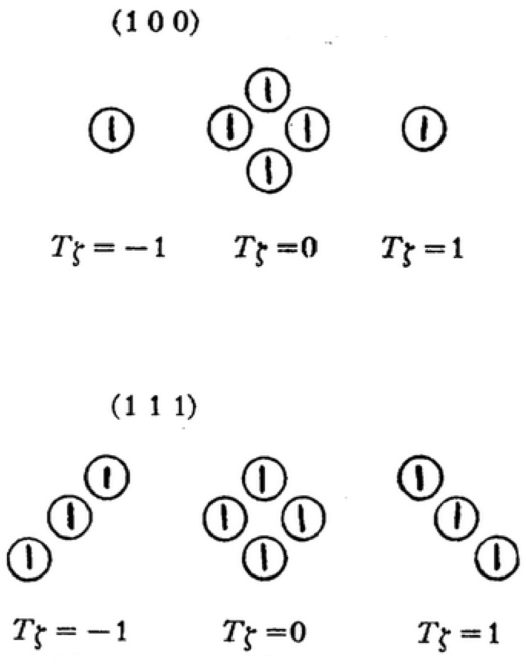


Irreducible representations of SU(4)

E. Wigner, Phys. Rev. 51, 106–119 (1937)

S.S.LM, T. R. Djärv, C. Forssén, D. R. Phillips, arXiv:2503.16372v2

Two-particle states



Many-particle states

Particles	[n]	$\langle C_2 \rangle$	Young tableau	Dual tableau
2	[**6**]	5	[1,1,0,0]	[2,0,0,0]
	[**10**]	9	[2,0,0,0]	[1,1,0,0]
3	[**4**]	3.75	[1,1,1,0]	[3,0,0,0]
	[**20**]	9.75	[2,1,0,0]	[2,1,0,0]
	[**20**]	15.75	[3,0,0,0]	[1,1,1,0]
4	[**1**]	0	[1,1,1,1]	[4,0,0,0]
	[**15**]	8	[2,1,1,0]	[3,1,0,0]
	[**20**]	12	[2,2,0,0]	[2,2,0,0]
6	[**6**]	5	[2,2,1,1]	[4,2,0,0]
	[**10**]	9	[3,1,1,1]	[4,1,1,0]
	[**64**]	15	[3,2,1,0]	[3,2,1,0]
7	[**4**]	3.75	[2,2,2,1]	[4,3,0,0]
	[**20**]	9.75	[3,2,1,1]	[4,2,1,0]
	[**36**]	13.75	[3,2,2,0]	[3,3,1,0]
8	[**1**]	0	[2,2,2,2]	[4,4,0,0]
	[**15**]	8	[3,2,2,1]	[4,3,1,0]
	[**20**]	12	[3,3,1,1]	[4,2,2,0]

Back to Physics

- **Are nuclei approximately SU(4) symmetric?**

- **What consequences SU(4) symmetry introduces to nuclear observable?**

SU(4) decomposition of Nuclear States

- Solve the many-body Schrödinger equation (here with chEFT interactions and ncsm)

$$\mathrm{H}\,|\Psi_A\rangle = \mathrm{E}\,|\Psi_A\rangle$$

- The obtained wavefunctions can then be decomposed into SU(4) irreducible representations

$$|\Psi_A\rangle = \sum_{C_2^A} d(C_2^A)\,|\Phi(C_2^A)\rangle \otimes |C_2^A\rangle$$

using the Lanczos alghoritm with the nuclear wavefuction as the first Krylov pivot

C. W. Johnson, Phys. Rev. C 91, 034313 (2015)
A. E. McCoy et al, Phys. Rev. Lett. 125, 102505 (2020)

$$C_2\,|\Psi_A\rangle = \alpha_1\,|\Psi_A\rangle + \beta_1\,|K_2\rangle$$
$$C_2\,|K_2\rangle = \beta_1\,|\Psi_A\rangle + \alpha_2\,|K_2\rangle + \beta_2\,|K_3\rangle$$
$$C_2\,|K_3\rangle = \beta_2\,|K_2\rangle + \alpha_3\,|K_3\rangle + \beta_3\,|K_4\rangle$$
$$\dots$$

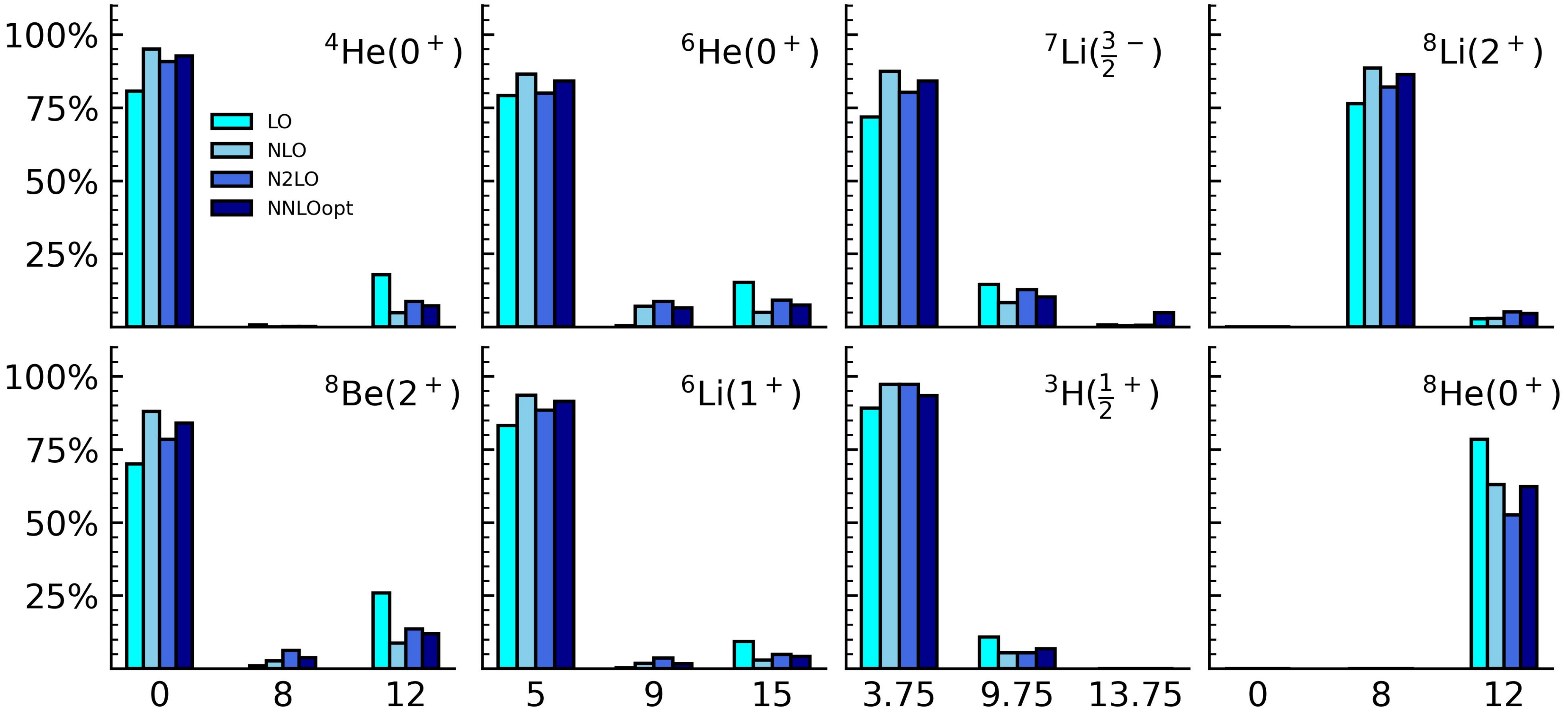
- Diagonalization of the tridiagonal Lanzos matrix gives the representation of the eigenvalues of the quadratic Casimir in the Krylov space.

SU(4) decomposition of light nuclei

S.S.LM, T. R. Djärv, C. Forssén, D. R. Phillips, arXiv:2503.16372v2

S. Wesolowski et al, Phys. Rev. C 104, 064001 (2021)

A. Ekström et al, Phys. Rev. Lett. 110, 192502 (2013)



Axial-current operator

The **Gamow-Teller operator** is **a combination of generators of** SU(4).

$$\mathrm{G}_{ab} = \frac{1}{2}\sum_i \tau_a^{(i)}\sigma_b^{(i)}$$

It does not connect different irreducible representations. It is **block-diagonal** in SU(4) irreps.

E. Wigner, Phys. Rev. 56, 519–527 (1937)

P. T. Nang, Nucl. Phys. A 185 413-432 (1971)

N.C. Mukhopadhyay, F. Cannata, Phys. Lett. B 51 225-228 (1974)

We take the axial-current operator, up to N3LO in the counting of **A. Baroni et al, Phys. Rev. C 98, 044003 (2018)** and study the block structure of the current in the long-wavelength limit

We show that, up to small terms, the full axial current is also block-diagonal in SU(4) irreps

S.S.LM, T. R. Djärv, C. Forssén, D. R. Phillips, arXiv:2503.16372v2

Option 1

- $$[\mathrm{C}_2, \mathcal{O}_{ST}] = 0$$

Axial-current operator

The **Gamow-Teller operator** is **a combination of generators of** SU(4).

$$\mathrm{G}_{ab} = \frac{1}{2} \sum_i \tau_a^{(i)} \sigma_b^{(i)}$$

It does not connect different irreducible representations. It is **block-diagonal** in SU(4) irreps.

E. Wigner, Phys. Rev. 56, 519–527 (1937)

P. T. Nang, Nucl. Phys. A 185 413-432 (1971)

N.C. Mukhopadhyay, F. Cannata, Phys. Lett. B 51 225-228 (1974)

We take the axial-current operator, up to N3LO in the counting of **A. Baroni et al, Phys. Rev. C 98, 044003 (2018)** and study the block structure of the current in the long-wavelength limit

We show that, up to small terms, the full axial current is also block-diagonal in SU(4) irreps

S.S.LM, T. R. Djärv, C. Forssén, D. R. Phillips, arXiv:2503.16372v2

Option 2

$$(\tau^{(1)} \wedge \tau^{(2)})_a (\sigma^{(1)} \wedge \sigma^{(2)})_b = 2\epsilon_{ace}\epsilon_{bdf} G_{cd} G_{ef} + 4G_{ab}$$

- $$(\tau^{(1)} \wedge \tau^{(2)})_a \epsilon_{ijk} \sigma_j^{(1)} \sigma_l^{(2)} + (1 \leftrightarrow 2) = \epsilon_{ijk}\epsilon_{abc} G_{bj} G_{cl} - 2\delta_{il} G_{ka}$$

SU(4) predictions of axial nuclear matrix elements

Transition	*SU(4)*	*ab initio* this work	*ab initio* GFMC
$^3\mathrm{H}(\frac{1}{2}^+)\ [C_2 = 3.75] \to$ $^3\mathrm{He}(\frac{1}{2}^+)\ [C_2 = 3.75]$	2.449	2.313	//
$^6\mathrm{He}(0^+)\ [C_2 = 5] \to$ $^6\mathrm{Li}(1^+)[C_2 = 5]$	2.449	2.260	2.200
$^7\mathrm{Be}(\frac{3}{2}^-)\ [C_2 = 3.75] \to$ $^7\mathrm{Li}(\frac{3}{2}^-)\ [C_2 = 3.75]$	2.582	2.357	2.317
$^7\mathrm{Be}(\frac{3}{2}^-)\ [C_2 = 3.75] \to$ $^7\mathrm{Li}(\frac{1}{2}^-)\ [C_2 = 3.75]$	2.309	2.175	2.157
$^8\mathrm{Li}(2^+)\ [C_2 = 8] \to$ $^8\mathrm{Be}(2^+)\ [C_2 = 0]$	0.0	0.093	0.147
$^8\mathrm{He}(0^+)\ [C_2 = 12] \to$ $^8\mathrm{Li}(1^+)\ [C_2 = 8]$	0.0	0.335	0.386

G.King et al, Phys. Rev. C 102, 025501 (2020)

S.S.LM, T. R. Djärv, C. Forssén, D. R. Phillips, arXiv:2503.16372v2

Conclusions

- **Are nuclei approximately SU(4) symmetric?**

  SU(4) symmetry is broken, but not to the point to be irrelevant for the structure of nuclei.

- **What consequences SU(4) symmetry introduces to nuclear observable?**

  Selection rules in nuclear beta decays, based on SU(4) symmetry and block-diagonal structure of the axial current.