Polarization phenomena in 2N and 3N systems

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Outline

Introduction

Part I: 3N system - beyond double polarization experiments

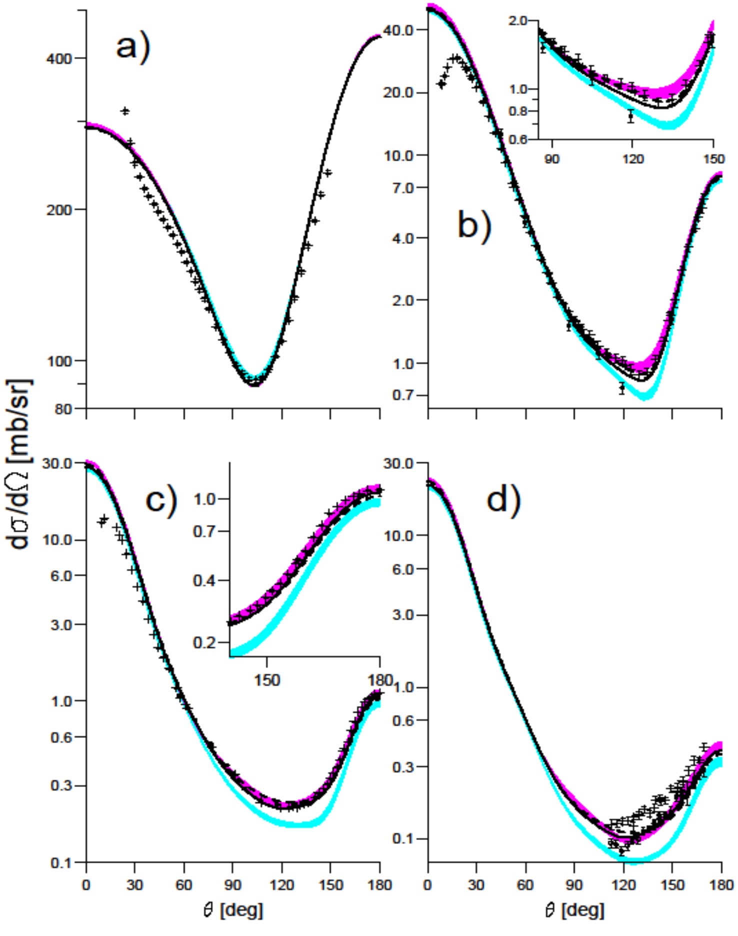
- Deriving polarization transfer coefficients in elastic nd scattering from doubly spin-polarized initial nd state to outgoing neutron.
- 3NF effects in triple polarizations in Nd scattering

Part II: Entanglement in NN scattering

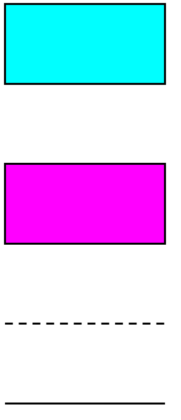
- Final spin state from initial spin state
- Indicators of entangled states

Conclusions

Nd elastic scattering

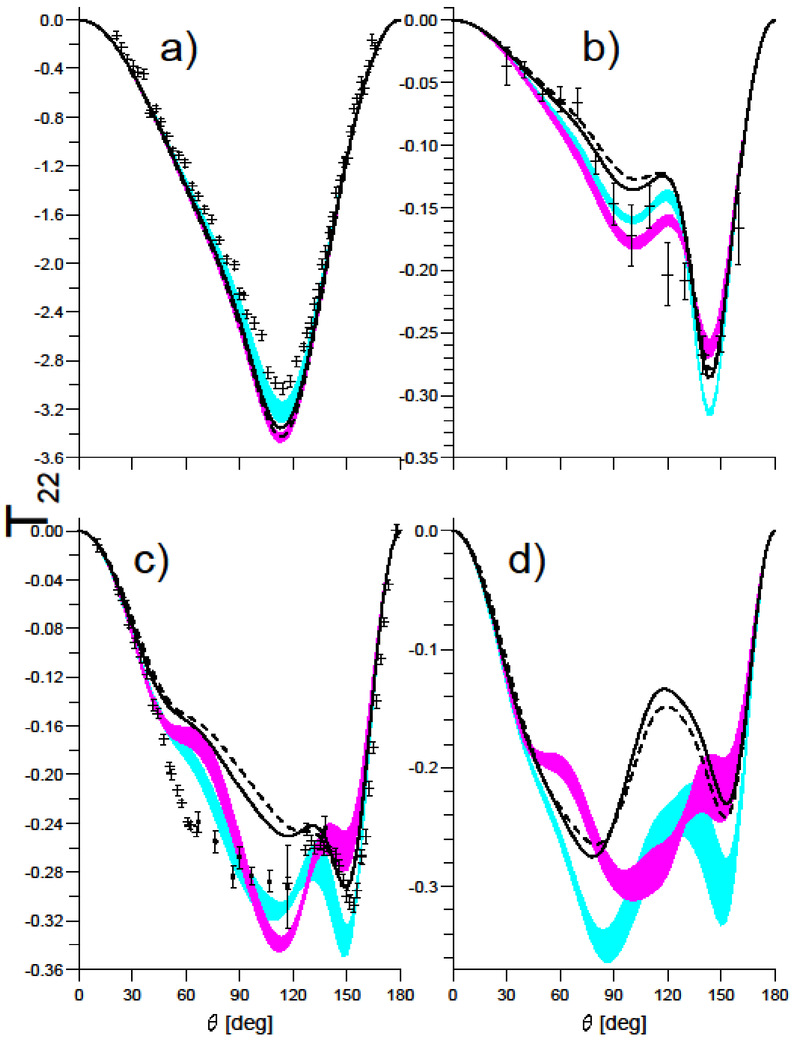


a) E=3 MeV
b) E=65 MeV
c) E=135 MeV
d) E=190 MeV



H.Witała et al., Phys. Rev. C63 (2001) 024007

Nd elastic scattering



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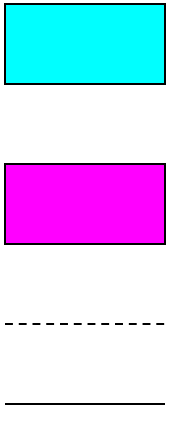

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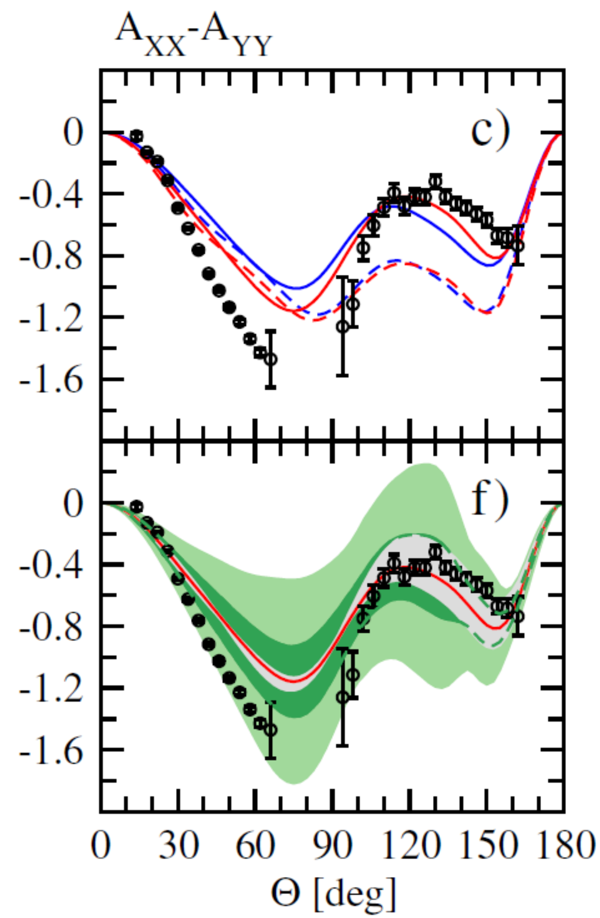
Available data for Nd elastic scattering 50-250 MeV

- So far, in addition to the unpolarized proces the following reactions have been measured:

- $\vec{p}$ + *d* (RCNP, KVI)
- $\vec{n}$ + *d* (RCNP)
- $\vec{d}$ + *p* (RIKEN, KVI)
- $\vec{p}$ + $\vec{d}$ (IUCF)
- $\vec{d}$ + *p* → *d* + $\vec{p}$ (RIKEN)

- cross section, the nucleon analyzing powers, the deuteron vector and tensor analyzing powers, the deuteron to proton polarization transfer coeff. induced proton polarization, spin correlation coefficients



- However, more systematics is needed

Triple polarization $\vec{b}(\vec{a},\vec{c})d$

- Density matrix of the initial state is defined via polarization tensor t_{kq} of particles *a* and *b* and spherical tensor operators τ_{kq}

$$\rho^{in} = \frac{1}{(2s_a+1)(2s_b+1)} \sum_{k_b q_b} t_{k_b q_b} \tau^{\dagger}_{k_b q_b} \sum_{k_a q_a} t_{k_a q_a} \tau^{\dagger}_{k_a q_a}$$

where $t_{kq} \overset{\text{def}}{=} \sqrt{2s+1} \sum_{\mu,\mu'} (-1)^{s-\mu} (s,\mu',s,-\mu;k,q) \rho_{\mu,\mu'}$

$$\langle s\mu' | \tau_{kq} | s\mu \rangle \overset{\text{def}}{=} \sqrt{2s+1} (-1)^{s-\mu} (s,\mu',s,-\mu;k,q)$$

- For the final state (T is scattering amplitude U or U_0)

$$\rho^{out} = T\rho^{in}T^{\dagger}$$

- Where T is scattering amplitude U or U_0

Polarization tensor of particle c

$$t^c_{k_c q_c}(t^a_{k_a q_a}, t^b_{k_b q_b}) = \frac{Tr(\rho^{out}\tau_{k_c q_c})}{Tr(\rho^{out})} = \frac{\sigma^0}{\sigma} \sum_{k_a q_a, k_b q_b} t_{k_a q_a} t_{k_b q_b} \frac{Tr(T\tau^\dagger_{k_b q_b}\tau^\dagger_{k_a q_a} T^\dagger \tau_{k_c q_c})}{Tr(TT^\dagger)}$$

- Tensors of polarization transfer:

$$t^{k_c q_c}_{k_b q_b, k_a q_a}(\vec{b}(\vec{a},\vec{c})d) \equiv \frac{Tr(T\tau^\dagger_{k_b q_b}\tau^\dagger_{k_a q_a} T^\dagger \tau_{k_c q_c})}{Tr(TT^\dagger)}$$

- Altogether

$$\sigma t^c_{k_c q_c}(t^a_{k_a q_a}, t^b_{k_b q_b}) = \sigma^0 \sum_{k_a q_a, k_b q_b} t_{k_a q_a} t_{k_b q_b} t^{k_c q_c}_{k_b q_b, k_a q_a}(\vec{b}(\vec{a},\vec{c})d)$$

Polarization tensor of particle c

- Separation of unpolarized particles in initial state

$$\sigma t^{c}_{k_c q_c}(t^{a}_{k_a q_a}, t^{b}_{k_b q_b}) = \sigma^0 [t^{k_c q_c}_{00,00}(b(a,\vec{c})d) + \sum_{k_b \neq 0 q_b} t_{k_b q_b} t^{k_c q_c}_{k_b q_b,00}(\vec{b}(a,\vec{c})d)$$

$$+ \sum_{k_a \neq 0 q_a} t_{k_a q_a} t^{k_c q_c}_{00,k_a q_a}(b(\vec{a},\vec{c})d) + \sum_{\substack{k_a \neq 0 q_a \\ k_b \neq 0 q_b}} t_{k_a q_a} t_{k_b q_b} t^{k_c q_c}_{k_b q_b, k_a q_a}(\vec{b}(\vec{a},\vec{c})d)]$$

- 1st term: induced polarization of particle c
- 2nd and 3rd terms: single polarization transfer
- 4th term: double polarization transfer

Polarization tensor of particle c

- Explicit expressions:

$$t^{k_c q_c}_{k_a q_a}(b(\vec{a},\vec{c})d) = \frac{1}{\sum_m |T^{m_a m_b}_{m_c m_d}|^2}[\sum_{\substack{m_c m_{c'} m_d \\ m_a m_b m_{a'}}} T^{m_a m_b}_{m_c m_d}(-1)^{q_a}$$
$$\hat{s}_a(-1)^{s_a - m_{a'}} < s_a m_a s_a - m_{a'}|k_a - q_a > T^{*m_{a'} m_b}_{m_{c'} m_d}$$
$$\hat{s}_c(-1)^{s_c - m_c} < s_c m_{c'} s_c - m_c|k_c q_c >] \; ,$$

- To compare with experiment (convention)

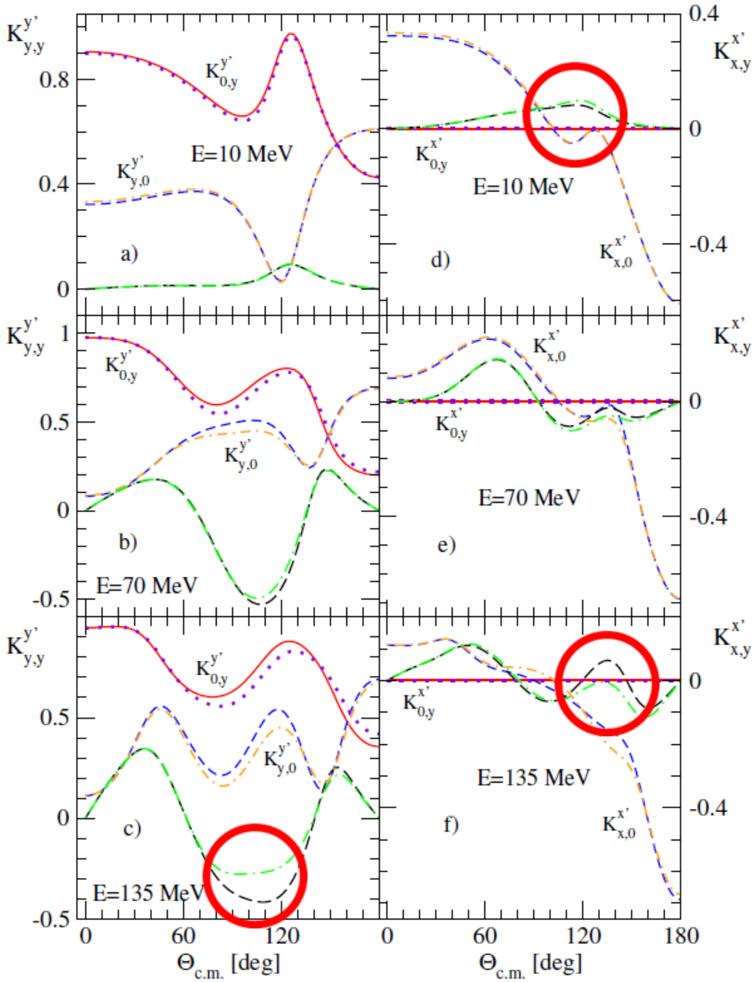
$$t^{k_c q_c}_{k_b q_b, k_a q_a}(\vec{b}(\vec{a},\vec{c})d) = \sum_{q'_c} D^{k_c}_{q'_c, q_c}(0\theta^{lab}_c 0) t^{k_c q'_c}_{k_b q_b, k_a q_a}(\vec{b}(\vec{a},\vec{c})d)$$

- Transformation from spherical to Cartesian tensors, eg

$$K^{z'}_{yy,z} = \tfrac{1}{2}[-\sqrt{2} t^{10}_{20,10} - \sqrt{3}(t^{10}_{2-2,10} + t^{10}_{2+2,10})]$$

$$K^{y'}_{y,y} = -\tfrac{2}{3} \times \frac{i\sqrt{3}}{4}(t^{1-1}_{1-1,1-1} + t^{1+1}_{1-1,1-1} + t^{1-1}_{1-1,1+1} + t^{1+1}_{1-1,1+1} + t^{1-1}_{1+1,1-1} +$$
$$+ t^{1+1}_{1+1,1-1} + t^{1-1}_{1+1,1+1} + \; t^{1+1}_{1+1,1+1})$$

Polarization transfer coefficients $K_{y,y}^{y'}$ and $K_{x,y}^{x'}$ ($K_{d,n}^{n'}$)

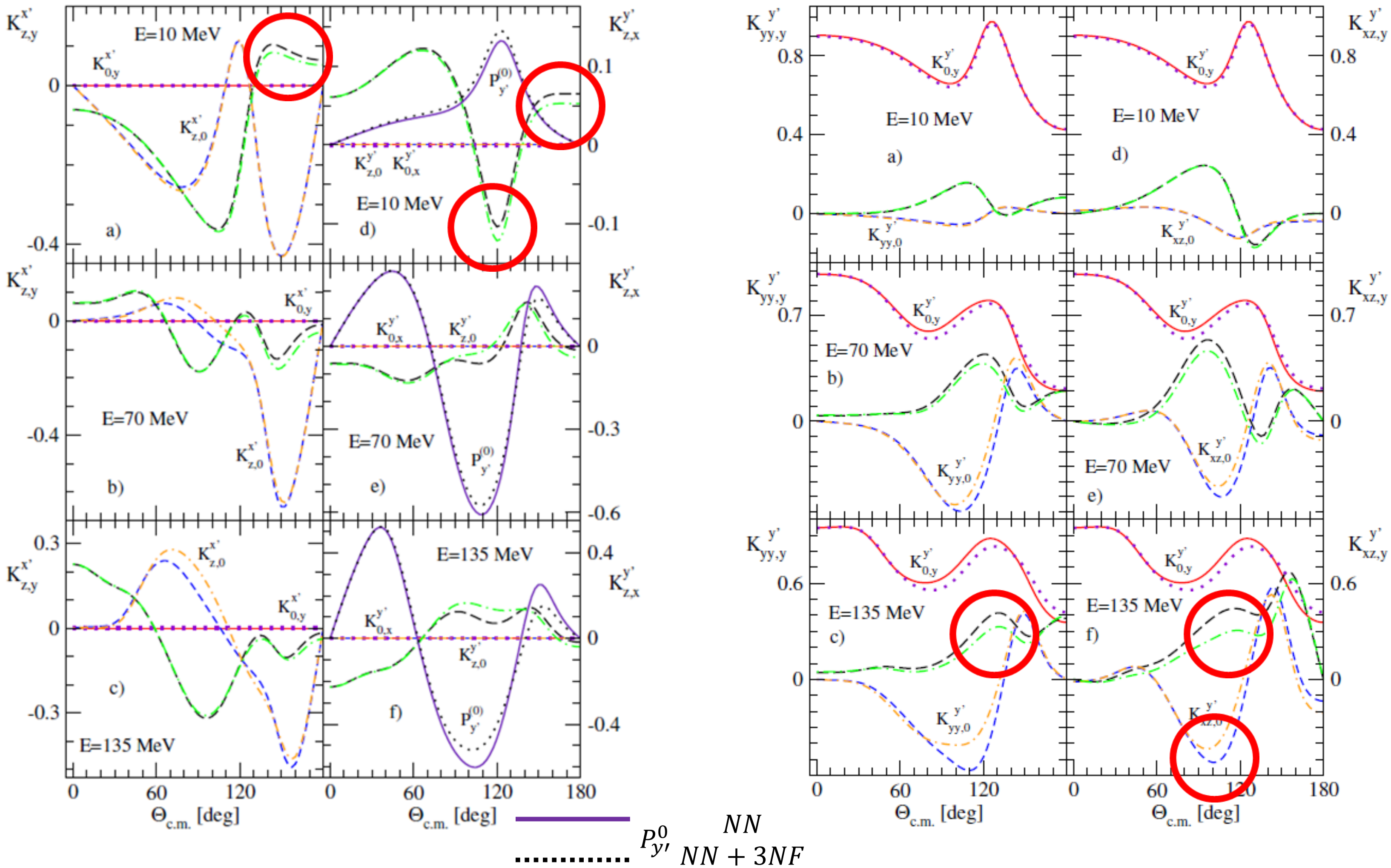


- NN: N4LO+ +N2LO 3NF

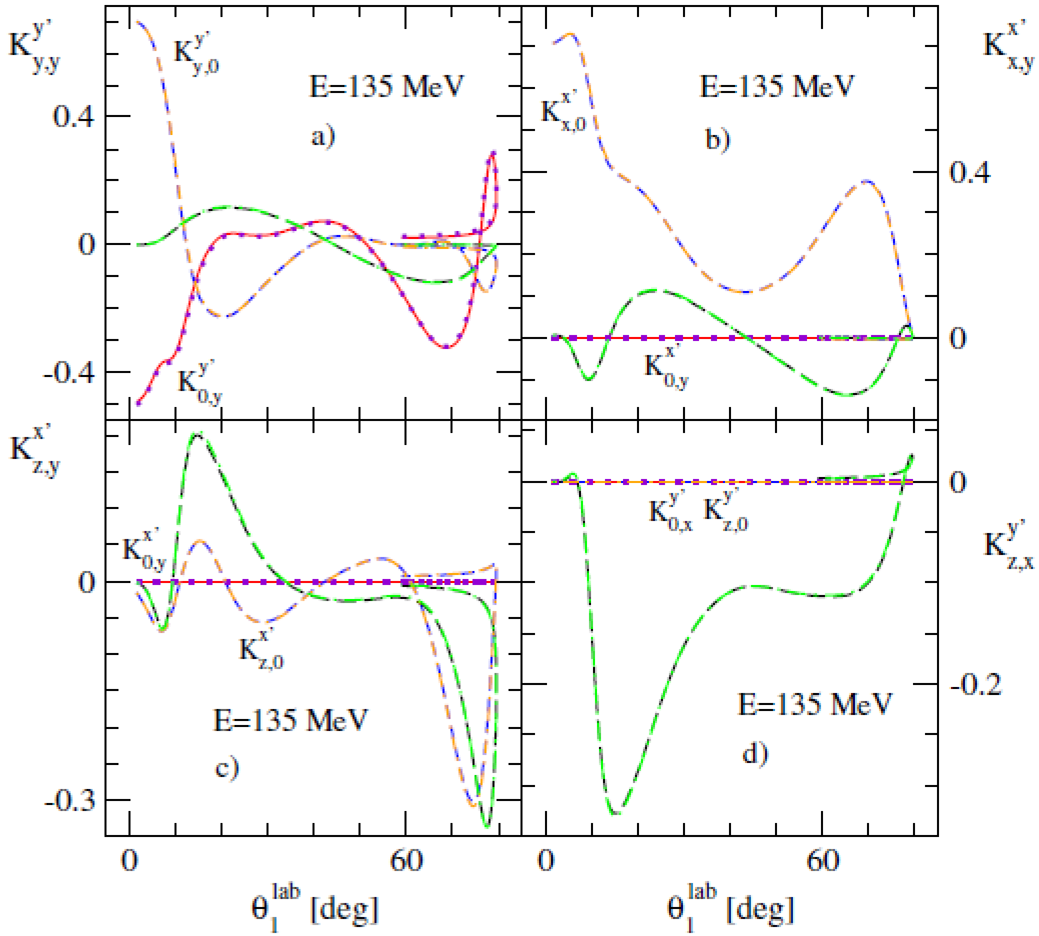
NN: N4LO+		+N2LO 3NF
solid red	$K_{o,n}^{n'}$	dotted violet
dashed blue	$K_{d,0}^{n'}$	dash-dot orange
long-dashed black	$K_{d,n}^{n'}$	dash-dot green

- Potentially accessible for current experiment
- $K_{0,y}^{x'}$=0
- 3NF effects grows with energy
- Magnitude below 0.5 at interesting angles
- Small 3NF effects even at 10 MeV for $K_{x,y}^{x'}$

Polarization transfer coefficients $K_{z,y}^{x'}$ $K_{z,x}^{y'}$ $K_{yy,y}^{y'}$ and $K_{xz,y}^{y'}$



Deuteron breakup $K_{d,n}^{n_1}$, QFS(1-2) geometry



- NN: N4LO+ +N2LO 3NF

NN: N4LO+		+N2LO 3NF
red solid line	$K_{o,n}^{n\prime}$	violet dotted line
blue dashed line	$K_{d,0}^{n\prime}$	orange dash-dotted line
black long-dashed line	$K_{d,n}^{n\prime}$	green dash-dotted line

- at some Θ_1 two solutions for Θ_2 and energies exist
- No 3NF effects
- The same is observed for FSI geometry

Summary I

- In 3-body systems we derived expressions for the polarization transfer coefficients from doubly spin-polarized initial state of the pd elastic scattering to the outgoing proton
- checked for the 3NF effects in triple polarized experiments (soon) possible to perform
- At E=135 MeV the double spin-polarization transfer coefficient $K_{y,y}^{y'}$ shows big 3NF effects, but also other coefficients are interesting. However, the measurement will be challenging
- For deuteron breakup 3NF effects are found for that observables in FSI1-3 geometry. But nevertheless, for configurations (in scattering plane) with the highest cross section role of 3NF is marginal

Neutron-proton scattering and quadruple polarization $\vec{b}(\vec{a},\vec{c})\vec{d}$

- Density matrix of the outgoing two-nucleon state

$$\rho = \frac{1}{4}[I + \sum_{i=1}^{3}\langle\sigma_i^n\rangle\sigma_i^n \otimes I^p + \sum_{i=1}^{3}\langle\sigma_i^p\rangle I^n \otimes \sigma_i^p + \sum_{i,j=1}^{3}\langle\sigma_i^n\sigma_j^p\rangle\sigma_i^n \otimes \sigma_j^p]$$

$$\langle O\rangle \equiv \mathrm{Tr}(\rho O)$$

$\rightarrow$ we need to know the polarizations of the outgoing neutron $\langle\sigma_i^n\rangle$, the polarizations of the outgoing proton $\langle\sigma_i^p\rangle$, and their spin correlation coefficients $\left\langle\sigma_i^n\sigma_j^p\right\rangle$

- As in previous part we build the spin correlation tensor of the outgoing particles *c* and *d*

$$t_{k_c q_c, k_d q_d}(t^a_{k_a q_a}, t^b_{k_b q_b}) \equiv \frac{\mathrm{Tr}(\rho^{out}\tau_{k_c q_c}\tau_{k_d q_d})}{\mathrm{Tr}(\rho^{out})}$$

Neutron-proton scattering and quadruple polarization

... and introduce tensors of spin correlations we obtain

$$\sigma t_{k_c q_c, k_d q_d}(t^a_{k_a q_a}, t^b_{k_b q_b}) = \sigma^0 [t^{k_c q_c, k_d q_d}_{00,00}(b(a, \vec{c})\vec{d}) + \sum_{k_b \neq 0 q_b} t_{k_b q_b} t^{k_c q_c, k_d q_d}_{k_b q_b, 00}(\vec{b}(a, \vec{c})\vec{d})$$
$$+ \sum_{k_a \neq 0 q_a} t_{k_a q_a} t^{k_c q_c, k_d q_d}_{00, k_a q_a}(b(\vec{a}, \vec{c})\vec{d}) + \sum_{\substack{k_a \neq 0 q_a \\ k_b \neq 0 q_b}} t_{k_a q_a} t_{k_b q_b} t^{k_c q_c, k_d q_d}_{k_b q_b, k_a q_a}(\vec{b}(\vec{a}, \vec{c})\vec{d})]$$

with various contributions to the spin correlation of outgoing nucleons.

- Further steps are the same as in the 1st part:
  explicit computing via transition operator matrix elements,
  rotations,
  transfer to Cartesian single and double spin-polarization transfer coefficients to the outgoing nucleon as well as single and double spin correlation transfer coefficients
- At the end of the day, polarization we have all terms involved in polarizations and correlation transfers

Neutron-proton scattering and quadruple polarization (on the top of induced polarizations)

Notation: $K_{p,n}^{n',p'}$

- Single polarization transfers $K_{0,n}^{n',0}$ and $K_{0,p}^{n',0}$ (+ transfer to p)

$$K_{0,x}^{z',0} = \frac{\sqrt{2}}{2}(t_{00,1-1}^{10,00} - t_{00,1+1}^{10,00})$$

- Double polarization transfers $K_{p,n}^{n',0}$ (+ transfer to p)

$$K_{z,y}^{z',0} = -\frac{i\sqrt{2}}{2}(t_{10,1-1}^{10,00} + t_{10,1+1}^{10,00})$$

- Single spin correlation transfers $K_{0,n}^{n',p'}$ and $K_{p,0}^{n',p'}$

$$K_{0,y}^{x',z'} = -\frac{i}{2}(t_{00,1+1}^{1+1,10} - t_{00,1+1}^{1-1,10} + t_{00,1-1}^{1+1,10} - t_{00,1-1}^{1-1,10})$$

- Double spin correlation transfers $K_{p,n}^{n',p'}$

$$K_{x,y}^{z',y'} = \frac{\sqrt{2}}{4}(-t_{1+1,1+1}^{10,1+1} - t_{1+1,1-1}^{10,1+1} - t_{1+1,1+1}^{10,1-1} - t_{1+1,1-1}^{10,1-1} + t_{1-1,1+1}^{10,1+1} + t_{1-1,1-1}^{10,1+1} +$$

$$+ t_{1-1,1+1}^{10,1-1} + t_{1-1,1-1}^{10,1-1})$$

Entanglement in final polarization states of the neutron-proton scattering: pure state

- Pure state

$$|\psi_{np}^{spin}\rangle = \alpha_{+\frac{1}{2}+\frac{1}{2}}|+\frac{1}{2}+\frac{1}{2}\rangle + \alpha_{+\frac{1}{2}-\frac{1}{2}}|+\frac{1}{2}-\frac{1}{2}\rangle + \alpha_{-\frac{1}{2}+\frac{1}{2}}|-\frac{1}{2}+\frac{1}{2}\rangle + \alpha_{-\frac{1}{2}-\frac{1}{2}}|-\frac{1}{2}-\frac{1}{2}\rangle$$

- Density matrix

$$\rho^{np} = |\psi_{np}^{spin}\rangle\langle\psi_{np}^{spin}|$$

has a unit trace and is idempotent $\mathrm{Tr}(\rho^{np}) = 1 \quad (\rho^{np})^2 = \rho^{np}$

- Parameters are related to polarizations and spin correlations

$$\langle\sigma_z^p\rangle = |\alpha_{+\frac{1}{2}+\frac{1}{2}}|^2 + |\alpha_{-\frac{1}{2}+\frac{1}{2}}|^2 - |\alpha_{+\frac{1}{2}-\frac{1}{2}}|^2 - |\alpha_{-\frac{1}{2}-\frac{1}{2}}|^2$$

$$\langle\sigma_x^n\sigma_x^p\rangle = 2\mathfrak{Re}(\alpha_{+\frac{1}{2}+\frac{1}{2}}\alpha^*_{-\frac{1}{2}-\frac{1}{2}} + \alpha_{+\frac{1}{2}-\frac{1}{2}}\alpha^*_{-\frac{1}{2}+\frac{1}{2}})$$

$$\langle\sigma_y^n\sigma_y^p\rangle = -2\mathfrak{Re}(\alpha_{+\frac{1}{2}+\frac{1}{2}}\alpha^*_{-\frac{1}{2}-\frac{1}{2}} - \alpha_{+\frac{1}{2}-\frac{1}{2}}\alpha^*_{-\frac{1}{2}+\frac{1}{2}})$$

Entanglement in final polarization states of the neutron-proton scattering: entangled state

- By entanglement we mean strong (maximal) correlation between results of measurement of neutron and proton spin projections

- Bell states

$$
\begin{aligned}
|\psi_I\rangle &= \frac{1}{\sqrt{2}}(|+\frac{1}{2}+\frac{1}{2}\rangle + |-\frac{1}{2}-\frac{1}{2}\rangle) \\
|\psi_{II}\rangle &= \frac{1}{\sqrt{2}}(|+\frac{1}{2}+\frac{1}{2}\rangle - |-\frac{1}{2}-\frac{1}{2}\rangle) \\
|\psi_{III}\rangle &= \frac{1}{\sqrt{2}}(|-\frac{1}{2}+\frac{1}{2}\rangle + |+\frac{1}{2}-\frac{1}{2}\rangle) \\
|\psi_{IV}\rangle &= \frac{1}{\sqrt{2}}(|+\frac{1}{2}-\frac{1}{2}\rangle - |-\frac{1}{2}+\frac{1}{2}\rangle)
\end{aligned}
$$

- For Bell states the spin correlation $\langle \sigma_i^n \sigma_i^p \rangle = \pm 1$ and $\left\langle \sigma_i^{n(p)} \right\rangle = 0$

Entanglement in final polarization states of the neutron-proton scattering: entangled state

We will check:

1) If spin correlations are close to -1, 0 or +1

$\langle \sigma_x^n \sigma_x^p \rangle$, $\langle \sigma_y^n \sigma_y^p \rangle$, $\langle \sigma_z^n \sigma_z^p \rangle$, $\langle \sigma_x^n \sigma_z^p \rangle$, $\langle \sigma_z^n \sigma_x^p \rangle$

2) If nucleon's polarization $\left\langle \sigma_y^{n(p)} \right\rangle$ close to 0

3) Idempotent condition for density matrix

Assumptions: proton and neutron in initial state prepared separately (no spin correlation), both have polarization only in y-direction.

Using LS eq. we compute induced neutron-proton spin correlations, induced nucleons polarizations and spin correlation transfers and double spin correlation transfers.

Entanglement in final polarization states of the neutron-proton scattering: unpolarized initial state

- magenta curve shows for the final state

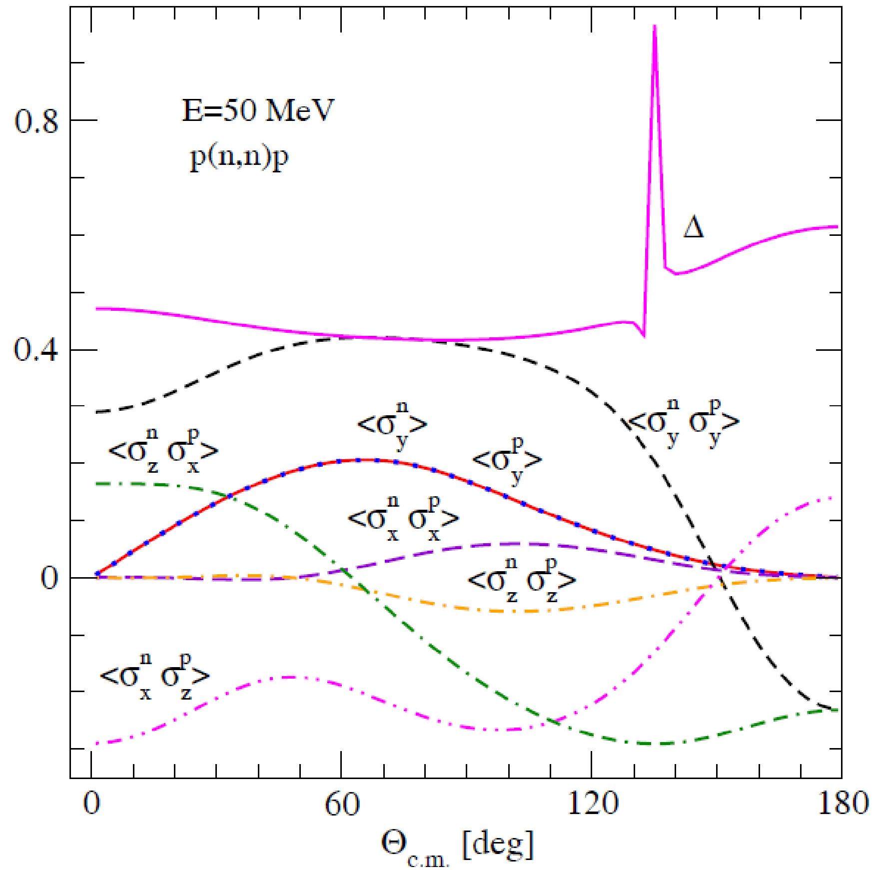
$$\triangle = \frac{1}{16} \sum_{i,j=1}^{4} \frac{|\rho_{i,j} - (\rho^2)_{i,j}|}{|\rho_{i,j}|}$$

which measures idempotent condition.

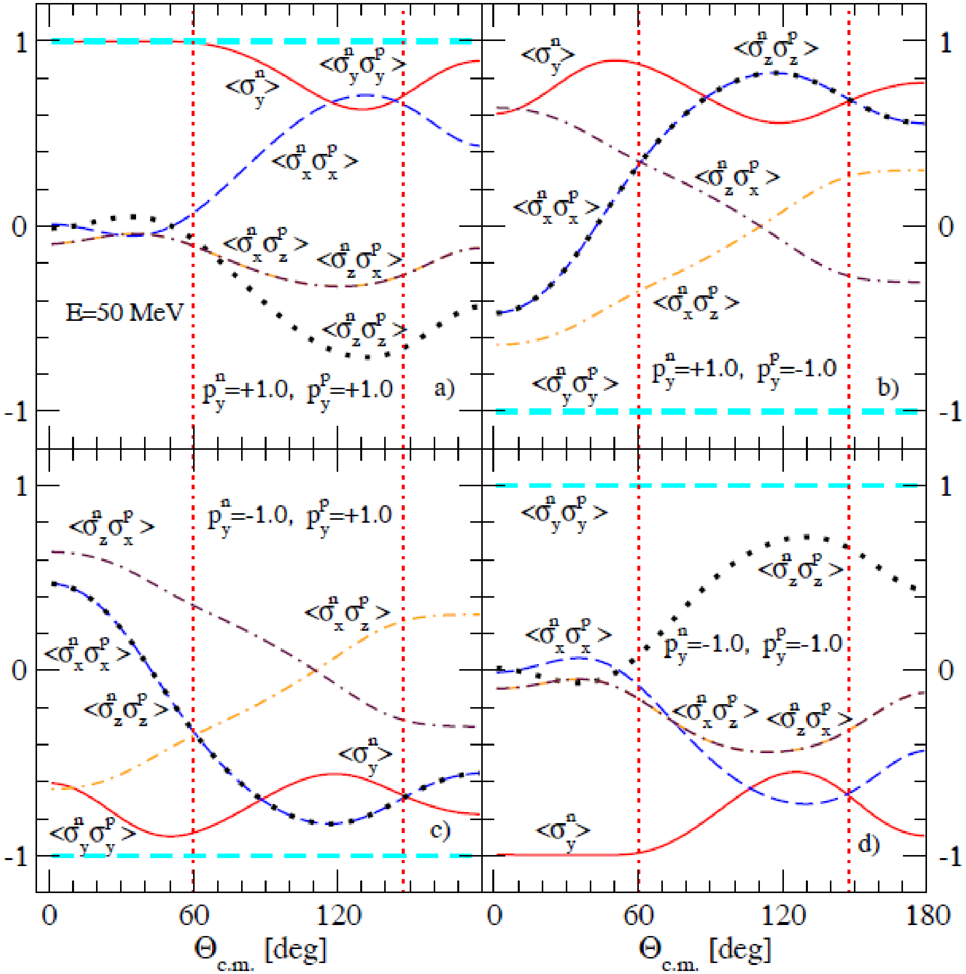
→ No entanglement

- But how does it depend on initial polarizations?

→ only for initial σ_y^n, σ_y^p = ± 1 we see something interesting (but not entanglement)



Entanglement in final polarization states of the neutron-proton scattering: polarized initial state $\sigma_y^n, \sigma_y^p = \pm 1$

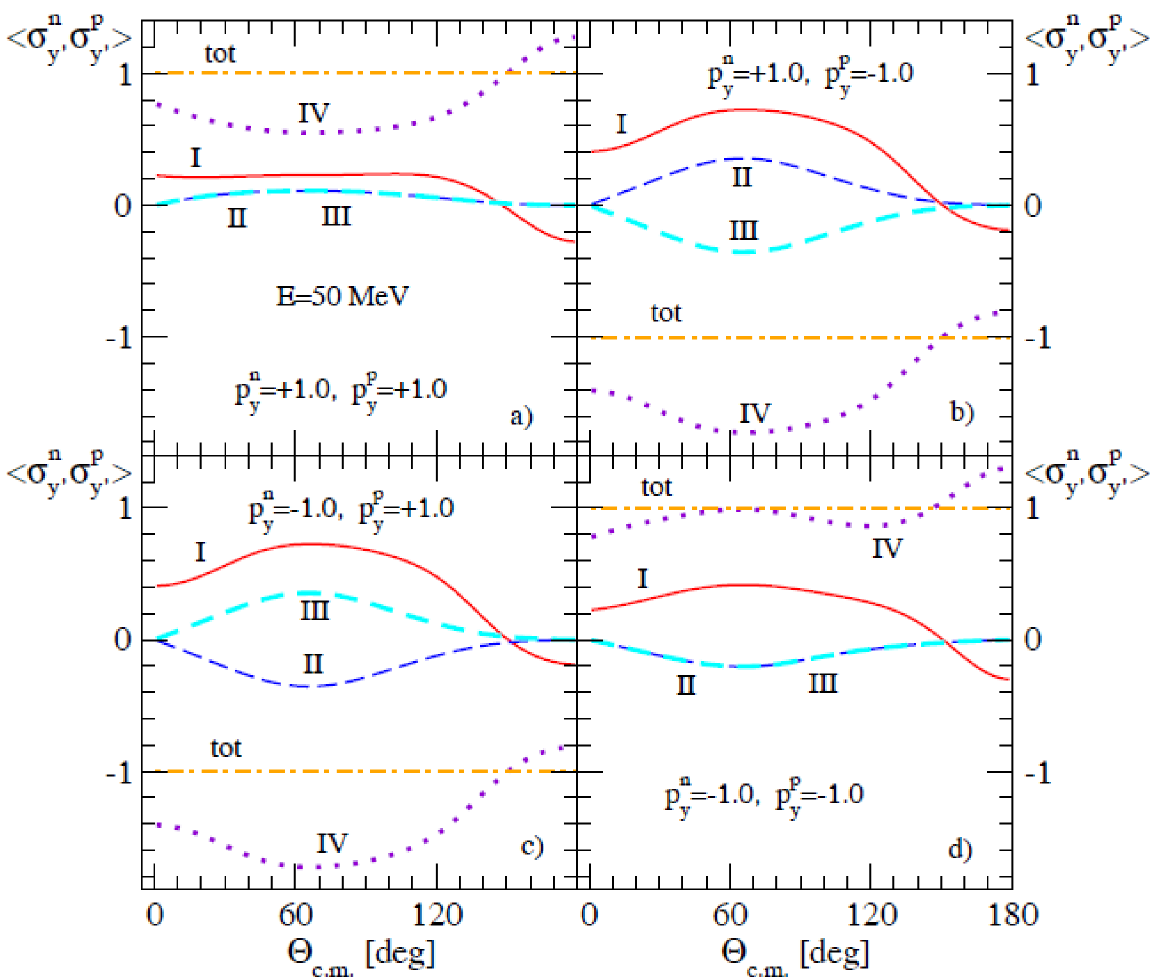


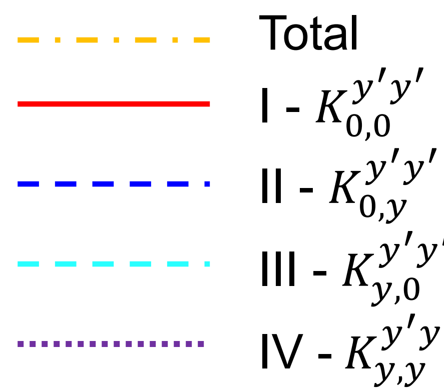
- Remaining polarizations and spin correlations vanish.
- Neutron polarization is far from zero

→ state is not entangled

(vertical dotted curve at $\Theta_{c.m.=}60^o$ indicates angle from the previous slide)

Entanglement in final polarization states of the neutron-proton scattering: polarized initial state $\sigma_y^n, \sigma_y^p = \pm 1$





- dominant contribution is from IV (spin correlation transfer from double polarized initial state)
- for other initial polarizations picture is more complex

Summary II

- We extended our research to the process with four polarized particles.
- We consider all contributions to the final spin state, including spin correlation transfer from a doubly polarized initial state.
- In 2-body subsystem this allows to search for the entanglement (Bell states).
- assuming initial polarizations at y-direction we find that final spin states are statistical mixtures of states.
- only when incoming neutron and proton polarizations are ±1 (pure initial state) also final state is pure
- outgoing neutron and proton polarizations as well as their spin correlations differ of what expected for the Bell states -> no entanglement (Bell states) found.

Conclusions

- Progress in experimental techniques opens possibility to study processes with more than two particles polarized

- We answered this challenge and extended the study of final polarizations by including the spin transfer from double polarization in initial state.

- The new observables and phenomena are worth studying.

- Some of triple polarized observables in Nd scattering can be useful in fixing free parameters of many-nucleon forces or in testing quality of theoretical models.

- Our findings so far do not close the possibility to find entangled states in many-nucleon systems (A -> N+N+(A-2) or n+X -> n+p+…

More in: H.Witała et al., Phys. Rev. C 111 (2025), 044003
H.Witała et al., arXiv:2505.14401 [nucl-th]

THANK YOU!