

Impact of ground-state correlations on the nuclear response

ECT* workshop
Next generation *ab initio* nuclear theory

Trento, July 16th, 2025

Andrea Porro

Technische Universität Darmstadt



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DFG



Introduction

- Physics case
- Existing ab initio methods

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IMSRG multipole moments

- Moments of the strength
- IMSRG implementation
- Model-space convergence

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- Interaction sensitivity
- Comparison to experiment
- Comparison to sum rules

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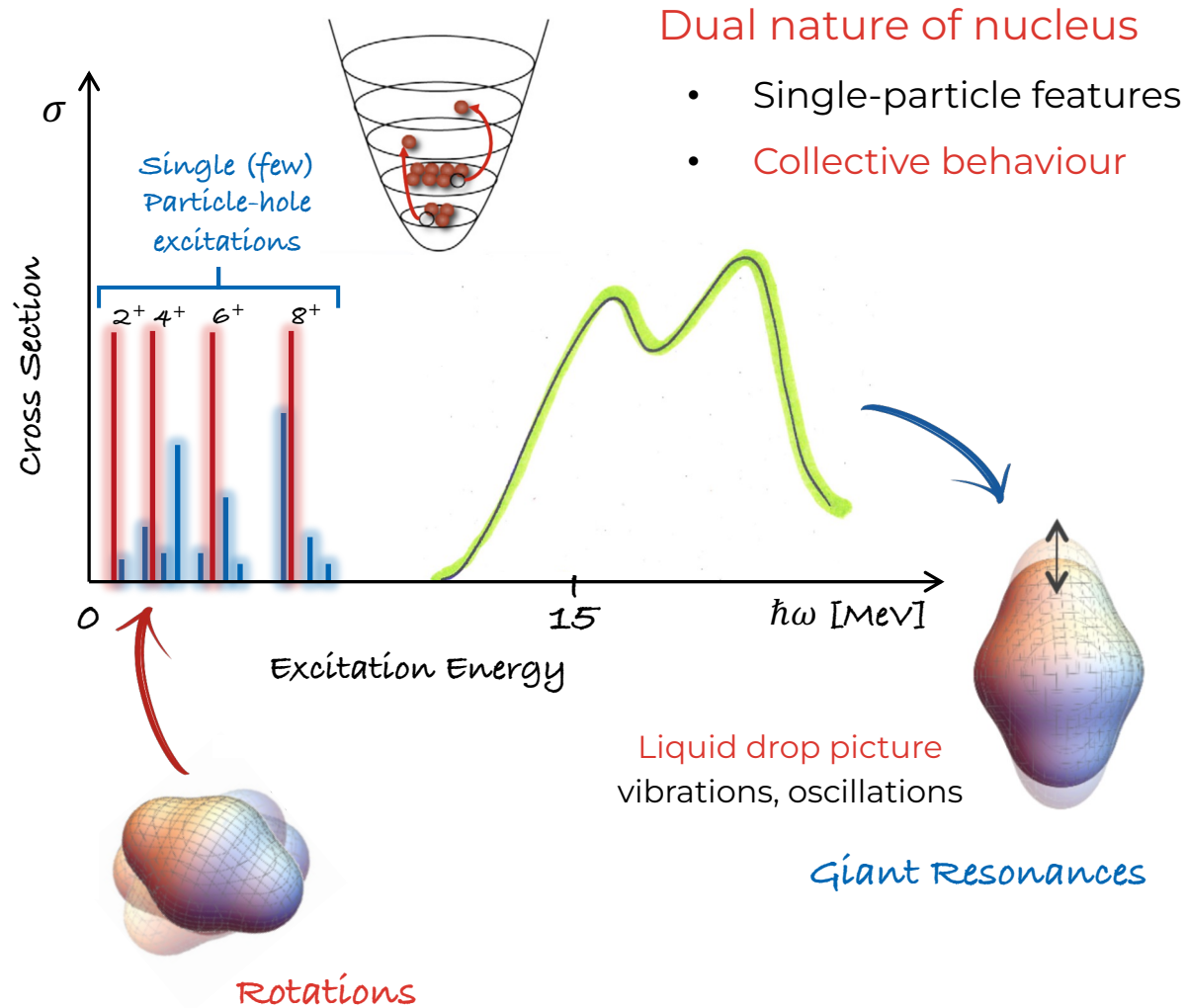
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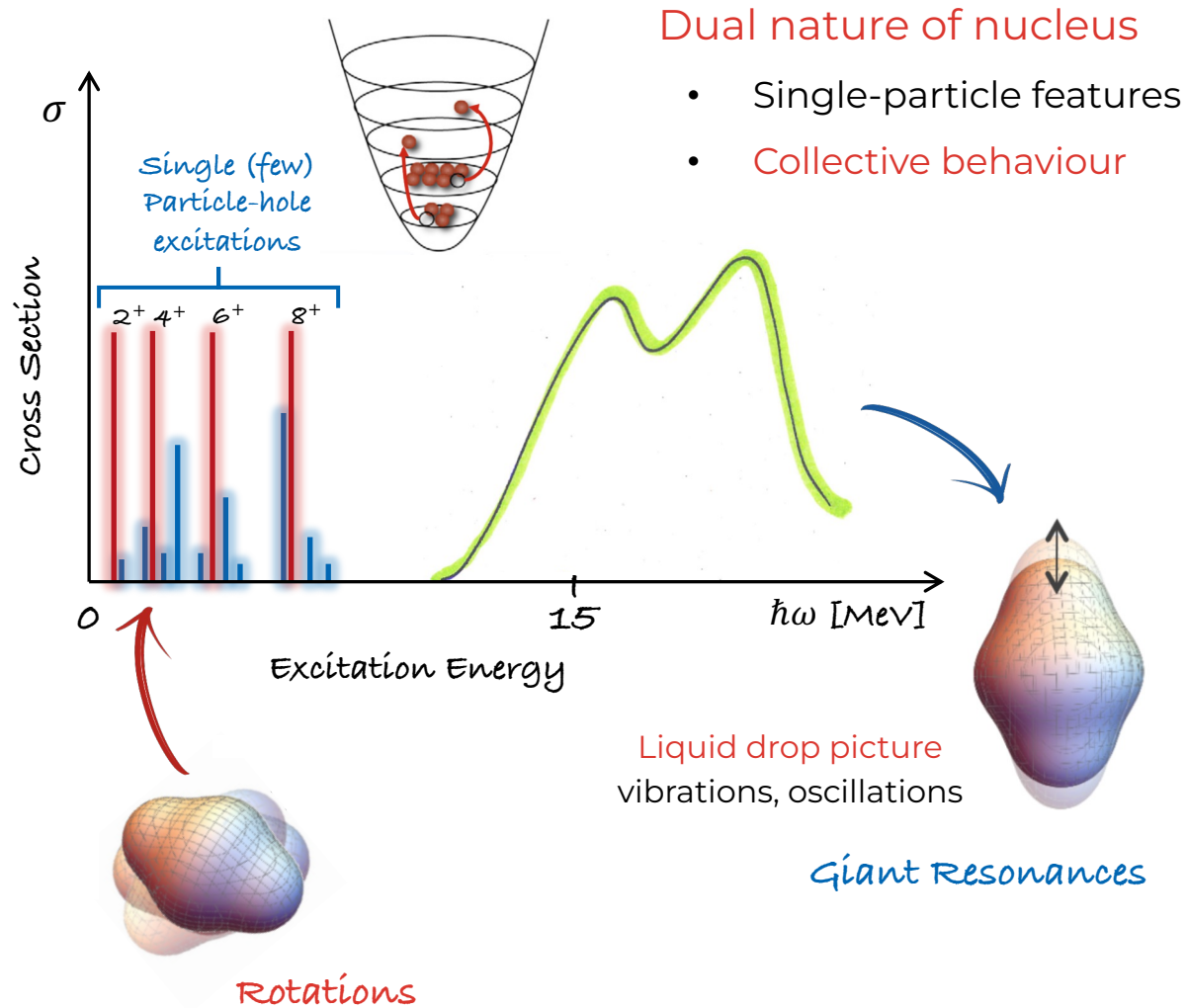
Nuclear spectroscopy

3



Nuclear spectroscopy

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Response function

Fully characterise linear response

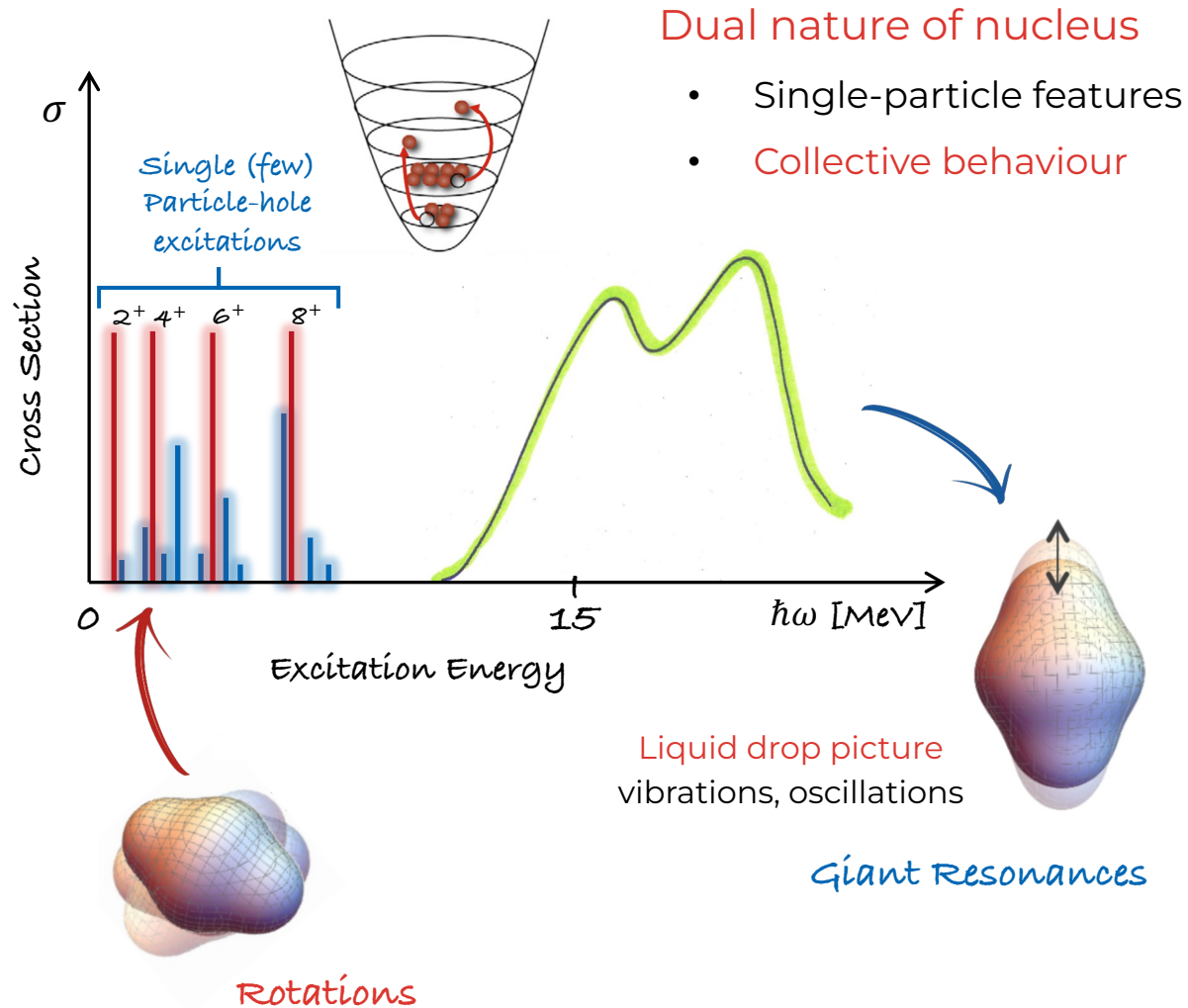
$$S(Q_\lambda, E) \equiv \sum_{\mu\nu} |\langle \Psi_\nu | Q_{\lambda\mu} | \Psi_0 \rangle|^2 \delta(E_\nu - E_0 - E)$$

Transition probability

Excitation energy

Nuclear spectroscopy

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Response function

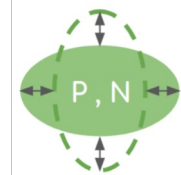
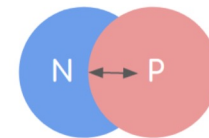
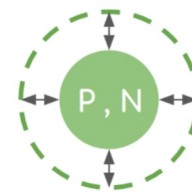
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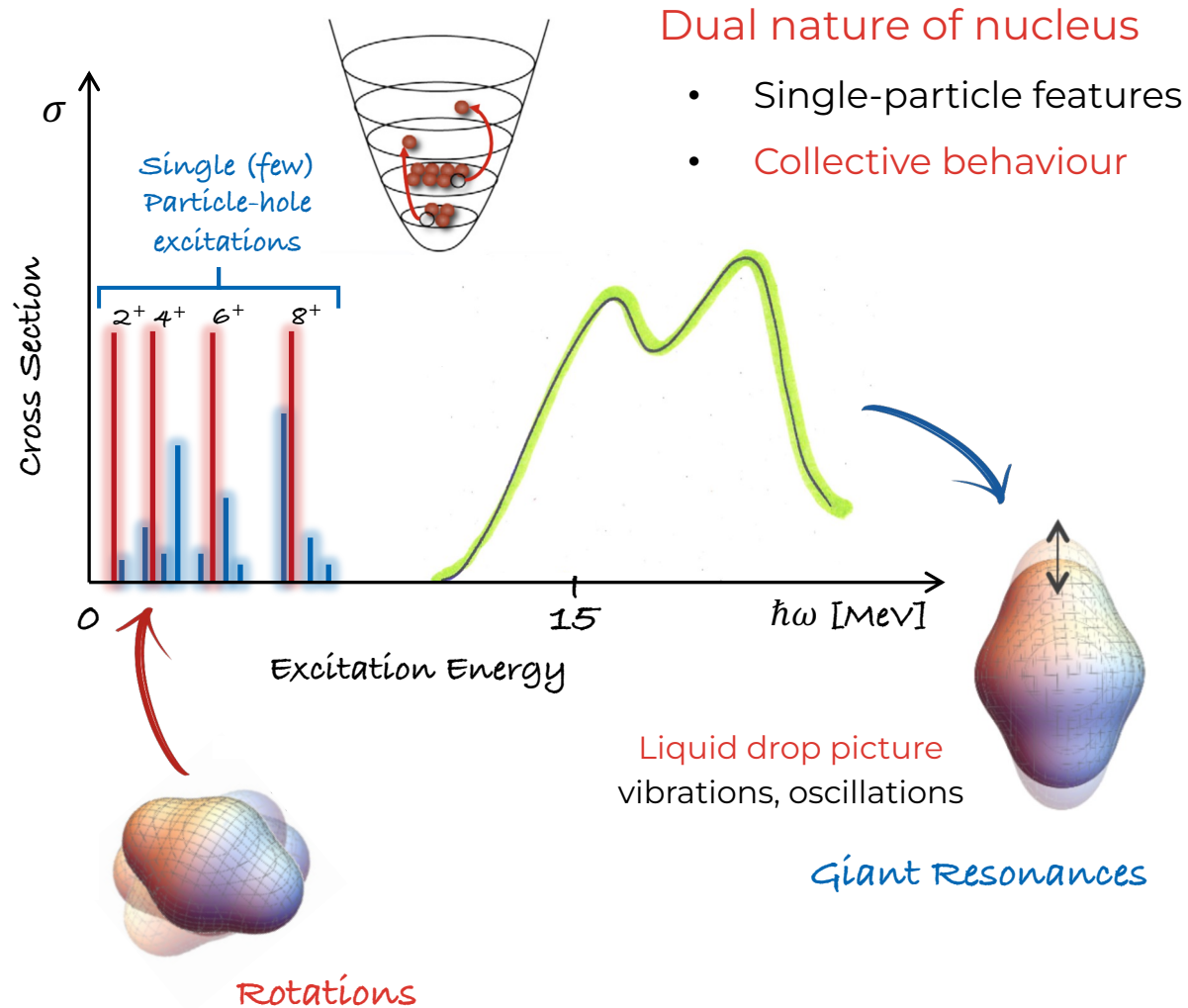
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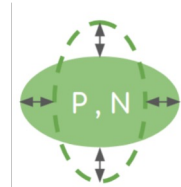
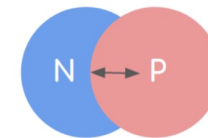
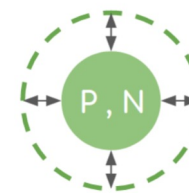
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Different excitations

Different methods

Established techniques*

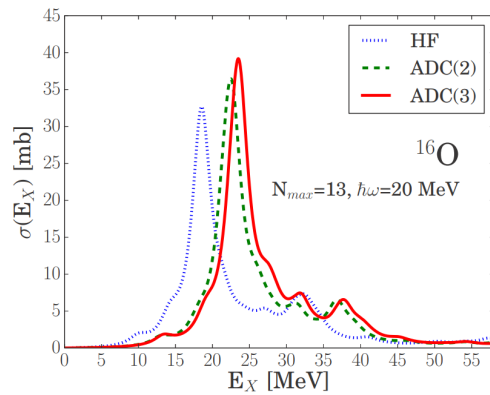
*Excluding virtually exact methods

Established techniques*

4

SC(G)GF

- Polarisation
- Low-lying states (neighbouring)



[Barbieri, Raimondi, PRC, 2019]

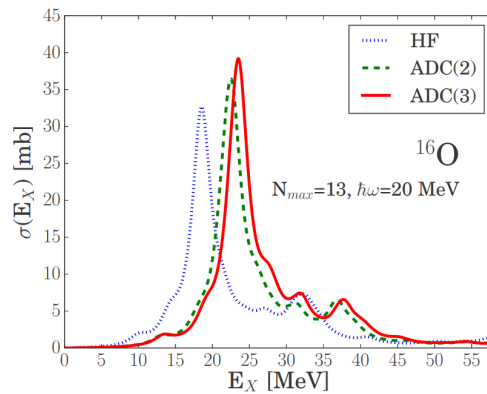
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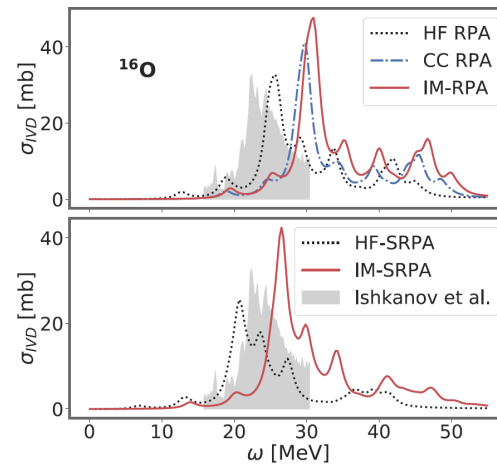
RPA inspired

HF based RPA

- U(1) breaking
- SU(2) breaking

IMSRG based

- TDA equivalent
- npnh excitations



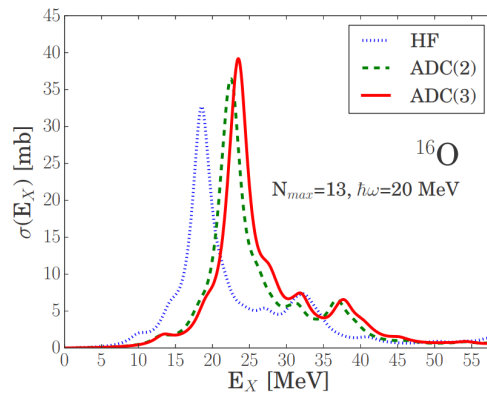
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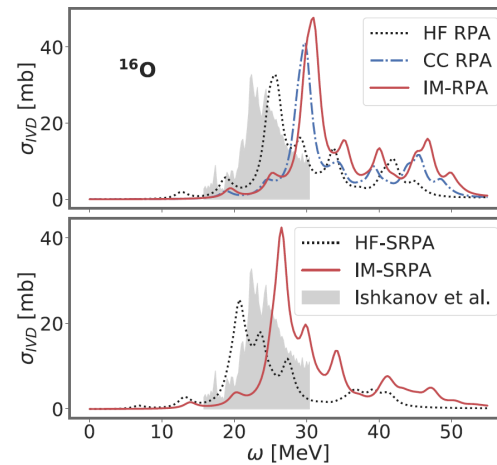
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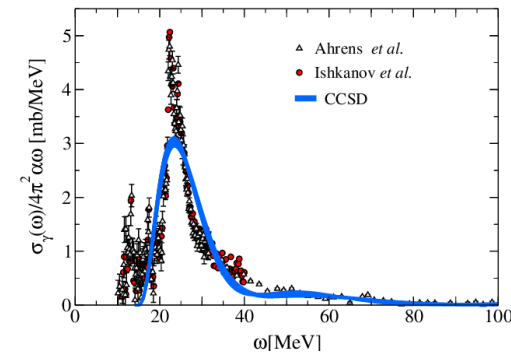
EOM based

CC-LIT

- Continuum
- 2p2h complete

EOM-IMSRG

- 2p2h (+ corr)



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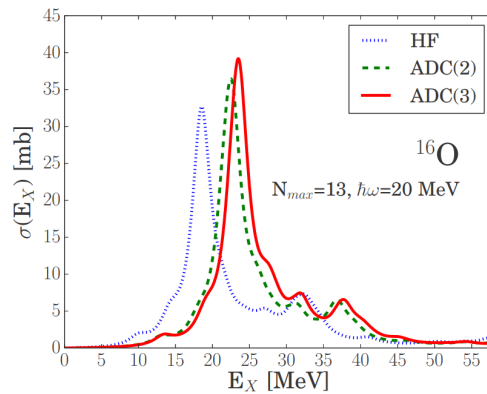
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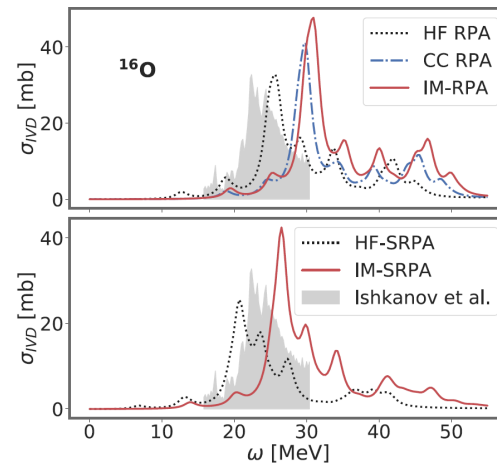
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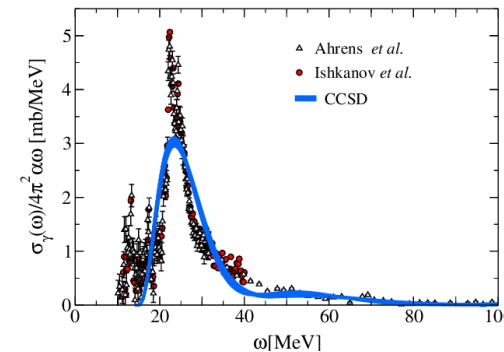
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VS-IMSRG

- Effective SM
- Low-lying states

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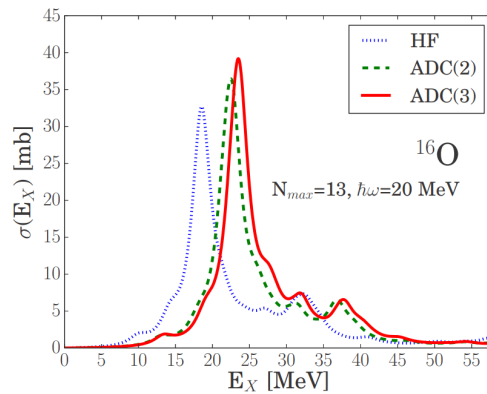
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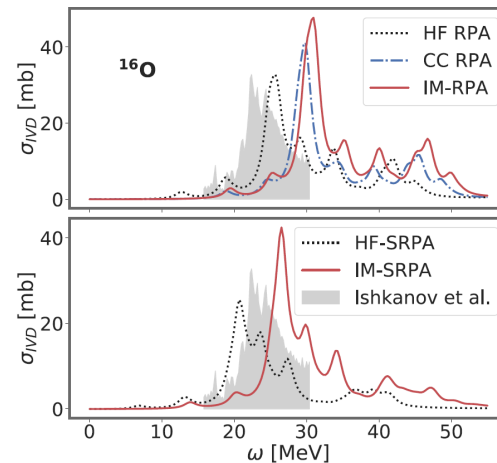
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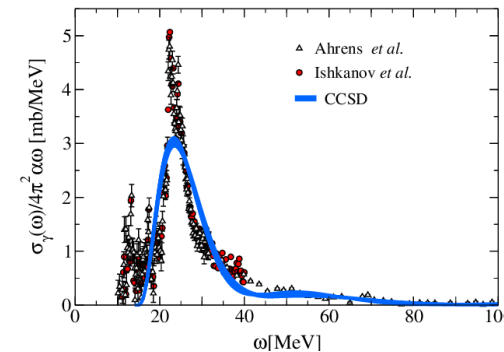
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- Collective space
- Rotational states

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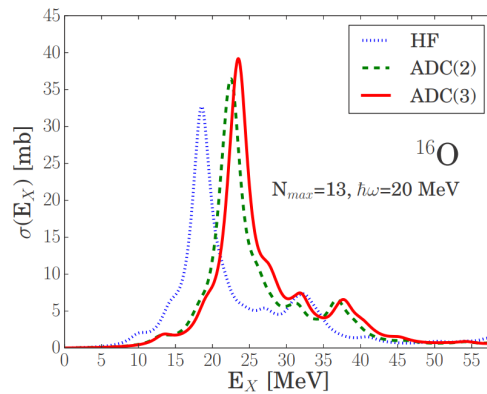
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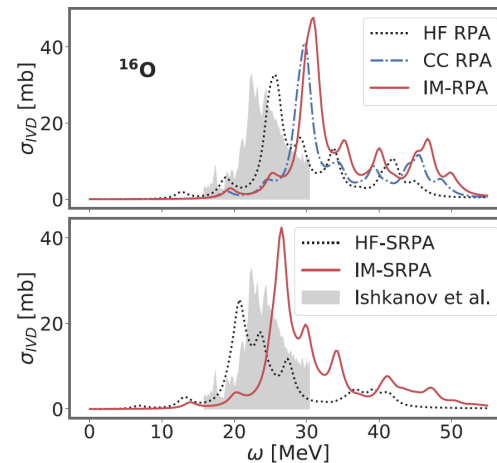
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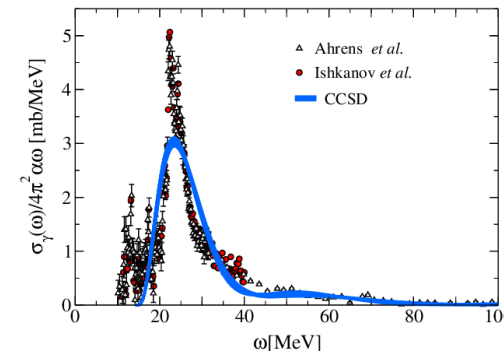
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How complete is the excited states description ?

Uncertainty quantification / benchmark

*Excluding virtually exact methods



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Moments of the strength

Studied quantity: multipole response

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Integrated properties

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Must know excited states

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Integrated properties

Must know excited states

Ground state only

Identity resolution

$$\mathbb{1} = \sum_\nu |\Psi_\nu\rangle \langle \Psi_\nu|$$

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- Exact treatment for exc states
- Many-body truncation only GS

“Exact sum rules with approximate ground states”

Moments of the strength

6

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Integrated properties



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Ground state only



Identity resolution

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Exact implementation up to m_1

Effective two-body Hamiltonian

$$H = H^{[1]} + H^{[2]}$$

Spherical tensor operators

$$Q_{\lambda\mu}^\dagger = (-1)^\mu Q_{\lambda, -\mu}$$

- **Exact** treatment for **exc** states
- Many-body truncation only **GS**

“Exact sum rules with approximate ground states”

Previous PGCM study

7



Eur. Phys. J. A (2024) 60:155
<https://doi.org/10.1140/epja/s10050-024-01377-5>

THE EUROPEAN
PHYSICAL JOURNAL A



Regular Article - Theoretical Physics

Ab initio description of monopole resonances in light- and medium-mass nuclei

III. Moments evaluation in ab initio PGCM calculations

A. Porro^{1,2,3,a}, T. Duguet^{3,4}, J.-P. Ebran^{5,6}, M. Frosini⁷, R. Roth^{1,8}, V. Somà³

- I. [EPJA (2024) 60, 133]
- II. [EPJA (2024) 60, 134]
- III. [EPJA (2024) 60, 155]
- IV. [EPJA (2024) 60, 233]

Previous PGCM study

The European Physical Journal

volume 60 · number 6 · june · 2024

EPJ A

Recognized by European Physical Society

Hadrons and Nuclei

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I. Technical aspects and uncertainties of ab initio PGCM calculations by A. Porro et al.

Many-Body truncation

- Comparison to PGCM-PT
- Only tested for low-lying etc.
- Correlated to SRG and generator coordinate

SRG dependence

- Strong centroid dependence ~ 10 %
- Dispersion relative error ~ 20 %
- Truncates both N and many-body

Generator coordinates choice

- Empirical knowledge, two coords r and β_2
- More systematic choice needed

Three-body treatment

- N³B approximation
- 1-2 % uncertainty in low-lying etc.
- Not tested for giant resonances

Hamiltonian parameters

- LEC dependence of g forces
- Few interactions compared
- Correlated to SRG
- Need for emulators (ITS)

Uncertainty Budget

Chiral Order

- Good overall convergence
- Centroid relative error ~ 2 %
- Dispersion relative error ~ 10 %

Harmonic Oscillator width

- Good overall convergence
- Centroid relative error ~ 2 %
- Dispersion relative error ~ 5 %

Finite Basis Size

- Good overall convergence
- Centroid relative error ~ 1 %
- Dispersion relative error ~ 2 %
- n_{max} not studied (14 safe for G3)

Summary of the uncertainty budget. In green are indicated the uncertainties that were thoroughly investigated. In yellow are those that could only be touched upon. Eventually, boxes in red correspond to those that could at best be estimated from previous but somewhat different works or not estimated at all

Società Italiana di Fisica

Springer

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Check for updates

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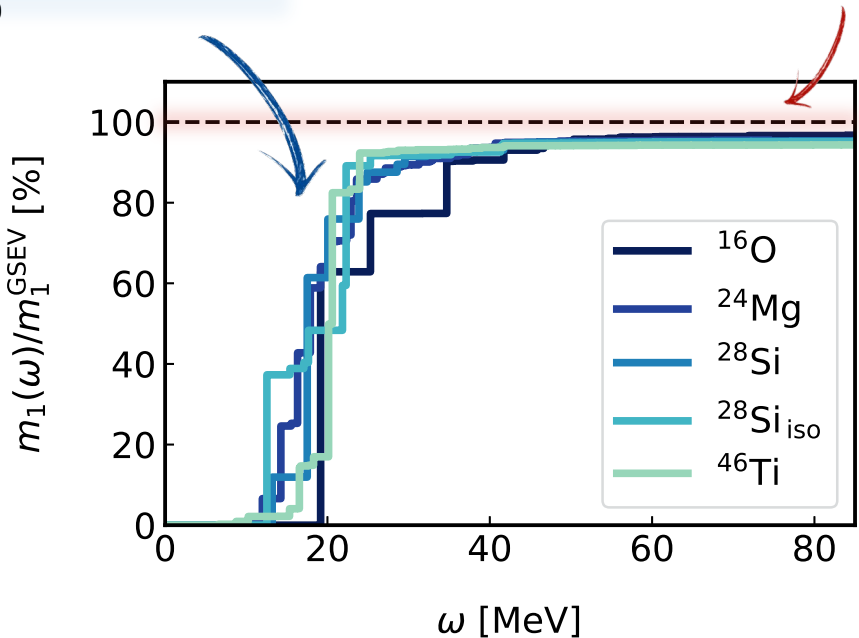
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6-7 % difference in PGCM

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Strategy in the IMSRG framework

8

Unitary transformation

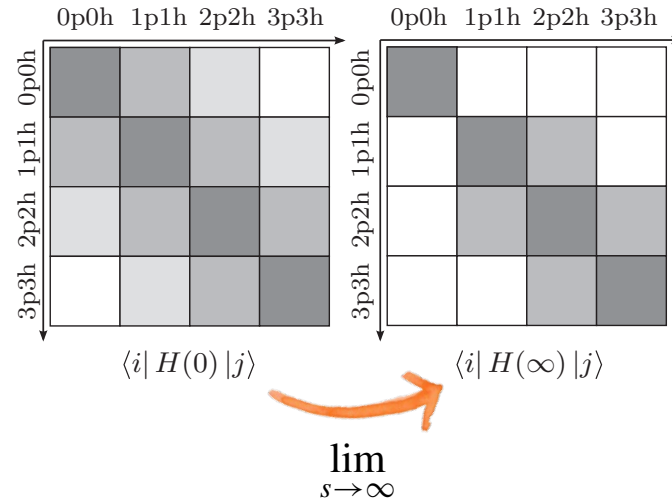
$$H(s) = U(s) H U^\dagger(s) \\ \equiv H^d(s) + H^{\text{od}} \rightarrow H^d(\infty)$$

Diagonal

Off-diagonal

$$E_{\text{gs}} = \lim_{s \rightarrow \infty} E_0(s) = \langle \Phi | H(s) | \Phi \rangle$$

Slater determinant



[Tsukiyama, Bogner and Schwenk, PRL, 2011]

[Hergert, Bogner, Morris, Schwenk, Tsukiyama, Phys. Rept., 2016]

Strategy in the IMSRG framework

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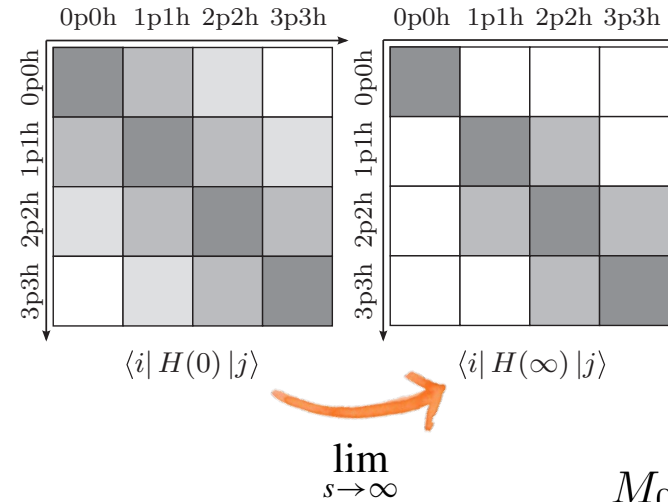
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Steps

- Start from the moment operator in the **HO basis**



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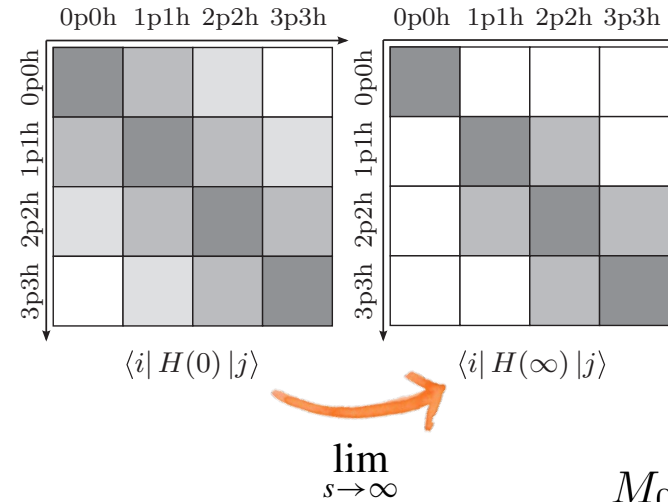
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J-scheme expressions of m_0 and m_1

[Lu and Johnson, PRC 97 (2018) 3, 034330]

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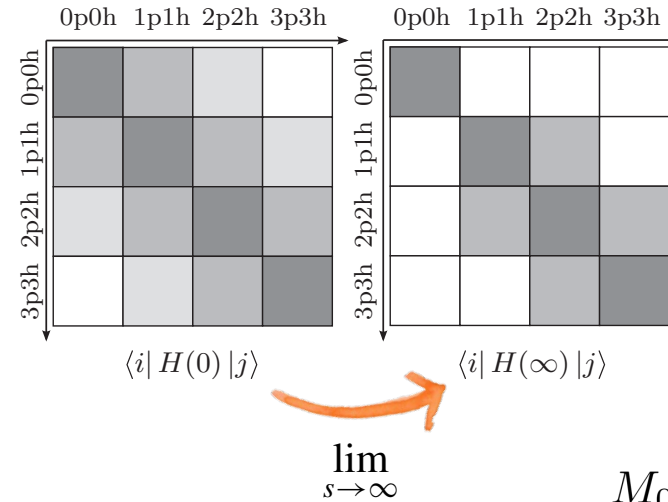
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Implemented within **imsrg++** code

[github.com/ragnarstroberg/imsrg]

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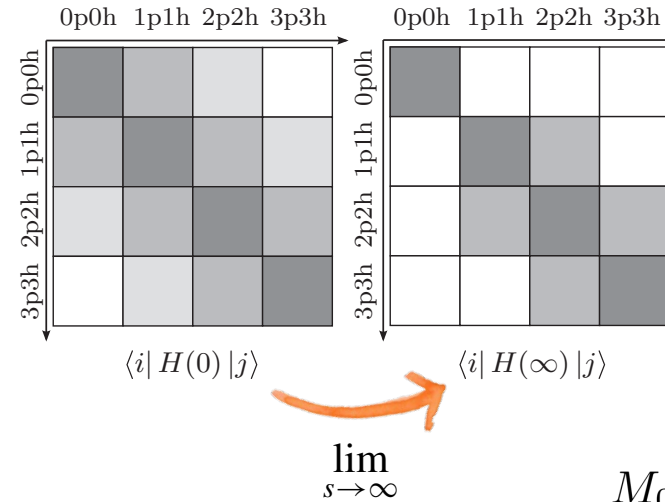
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Slater determinant

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- Start from the moment operator in the **HO basis**
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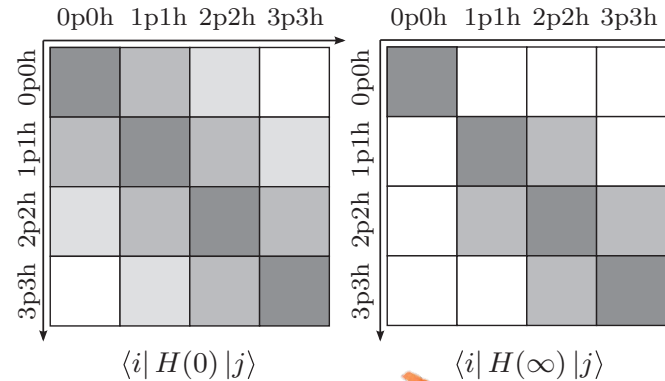
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- Evolve** moment operators using **Magnus**

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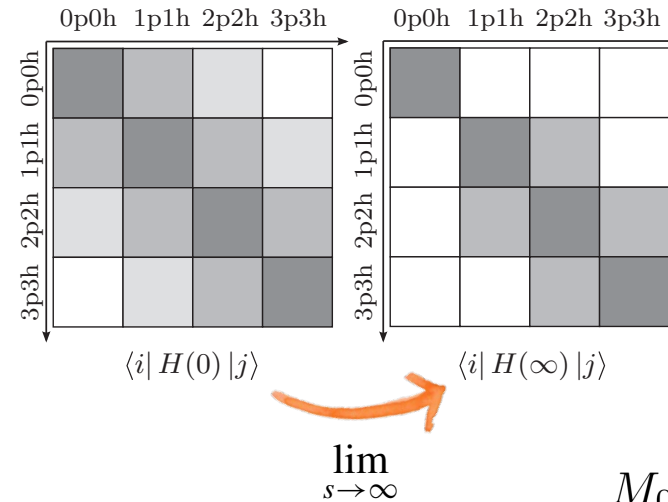
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Benchmarks

- HF value of **m₀** against **TDA**
- HF value of **m₁** against **RPA**



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Model-space convergence

Dipole response

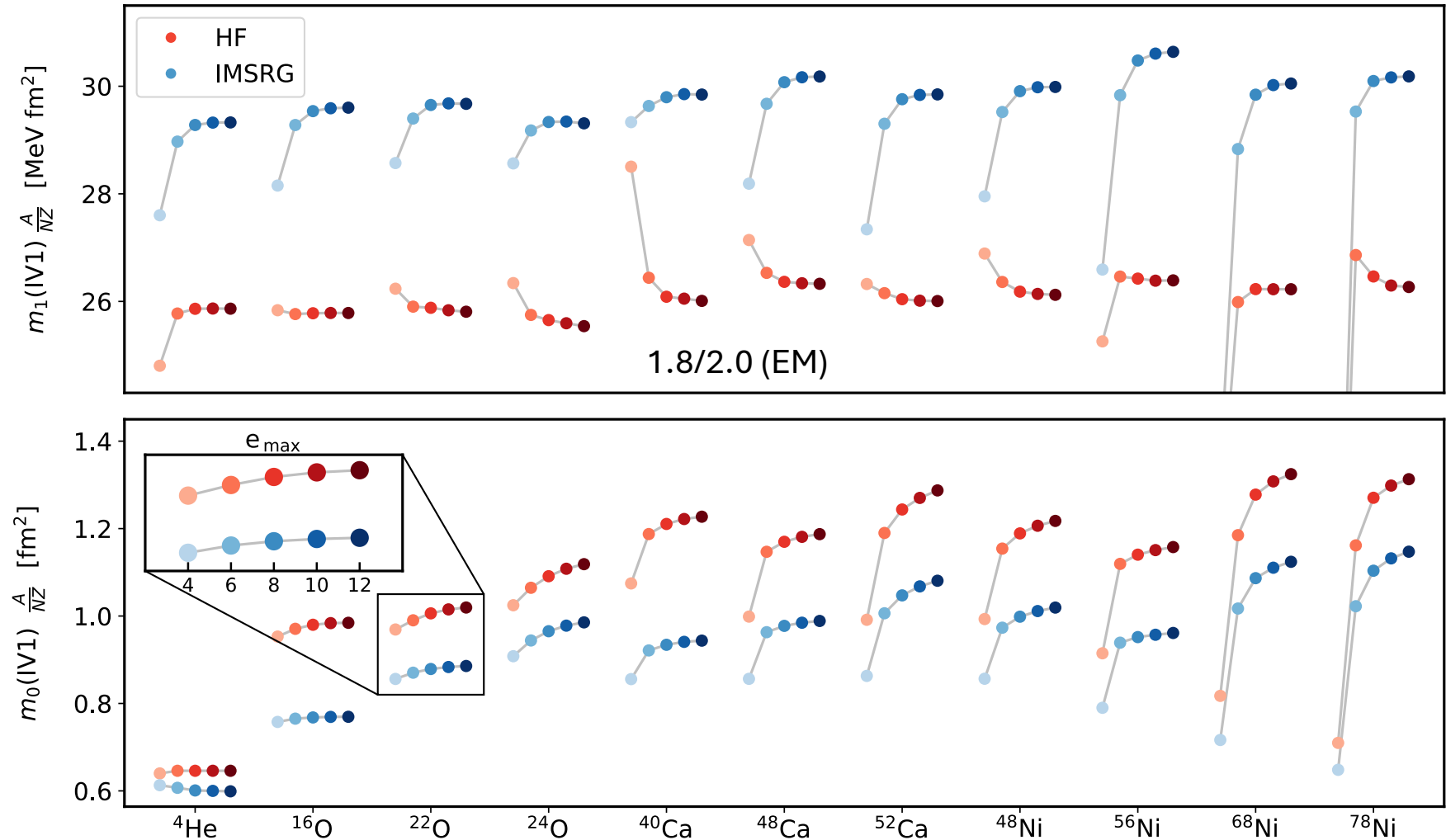
$$Q_{1\mu}^{\text{IV}} = \frac{N}{A} \sum_{i=1}^Z r_i Y_{1\mu}(\hat{r}_i) - \frac{Z}{A} \sum_{i=1}^N r_i Y_{1\mu}(\hat{r}_i)$$

Model-space convergence

9

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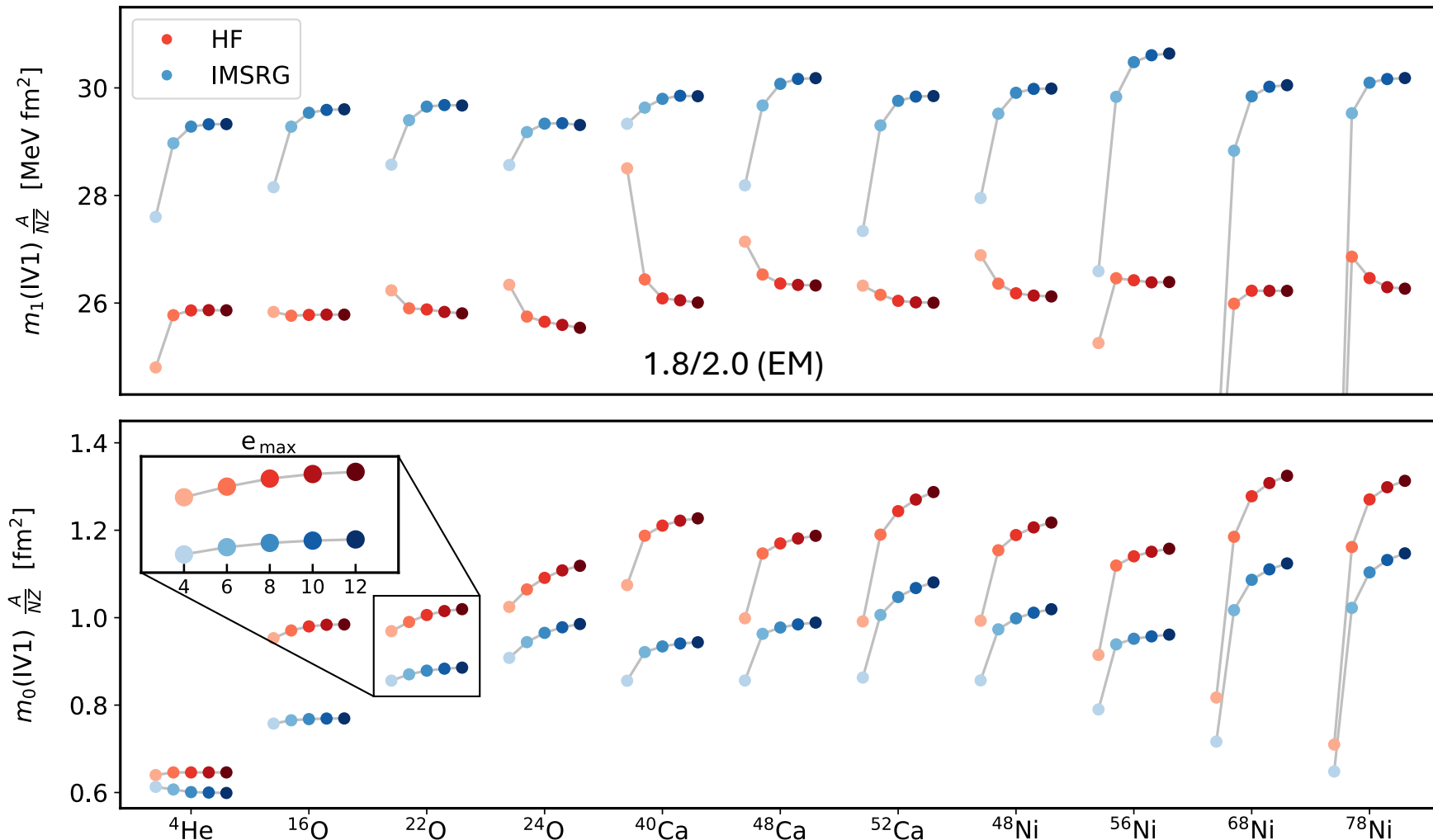
9

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- Large correlation impact
- Relative difference $\sim 0.2\%$
- Similar error for $\hbar\omega$ variations



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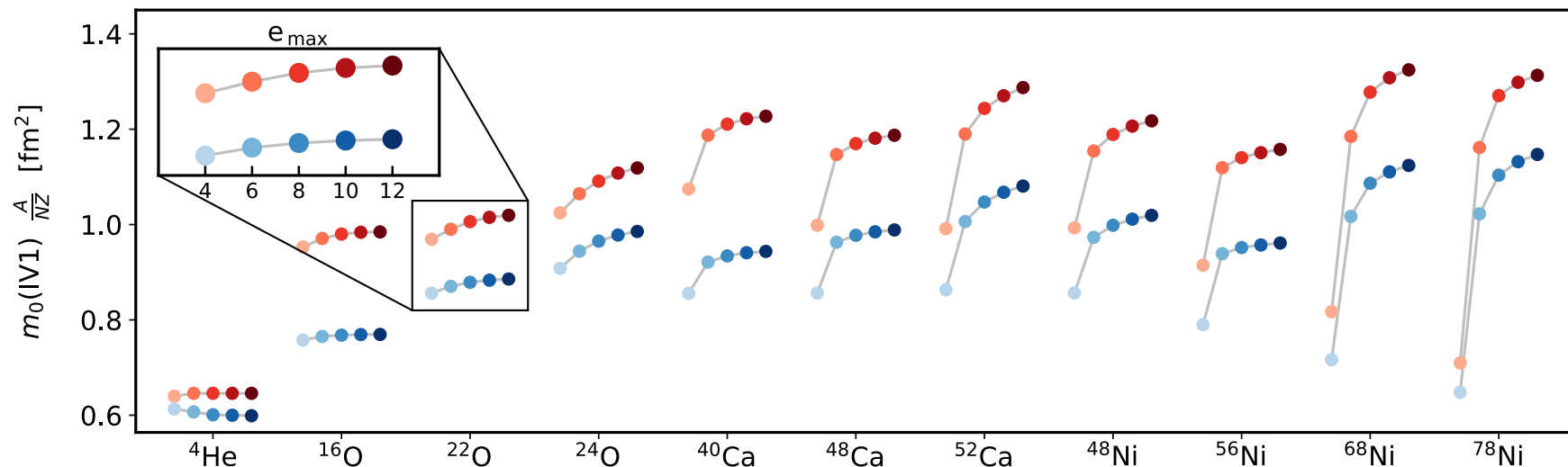
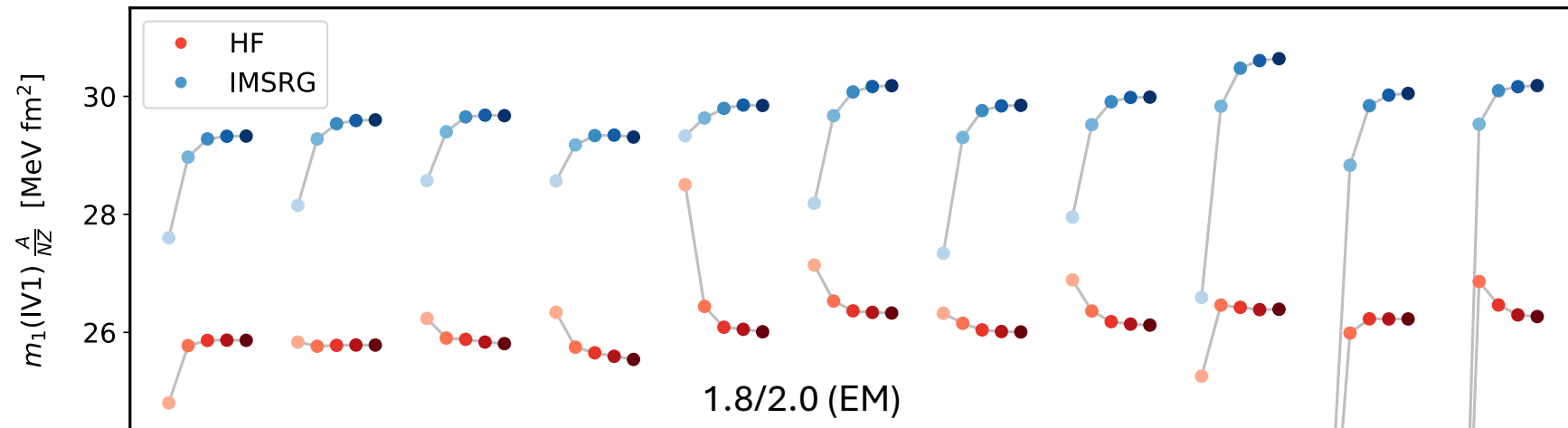
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$$M_0(Q_\lambda) \equiv \sum_{\mu} (-1)^{\mu} Q_{\lambda, -\mu} Q_{\lambda\mu}$$

- Slower convergence
- Relative difference $\sim 1.3\%$
- 2% error for $\hbar\omega$ variations





Introduction

- Physics case
- Existing ab initio methods

IMSRG multipole moments

- Moments of the strength
- IMSRG implementation
- Model-space convergence

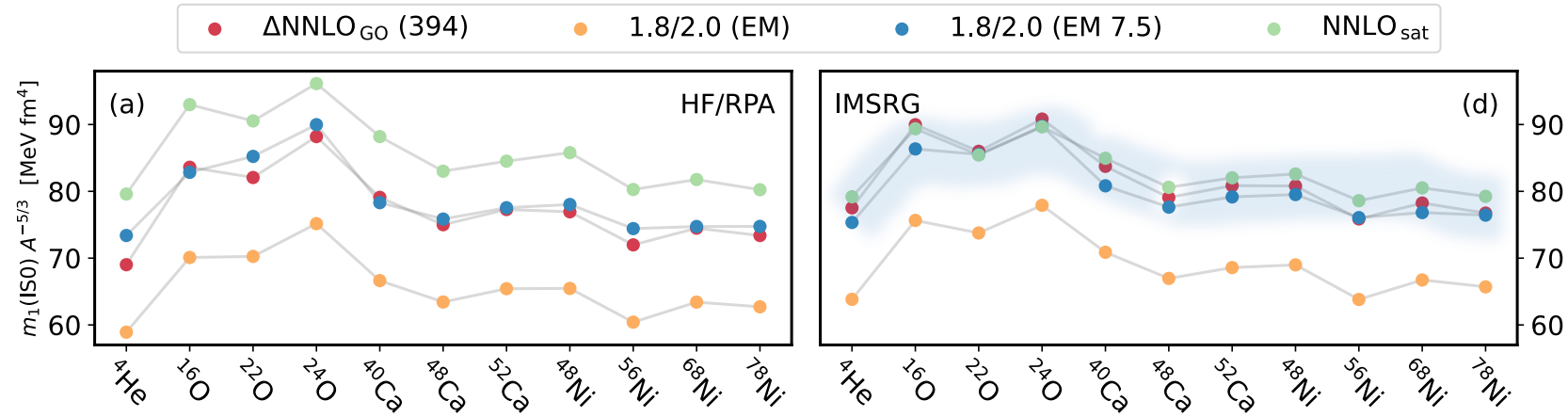
Numerical results

- Interaction sensitivity
- Comparison to experiment
- Comparison to sum rules

Challenges and opportunities

Interaction sensitivity

11

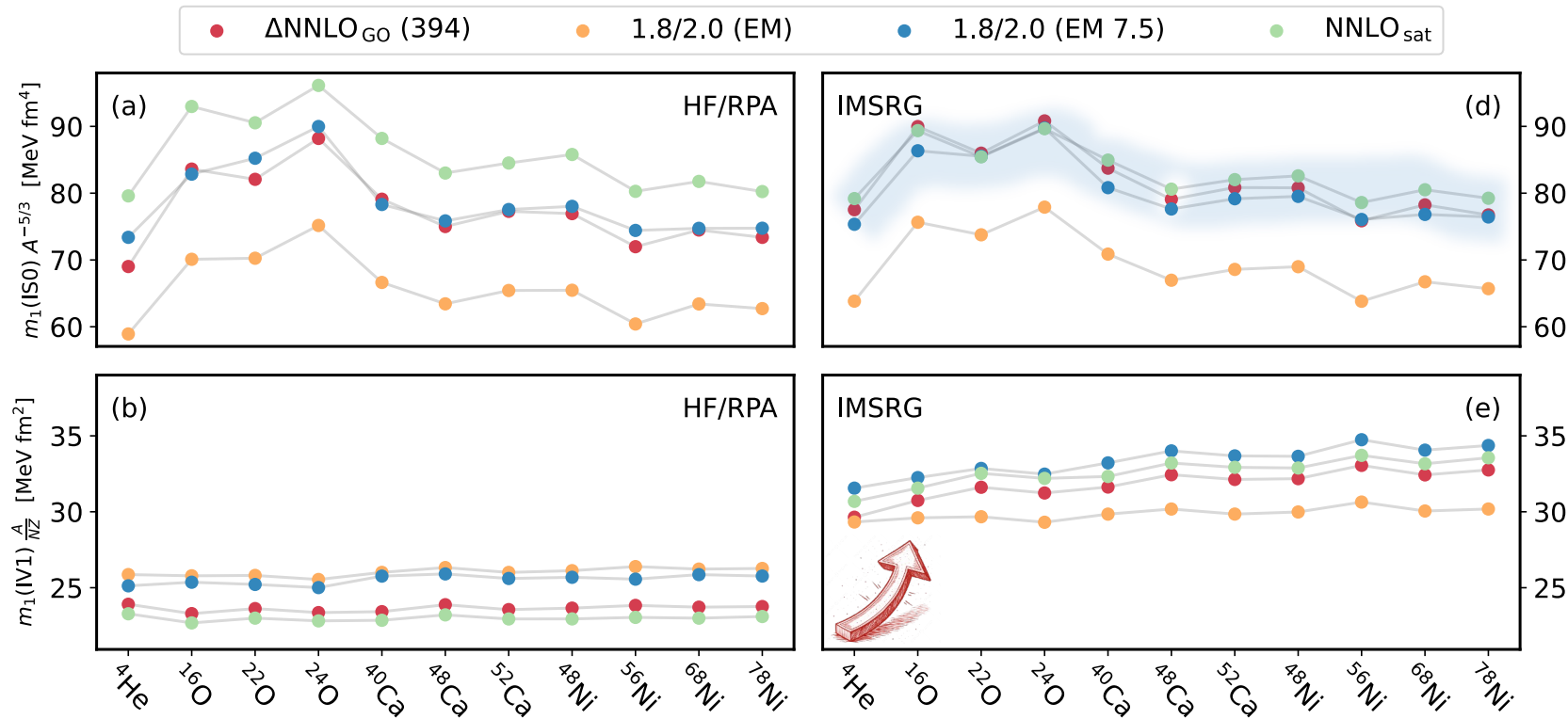


Monopole

- Reduced spread
- ~5% correlations effect

Interaction sensitivity

12



Monopole

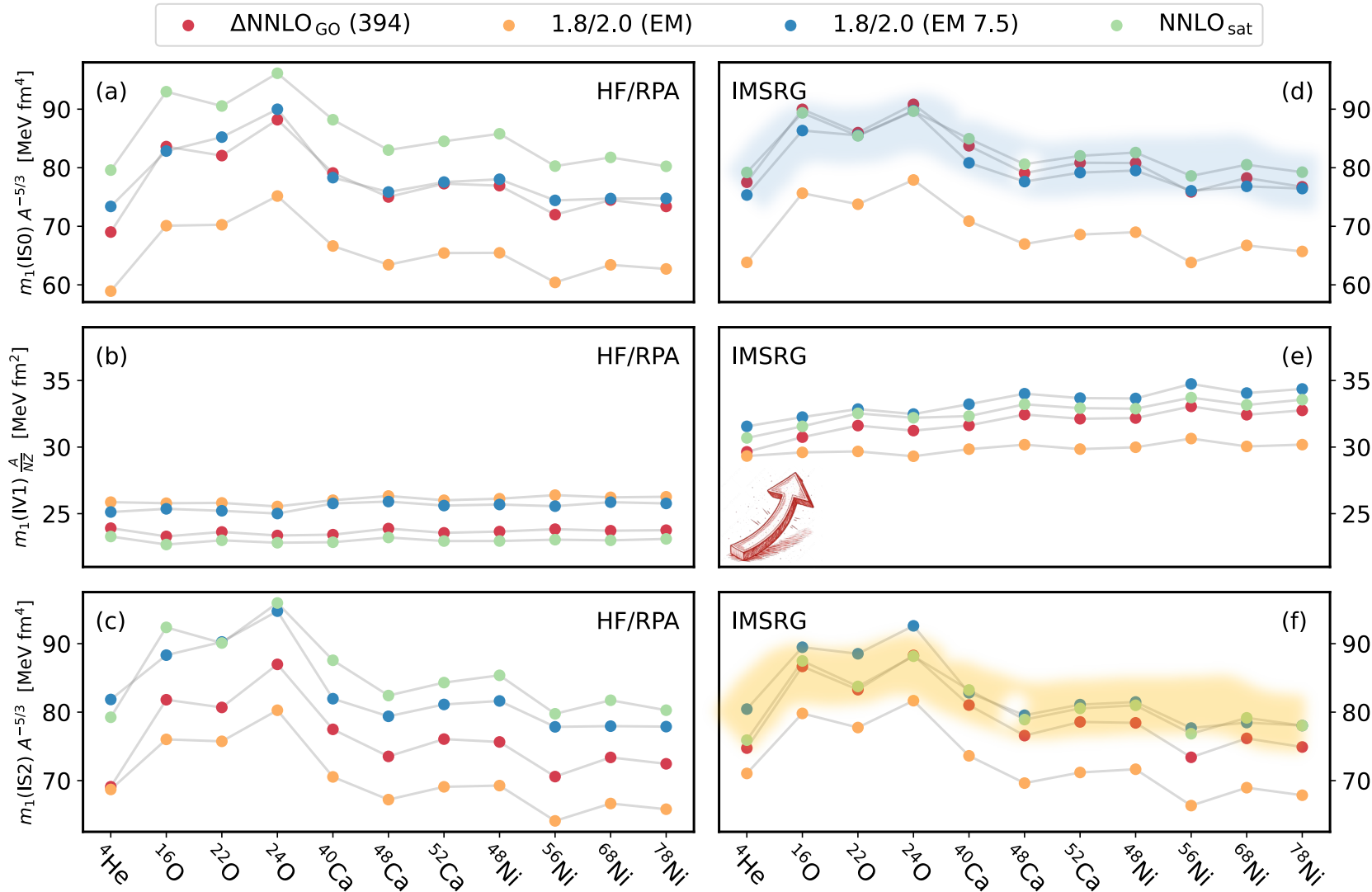
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Dipole

- Increase up to 40%
- 2% spread (w/o 1.8/2.0(EM))

Interaction sensitivity

13



Monopole

- Reduced spread
- ~5% correlations effect

Dipole

- Increase up to 40%
- 2% spread (w/o 1.8/2.0(EM))

Quadrupole

- Reduced spread
- ~5% correlations effect

Photoabsorption cross section

Comparison to exp only makes sense for integrated quantities

Photoabsorption cross section

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$$\int_0^\infty \sigma(E) dE = \frac{2\pi^2}{\hbar c} \langle \Psi_0 | [D, [H, D]] | \Psi_0 \rangle = \frac{16\pi^3}{9} \alpha m_1(\text{IV1})$$

$$\approx 60 \frac{NZ}{A} (1 + \kappa) \text{ mb} \cdot \text{MeV} \quad \text{TRK sum rule}$$


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 Pion-production threshold

[Ahrens et al., NPA, 1975]

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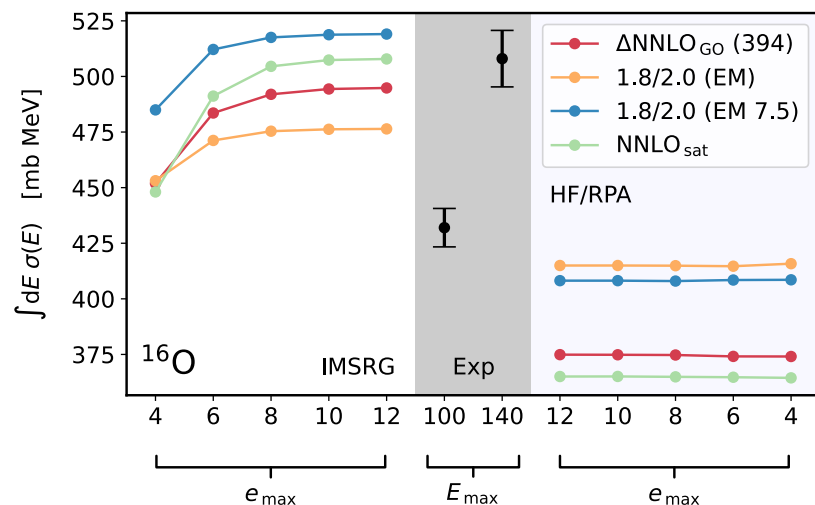
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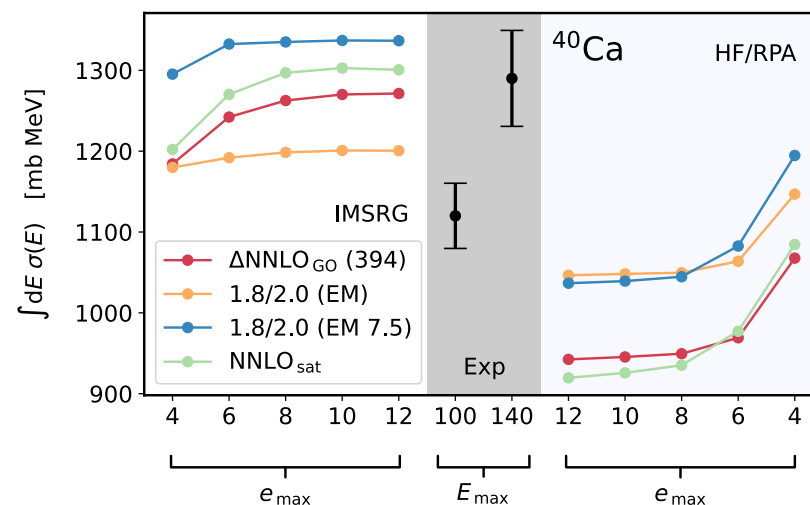
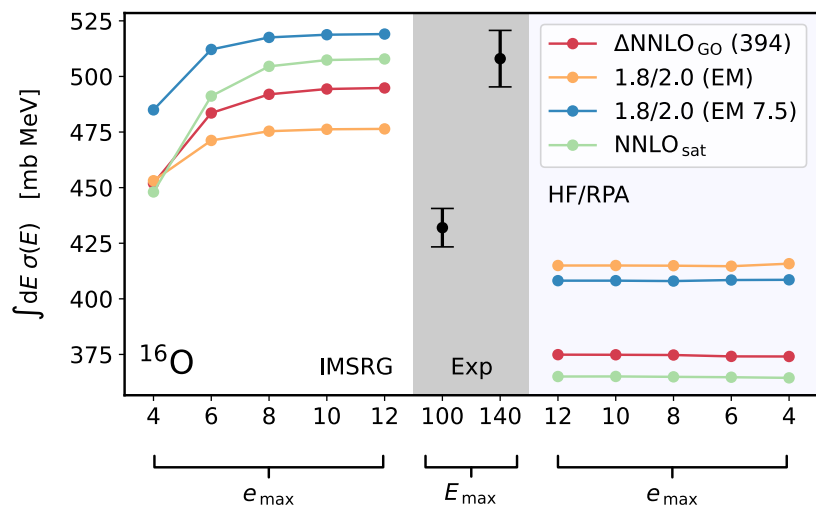
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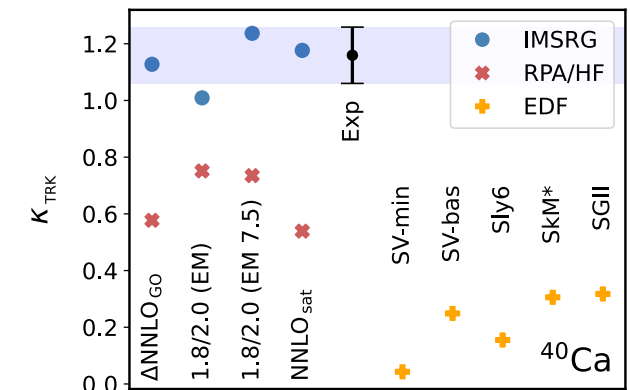
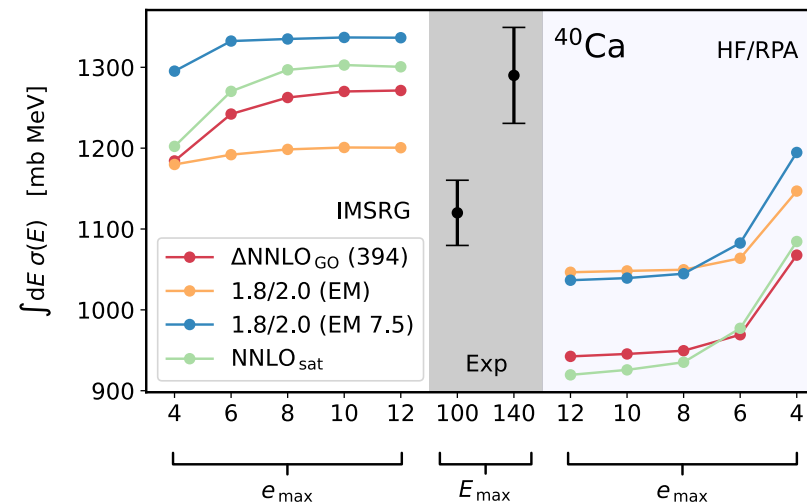
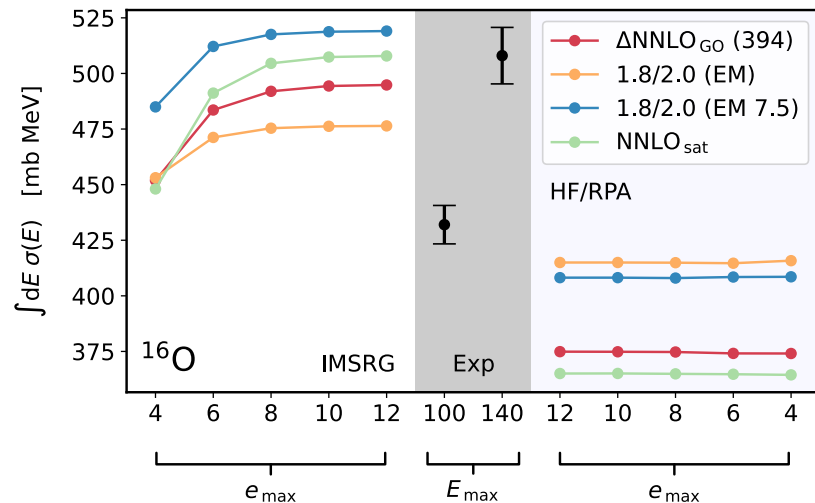
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Pion-production threshold



Comparison to EDF calculations

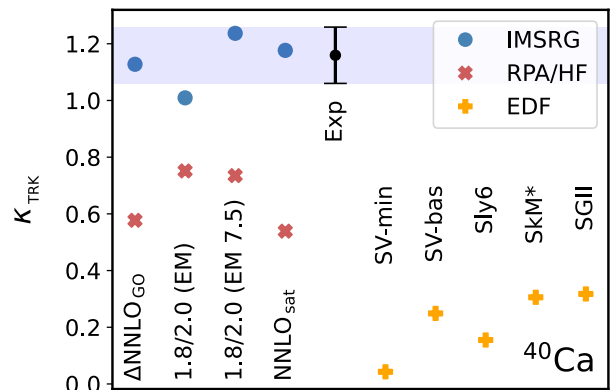
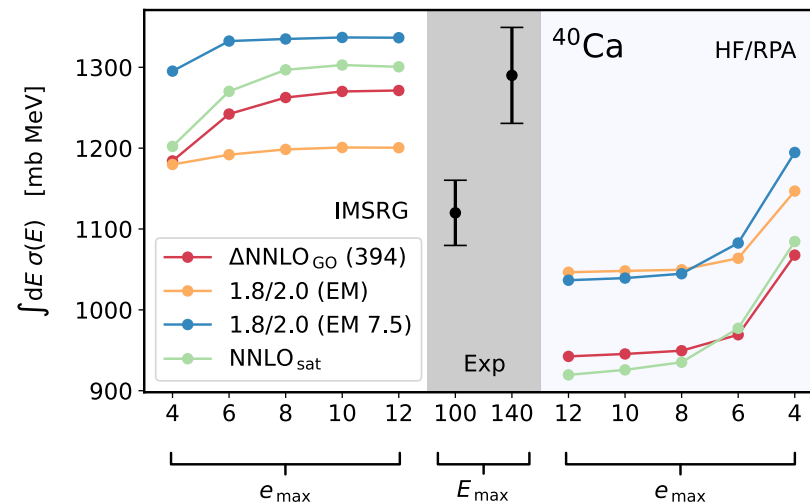
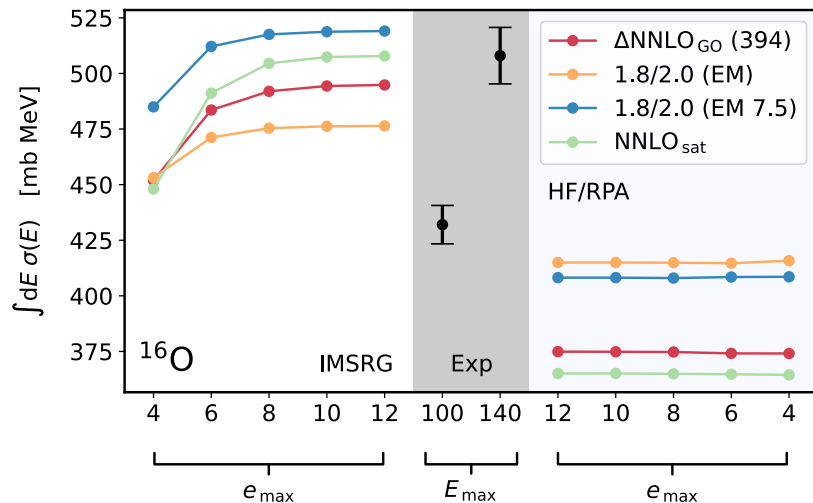
[Courtesy of P.-G. Reinhard]

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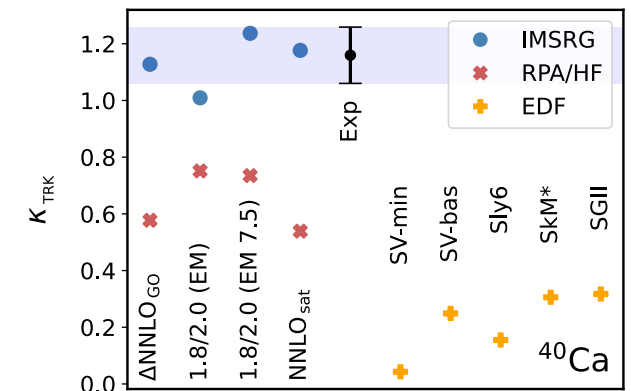
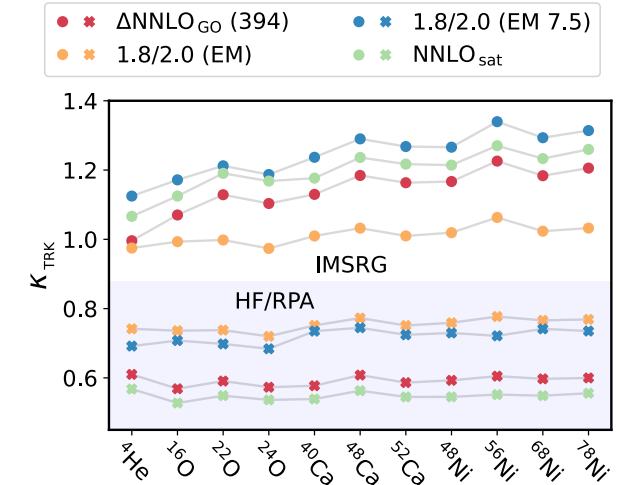
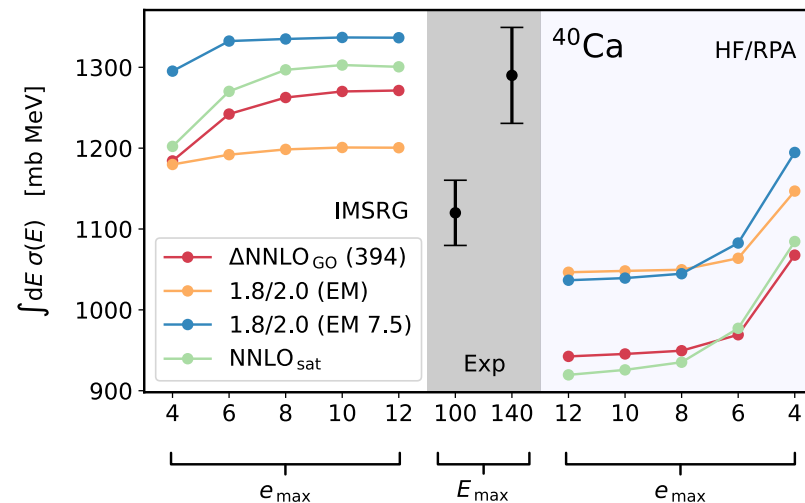
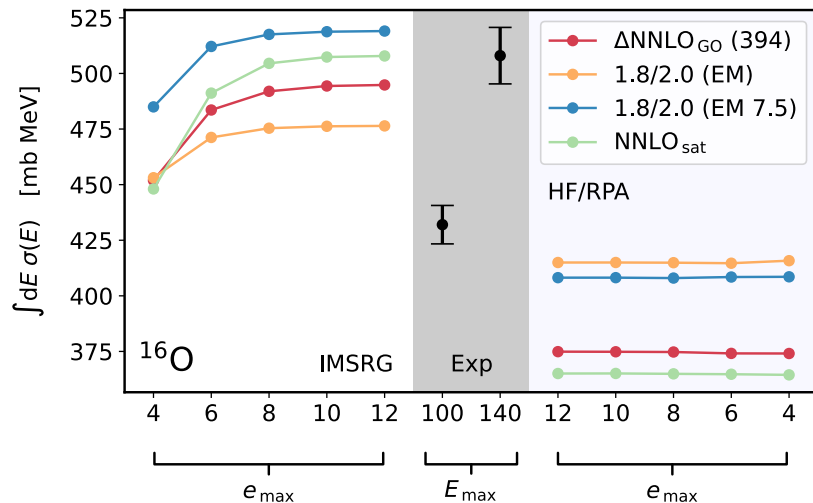
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14

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Both needed for consistent description

- Ground-state correlations
- Commutator expression generates 2-body currents


Comparison to EDF calculations

[Courtesy of P.-G. Reinhard]

Comparison to sum rules

Sum rules extensively studied in the past

$$\begin{aligned} \text{EWSR}(Q_\lambda(\vec{r})) &= \frac{1}{2} \langle \Psi_0 | [Q_\lambda^\dagger(\vec{r}), [T + V(\vec{r}), Q_\lambda(\vec{r})]] | \Psi_0 \rangle \\ &= \frac{1}{2} \langle \Psi_0 | [Q_\lambda^\dagger(\vec{r}), [T, Q_\lambda(\vec{r})]] | \Psi_0 \rangle \end{aligned}$$

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$$m_1(Q_\lambda) \stackrel{?}{=} \text{EWSR}(Q_\lambda)$$

Comparison to sum rules

15

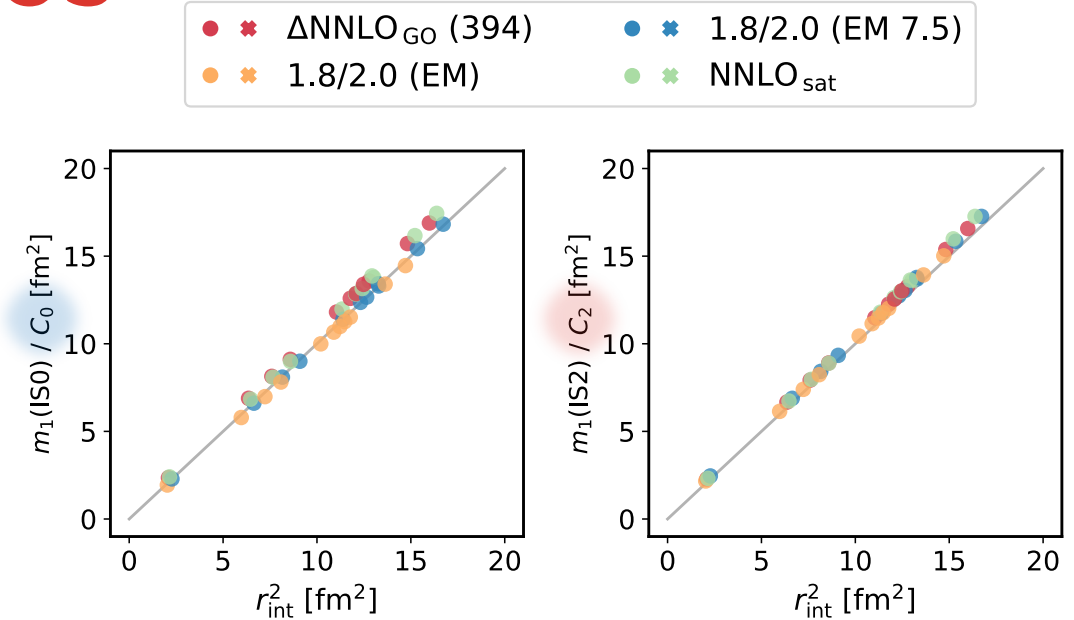
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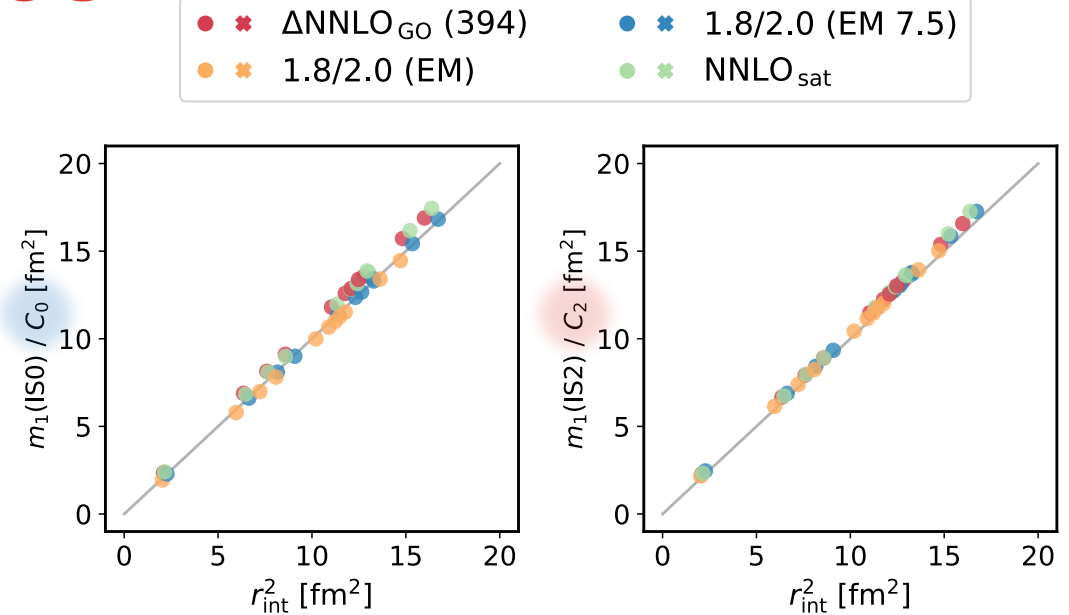
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True if continuity Eq is only 1B

$$\vec{\nabla} \cdot \vec{j}(\vec{r}) + \frac{i}{\hbar} [H, \rho(\vec{r})] = 0$$

if V local

$$\vec{\nabla} \cdot \vec{j}_{[1]}(\vec{r}) + \frac{i}{\hbar} [T, \rho_{[1]}(\vec{r})] = 0$$



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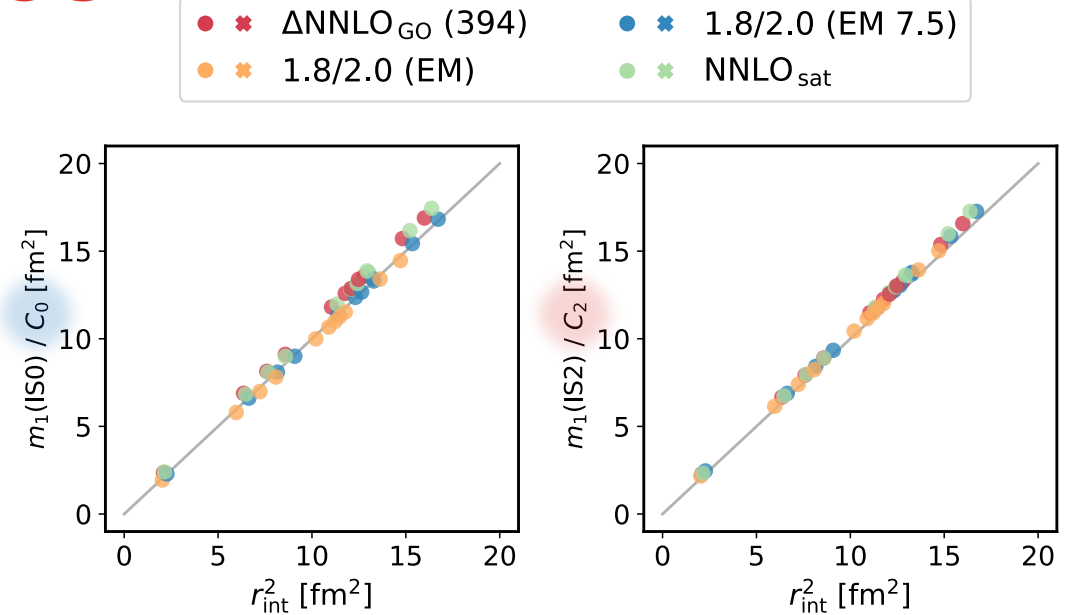
$$m_1(Q_\lambda) \stackrel{?}{=} \text{EWSR}(Q_\lambda)$$

True if continuity Eq is only 1B

$$\vec{\nabla} \cdot \vec{j}(\vec{r}) + \frac{i}{\hbar} [H, \rho(\vec{r})] = 0$$

if V local

$$\vec{\nabla} \cdot \vec{j}_{[1]}(\vec{r}) + \frac{i}{\hbar} [T, \rho_{[1]}(\vec{r})] = 0$$



But many-body currents can be there !

$$\vec{\nabla} \cdot \vec{j}_{[\nu]}(\vec{r}) + \frac{i}{\hbar} \sum_{\mu=1}^{\nu} [H_{[\nu+1-\mu]}, \rho_{[\mu]}(\vec{r})]_{[\nu]} = 0$$

Comparison to sum rules

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Sum rules extensively studied in the past

$$\begin{aligned} \text{EWSR}(Q_\lambda(\vec{r})) &= \frac{1}{2} \langle \Psi_0 | [Q_\lambda^\dagger(\vec{r}), [T + V(\vec{r}), Q_\lambda(\vec{r})]] | \Psi_0 \rangle \\ &= \frac{1}{2} \langle \Psi_0 | [Q_\lambda^\dagger(\vec{r}), [T, Q_\lambda(\vec{r})]] | \Psi_0 \rangle \quad \text{if } V \text{ local} \end{aligned}$$

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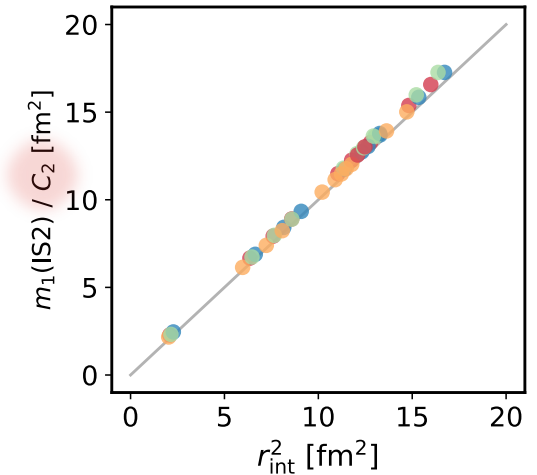
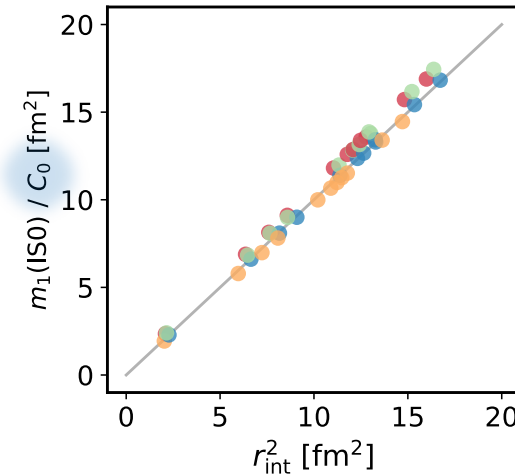
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E.g. leading order exchange currents (Siegert's limit)

$$e\vec{J} = i[V_\pi(q), \vec{D}] \propto \text{diagram a} + \text{diagram b}$$

[Christillin, Physics Reports, 1990]

Comparison to sum rules

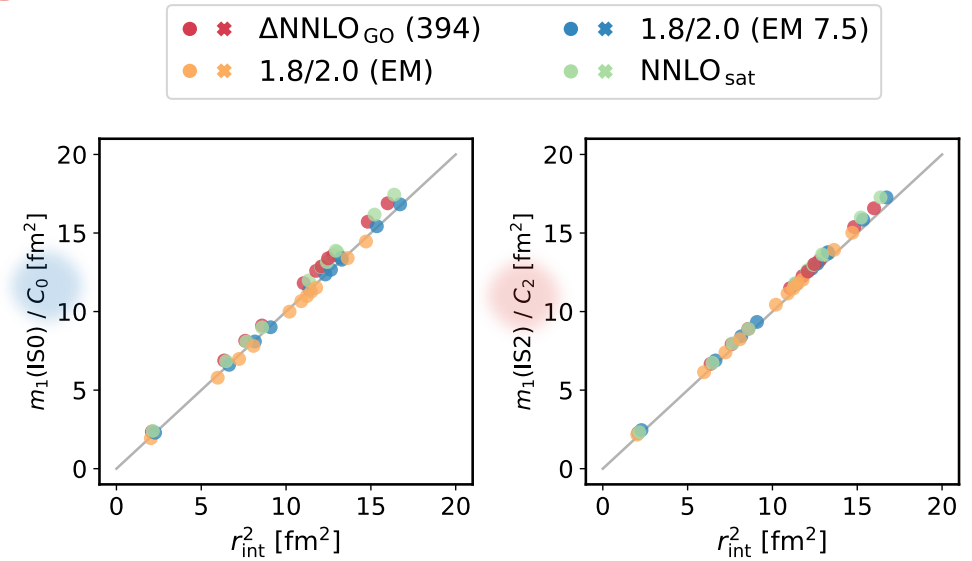
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Let's look more closely



Comparison to sum rules

16

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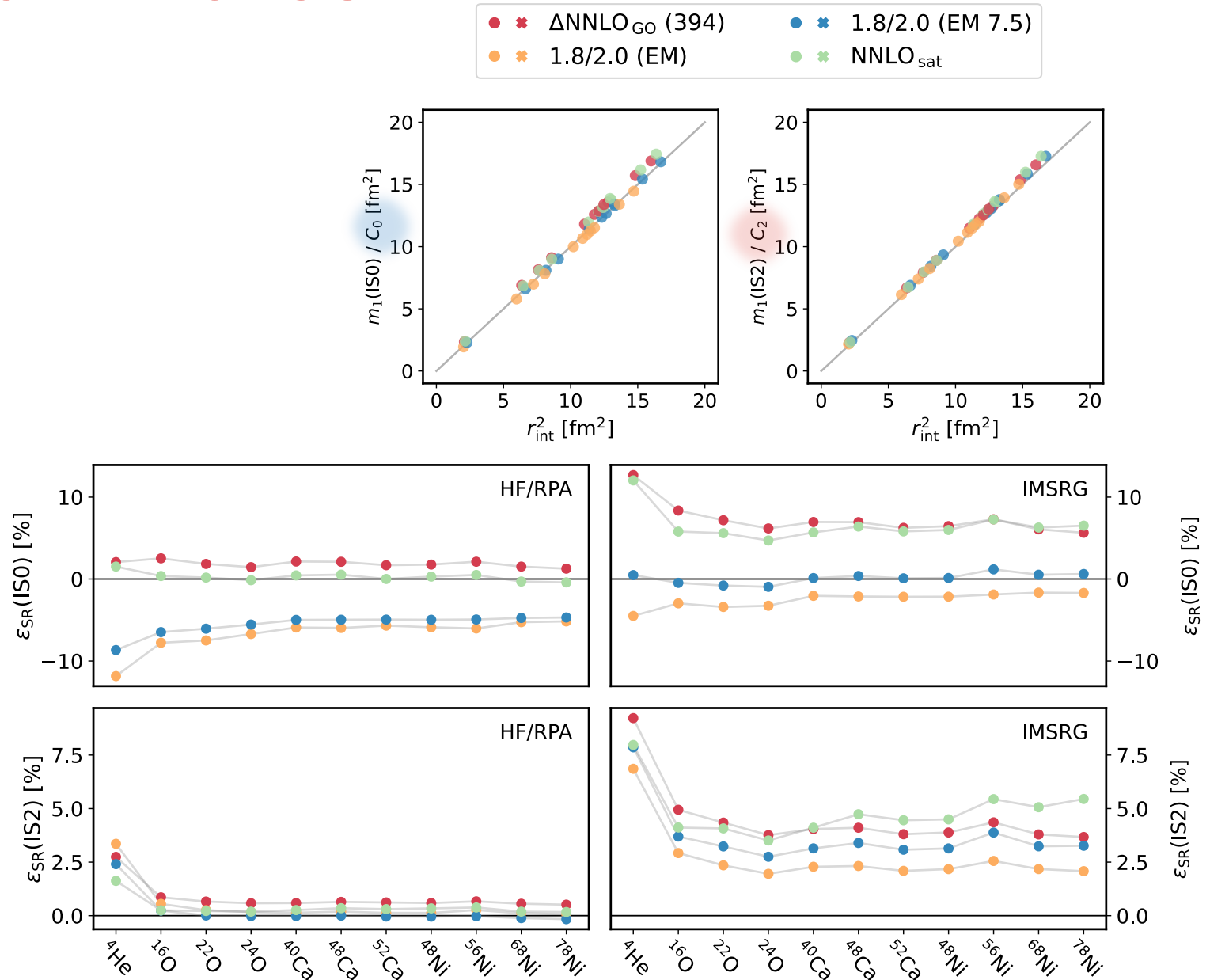
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Relative difference

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Comparison to sum rules

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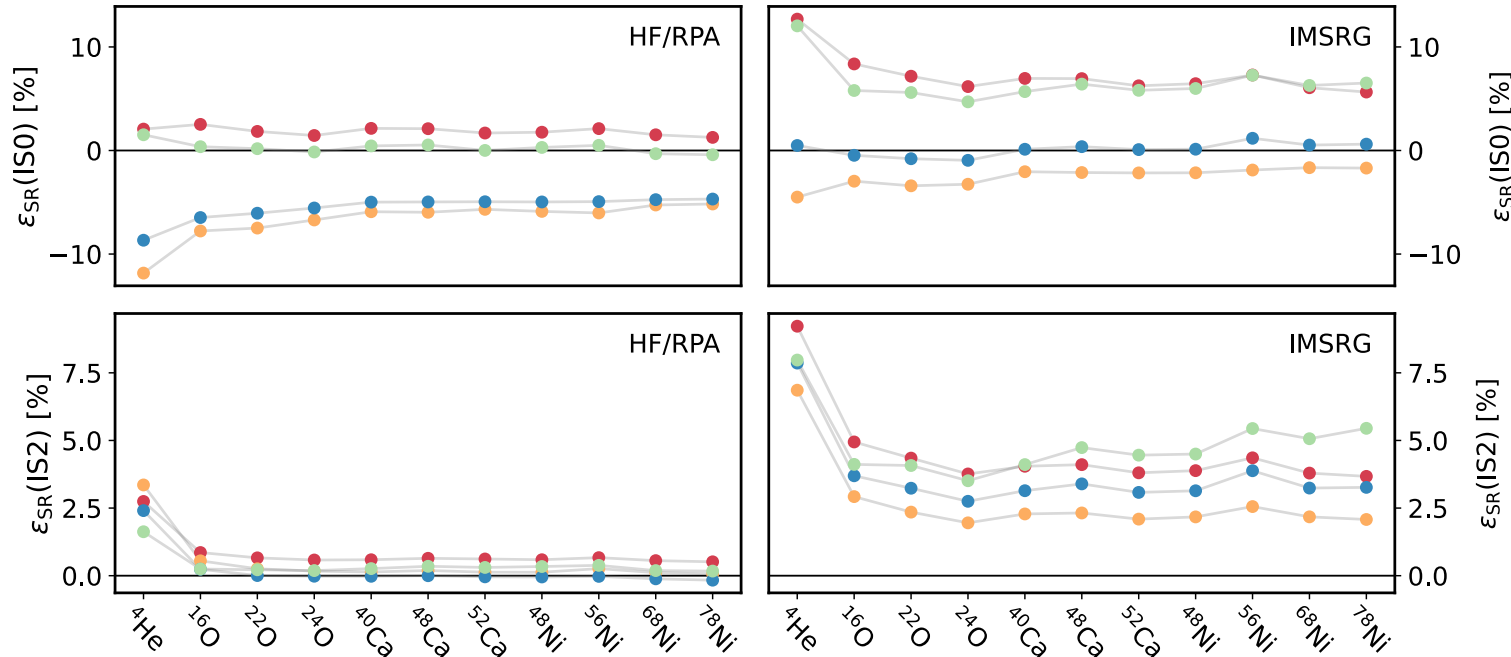
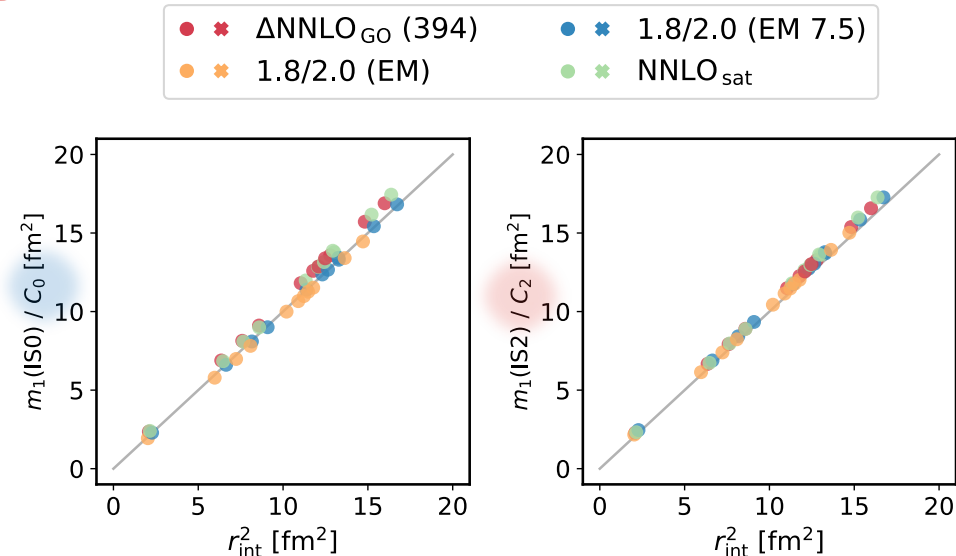
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Differences from EWSR

- Nonlocalities
- Two-body currents



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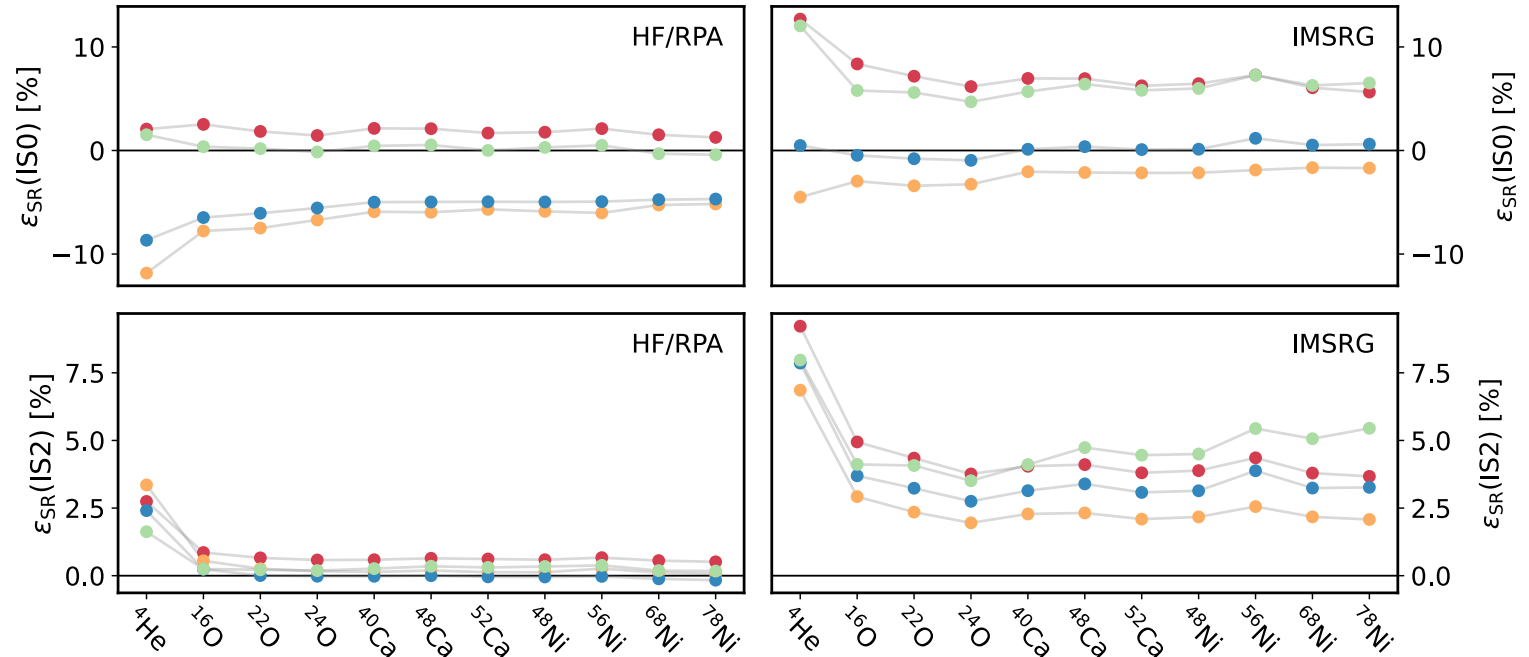
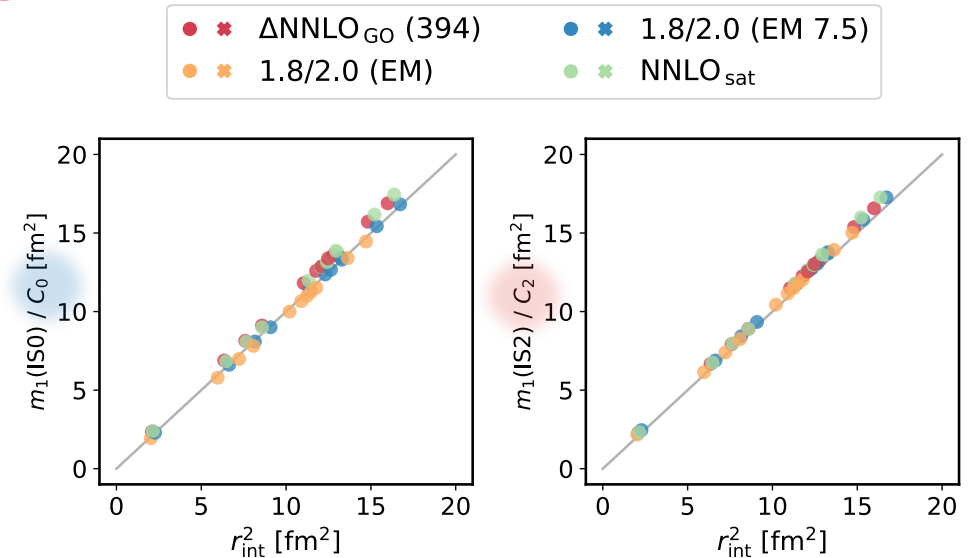
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Differences from EWSR

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m1 from moments is better !





Introduction

- Physics case
- Existing ab initio methods

IMSRG multipole moments

- Moments of the strength
- IMSRG implementation
- Model-space convergence

Numerical results

- Interaction sensitivity
- Comparison to experiment
- Comparison to sum rules

Challenges and opportunities

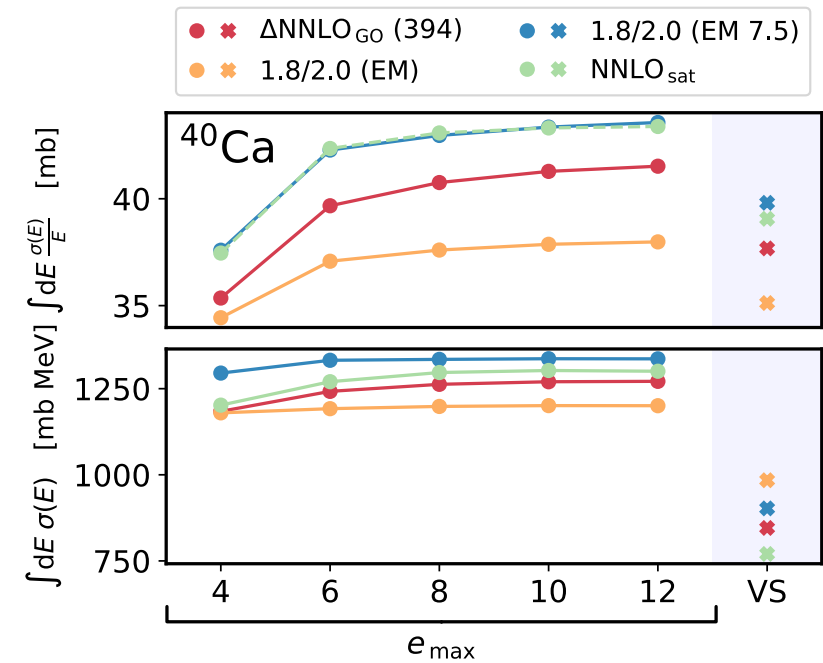
Challenges and opportunities

18

Going open-shell

Comparison to VS calculation for ^{40}Ca with ^{28}Si core

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Challenges and opportunities

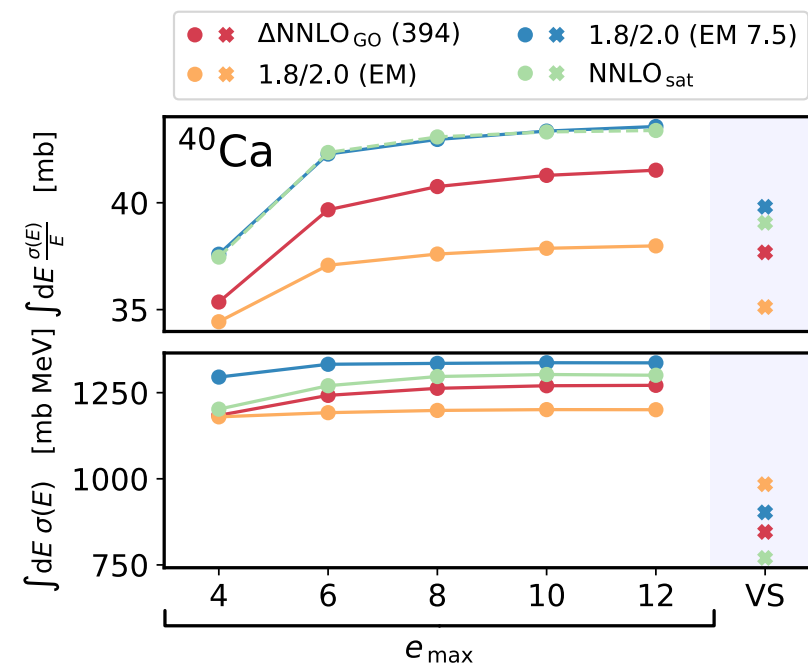
Going open-shell

Comparison to **VS calculation** for ^{40}Ca with ^{28}Si core

- Large **uncertainties** for m_1 and m_0
- Two-step decoupling
- Is the core well described ? (deformation)

Other possibilities within the IMSRG

- **Multi-reference** formulation
- **Symmetry-breaking** calculations



Challenges and opportunities

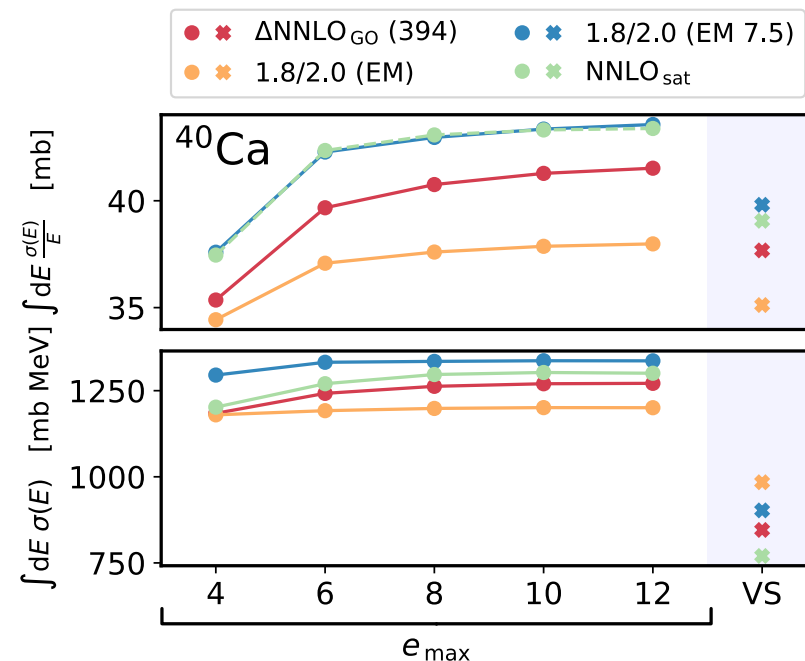
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Benchmarks and uncertainty quantification

No limitation on the many-body method (and operators) of choice

- Exact treatment of excited states
- Can **benchmark** response calculations going through **exc states EOM, LIT** etc. (can send matrix elements)

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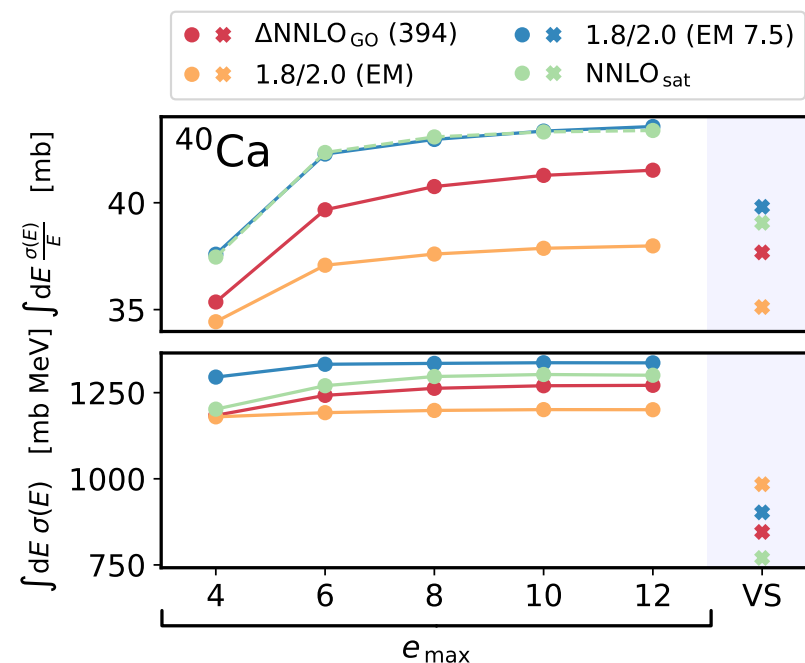
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Systematic studies of **H** properties

Thank you for the attention



Robert Roth
Achim Schwenk
Alexander Tichai

Summary

- Moment method useful for **uncertainty quantification** /benchmark of exc states
- Can be used for systematic studies of **H properties**
- Small finite-basis uncertainty
- Correlation effect ~5% in monopole and quadrupole
- Qualitative change (~40%) for dipole
- Better agreement with **data** and **smaller interaction spread**
- Comparison to EWSR diagnostic of **non-localities** and **two-body currents**



Thomas Duguet
Jean-Paul Ebran
Mikael Frosini
Vittorio Somà



Francesca Bonaiti



Sonia Bacca