Reaction4Exp example for transfer reactions: the ^{208}Pb(d,p)^{209}Pb reaction

1 Introduction

This document describes briefly the use of the Reaction4Exp website for the case of transfer reaction. We will use as example the ^{208}Pb(d,p)^{209}Pb stripping reaction. This physics case is taken from the reference of Kovar, Nuclear Physics A231 (1974) 266, referred to as Ref. I hereafter.

2 Reminder of the DWBA transfer formula

Consider the transfer process of the form $d+A \to p+B$ (with $B = A+n$) as illustrated in Fig. 1. Using the *post-form* representation, the DWBA scattering amplitude for this process is given by:

$$f_{d,p}(\theta) = \frac{\mu_\beta}{2\pi\hbar^2} \int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\Phi_B(\xi, \mathbf{r}')\, \Delta V(\mathbf{r}, \mathbf{R})\chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r})\phi_A(\xi)\, d\xi_\beta d\mathbf{R}' \tag{1}$$

where

- $\phi_A(\xi)$, $\Phi_B(\xi, \mathbf{r}')$ are the target (A) and residual (B) nuclei wavefunctions.
- $\varphi_d(\mathbf{r})$ is the deuteron wavefunction.
- $\chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})$, $\chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')$ are distorted waves describing the $d + A$ and $p + B$ relative motion, respectively.
- $V(\mathbf{r}, \mathbf{R}) = V_{pn} + U_{pA} - U_{pB}$ is the residual interaction responsible for the transfer process. When A is a medium-mass or heavy nucleus, one can make the approximation $V(\mathbf{r}, \mathbf{R}) \approx V_{pn}(\mathbf{r})$.
- ξ_β is the set of internal coordinates of the final state, i.e., $\xi_\beta \equiv \{\xi, \mathbf{r}'\}$ (see Fig. 1).

The deuteron and proton distorted waves are to be obtained by solving the corresponding Schrödinger equations with appropriate optical potentials $U_d(\mathbf{R})$, $U_p(\mathbf{R}')$:

$$\left[E_{\text{c.m.}} - \hat{T}_{\mathbf{R}} - U_d(\mathbf{R})\right] \chi_d^{(+)}(\mathbf{K}_d, \mathbf{R}) = 0$$
$$\left[E'_{\text{c.m.}} - \hat{T}_{\mathbf{R}'} - U_p(\mathbf{R}')\right] \chi_p^{(+)}(\mathbf{K}_p, \mathbf{R}') = 0$$

where $E_{\text{c.m.}}$ and $E'_{\text{c.m.}}$ are the total kinetic energy in the entrance and exit channel, respectively. Recall that $E'_{\text{c.m.}} = E_{\text{c.m.}} + Q$.

The integral in the coordinates of the target nucleus (ξ) can be performed by expanding the B wavefunction in terms of $A + n$ states (parentage decomposition):

$$\Phi_B(\xi', \mathbf{r}') = \mathcal{A}_{BA}^{\ell j}\phi_A(\xi)\varphi_{nA}^{\ell j I}(\mathbf{r}') + \sum_{A' \neq A} \mathcal{A}_{BA'}^{\ell' j'}\phi_{A'}(\xi)\varphi_{nA}^{\ell' j' I'}(\mathbf{r}') \tag{2}$$

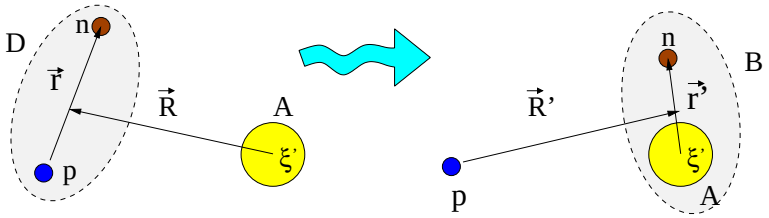

Figure 1: Schematic representation of a (d, p) transfer reaction showing the relevant coordinates in the entrance and exit channels.

where ℓ, j the orbital and total angular momentum of the transferred particle and I the target spin. the function $\varphi_{nA}^{\ell j I}(\mathbf{r}')$ represents a unit-normalized single-particle wavefunction. The first term of this expansion contains the A nucleus ground state, whereas the other term involve its excited states. Since (1) contains the target ground state, only the first term contributes to the integral in ξ, giving

$$\int d\xi\ \phi_B^*(\xi, \mathbf{r}')\phi_A(\xi) = \mathcal{A}_{BA}^{\ell j}\varphi_{nA}^{\ell j I}(\mathbf{r}').$$

The coefficient $\mathcal{A}_{BA}^{\ell j}$ is the so-called **spectroscopic amplitude** and its square, $|\mathcal{A}_{BA}^{\ell j}|^2 = S_{BA}^{\ell j I}$, the **spectroscopic factor**. The spectroscopic factor is related to the occupation of the orbital $\ell s j$ with the core in the state with spin I.

Using the above results, the DWBA scattering amplitude results

$$f_{d,p}(\theta) = \mathcal{A}_{BA}^{\ell j}\frac{\mu_\beta}{2\pi\hbar^2}\int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\varphi_{nA}^{\ell j I,*}(\mathbf{r}')\ V_{pn}(\mathbf{r})\ \chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r})d\mathbf{r}'d\mathbf{R}' \quad (3)$$

and the associated differential cross section:

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{d,p} = \frac{\mu_\alpha\mu_\beta}{(2\pi\hbar^2)^2}S_{BA}^{\ell j I}\left|\int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\varphi_{nA}^{\ell j I,*}(\mathbf{r}')\ V_{pn}(\mathbf{r})\ \chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r})d\mathbf{r}'d\mathbf{R}'\right|^2}$$

Thus, in DWBA, the transfer cross section is proportional to the product of the projectile and target spectroscopic factors[1]. If the approximations leading to the DWBA are valid, one can extract the values of $S_{BA}^{\ell j I}$ by comparing the data with DWBA calculations.

Moreover, the angular distribution turns out to be very sensitive to the value of ℓ which, in a (d, p) reaction, corresponds to the orbital angular momentum transferred to the target nucleus. Consequently, the shape of the angular distribution can be used to infer the value of ℓ.

[1]In this case, we assume that the projectile spectroscopic factor, describing the $d \to p + n$ decomposition, is just one.

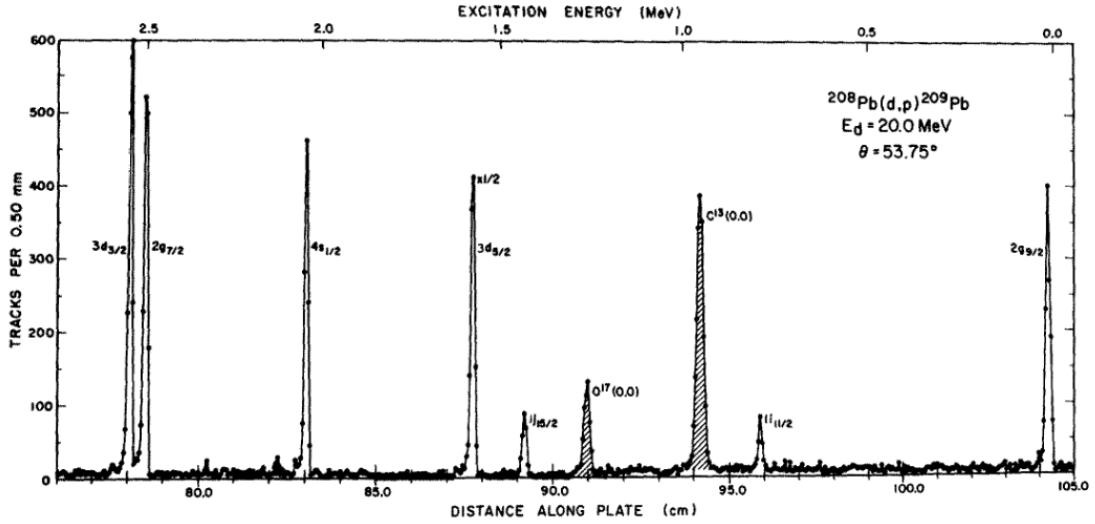

Figure 2: Proton spectrum showing the strongly populated single-particle states in ^{209}Pb. The upper axis indicates the excitation energy relative to the ^{209}Pb ground state. Quoted from Fig. 1 of Ref. I.

3 Application to the ^{208}Pb(d,p)^{209}Pb reaction

The goal of this hands-on exercise is to reproduce some of the results of Ref. I, and understand their physical significance. In this measurement, the d+^{208}Pb reaction was measured at an incident deuteron energy of 20 MeV. The elastically scattered deuterons and the protons were recorded at different scattering angles, thus providing differential cross sections for the elastic and (d, p) stripping reaction channels.

Figure 2 shows the proton energy spectrum at a given angle ($\theta = 53.75°$). This spectrum shows definite peaks corresponding to bound states of ^{209}Pb. The single-particle assignment indicated besides each peak can be inferred from the ordering predicted by the independent particle model, as shown in Fig. 3. For each of these peaks, the corresponding angular distribution was obtained, some of which are shown in Fig. 5. We will analyze these distributions within the DWBA framework.

According to the formulae presented in the previous section, these calculations require the following ingredients:

- Distorted waves for the entrance and exit channels. These distorted waves will be obtained with the deuteron and proton optical potentials reported in Ref. I, reproduced in Fig. 4 of this document for completeness.

- The deuteron ground state wavefunction, for which we will employ the simple Gaussian potential

$$V_{pn}(r) = -72.15 \exp[-(r/1.484)^2] \tag{4}$$

- The single-particle wavefunction describing the ^{209}Pb bound states, for which we will employ a standard Woods-Saxon parametrization with $r_0 = 1.25$ fm and

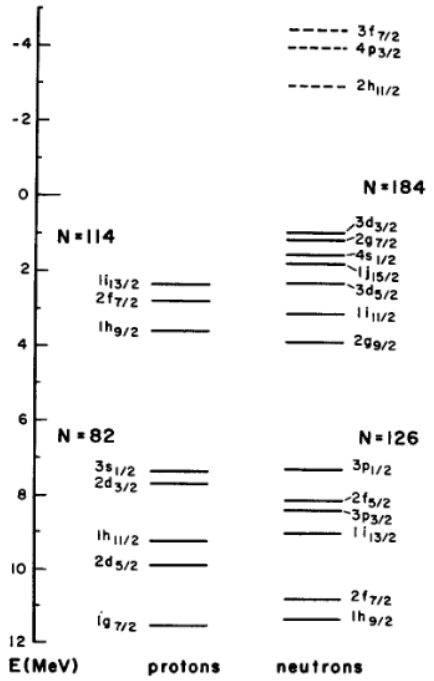

Fig. 7. Predicted ordering of the proton and neutron shell-model orbits in the mass region $A = 208$.

Figure 3: Single-particle levels in ^{208}Pb.

$a_0 = 0.65$ fm. The depth of this potential will be adjusted automatically by the code to yield the correct separation energy, so we can just provide a reasonable starting value ($V_0 \sim 50$ MeV).

In Fig. 5 we show the comparison of the DWBA calculations with the data from Ref. I. for simplicity, a spectroscopic factor of 1 has been assumed in all cases. From this comparison, the following conclusions can be extracted:

- Overall, the calculations reproduce well the magnitude of the data, which suggests that our assumption of unit spectroscopic factor is reasonable. This is not unexpected, given the double-magic nature of the ^{208}Pb.
- The calculations provide also a good description of the shape of the data. We see that, the higher the orbital angular momentum of the populated state, the larger is the angle at which the maximum of the cross section appears. In fact, in a classical description of the process, the first maximum (in LAB) appears as

$$\theta_{\max} \approx \arcsin\left(\frac{\sqrt{\ell(\ell+1)}\hbar}{pR}\right) \tag{5}$$

where p is the incident momentum (also in LAB). This expression indicates that the angle for the first maximum increases with increasing ℓ.

TABLE 2
Optical-model and bound-state parameters used in distorted-wave calculations

Parameter set	V (MeV)	r_0 (fm)	a_0 (fm)	W (MeV)	W_D (MeV)	r_0' (fm)	a_0' (fm)	r_c (fm)
deuterons	100.0	1.14	0.89	0	13.80	1.33	0.75	1.30
protons	52.0	1.25	0.65	0	10.0	1.25	0.76	1.25
neutrons	V_n [a])	1.23	0.65	($\lambda = 25$)[b])				
deuteron (break-up)	112	1.25	0.682	0	19.4	1.25	0.783	1.30

Figure 4: Optical potential parameters used in the DWBA calculations (corresponds to Table II of Ref. I). For the core-core interaction use the same interaction as for $p-^{209}$Pb, using $A_p = 208$. The spin-orbit term for the binding potential in ^{209}Pb has the same geometry as the central term and a depth of 6.25 MeV.

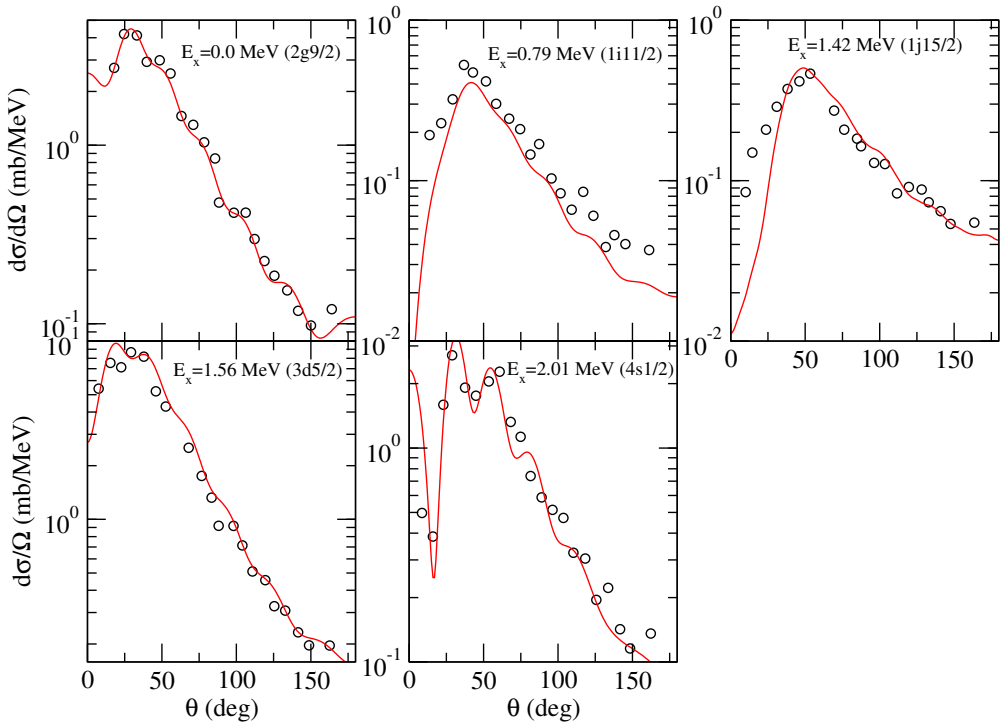

Figure 5: Comparison of experimental data from Ref. I and DWBA calculations. A spectroscopic factor of one was assumed.

4 Questions

- Compute the transfer (d, p) angular differential cross section for the different states of ^{209}Pb shown in Fig. 2 and obtain the angle of the first maximum. Assuming a radius of $R = 1.2 \times A^{1/3}$ fm, are the results consistent with the classical prediction?
- Compute the spectroscopic factor for the $1i_{11/2}$ state adjusting for the peak of the distribution.