UNIVERSAL IMPLICATIONS OF S-MATRIX UNITARITY

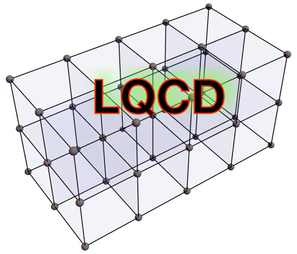
*Universality WS @ ECT**
June 2025

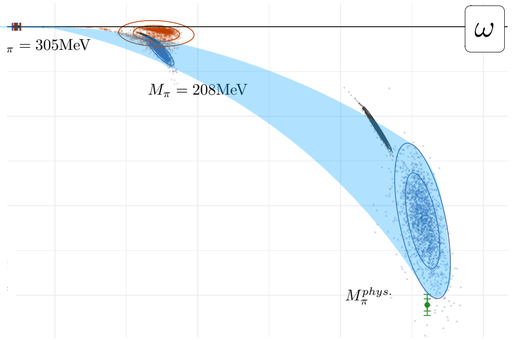
MAXIM MAI

ALBERT EINSTEIN CENTER (BERN)

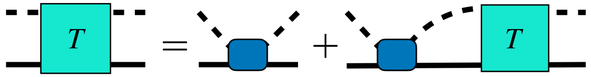
TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076 DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907 , NSF: PHY-2012289

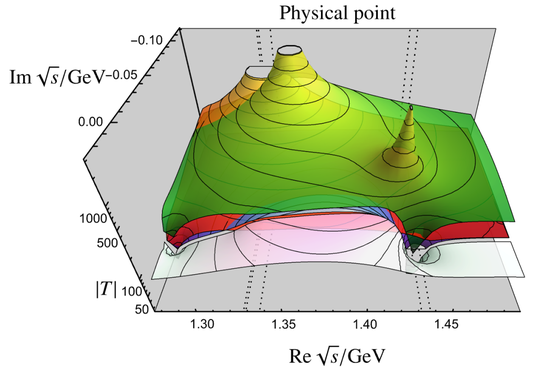
… rediscover 3-hadron resonances from lattice QCD



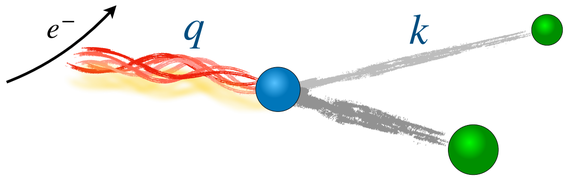


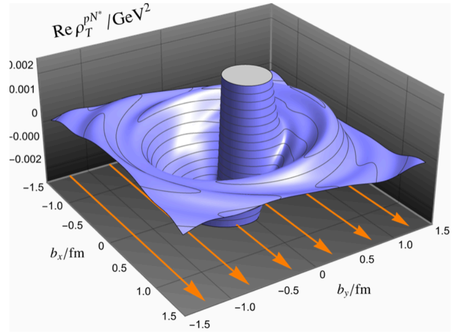
… discover new resonances from Chiral Dynamics





… clues about nature of states





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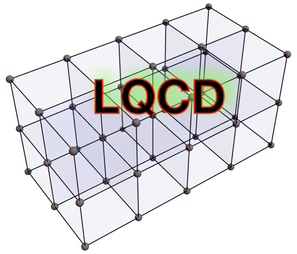

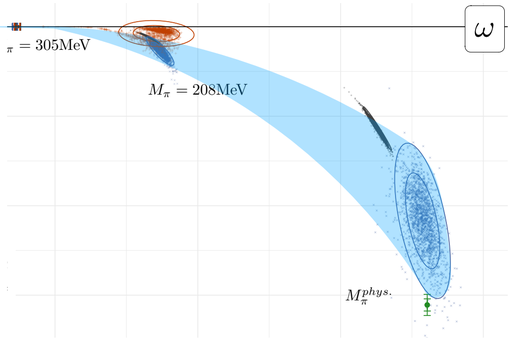

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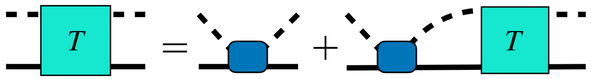

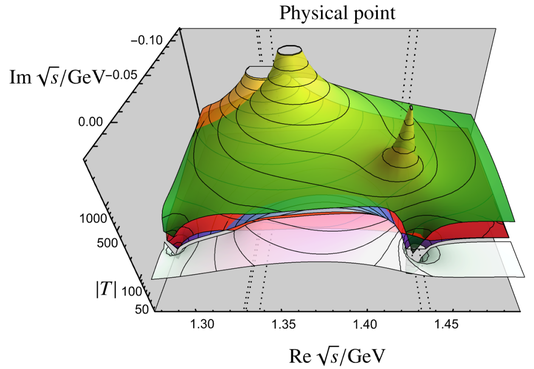

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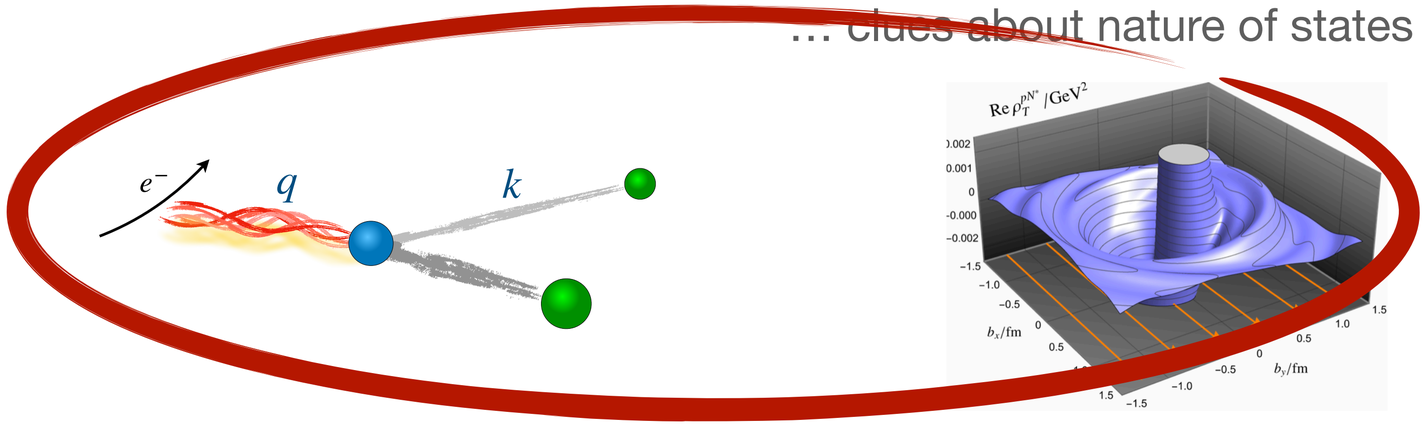

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HADRON SPECTRUM

Experimental progress

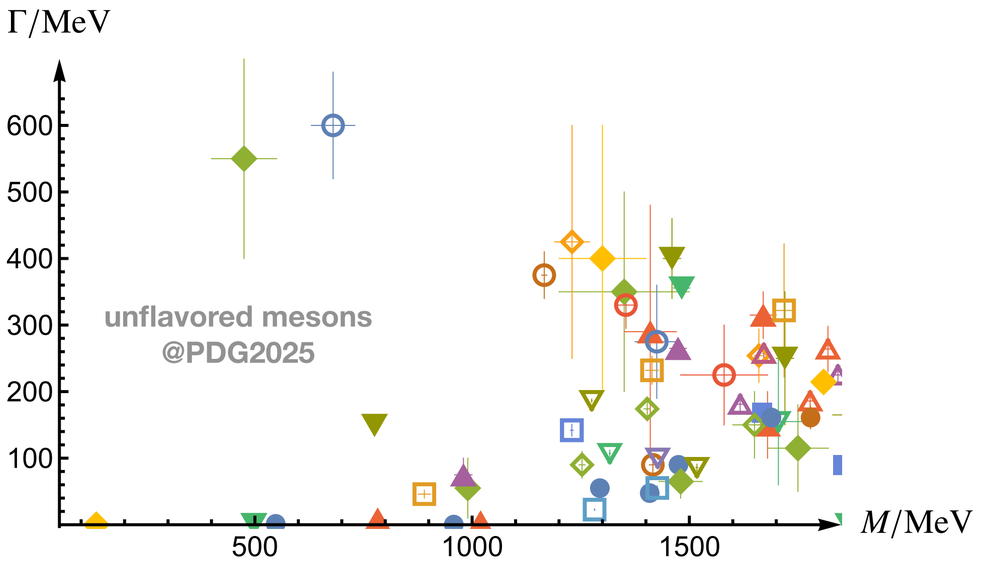
- 70y research: $\Delta(1232), \rho(770), \omega(782), \ldots$
- ongoing progress, new techniques and experiments
- mostly excited states

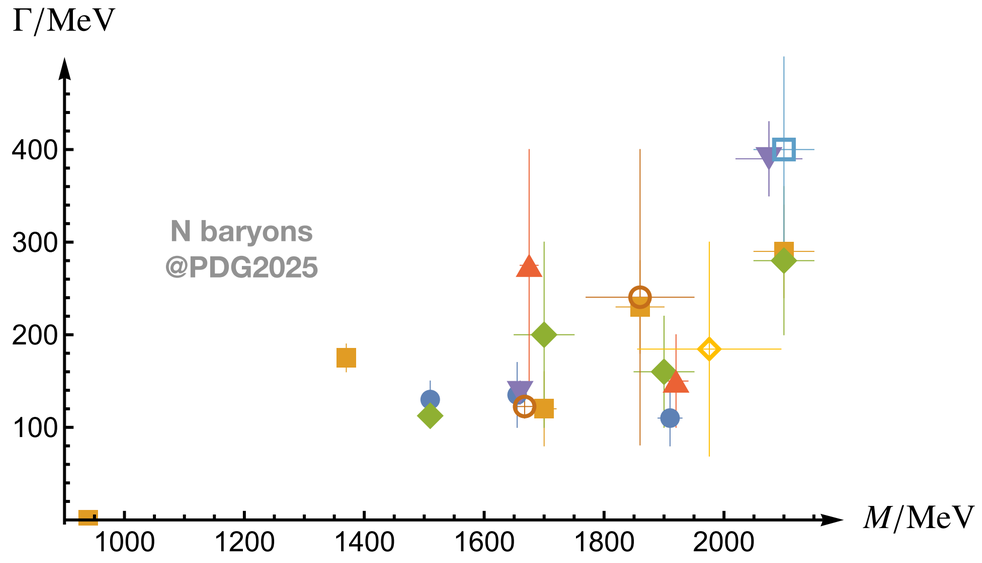
≈ 100 mesons + 50 baryons (****)

Key questions

🦅 “what is the pattern of these states?“

🐸 “how are they formed?”

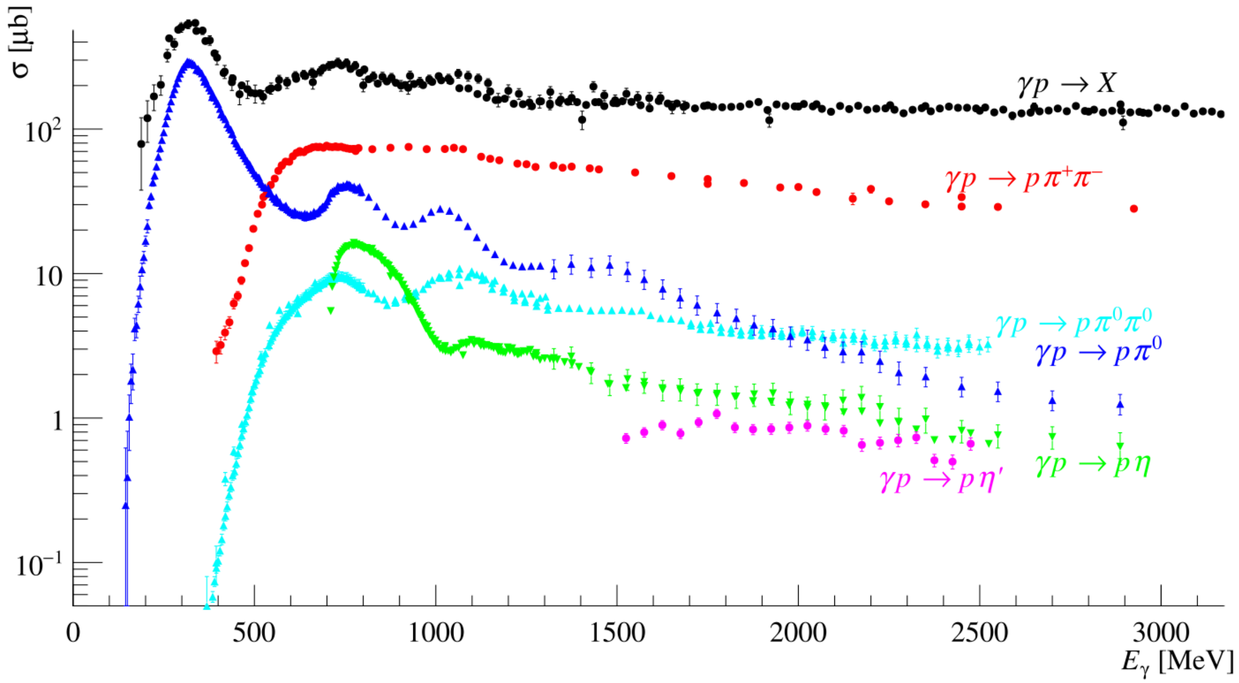




Review: MM/Meißner/Urbach Phys.Rept. 1001 (2023) 1-6

EXPERIMENTS / RESONANCES

Data: JLAB, ELSA, MAMI CLAS12, GlueX, …
Plot: Thiel, Afzal, Wunderlich, Light Baryon Spectroscopy, Prog. Part. Nucl. Phys. 125 (2022) 103949



Observations

- many available data and ongoing experiments
  BESII, GlueX, LHCb, CLAS…
- Photon-induced excitation via meson photo-/ electroproduction
  - large amount of data: $N_{\gamma^* p \to MB} = \mathcal{O}(10^5)$
  - many more data $Q^2 = 5 - 12\,\mathrm{GeV}^2$

  CLAS12@Jlab, Carman et al. arXiv:2505.12030, …

Resonances:

- increased interaction rates (bumps)

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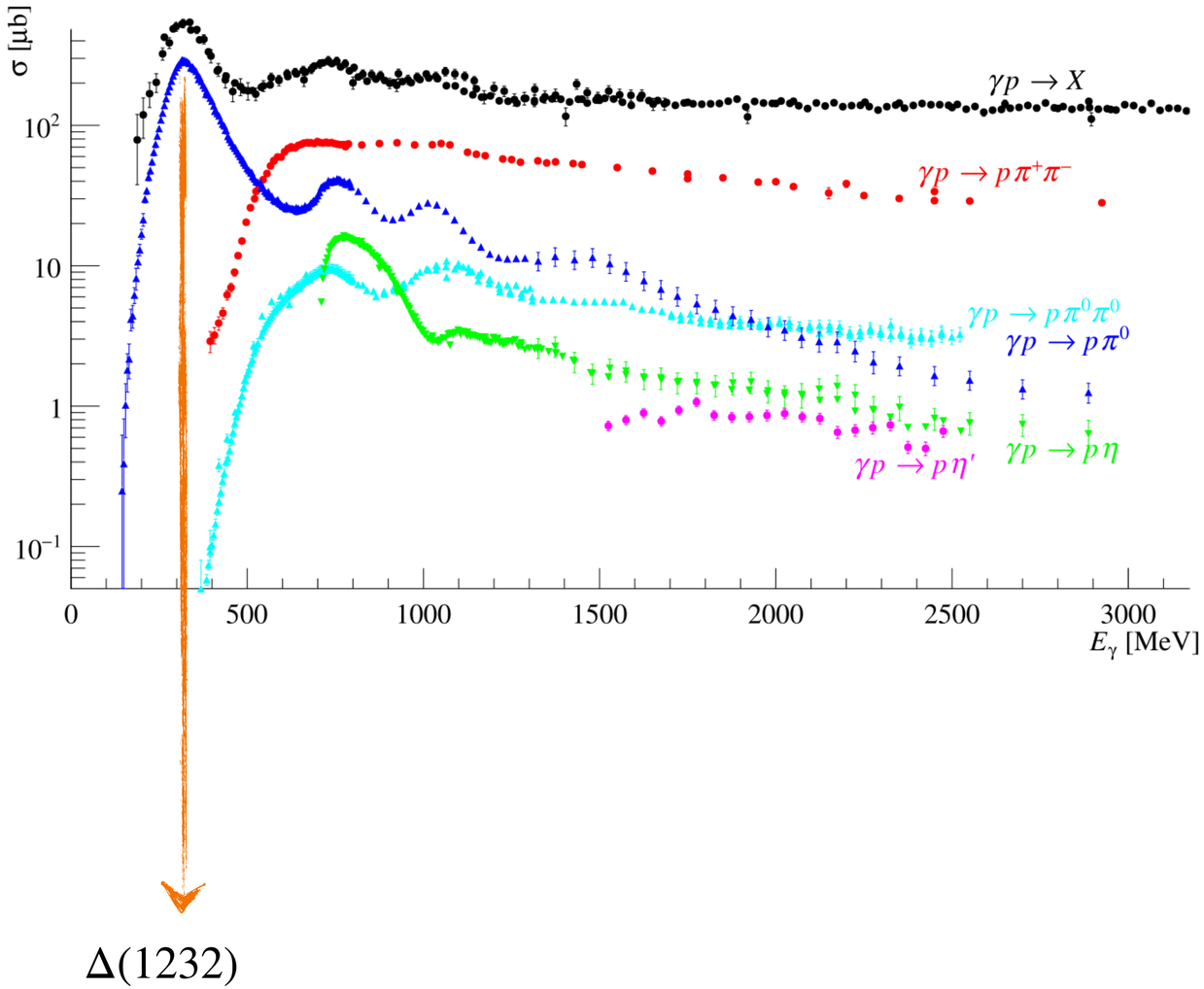

Anderson/Fermi/…PhysRev.85.934

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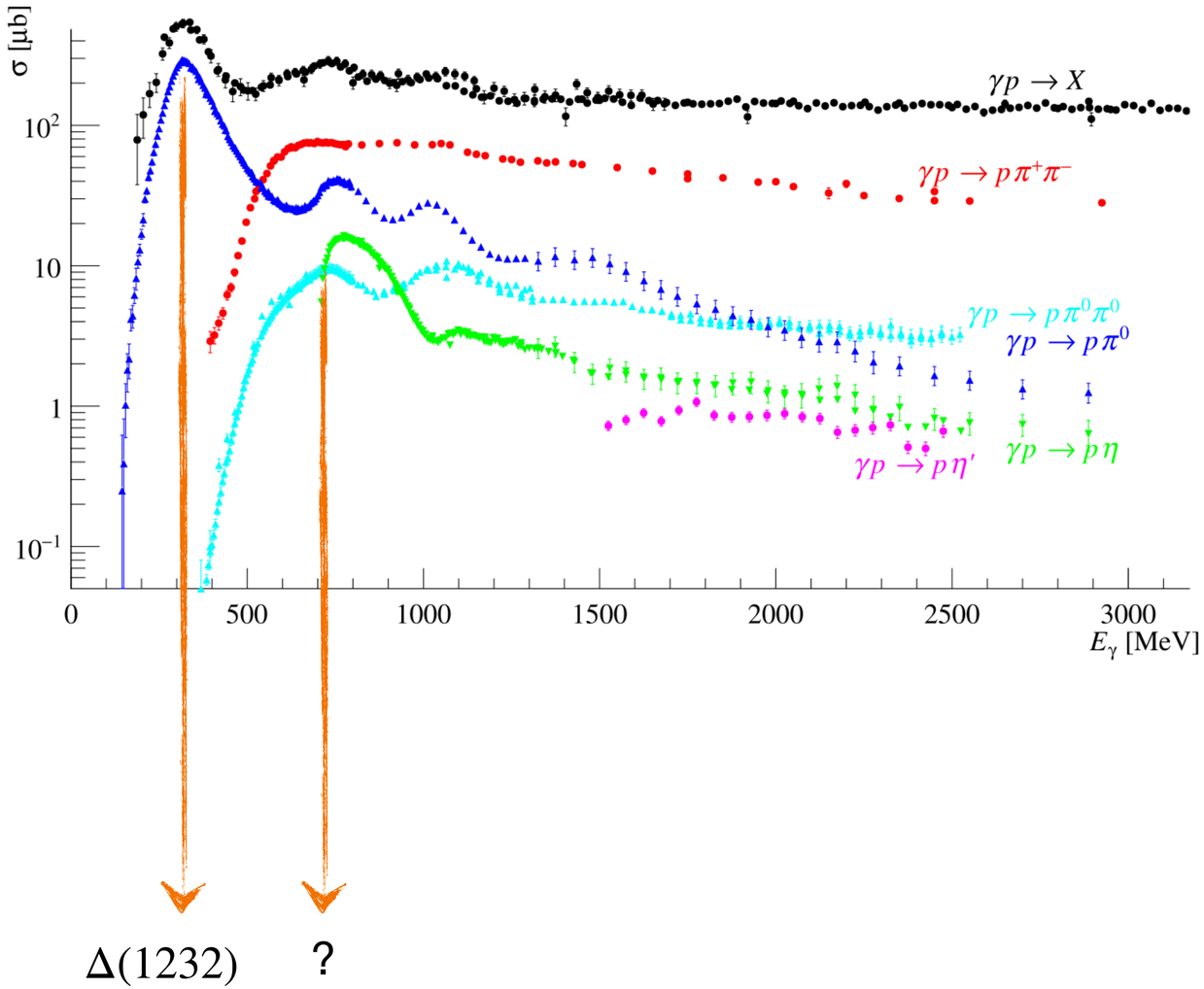

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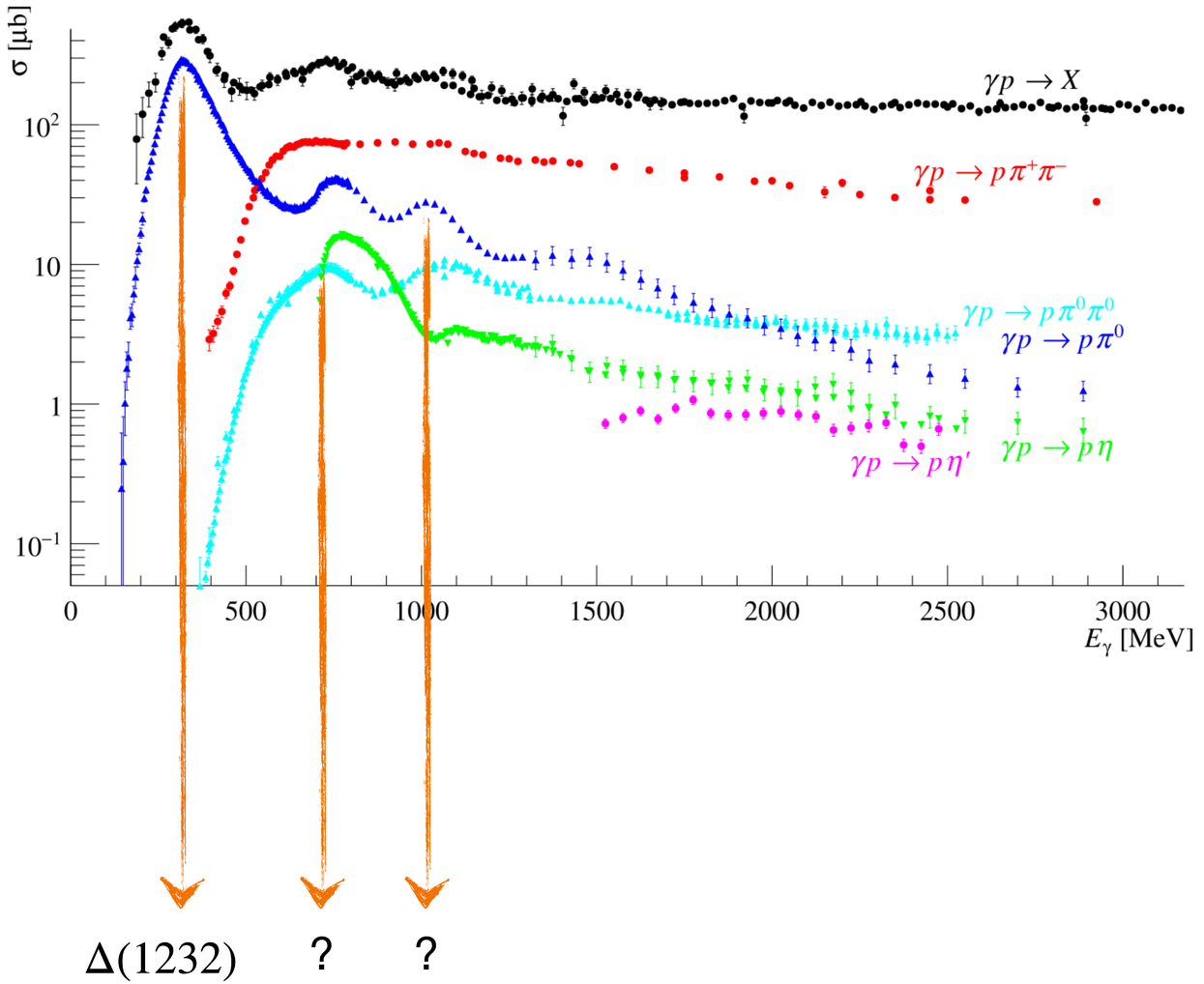

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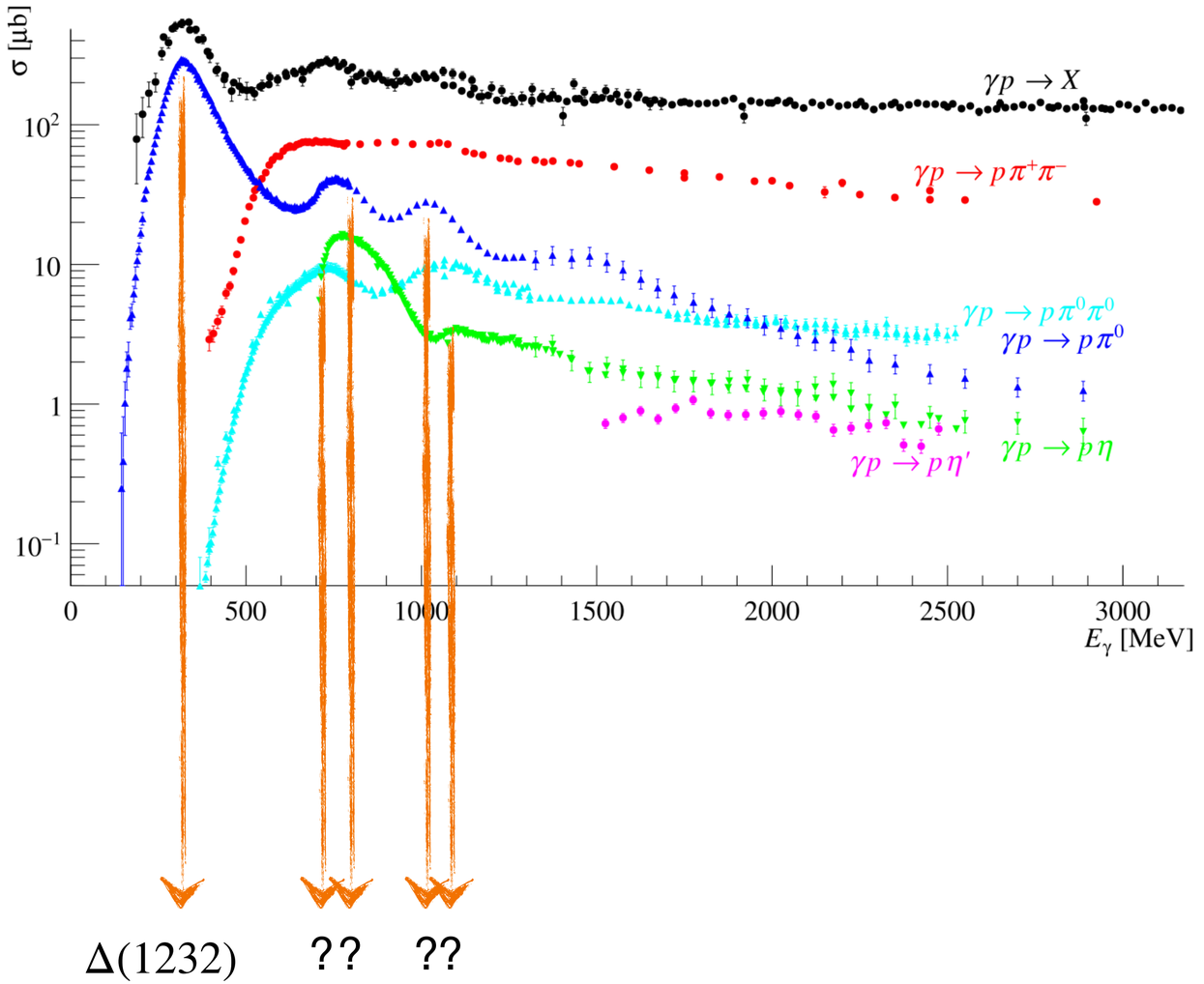

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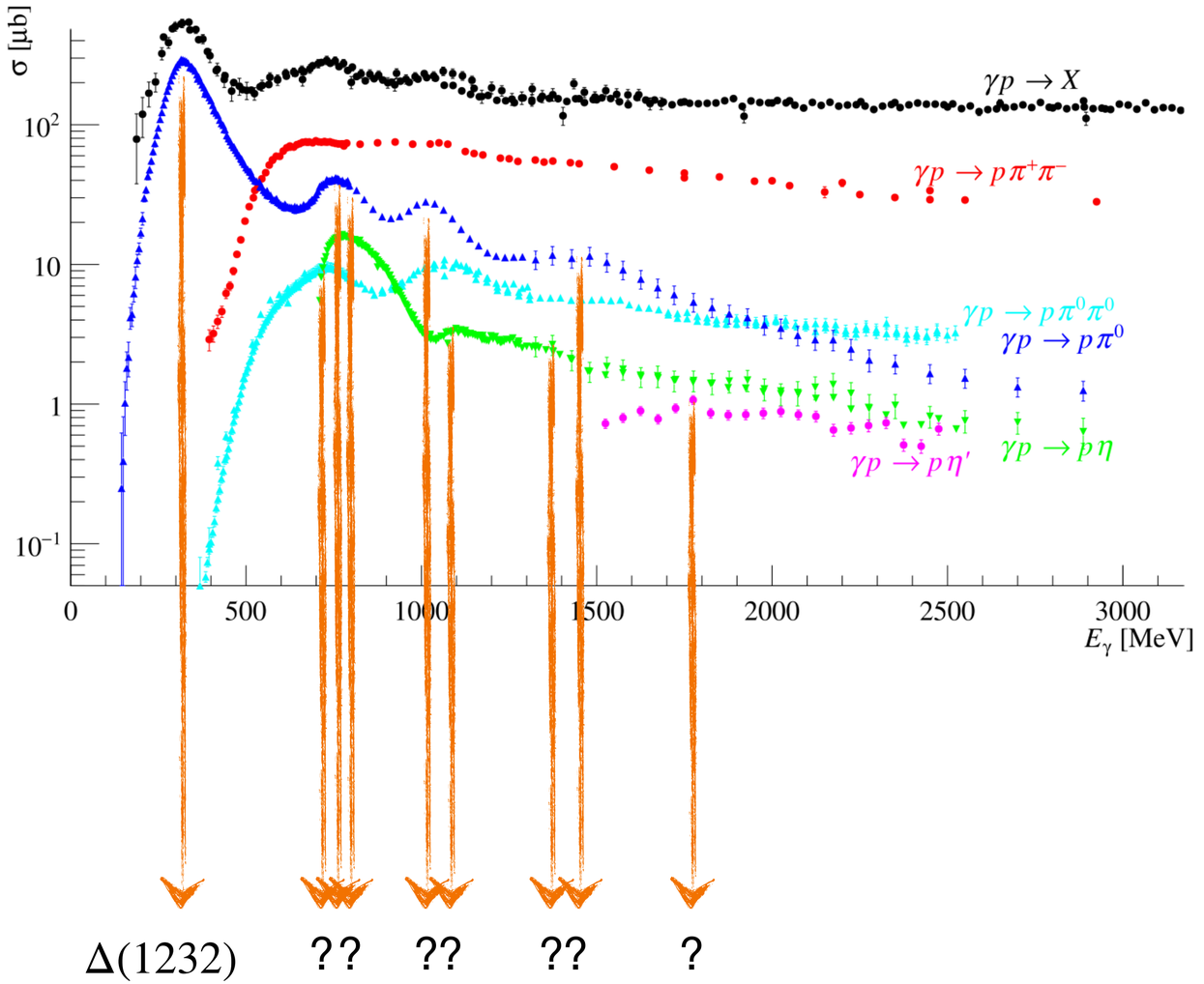

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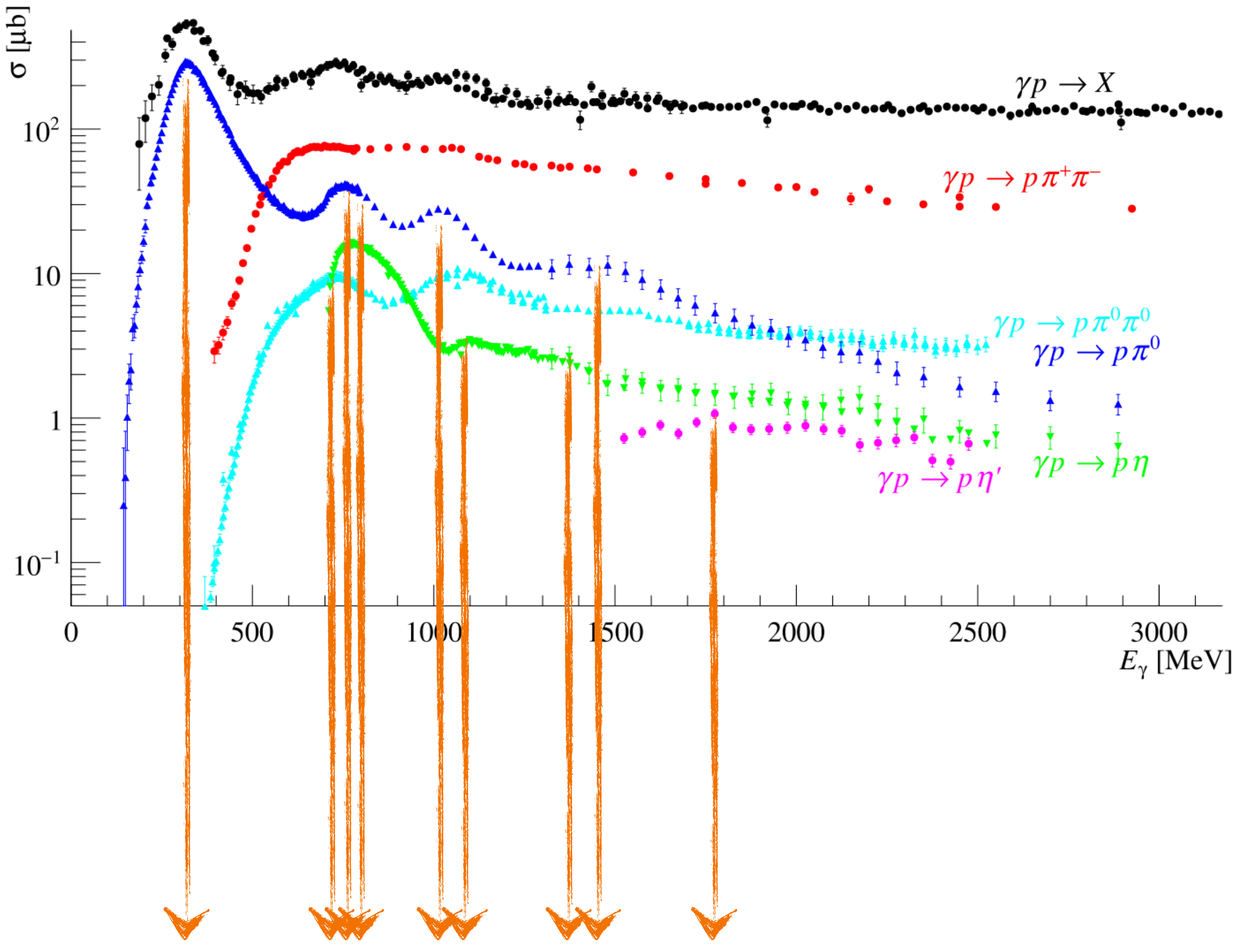

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Resonances:

- increased interaction rates (bumps)

- reaction-type dependence
- overlapping resonances
- kinematical effects (cusps/triangle singularities/…)

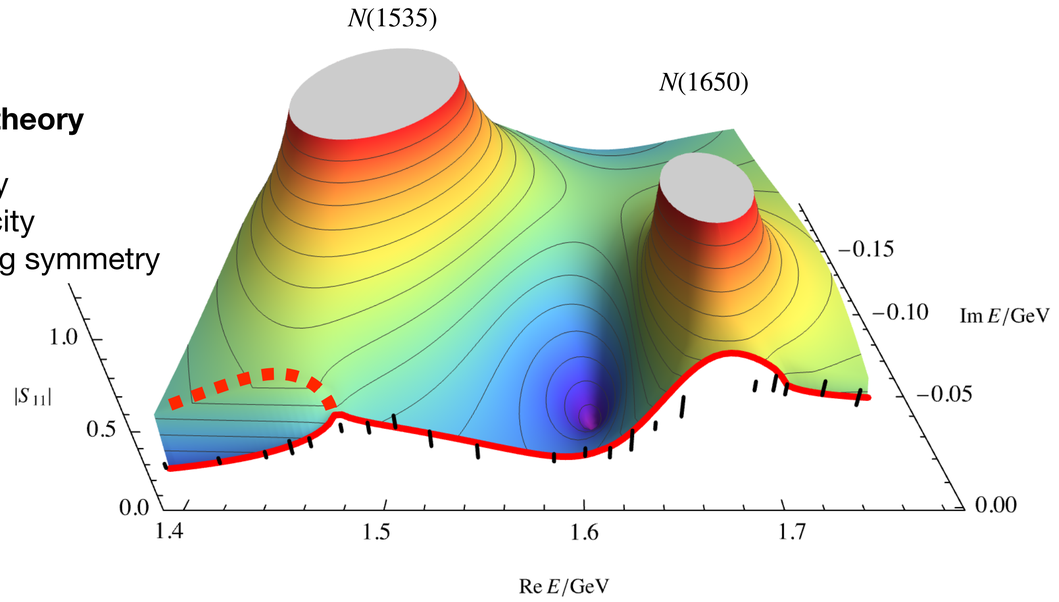
B
P ~~D~~ G=*“particle bump group”*

TRANSITION AMPLITUDES

Data: SAID: Phys. Rev. C 74 (2006) 045205
Model: MM et al. Phys.Rev.D 86 (2012) 094033

S-matrix theory

- Unitarity
- Analyticity
- Crossing symmetry



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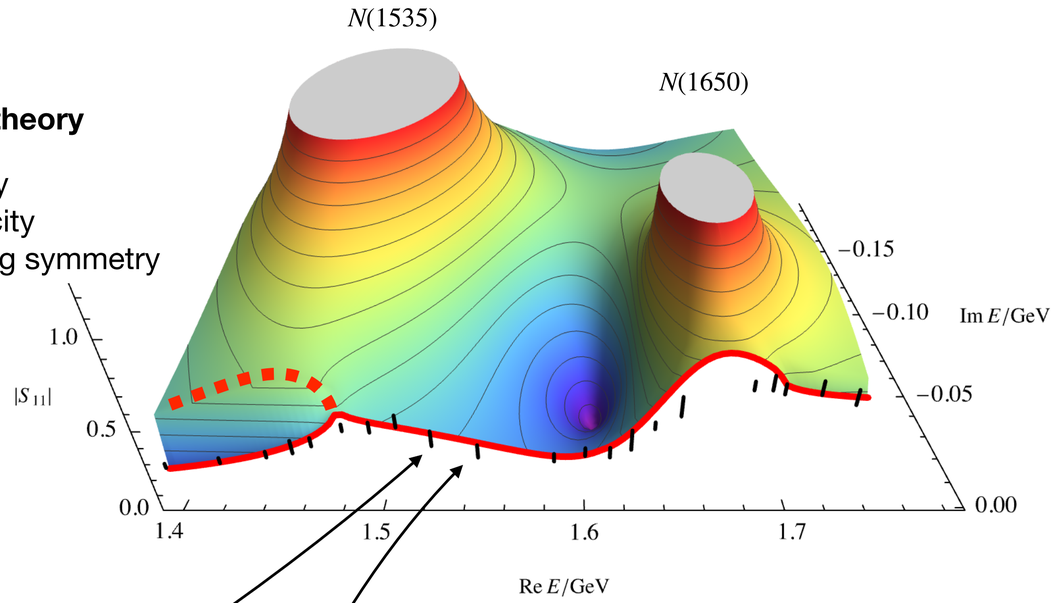
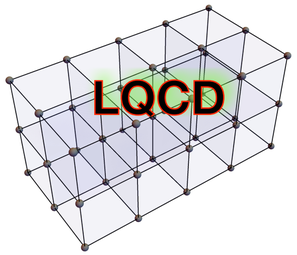
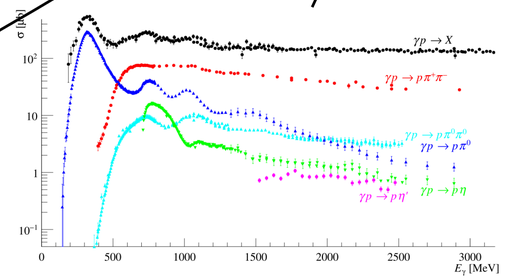


Boundary ($E \in \mathbb{R}$):

- Experiment
- Lattice QCD
- Effective Field Theories





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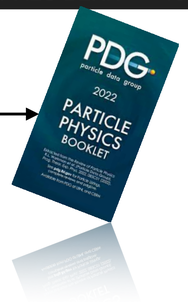
Poles on unphysical Riemann Sheets

- Universal resonance parameter
- Overlapping singularities
- Excludes triangle singularities
- …



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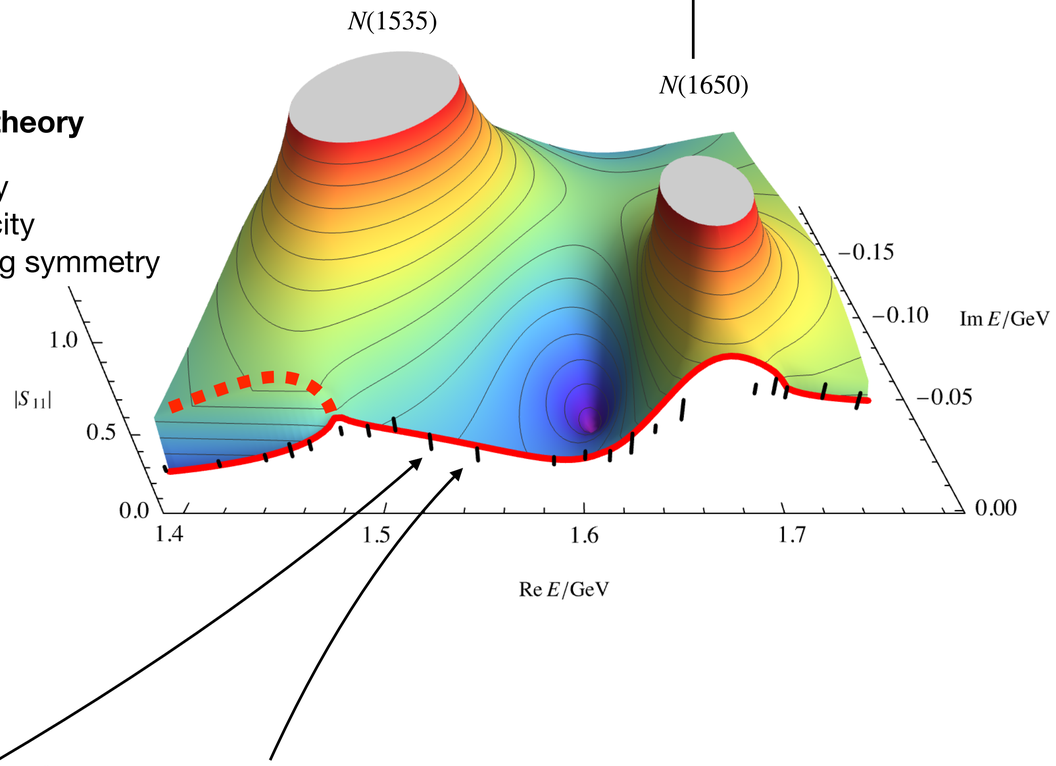

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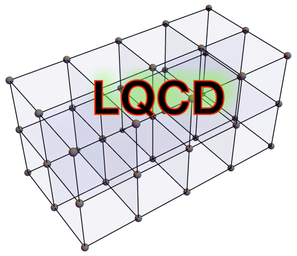

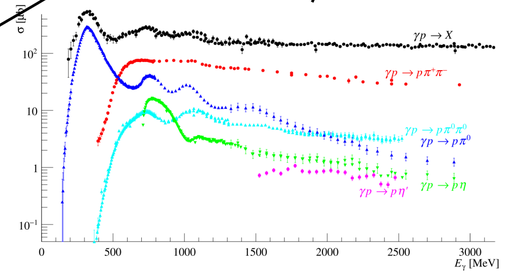

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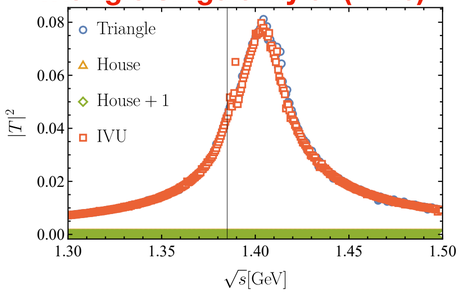
Triangle singularity a1(1420)



Sakthivasan/MM/… *JHEP* 10 (2024) 246

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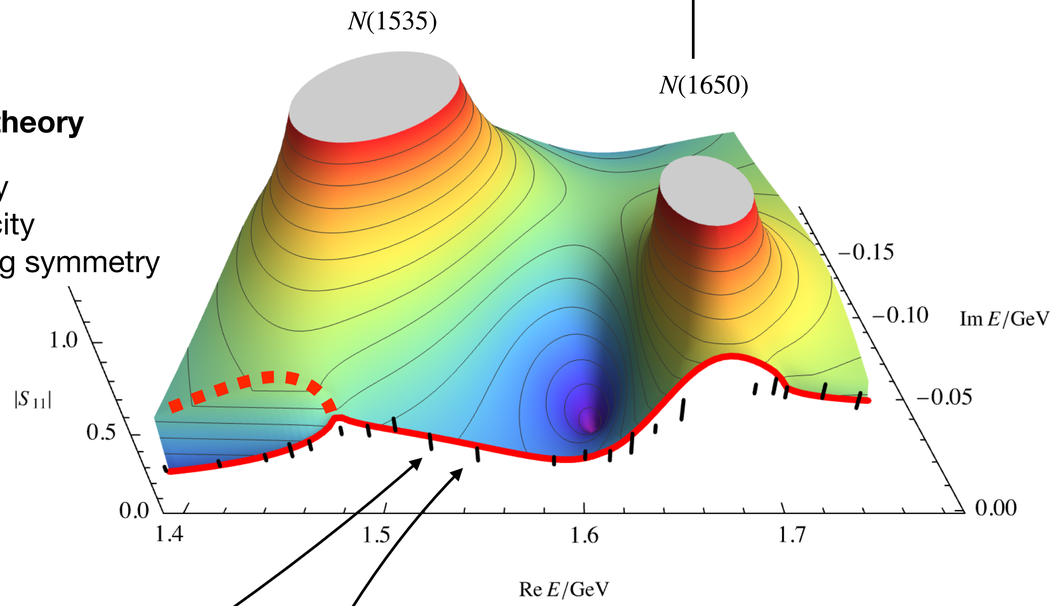

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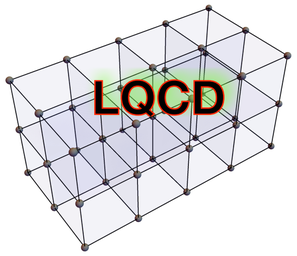
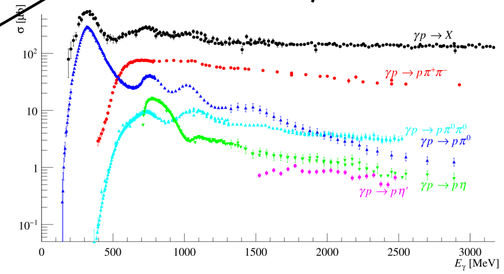
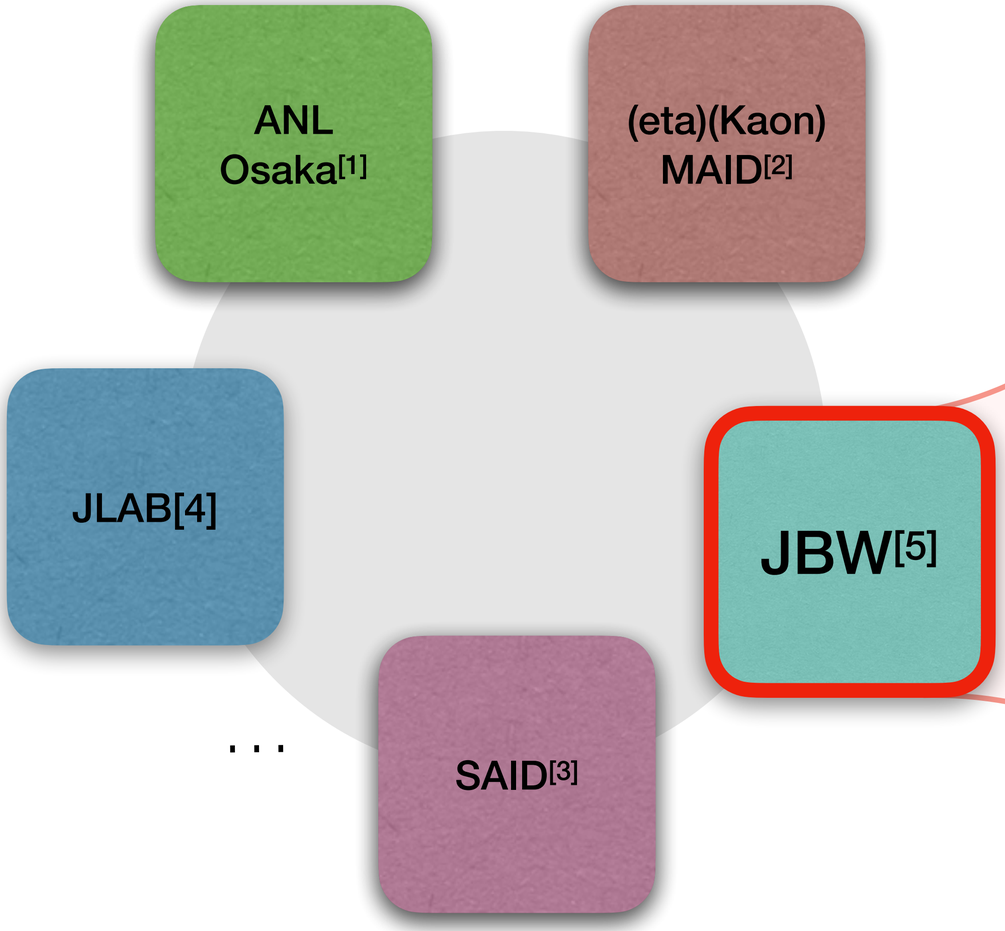




DYNAMICAL COUPLED-CHANNEL MODELS



Jülich-Bonn-Washington

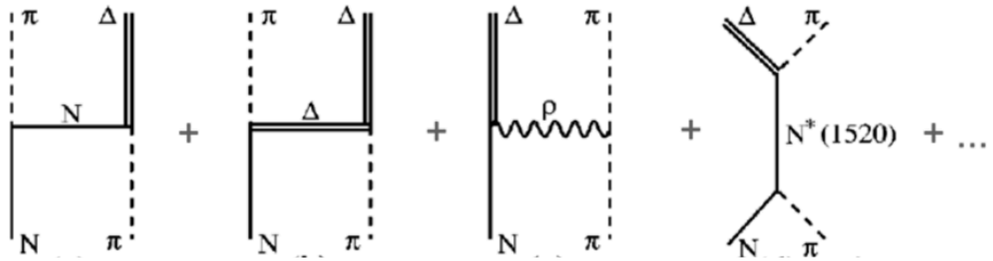
- coupled-channel approach $\pi N / \eta N / K\Lambda / \ldots$
  - → gauge invariance
  - → (pseudo)-threshold behavior
  - → Siegert's theorem
  - → **Final-state unitarity**
- web-interface: https://jbw.phys.gwu.edu/

[1] ANL-Osaka PRC 80(2009), Few-Body Syst. 59(2018),... [2] MAID2007, EPJA 34(2007) EtaMAID2018, EPJA 54(2018) [3] SAID, PiN Newsletter 16(2002) [4] Aznauryan et al., PRC 80(2009), IJMP(2013),... Gent group PRC 89(2014),...

FINAL-STATE UNITARITY

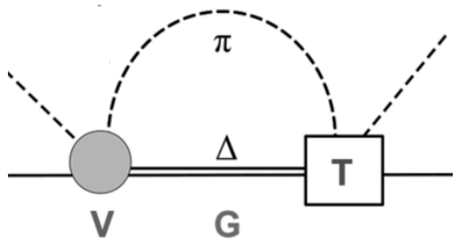
Phenomenological Lagrangian

Wess/Zumino Phys. Rev. 163 (1967)



- interaction patterns
- symmetry relations
- Many unknown couplings (overcounted?)

Non-perturbative dynamics



- Lippmann-Schwinger equation with relativistic energies
- Coupled-channel equation
- Extension to photo/electroproduction reactions

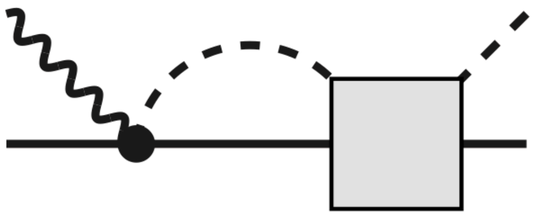
- ridiculously many parameters (~1000)

… but many scattering and photo-production data

πN→xX (~20k)

γN→xX (~40k data)

γ^*N→xX (~200k data)



Rönchen et al. Eur. Phys. J. A 50, 101 (2014)

Review: Döring, Haidenbauer, MM, Sato PPNP2025

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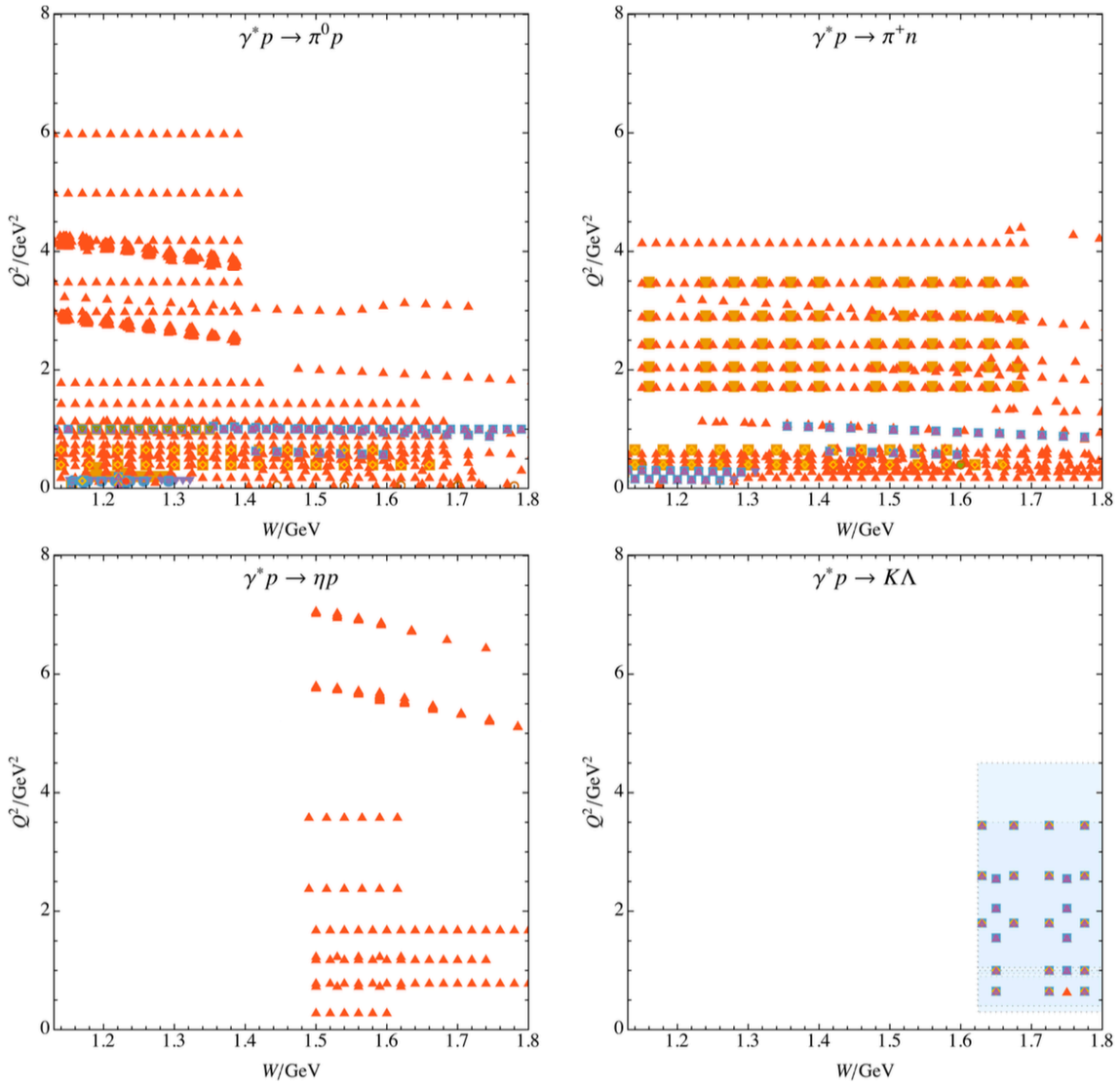

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TRANSITION FORM FACTORS

Resonance parameters

- one of the major tools to determine mass, width, couplings, … [cf PDG]

Transition Form factors[1] — Hadron structure probe

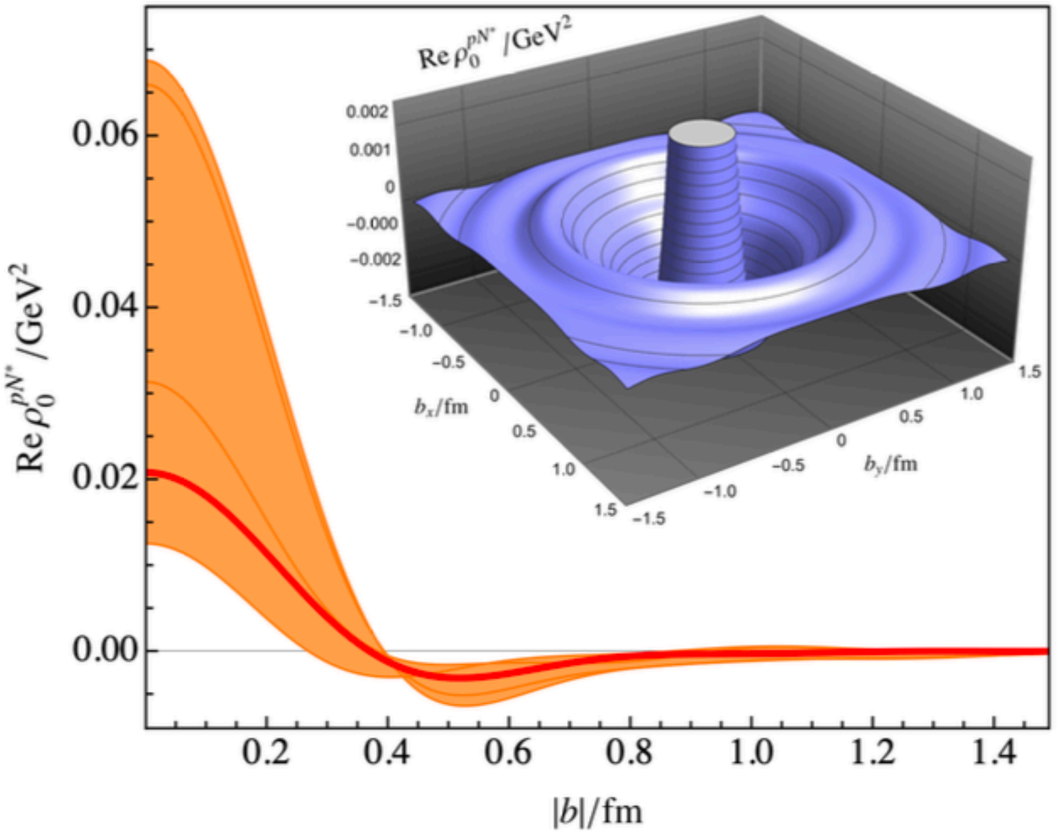
- transition between excited and ground state baryons[2]

$$H_h^{l\pm,I}(Q^2) = C_I \sqrt{\frac{p_{\pi N}}{\omega_0} \frac{2\pi(2J+1)z_p}{m_N \widetilde{R}^{l\pm,I}}} \widetilde{\mathcal{H}}_h^{l\pm,I}(Q^2)\,,$$

- 12 N/Δ states are determined

- Charge distribution in light front RF[3]

$$\rho_0^{NN^*}(\vec{b}) = \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(bQ) F_1^{NN^*}(Q^2),$$



[JBW] MM et al. Phys.Rev.C 103 (2021) 6
[JBW] MM et al. *Phys.Rev.C* 106 (2022) 015201
[JBW] MM et al. *Eur.Phys.J.A* 59 (2023) 12
[JBW] Wang et al. PRL 133 (10) 2024

[1] Aznauryan and V. D. Burkert, Prog. Part. Nucl. Phys. 67, 1 (2012), arXiv:1109.1720 [hep-ph]; G. Ramalho and M. T. Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024), arXiv:2306.13900 [hep-ph].
[2] Workman, L. Tiator, and A. Sarantsev, Phys. Rev. C 87, 068201 (2013), arXiv:1304.4029 [nucl-th].
[3] Tiator et al. CPC(HEP & NP), 2009, 33(X)

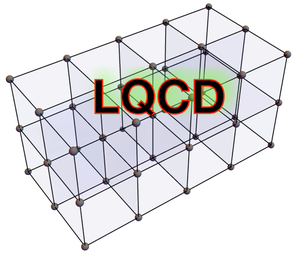
UNIVERSAL IMPLICATIONS OF S-MATRIX UNITARITY

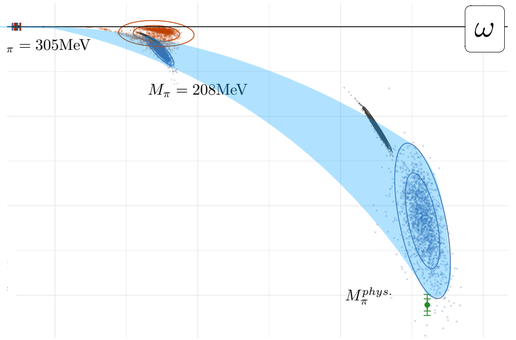
*Universality WS @ ECT**
June 2025

MAXIM MAI

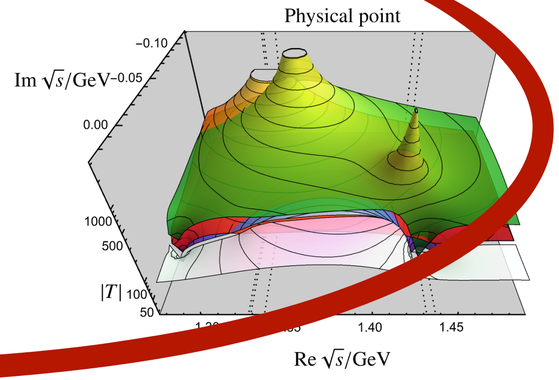
ALBERT EINSTEIN CENTER (BERN)

… rediscover 3-hadron resonances from lattice QCD

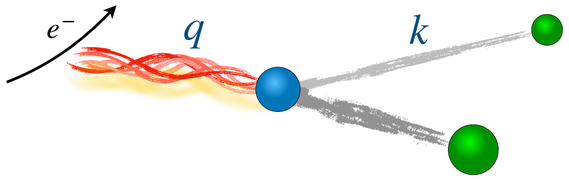


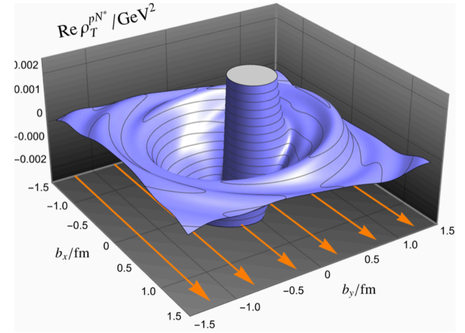


… discover new resonances from Chiral Dynamics



… clues about nature of states

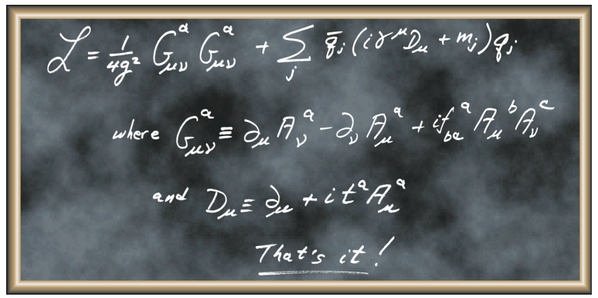




Heisenberg-Programm

TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076 DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907 , NSF: PHY-2012289

THEORY



http://frankwilczek.com/Wilczek_Easy_Pieces/298_QCD_Made_Simple.pdf

Low-energy regime of QCD = double trouble

- small relative momenta
- non-perturbative energy regime
- need to evaluate infinitely many diagrams

Further approaches: Functional methods, holography, K-matrix, dynamical models, …

Review: Eichmann/Sanchis-Alepuz/Alkofer/Fischer Prog.Part.Nucl.Phys. 91 (2016) 1-100
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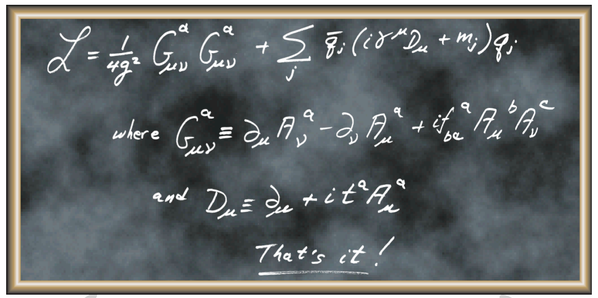

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$$Z[J] = \int [DU] e^{\int id^4x \, \mathscr{L}_{\text{eff}}(U,v,a,s,p)}$$

Effective Field Theory (CHPT)

- Effective/Hadronic degrees of freedom
- <u>Infinitely</u> many low-energy constants
- Well-defined power counting
- Benchmark for many low-energy hadronic interactions

Weinberg (1979) Gasser, Leutwyler (1981), …
Reviews:
Bernard, Meißner, Ann. Rev. Nucl. Part. Sci. 57, 33 (2007),
Scherer, Adv. Nucl. Phys. 27, 277 (2003), …

$$Z[J] = \int [DU] e^{-S_E} \det[M[U]]$$

Lattice Gauge Theory

- QCD degrees of freedom
- discretized (Euclidean) space-time
- finite volume
- unphysical quark mass

Wilson, Phys. Rev. D10 (1974) 2445 , …
Reviews:
Gupta hep-lat/9807028 [hep-lat] —
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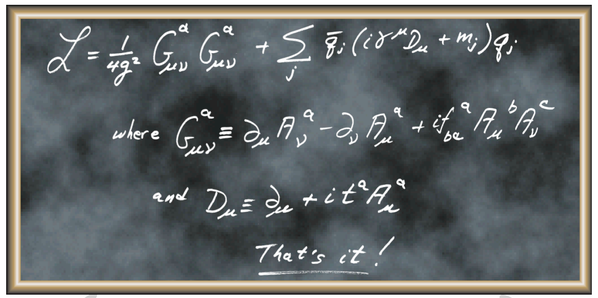

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quark mass dependence

first principle non-perturbative input

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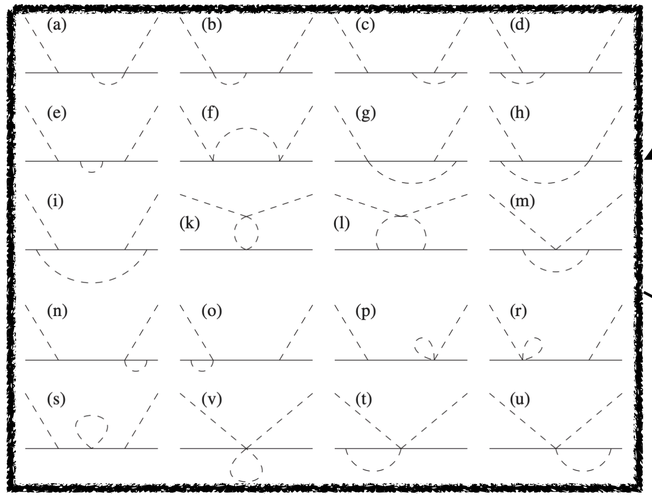
EXAMPLE: BARYON CHPT

Meson-baryon scattering from CHPT

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

- full SU(3) dynamics near threshold
- agrees with experiment in many cases
- provides predictions for not measured channels

$$\begin{aligned}\mathcal{L}^{(2)}_{\phi B} = {} & b_{D/F}\langle\bar{B}[\chi_+, B]_\pm\rangle + b_0\langle\bar{B}B\rangle\langle\chi_+\rangle + b_{1/2}\langle\bar{B}[u_\mu, [u^\mu, B]_\mp]\rangle + b_3\langle\bar{B}\{u_\mu, \{u^\mu, B\}\}\rangle + b_4\langle\bar{B}B\rangle\langle u_\mu u^\mu\rangle \\ & + i\sigma^{\mu\nu}(b_{5/6}\langle\bar{B}[[u_\mu, u_\nu], B]_\mp\rangle + b_7\langle\bar{B}u_\mu\rangle\langle u_\nu B\rangle) + \frac{ib_{8/9}}{2m_0}(\langle\bar{B}\gamma^\mu[u_\mu, [u_\nu, [D^\nu, B]]_\mp]\rangle + \langle\bar{B}\gamma^\mu[D_\nu, [u^\nu, [u_\mu, B]]_\mp]\rangle) \\ & + \frac{ib_{10}}{2m_0}(\langle\bar{B}\gamma^\mu\{u_\mu, \{u_\nu, [D^\nu, B]\}\}\rangle + \langle\bar{B}\gamma^\mu[D_\nu, \{u^\nu, \{u_\mu, B\}\}]\rangle) + \frac{ib_{11}}{2m_0}(2\langle\bar{B}\gamma^\mu[D_\nu, B]\rangle\langle u_\mu u^\nu\rangle \\ & + \langle\bar{B}\gamma^\mu B\rangle\langle[D_\nu, u_\mu]u^\nu + u_\mu[D_\nu, u^\nu]\rangle), \end{aligned} \tag{8}$$



Channel =	$\mathcal{O}(q^1)$	$+\mathcal{O}(q^2)$	$+\mathcal{O}(q^3)_{\rm HB}$	$\sum_{\rm HB}$
$a^{(3/2)}_{\pi N}$ =	-0.12	$+0.05^{+0.02}_{-0.03}$	$-0.06^{+0.00}_{+0.00}$	$-0.13^{+0.03}_{-0.03}$
$a^{(1/2)}_{\pi N}$ =	$+0.21$	$+0.05^{+0.02}_{-0.03}$	$+0.00^{+0.00}_{+0.00}$	$+0.26^{+0.03}_{-0.03}$
$a^{(3/2)}_{\pi\Xi}$ =	-0.12	$+0.04^{+0.03}_{-0.03}$	$-0.09^{+0.00}_{+0.00}$	$-0.17^{+0.03}_{-0.03}$
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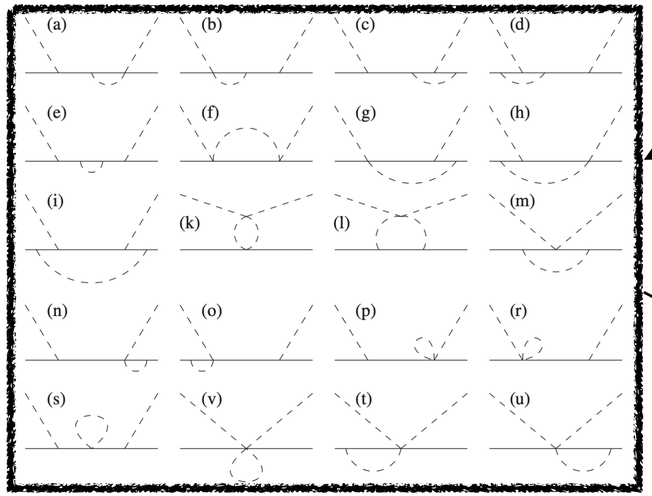

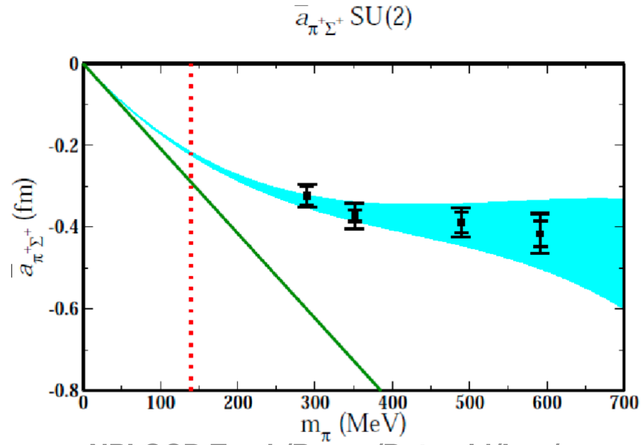

NPLQCD Torok/Beane/Detmold/Luu/…
Phys.Rev.D 81 (2010) 074506

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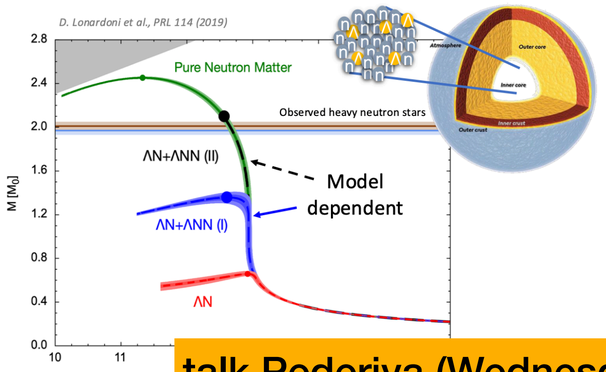
- Fails for resonant (strangeness) channel
  - Kaon mass is large → convergence
  - Relevant thresholds are widely separated → convergence
  - Resonance just below $\bar{K}N$ threshold → non-perturbative effect



talk Pederiva (Wednesday)

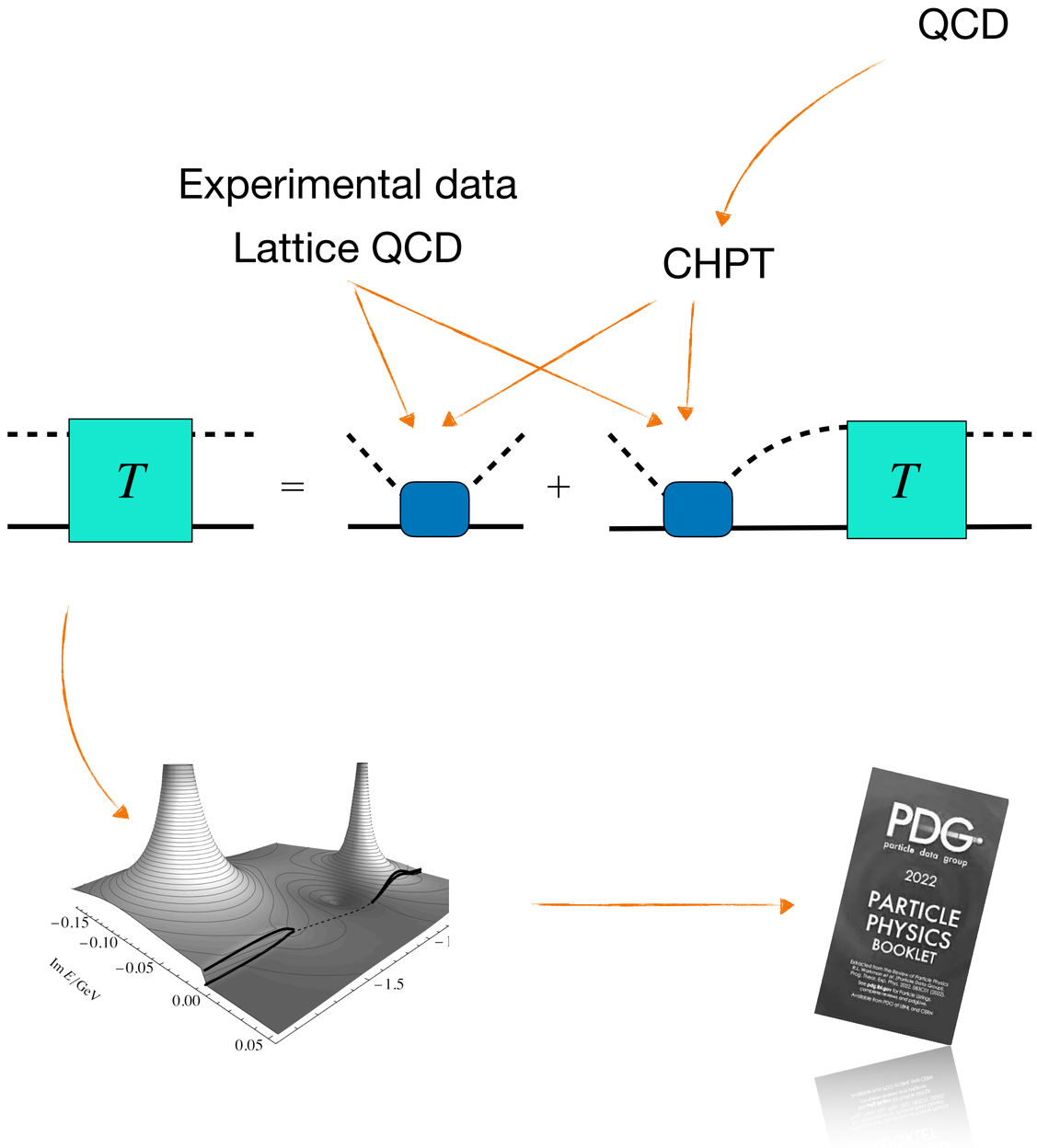
$$a^{I=0}_{\bar{K}N} = \Big((+0.53)_{\rm LO} + (+0.97)_{\rm NLO} + (-0.40+0.22i)_{\rm NNLO} + \ldots\Big)\ \text{fm}\,,$$
$$a^{I=1}_{\bar{K}N} = \Big((+0.20)_{\rm LO} + (+0.22)_{\rm NLO} + (-0.26+0.18i)_{\rm NNLO} + \ldots\Big)\ \text{fm}\,.$$

UNITARISATION

Extension to higher energies — Chiral Unitary Approach

- **Good**
  - Non-perturbative scheme
  - Record complex pole-positions (*II Riemann Sheet*)
  - Often works: $N(1535), N(1650), \Lambda(1405), \Lambda(1380), \ldots$

Kaiser/Siegel/Weise Phys.Lett.B 362 (1995)
Lutz/Soyeur Nucl.Phys.A 773 (2006);
MM et al. Phys.Lett.B 697 (2011); ...



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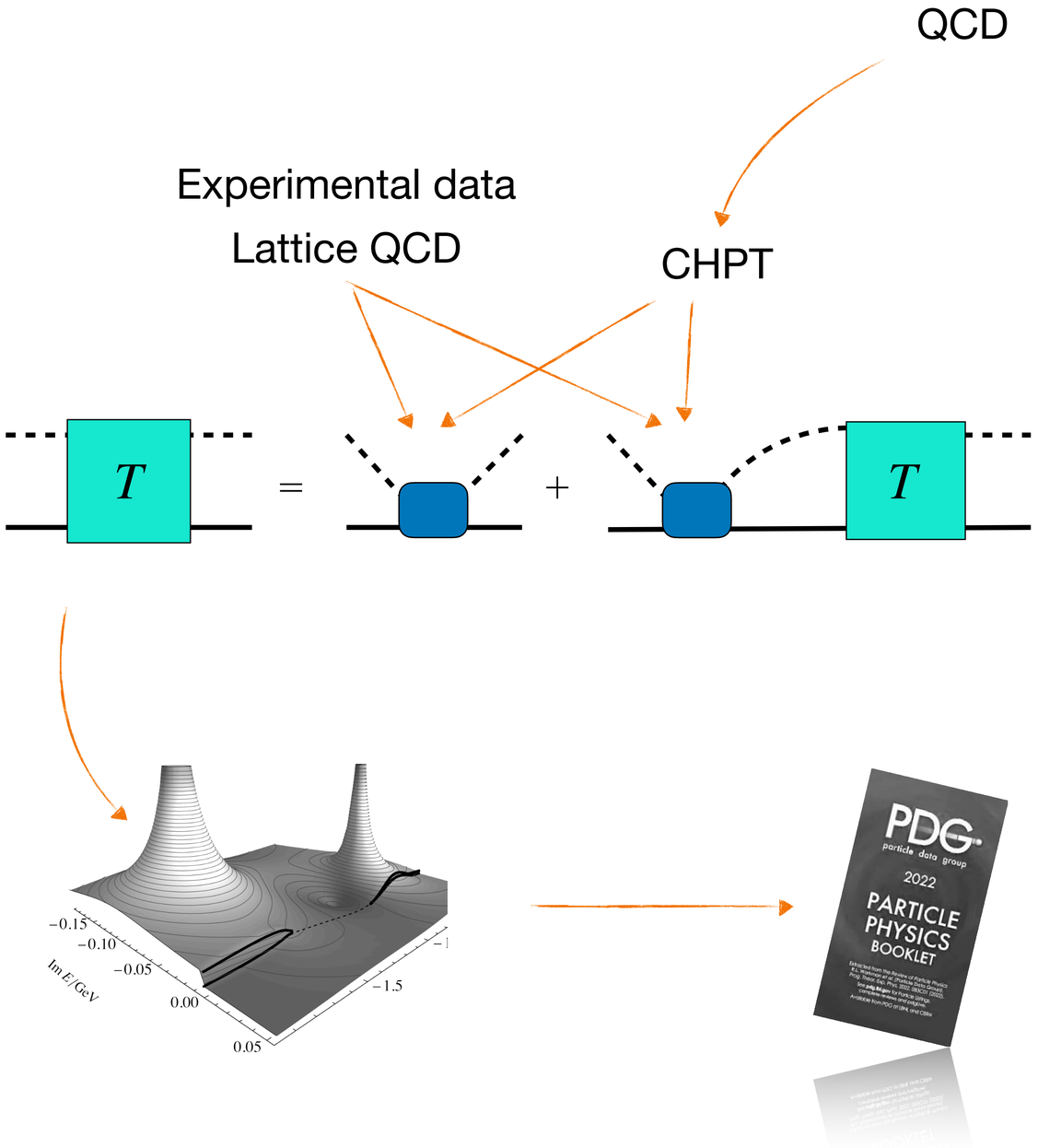
    MM et al. Phys.Lett.B 697 (2011); ...

- **Attention (model dependence)**

  Review: MM, Eur.Phys.J.ST 230 (2021) 6, 1593-1607
  - Renormalisation/Crossing symmetry/Power counting
  - Choice of the interaction kernel



STATUS: $\Lambda(1405)$, $\Lambda(1380)$ S=-1 I=0

"A curious case of a strangeness resonance"

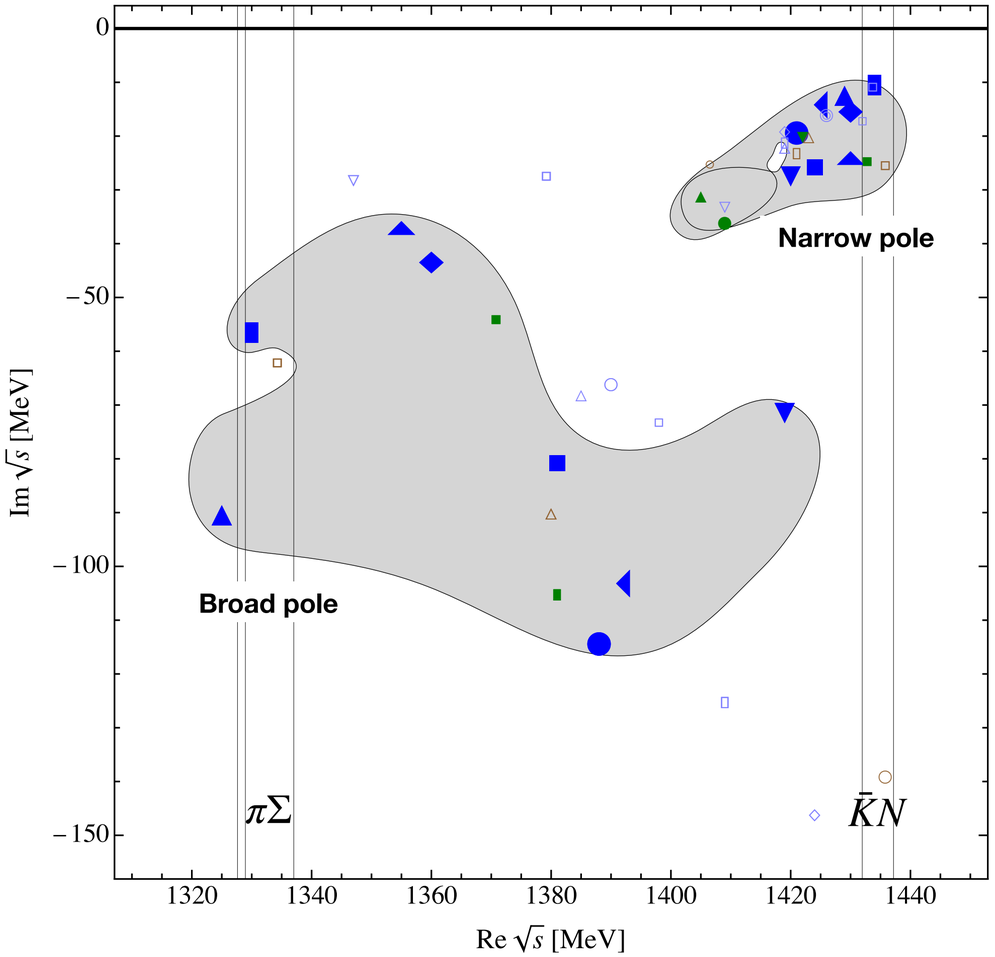
MM, *Eur.Phys.J.ST* 230 (2021) 6, 1593-1607

- Sub-($\bar{K}N$)-threshold $\Lambda(1405)$ resonance
- second state $\Lambda(1380)$ predicted from UCHPT
  - no direct experimental verification
  - indirectly through photoproduction experiments

    [CLAS] Moriya et al. Phys.Rev.Lett. 112 (2014) 8
    MM/Meißner *Eur.Phys.J.A* 51 (2015) 3, 30
  - confirmed by many critical tests & LQCD

    Lu/Geng/Döring/MM PRL (2021)
    Bulava et al. [BaSc] *Phys.Rev.Lett.* 132 (2024) 5, 051901

MM, *Eur.Phys.J.ST* 230 (2021) 6, 1593-1607



Models: Ikeda/Weise/Feijoo/MM/Meißner/Ramos/Hyodo/…

QUARK MASS DEPENDENCE

CHPT encodes quark mass dependence

- SU(3) limit provides a simpler resonance structure

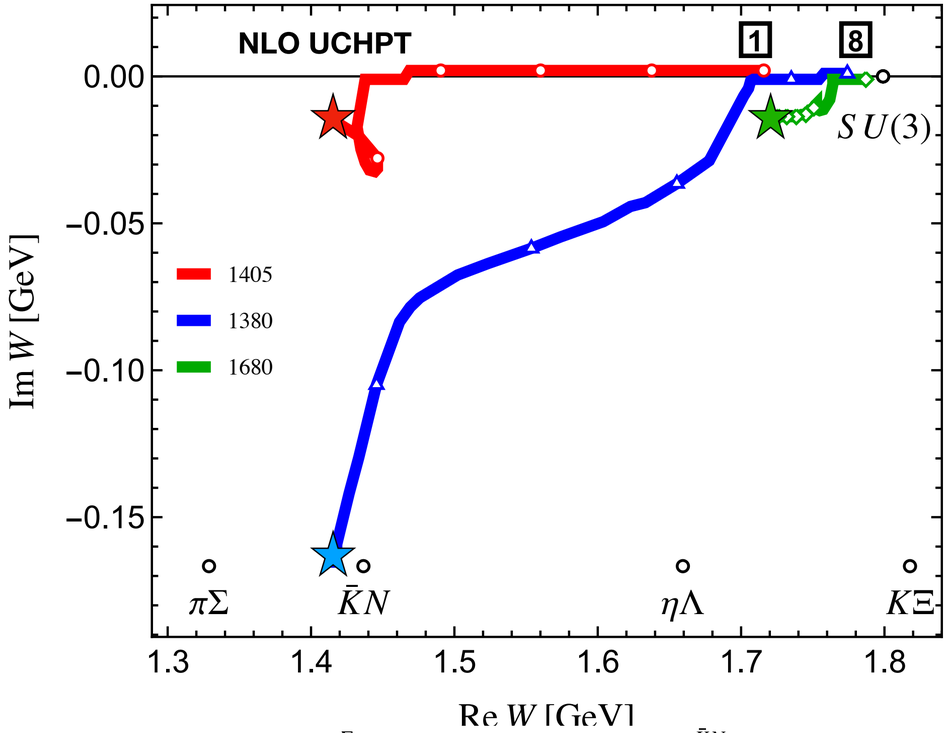
  Jido et al. Nucl.Phys.A 725 (2003); Garcia-Recio/Lutz/Nieves Phys.Lett.B 582 (2004) 49-54;

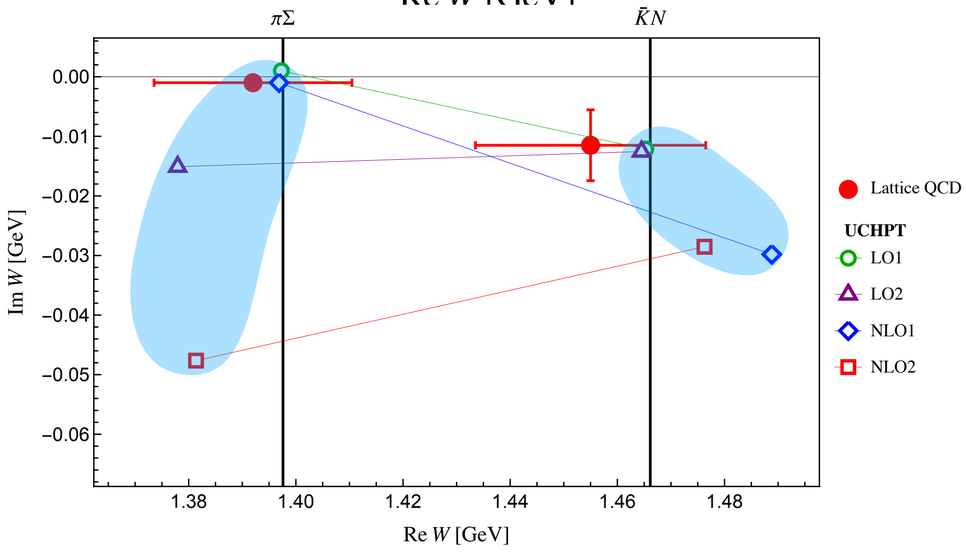
  - 1 singlet + 2 octet poles
  - LO/NLO UCHPT *pole-"tracks"* differ

    Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)

- Resonance ⇜⇝ virtual bound state ⇜⇝ bound state

Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)





UNIVERSAL IMPLICATIONS OF S-MATRIX UNITARITY

*Universality WS @ ECT**
June 2025

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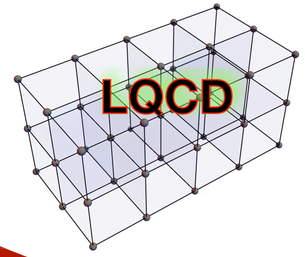

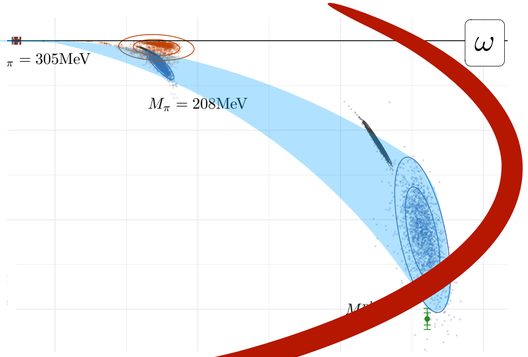

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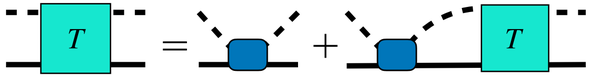
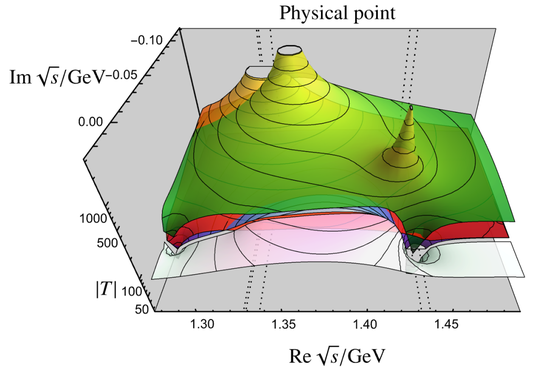
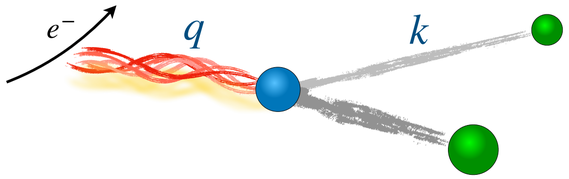
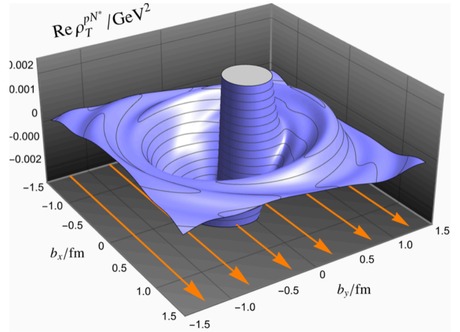




… clues about nature of states





Heisenberg-Programm
DFG Deutsche Forschungsgemeinschaft

TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076 DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907 , NSF: PHY-2012289

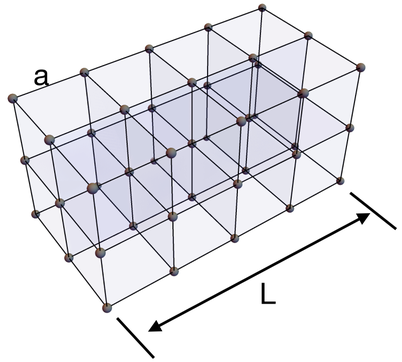
LATTICE QCD (SPECTROSCOPY)

Advantages

- QCD degrees of freedom (first principles)
- Experimentally inaccessible scenarios:
  - → Unconventional quantum numbers
  - → **Three-body scattering/…**
  - → Chiral trajectory

Roadblocks

- discretized (Euclidean) space-time — continuum extrapolation
- unphysical quark mass — extrapolations tools from CHPT
- **finite volume — quantization conditions needed**



Discretization of space-time

Euclidean space-time

Boundary conditions

Gauge and fermion degrees of freedom

links

plaquettes

Nielsen–Ninomiya theorem

Fermion doublers

Lattice QCD action

Construction of the action

Wick's theorem

measure of integration in the path integral.

Generating functional

Hybrid Monte-Carlo simulation

The transcription of the operators used to probe the physics

Operator construction

Correlation functions

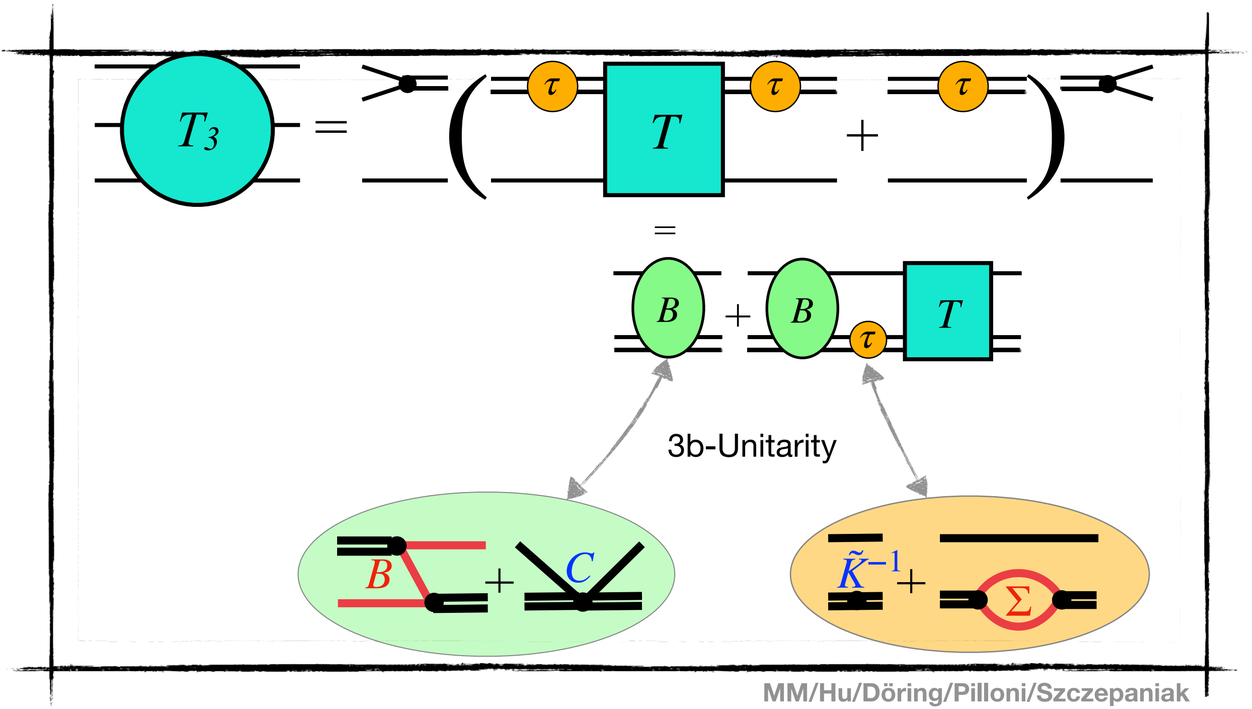
Generalized eigenvalue problem

scale setting

Energy eigenvalues ($\mathbb{R}$)

K. Wilson, Phys. Rev. D10 (1974) 2445 , …
Introduction to lattice QCD: Course Rajan Gupta hep-lat/9807028 [hep-lat] ….

3-BODY QUANTIZATION CONDITION



MM/Hu/Döring/Pilloni/Szczepaniak
Eur.Phys.J.A 53 (2017)

- 3-body unitarity accounts for all on-shell states

talk: Sebastian (Tuesday)
talk: Andrew (Thursday)

Finite Volume Unitarity (FVU) approach

$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}^{-1} - \Sigma^L\right) - B - C \,\right]^{\Lambda} \equiv 0$$

MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

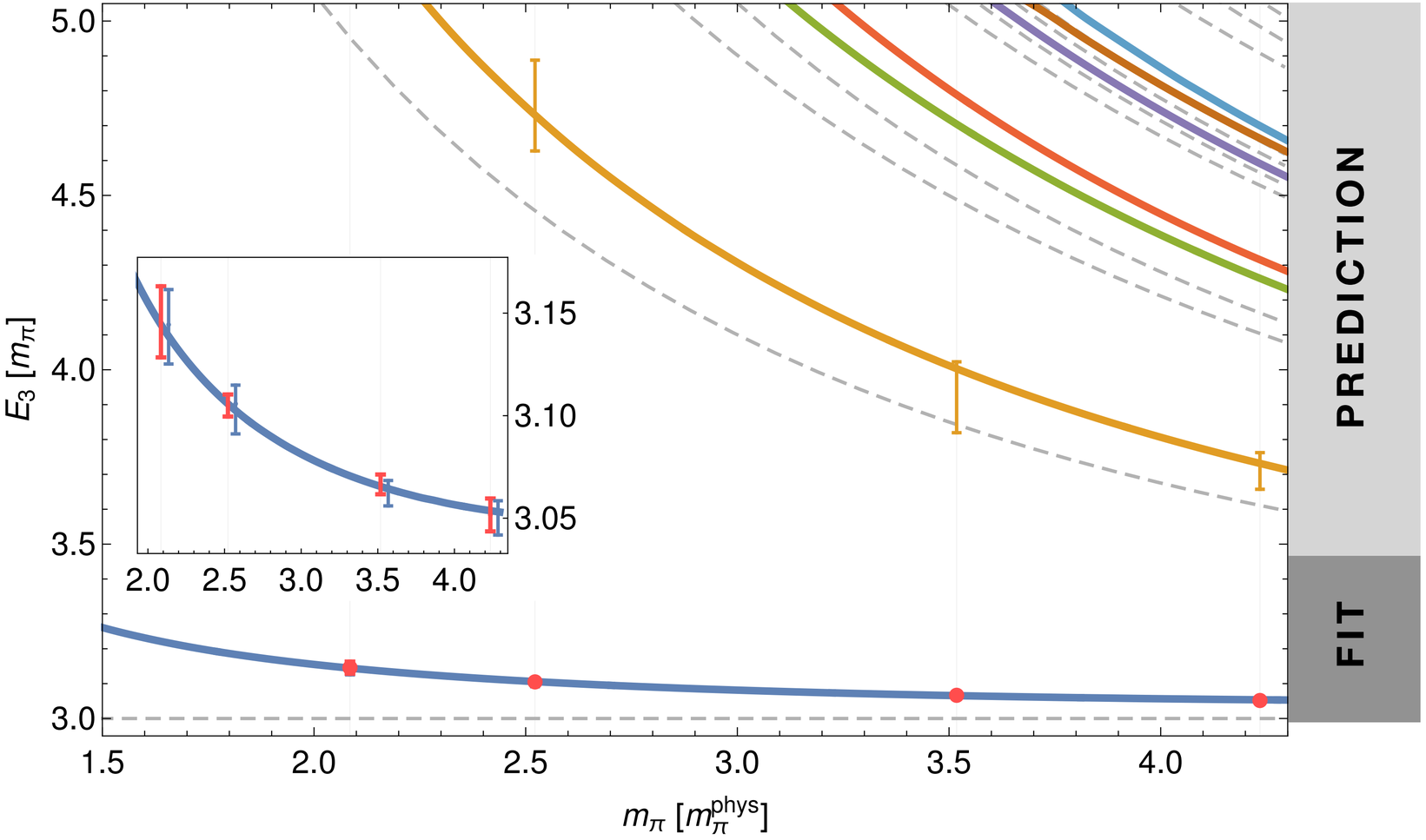
- genuine determinant condition
- Alternatives: RFT, NREFT
  RFT(Hansen/Sharpe 2014) NREFT(Rusetsky/Hammer/Pang 2017)
- equivalence:
  Jackura et al. *Phys.Rev.D* 100 (2019) 3, 034508, Garofalo et al. *JHEP* 02 (2023) 252

Many new applications

- proof of concepts and spin-less repulsive systems
  MM/Doring Phys.Rev.Lett. 122 (2019) 6, Fischer et al. Eur.Phys.J.C 81 (2021) 5, Blanton, Romero-Lopez, Hansen, Briceno, …
  Dawid et al. 2502.17976
- systems with left-hand cut
  Hansen et al. *JHEP* 06 (2024) 051, Dawid et al. *JHEP* 01 (2025) 060, Rusetsky, …
- 3-body resonant systems
  MM/Culver Phys.Rev.Lett. 127 (2021) 22
  Yan et al. Phys.Rev.Lett. 133 (2024) 21

SIMPLEST EXAMPLE: $\pi^+\pi^+\pi^+$

MM, Doring PRL 122 (2019) arXiv: 1807.04746



FVU 3-body quantization condition

- ground level
- 1st excited level
- 2nd excited level
- …

LQCD: results.

- Beane et al. [NPLQCD] PRL100 (2008) Detmold et al. [NPLQCD] PRD78 (2008)

- Newer results including physical pion mass calulations

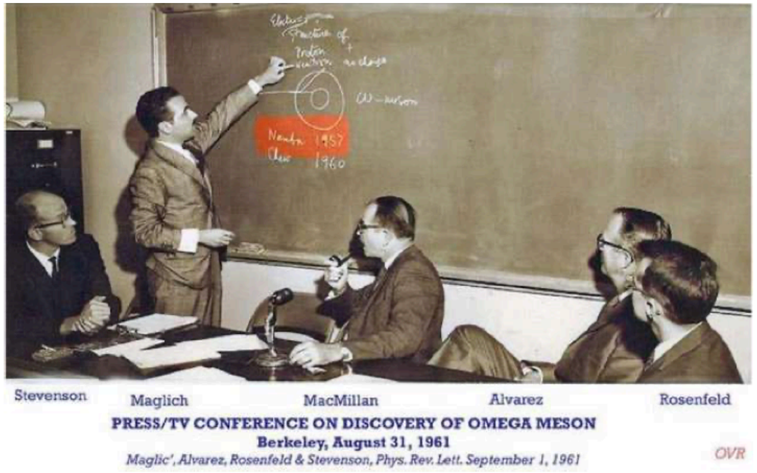
Fischer et al. Eur.Phys.J.C 81 (2021) 5, Blanton, Romero-Lopez, Hansen, Briceno, …
Dawid et al. 2502.17976

VECTOR MESON

Experiment

- lightest hadron decaying into three particles

  Maglich/Alvarez/Rosenfeld/Stevenson Phys.Rev.Lett. 7 (1961) 178-182



- soscalar response in the VMD picture of the photon-nucleon interactions

  Sakurai (1960); Erkelenz (1974); Brown and Jackson (1976); Barkov et al., 1985; Connell et al. (1997); Bazavov et al. (2021)

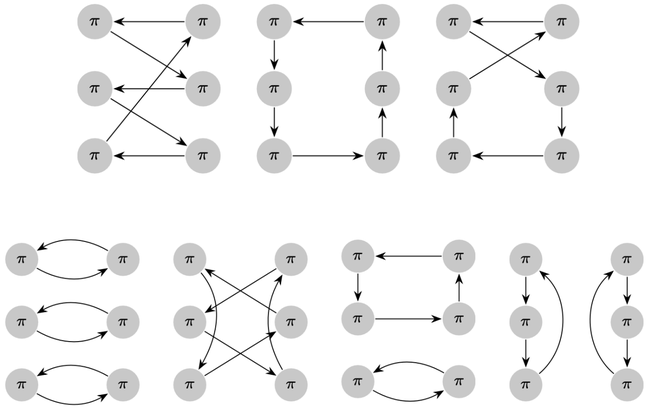
- generates the observed repulsion at < 1 fm in the one-boson-exchange picture of the NN

VECTOR MESON

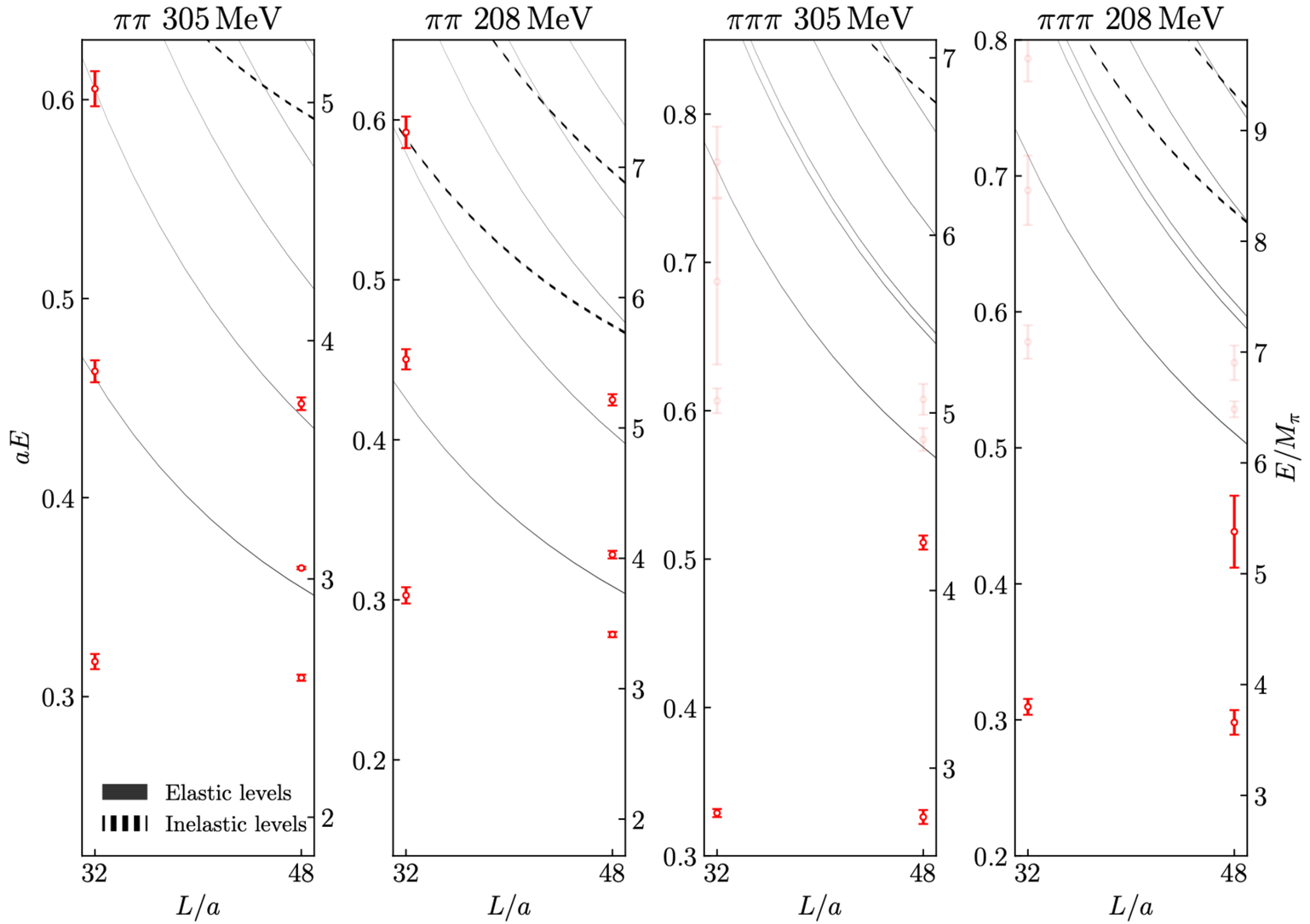
Yan/MM/Garofalo/Liu/Liu/Meißner/Urbach PRL133 (2024)

Lattice QCD setup

- Nf = 2 + 1 Clover fermions
  CLQCD, 2024
- 2/3 particle operators
  OpTion package @HaoboYan
- 2 pion masses ($\approx 210, 305$ MeV)
- 2 volumes ($L^3 = 32^3, 48^3$)



Finite-volume spectrum = Energy eigenvalues



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Yan/MM/Garofalo/Liu/Liu/Meißner/Urbach PRL133 (2024)

Mapping to infinite volume

- 3-body quantization condition

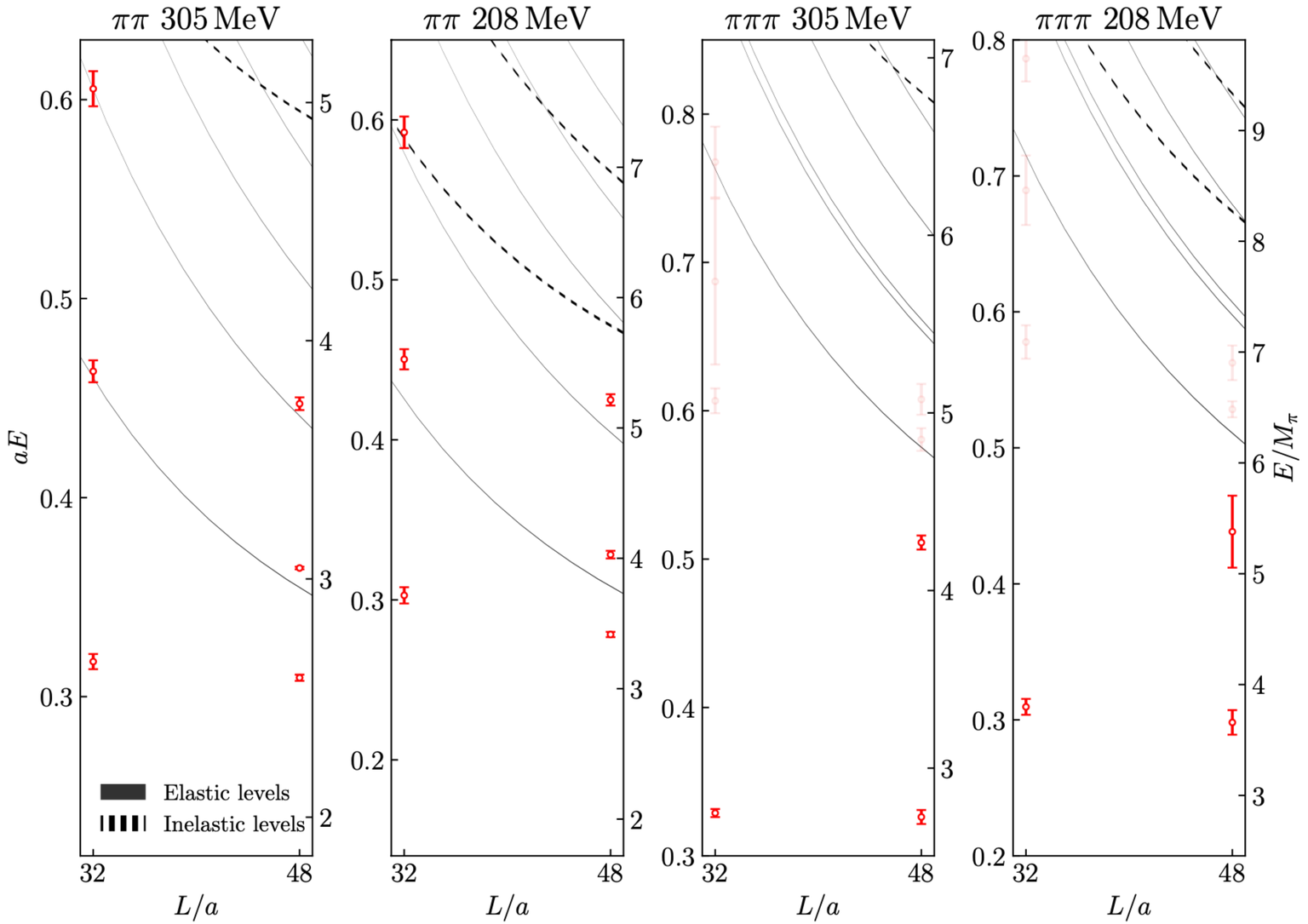
FVU

$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}^{-1} - \Sigma^L\right) - B - C \,\right]^{\Lambda} \equiv 0$$

MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

- Volume-independent 2-,3-body force $C, \tilde{K}$

Finite-volume spectrum = Energy eigenvalues



H.Yan/MM/Garofalo/Meißner/Lui/Liu/Urbach: 2407.16659 [hep-lat]

VECTOR MESON

Yan/MM/Garofalo/Liu/Liu/Meißner/Urbach PRL133 (2024)

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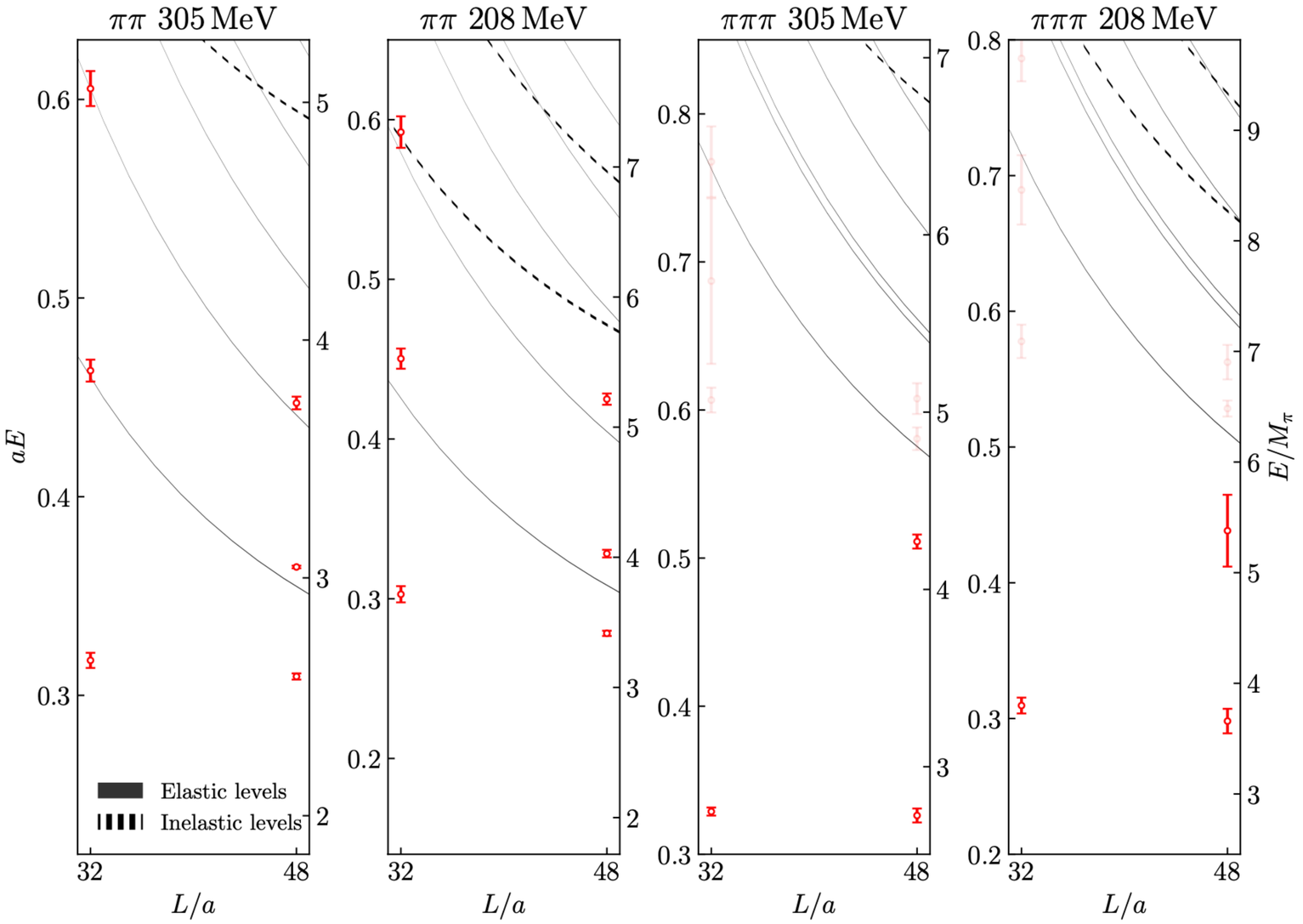
MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

- Volume-independent 2-,3-body force: **GEN form** (Laurent series)

$$\tilde{K}(s)^{-1} = a_0 + a_1 \sigma_{\vec{p}}(s) + \dots$$

$$C = \frac{c_0}{s - m_\omega^2} + c_1 + \dots$$

Finite-volume spectrum = Energy eigenvalues



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Yan/MM/Garofalo/Liu/Liu/Meißner/Urbach PRL133 (2024)

Mapping to infinite volume

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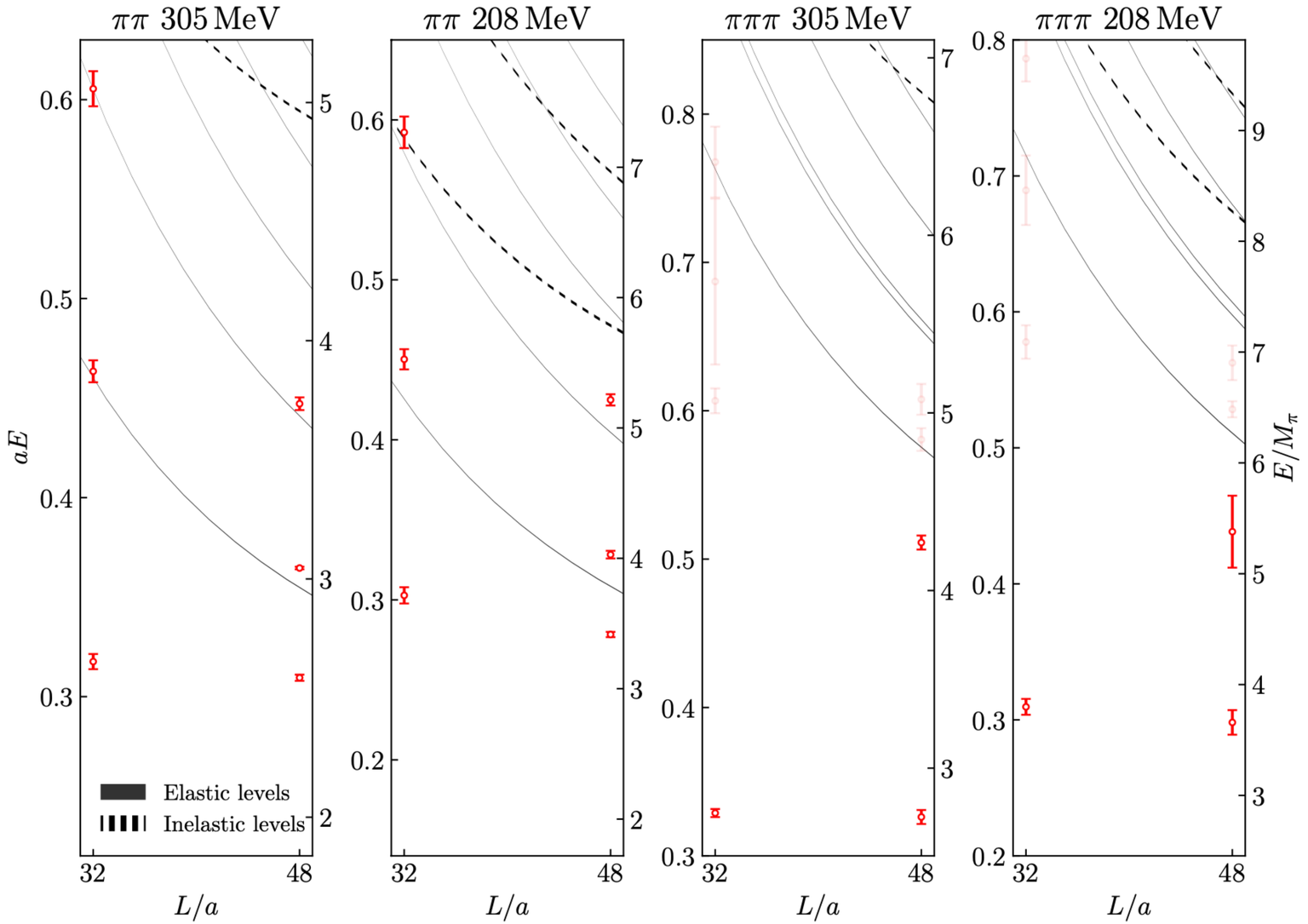
MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

- Volume-independent 2-,3-body force: **EFT form** (saturated by meson s-channel interaction)

$$C = \frac{6s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{64 g^2 \pi^3 f_\pi^6 (s - M_\omega^2)}$$

$$\tilde{K}^{-1} = \delta_{\lambda'\lambda}\delta_{\mathbf{p}'\mathbf{p}} \frac{\sigma_p - M_\rho^2}{2g^2}$$

Finite-volume spectrum = Energy eigenvalues



Meissner — *Phys.Rept.* 161 (1988) 213

VECTOR MESON

Yan/MM/Garofalo/Liu/Liu/Meißner/Urbach PRL133 (2024)

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MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

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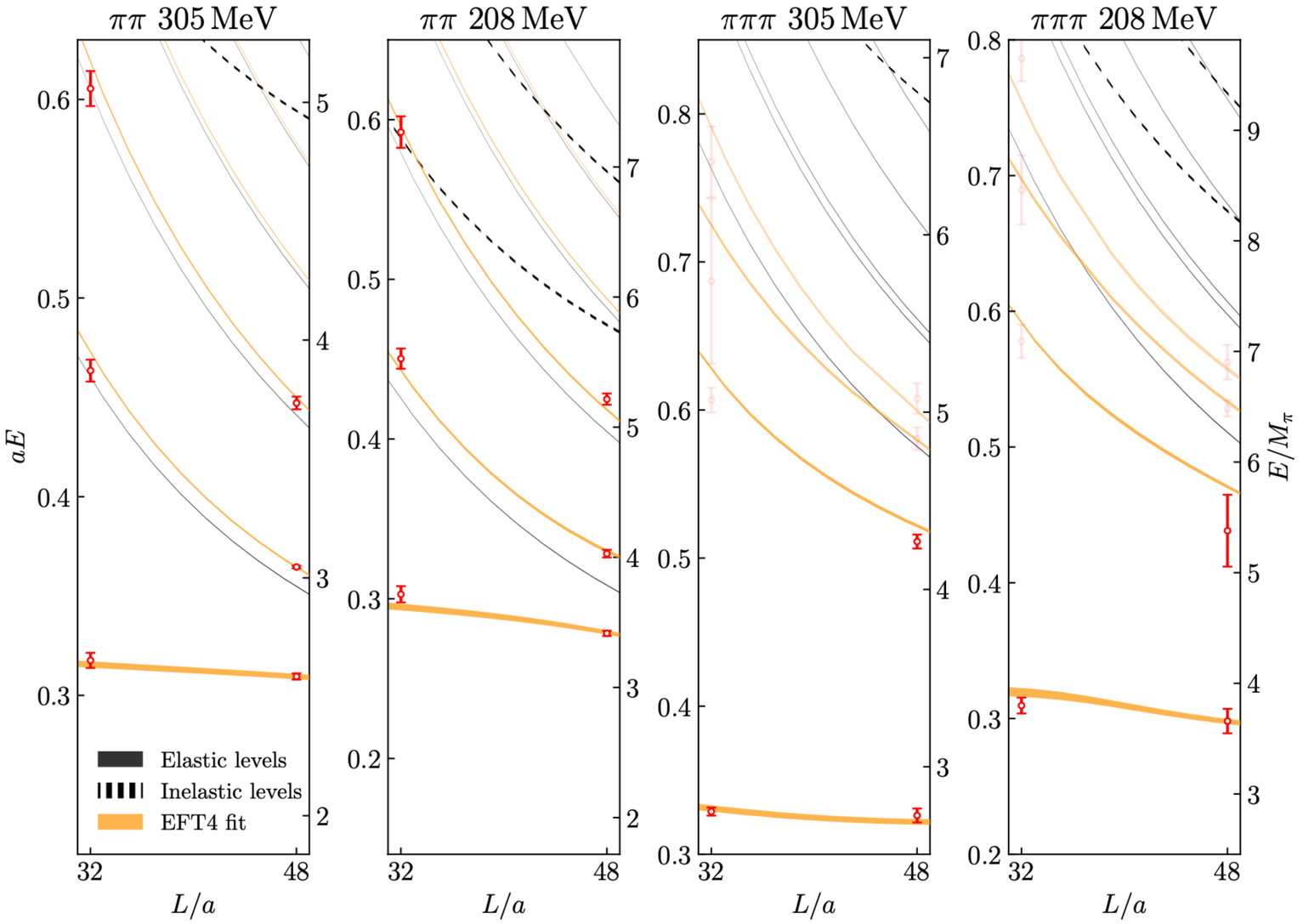
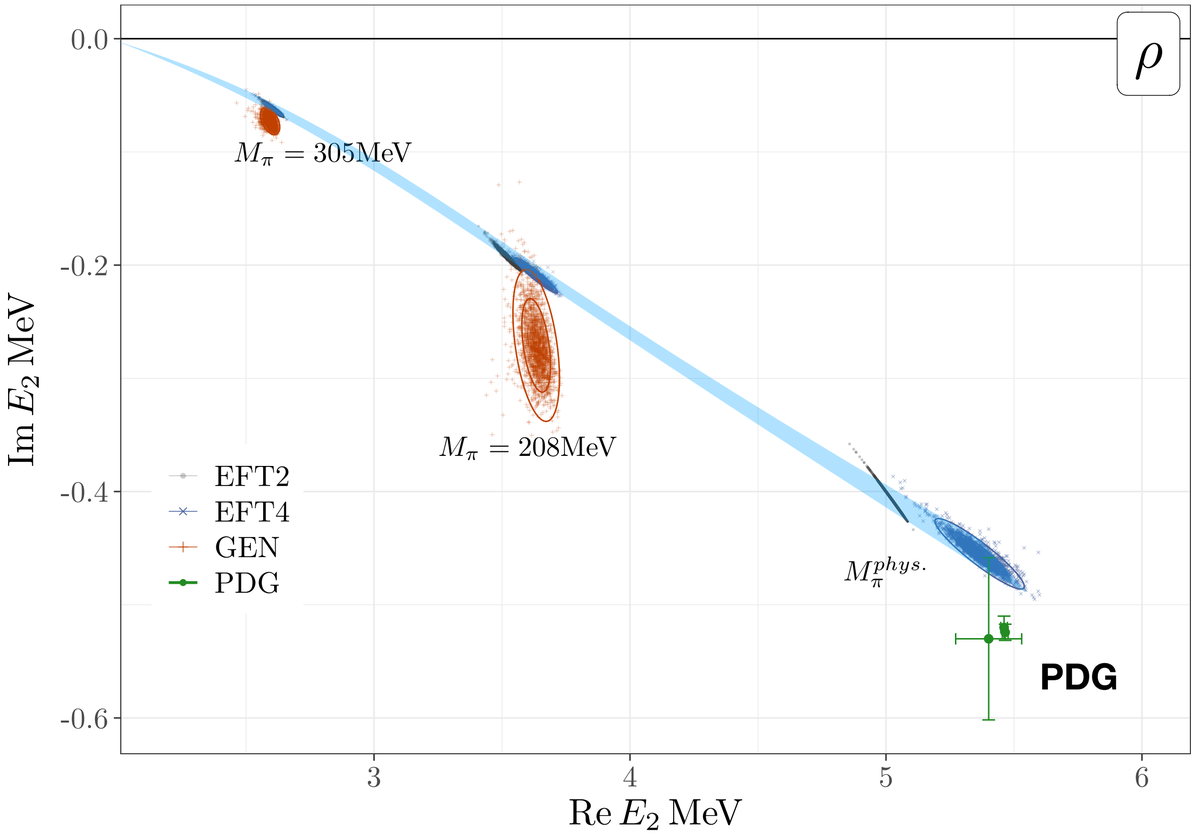


VECTOR MESON

Yan/MM/Garofalo/Liu/Liu/Meißner/Urbach PRL133 (2024)

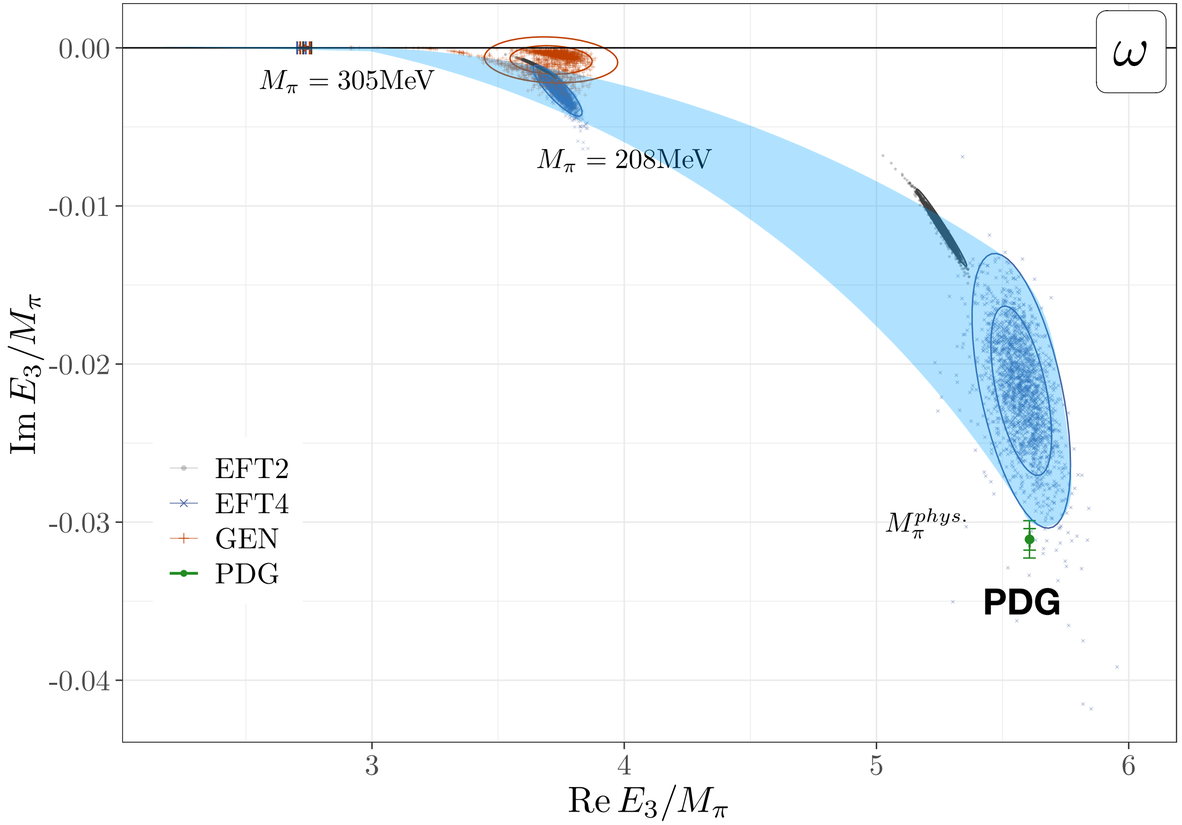
Final step

- Volume-independent 2-,3-body force $C, \tilde{K}$
- solve intergral equation
- analytically continue to the II Riemann Sheet



Result

- GEN/EFT2/EFT4 ansatzes consistent
- $\omega(782)$ becomes abound state at ~300 MeV
- at the physical point very close to the EXP value



… similar program carried out for $a_1(1260)$
MM/Culver+ PRL127 (2021)

Effective Field Theories

- quark-mass dependence
- analytical tools
- dynamically generated resonances

S-matrix

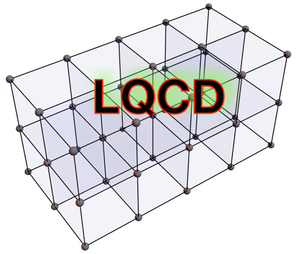
- Mathematical constraints on transitions
- Universal resonance parameter

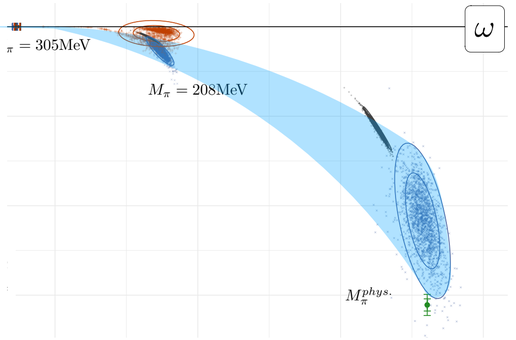
Lattice QCD:

- ab-initio calculations
- universal tool for physical und unphysical scenarios
- many new advances and results

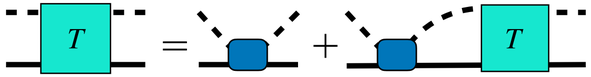
Thank you

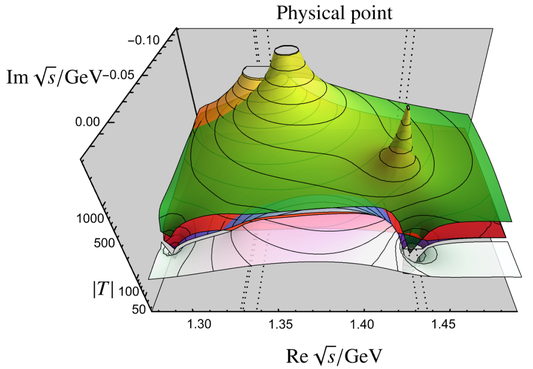
… rediscover 3-hadron resonances from lattice QCD





… discover new resonances from Chiral Dynamics





… clues about nature of states