Mirroring nuclei at the unitary limit

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Next generation ab initio nuclear theory

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- G.B. King - *LANL & Los Alamos, (USA)*
- R. Wiringa - *ANL & Argonne, (USA)*

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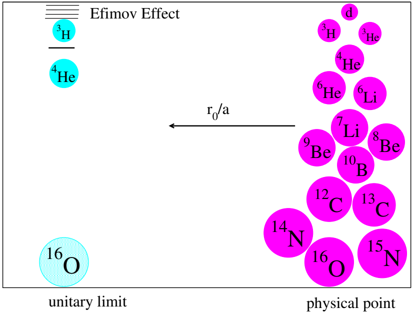

The trip

- The trip follows a particular path, it is determined by the ratio r_0/a
- a is the scattering length, it characterizes each point of the path
- r_0 is a characteristic length, constant along the path. It is different for each nucleus, $r_0^{(N)}$. Its determination is part of this talk.
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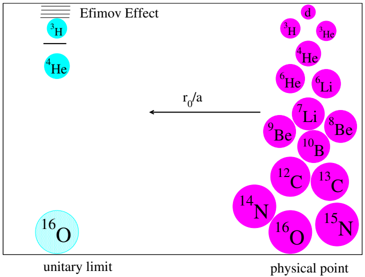


The two-body characteristic length $r_0^{(2)}$

- The path from the physical point to the unitary point is characterized by the two-pole S-matrix representing one shallow state, virtual or bound

$$S(k) = \frac{k + i/a_B}{k - i/a_B} \frac{k + i/r_B}{k - i/r_B}$$

- The energy pole is described by the energy length a_B

$$1/\kappa = a_B \longrightarrow E_2 = -\hbar^2\kappa^2/m = -\hbar^2/ma_B^2$$

- E_2 is a bound or virtual state when $a_B > 0$ or $a_B < 0$
- the second pole is described by the length $r_B = a - a_B$
- For example, for the deuteron at the physical point
  $a = 5.4\,\text{fm},\ a_B = 4.3\,\text{fm}$
  $r_B = a - a_B = 1.1\,\text{fm}$

The two-body characteristic length $r_0^{(2)}$

- The two-pole S-matrix

$$S(k) = e^{2i\delta} = \frac{e^{i\delta}}{e^{-i\delta}} = \frac{k\cot\delta + ik}{k\cot\delta - ik} = \frac{k + i/a_B}{k - i/a_B}\frac{k + i/r_B}{k - i/r_B}$$

  is equivalent to the second-order effective range expansion
  $k\cot\delta_0 = -1/a + r_e k^2/2$

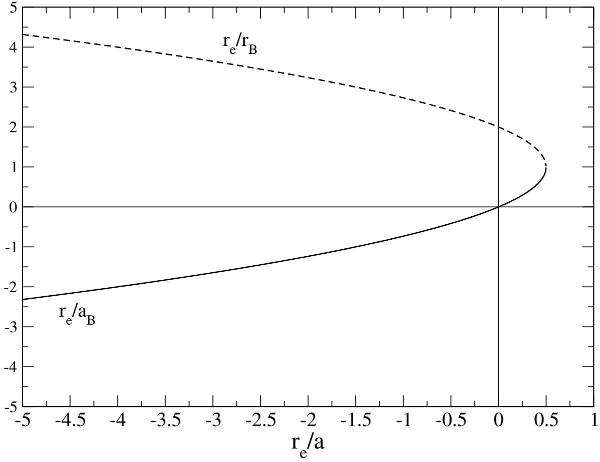
- The two-poles are in the immaginary axes $k = i\kappa$, verifying the pole equation
  $\kappa = 1/a + r_e\kappa^2/2$

- they are:

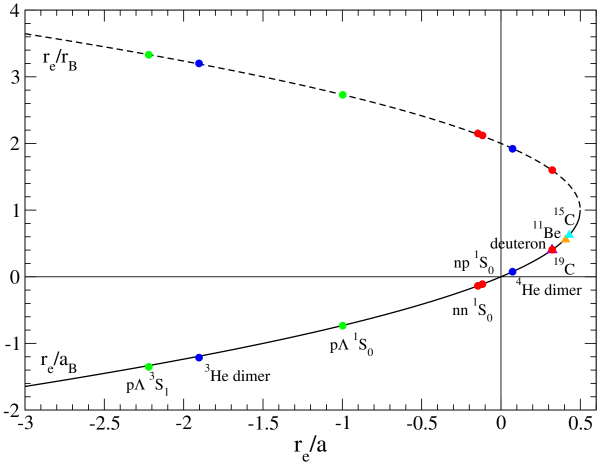
  $\frac{r_e}{a_B} = 1 - \sqrt{1 - 2r_e/a}$ $\quad\quad \rightarrow r_e/a < 0.5$

  $\frac{r_e}{r_B} = 1 + \sqrt{1 - 2r_e/a}$

The two poles form the universal window

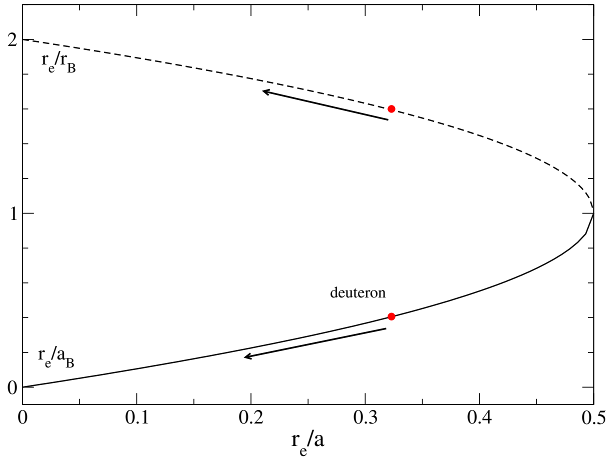


Physical systems inside the universal window



Systems are put on the figure by their experimental data

Our trip: $r_0^{(2)} \equiv$constant $\Longrightarrow$ $r_B \equiv$constant



Effective description (scale invariance)

- The S-matrix

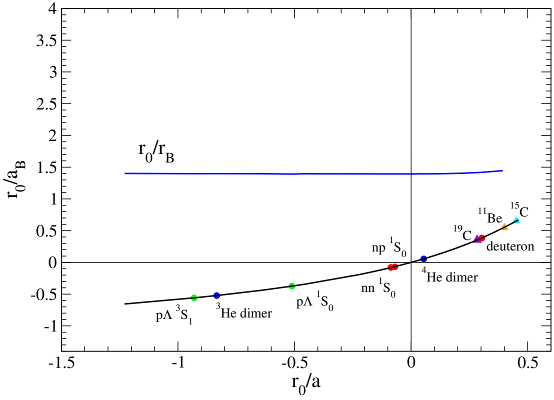
$$S(k) = \frac{k + i/a_B}{k - i/a_B}\frac{k + i/r_B}{k - i/r_B}$$

is exactly represented by the Eckart potential:

$$V(r) = -2\frac{\hbar^2}{mr_0^2}\frac{\beta e^{-r/r_0}}{(1 + \beta e^{-r/r_0})^2}$$

$$\begin{cases} a = 4r_0\frac{\beta}{\beta-1} \\ a_B = 2r_0\frac{\beta+1}{\beta-1} \end{cases} \qquad \begin{cases} r_e = 2r_0\frac{\beta+1}{\beta} \\ r_B = 2r_0 \to \text{the second pole!} \end{cases}$$

The universal window in terms of the Gaussian parameters



$$V(r) = -\frac{\hbar^2}{mr_0^2}\beta e^{-(r/r_0)^2}$$

Motivation of the trip

- The nuclear system, as well many other systems, are inside the universal window
- The universal window is characterized by scale invariance
- Scale invariance is not a symmetry of the underlying theory but appears for particular values of the interaction parameters

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- In this workshop, Next Generation Ab initio Nuclear Theory, we can consider the following two main ingredients
  - The microscopic theory for the nuclear interaction
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- In this workshop, Next Generation Ab initio Nuclear Theory, we can consider the following two main ingredients
  - The microscopic theory for the nuclear interaction
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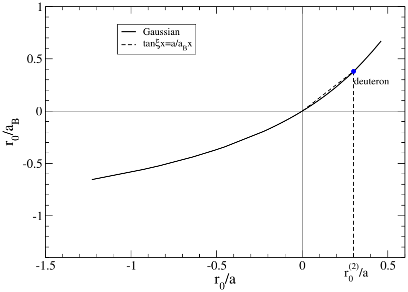
- The trip will help to see how scale invariance manifests and, hopefully, how to incorporate it in the effective description of nuclei.

The two-body scale $r_0^{(2)}$ → assigning dimensions → the deuteron trip



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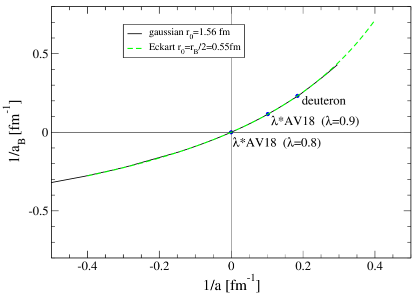

$\tan\xi = \frac{a}{a_B}$ $\qquad$ $r_0^{(2)} = 1.56\,\mathrm{fm}$ $\qquad$ The $T=0$ np plot ($r_0 = 1.56\,\mathrm{fm}$)

$$V_{r_0}(r) = -\frac{\hbar^2}{m r_0^2}\beta e^{-r^2/r_0^2}$$

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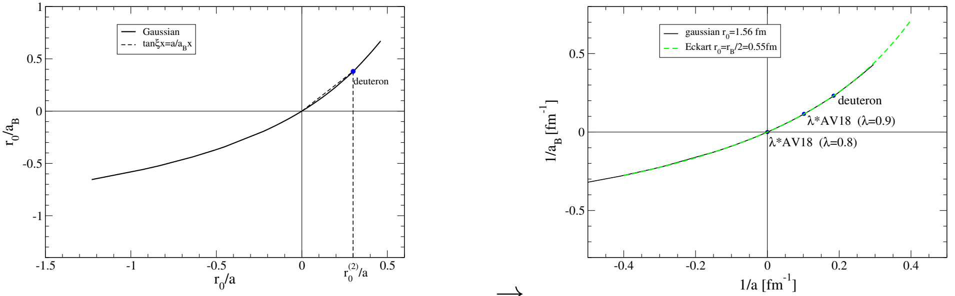

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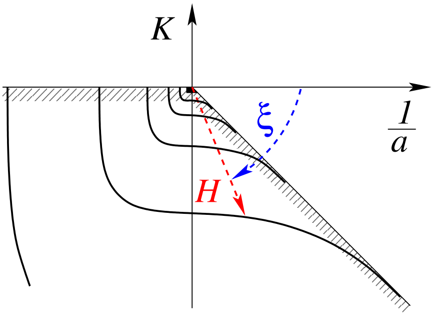
The $T = 0$ np plot ($r_0 = 1.56\,\text{fm}$)

$$V_{r_0}(r) = -\frac{\hbar^2}{m r_0^2}\beta e^{-r^2/r_0^2}$$

The *np* data, represented by AV18 have been moved from the physical point to the unitary point

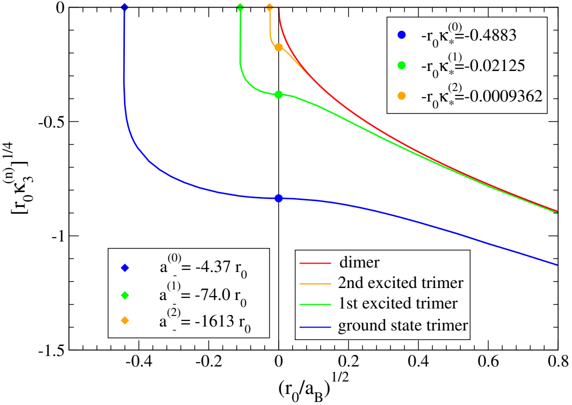
The three-body scale $r_0^{(3)}$ and K_*, the three-body parameter



The Efimov plot: The three-body sector is scale invariant, K_*, is the binding momentum at the unitary limit. It fixes the branch in which the system is located

The three-body scale $r_0^{(3)}$ using the gaussian characterization

The case of three bosons: $V = \sum_{ij} V_0 e^{-(r_{ij}/r_0)^2}$

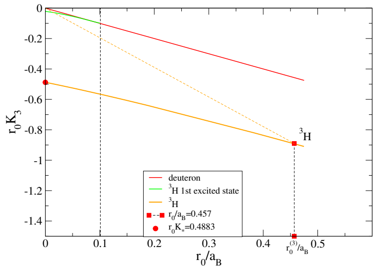


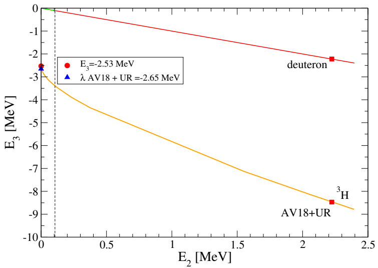
$$a_-^{(0)}\kappa_*^{(0)} = -2.14$$
$$a_-^{(1)}\kappa_*^{(1)} = -1.57$$
$$a_-^{(2)}\kappa_*^{(2)} = -1.51$$

The three-body scale $r_0^{(3)}$ using the gaussian characterization

$$V(1,2,3) = \sum_{i<j} V(i,j) = \sum_{i<j} \left(V_0 e^{-(r/r_0)^2} \mathcal{P}_{01} + V_1 e^{-(r/r_0)^2} \mathcal{P}_{10} \right)$$

To construct the plot we follow the nuclear path defined as ${}^0a_{np}/{}^1a_{np} = -4.38$

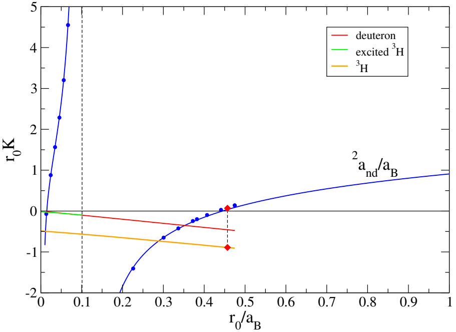




$\tan\xi = K a_B$ $\qquad$ $r_0^{(3)} = 1.98$ fm $\qquad$ The ^{3}H plot $(\lambda_1 = 0.8,\ \lambda_0 = 1.06)$

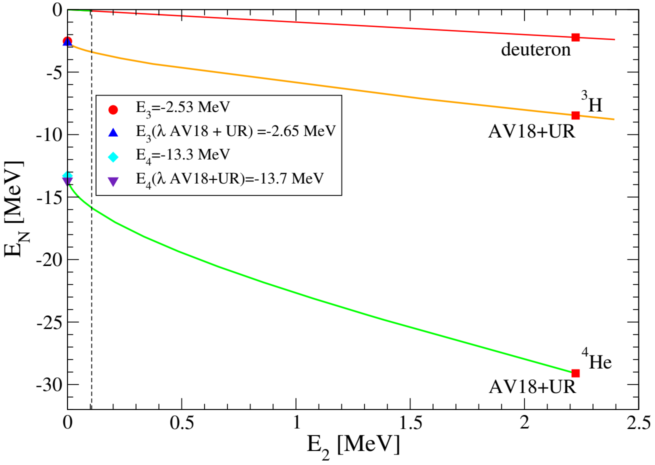
The three-nucleon system: correlations

$$V(1,2,3)=\sum_{i<j}V(i,j)=\sum_{i<j}\left(V_0 e^{-(r/r_0)^2}\mathcal{P}_{01}+V_1 e^{-(r/r_0)^2}\mathcal{P}_{10}\right)$$

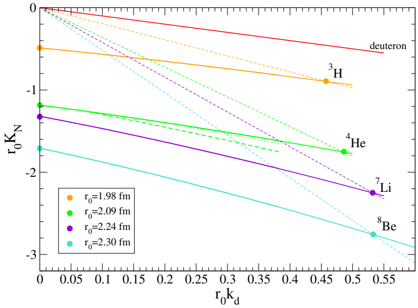


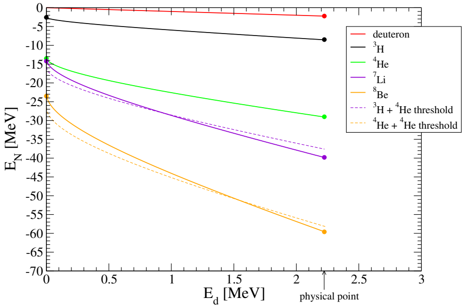
$$a_{AD}/a_B = d_1 + d_2\tan[s_0\ln(K_* r_0(a_B/r_0)+\Gamma_3)+d_3]$$

The three- and four-body scales, $r_0^{(3)}$ and $r_0^{(4)}$

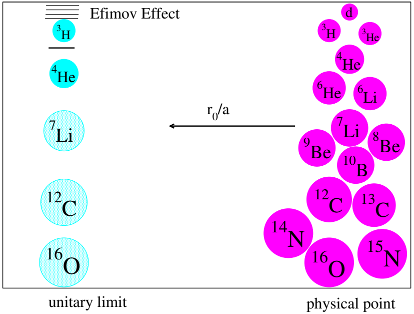


The N-body scales, $r_0^{(N)}$, for $A \leq 8$





Unifying the scales



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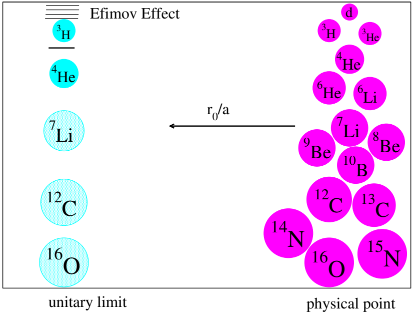


$$V_N = \sum_{i<j} V(i,j,r_0^{(N)},\beta_0)\mathcal{P}_{01} + \sum_{i<j} V(i,j,r_1^{(N)},\beta_1)\mathcal{P}_{10}$$

Unifying the scales: the LO potential

- The scale invariance is encoded in the two-pole S-matrix
- The trip to the unitary point has shown that two nuclear structures. They form the thresholds from which the other nuclei emerge

Unifying the scales: the LO potential

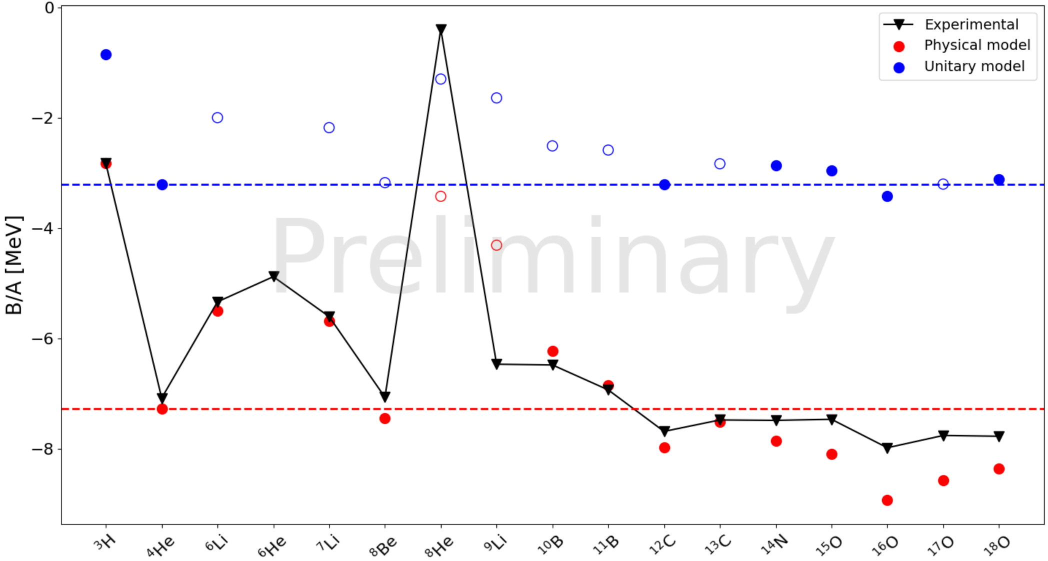
- The scale invariance is encoded in the two-pole S-matrix
- The trip to the unitary point has shown that two nuclear structures. They form the thresholds from which the other nuclei emerge
- Accordingly we propose the following potential to be considered at the lowest order

$$V_N = \sum_{i<j} V(i,j,r_0^{(N)},\beta_0)\mathcal{P}_{01} + \sum_{i<j} V(i,j,r_1^{(N)},\beta_1)\mathcal{P}_{10} \rightarrow$$

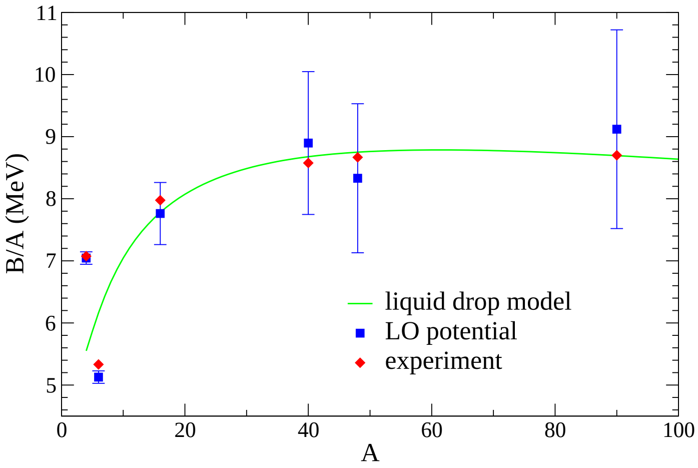
$$V_N = \sum_{i<j} V(i,j,r_0^{(2)},\beta_0)\mathcal{P}_{01} + \sum_{i<j} V(i,j,r_1^{(2)},\beta_1)\mathcal{P}_{10} + \sum_{i<j<k} W(i,j,k,r_3,\beta_3)$$

with β_3, r_3 fixed to reproduce $E(^3\mathrm{H})$ and $E(^4\mathrm{He})$

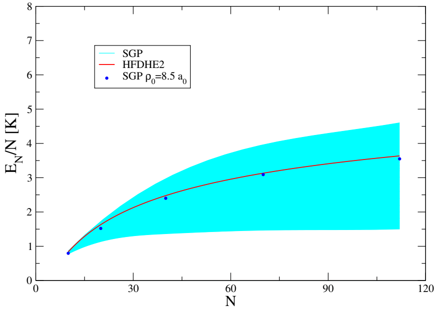
Mirroring the nuclear chart at the unitary limit



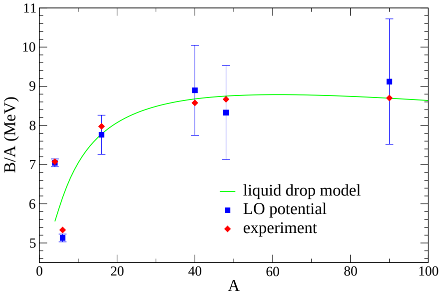
The LO potential at the physical point



E/A curves in helium and in nuclei at the physical point



A.K et al, Phys.Rev.A96, 040501(R)(2017)



R. Schiavilla et al. Phys.Rev.C103, 054003 (2021)

Conclusions

- The nuclear system is well inside the universal window, accordingly it shows scale invariance
- Scale invariance manifests in particular correlations not well explained otherwise
- Moreover, this symmetry is independent of the microscopic theory as many different systems are located inside this window
- It will be important to incorporate this symmetry in the Ab Initio description of the nuclear structure
- From our trip we have seen important structures suggesting a modification in the power counting that organizes the perturbative series
- We refer here either to chiral or to pionless EFT
- From our point of view the nuclear potential at lowest order should decribe the two-pole S-matrix plus the triton and alpha-particle binding energies