THE REALISTIC SHELL MODEL AND THE KSHELL IMPLEMENTATION

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Theory Service for the Low Energy Nuclear Physics Community: a Hands-on Workshop

07 July 2025 — 09 July 2025

OUTLINE

- **The Shell Model**
  - ➢ The basic framework
  - ➢ The shell-model effective Hamiltonian
  - ➢ Dimensionality of the SM basis and SM codes
  - ➢ Calculations of physical observables

- **Some examples of calculations within SM**

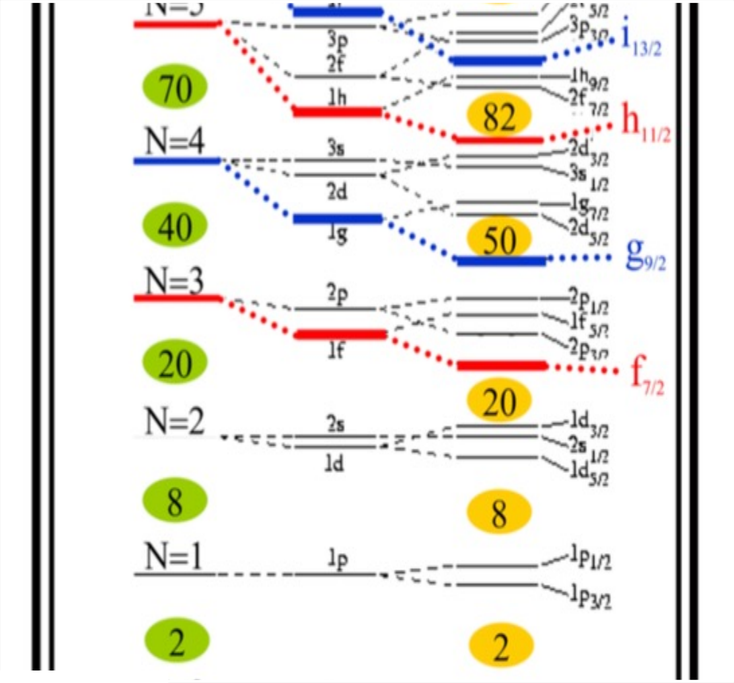
SHELL MODEL (non interacting)

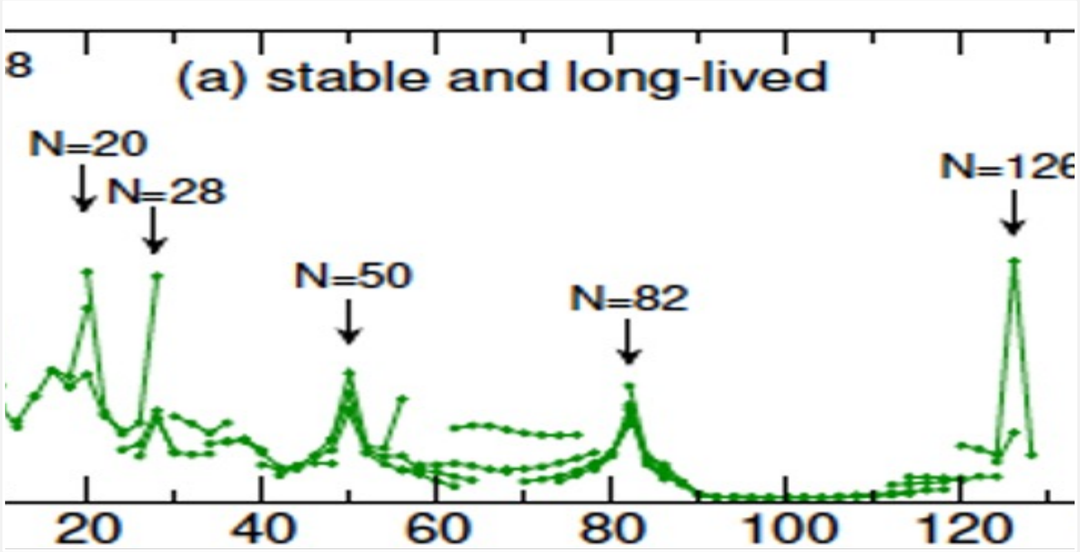
Each nucleon is assumed to be moving in an external field created by the remaining nucleons, with the mean field consisting in an isotropic harmonic oscillator plus a strongly attractive spin-orbit potential and an orbit-orbit term

$$U = U(r) + U_{ls}\, \boldsymbol{l} \cdot \boldsymbol{s} + U_b \boldsymbol{l}^2$$

J. Hans D. Jensen
Maria Goeppert Mayer

Noble Prize in 1963

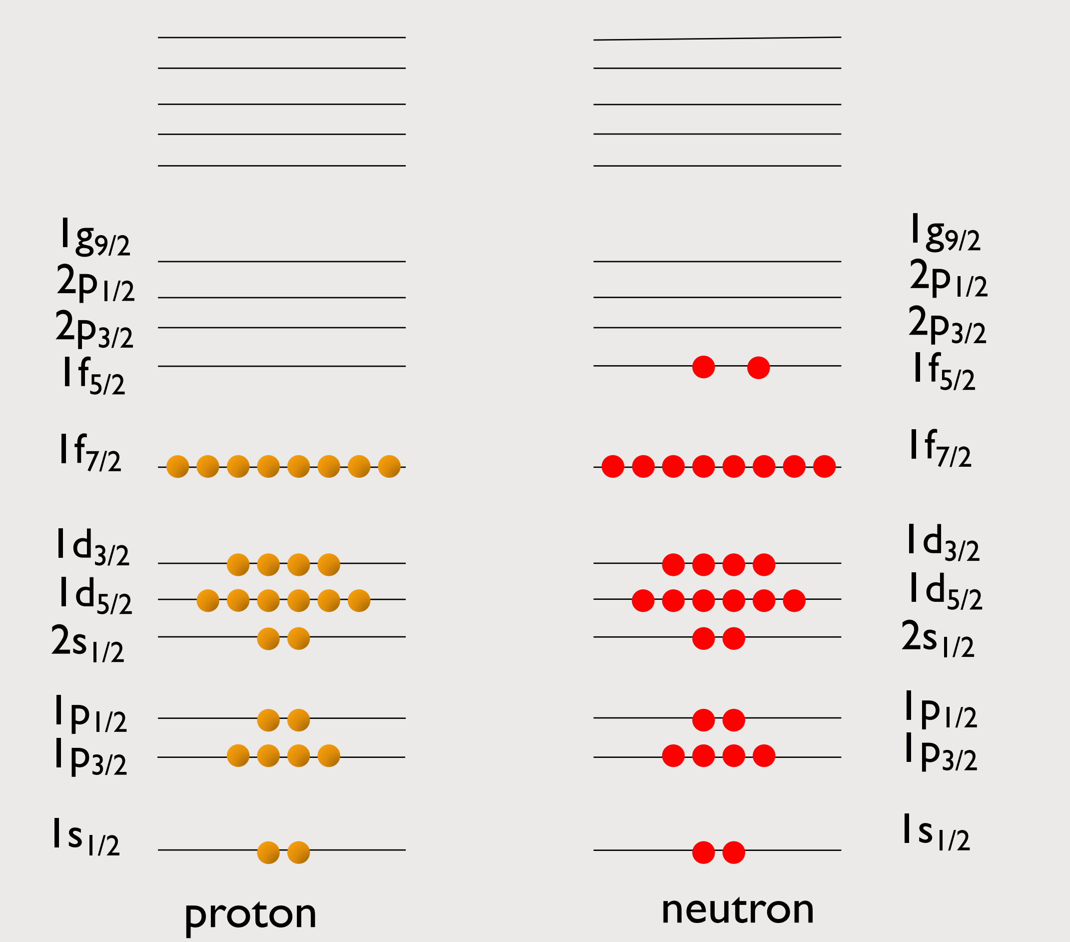




n≡radial quantum number
l ≡ orbital angular momentum
j ≡ total angular momentum

TWO OR MORE VALENCE NUCLEONS

^{58}Ni with 28 protons and 30 neutrons

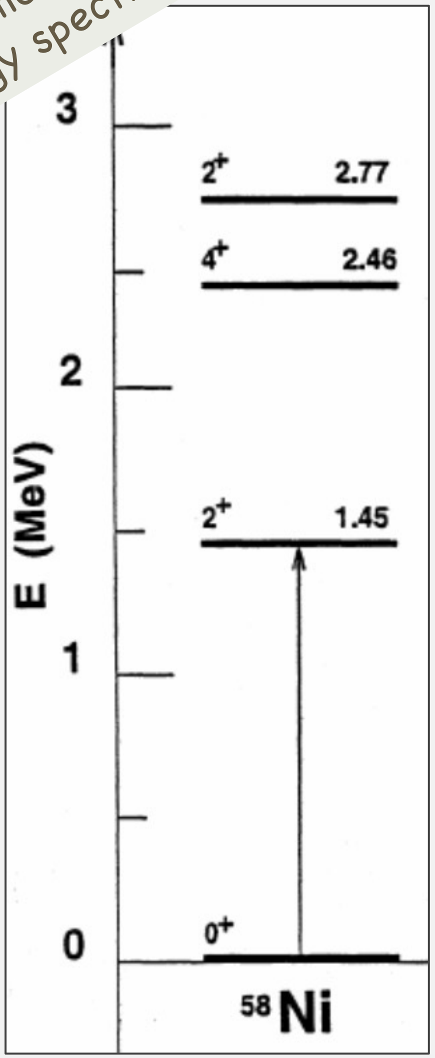


Non interacting shell model

$(f_{5/2}p_{3/2})\ J^{\pi} = 1^+, 2^+, 3^+, 4^+$

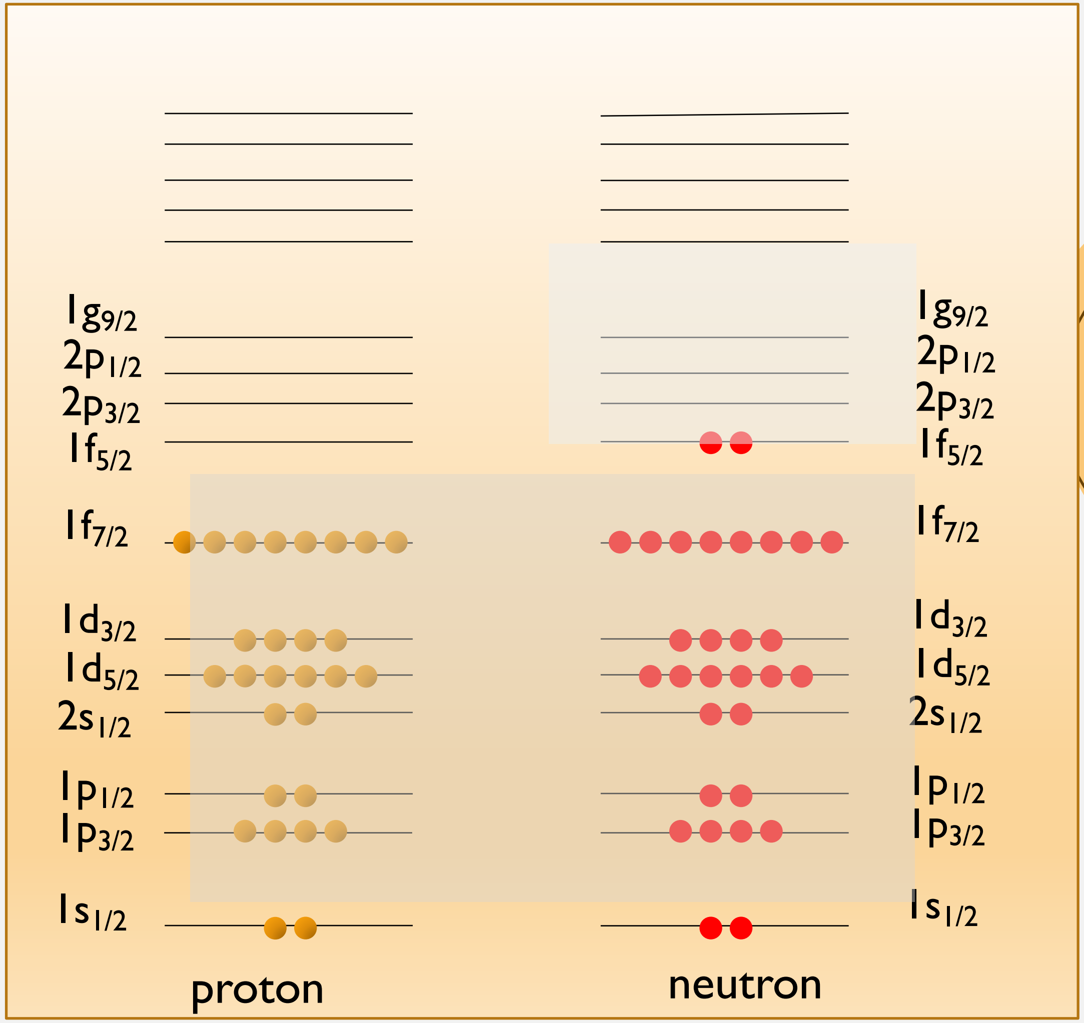
$(f_{5/2})^2\ J^{\pi} = 0^+, 2^+, 4^+$

Experimental low-enegy spectrum



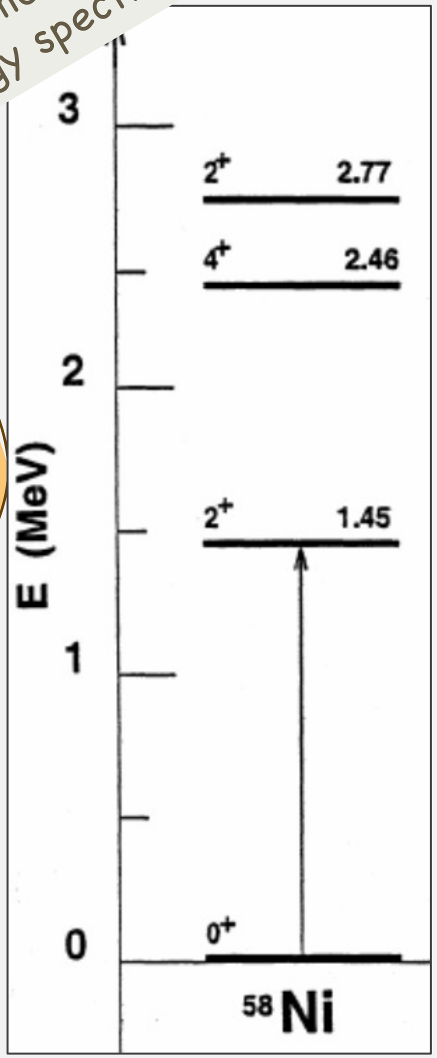
TOWARDS THE INTERACTING SHELL MODEL

^{58}Ni with 28 protons and 30 neutrons



The 2 valence neutrons interact in a truncated space
→ mixing of the different configurations
→ degeneracy is removed

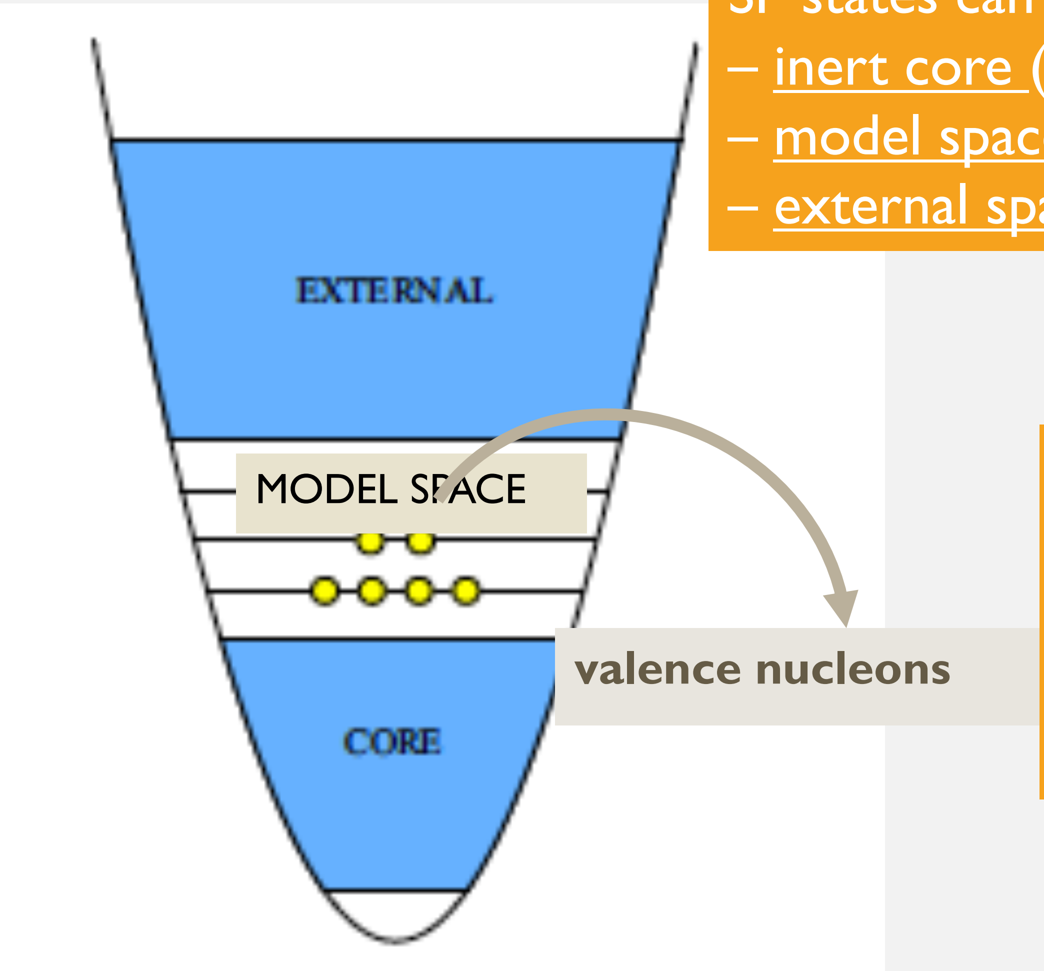
Experimental low-enegy spectrum



SHELL MODEL (interacting)

Mean-field ansatz → we obtain a set of single-particle states, where all nucleons are distributed starting from the lowest energy level

SP states can be grouped into 3 sub-spaces, well separated in energy:
- inert core (*completely filled levels with neutrons and protons paired to J=0*)
- model space (*partially filled levels*)
- external space (*empty levels*)



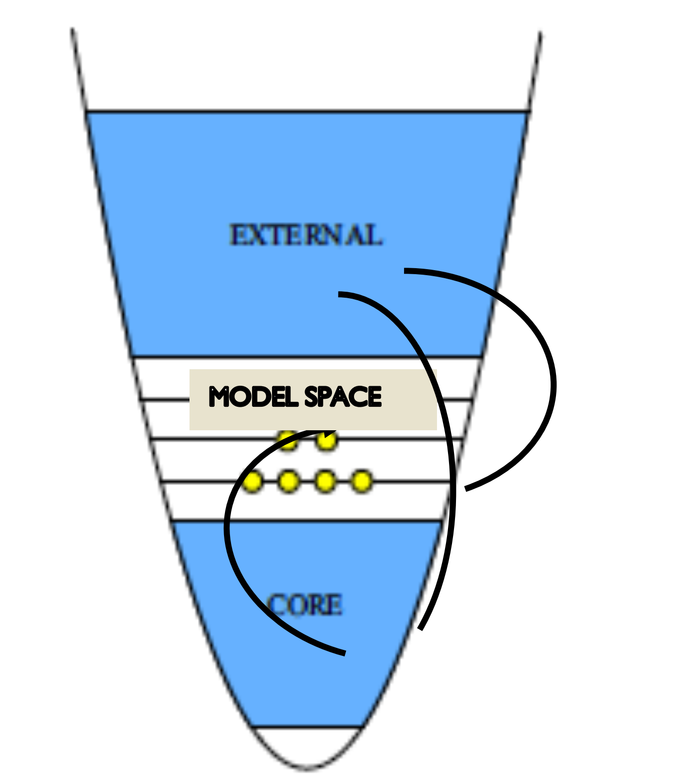
the only active degrees of freedom are given by nucleons inside the model space (*valence nucleons*) while excitations of core nucleons and valence nucleons in the external space are "*frozen*" or, more properly, "not taken into account explicitly"

SHELL MODEL HAMILTONIAN

$$H_{\text{eff}} = \sum_i U_i + \sum_{i<j} V_{ij}$$

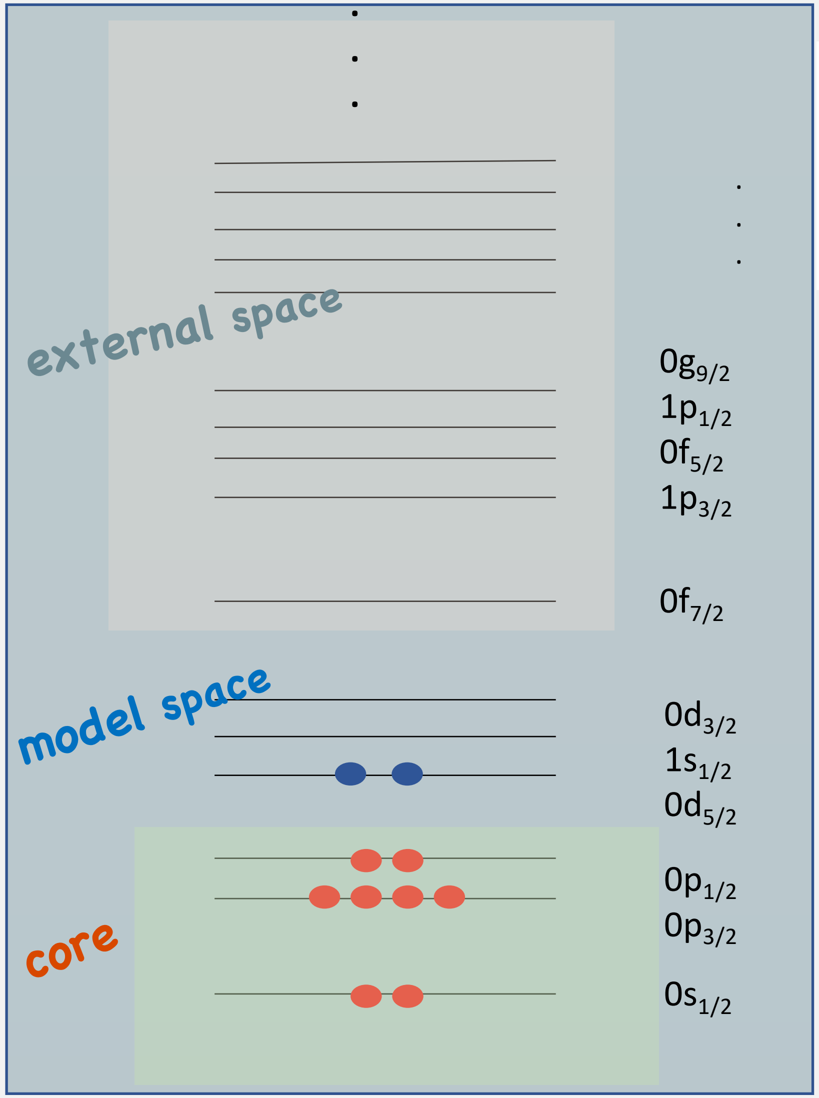
<u>defined</u> in the model space for only valence nucleons

H_{eff}
should take into account
<u>in an effective way</u>
all the degrees of freedom not considered explicitly



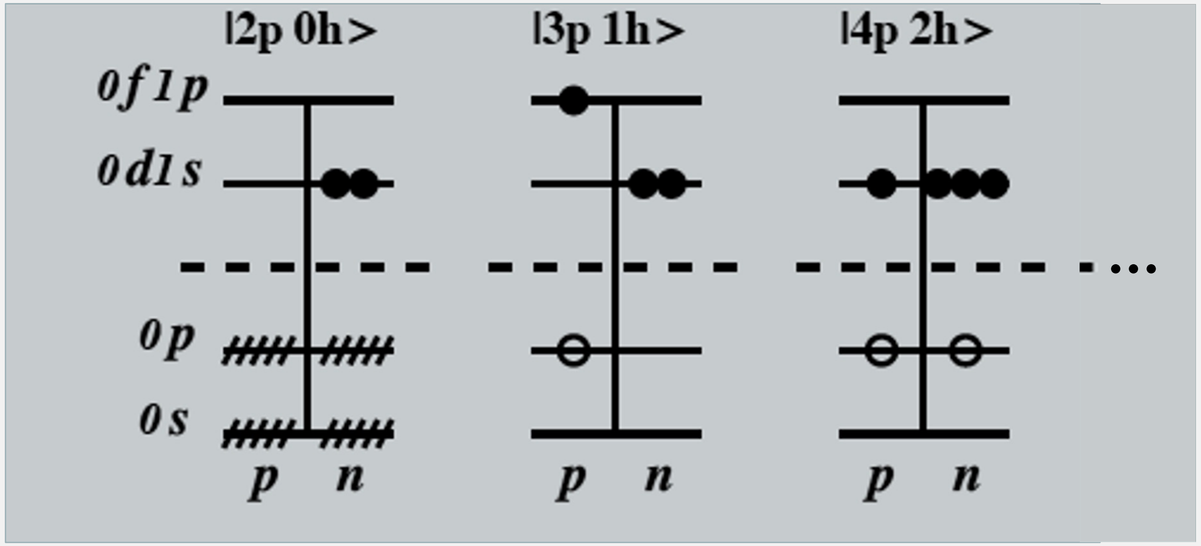
EXAMPLE

$$ {}^{18}_{8}\mathrm{O} \Longrightarrow {}^{16}_{8}\mathrm{O} + 2n $$



$$ |\Phi_i\rangle = \sum_{\alpha\beta \in \text{ valence space}} C^i_{\alpha\beta}[a^\dagger_\alpha a^\dagger_\beta]_i |c\rangle \ , \quad i = 1, \ldots, d, $$

$$ |c\rangle = \prod_{\alpha \in \text{ filled shells}} a^\dagger_\alpha |0\rangle $$



EXAMPLE

$$^{18}_{8}\mathrm{O} \Longrightarrow \; ^{16}_{8}\mathrm{O} + 2n$$

$$\begin{array}{c|ccc}
 & |2p\,0h\rangle & |3p\,1h\rangle & |4p\,2h\rangle \;\ldots \\
\hline
\langle 2p\,0h| & & & \\
\langle 3p\,1h| & & H & \\
\langle 4p\,2h| & & & \\
\vdots & & &
\end{array}$$

$$\begin{array}{cc}
 & |2p'\,0h\rangle \\
|2p'\,0h\rangle & \boxed{H_{\mathrm{eff}}}
\end{array}$$

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

$$PH_{\mathrm{eff}}P|\Psi_\nu\rangle = (E_\nu - E_c)P|\Psi_\nu\rangle$$

$$P = \sum_{i=1}^{d} |\Phi_i\rangle\langle\Phi_i|$$

SHELL-MODEL HAMILTONIAN

$$H = H_0 + V = \sum_{i=1}^{\mathcal{N}} h_i + \sum_{i,j=1}^{\mathcal{N}} V_{ij}$$

$$H_0 = \sum_a \varepsilon_a N_a$$

$$N_j = \sum_a a_{jm}^{+} a_{jm}$$

$$V = -\tfrac{1}{4} \sum_J \sum_{abcd} [\mathcal{N}_{ab}(J)\mathcal{N}_{cd}(J)]^{-1} \hat{J} \langle a\, b;\, J|V|c\, d;\, J\rangle \left[[a_a^+ a_b^+]_J [\tilde{a}_c \tilde{a}_d]_J \right]_{00}$$

$$V = -\tfrac{1}{4} \sum_{\substack{J \\ T=0,1}} \sum_{abcd} [\mathcal{N}_{ab}(JT)\mathcal{N}_{cd}(JT)]^{-1} \hat{J}\hat{T} \langle a\, b;\, J\, T|V|c\, d;\, J\, T\rangle \times \left[[a_a^+ a_b^+]_J^T [\tilde{a}_c \tilde{a}_d]_J^T \right]_{00}^{00}$$

$$A_{JM}^{+}(ab) = [a_a^+ a_b^+]_{JM}$$
$$A_{JM}(ab) = [a_b a_a]_{JM}$$
$$\tilde{A}_{JM}(ab) = (-1)^{J-M} A_{J-M}(ab)$$

SHELL-MODEL HAMILTONIAN

Two alternative approaches:

1. **Phenomenological**
2. **Microscopic**

PHENOMENOLOGICAL SHELL-MODEL HAMILTONIAN

SPE and interaction TBMEs are considered as adjustable parameters and they - or their linear combinations - are fitted to experimental data in order reproduce experimental low-energy spectra by a least-square fit procedure

<u>A very simple case:</u>

two valence nucleons in the $0f_{7/2}$ orbit: $(0f_{7/2})^2$
8 TBME
J=0, 2 ,4, 6 T=1 & J=1, 3, 5, 7 T=0
1 SPE

0p	1s0d	1s0d	1p0f
15 TBME + 2 SPE A=5-16 Cohen-Kurath (1965)	63 TBME + 3 SPE A=17-40 (389 data) USD-Brown et al (1980)	63 TBME + 3 SPE A=17-40 (608 nuclei) USDA-Brown et al (2006)	195 TBME + 4 SPE A=47-66 (699 data) GXPF1-Honma et al (2002)

the number of ME to be fitted becomes too large for increasing spaces

MONOPOLE MODIFIED SHELL-MODEL HAMILTONIAN

Start from interactions derived from the free nuclear potentials and use a readjusting procedure in which only the monopole component is modified

$$H = H_{mon} + H_{multi}$$

Monopole: spherical mean field as extracted from the interacting shell model
Multipole: correlations like pairing, qradrupole, octupole,...

Monopole component

$$H_{\mathrm{mon}} = \sum_{a\tau} \epsilon_{a\tau} N_{a\tau} + \frac{1}{2} \sum_{ab\tau\tau'} \frac{\bar{V}_{ab}^{\tau\tau'} N_{a\tau}(N_{b\tau'} - \delta_{ab}\delta_{\tau\tau'})}{\sum_J \hat{J}^2}$$

Centroid

$$\bar{V}_{ab}^{\tau\tau'} = \frac{\sum_J \hat{J} \langle a\tau b\tau'; J \mid V_{\mathrm{eff}} \mid a\tau b\tau'; J\rangle}{\sum_J \hat{J}}$$

$\Rightarrow$

$$\mathrm{ESPE}(a\tau) = \epsilon_{a\tau} + \sum_{b\tau'} \bar{V}_{ab}^{\tau\tau'} n_b^{\tau'}$$

Responsible for the evolution of the SP energies

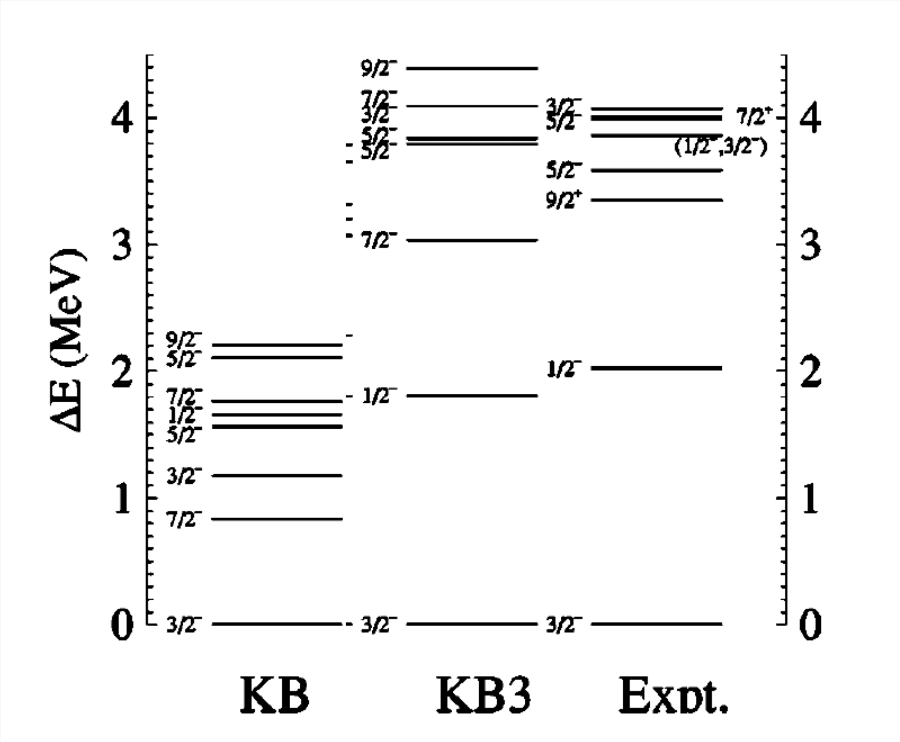
$n_b^\tau \equiv$ number of protons or neutrons n the *b* orbital

The case of ^{49}Ca

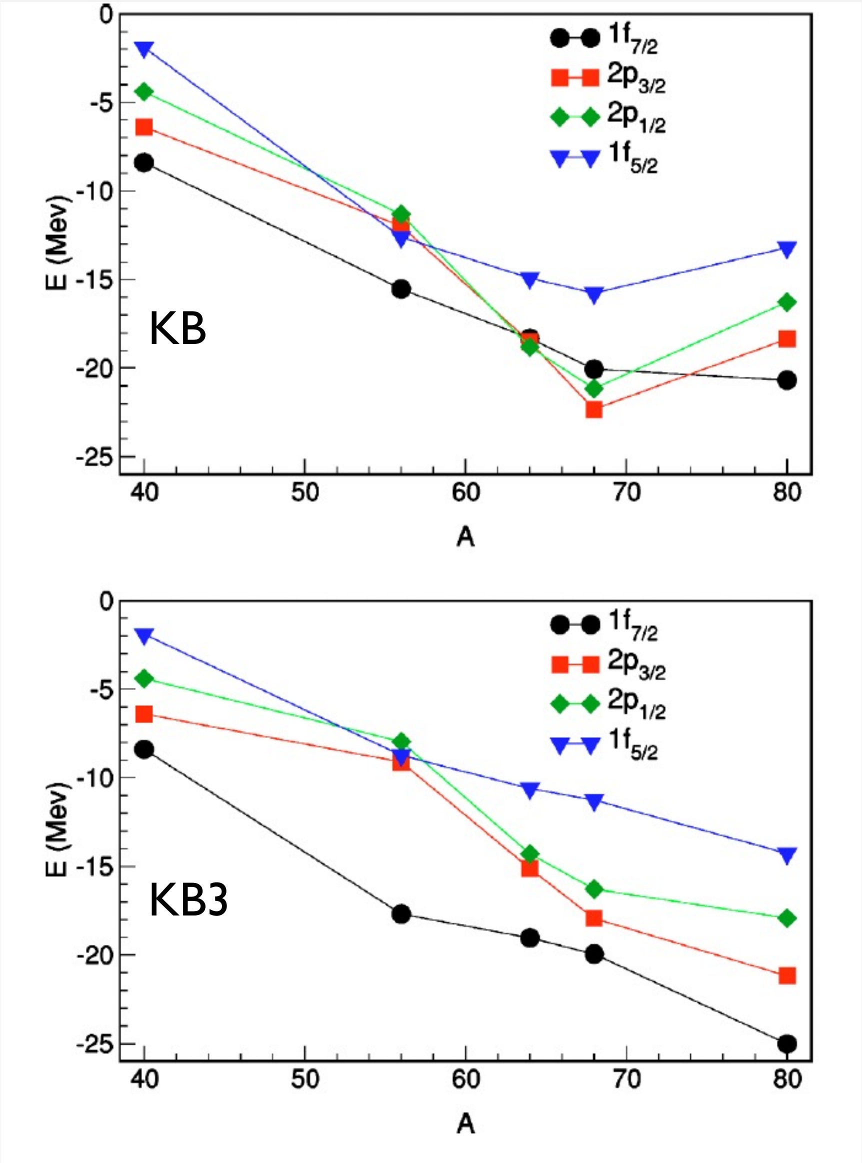
KB: Kuo-Brown interaction for the *fp* valence space

KB3: monopole modified interactions from KB

^{49}Ca



ESPE



E. Caurier, G. Martínez-Pinedo, F. Nowacki, A. Poves, A. P. Zuker, Rev. Mod. Phys. 77 (2005) 427

SOME SEMI-PHENOMENOLOGICAL SHELL-MODEL HAMILTONIANS

Label	Model space	Ref
KB3	1p0f	A. Poves, A.P. Zuker. Phys. Rep, 70, 235 (1981)
SDPF-U	1s0d-1p0f	F. Nowacki, A. Poves. PRC 79, 014310 (2009)
LNPS	$0f_{5/2}1p0g_{9/2}1d_{5/2}$	S. M. Lenzi et al, PRC 82, 054301 (2010)
SVD GCN5082	$0g_{7/2}1d2s0h_{11/2}$	C. Qi, Z.X.Xu, PRC,86, 044323 (2012) J. Menéndez et al, NPA 818, 139 (2009)
---	above 78Ni	K. Sieja et al, PRC 79, 064310 (2009)
…	…	…

MICROSCOPIC SHELL-MODEL HAMILTONIAN

$$V_{NN}(+V_{NNN}) \Longrightarrow many\ body\ theory \Longrightarrow H_{\mathrm{eff}}$$

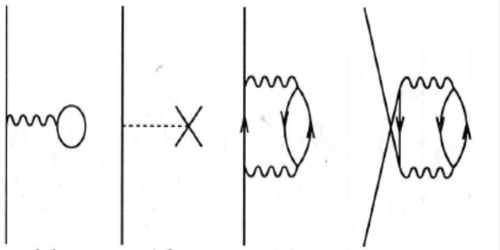
Hilbert space $H = T + V_{NN} = (T + U) + (V_{NN} - U) = H_0 + H_1 \Longrightarrow |\psi_\alpha\rangle;\ \ E_\alpha$

Model space $H_{\mathrm{eff}} = H_0 + H_1^{\mathrm{eff}} \Longrightarrow |\psi_\beta\rangle = P|\psi_\beta\rangle;\ \ E_\beta \in \{E_\alpha\}$

A well-established approach ”folded-diagrams expansion” is based on the many-body perturbation theory and has started with the seminal paper by Tom Kuo and Gerry Brown on sd-shell nuclei [Nucl. Phys. **85,** 40 (1966)]

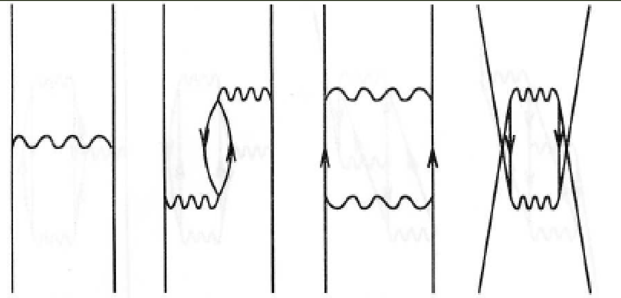
1-body diagrams up to 2nd order

S-box



2-body diagrams up to 2nd order:

V V_{1p1h} V_{2p} V_{2p2h}



+ 1 and 2-body third order diagrams (~200) + . . .

MODEL SPACE & DIMENSIONALITY OF SHELL-MODEL CALCULATIONS

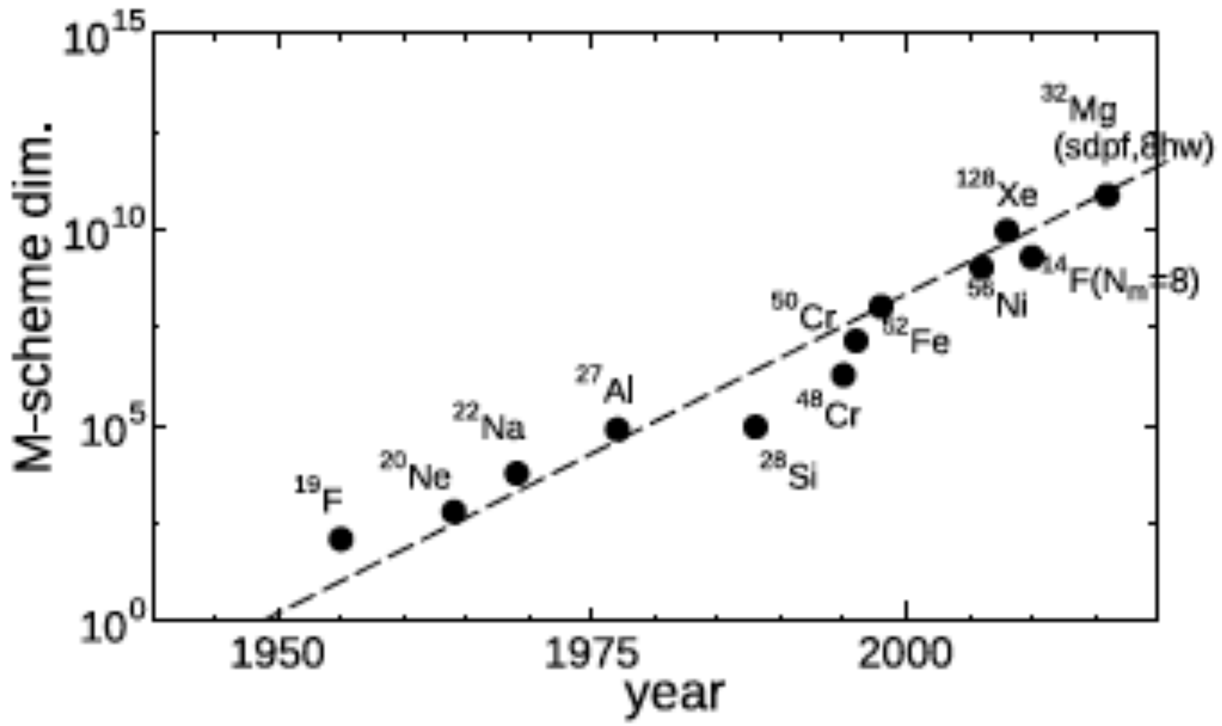
Table 1. *M*-scheme and *J*-scheme dimensions of several nuclei. The dimension of the $M^\pi = 0^+$ ($J^\pi = 0^+$) subspace is shown. See text for details.

Nuclide	Model Space	*M*-Scheme Dim	*J*-Scheme Dim
^{12}C	$2 \leq N, Z \leq 8$ (p shell)	51	9
^{28}Si	$8 \leq N, Z \leq 20$ (sd shell)	93,710	3372
^{56}Ni	$20 \leq N, Z \leq 40$ (pf shell)	1.08×10^9	1.54×10^7
^{78}Y	$28 \leq N, Z \leq 50$ (f_5pg_9, $jj44$)	1.31×10^{10}	1.11×10^8
^{130}Sm	$50 \leq Z, N \leq 82$ ($jj55$)	2.06×10^{15}	9.58×10^{12}
^{172}Dy	$50 \leq Z \leq 82, 82 \leq N \leq 126$ ($jj56$)	1.71×10^{19}	5.54×10^{16}

N. Shimizu,, Physics 2022, 4, 1081–1093

PERFORMANCE OF SHELL MODEL CODES

Manageable Hamiltonian size as a function of the year



Current limit of the Hamiltonian size $\sim 10^{11}$

N. Shimizu, T. Mizusaki, Y. Utsuno et al. Computer Physics Communications 244 (2019) 372–384

SHELL MODEL CODES

OXBASH→NuShellX@MSU

B.A. Brown et al., Report 524,1289; National Superconducting Cyclotron Laboratory: East Lansing, MI, USA, 1988;,2004;; Nucl. Data Sheets 2014, 120, 115–118.

ANTOINE (M scheme)
NATHAN (J scheme)

E. Caurier et al., Centre de Recherches Nucléaires: Strasbourg, France, 1989;1995

BIGSTICK

C.W. Johnson, et al., Comput. Phys. Commun. 2013,184, 2761–2774

KSHELL

N. Shimizu e al., Comput. Phys. Commun. 2019, 244, 372–384

PHYSICAL OBSERVABLES

$$\langle \phi_f \| \hat{O}^{\lambda} \| \phi_i \rangle$$

ϕ_f and ϕ_i are many-body wave functions obtained from the SM diagonalization

$$\langle \phi_f \| \hat{O}^{\lambda} \| \phi_i \rangle = \sum_{k_\alpha k_\beta} \mathrm{OBTD}(f i k_\alpha k_\beta \lambda) \langle k_\alpha \| \hat{O}^{\lambda} \| k_\beta \rangle$$

$$\mathrm{OBTD}(f i k_\alpha k_\beta \lambda) = \frac{\langle f \left\| \left[a^*_{k_\alpha} \otimes \tilde{a}_{k_\beta} \right]^{\lambda} \right\| i \rangle}{\sqrt{(2\lambda + 1)}}$$

depends on the eigenstates of H

$$\langle k_\alpha \| \hat{O}^{\lambda} \| k_\beta \rangle$$

depends on the operator structure calculated between SP states

We need

- ✓ the value of the operator between SP states
- ✓ the one body transition density matrix elements

ELECTROMAGNETIC OBSERVABLES

$$\langle \phi_f \| \hat{O}^\lambda \| \phi_i \rangle = \sum_{k_\alpha k_\beta} \mathrm{OBTD}(f i k_\alpha k_\beta \lambda) \langle k_\alpha \| \hat{O}^\lambda \| k_\beta \rangle$$

- **electric**

$$O(E\lambda) = r^\lambda Y^\lambda_\mu(\hat{r}) \, e_{t_z} e,$$

Free charges e_n=0, e_p=1

- **magnetic**

$$O(M\lambda) = \sqrt{\lambda(2\lambda+1)} \left[[Y^{\lambda-1} \otimes \vec{\ell}]^\lambda_\mu \frac{2g^\ell_{t_z}}{(\lambda+1)} + [Y^{\lambda-1} \otimes \vec{s}]^\lambda_\mu \, g^s_{t_z} \right] r^{\lambda-1} \mu_N,$$

Free g factors $g^\ell_p = 1,\ g^\ell_n = 0,\ g^s_p = 5.586$ and $g^s_n = -3.826.$

EFFECTIVE OPERATORS

$$\langle \phi_f | \hat{O}^{\lambda}_{\text{eff}} | \phi_i \rangle = \langle \psi_f | \hat{O}^{\lambda} | \psi_i \rangle$$

- $\phi_\nu \equiv$ eigenstate of the H_{eff} corresponding to the eigenvalue E_ν:

$$H_{\text{eff}} |\phi_\nu\rangle = E_\nu |\phi_\nu\rangle$$

- $\psi_\nu \equiv$ eigenstate of the full Hamiltonian corresponding to the eigenvalue E_ν:

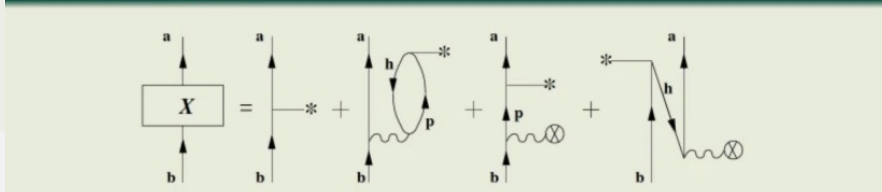
$$H |\psi_\nu\rangle = E_\nu \ |\psi_\nu\rangle$$

$$\langle k_\alpha || \hat{O}^{\lambda} || k_\beta \rangle \Longrightarrow \langle k_\alpha || \hat{O}^{\lambda}_{\text{eff}} || k_\beta \rangle$$

- empirical effective charges and empirical gyromagnetic factors

- microscopic effective operators derived consistently with SM Hamiltonian

One-body operator



GLOSSARY

- **STATE:** a solution of the Schrödinger equation for the effective SM Hamiltonian
- **ORBITAL:** the ensemble of SP states with the same quantum numbers nlj, e.g. 0d5/2. with degeneracy is (2j+1)
- **SHELL:** an ensemble of orbitals quasi-degenerated in energy, e.g. the sd shell
- **MAGIC NUMBERS:** the numbers of protons or neutrons that fill sequentially a certain number of shells
- **SHELL GAP:** the energy difference between two shells
- **MODEL SPACE:** the ensemble of orbitals included in the SM calculation
- **SPE**: SP energies, namely the eigenvalues of the one-body component of the Hamiltonian
- **TBME:** two-body matrix elements of the effective interaction

FLOWCHART OF SHELL-MODEL CALCULATIONS

Identify the best model space for the nucleus under investigation: the set of orbitals that are accessible by the valence nucleons

Fix the Hamiltonian: the one-body term U_j and the V_{ij}

Construct the basis, and the ME of the Hamiltonian

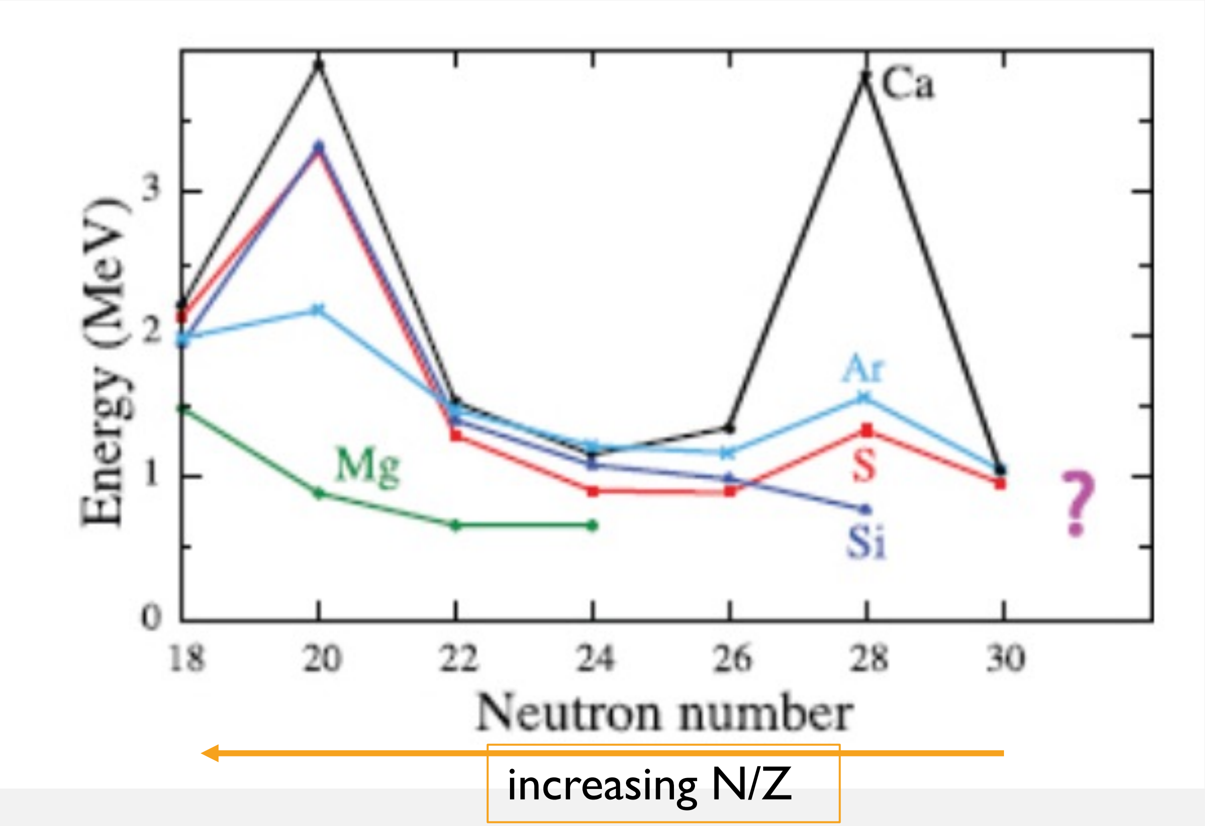
Diagonalize the shell-model Hamiltonian → energies, wave functions

Calculate physical observables by using the SM wave functions and effective operators

SOMETHING TO CONSIDER

The shell structure known so far is based on stable nuclei

New data on light nuclei with N >> Z show significant modifications → in neutron-rich exotic nuclei, traditional magic numbers (N=8, 20, etc.) can disappear, while new ones (N=6, 16, etc.) may emerge



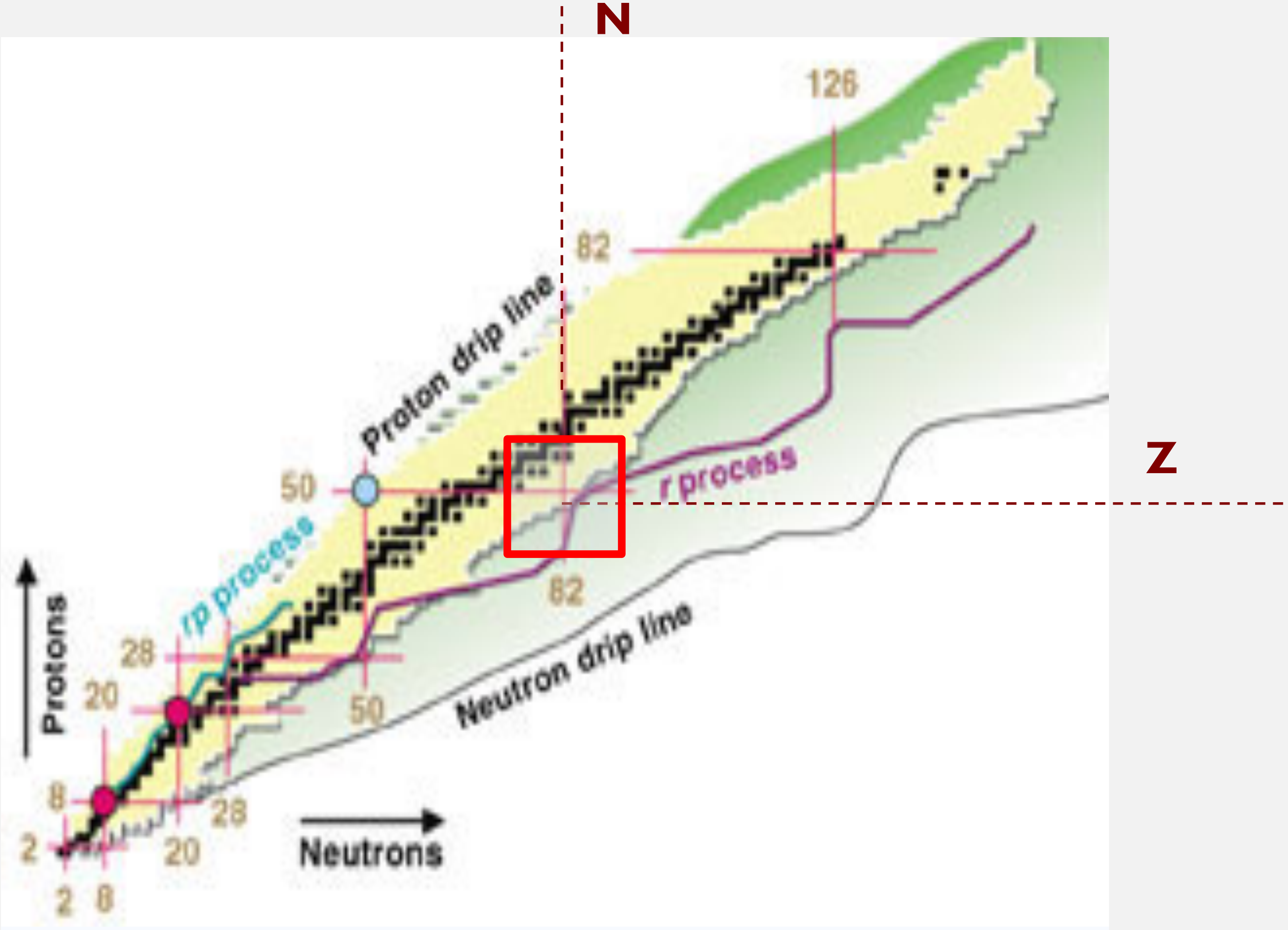
	Mg	Si	S	Ar	Ca
Z	12	14	16	18	20

How do magic numbers evolve as a function of the neutron-to-proton ratio and what are the underlying mechanisms?

- Large-scale shell-model calculations (LSSM) including more than one major shell
- 3 body forces
- …

SOME STUDY CASES

^{132}Sn REGION



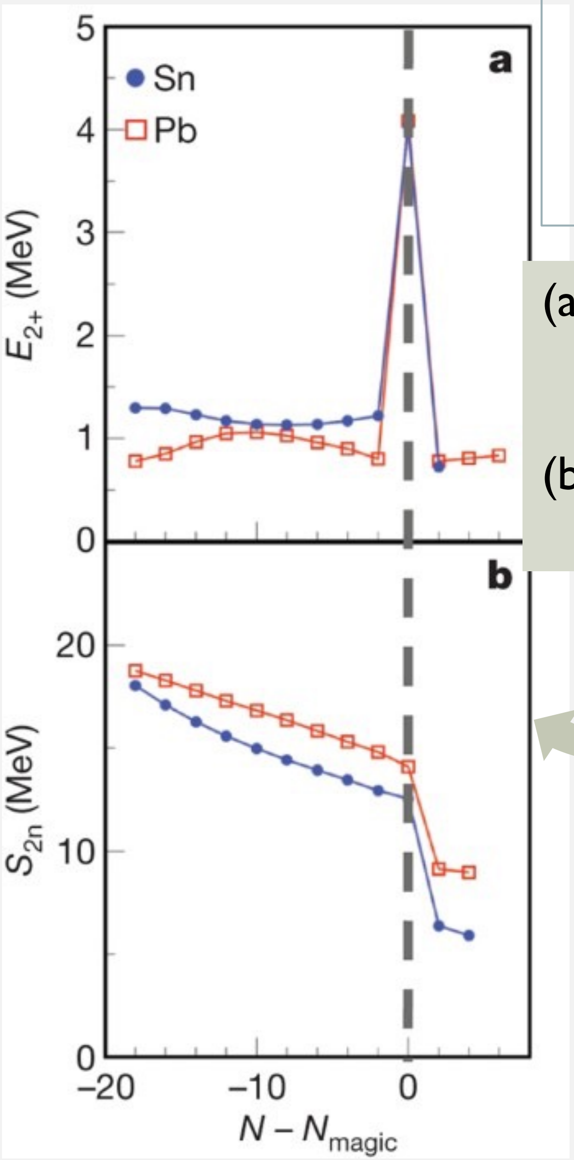
The only region around a heavy, neutron rich doubly-closed shell nucleus far-off stability experimentally accessible today

- ➢ shell evolution & underlying driving forces to search for effects similar to those observed in lighter nuclei

- ➢ provide predictions for nuclei
- ✓ still inaccessible for present experiments
- ✓ involved in $0\nu\beta\beta$ decay (^{130}Te, ^{136}Xe)
- ✓ of great relevance for the rapid neutron-capture process,

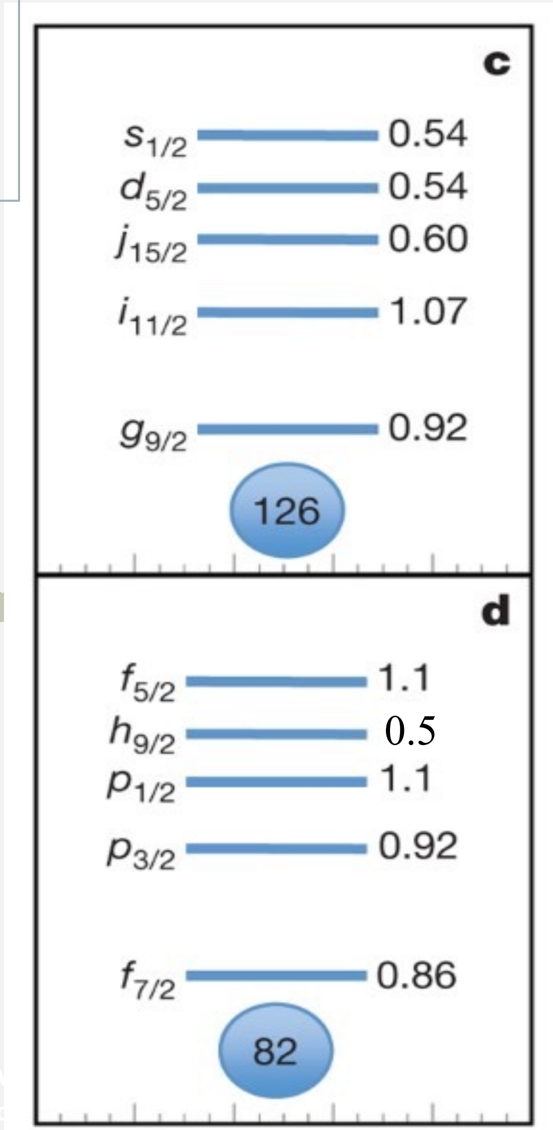
THE MAGICITY OF ^{132}Sn

^{132}Sn can be considered a very robust doubly magic core, as ^{208}Pb, by mass measurement and transfer reactions experiments, which illustrate the single-particle nature of the levels in ^{133}Sn



(a) The excitation energy of the first 2^+ state in ^{132}Sn and ^{208}Pb show a large discontinuity in comparison with those of the neighbouring isotopes.

(b) A sudden decrease is observed in the 2-neutron separation energies just beyond the shell closure in both cases.



Spectroscopic factors of the single-neutron orbitals in (c) ^{209}Pb (d) and (d) ^{133}Sn.
Quite large spectroscopic factors in ^{133}Sn , like those of ^{209}Pb, indicate little fragmentation of the single-neutron strength.

K. Jones et al, Nature 465, 454 (2010).
J. M. Allmond et al, Phys. Rev. Lett. 112, 172701 (2014)

RSM CALCULATIONS FOR NUCLEI IN ^{132}Sn REGION

model space for $Z \geq 50, N \geq 82$

π: $h_{11/2}$, $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$

ν: $i_{13/2}$, $f_{5/2}$, $p_{1/2}$, $h_{9/2}$, $p_{3/2}$, $f_{7/2}$

model space for $Z \geq 50, N < 82$

π, ν: $h_{11/2}$, $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$

Nuclei	Valence neutrons	Valence protons
^{134}Sn	2	0
^{134}Te	0	2
^{134}Sb	1	1

to direct test
- SPE
- TBME

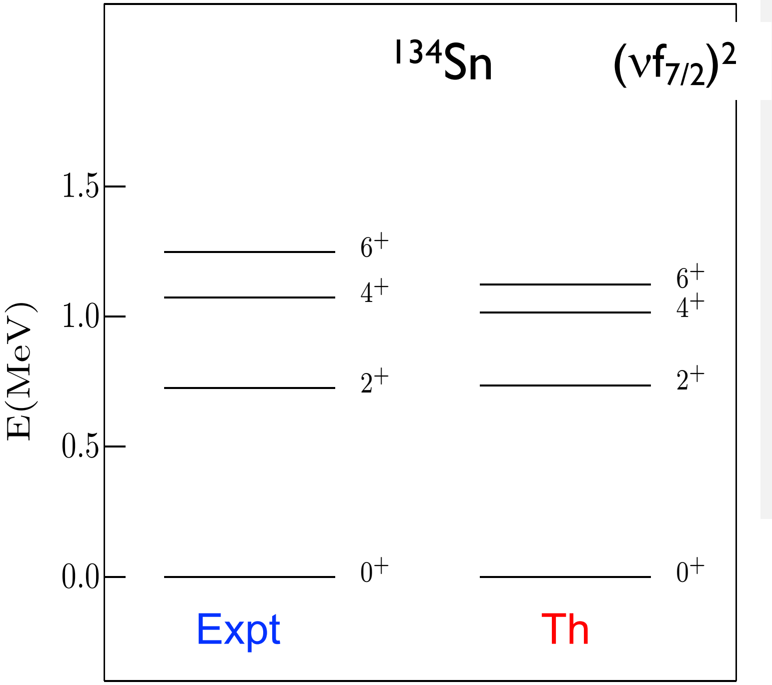
CD-Bonn NN potential

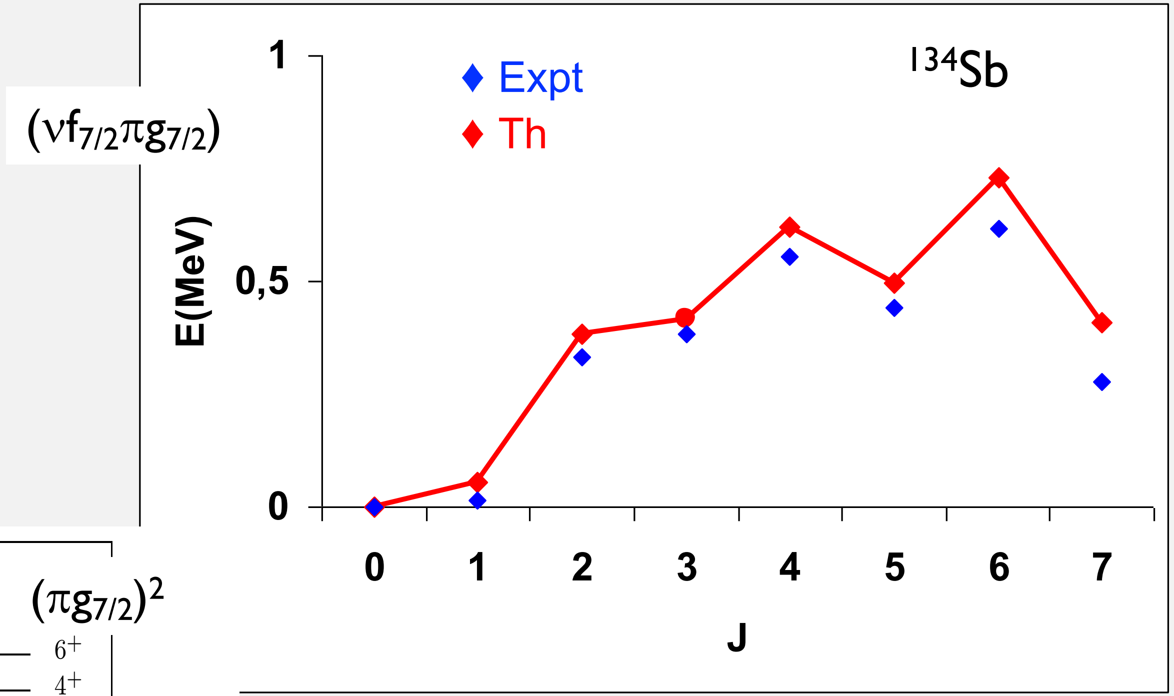
$V_{\text{low-k}}$ with Λ=2.2 fm^{-1}

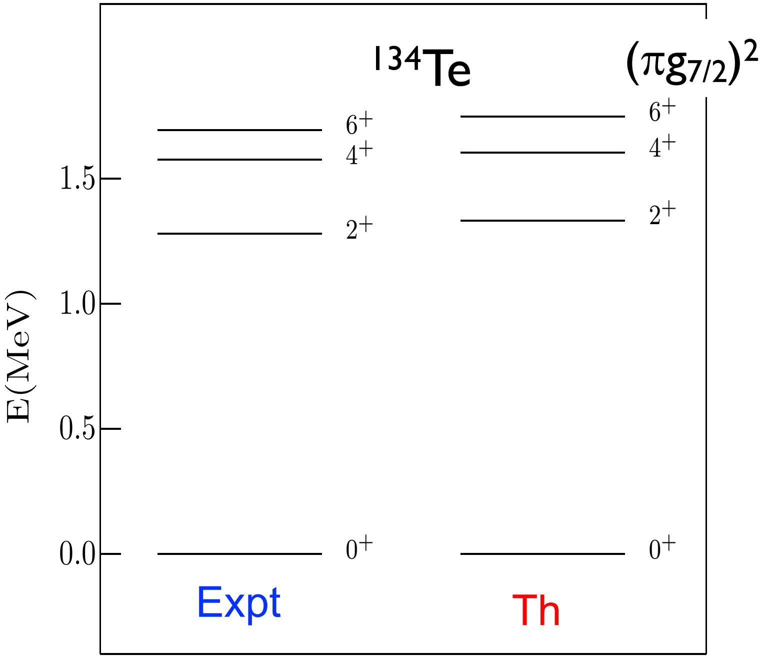
TBME from folded diagram approach

Nuclei	
$^{134\text{-}140}$Sn	Neutron shell evolution
N=83, even Z>50 N=82, odd Z>50	Magicity of ^{132}Sn via electromagnetic moments
^{129}Sb	Emerge of collectivity
^{132}Te	Mixed symmetry states

^{134}Sn - ^{134}Te - ^{134}Sb

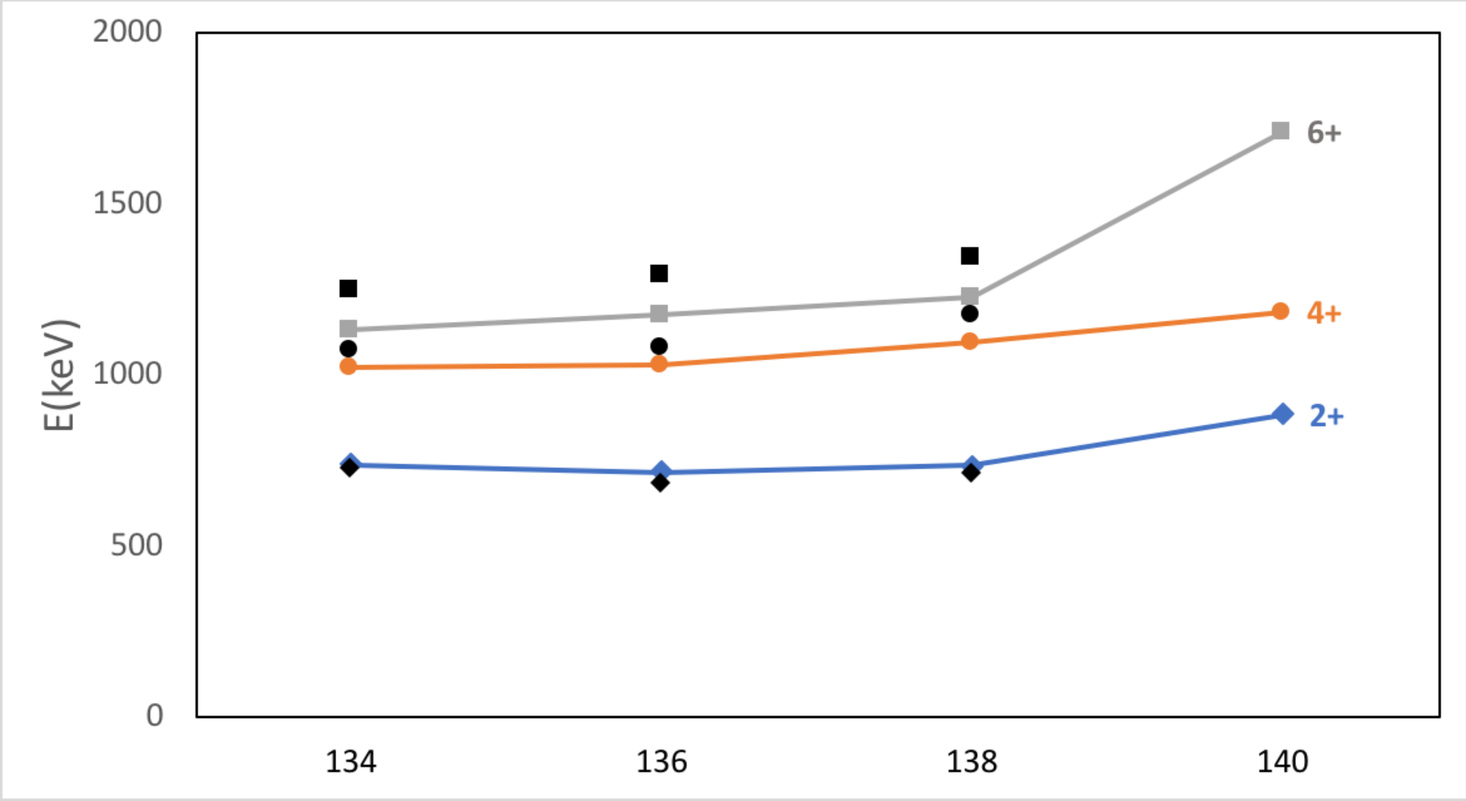






pairing reduction in ^{134}Sn as compared to ^{134}Te

SHELL EVOLUTION IN Sn ISOTOPES FOR N>82

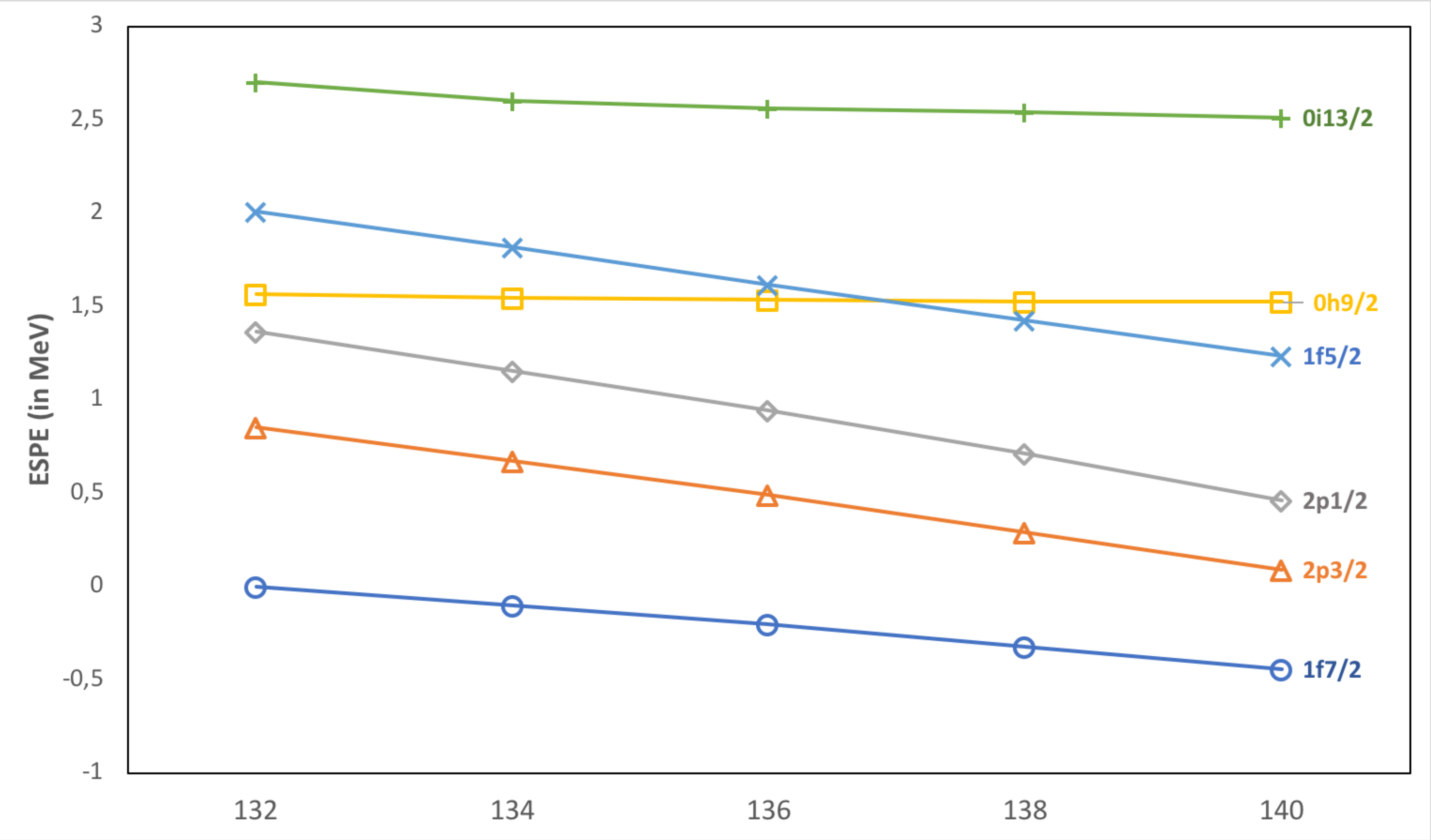


		^{134}Sn n=2	^{136}Sn n=4	^{138}Sn n=6	^{140}Sn n=8
$(f_{7/2})^n$	0^+	80	64	60	35
	2^+	85	66	60	
	4^+	94	75	64	
	6^+	98	83	71	
$(f_{7/2})^{n-1}\, p_{3/2}$	0^+				
	2^+	11	12	13	27
	4^+				62
	6^+				
$(f_{7/2})^{n-2}(p_{3/2})^2$	0^+		10	15	21
	2^+				21
	4^+				
	6^+				56

No significant upshift of the 2+ state in ^{140}Sn → No significant changes of the $1f_{7/2}$-$2p_{3/2}$ gap

→No shell gap @N=90

NEUTRON ESPE IN Sn ISOTOPES FOR N>82



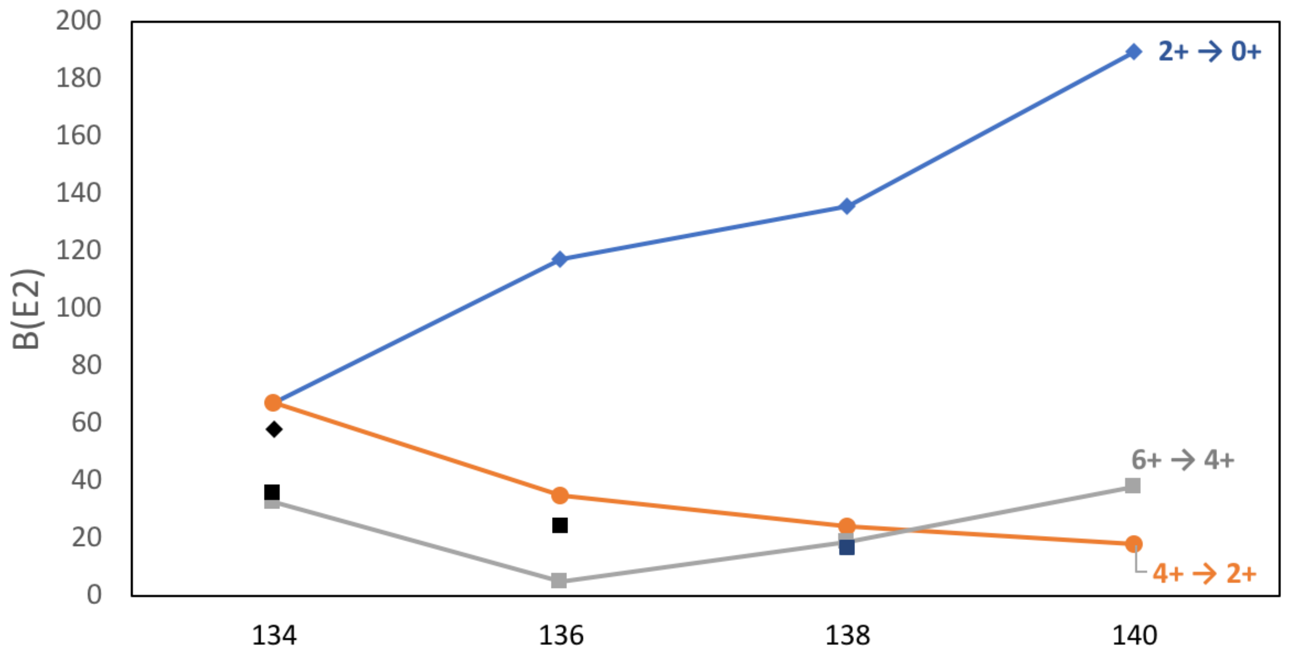
Neutron ESPE as a function of the valence neutron number

$$\mathrm{ESPE}(a\tau) = \epsilon_{a\tau} + \sum_{b\tau'} \bar{V}_{ab}^{\tau\tau'} n_b^{\tau'}$$

$$\bar{V}_{ab}^{\tau\tau'} = \frac{\sum_J \hat{J} \langle a\tau b\tau'; J \mid V_{\mathrm{eff}} \mid a\tau b\tau'; J \rangle}{\sum_J \hat{J}}$$

$n_b^{\tau} \equiv$ number of neutrons in the *b* orbital

E2 PROPERTIES IN Sn ISOTOPES FOR N>82



Expt G S Simpson et al, Phys.Rev. Lett.113, 132502 (2014)

Theory deviates from expt at A=136.

^{136}Sn

E	calc	expt
0+	0	0
2+	737	688
4+	1020	1079
6+	1133	1295
4+	1180	

B(E2)	calc	expt
$2^+ \rightarrow 0^+$	117	
$4^+ \rightarrow 2^+$	34	
$6^+ \rightarrow 4^+_1$	5	24(4)
$4^+_2 \rightarrow 2^+$	74	
$6^+ \rightarrow 4^+_2$	66	

E2 PROPERTIES IN Sn ISOTOPES FOR N>82

Small changes (~50/100 keV) in the $(f7/2)^2$ J=2,4,6 diagonal MEs to favor the mixing between the two 4^+ states

E	calc	expt
0+	0	0
2+	737	688
4+	1020	1079
6+	1133	1295
4+	1180	

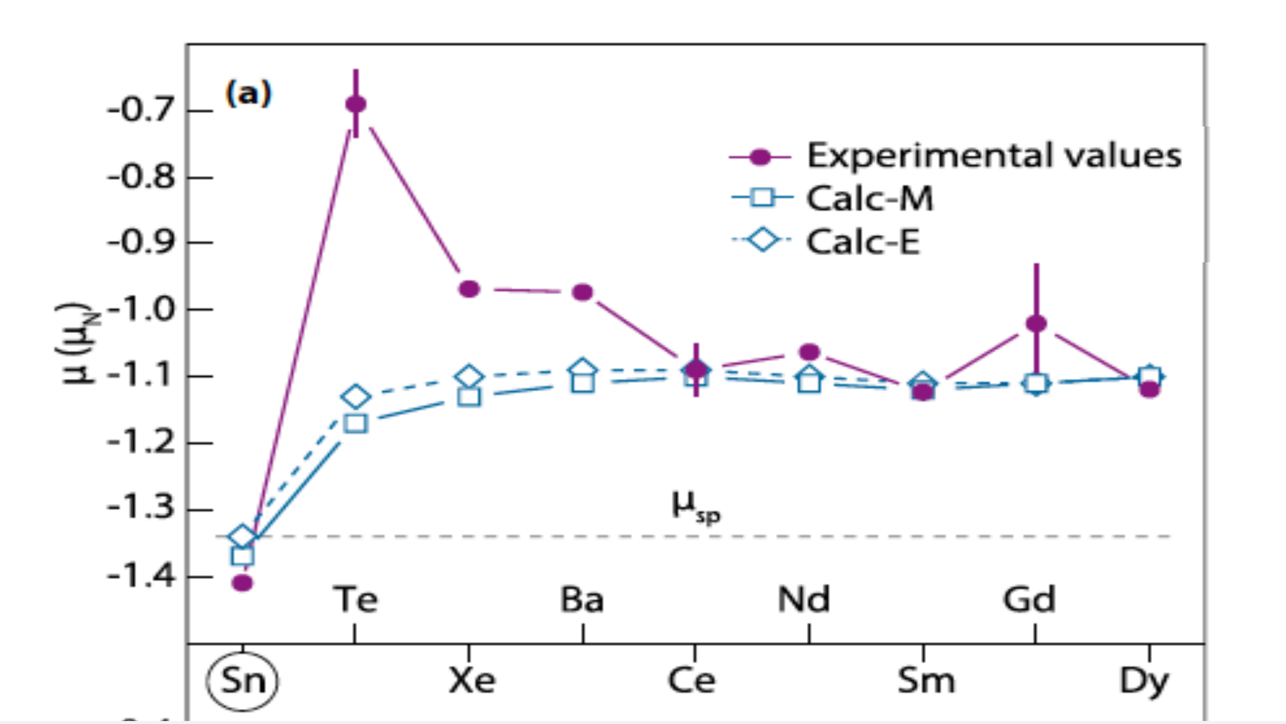
E	calc	expt
0+	0	0
2+	658	688
4+	1054	1079
4+	1176	
6+	1280	1295

B(E2)	calc	expt
$2^+ \rightarrow 0^+$	117	
$4^+ \rightarrow 2^+$	34	
$6^+ \rightarrow 4^+$	5	24(4)
$6^+ \rightarrow 4^+_2$	66	

B(E2)	calc	expt
$2^+ \rightarrow 0^+$	119	
4+ --> 2+	90	
6+ -->4+	32	24(4)
$6^+ \rightarrow 4^+_2$	41	

THE MAGICITY OF ^{132}Sn VIA ELECTROMAGNETIC MOMENTS

μ moments in 7/2^{-} ground state of even Z≥50, N = 83 nuclei



g^l_{eff}(p)=1 g^l_{eff}(n)=0
g^s_{eff}(p,n)= 0.7 g_s(p,n)

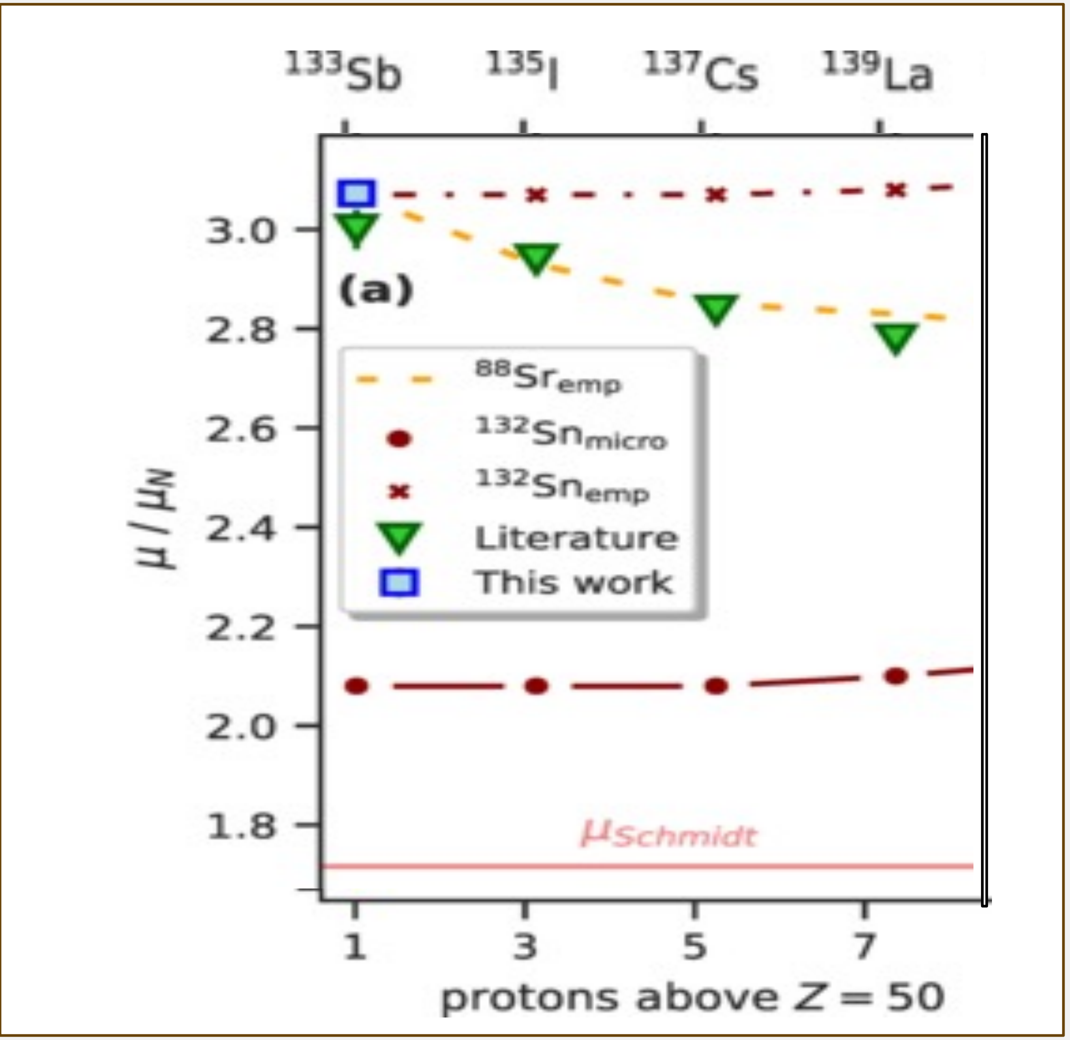
emprical g factors

$$|7/2^-\rangle = \approx 85\% \, |\pi 0^+_1 \otimes \nu f_{7/2}\rangle + (5-6)\% |\pi 2^+_1 \otimes \nu f_{7/2}\rangle + \cdots$$ for Z>50

L.V. Rodríguez *et al.*, PRC **102**, 051301(R) (2020)

THE MAGICITY OF ^{132}Sn VIA ELECTROMAGNETIC MOMENTS

μ moments in 7/2$^+$ ground state of odd Z>50, N = 82 nuclei

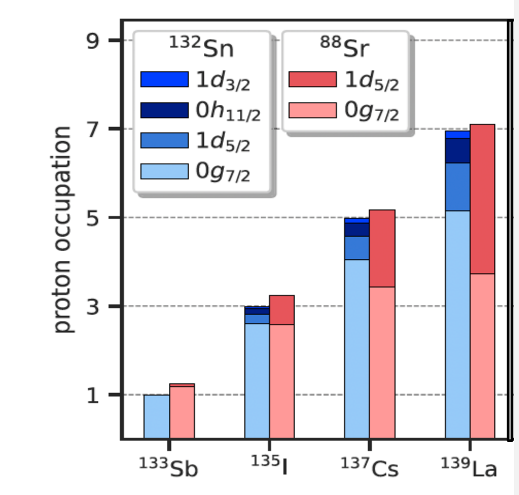


g^l_{eff}(p)=1.18 g^l_{eff}(n)=0

g^s_{eff}(p,n)= 0.7 g_s(p,n)

emprical g factors

→ Relevance of first-order core polarization renormalization induced by spin-flip $\pi 0g_{9/2} \rightarrow \pi 0g_{7/2}$ transitions. Need to explicitly include proton cross shell excitations (see calculations with ^{88}Sr core



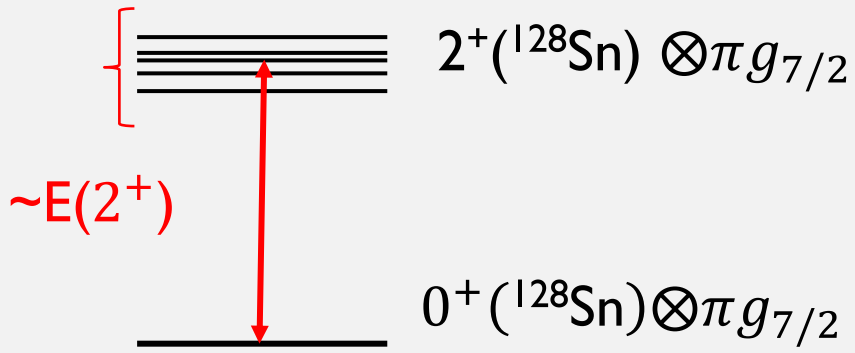
S. Lechner et al, Phys. Rev. C 104, 014302 (2021)

EMERGE OF COLLECTIVITY NEAR ^{132}Sn: ^{129}Sb

$^{129}_{51}\mathrm{Sb} \Leftrightarrow {}^{128}\mathrm{Sn} + 1\pi$

Weak particle-core coupling scheme:

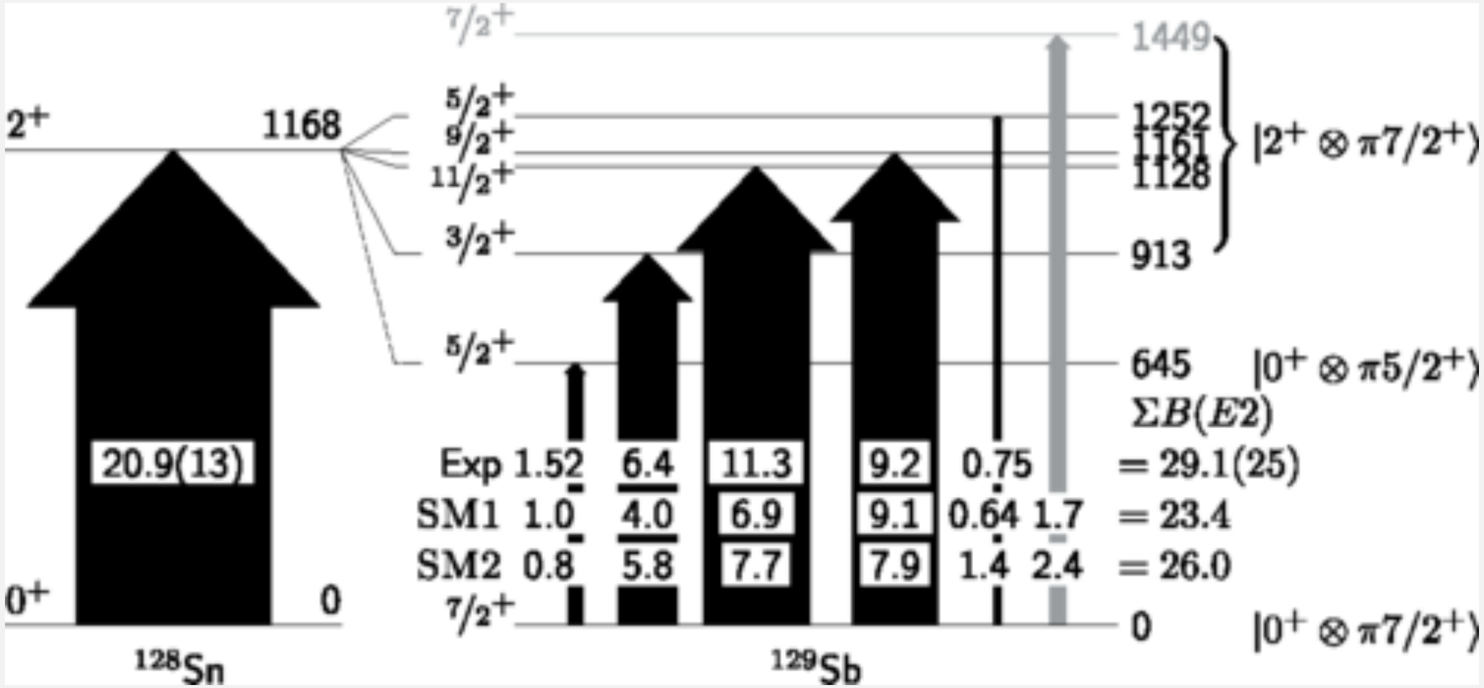
$3/2^{+}$, $5/2^{+}$ $7/2^{+}$ $9/2^{+}$ $11/2^{+}$



$$\sum B(E2;\uparrow) = B\left(E2;\ 0^{+}_{gs} \rightarrow 2^{+}\right)_{core}$$

When does this scheme break down and collectivity develop owing to the np interaction?

EMERGE OF COLLECTIVITY NEAR ^{132}Sn: ^{129}Sb



	n	p	np
^{129}Sb	19.6	2.2	4.3

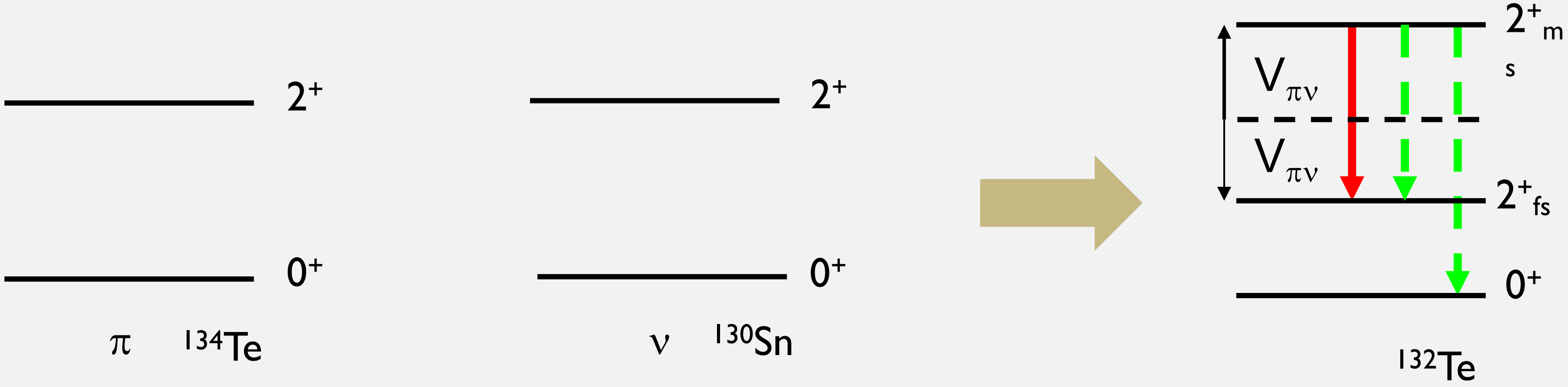
core	$B(E2;0^+_{gs} \rightarrow 2^+_1)$ In Wu	$\sum B(E2;\uparrow)$ Calc	$\sum B(E2;\uparrow)$ Expt
^{128}Sn	20.9(13)	26	29.1(25)

$$B(E2;\uparrow) = \overbrace{\left(e_p \frac{A_p}{\sqrt{2J+1}}\right)^2}^{\mathbf{p}} + \overbrace{\left(e_n \frac{A_n}{\sqrt{2J+1}}\right)^2}^{\mathbf{n}} + \overbrace{2e_p\, e_n \frac{A_p\, A_n}{(2J+1)}}^{\mathbf{np}}$$

T. J. Gray et al, Phys. Rev. Lett. 124, 032502 (2020)

ONE-PHONON 2^+ VIBRATIONAL STATES: ^{132}TE

$$^{132}\mathrm{Te} \Leftrightarrow {}^{132}\mathrm{Sn} + 2p + 2n^{-1} \Leftrightarrow {}^{134}\mathrm{Te} \otimes {}^{130}\mathrm{Sn}$$



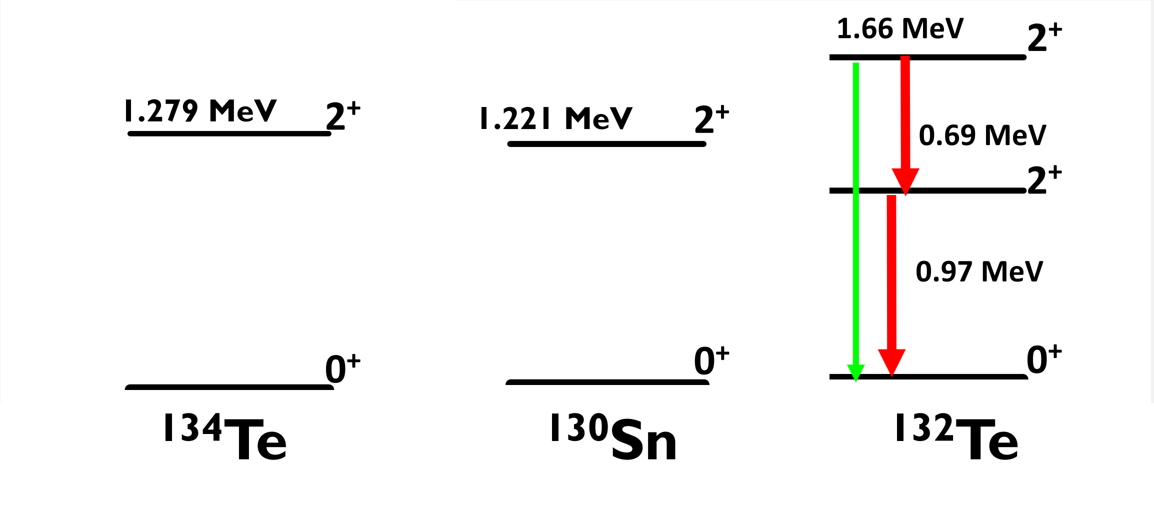
Two one-phonon 2^+ states may appear as a symmetric or antisymmetric combinations of the π & ν configurations:

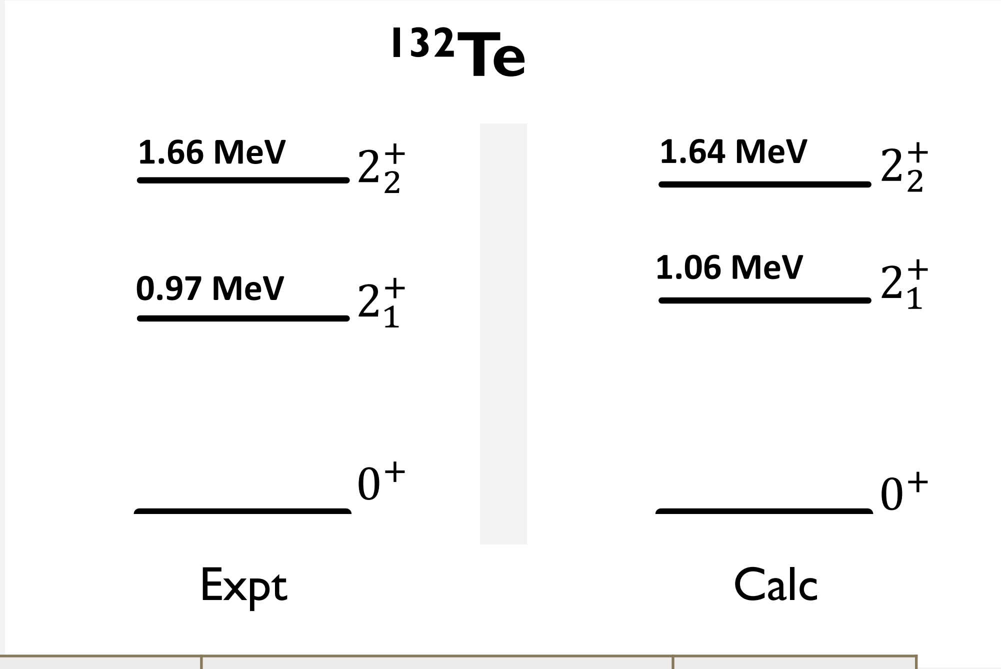
$$|2^+_{FSS}\rangle = \alpha|0^+(\pi)\rangle|2^+(\nu)\rangle + \beta|0^+(\nu)\rangle|2^+(\pi)\rangle \qquad |2^+_{MS}\rangle = \alpha|0^+(\pi)\rangle|2^+(\nu)\rangle - \beta|0^+(\nu)\rangle|2^+(\pi)\rangle$$

$|2^+_{MS}\rangle$ characterized by

- strong M1 to 2^+_{FSS} due its isovector nature
- weak E2 to 0^+ due to the partial cancellation of the neutron and proton contributions

ONE-PHONON 2^+ VIBRATIONAL STATES: ^{132}TE





$B(E2;\ 2_1^+ \to 0^+)$	10(1) Wu
$B(E2;\ 2_2^+ \to 0^+)$	0.5(1) Wu
$B(\mathrm{M1};\ 2_2^+ \to 2_1^+)$	5.4(35) - >0.23 μ_N^2

	Expt	Calc
$B(E2;\ 2_1^+ \to 0^+)$	10(1) Wu	7.8 Wu
$B(E2;\ 2_2^+ \to 0^+)$	0.5(1) Wu	0.21 Wu
$B(\mathrm{M1};\ 2_2^+ \to 2_1^+)$	5.4(35) - > 0.23 μ_N^2	0.20 μ_N^2

$$|0_1^+\rangle = 0.94|0_1^+\rangle_\nu|0_1^+\rangle_\pi + \cdots,$$

$$|2_1^+\rangle = 0.62|2_1^+\rangle_\nu\ |0_1^+\rangle_\pi + 0.66|0_1^+\rangle_\nu\ |2_1^+\rangle_\pi + \cdots,$$

$$|2_2^+\rangle = 0.63|2_1^+\rangle_\nu\ |0_1^+\rangle_\pi\ - 0.58|0_1^+\rangle_\nu\ |2_1^+\rangle_\pi + \cdots$$

M. Danchev, Phys. Rev. C 84, 061306(R) (2011)

THANK YOU FOR YOUR ATTENTION

&

ENJOY EXPLORING SHELL-MODEL CALCULATIONS