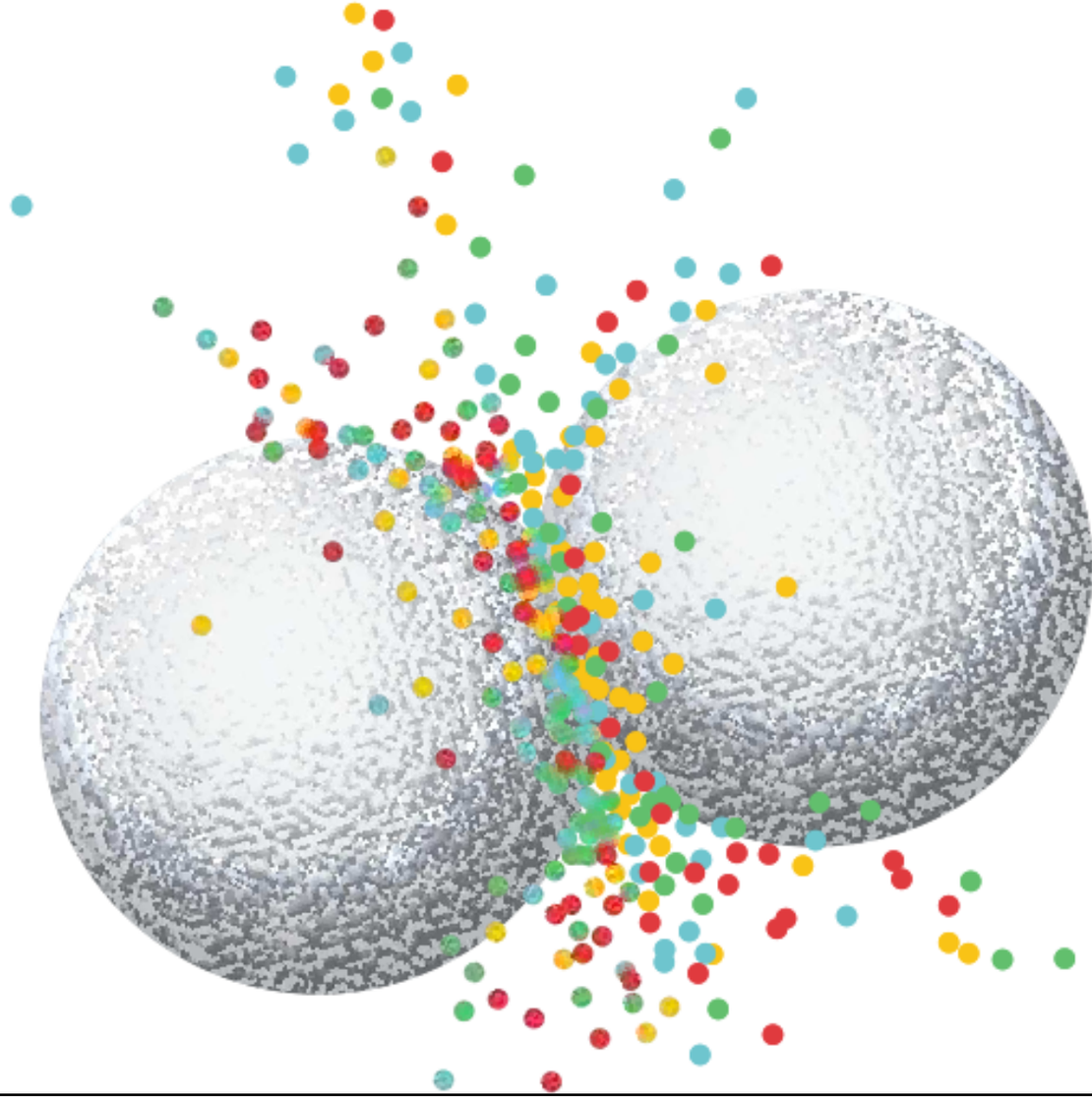

ECT* workshop on:
Universality in strongly-interacting systems: from QCD to atoms
Trento, June 11, 2025

Toward quantum computing hadron scattering

Zohreh Davoudi
University of Maryland, College Park

Joint Center for Quantum Information and Computer Science

MCFP MARYLAND CENTER FOR FUNDAMENTAL PHYSICS

Institute for Robust Quantum Simulation

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SCATTERING

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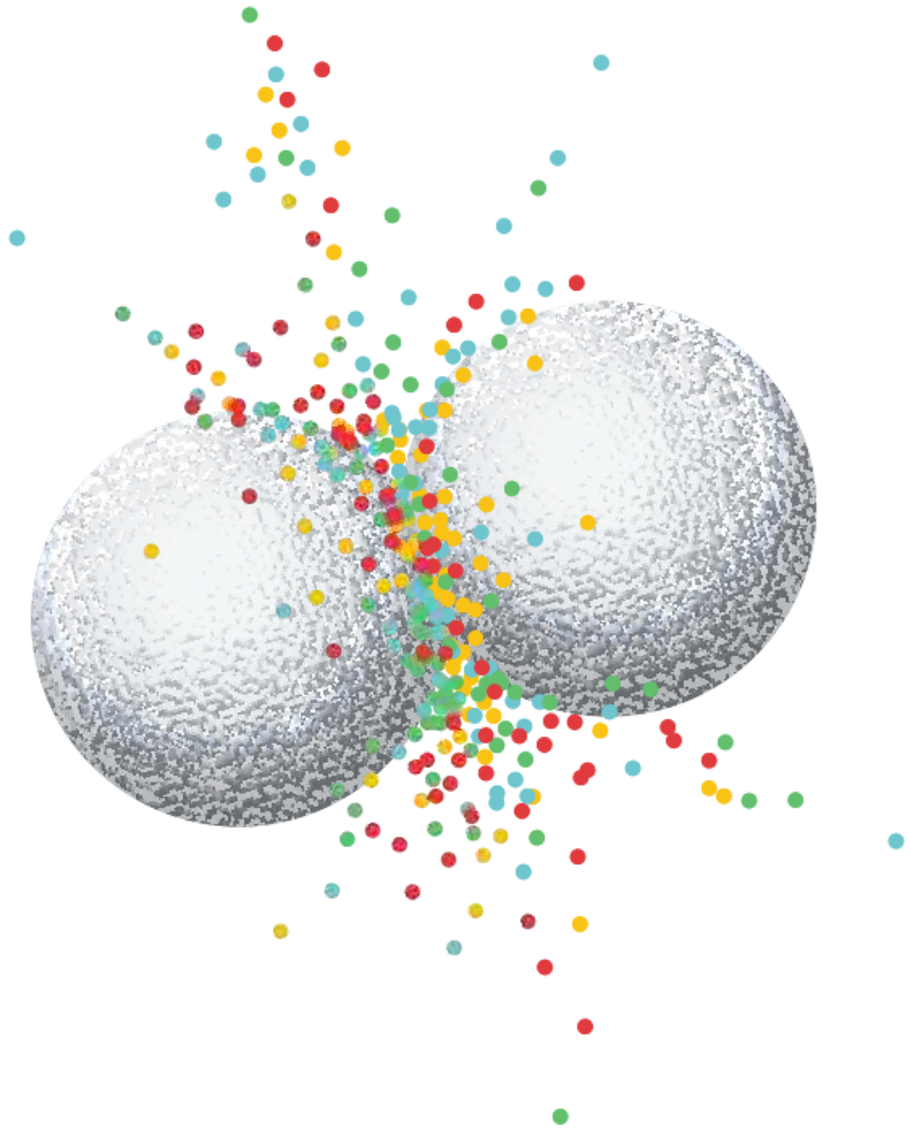
Classic Computing

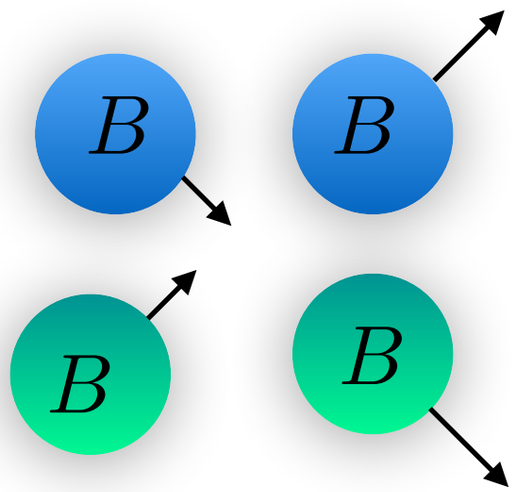
Can we classically simulate scattering of composite particles from the Standard Model?

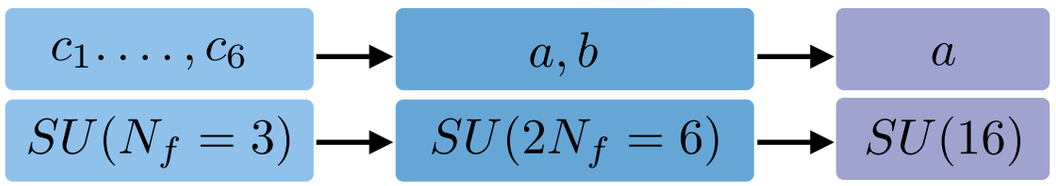


Lattice-gauge-theory methods based on Monte-Carlo sampling in Euclidean (imaginary) time have enabled this…but only at low energies so far…

SOME EXAMPLES FROM MY PAST AND CURRENT WORK WITHIN NPLQCD

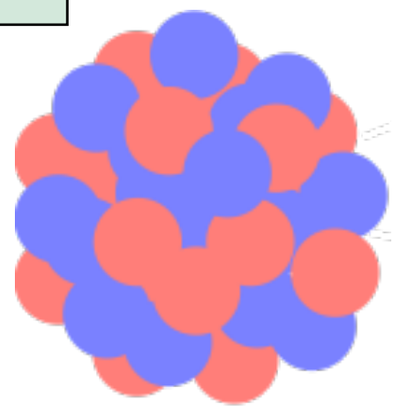
Wagman, ZD et al (NPLQCD), Phys.Rev.D96,114510 (2017).
Illa, ZD et al (NPLQCD), *Phys. Rev. D*103, 5, 054508 (2021).
Amarasinghe, ZD et al (NPLQCD), Phys. Rev. D 107, 094508 (2023).





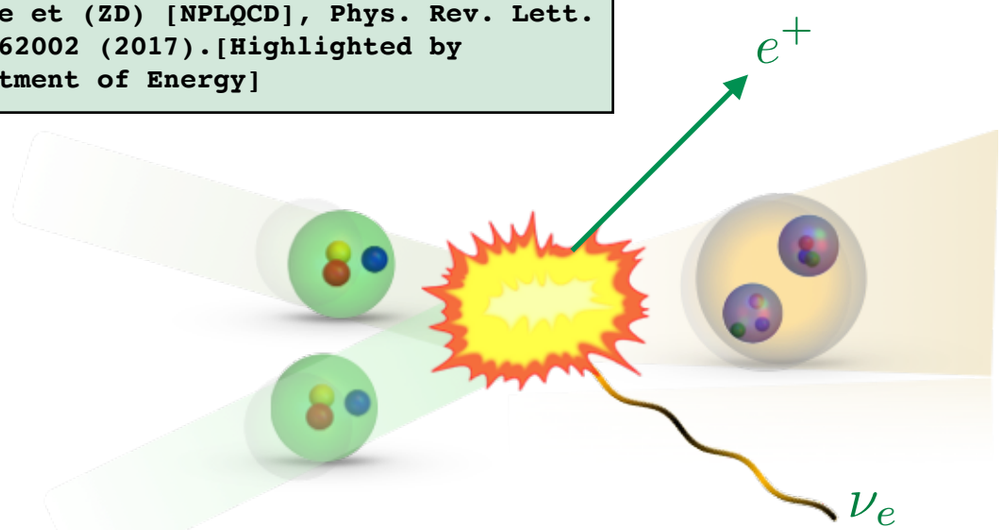
Chang, ZD et al (NPLQCD), Phys. Rev. Lett. 120, 5, 152002 (2018).

$$g_X^{(f)}(A) = \langle A|\bar{q}_f \Gamma_X q_f|A\rangle$$



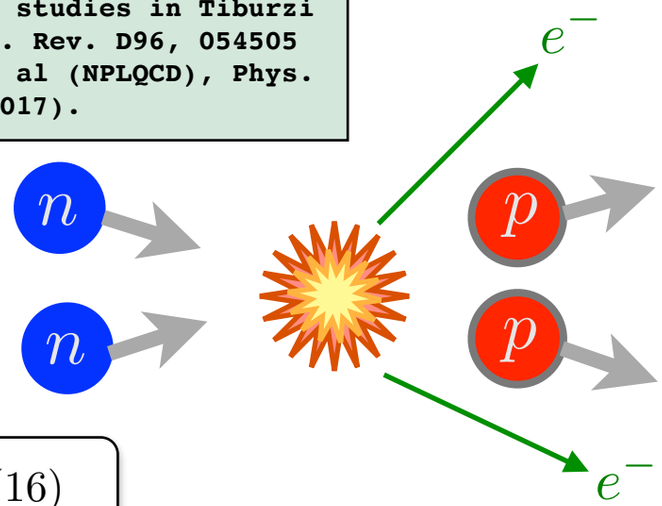
$$= \sum \; , \; ?$$

Savage et (ZD) [NPLQCD], Phys. Rev. Lett. 119,062002 (2017).[Highlighted by Department of Energy]



$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \text{fm}^3 \ @\ \mu = m_\pi^{\text{phys.}} = 140\ \text{MeV}$$

ZD, Detmold, Fu, Grebe, Jay, Murphy, Oare, Shanahan, Wagman, arXiv:2402.09362 [hep-lat]. See also our two-neutrino studies in Tiburzi et al (ZD) (NPLQCD), Phys. Rev. D96, 054505 (2017)and Shanahan, ZD et al (NPLQCD), Phys. Rev. Lett. 119, 062003 (2017).



$$a^2\mathcal{A}^{nn\to pp} = 0.078(16)$$
$$@\ m_\pi = 806\ \text{MeV}$$

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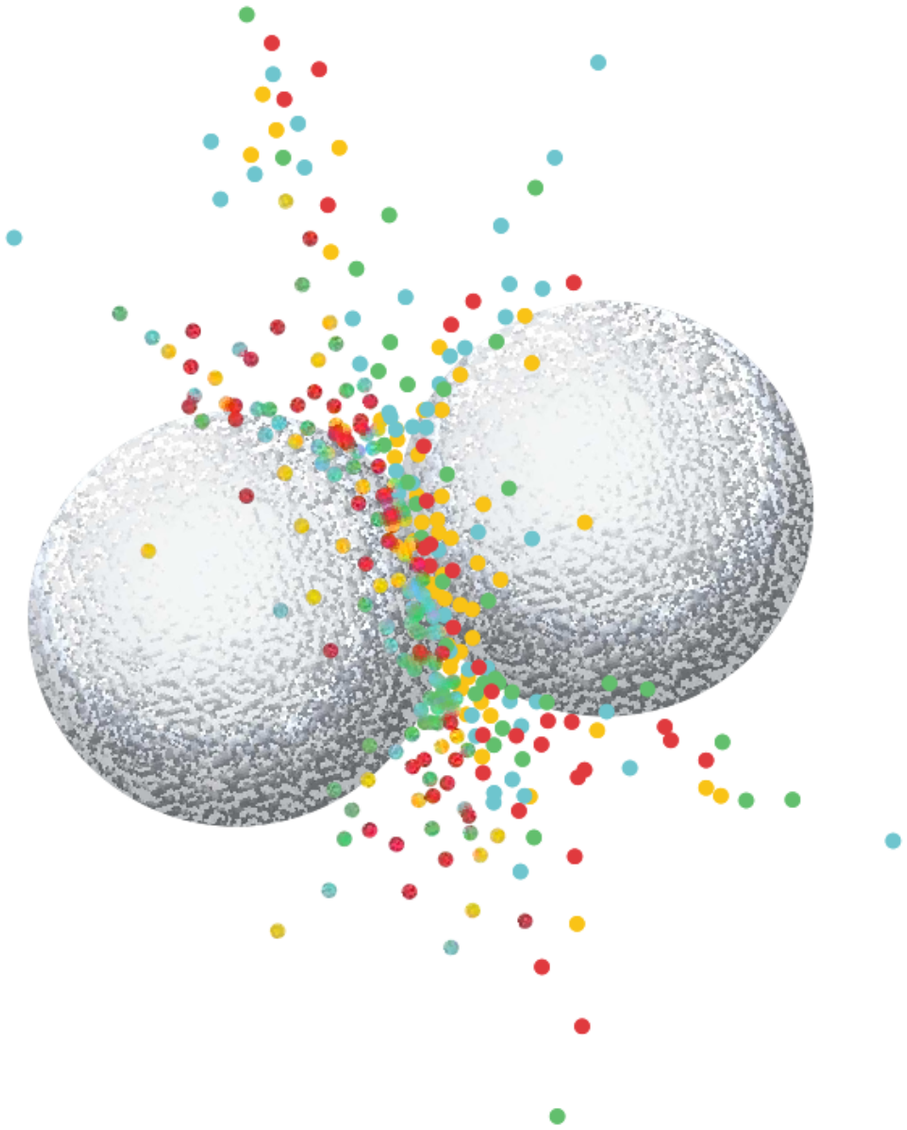
Classic Computing

What about high energies, like events at the Large Hadron Collider or the Relativistic Heavy-Ion Collider?

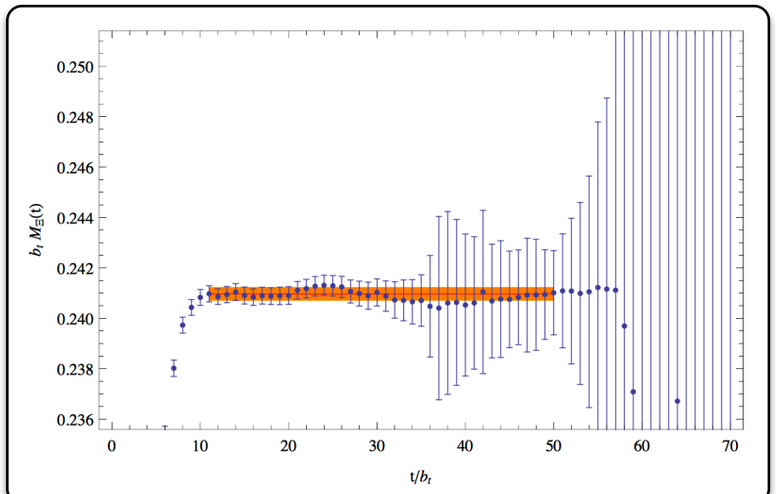


There are mainly two issues…i) making complicated states, i.e., high-energy protons, or heavy ions, etc, and ii) imaginary-time nature of the classical Monte-Carlo calculations…no access to states as a function of Minkowski time elapsed after the collision!

THREE FEATURES MAKE LATTICE-QCD CALCULATIONS OF NUCLEI HARD:

i) The complexity of systems grows factorially with the number of quarks.

Detmold and Orginos (2013)
Detmold and Savage (2010)
Doi and Endres (2013)

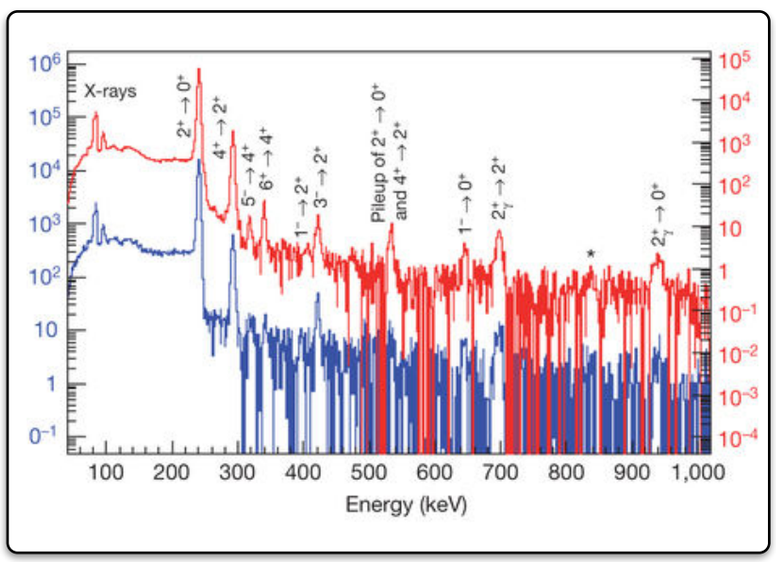


ii) There is a severe signal-to-noise degradation.

Paris (1984) and Lepage (1989)
Wagman and Savage (2017, 2018)

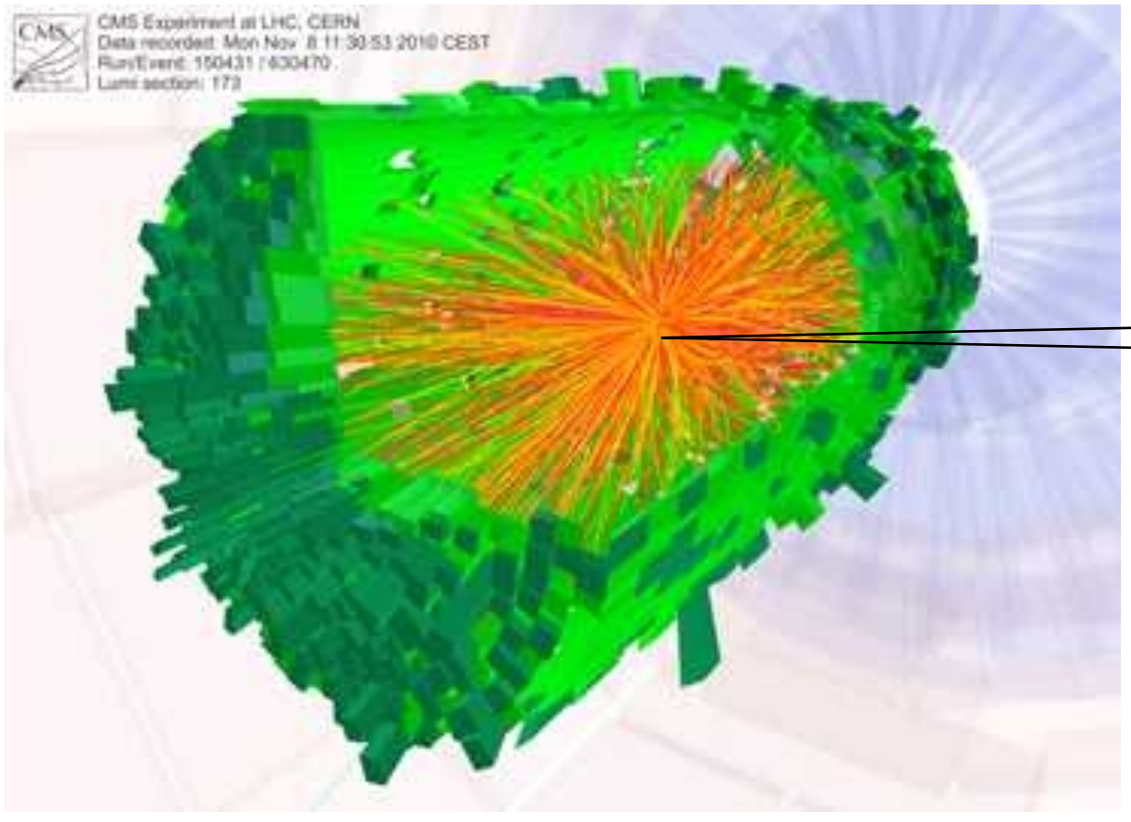
iii) Excitation gaps of nuclei are much smaller than the QCD scale.

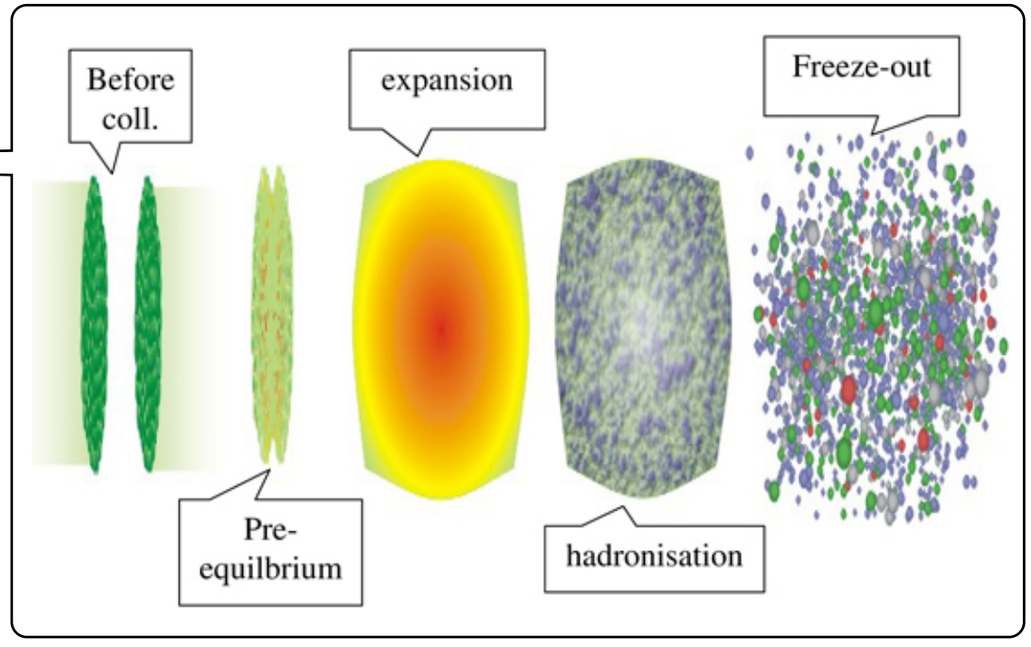
Beane at al (NPLQCD) (2009)
Beane, Detmold, Orginos, Savage (2011)
ZD (2018)
Briceno, Dudek and Young (2018)



SIGN PROBLEM MAKES CONVENTIONAL LATTICE-GAUGE-THEORY METHODS INTRACTABLE.

No access to real-time non-equilibrium dynamics of matter in heavy-ion collisions or after the Big Bang…





…and to a wealth of dynamical response functions, transport properties, parton distribution functions, etc.

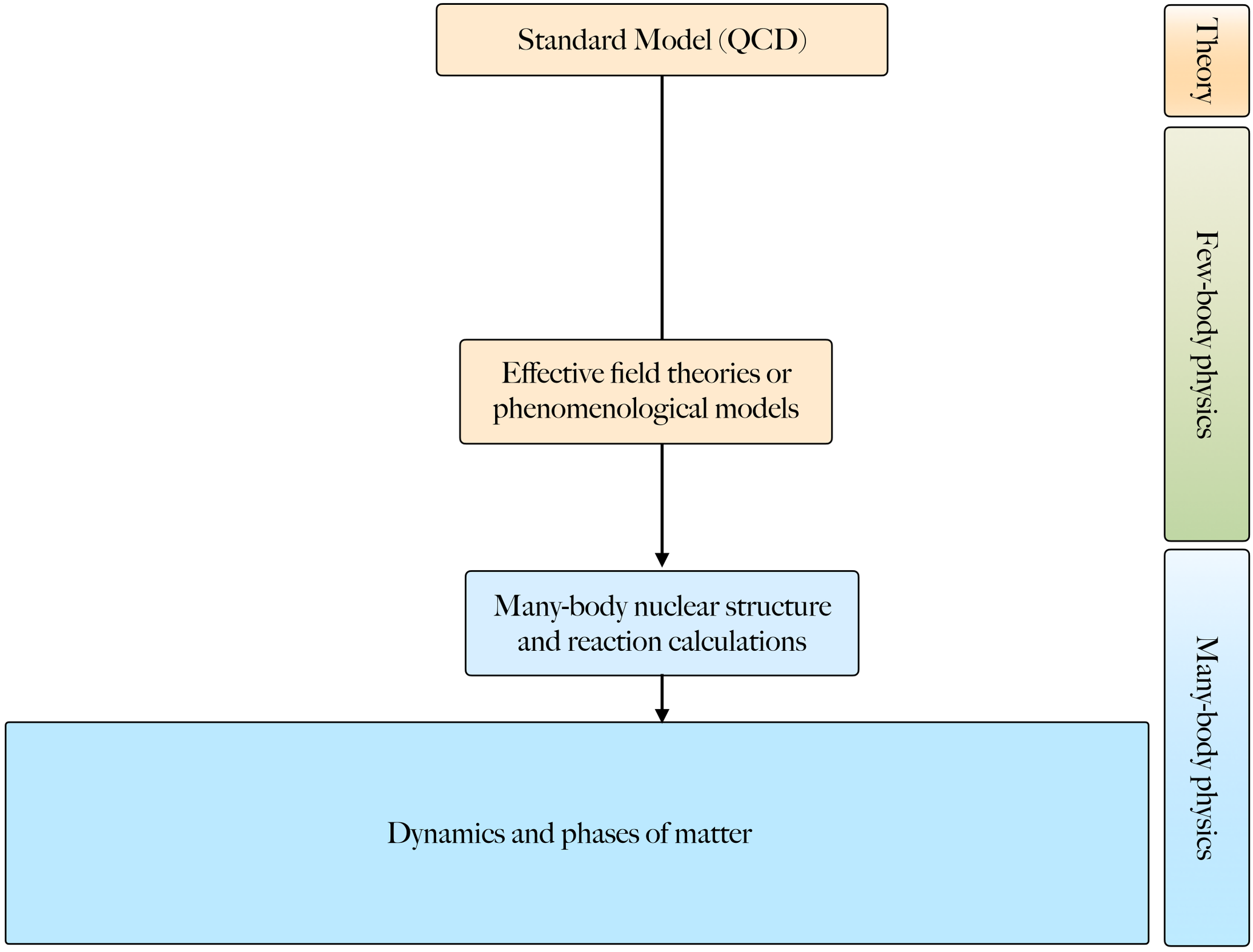
Path integral formulation:

$$e^{iS[U,q,\bar{q}]}$$

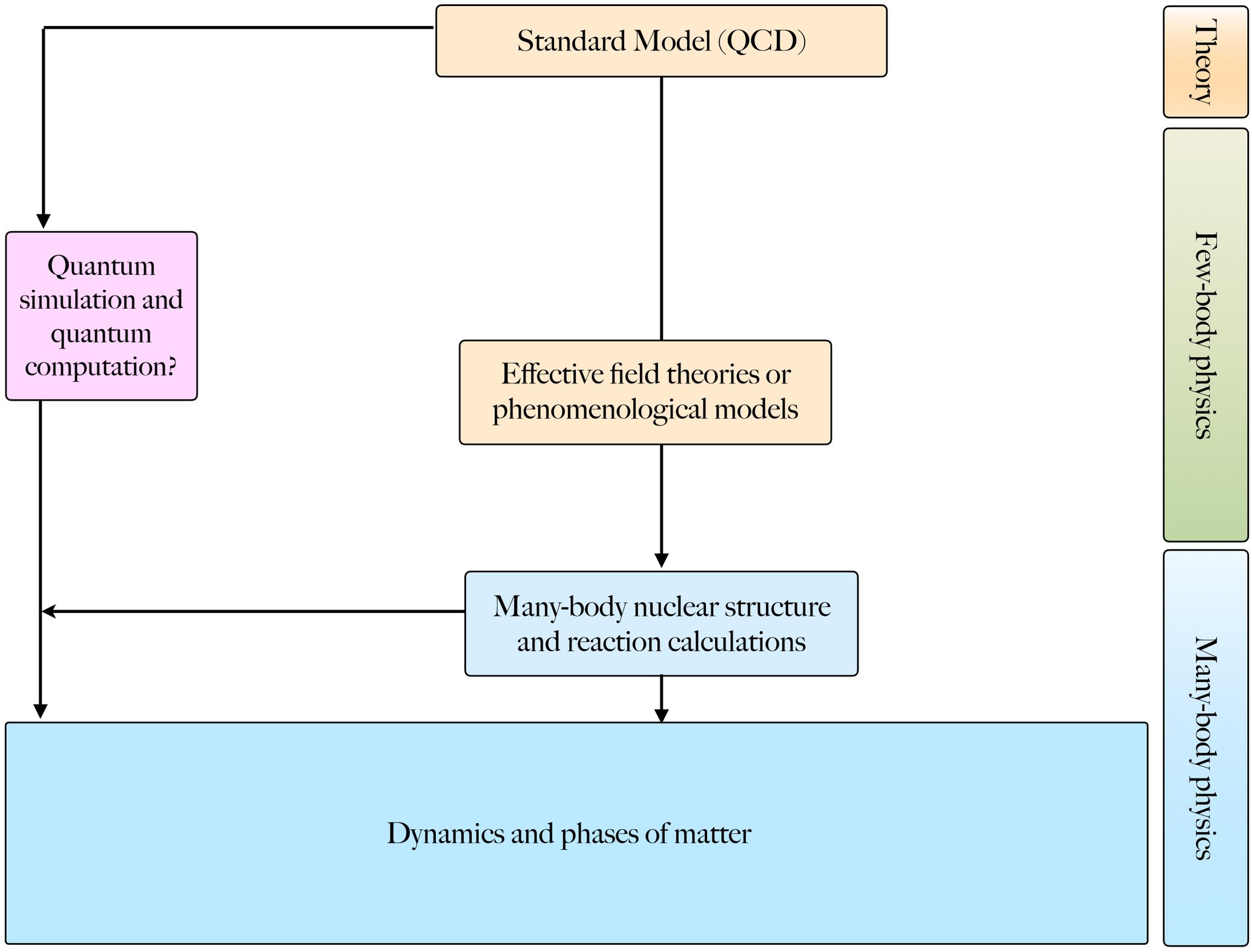
Hamiltonian evolution:

$$U(t) = e^{-itH}$$

A NUCLEAR/PARTICLE PHYSICS ROADMAP



A NUCLEAR/PARTICLE PHYSICS ROADMAP FOR LEVERAGING QUANTUM TECHNOLOGIES



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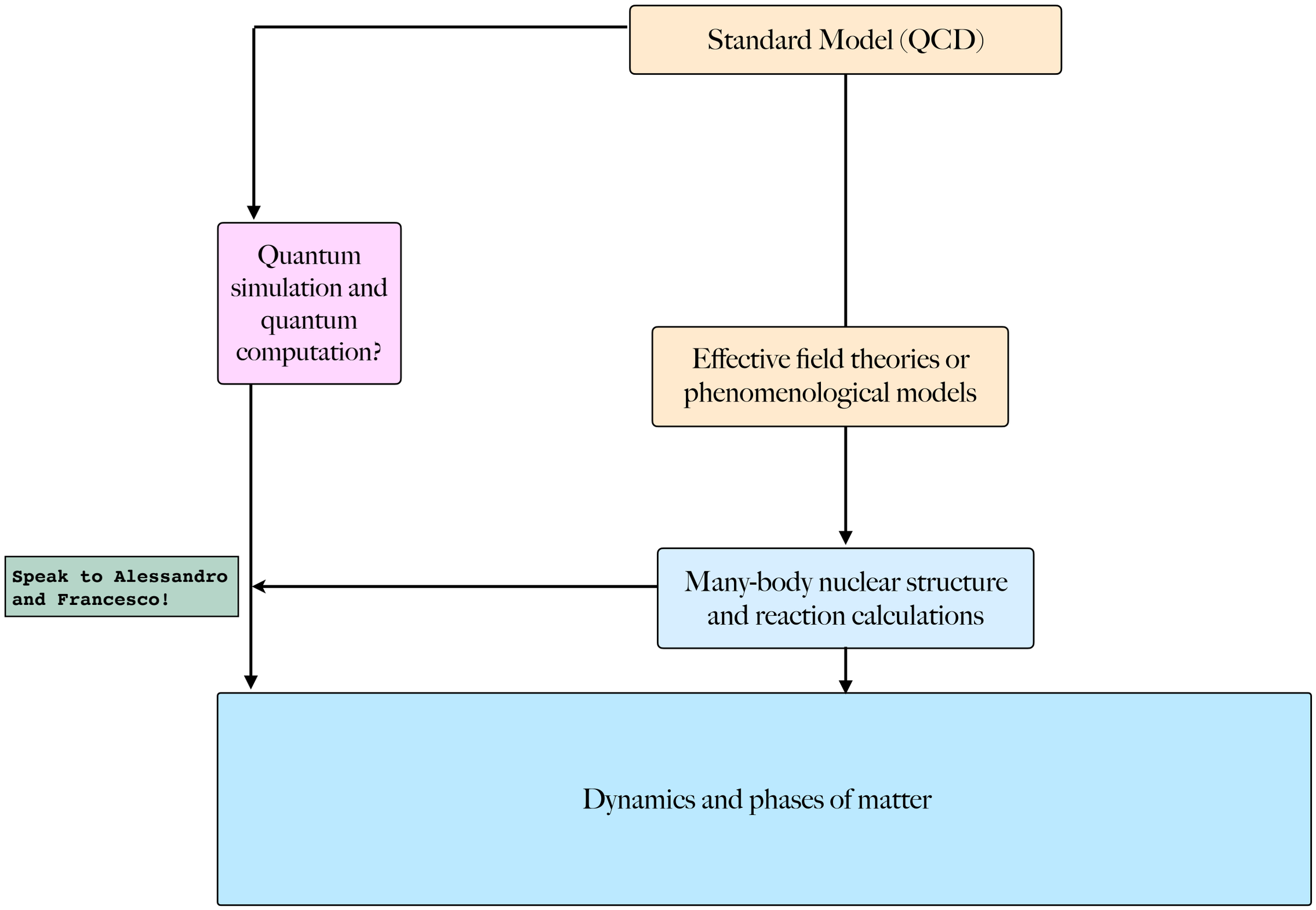

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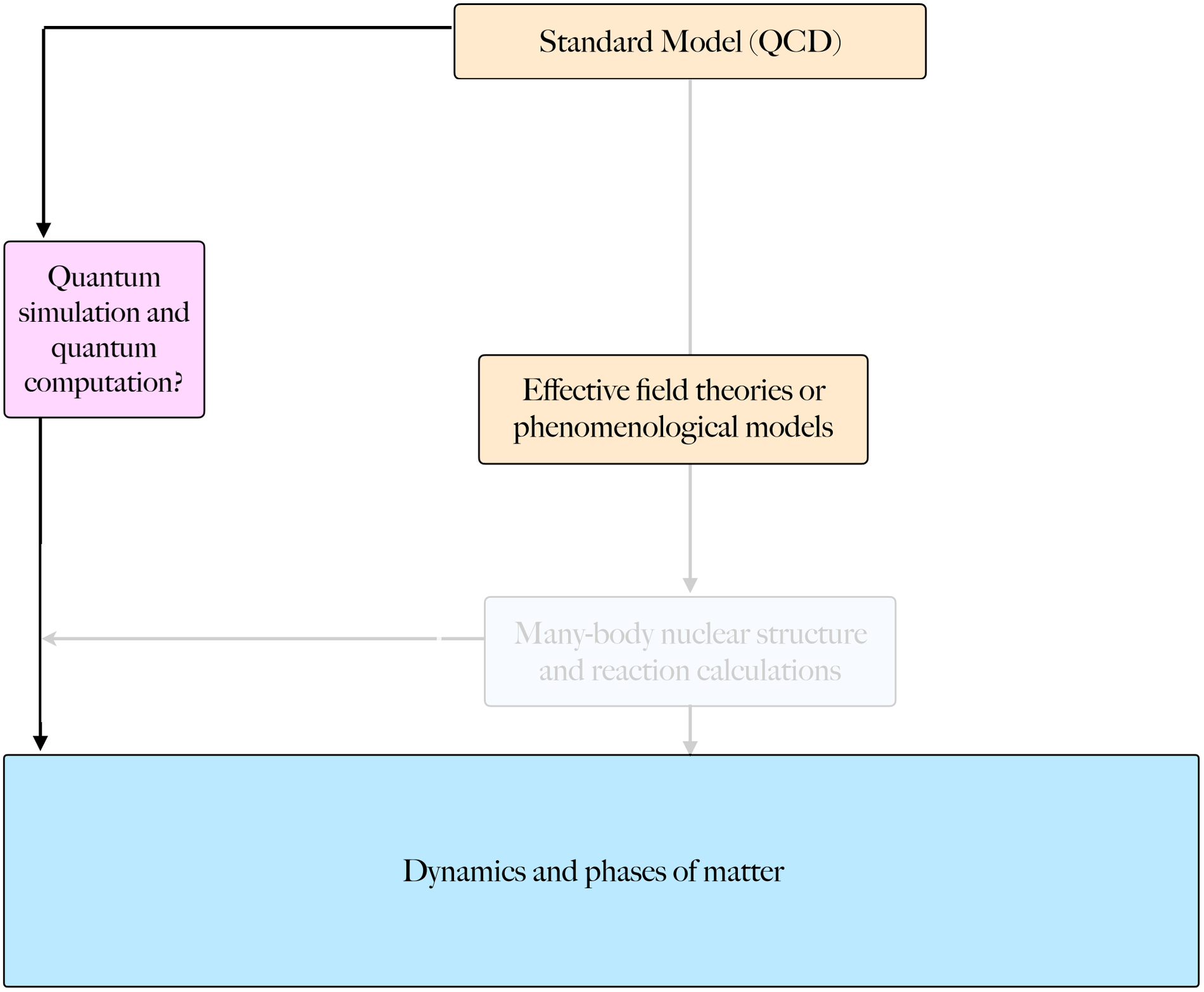


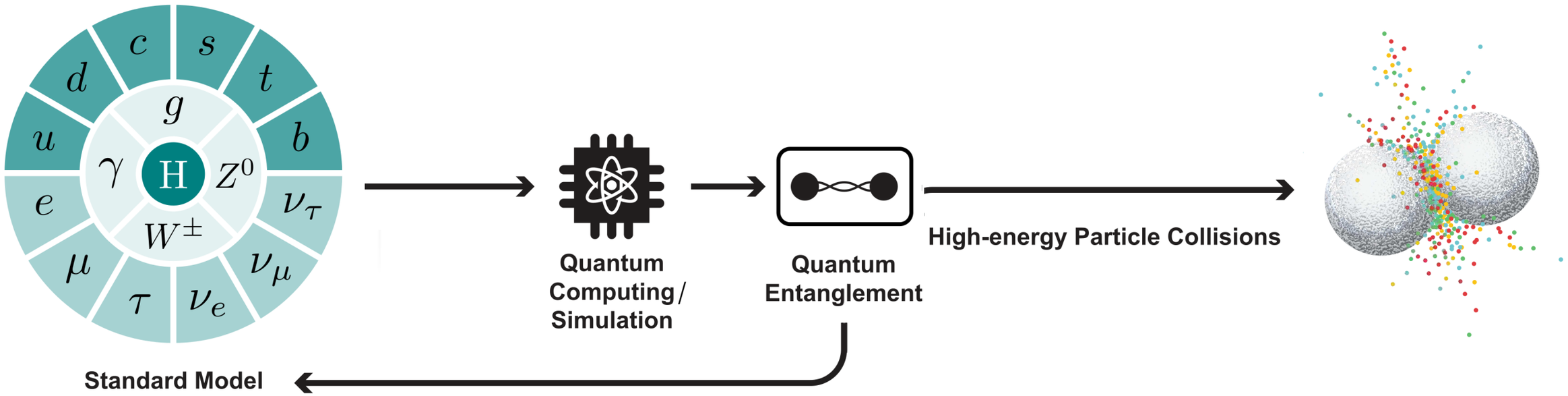
Quantum simulation and quantum computation?

We turn to **quantum simulation/computation** since:

i) Hilbert spaces can be encoded exponentially more compactly.

ii) Operations can be highly parallelized using quantum superposition and entanglement!

SO CAN WE STUDY HIGH-ENERGY SCATTERING VIA QUANTUM SIMULATION?



Graphic adopted from Bauer, ZD, Klco, and Savage, *Nature Rev. Phys.* 5 (2023) 7, 420-432.

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QUANTUM SIMULATION AND QUANTUM-COMPUTING BASICS

i) Quantum-simulation steps

ii) Various modes of quantum simulation: Digital, analog, hybrid

iii) Digital-quantum-simulations basics:

- qubits and gates
- Encoding fermions and bosons onto qubits
- State-preparation strategies
- Time evolution (via product formulas)
- Measurement strategies and observables

QUANTUM-SIMULATION STEPS:

QUANTUM SUBPROCESS

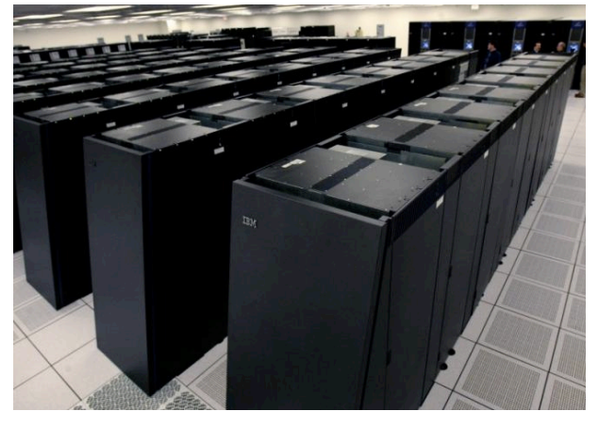
Prepare the initial state

Evolve with e^{-itH}

Measure observables

?

Conventional lattice QCD



?

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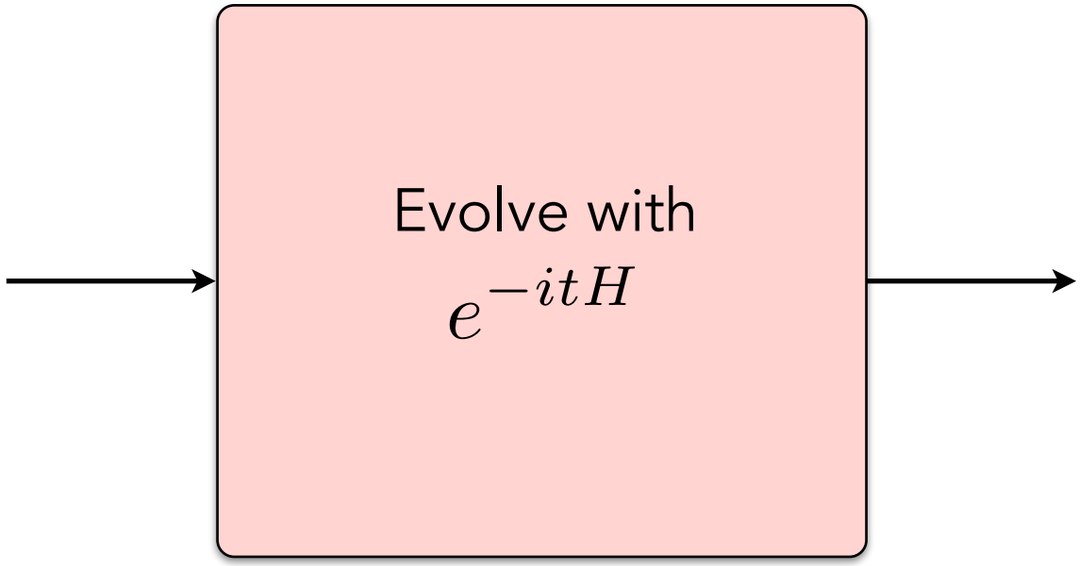
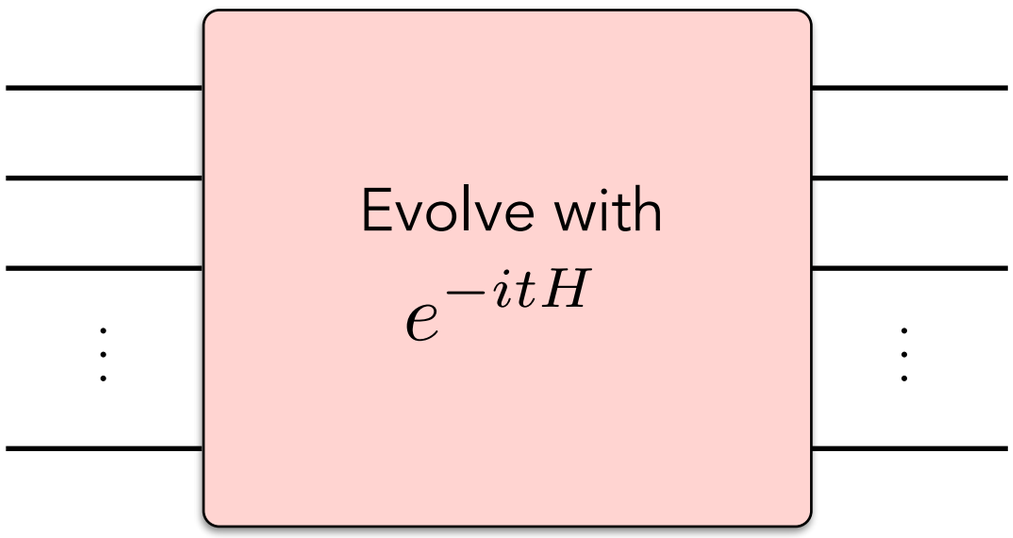
DIFFERENT APPROACHES TO QUANTUM SIMULATION



DIFFERENT APPROACHES TO QUANTUM SIMULATION

Analog



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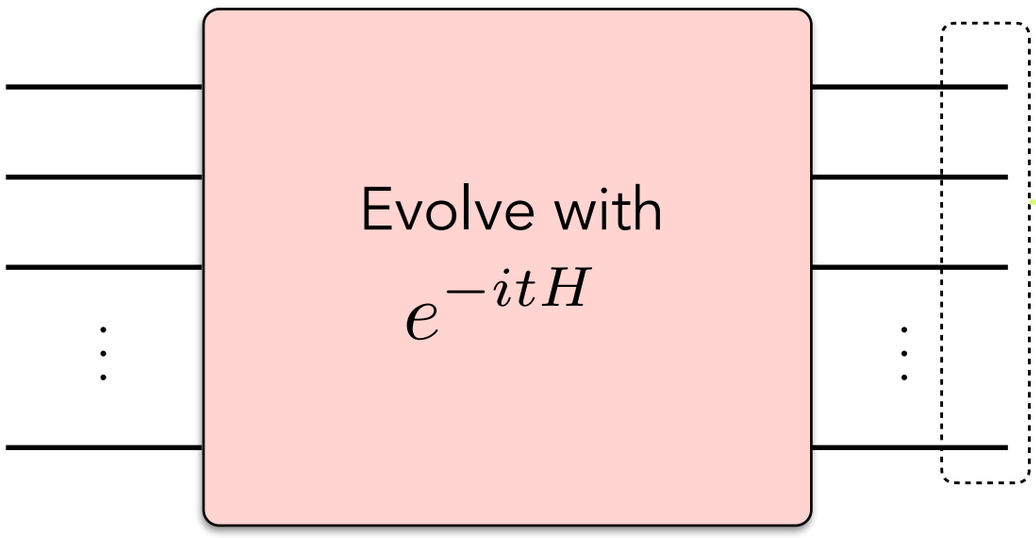
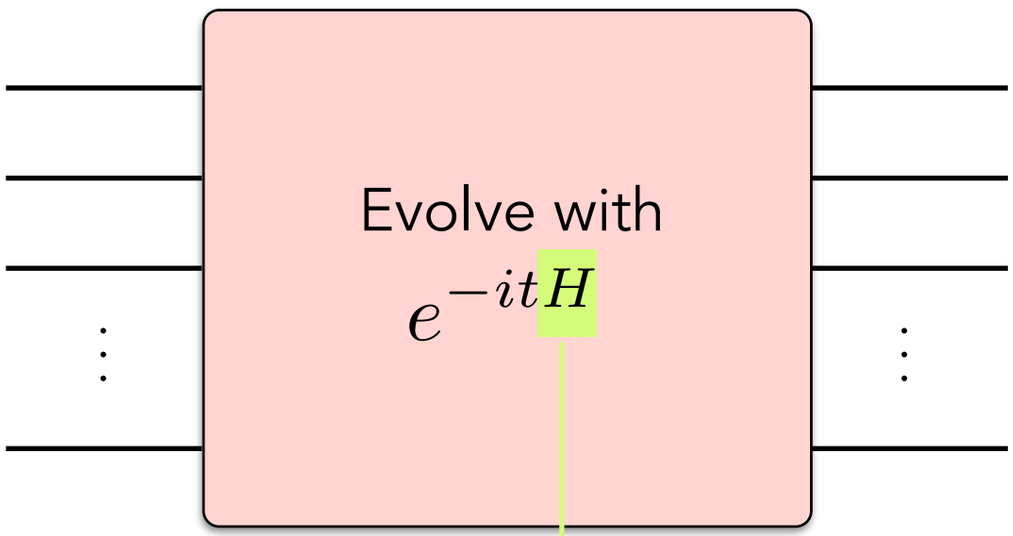


Degrees of freedom in the simulator: fermions, bosons, spins (of various dimensions), etc.

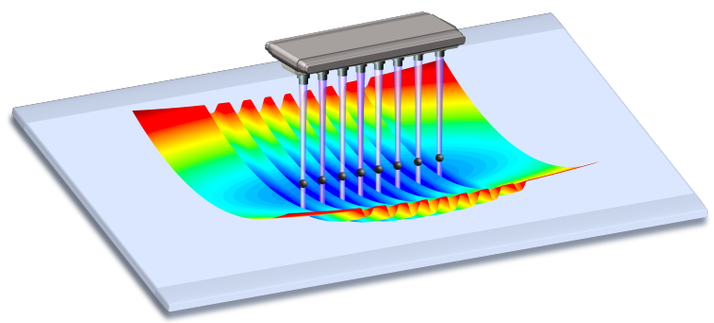
DIFFERENT APPROACHES TO QUANTUM SIMULATION

Analog

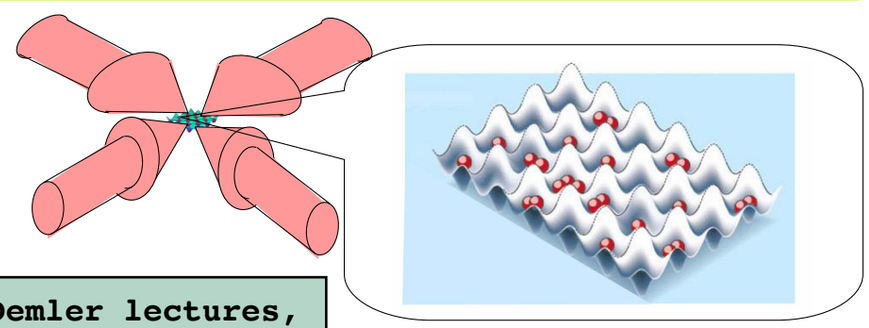


The engineered simulator Hamiltonian that mimics the Hamiltonian of target system.

Some of the leading analog simulators are: cold-atoms in optical lattices, Rydberg atoms with optical tweezers, trapped ions, superconducting circuits (including when coupled to photonics systems), etc.



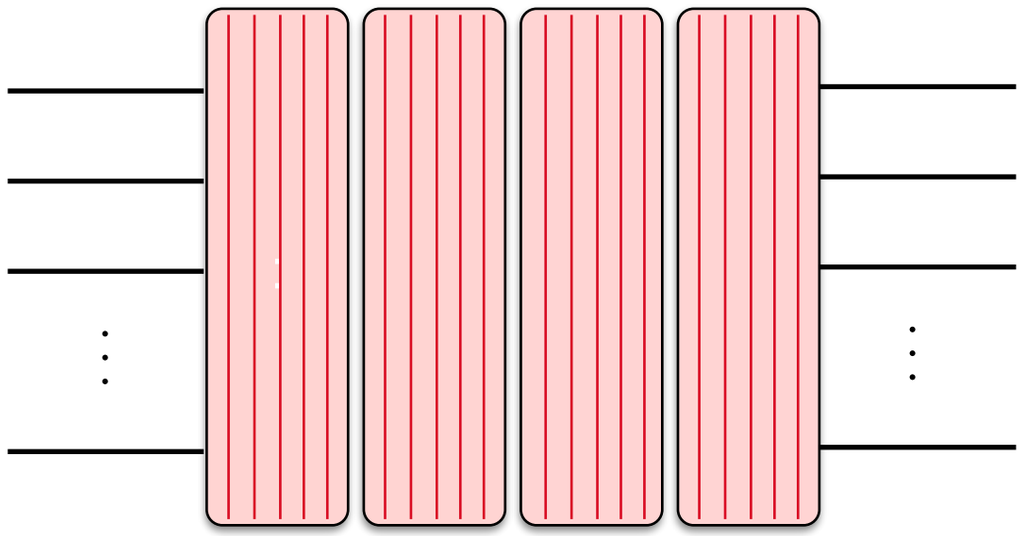
CREDIT: ANDREW SHAW, UNIVERSITY OF MARYLAND



Eugene Demler lectures, Harvard University.

DIFFERENT APPROACHES TO QUANTUM SIMULATION

Digital



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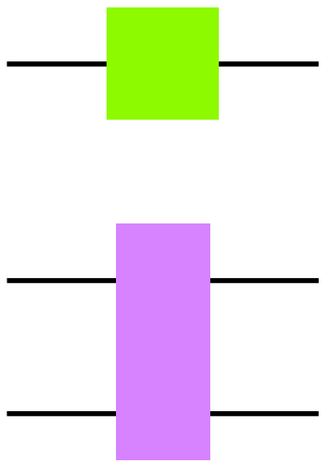
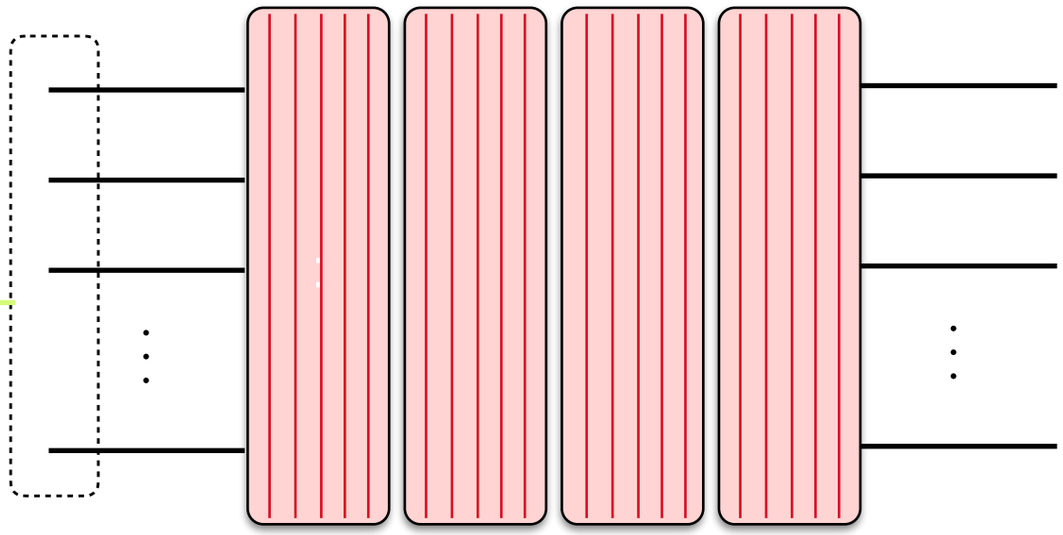


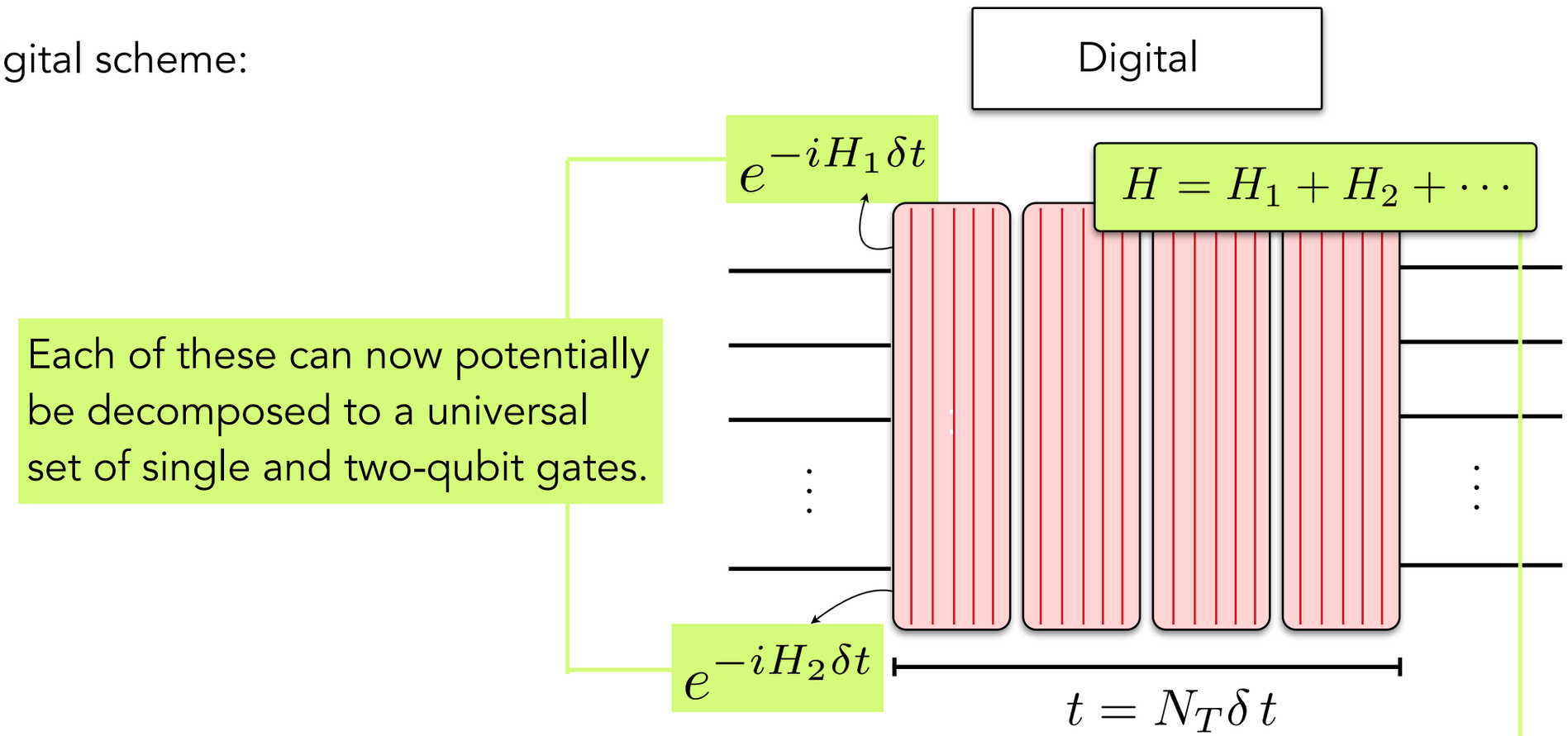
Only qubits as DOF. Only universal single- and two-qubit operations allowed.

Digital



DIFFERENT APPROACHES TO QUANTUM SIMULATION

Example of a digital scheme:



Each of these can now potentially be decomposed to a universal set of single and two-qubit gates.

Trotter-Suzuki expansion:

$$e^{-i(H_1+H_2+\cdots)t} = \left[e^{-iH_1\delta t}e^{-iH_2\delta t}\cdots\right]^{t/\delta t} + \mathcal{O}((\delta t)^2)$$

Other digitalization schemes also exist.

…other methods exist too.

Andrew Childs lectures on Quantum Simulation, University of Maryland.

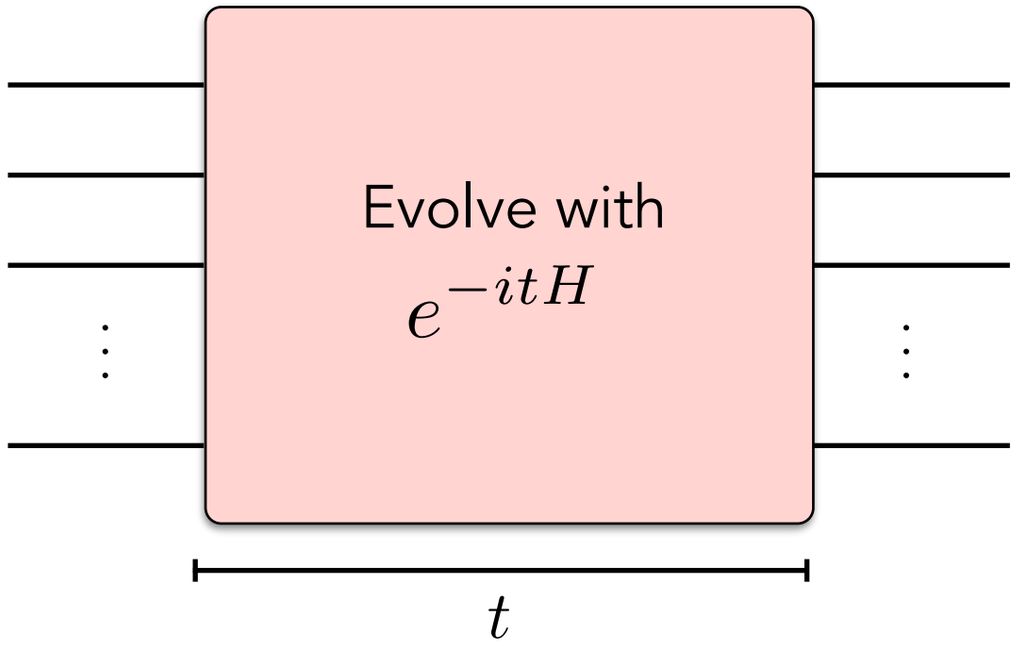
DIFFERENT APPROACHES TO QUANTUM SIMULATION

Analog

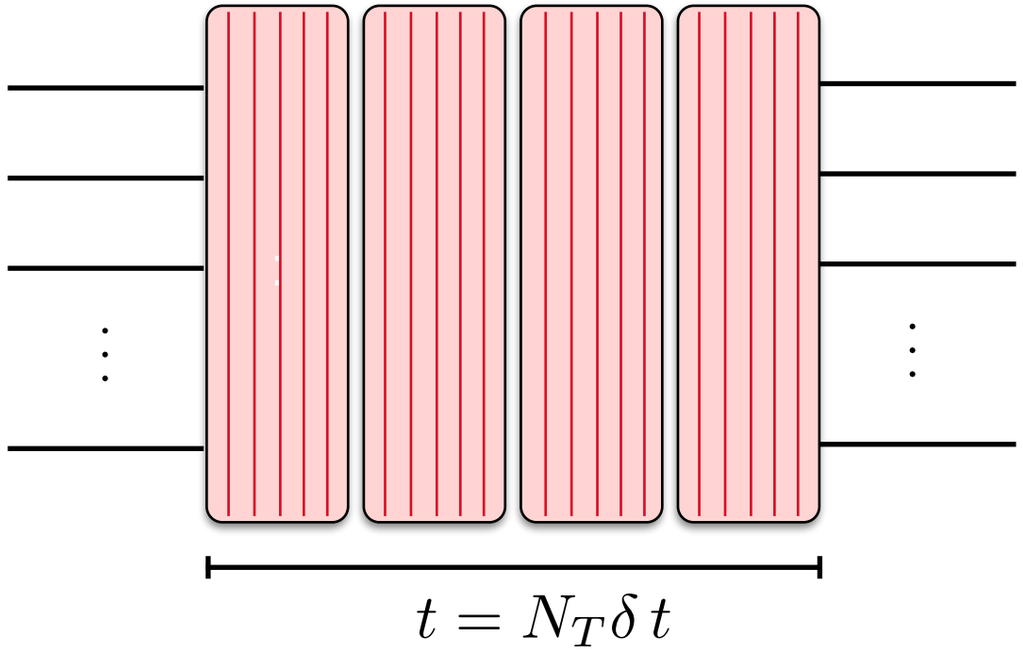
Digital

$\approx e^{-itH}$

Evolve with
e^{-itH}



t



$t = N_T \delta\, t$

Analog-Digital

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A textbook of extreme popularity:
Nielsen and Chuang, Quantum Computation and Quantum Information.
But some of the newer notions not there.

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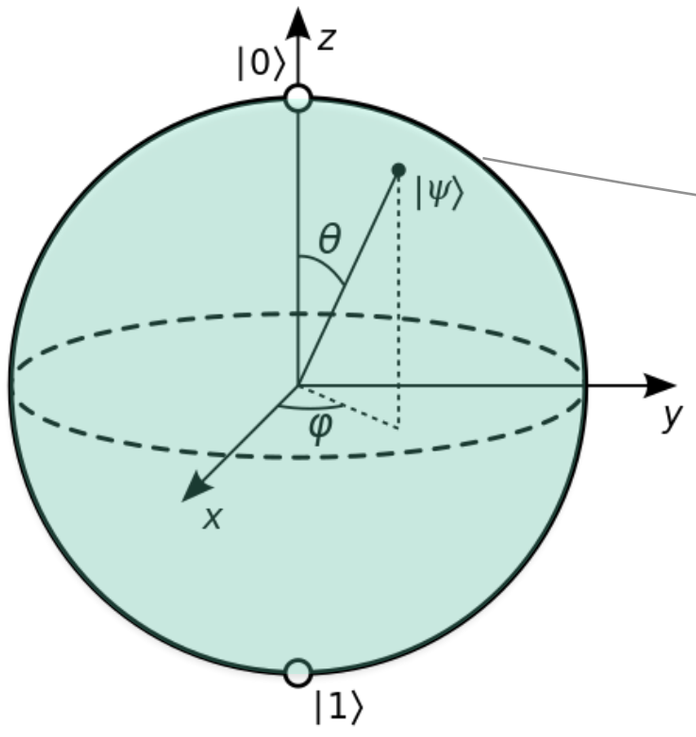
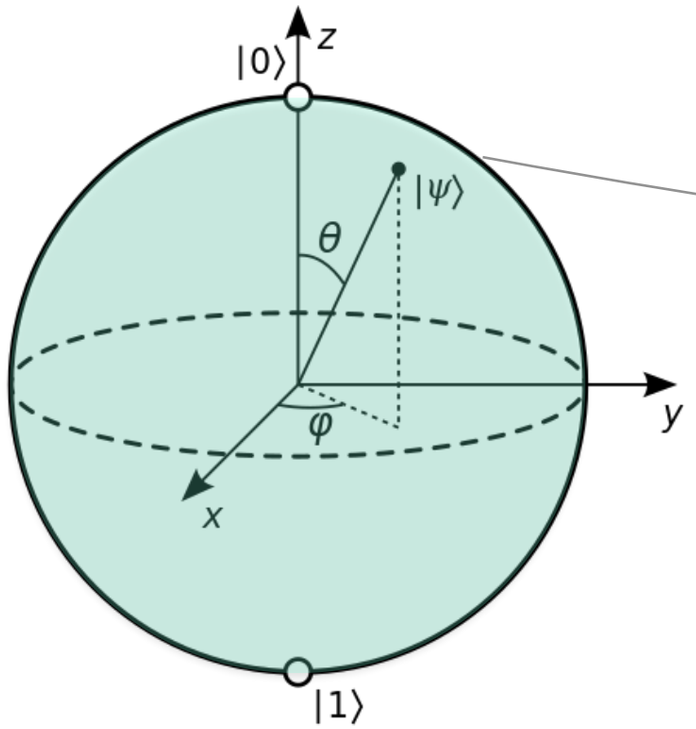


State of a single qubit: $|\psi\rangle = a|0\rangle + b|1\rangle \equiv a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}$

$\equiv \cos(\theta/2)|0\rangle + ie^{i\phi}\sin(\theta/2)|1\rangle$

State of two qubits: $|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

$$\equiv a\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + b\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + c\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + d\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$



State of a single qubit: $|\psi\rangle = a|0\rangle + b|1\rangle \equiv a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}$

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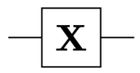
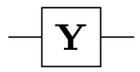
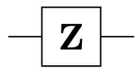
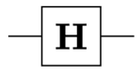
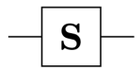
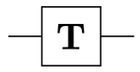
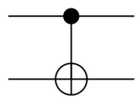
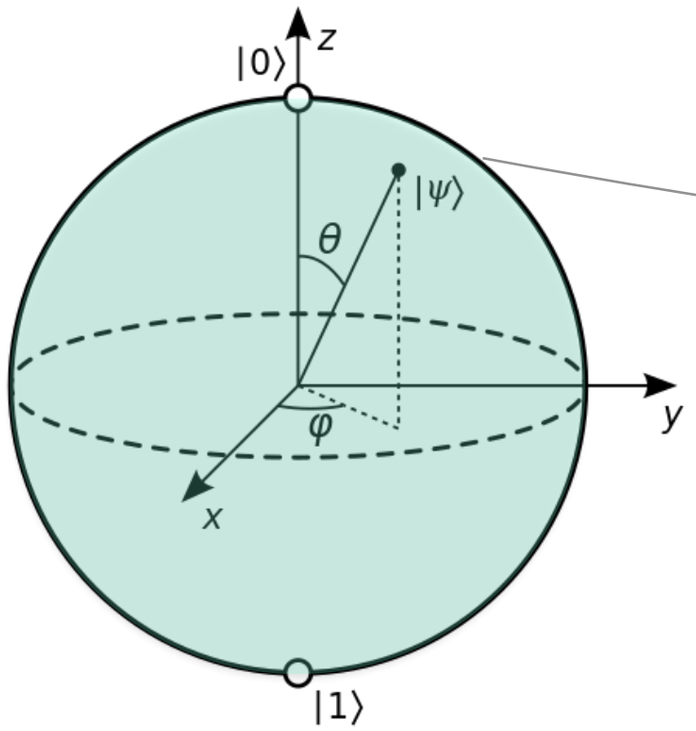
$$\equiv a\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + b\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + c\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + d\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

(Examples of) quantum logic gates

Operator	Gate(s)	Matrix
Pauli-X (X)		$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
Pauli-Y (Y)		$\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$
Pauli-Z (Z)		$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
Hadamard (H)		$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
Phase (S, P)		$\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$
π/8 (T)		$\begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$



State of a single qubit: $|\psi\rangle = a|0\rangle + b|1\rangle \equiv a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}$

$\equiv \cos(\theta/2)|0\rangle + ie^{i\phi}\sin(\theta/2)|1\rangle$

State of two qubits: $|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

$$\equiv a\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + b\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + c\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + d\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

(Examples of) quantum logic gates

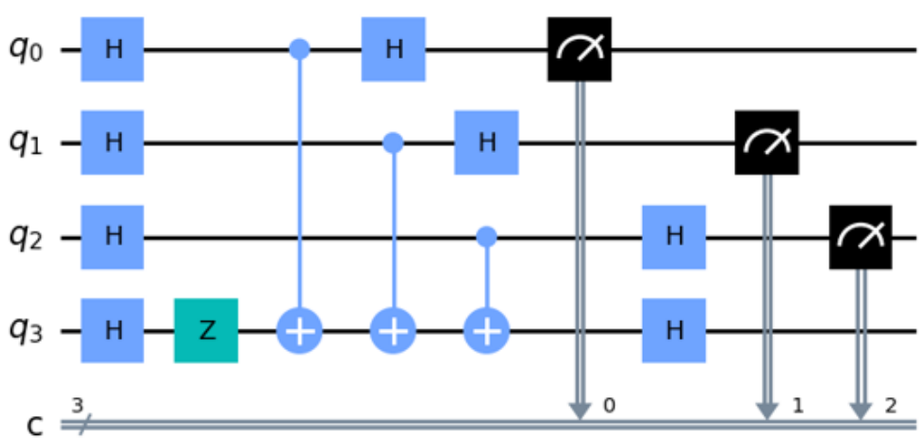
Operator	Gate(s)	Matrix
Pauli-X (X)	X	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
Pauli-Y (Y)	Y	$\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$
Pauli-Z (Z)	Z	$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
Hadamard (H)	H	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
Phase (S, P)	S	$\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$
π/8 (T)	T	$\begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$

Any unitary on a finite number of qubits can be approximated *efficiently* by a finite sequence of a universal gate set. **Solovay (1995) and Kitaev (1997).**

Two common choices for these gate sets are:

- $R^x(\theta) = e^{-i\theta\sigma^x/2}$, $R^y(\theta) = e^{-i\theta\sigma^y/2}$, $R^z(\theta) = e^{-i\theta\sigma^z/2}$, $P_\phi = \begin{pmatrix}1 & 0\\0 & e^{i\phi}\end{pmatrix}$, CNOT
- H, S, CNOT, T (S not strictly needed but more economical.)

Example of) a quantum circuit:



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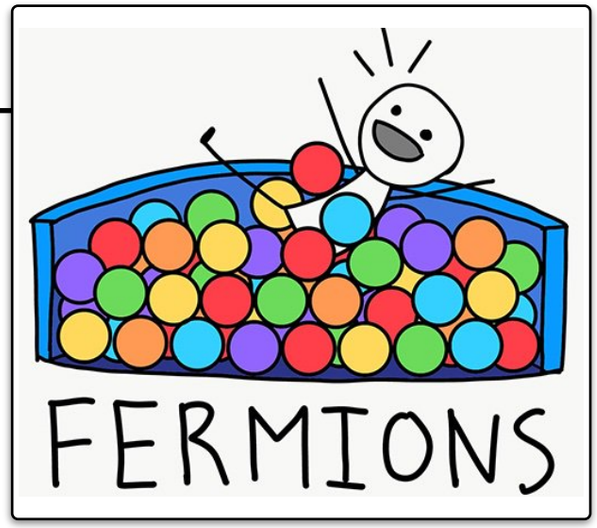
Fermions are **finite-dimensional** locally but obey **Fermi statistics**. Mapping a fermionic Hamiltonian into a qubit Hamiltonian can be done:

- using one qubit per fermion but at the cost of non-local qubit interactions using Jordan-Wigner transformation:

$$\psi_i = (\prod_{j<i} \sigma_j^z)\sigma_i^+, \;\; \psi_i^\dagger = (\prod_{j<i} \sigma_j^z)\sigma_i^-$$

- using more than one qubit per fermion to assist retaining any existing locality in the original fermionic Hamiltonian (e.g. Verstrate-Cirac, compact, superfast encodings).



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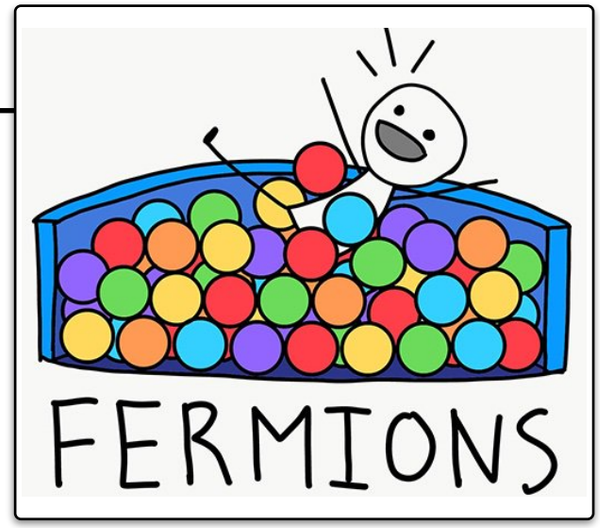
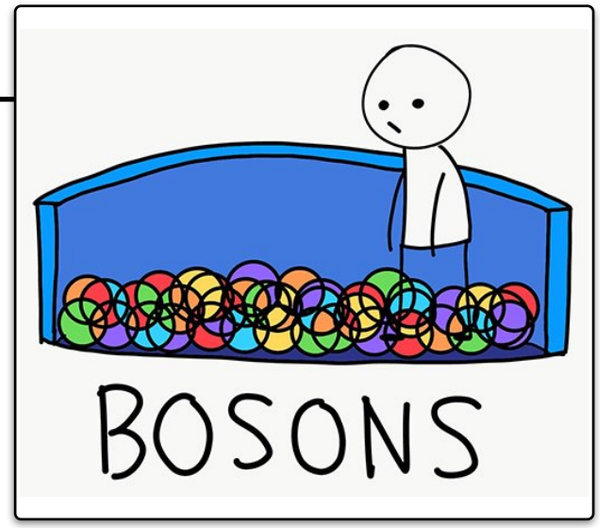


Bosons are **infinite-dimensional** locally but obey **Bose statistics**. Mapping a bosonic Hamiltonian into a qubit Hamiltonian can be done, e.g.,

- using binary encoding, requiring $\eta = \lceil \log(\Lambda + 1) \rceil$ qubits per boson, where Λ is the cutoff on boson occupation per site:

$$\hat{N}_p |p\rangle = p\,|p\rangle \text{ where } |p\rangle = \otimes_{j=0}^{\eta-1} |p_j\rangle \text{ with } p = \sum_{j=0}^{\eta-1} 2^j p_j$$

- using unary encoding, requiring Λ qubits per boson.



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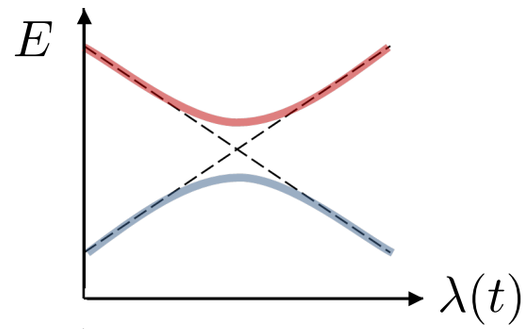
- **Adiabatic state preparation:** Prepare the ground state of a simple Hamiltonian, then adiabatically turn the Hamiltonian to that of the target Hamiltonian. Requires a non-closing energy gap.



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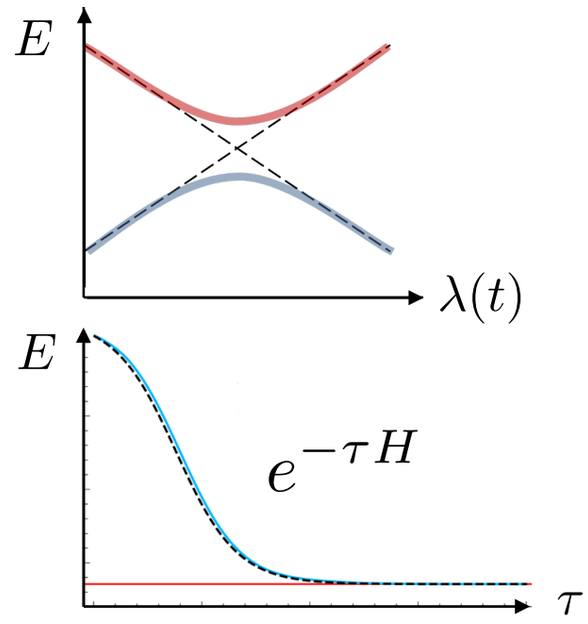

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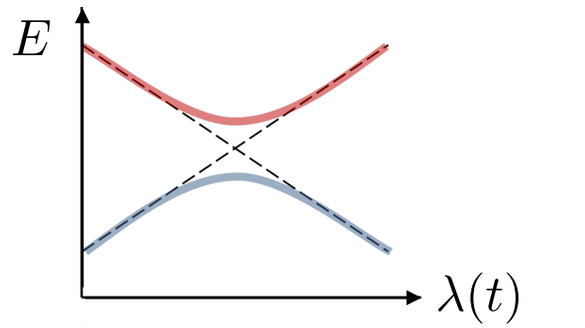
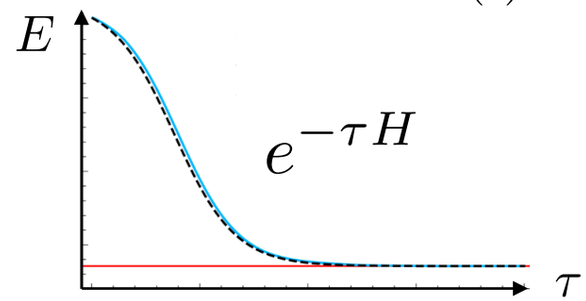
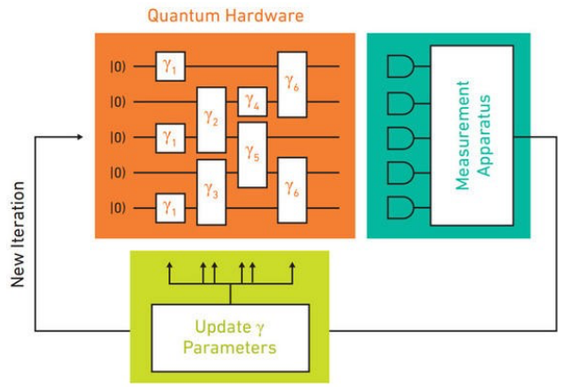
- **Variational quantum eigensolver (VQE):** Use the variational principle of quantum mechanic and classical processing to minimize the energy of a non-trivial ansatz wavefunction generated by a quantum circuit. The optimized circuit corresponding to the minimum energy generates an approximation to ground-state wavefunction. Can fail if stuck in local minima manifolds or manifolds with exponentially small gradients in qubit number.







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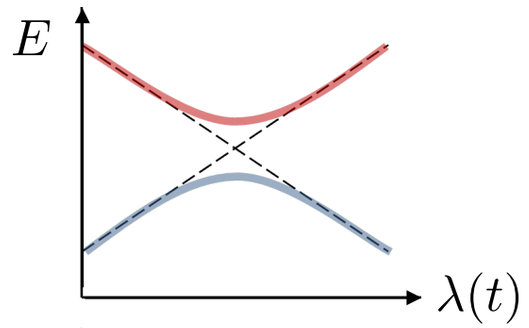

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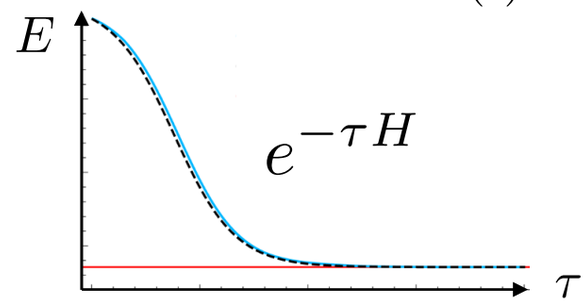

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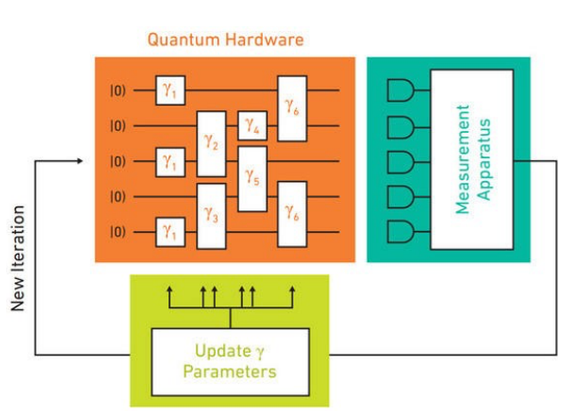

- **Classically computed states:** Use classical computing such as Monte Carlo, Tensor Networks, Neural Networks to learn the state or features of the state when possible, for a direct implementation of the state as a quantum circuit, or as close enough state to the ground state as a starting point of the above algorithms so as to achieve more efficient implementations.

Gupta, White, ZD, arXiv:2506.02313 [quant-ph].

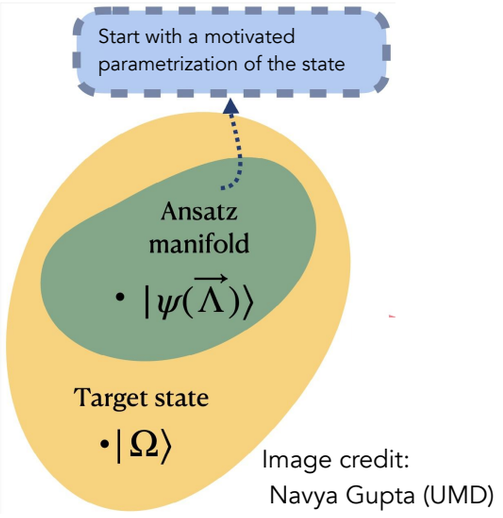

Image credit: Navya Gupta (UMD)

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(IMPROVED) THEORY OF PRODUCT FORMULAS

Consider the Hamiltonian $$H = \sum_{i=1}^{\Gamma} H_i$$

First-order product formula

$$V_1(t) = e^{-itH_1} e^{-itH_2} \cdots e^{-itH_\Gamma}$$

is bounded by:

$$\| V_1(t) - e^{-itH} \| \leq \frac{t^2}{2} \sum_{i=1}^{\Gamma} \left\| \left[\sum_{j=i+1}^{\Gamma} H_j, H_i \right] \right\|$$

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Second-order formula

$$V_2(t) = (e^{-itH_\Gamma/2} \cdots e^{-itH_2/2} e^{-itH_1/2})(e^{-itH_1/2} e^{-itH_2/2} \cdots e^{-itH_\Gamma/2})$$

is bounded by:

$$\| V_2(t) - e^{-itH} \| \leq \frac{t^3}{12} \sum_{i=1}^{\Gamma} \left\| \left[\sum_{k=i+1}^{\Gamma} H_k, \left[\sum_{j=i+1}^{\Gamma} H_j, H_i \right] \right] \right\| + \frac{t^3}{24} \sum_{i=1}^{\Gamma} \left\| \left[H_i, \left[H_i, \sum_{j=i+1}^{\Gamma} H_j \right] \right] \right\|$$

A general bound also exist, see: **Childs, Su, Tran, Wiebe, Zhu, Phys. Rev. X 11, 011020 (2021).**

RESOURCE ANALYSIS

$$\left\| V_p(T) - e^{-iTH} \right\| \leq \varepsilon$$

Given the accuracy ε on the time-evolution operator, how many **qubits and costly gates** are needed for simulating a Hamiltonian with given parameters for time T using the p^{th}-order product formula?

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i) Trotter error,
ii) function-evaluation approximation error,
iii) gate-synthesis error,
iv) measurement error, and
v) theoretical errors (finite-volume, discretization, truncation, etc.).

For FV effects, see, e.g.,: Burbano, Carrillo, Urek, Ciavarella, Briceño, arXiv:2506.06511 [hep-lat].

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The p^{th}-order product formula requires $O\left(\frac{T^{(p+1)/p}}{\epsilon^{1/p}}\right)$ Trotter steps. Near-optimal algorithms based on completely different digitization strategies achieve $O\left(T, \log\left(\frac{1}{\varepsilon}\right)\right)$.

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EXAMPLES OF ACCESSIBLE OBSERVABLES

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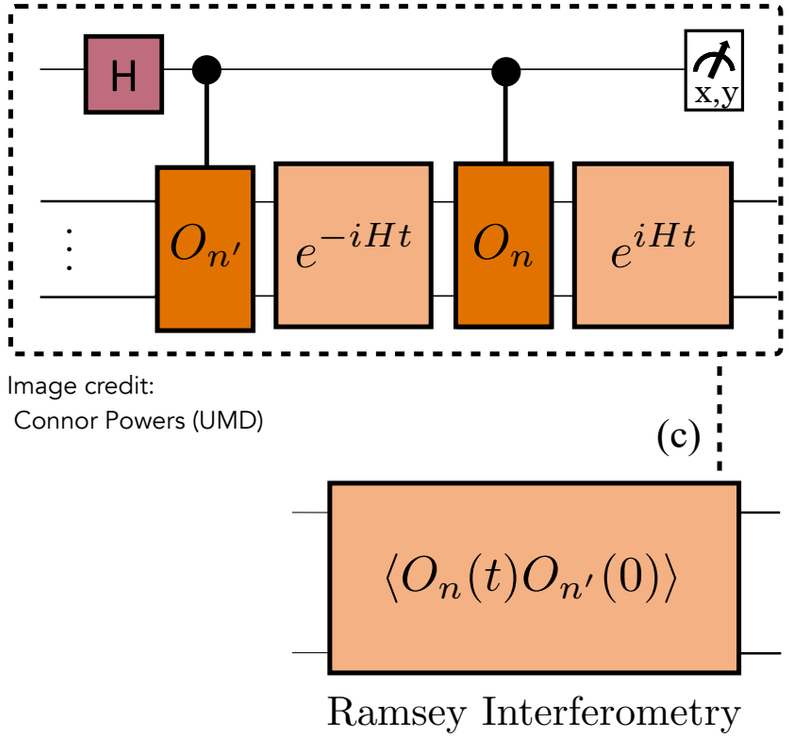

Image credit:
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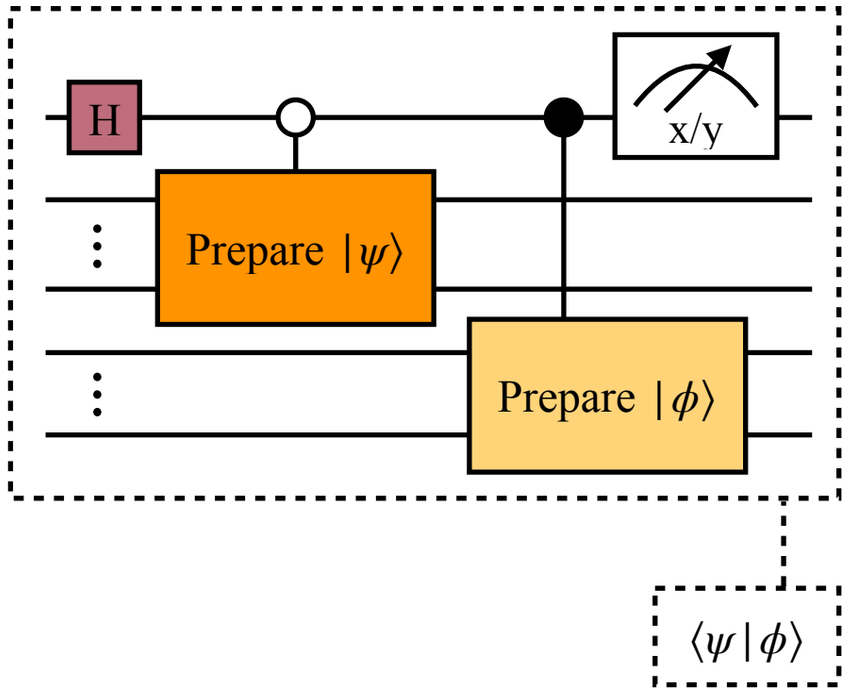
  - Inclusive processes: can be obtained from two-current correltor via optical theorem
  - Semi-inclusive processes: can be obtained using projectors



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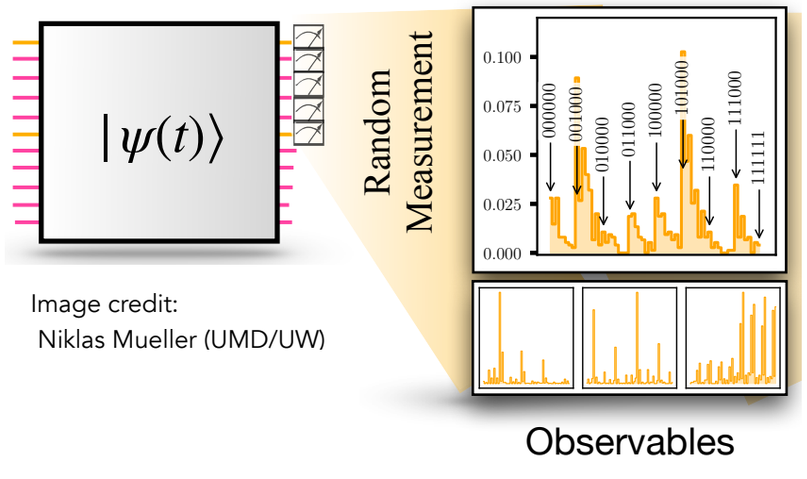

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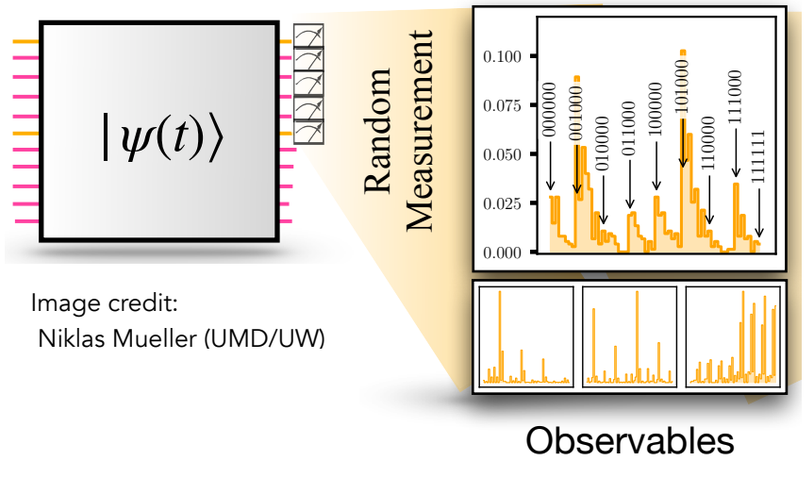

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Fidelities and full state tomography are hard (they demand exponentially large number of measurements).

PART I:
BACKGROUND AND MOTIVATION

PART II:
QUANTUM-COMPUTATION/SIMULATION BASICS

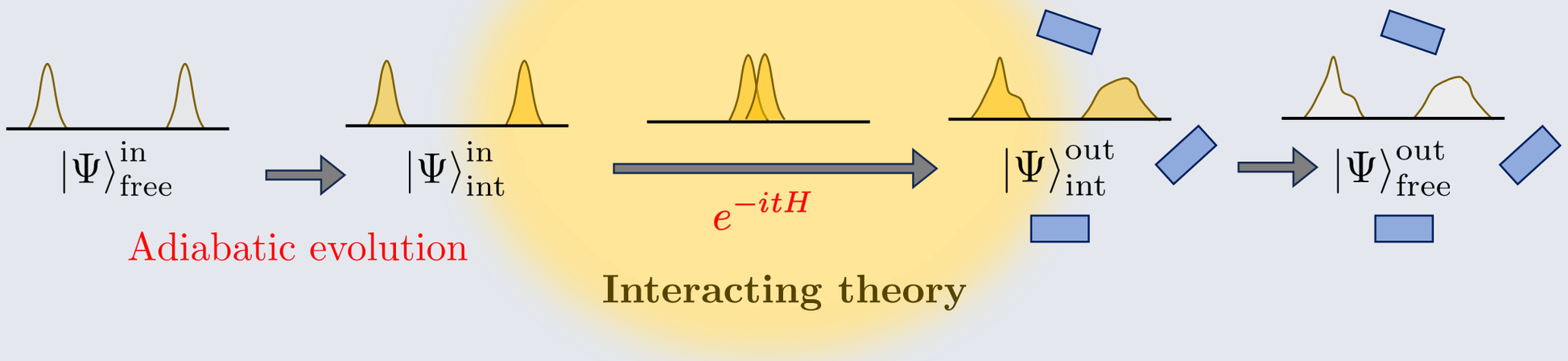
ZD, Hsieh, and Kadam, Quantum 8, 1520 (2024) and arXiv:2505.20408 [quant-ph].

PART III:
EXAMPLE: DIGITAL QUANTUM COMPUTING OF HADRON SCATTERING

PART IV:
EXAMPLE: ANALOG QUANTUM SIMULATION OF HADRON SCATTERING

QUANTUM SIMULATION OF SCATTERING CAN PROCEED WITH THE JORDAN-LEE-PRESKILL STRATEGY

1. State (wave-packet) preparation 2. Time evolution: scattering 3. Measurement of the final state



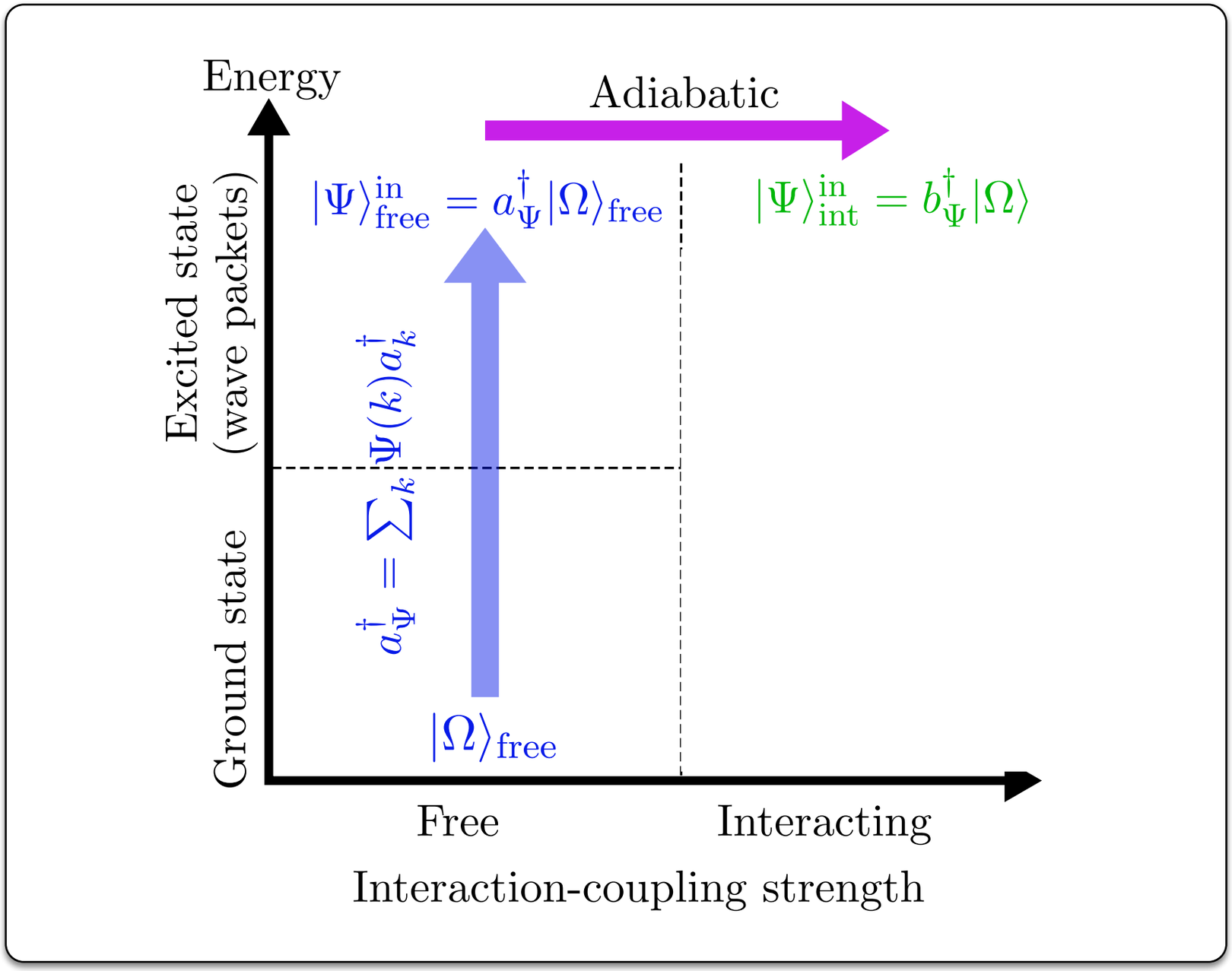
Jordan, Lee, Preskill, Science 336, 1130-1133 (2012).

Figure from: ZD, Hseih, and Kadam, Quantum 8, 1520 (2024).

Beside Jordan, Lee, Preskill, Science 336, 1130-1133 (2012), check out other hadronic wave-packet creation in: Turco et al, arXiv:2305.07692 [quant-ph], Kreshchuk et al, arXiv:2310.13742 [quant-ph], Farrell et al, arXiv:2401.08044 [quant-ph].

See also progress in scattering in: Schuhmacher et al, arXiv: 2505.20387 [quant-ph], Chai et al, arXiv:2505.21240 [quant-ph], Chai et al, Quantum 9, 1638 (2025), Farrell et al, arXiv:2505.03111 [quant-ph], Zemlevskiy, arXiv:2411.02486 [quant-ph],

OUR STRATEGY COMPARED WITH JORDAN-LEE-PRESKILL



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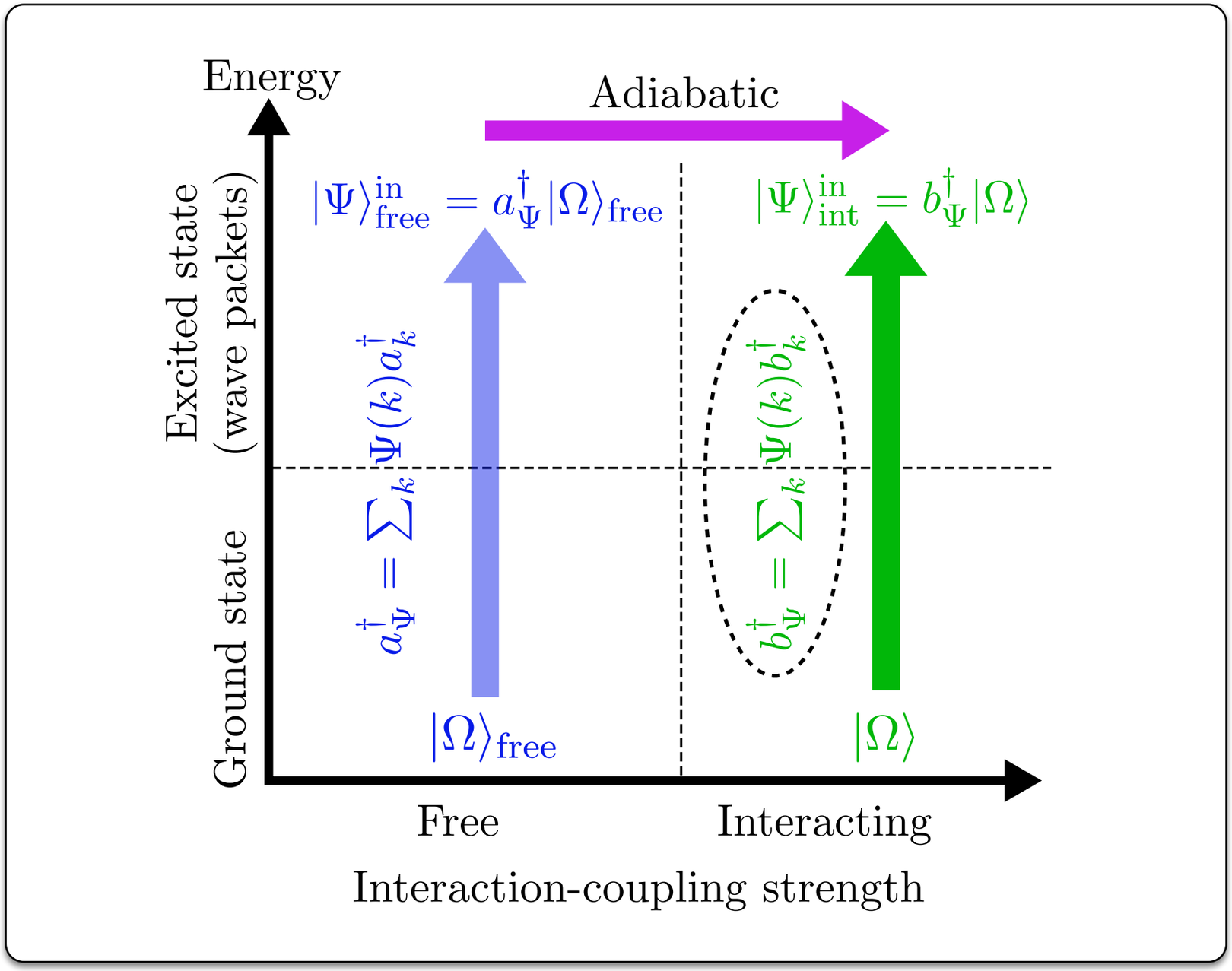

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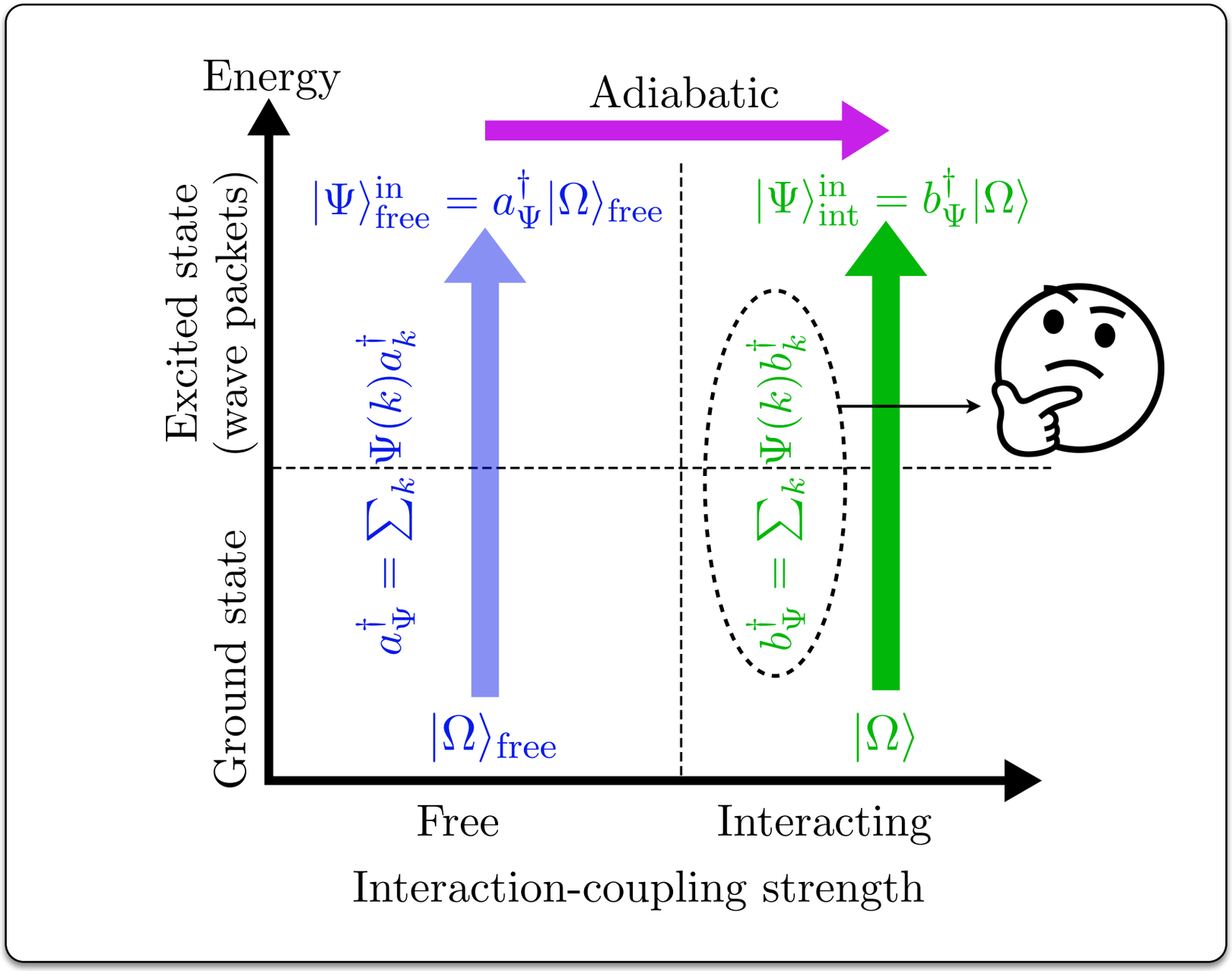
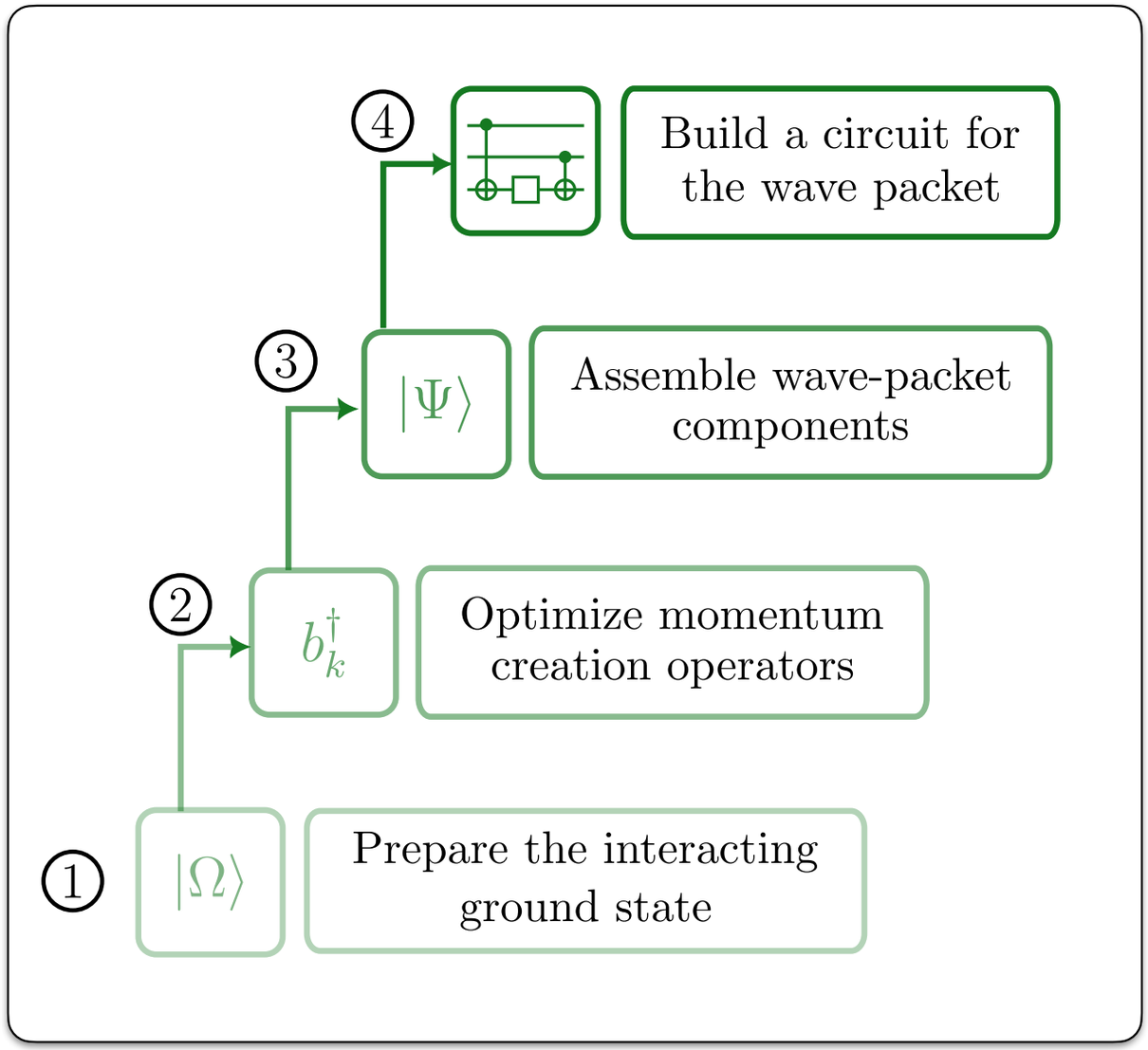


ZD, Hsieh, and Kadam, Quantum 8, 1520 (2024) and arXiv:2505.20408 [quant-ph].

Beside Jordan, Lee, Preskill, Science 336, 1130-1133 (2012), check out other recent interesting digital algorithms for hadronic wave-packet creation in: Turco, Quinta, Seixas, and Omar, arXiv:2305.07692 [quant-ph], Kreshchuk, Vary, Love, arXiv:2310.13742 [quant-ph], Chai, Crippa, Jansen, Kühn, Pascuzzi, Tacchino, and Tavernelli, arXiv:2312.02272 [quant-ph], Farrell, Illa, Ciavarella, and Savage, arXiv:2401.08044 [quant-ph].

THE STEPS OF OUR WAVE-PACKET PREPARATION ALGORITHM



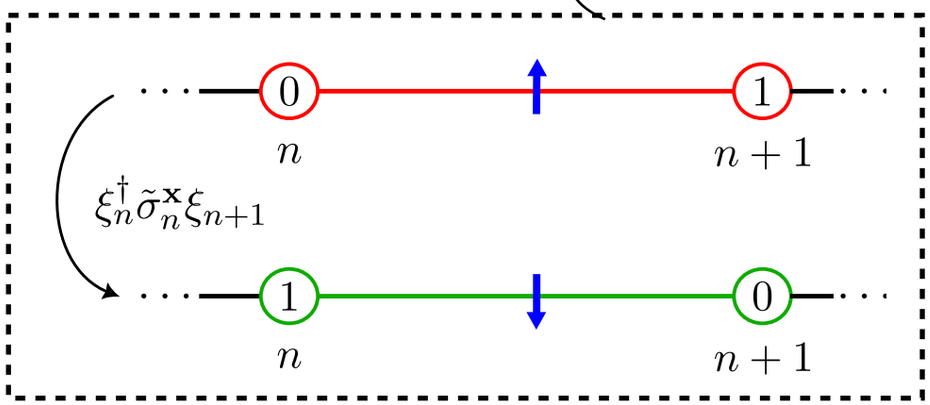
ZD, Hsieh, and Kadam, Quantum 8, 1520 (2024) and arXiv:2505.20408 [quant-ph].

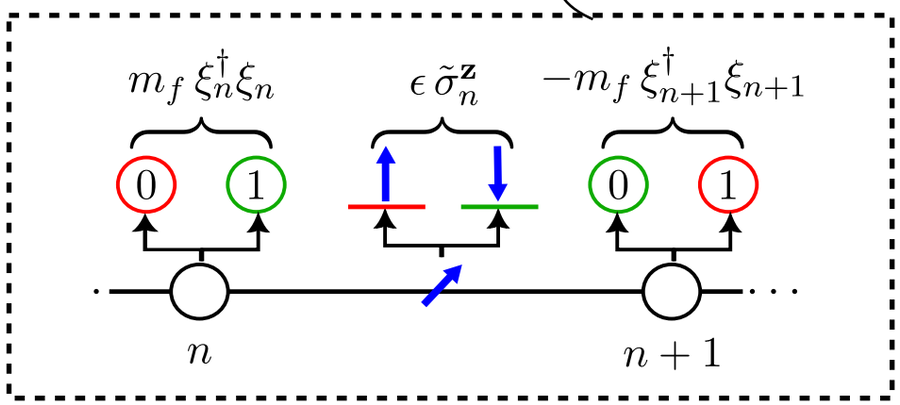
OUR TESTING GROUND:
Z_2 LATTICE GAUGE THEORY COUPLED TO FERMIONS IN 1+1 D

Hamiltonian: $$aH = \frac{1}{2}\sum_{n\in\Gamma}\left(\xi_n^\dagger \tilde{\sigma}_n^{\mathbf{x}} \xi_{n+a} + \text{H.c.}\right) + am_f \sum_{n\in\Gamma}(-1)^{n/a}\xi_n^\dagger\xi_n + a\epsilon\sum_{n\in\Gamma}\tilde{\sigma}_n^{\mathbf{z}}$$

Fermions

Gauge bosons





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Fermions

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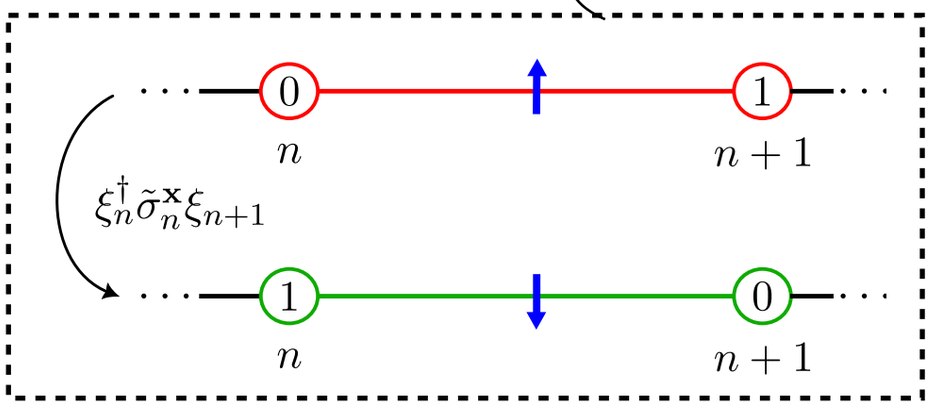
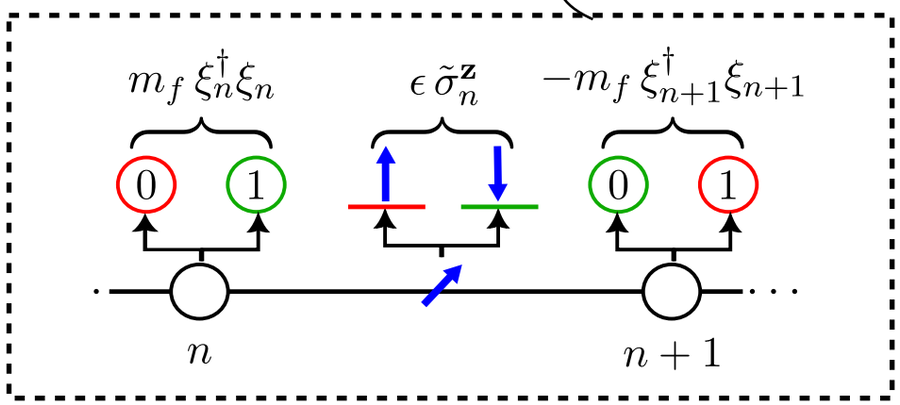
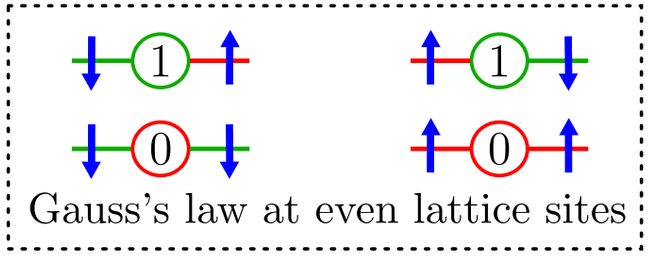
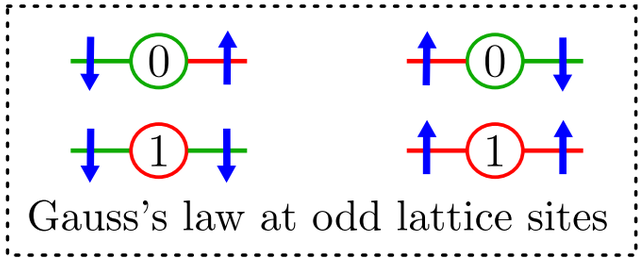




Gauss's law: $G_n\,|\psi_{\text{phys}}\rangle = g\,|\psi_{\text{phys}}\rangle \;\; \forall n$ with $G_n = \tilde{\sigma}^{\mathbf{z}}\tilde{\sigma}_{n-1}^{\mathbf{z}} e^{i\pi\left[\xi_n^\dagger\xi_n - \frac{1-(-1)^n}{2}\right]}$





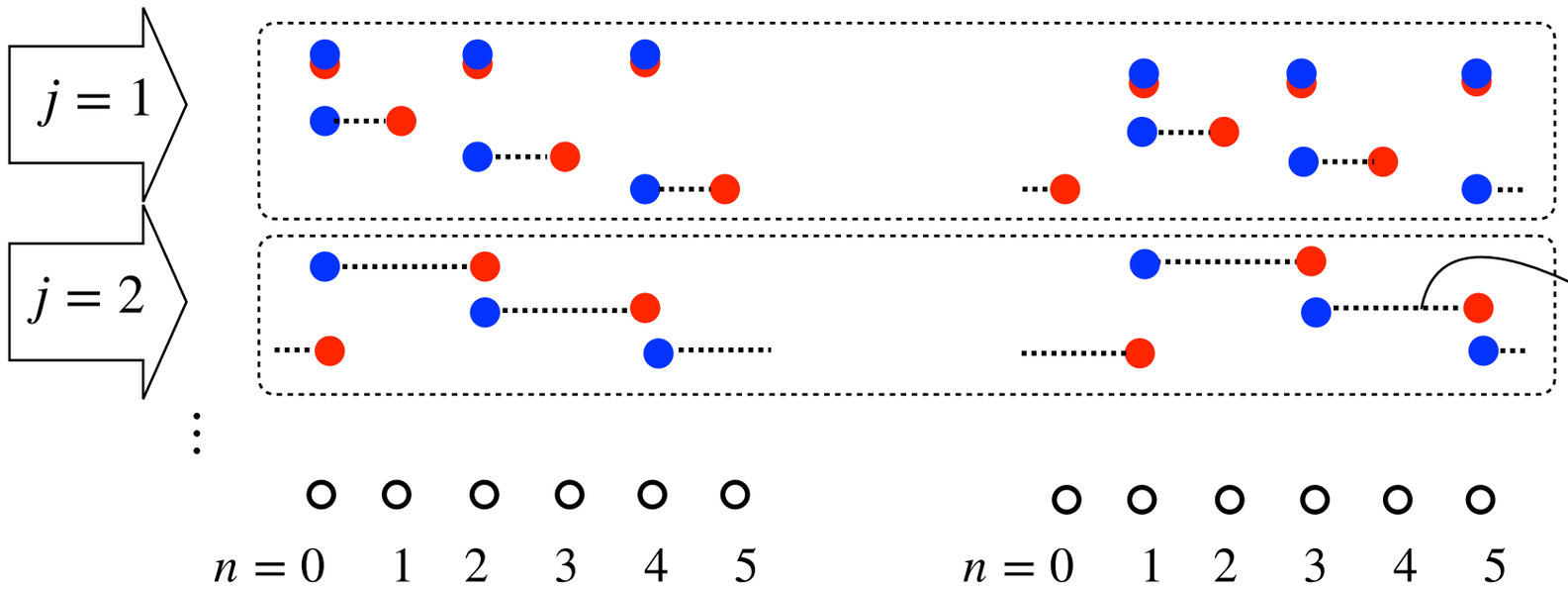
AN ANSATZ FOR THE MESON WAVE PACKET

$$\Psi(k) = \mathcal{N}_\Psi \exp\left(-ik\mu\right) \exp\left(-\frac{(k-k_0)^2}{4\sigma^2}\right)$$

$$b^\dagger_\Psi = \sum_k \Psi(k) b^\dagger_k$$

$$\Psi(k=-\tfrac{\pi}{3})\, b^\dagger_{k=-\frac{\pi}{3}} \qquad \Psi(k=0)\, b^\dagger_{k=0} \qquad \Psi(k=\tfrac{\pi}{3})\, b^\dagger_{k=\frac{\pi}{3}}$$

$\Gamma_0^{(j)}$ $\Gamma_1^{(j)}$



Bare meson operators $\mathcal{M}_{m,n}$

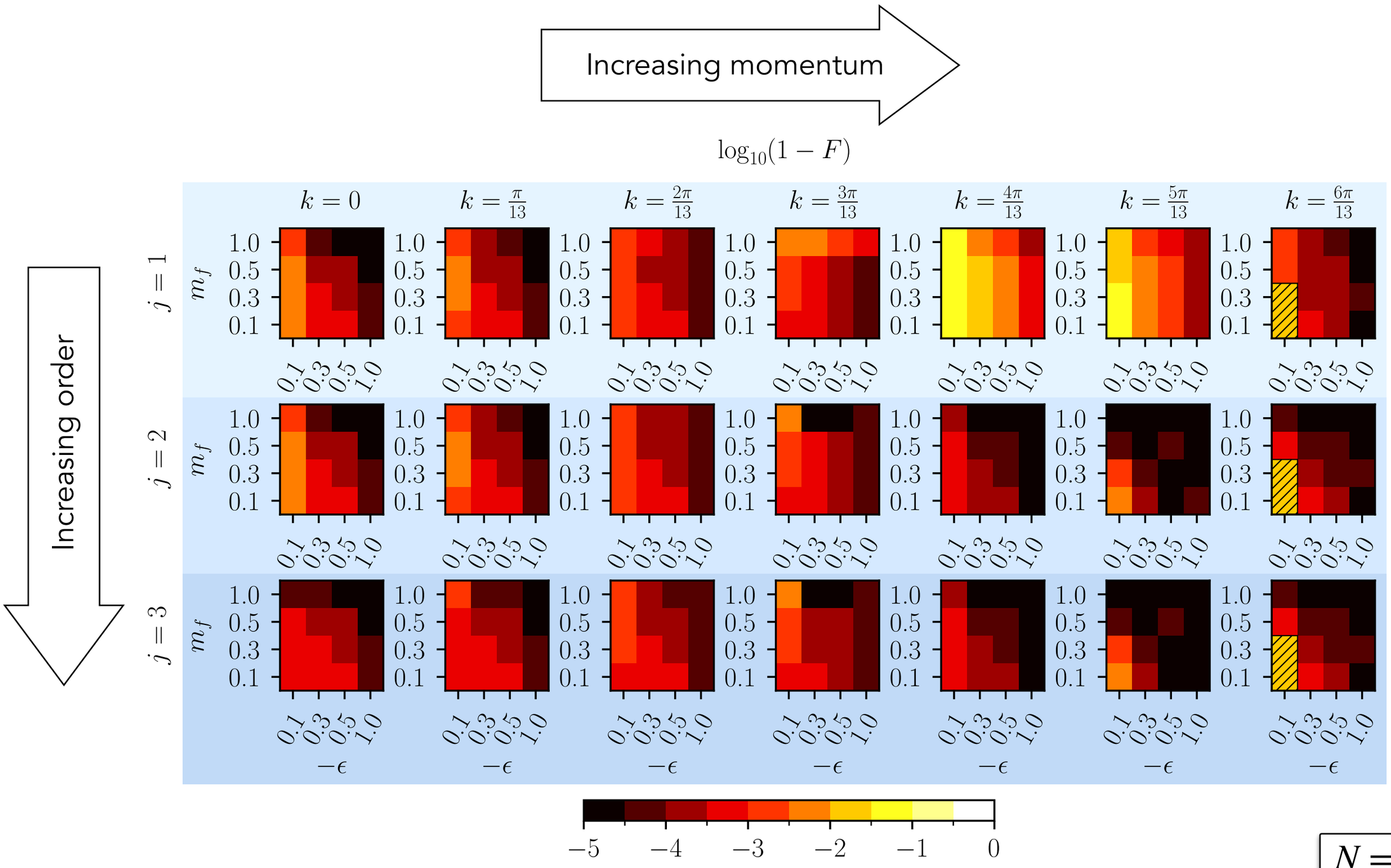
Single-particle k-momentum meson creation operator:

$$b_k^{(j)\dagger} = \sum_{j'=0}^{j} \sum_{m,n\in\Gamma^{(j')}} C_{m,n}^{(j'),k} \mathcal{M}_{m,n}$$

Coefficients such that short bare mesons are favored:

$$C_{m,n}^{(j),k} = \frac{1}{\mathcal{N}} e^{-\alpha_i^{(j),k}|m-n|^2} \overline{C}^k_{m,n} \quad \text{for} \quad m,n \in \Gamma_i^{(j)}$$

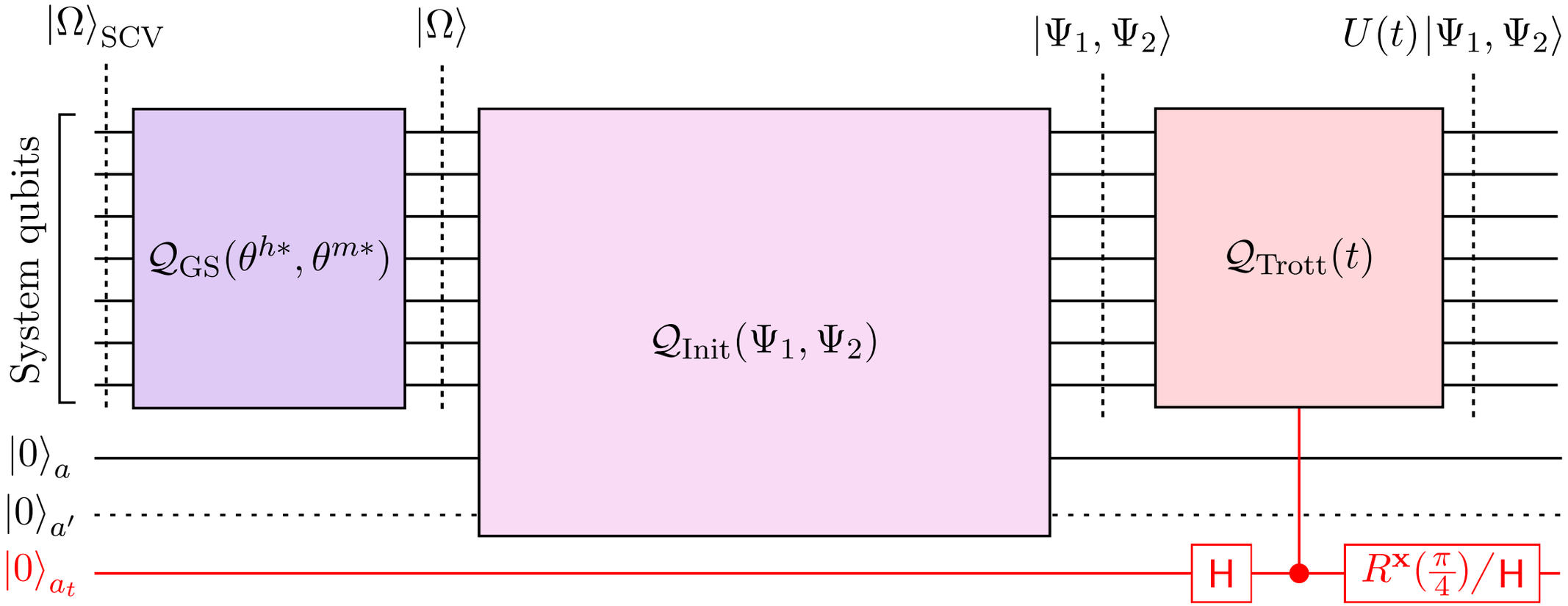
HOW WELL DOES THE ANSATZ WORK?



Fidelities are measured by comparing to MPS results.

ZD, Hsieh, and Kadam, arXiv:2402.00840 [quant-ph].

OUR QUANTUM CIRCUIT FOR HADRON SCATTERING

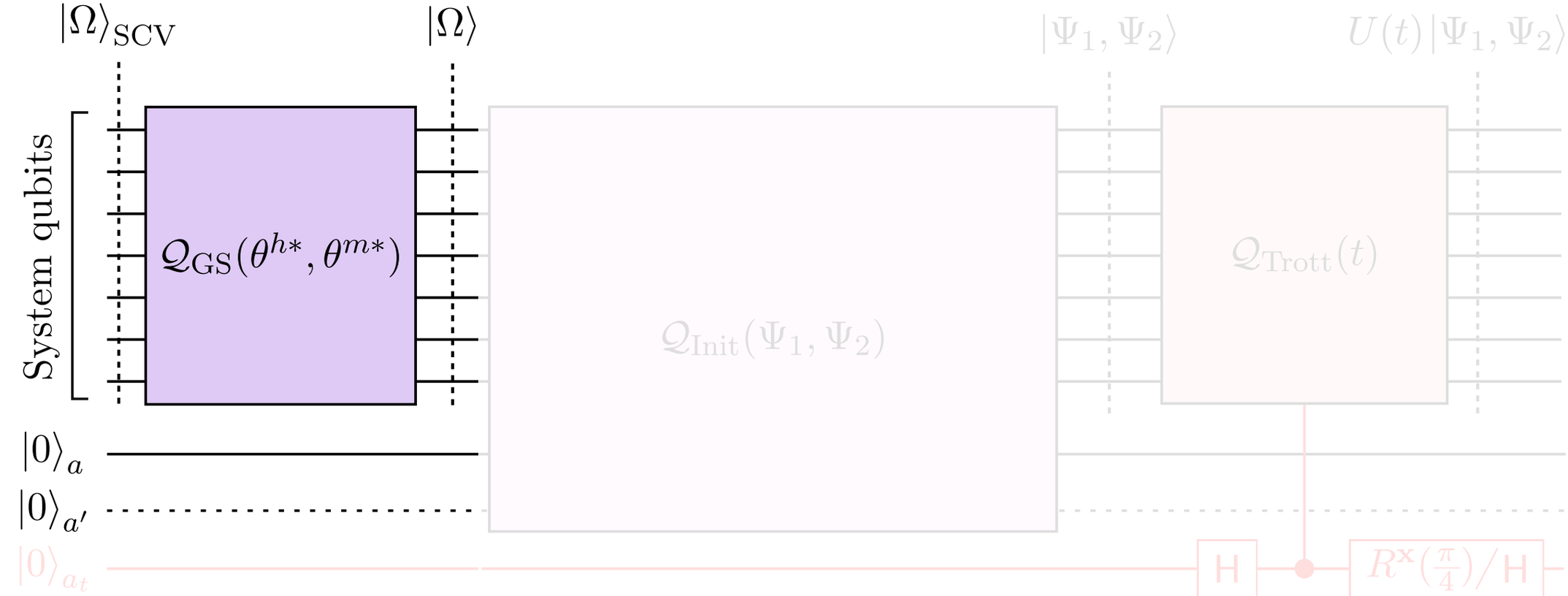


OUR QUANTUM CIRCUIT FOR HADRON SCATTERING

Creates the interacting vacuum

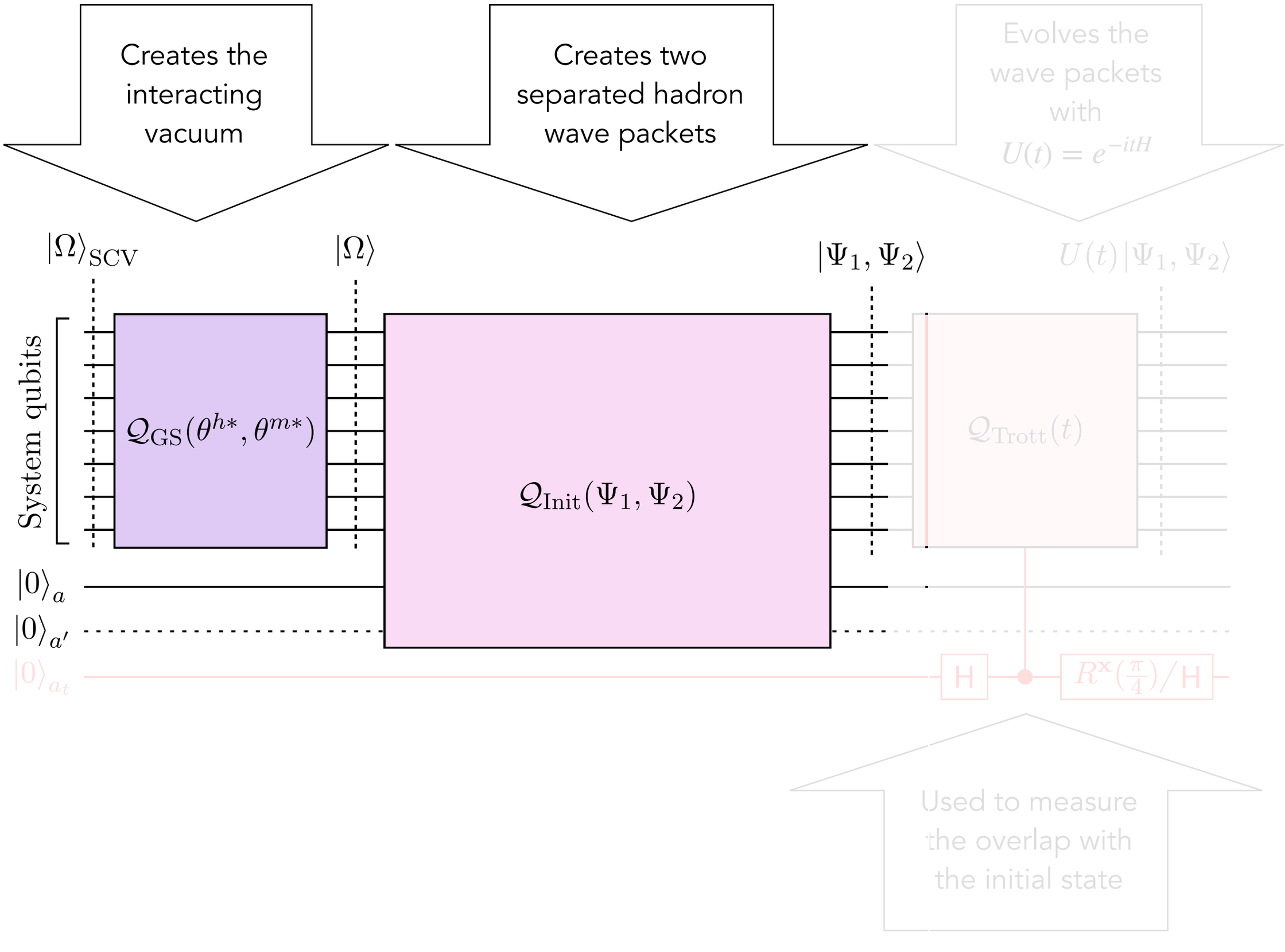
Creates two separated hadron wave packets

Evolves the wave packets with $U(t) = e^{-itH}$



Used to measure the overlap with the initial state

OUR QUANTUM CIRCUIT FOR HADRON SCATTERING



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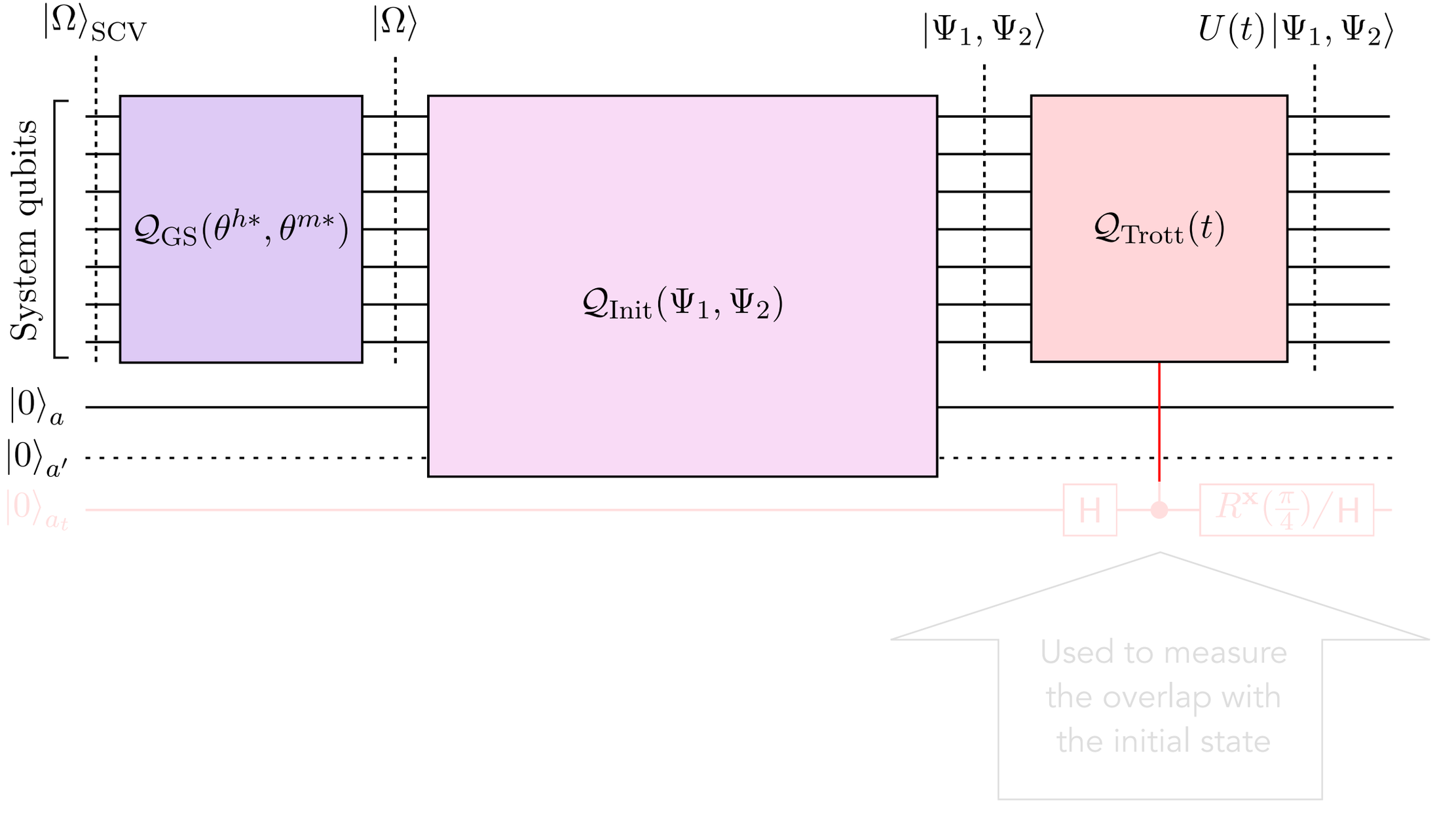

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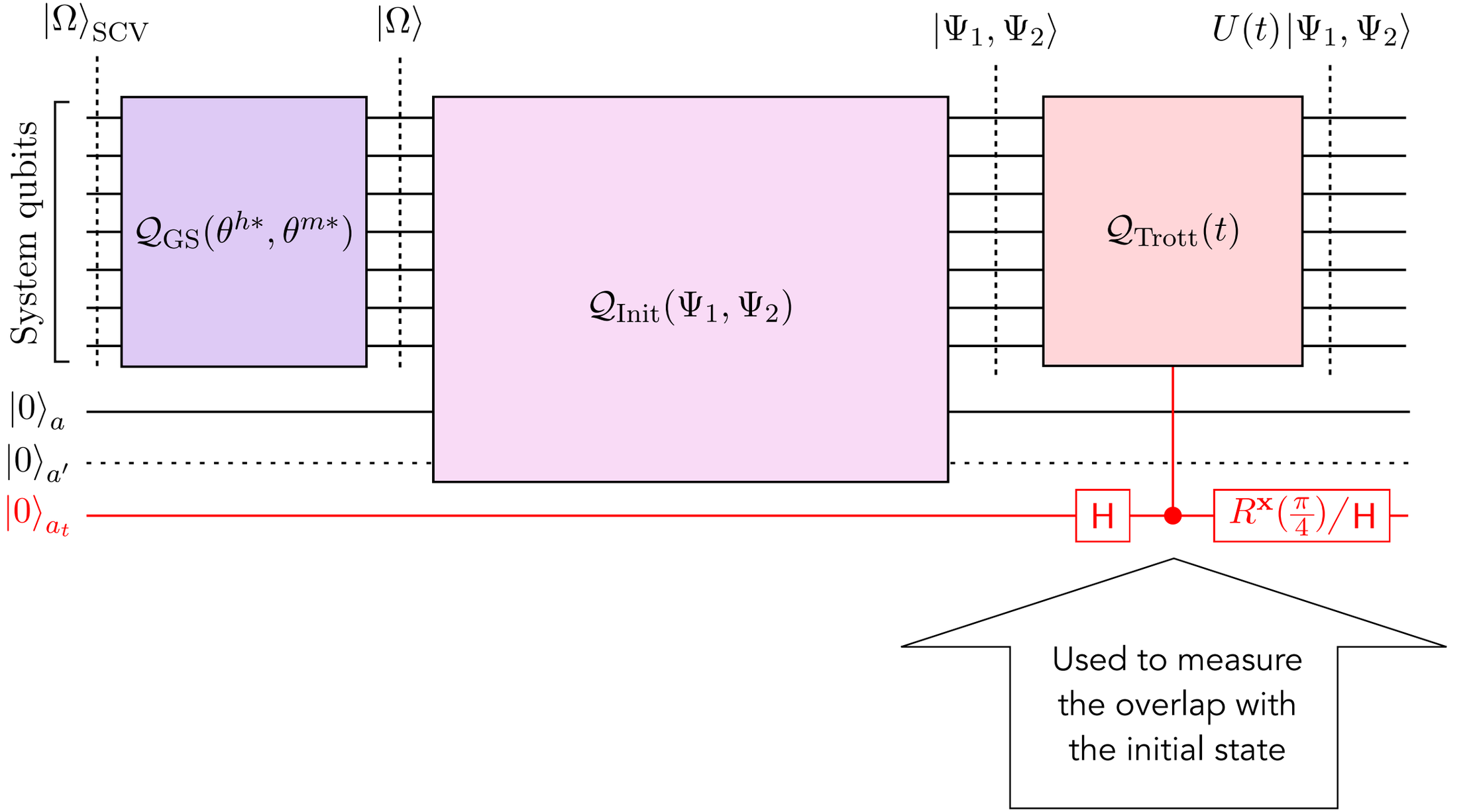
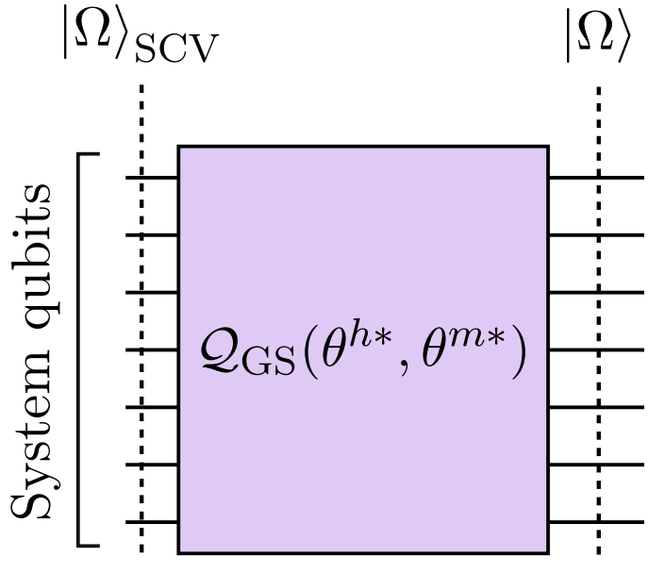


Used to measure the overlap with the initial state

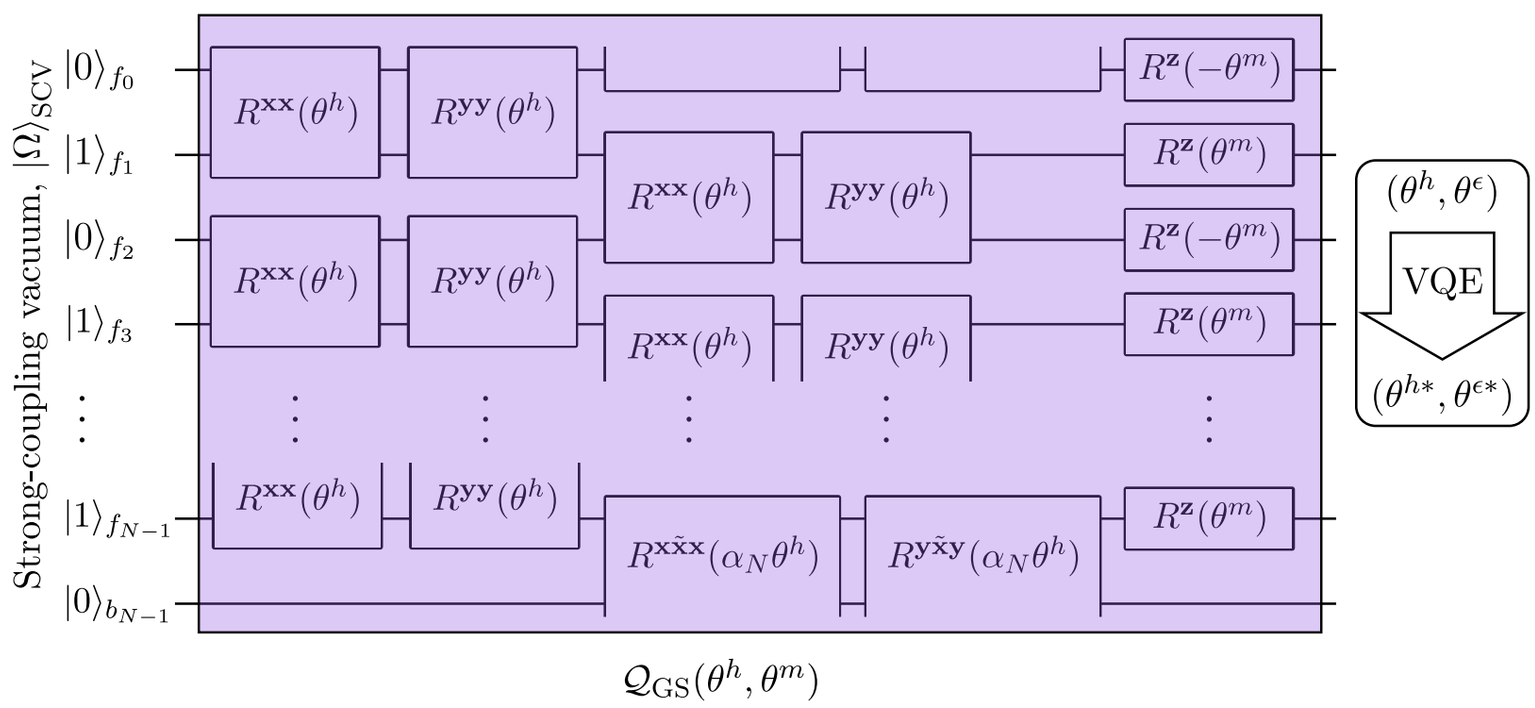
FIRST ELEMENT: INTERACTING VACUUM PREPARATION

Creates the interacting vacuum



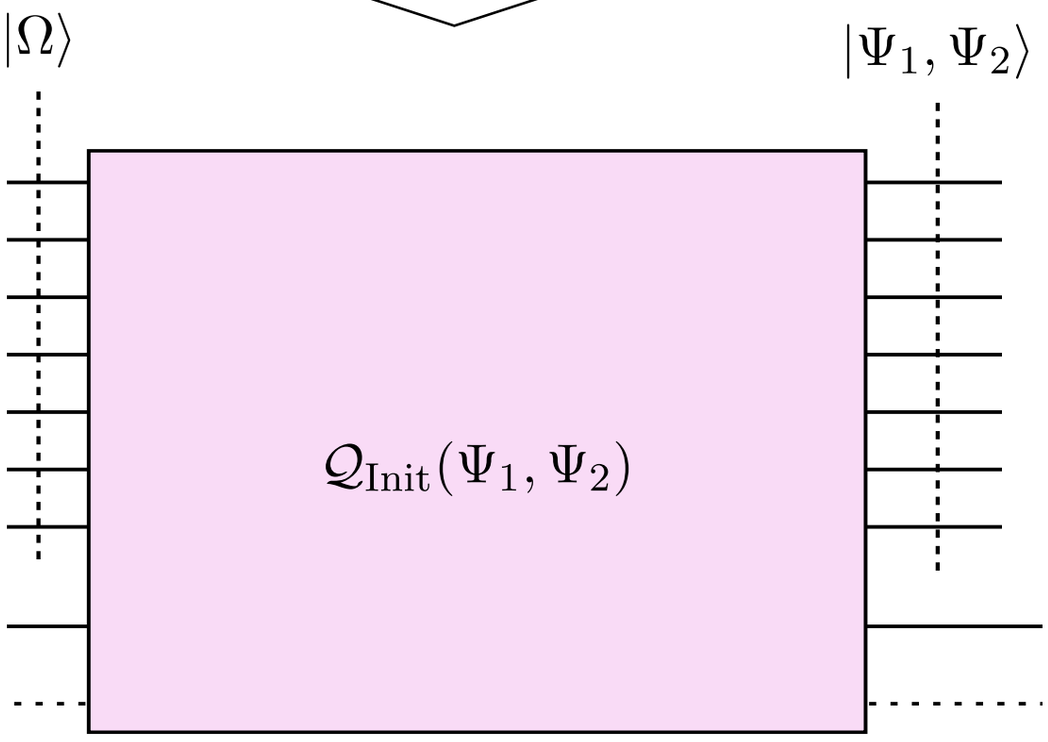
A variational quantum circuit:

Using similar ideas as in Lumia et al, PRX Quantum 3, 020320 (2022).



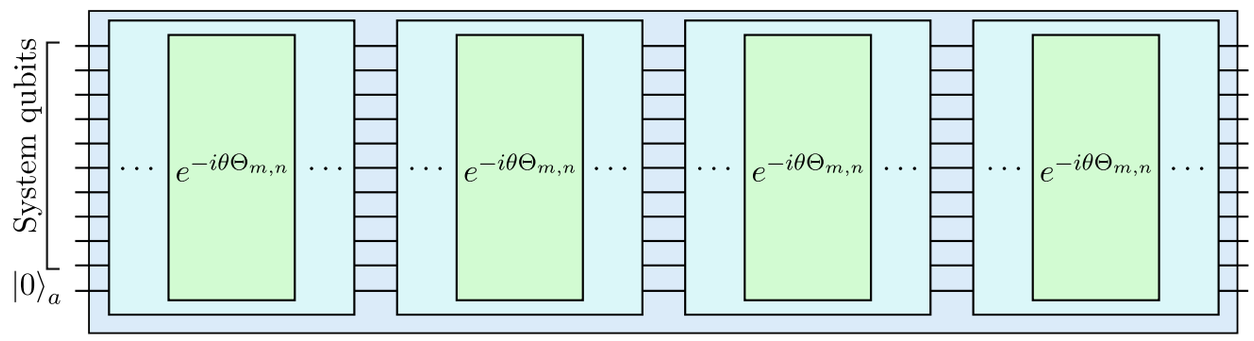
SECOND ELEMENT: WAVE-PACKETS PREPARATION

Creates two separated hadron wave packets

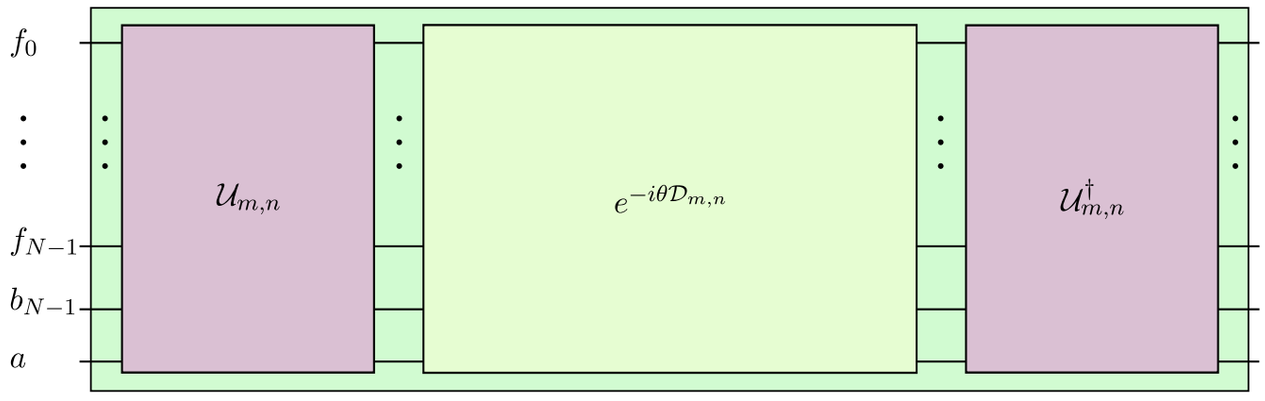


$\equiv$

Needs two similar modules, for each wave packet, of the form:



(a) $\mathcal{Q}_{\rm WP}(\Psi)$

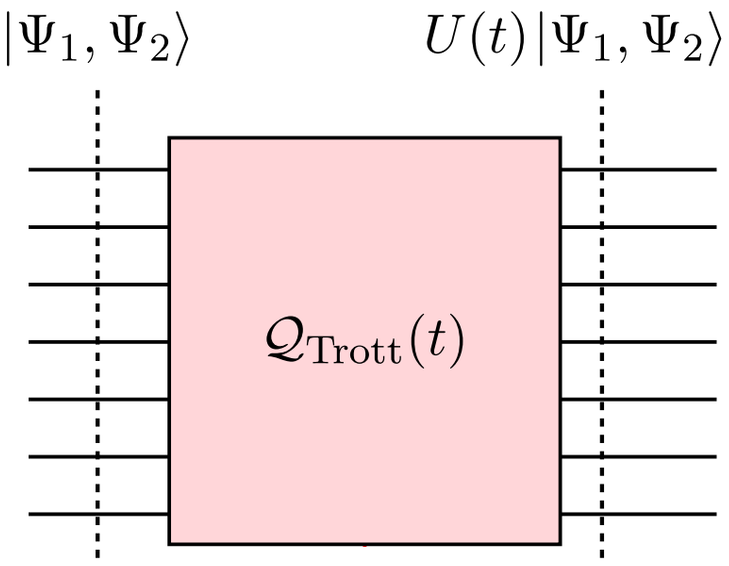


$[4(j^2+9j+1)N_P + 2j^2 + 2j] \times 2\tilde{n}_t$ CNOT gates

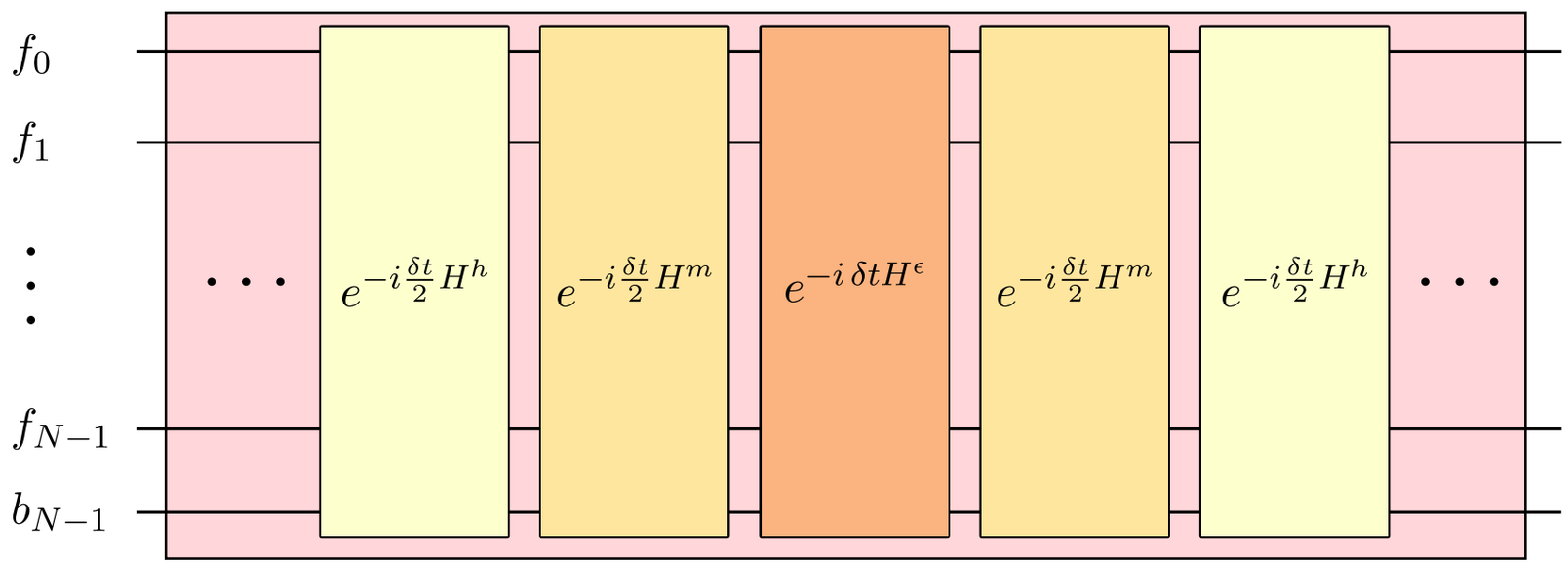
Using ideas in Jordan, Lee, Preskill, Science 336, 1130-1133 (2012) and ZD, Shaw, and Stryker, Quantum 7, 1213 (2023).

THIRD ELEMENT: WAVE-PACKETS EVOLUTION

Evolves the wave packets with $U(t) = e^{-itH}$

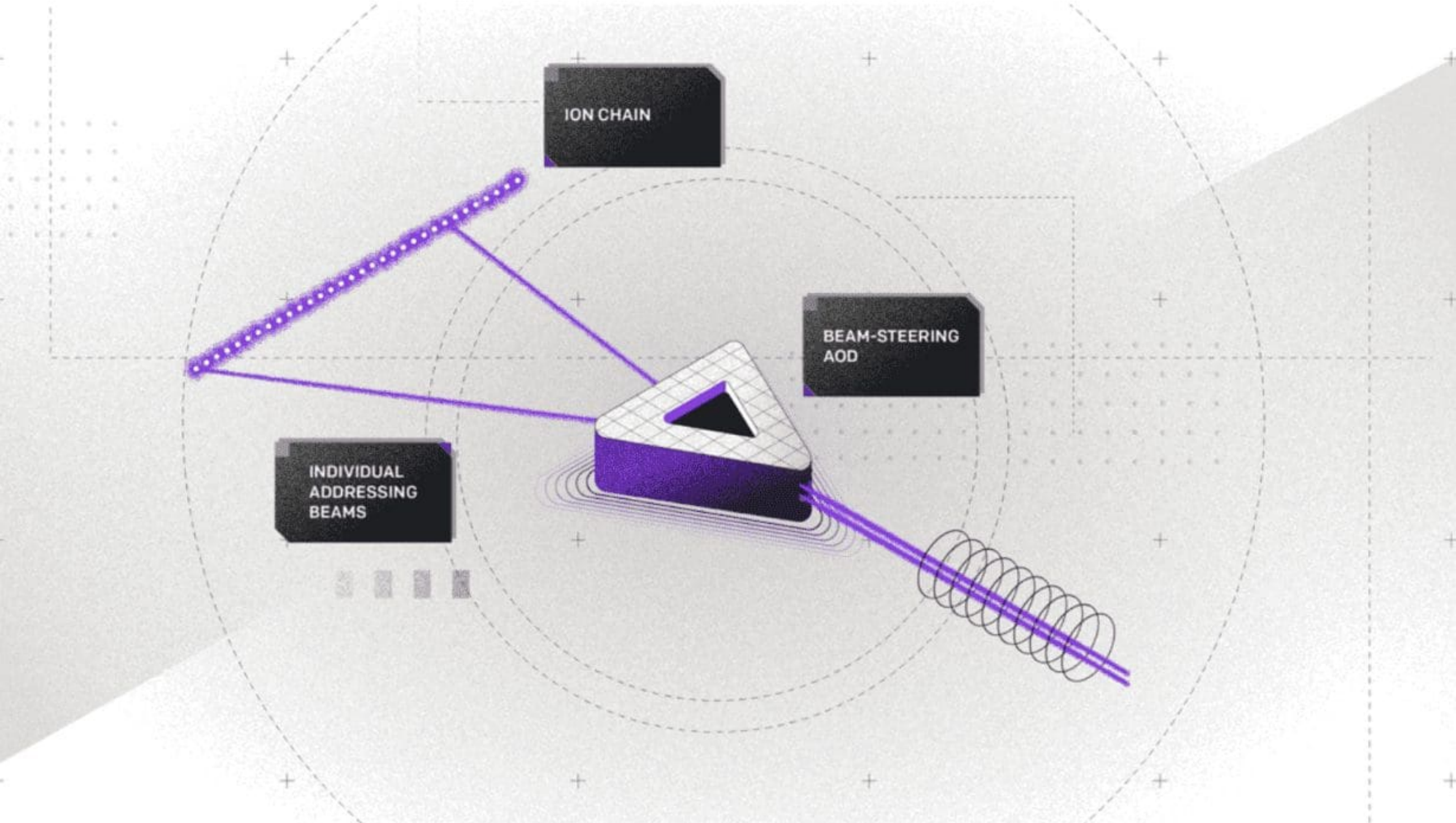


$\equiv$



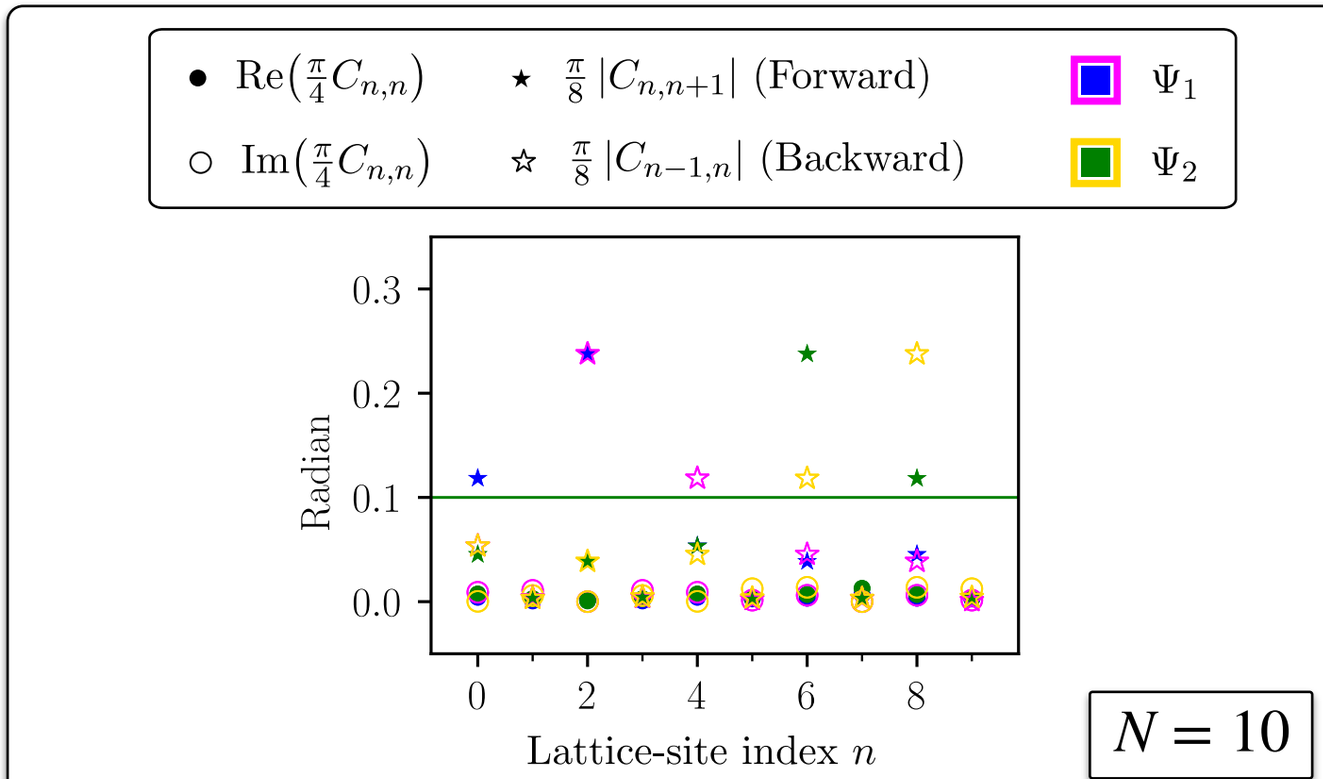
Each layer of Trotter evolution: $18N_P + 8$ CNOT gates

SOME EMULATOR AND HARDWARE RESULTS



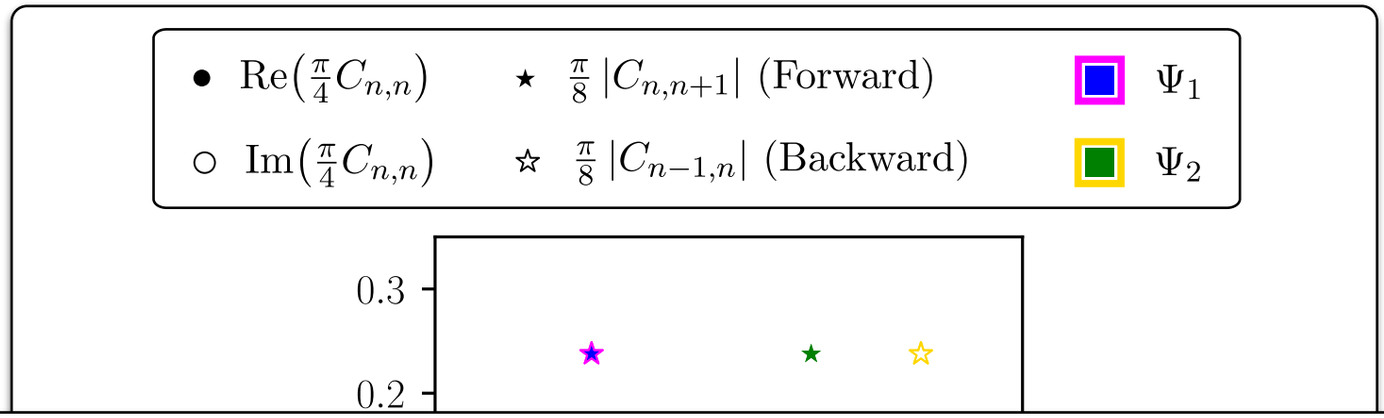
IonQ Forte quantum processor with 32 qubits

HOW WELL CAN WE PREPARE TWO WAVE PACKETS?

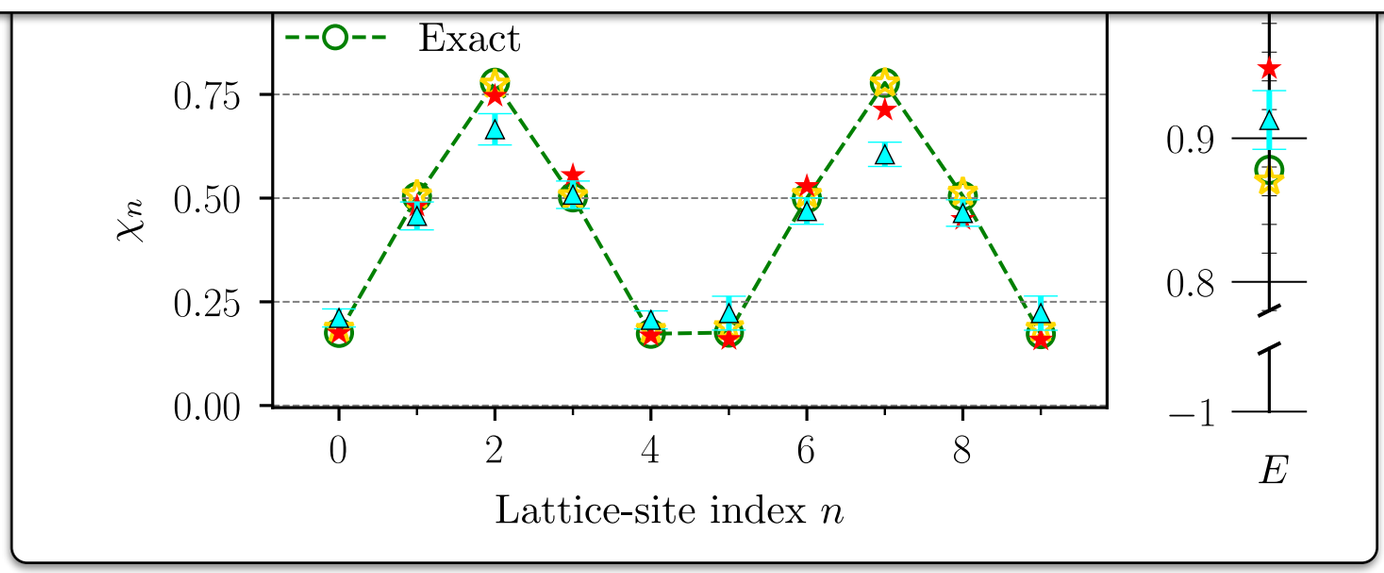


$$\chi_n = \begin{cases} \langle \psi_n^\dagger \psi_n \rangle & \text{if} \quad n \in \text{even}, \\ 1 - \langle \psi_n^\dagger \psi_n \rangle & \text{if} \quad n \in \text{odd} \end{cases}$$

HOW WELL CAN WE PREPARE TWO WAVE PACKETS?



N_P	Appx	Qubits	Single-qubit gates (raw/transpiled)	CNOT gates (raw/transpiled)
5	I	13	12757/13343	11324/7315
	II	12	310/369	236/167



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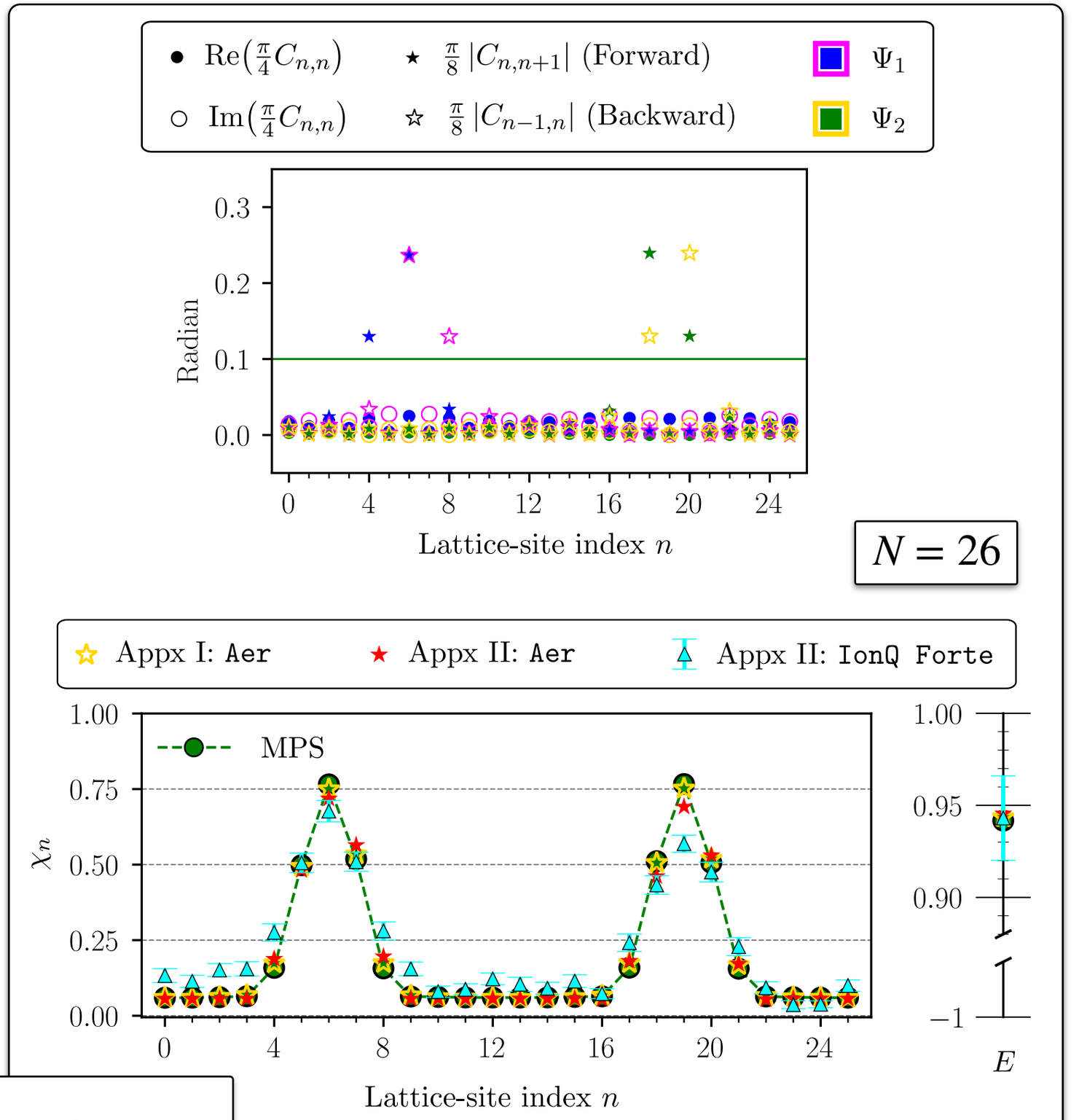

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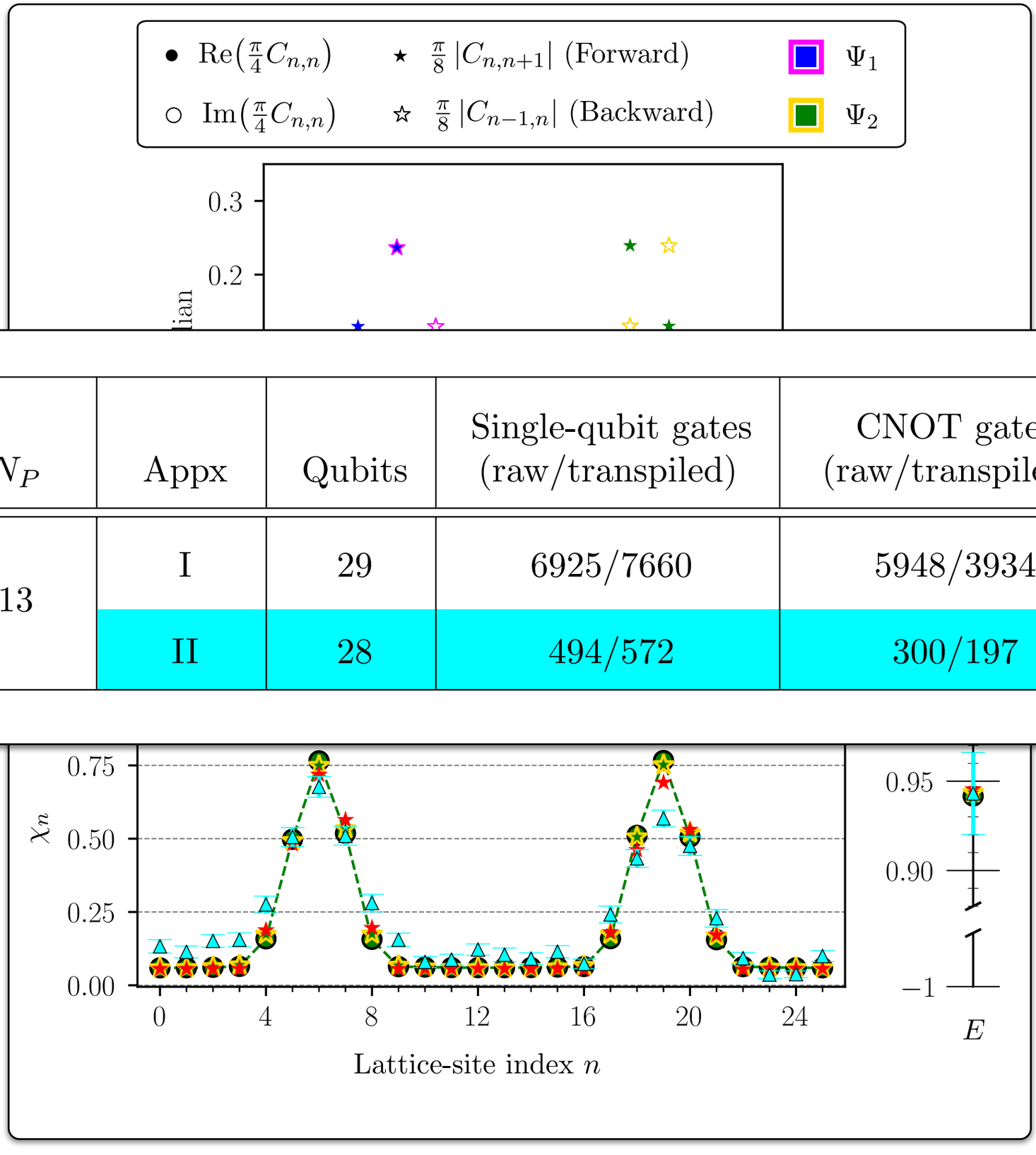
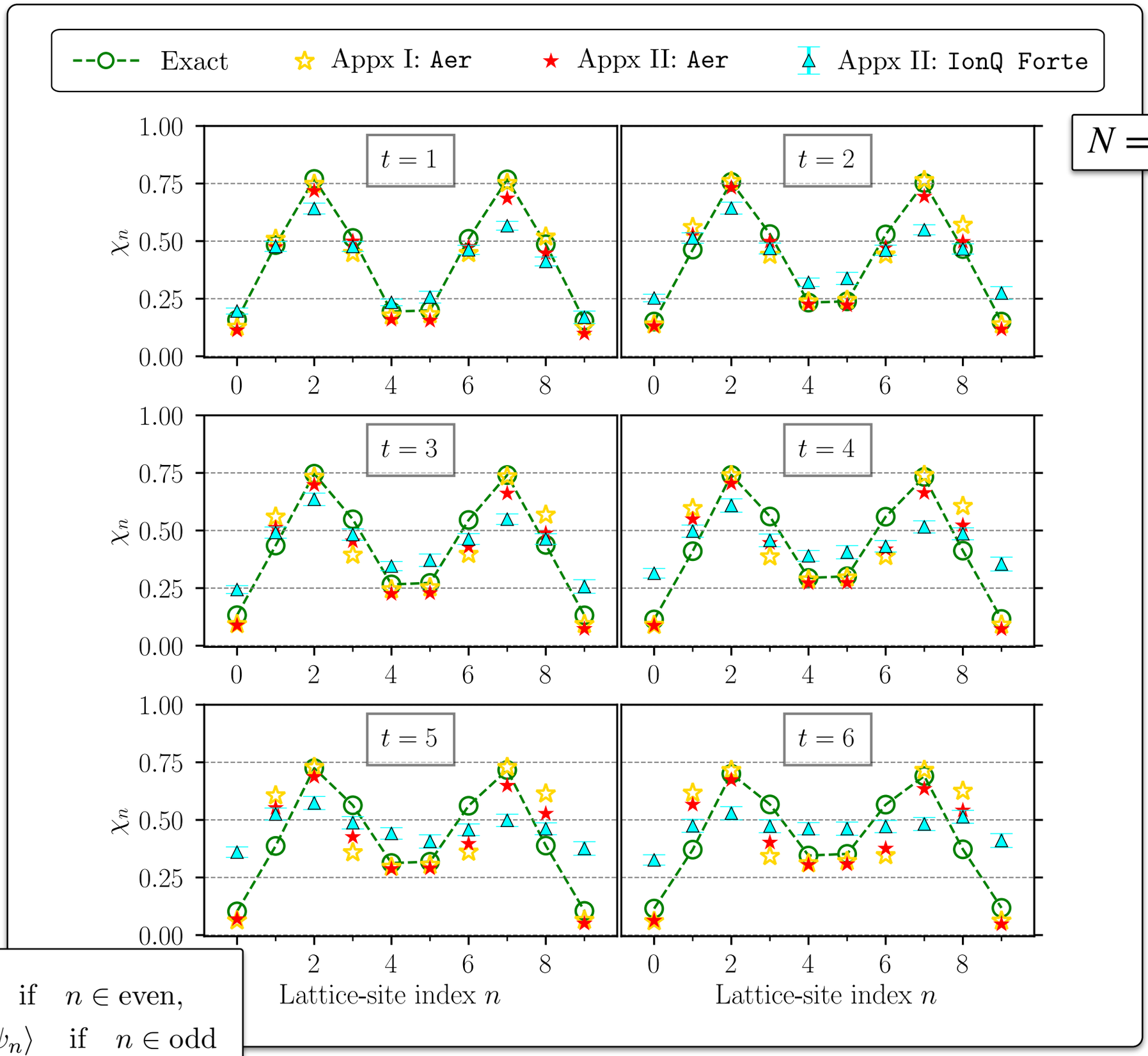
$$E = \langle \tilde{\sigma}^{\mathbf{z}} \rangle$$

HOW WELL CAN WE PREPARE TWO WAVE PACKETS?



N_P	Appx	Qubits	Single-qubit gates (raw/transpiled)	CNOT gates (raw/transpiled)
13	I	29	6925/7660	5948/3934
	II	28	494/572	300/197

HOW FAR CAN WE EVOLVE THE TWO WAVE PACKETS?



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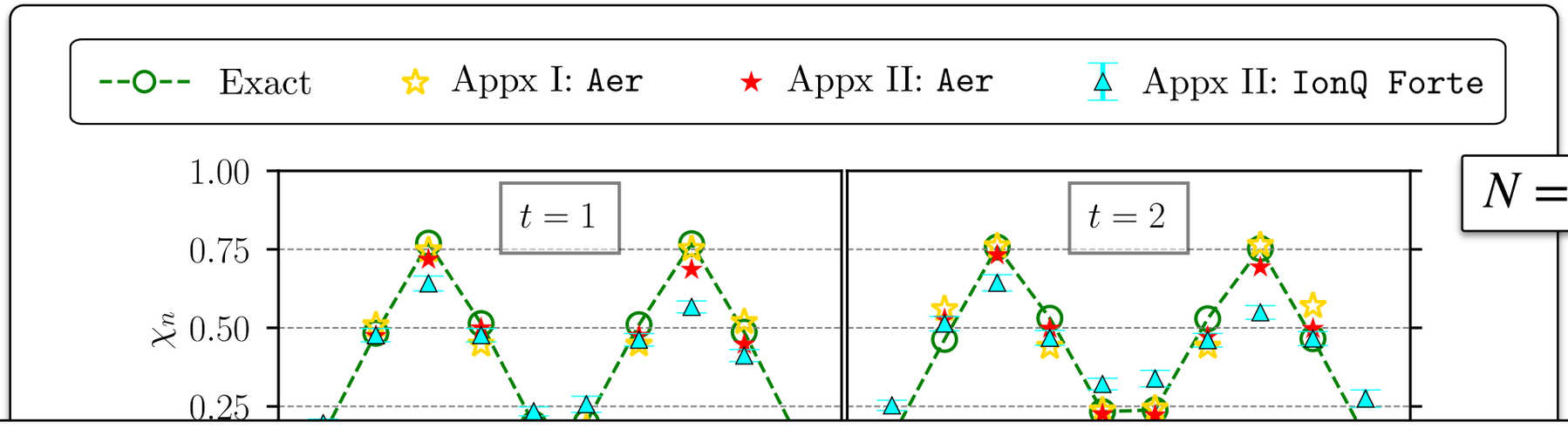
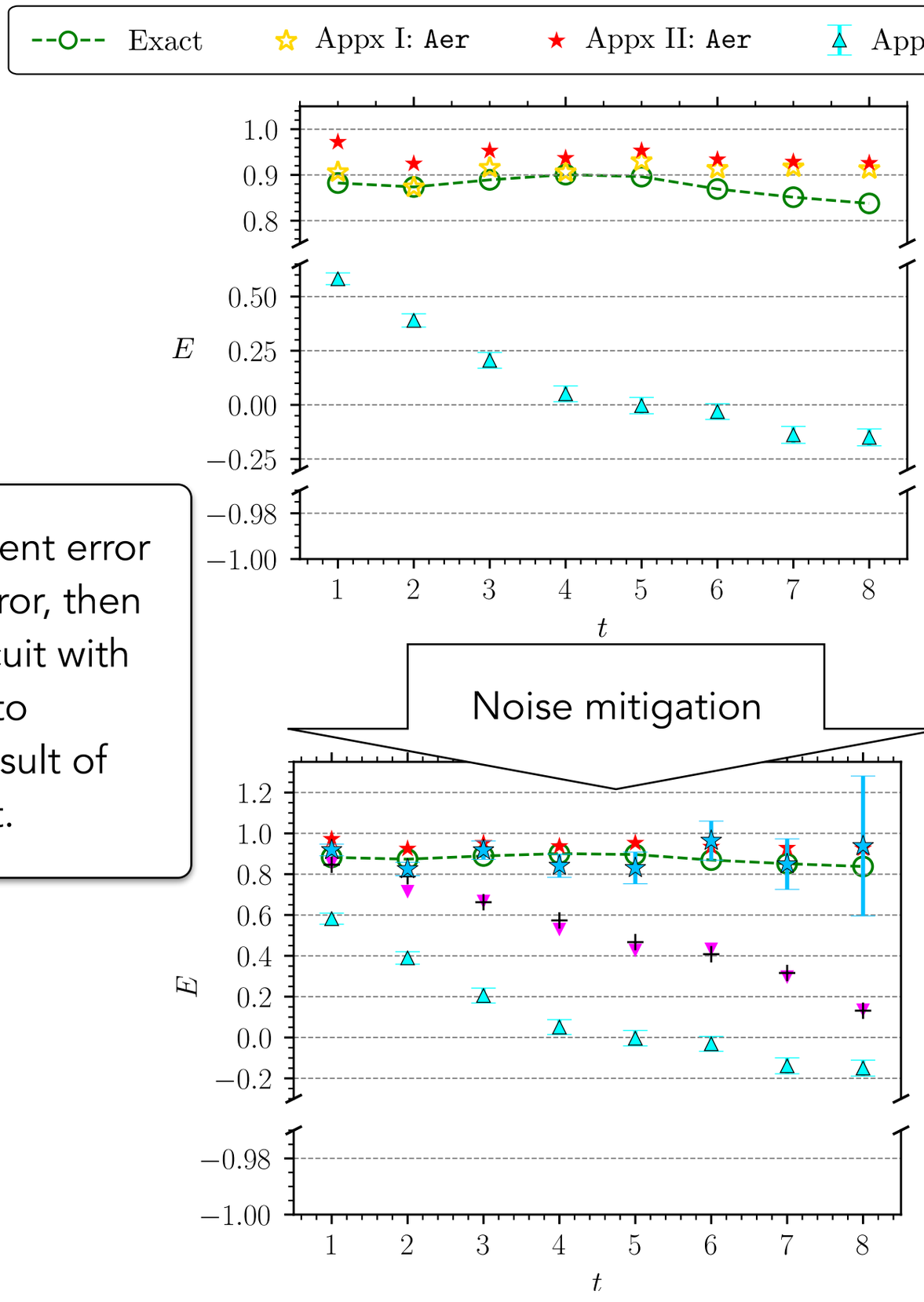


t	1	2	3	4	5	6	7	8
Single-qubit gates	573	771	969	1167	1365	1563	1761	1959
CNOT gates	227	287	347	407	467	527	587	647

HOW FAR CAN WE EVOLVE THE TWO WAVE PACKETS?

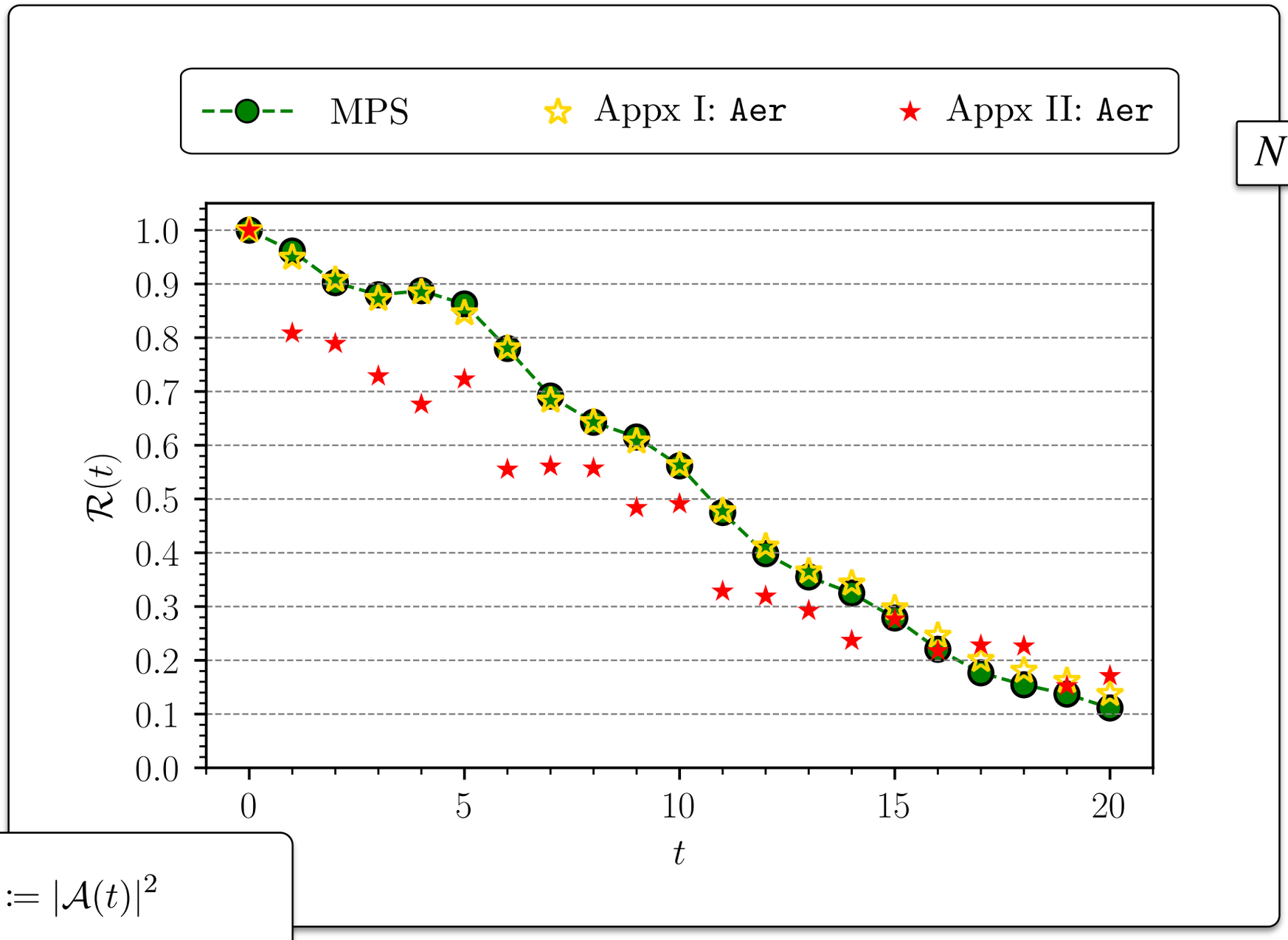


$N = 10$

* Turn the coherent error to incoherent error, then use a similar circuit with known solution to normalize the result of the target circuit.

$E = \langle \tilde{\sigma}^{\mathbf{z}} \rangle$

WHAT ABOUT RETURN PROBABILITY (A DIAGONAL ENTRY OF S-MATRX)?



$$\mathcal{R}(t) := |\mathcal{A}(t)|^2$$

$$\mathcal{A}(t) := \langle \Psi_1, \Psi_2 | U(t) | \Psi_1, \Psi_2 \rangle$$

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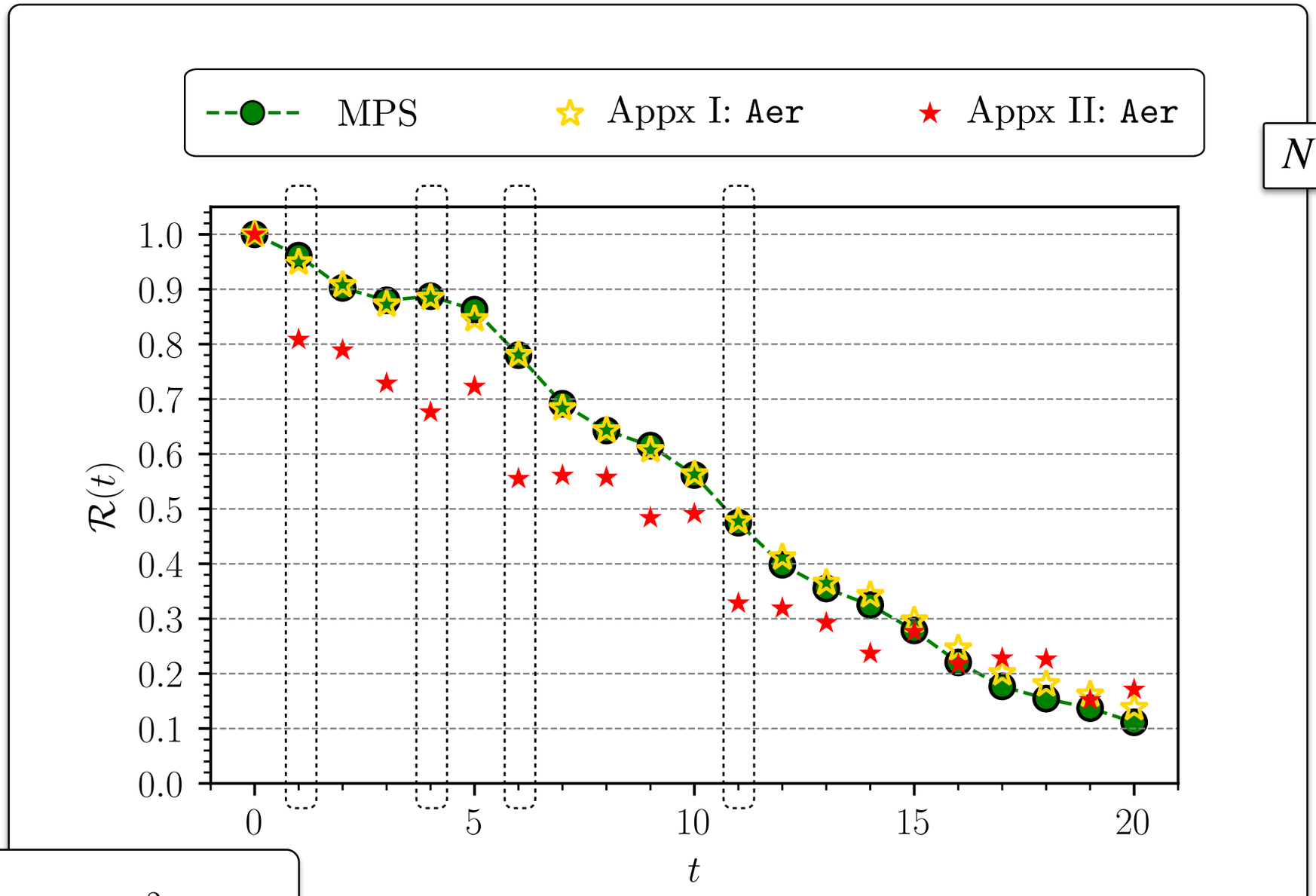
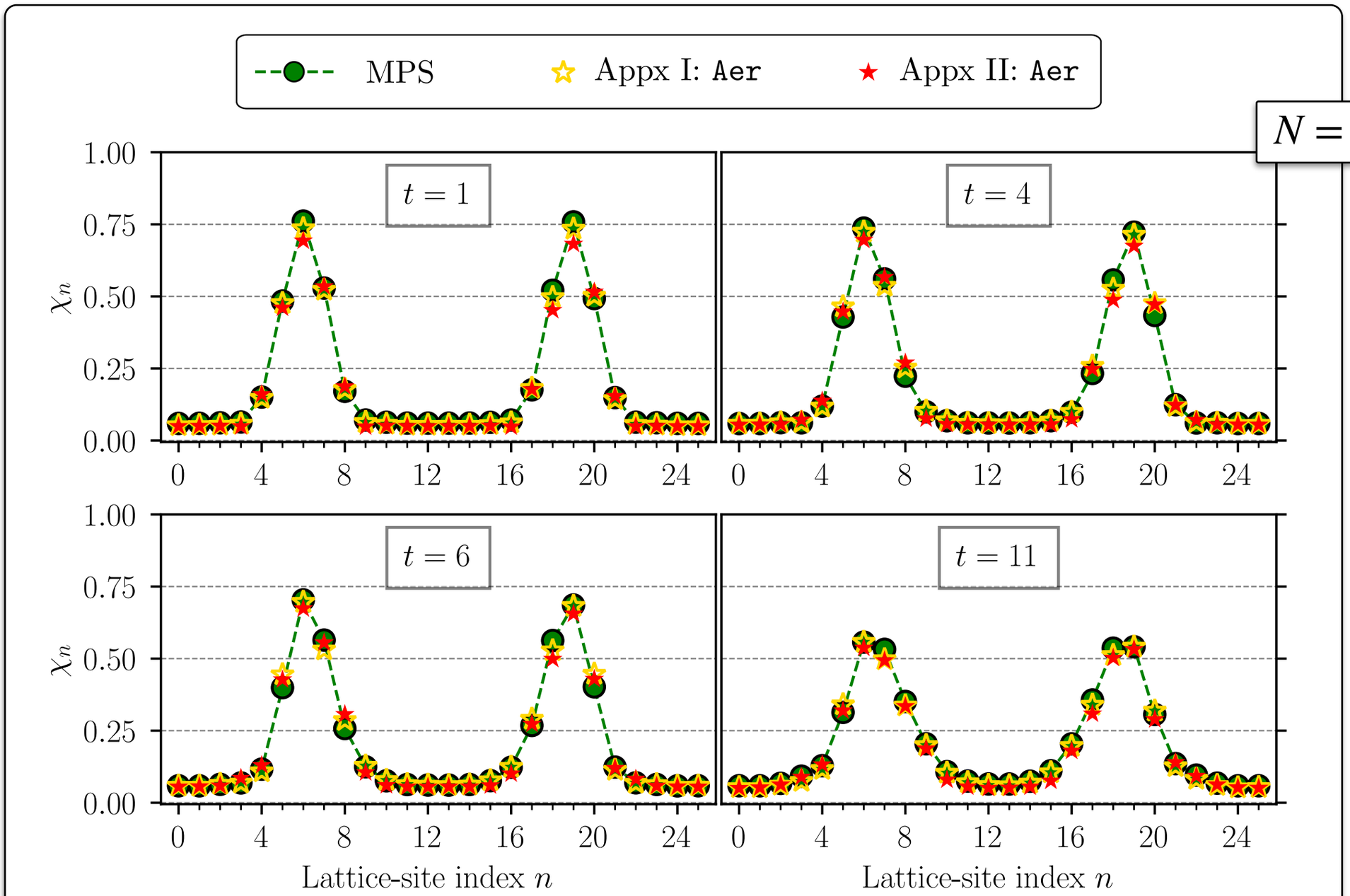


$$\mathcal{R}(t) := |\mathcal{A}(t)|^2$$

$$\mathcal{A}(t) := \langle \Psi_1, \Psi_2 | U(t) | \Psi_1, \Psi_2 \rangle$$

WHAT ABOUT LOCAL QUANTITIES AT TIMES $\mathcal{R}(t)$ DEVIATES?



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TAKE-HOME MESSAGES FOR PART III:

- Costly adiabatic state preparation can be avoided by **resorting to well-motivated composite-particle ansatzes** generated out of the interacting vacuum, which can be constrained by hybrid classical-quantum methods such as VQE.

- Our algorithm has **polynomial scaling with system size**, and can be **systematically improved** using truncated bare-meson operators, which is justified given the finite correlation length in the system.

- **Local quantities are far less sensitive to infidelities in the initial state** than non-local quantities. In the future simulations, therefore, **accurate state preparation for extracting S-matrix elements** is essential.

- Our ansatz works likely for all 1+1 D gauge theories, but need to be **generalized to higher dimensions** following the same logic.

- One may need to think about how to **incorporate a wealth information on hadronic states from lattice-QCD classical computations** to construct good ansatzes for scattering states on quantum computers.

See, e.g., Gupta, White, ZD, arXiv:2506.02313 [quant-ph].

PART I:
BACKGROUND AND MOTIVATION

PART II:
QUANTUM-COMPUTATION/SIMULATION BASICS

PART III:
EXAMPLE: DIGITAL QUANTUM COMPUTING OF
HADRON SCATTERING

Belyansky, Whitsitt, Mueller, Fahimniya, Bennewitz, ZD, Gorshkov, Phys. Rev. Lett. 132, 091903 (2024).

PART IV:
EXAMPLE: ANALOG QUANTUM SIMULATION OF HADRON
SCATTERING

THE MODEL: U(1) LATTICE GAUGE THEORY COUPLED TO FERMIONS IN 1+1 D

Lagrangian in the continuum:

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - e\gamma^u A_\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Fermions

Gauge bosons

$$\overline{\psi}\gamma^\mu\psi = -\frac{\beta}{2\pi}\epsilon^{\mu\nu}\partial_\nu\phi$$

$$\tfrac{e\beta}{2\pi}\phi = E + \tfrac{e\theta}{2\pi} \equiv E_T$$

Bosonized Hamiltonian on the lattice [Coleman, etc.]

$$H = \chi \sum_x \left[\frac{\pi_x^2}{2} + \frac{(\phi_x - \phi_{x-1})^2}{2} + \frac{\mu^2\phi_x^2}{2} - \lambda\cos(\beta\phi_x - \theta)\right]$$

New boson DOF

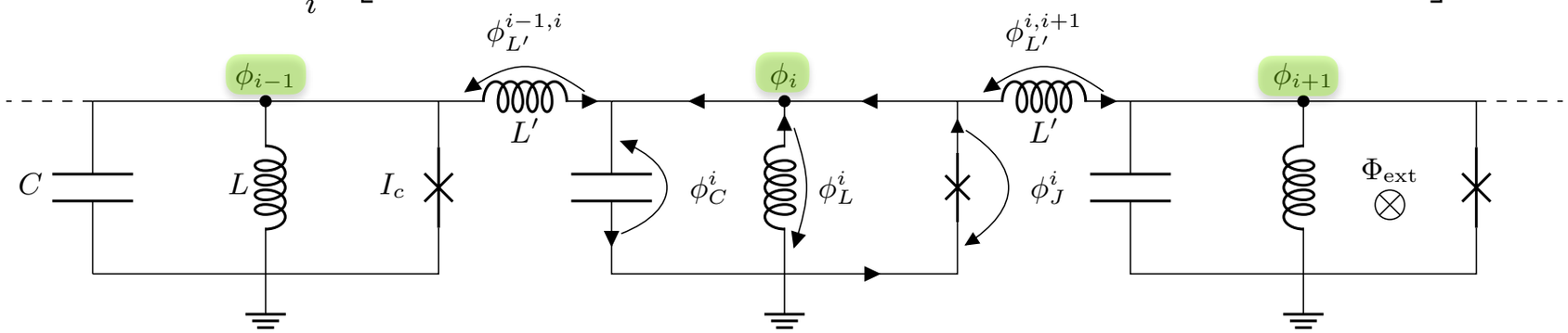
$$\chi = 1/a,\ \beta = \sqrt{4\pi},\ \mu^2 = a^2e^2/\pi,\ \lambda = a^2 e^\gamma\, m\, e \exp[2\pi\Delta(a)]/2\pi^{3/2}$$

Lattice fermion propagator at the origin

$$\chi = \frac{8E_C}{\beta^2}, \quad \frac{E_{L'}\beta^4}{8E_C} = 1, \quad \mu^2 = \frac{E_L\beta^4}{8E_C}, \quad \lambda = \frac{E_J\beta^2}{8E_C}, \quad \theta = \Phi_{\text{ext}} - \pi$$

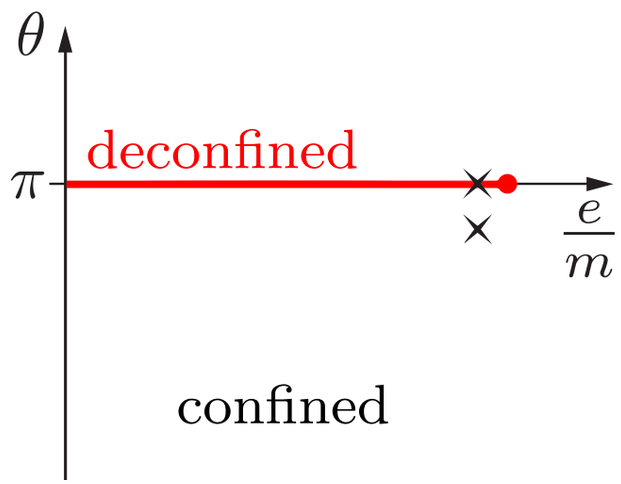
Hamiltonian of circuit QED:

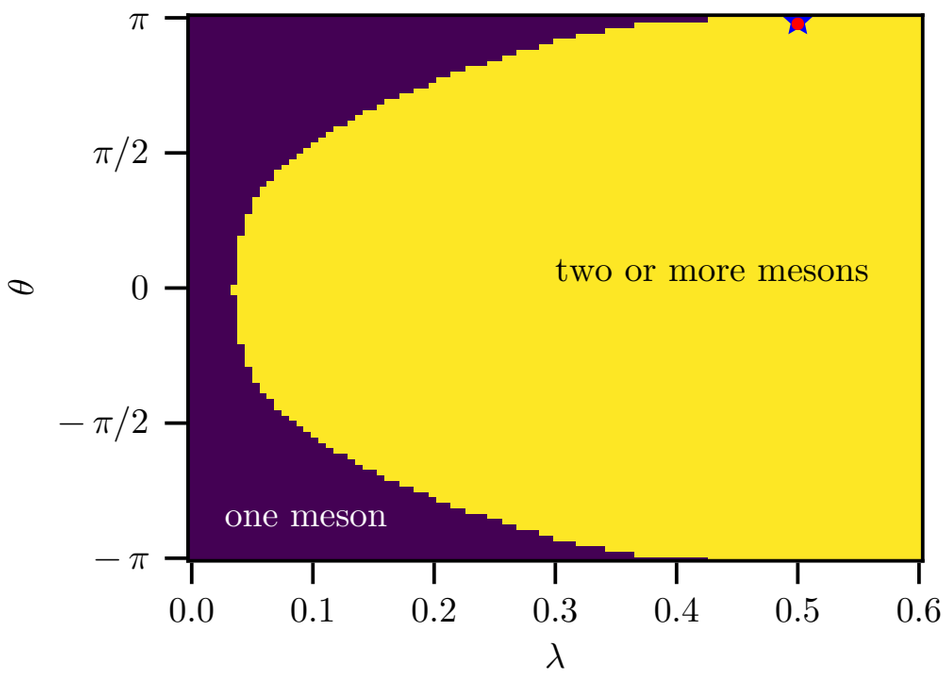
$$H = \sum_i \left[4E_C\pi_i^2 + \frac{E_{L'}(\phi_i - \phi_{i-1})^2}{2} + \frac{E_L\phi_i^2}{2} + E_J\cos(\phi_i - \Phi_{\text{ext}})\right]$$



PHENOMENOLOGY OF THE MODEL: SPECTRUM AND (DE)CONFINED EXCITATIONS

Phase diagram (2π periodic in θ)

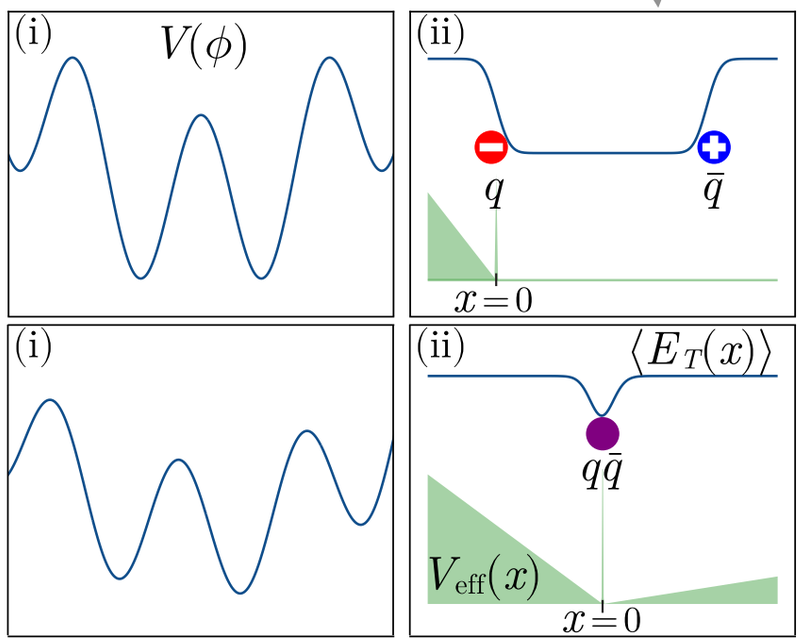




NR potential by Coleman

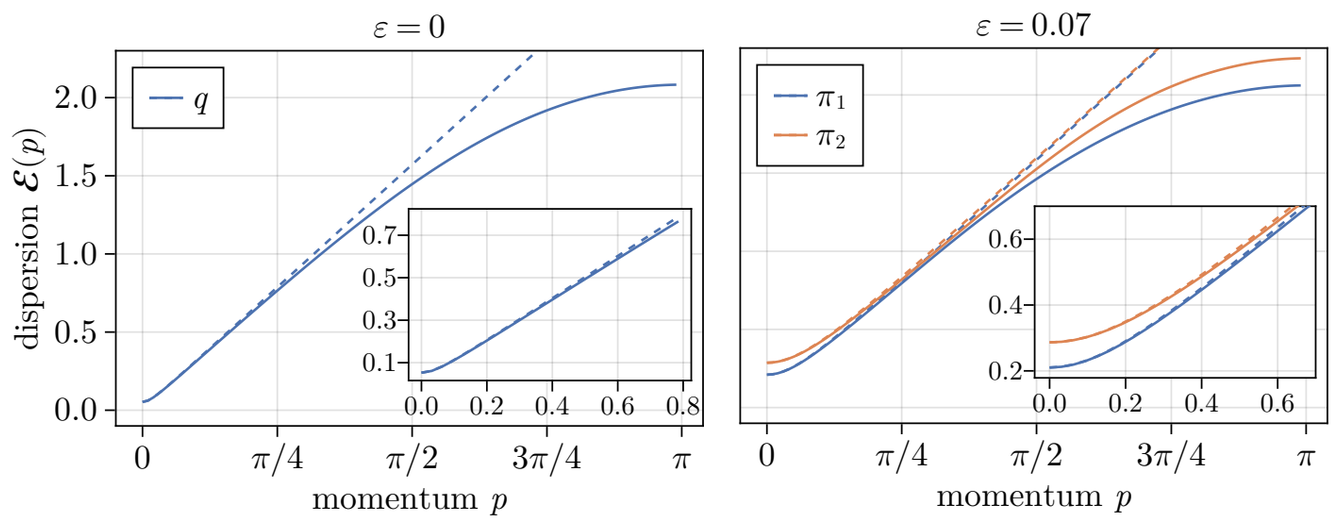
$$V_{\text{eff}}(x) = \frac{e^2}{2}\left(|x| - \frac{\theta}{\pi}x\right) + \frac{e^2}{4m^2}\delta(x)$$

$$V(\phi) = \tfrac{1}{2}\mu^2\phi^2 - \lambda\cos\left(\sqrt{4\pi}\phi - \theta\right)$$

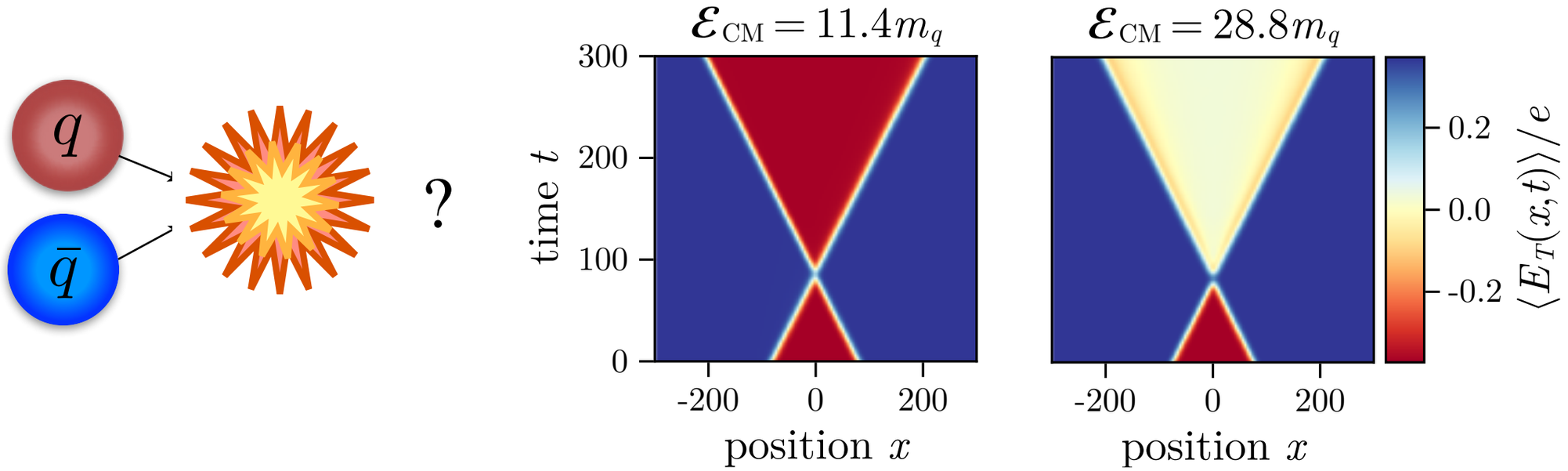


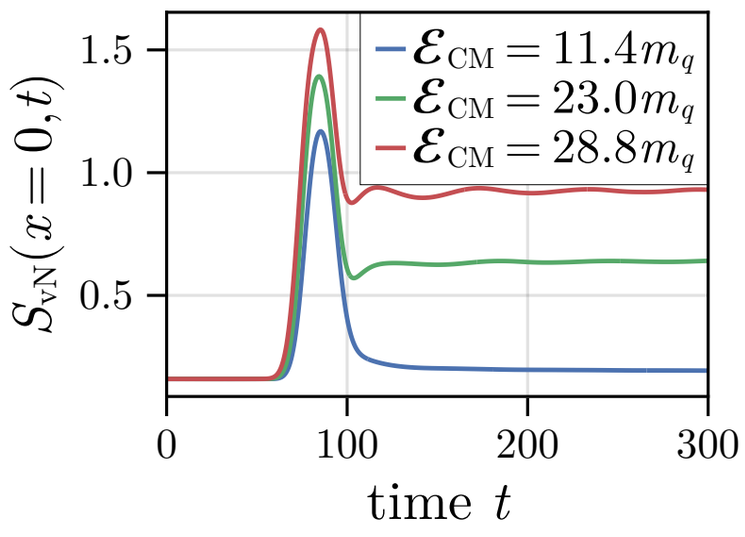
Momentum bands

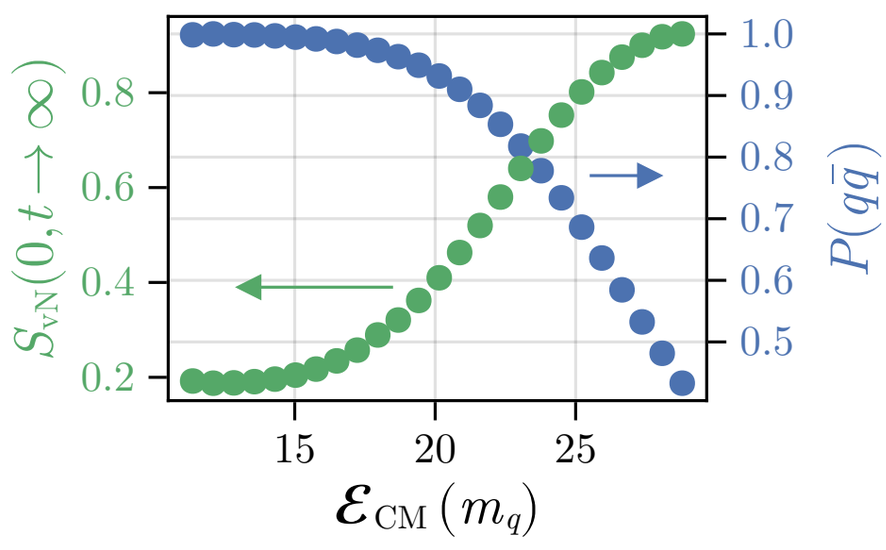
$\theta = \pi - \varepsilon$



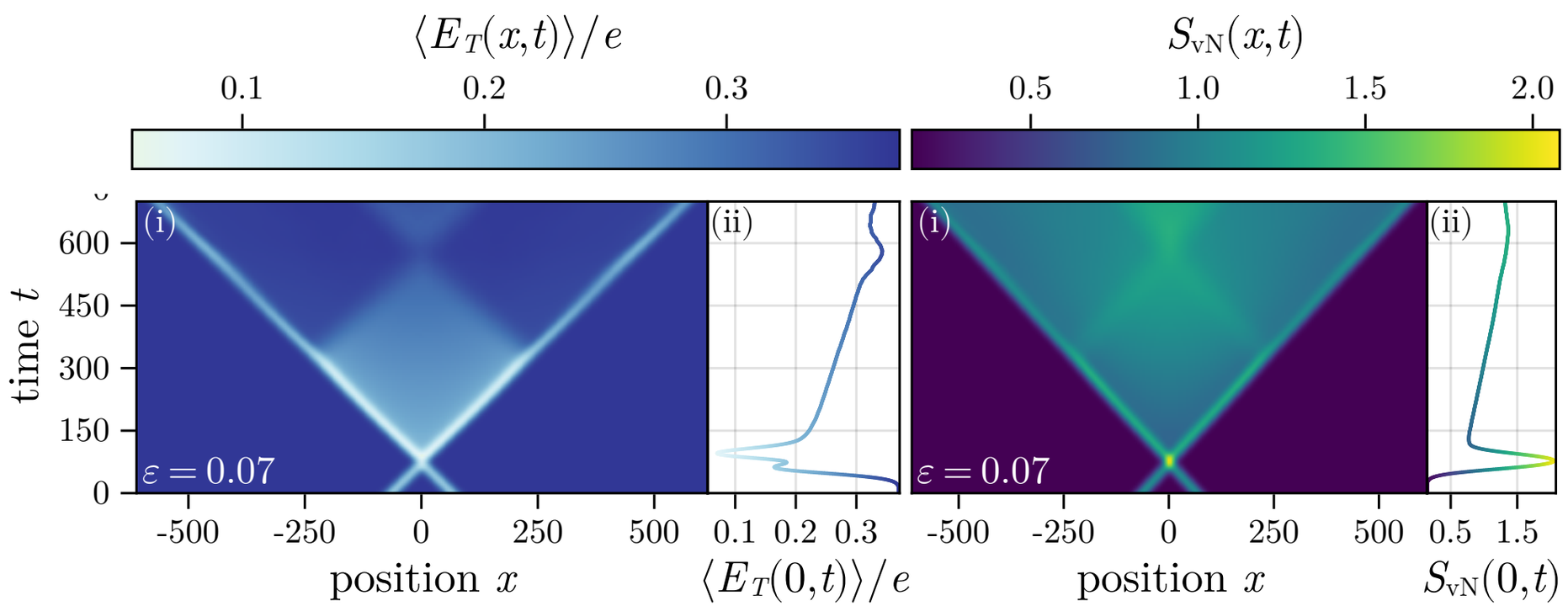
Numerical iMPS simulation of scattering: Deconfined phase

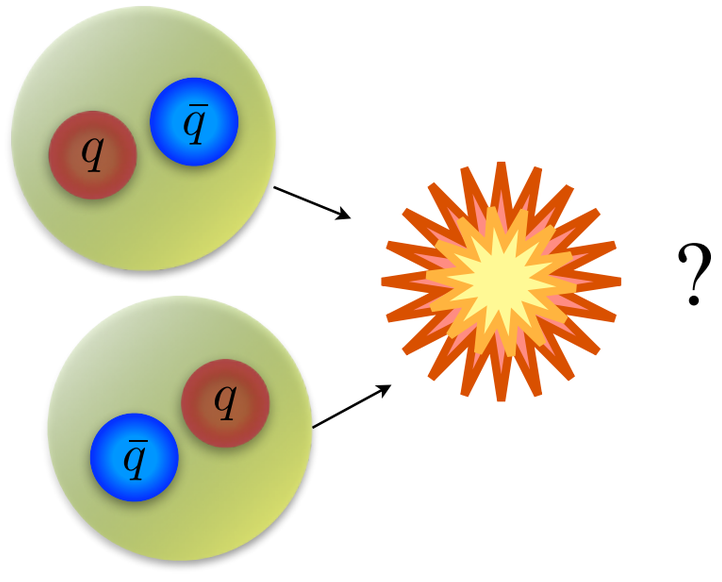


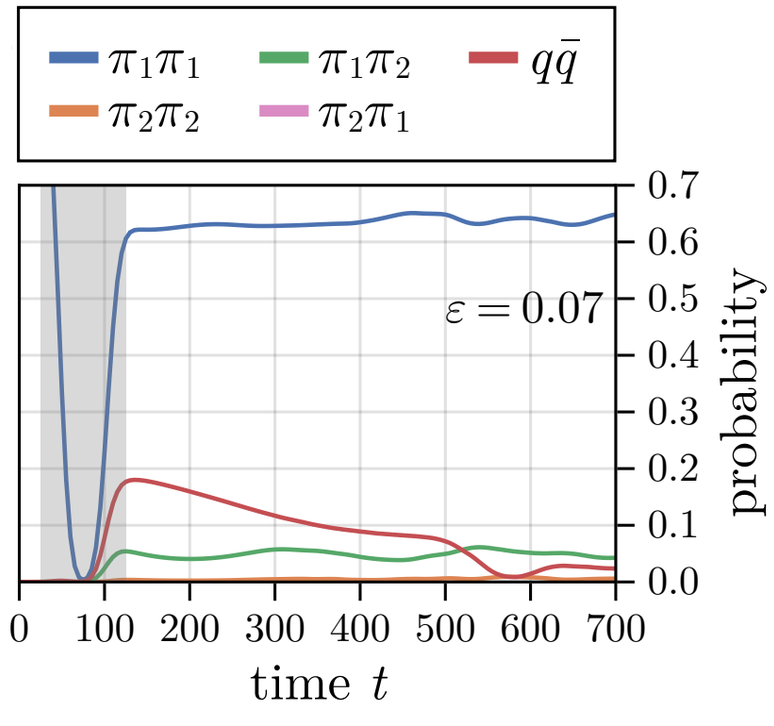




Numerical iMPS simulation of scattering: Confined phase







TAKE-HOME MESSAGES FOR PART IV:

- Circuit QED platforms is a natural analog simulator for certain bosonic quantum field theories. We show how it can be used to simulate the Schwinger model with a CP violating θ-term in both confined and deconfined phases.

- This allows us to propose an analog scattering experiment on circuit-QED platforms, which would be an exciting possibility!

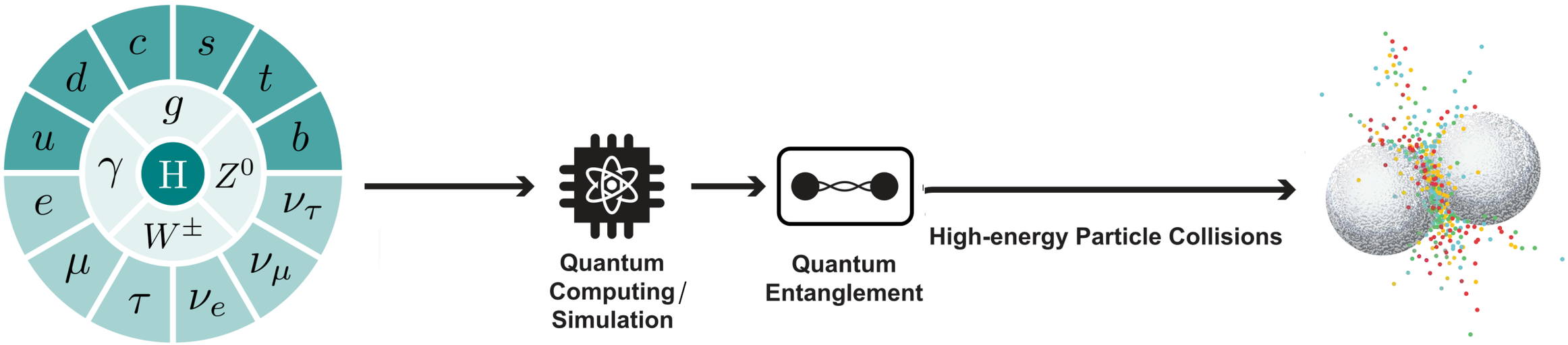
- Powerful iMPS tools allow creating bound and unbound particle excitations on top of interacting vacuum, generating wave packets, and scattering them, for an infinite system size with translational symmetry.

  For iMPS, see: Vanderstraeten, Haegeman, and Verstraete, SciPost Phys. Lect. Notes , 7 (2019), Haegeman, Pirvu, Weir, Cirac, Osborne, Verschelde, and Verstraete, Phys. Rev. B 85, 100408 (2012), Haegeman, Michalakis, Nachtergaele, Osborne, Schuch, and Verstraete, Phys. Rev. Lett. 111, 080401 (2013).

- We observe non-trivial dynamics: in the deconfined phase, quark-antiquark pairs get fragmented to two such pairs, and in the confined phase, meson-meson scattering generates other meson types, as well as fragmentation to quark-antiquark pairs. String formation and breaking clearly are in play!

For theoretical proposals for scattering in atomic analog simulators, see Surace and Lerose, New J. Phys. 23, 062001 (2021) and Su, Osborne, and Halimeh, arXiv:2401.05489 [cond-mat.quant-gas].

FINAL WORD

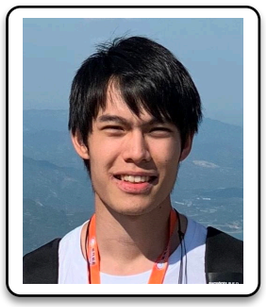


Bauer, ZD, Klco, and Savage, *Nature Rev. Phys.* 5 (2023) 7, 420-432.

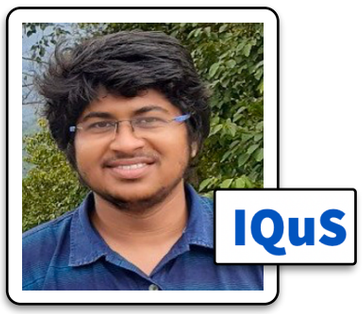
We have taken important steps in figuring out how to simulate hadron scattering using quantum computers but a lot more needs to be done to achieve quantum simulation of scattering in the Standard Model.

MANY THANKS TO MY COLLABORATORS IN THESE PROJECTS…

QIS/NP



C. HSIEH (S)



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Alessio Lerose (P/F) @Geneva→Leuven

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Niklas Mueller (RF/F) @IQuS→UNM

Alexander Schuckert (P) @UMD

Federica Surace (P) @Caltech

Brayden Ware (P) @UMD→Google

Christopher White @UMD→NRL

Seth Whitsitt (P) @UMD→Northrop Grumman

THANK YOU

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