Cluster EFT calculation of electromagnetic breakup reactions with the Lorentz Integral Transform method

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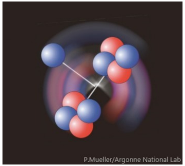
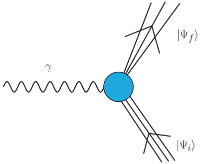
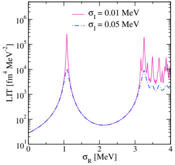
- 2024: PhD, University of Trento

  Thesis: "Cluster EFT calculation of electromagnetic breakup reactions with the LIT method"
- 2024–today: PostDoc, University of Salento

Cluster Effective Field Theory (EFT) calculation
of electromagnetic breakup reactions
with the Lorentz Integral Transform (LIT) method

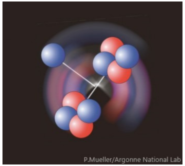






INTRODUCTION and OUTLINE

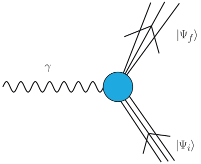
Cluster Effective Field Theory (EFT) calculation of electromagnetic breakup reactions with the Lorentz Integral Transform (LIT) method



MODEL

- **Effective particles**
  - nucleons and α-particles
- **Interaction**
  - potential models from Effective Field Theory (EFT)

Cluster Effective Field Theory (EFT) calculation of **electromagnetic breakup reactions** with the Lorentz Integral Transform (LIT) method

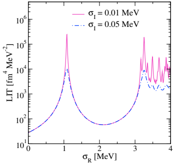


$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n$$

$$\gamma + {}^{12}\text{C} \rightarrow \alpha + \alpha + \alpha$$

- Study of the reactions of *astrophysical relevance* in a **three-body *ab initio*** approach and in the **low-energy regime**
  - three-body binding energies
  - cross sections
- Comparison of the results with the **experimental data**

Cluster Effective Field Theory (EFT) calculation of electromagnetic breakup reactions with the **Lorentz Integral Transform (LIT) method**



METHOD

- **Bound-state problem**
  - variational method
  - Non-Symmetrized Hyperspherical Harmonics (NSHH) method
- **Continuum-states problem**
  - integral transform approach: Lorentz Integral Transform (LIT) method

Cluster Effective Field Theory (EFT) approach

[Hammer *et al.* JPG **44** 103002 (2017), Hammer *et al.* RMP **92** 025004 (2020)]

$$^{6}\text{He} \sim \alpha n n\,,\; ^{9}\text{Be} \sim \alpha\alpha n\,,\; ^{10}\text{Be} \sim \alpha\alpha n n\,,\; ^{12}\text{C} \sim \alpha\alpha\alpha\,,\; ^{16}\text{O} \sim \alpha\alpha\alpha\alpha\,, \ldots$$

Borromean systems:

$$^{9}\text{Be} \sim \alpha\alpha n \quad S_3 \approx 1.573\ \text{MeV} \ll S_p(^{4}\text{He}) \approx 19.81\ \text{MeV}$$

$$^{12}\text{C} \sim \alpha\alpha\alpha \quad S_3 \approx 7.275\ \text{MeV} < S_p(^{4}\text{He}) \approx 19.81\ \text{MeV}$$

$\Rightarrow$ 3-body (or 4-body) *effective clustering* systems in the **low-energy regime**

Separation of energy scales $\rightarrow$ **Halo/Cluster EFT approach**

momentum scales: M_{low}, M_{high}

$\downarrow$

EFT expansion in $\left(\frac{M_{low}}{M_{high}}\right)^{\nu}$

$\downarrow$

error estimate

Cluster Effective Field Theory (EFT) approach

[Hammer *et al.* JPG **44** 103002 (2017), Hammer *et al.* RMP **92** 025004 (2020)]

$$^{6}\text{He} \sim \alpha nn\,,\; ^{9}\text{Be} \sim \alpha\alpha n\,,\; ^{10}\text{Be} \sim \alpha\alpha nn\,,\; ^{12}\text{C} \sim \alpha\alpha\alpha\,,\; ^{16}\text{O} \sim \alpha\alpha\alpha\alpha\,, \ldots$$

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$\downarrow$

EFT expansion in $\left(\frac{M_{low}}{M_{high}}\right)^{\nu}$

$\downarrow$

error estimate

Chiral EFT

$\downarrow$

Pionless EFT

$\downarrow$

Halo/Cluster EFT

$$\mathcal{L}^{\text{eff}} \rightarrow \mathcal{V}^{\text{eff}}$$

Two-body effective potentials

[Hammer *et al.* JPG **44** 103002 (2017), Ji]

Effective potential in momentum space and in the partial wave ℓ:

$$\mathcal{V}_\ell(p, p') = \left[\lambda_0 + \lambda_1 (p^2 + p'^2) \right] p^\ell p'^\ell \; g(p; \Lambda) \; g(p'; \Lambda)$$

- sum of **contact terms** parametrized by the LECs
- **momentum-regulator function** $g(p; \Lambda) = e^{-\left(\frac{p}{\Lambda}\right)^{2m}}$ $\quad m = 1, 2$

[✓]
We calculate the $\mathcal{T}$-matrix by solving the Lippmann-Schwinger eq.
(on-shell: $\boldsymbol{p} = \boldsymbol{p}' = \boldsymbol{k}$)

α–n : $\mathcal{T}_\ell(k)$
α–α : $\mathcal{T}_\ell^{SC}(k)$ Coulomb-distorted strong term

[✓]
We compare the calculated low-energy $\mathcal{T}_\ell$ with its ERE or Coulomb-modified ERE up to terms $O(k^2)$

$$\lambda_i = \lambda_i(a_\ell, r_\ell, \Lambda)$$

[✓]
The LECs are fixed on the experimental values of scattering length a_ℓ^{exp} and effective range r_ℓ^{exp}

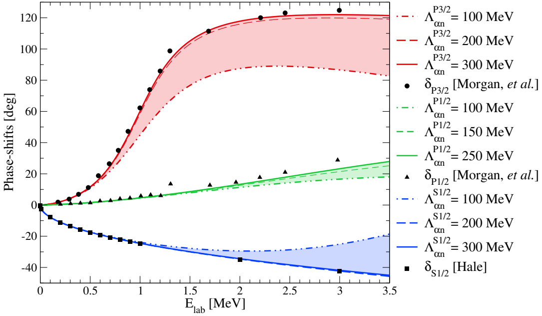
$$\lambda_i = \lambda_i(a_\ell^{\text{exp}}, r_\ell^{\text{exp}}, \Lambda)$$

The effective potentials $\mathcal{V}_\ell^{\alpha n}$ and $\mathcal{V}_\ell^{\alpha\alpha}$ reproduce the experimental low-energy α–n and α–α phase-shifts ➠

Low-energy phase-shifts

Calculated phase-shifts for different two-body cut-offs $\Lambda < \Lambda^{max}$ ($\Leftarrow$ Wigner bound)

$\alpha - n$

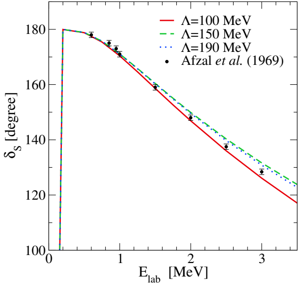


$P_{3/2}$ Resonance

$P_{1/2}$

$S_{1/2}$

$\alpha - \alpha$



S_0 Resonance

Power counting ➠

α–n Power counting α–α

$$\frac{k^{2\ell}}{\mathcal{T}_\ell(k)} = \frac{\mu}{2\pi}\left[\frac{1}{a_\ell} - \frac{1}{2} r_\ell k^2 + O(k^4) + ik^{2\ell+1}\right]$$

$$M_{hi} = \sqrt{2\mu S_p(^4\mathrm{He})} \approx 170 \text{ MeV}$$

$$\frac{1}{\mathcal{T}^{SC}_{\ell=0}(k)} \propto \frac{\mu}{2\pi}\left[\frac{1}{a_0} - \frac{r_0}{2} k^2 + \mathcal{O}(k^4) + 2k_C H(\eta)\right]$$

$$M_{hi} = \sqrt{2\mu S_p(^4\mathrm{He})} \approx 270 \text{ MeV}, \quad k_C = Z_\alpha^2 \alpha_{\text{em}} \mu \approx 60 \text{ MeV}$$

α–n Power counting α–α

$$\frac{k^{2\ell}}{\mathcal{T}_\ell(k)} = \frac{\mu}{2\pi}\left[\frac{1}{a_\ell} - \frac{1}{2}r_\ell k^2 + O(k^4) + ik^{2\ell+1}\right]$$

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$P_{3/2}$: resonance $\Rightarrow$ **enhanced** partial wave

$$M_{lo} = \sqrt{2\mu E_R(^5\mathrm{He})} \approx 30 \text{ MeV} \ll M_{hi}$$

$$\frac{1}{a_1} \sim M_{lo}^2 M_{hi}\,,\quad r_1 \sim M_{hi} \quad \Rightarrow \quad a_1, r_1 \quad \textbf{LO}$$

[Bedaque, *et al.* Phys.Lett.B **569** (2003)]

$$a_1^{\text{exp}}, r_1^{\text{exp}} \Rightarrow M_{lo}/M_{hi} \approx 30 \text{ MeV}/170 \text{ MeV} \approx 0.2$$

nonperturbative approach

$$\frac{1}{\mathcal{T}^{SC}_{\ell=0}(k)} \propto \frac{\mu}{2\pi}\left[\frac{1}{a_0} - \frac{r_0}{2}k^2 + \mathcal{O}(k^4) + 2k_C H(\eta)\right]$$

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S_0: resonance $\Rightarrow$ **enhanced** partial wave

$$M_{lo} = \sqrt{2\mu E_R(^8\mathrm{Be})} \approx 20 \text{ MeV} \ll M_{hi}$$

$$\frac{1}{a_0} \sim \frac{M_{\text{lo}}^3}{M_{hi}^2}\,,\quad r_0 \sim \frac{1}{M_{\text{hi}}} \sim \frac{1}{3k_C} \quad \Rightarrow \quad a_0, r_0 \quad \textbf{LO}$$

[Higa, *et al.* Nucl.Phys.A **809** (2008)]

$$a_0^{\text{exp}}, r_0^{\text{exp}} \Rightarrow M_{lo}/M_{hi} \approx 20 \text{ MeV}/180 \text{ MeV} \approx 0.1$$

nonperturbative approach

α–n Power counting α–α

$$\frac{k^{2\ell}}{\mathcal{T}_\ell(k)} = \frac{\mu}{2\pi}\left[\frac{1}{a_\ell} - \frac{1}{2}r_\ell k^2 + O(k^4) + ik^{2\ell+1}\right]$$

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nonperturbative approach

$S_{1/2}$ and **$P_{1/2}$**: **non-enhanced** partial waves

a_0 LO, r_0, a_1, r_1 subleading [Bedaque (2003)]

we use the same nonperturbative approach

$$\frac{1}{\mathcal{T}^{SC}_{\ell=0}(k)} \propto \frac{\mu}{2\pi}\left[\frac{1}{a_0} - \frac{r_0}{2}k^2 + \mathcal{O}(k^4) + 2k_C H(\eta)\right]$$

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nonperturbative approach

Two-body potentials

^{9}Be

LO

$$\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}^{\alpha n}_{P_{3/2}}(\Lambda^{\alpha n}_{P_{3/2}}) + \mathcal{V}^{\alpha n}_{S_{1/2}}(\Lambda^{\alpha n}_{S_{1/2}}) + \mathcal{V}^{\alpha n}_{P_{1/2}}(\Lambda^{\alpha n}_{P_{1/2}})$$

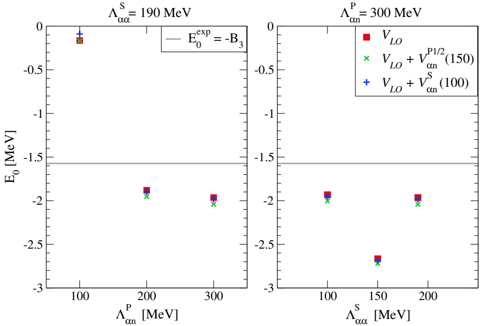
Two-body potentials

^{9}Be

LO	$\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}^{\alpha n}_{P_{3/2}}(\Lambda^{\alpha n}_{P_{3/2}})$
	$+ \mathcal{V}^{\alpha n}_{S_{1/2}}(\Lambda^{\alpha n}_{S_{1/2}})$
	$+ \mathcal{V}^{\alpha n}_{P_{1/2}}(\Lambda^{\alpha n}_{P_{1/2}})$

$$\Lambda^{\alpha\alpha}_{S_0} = 190 \text{ MeV}, \ \Lambda^{\alpha n}_{P_{3/2}} = 300 \text{ MeV}$$

LO	$E_0 = -1.965$ MeV	
LO $+ S_{1/2}$	$E_0 = -1.982$ MeV	subleading!
LO $+ P_{1/2}$	$E_0 = -2.041$ MeV	subleading!



Variation of $\Lambda^{\alpha n}_{P_{3/2}}$ at fixed $\Lambda^{\alpha\alpha}_{S_0}$

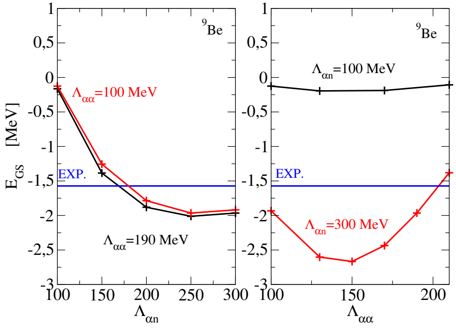
Variation of $\Lambda^{\alpha\alpha}_{S_0}$ at fixed $\Lambda^{\alpha n}_{P_{3/2}}$

Two-body potentials

^{9}Be

LO $\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}^{\alpha n}_{P_{3/2}}(\Lambda^{\alpha n}_{P_{3/2}})$

- Different behaviour
- Stronger dependence of the ground-state energy by varying $\Lambda^{\alpha n}_{P_{3/2}}$ at fixed $\Lambda^{\alpha\alpha}_{S_0}$



Variation of $\Lambda^{\alpha n}_{P_{3/2}}$ at fixed $\Lambda^{\alpha\alpha}_{S_0}$

Variation of $\Lambda^{\alpha\alpha}_{S_0}$ at fixed $\Lambda^{\alpha n}_{P_{3/2}}$

Three-body potential

^{9}Be

LO $\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}^{\alpha n}_{P_{3/2}}(\Lambda^{\alpha n}_{P_{3/2}}) + \mathcal{V}_3(\Lambda_3, \lambda_3)$

$[+\mathcal{V}^{\alpha n}_{S_{1/2}}(\Lambda^{\alpha n}_{S_{1/2}})]$

To avoid the dependence of the 3-body results on the 2-body cutoffs, we add a 3-body potential

$$\mathcal{V}_3(Q, Q') = \lambda_3 \; e^{-\left(\frac{Q}{\Lambda_3}\right)^2} \; e^{-\left(\frac{Q'}{\Lambda_3}\right)^2}$$

- we choose the two-body cutoffs $\Lambda^{\alpha n}$ and/or $\Lambda^{\alpha\alpha}$
- for each value of the 3-body cutoff Λ_3, the LEC λ_3 is fixed on a 3-body observable

Electromagnetic inclusive reactions

Cross section

$$\sigma_{\mathrm{EM}} \propto \mathcal{R}(\omega)$$

Response function

$$\mathcal{R}(\omega) = \int df \, |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2 \, \delta(E_f - E_0 - \omega)$$

1. calculation of the initial bound state $|\Psi_0\rangle$ ➛ bound-state method
2. calculation of the final states $|\Psi_f\rangle$ in the continuum ➛ integral transform approach
3. determination of the operator $\hat{\mathcal{O}}$ ➛ photodisintegration processes

Non-Symmetrized Hyperspherical Harmonics (NSHH) method

[Gattobigio, *et al.* PRC **83** (2011), Deflorian, *et al.* FBS **54** (2013)]

❶ $\mathcal{R}(\omega) \sim |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2$

Variational method + Non-Symmetrized HH basis

- potentials from EFT ← INPUT

- $\hat{\mathcal{H}}$ is represented on a Non-Symmetrized **basis in momentum space**

$$\Psi = \sum_\nu c_\nu \Psi_\nu$$

$$\equiv \sum_{m\{K\}} c_{m\{K\}} f_m(Q) \, \mathcal{Y}_{\{K\}}(\Omega_Q)$$

$f_m(Q) \propto$ Laguerre polynomials basis ($m = 1, \ldots, N_{\text{Lag}}$)

$\mathcal{Y}_{\{K\}}(\Omega_Q) =$ complete basis of the HH functions ($K = 1, \ldots, K^{\max}$)

- $\hat{\mathcal{H}}$ is diagonalized

$$\sum_{\nu'} \langle \Psi_\nu | \hat{\mathcal{H}} | \Psi_{\nu'} \rangle c_{\nu'} = E \, c_\nu \qquad E_0, \{ c_\nu^0 \} \Rightarrow \Psi_0$$

- Convergence is reached by enlarging the dimension of the basis

Lorentz Integral Transform (LIT) method

[Efros, *et al.* JPG **34** (2007)]

❷ $\mathcal{R}(\omega) \sim |\langle \Psi_f | \hat{O} | \Psi_0 \rangle|^2$

$\mathcal{R}(\omega)$: states in the continuum spectrum are involved $\hat{\mathcal{H}} |\Psi_f\rangle = E_f |\Psi_f\rangle \Rightarrow$ direct calculation is DIFFICULT

Integral transform approach

- Definition of an **Integral Transform** $\mathcal{L}(\sigma)$ of the response function $\mathcal{R}(\omega)$ with a **Lorentzian kernel**

$$\mathcal{L}(\sigma) = \int d\omega \frac{1}{(\omega - E_0 + \sigma_R)^2 + \sigma_I^2} \mathcal{R}(\omega) ,$$

$$\mathcal{L}(\sigma) \xrightarrow{\text{INVERSION}} \mathcal{R}(\omega)$$

- It can be demonstrated that $\mathcal{L}(\sigma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle$, with $|\tilde{\Psi}\rangle \equiv$ LIT states

  $|\tilde{\Psi}\rangle$ can be calculated using **bound-state methods**

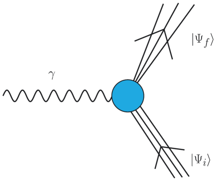
Continuum-states problem $\mathcal{R}(\omega)$ $\xrightarrow{\text{reformulation}}$ Bound-state-like problem $\mathcal{L}(\sigma)$

Photodisintegration reactions

[Bacca and Pastore JPG **41** (2014)]

❸ $\mathcal{R}(\omega) \sim |\langle\Psi_f|\hat{\mathcal{O}}|\Psi_0\rangle|^2$

Photon γ:
$\boldsymbol{A}(\boldsymbol{x})$, $\hat{\epsilon}_{\boldsymbol{q},\lambda}$
choice: $\boldsymbol{q} \parallel \hat{\boldsymbol{z}}$



Nucleus:
$\boldsymbol{J}(\boldsymbol{x})$ nuclear current operator
$\rho(\boldsymbol{x})$ nuclear charge operator

$\omega = |\boldsymbol{q}|$

$|\Psi_0\rangle \rightarrow |\Psi_f\rangle$
transition

$$\mathcal{R}_\gamma(\omega) \sim \left|\langle\Psi_f|\,\hat{\epsilon}_{\boldsymbol{q},\lambda}\cdot\boldsymbol{J}(\boldsymbol{q})\,|\Psi_0\rangle\right|^2$$

Nuclear current matrix element

Twofold calculation

[Bacca and Pastore JPG **41** (2014)]

$$\mathcal{R}(\omega) \sim \left| \langle \Psi_f | \, \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) \, | \Psi_0 \rangle \right|^2$$

1. One-body current $J_\lambda^{[1]}(q)$

- The nuclear current operator is a sum of terms

$$\boldsymbol{J} = \boldsymbol{J}^{[1]} + \boldsymbol{J}^{[2]} + \boldsymbol{J}^{[3]} + \ldots$$

← the nuclear current m.e. can be calculated by using only the **one-body term** i.e. the nuclear convection current

- Specifically with our EFT, the continuity equation is **NOT** fully satisfied

continuity equation: $\boldsymbol{q} \cdot \boldsymbol{J}(\boldsymbol{q}) = [\mathcal{H}, \rho(\boldsymbol{q})]\,, \qquad \mathcal{H} = T + \mathcal{V}_{2B} + \ldots$

$\boldsymbol{q} \cdot \boldsymbol{J}^{[1]}(\boldsymbol{q}) = [T, \rho(\boldsymbol{q})]$ always satisfied

$[\mathcal{V}_{2B}, \rho(\boldsymbol{q})] \neq 0 \;\Rightarrow\; \exists\; \boldsymbol{J}^{[2]}(\boldsymbol{q})$ such that $\boldsymbol{q} \cdot \boldsymbol{J}^{[2]}(\boldsymbol{q}) = [\mathcal{V}_{2B}, \rho(\boldsymbol{q})]$

$\ldots$

Twofold calculation

[Siegert PR **52** (1937)]

$$\mathcal{R}(\omega) \sim \left| \langle \Psi_f | \, \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) \, | \Psi_0 \rangle \right|^2$$

2. Siegert operator $\mathcal{T}^{el,S}_{J\lambda}(q;\rho)$

- Multipole decomposition: $\hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) = -\sum_J \sqrt{2\pi(2J+1)} \left[\mathcal{T}^{el}_{J\lambda}(q;\boldsymbol{J}) + \lambda \mathcal{T}^{mag}_{J\lambda}(q;\boldsymbol{J}) \right]$

$$\mathcal{T}^{el}_{J\lambda}(q;\boldsymbol{J}) = \mathcal{T}^{el,I}_{J\lambda}(q;\boldsymbol{J}) + \mathcal{T}^{el,II}_{J\lambda}(q;\boldsymbol{J})$$

Dominant: $EJ = E1, E2$

- Siegert theorem (**continuity equation**)

$$\mathcal{T}^{el,I}_{J\lambda}(q;\boldsymbol{J}) \propto \int \mathrm{d}\hat{q}' \, \boldsymbol{q}' \cdot \boldsymbol{J}(\boldsymbol{q}') \, Y_{J\lambda}(\hat{q}') \qquad \leftarrow \quad \omega\rho(\boldsymbol{q}) - \boldsymbol{q} \cdot \boldsymbol{J}(\boldsymbol{q}) = 0$$

$$\propto \int \mathrm{d}^3\boldsymbol{x} \, j_J(qx) \, \rho(\boldsymbol{x}) \, Y_{J\lambda}(\hat{x}) \ \propto \mathcal{T}^{el,S}_{J\lambda}(q;\rho)$$

- Long-wavelength approximation ($qR \ll 1$)

$\mathcal{T}^{el,S}_{J\lambda}(q;\rho) \propto \omega^J \int \mathrm{d}^3\boldsymbol{x} \, x^J \rho(\boldsymbol{x}) Y_{J\lambda}(\hat{x}) \leftarrow$ dipole $\boldsymbol{d}_\lambda$ or quadrupole $\boldsymbol{u}_\lambda$ op.

$\mathcal{T}^{el,II}_{J\lambda}(q;\boldsymbol{J}) \propto \omega^{J+1} \leftarrow$ correction

Twofold calculation

$$\mathcal{R}(\omega) \sim \left| \langle \Psi_f | \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) | \Psi_0 \rangle \right|^2$$

1. One-body current $J_\lambda^{[1]}(q)$

2. Siegert operator $\mathcal{T}_{J\lambda}^{el,S}(q;\rho)$

- With the **one-body current**, the *continuity equation* is not fully satisfied already at LO
- With the **Siegert theorem**, since the *continuity equation* is used explicitly, the contribution due to the many-body current operators is implicitly included in the calculation
- Comparison between the two calculations $\Rightarrow$ the contribution due to the many-body currents can be quantified

Practical calculation of the LIT

Eigenvalue method

[Efros, *et al.* JPG **34** (2007)]

$$\mathcal{L}(\sigma) = \sum_L \frac{|\langle \Psi_L | \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) | \Psi_0 \rangle|^2}{(E_L - E_0 - \sigma_R)^2 + \sigma_I^2}$$

$$\sum_{\nu'} \langle \Psi_\nu | \hat{\mathcal{H}} | \Psi_{\nu'} \rangle \, c_{\nu'} = E \, c_\nu$$

$$E_0, \{ \, c_\nu^0 \, \} \Rightarrow \Psi_0 \qquad E_L, \{ \, c_\nu^L \, \} \Rightarrow \Psi_L$$

1. One-body current [E. Filandri (2022)]

$$\langle \Psi_L | j_\lambda | \Psi_0 \rangle$$

momentum space calculation

$$\Psi_{0/L}(Q, \Omega_Q) = \sum_{m\{K\}} c_{m\{K\}}^{0/L} f_m(Q) \, \mathcal{Y}_{\{K\}}(\Omega_Q)$$

2. Siegert operator [YC (2024)]

$$E1 : \ \langle \Psi_L | d_\lambda | \Psi_0 \rangle \qquad E2 : \ \langle \Psi_L | u_\lambda | \Psi_0 \rangle$$

coordinate space calculation

$$\Psi^{0/L}(\rho, \Omega_\rho) = \sum_{m\{K\}} c_{m\{K\}}^{0/L} g_{m,K}(\rho) \, \mathcal{Y}_{\{K\}}(\Omega_\rho)$$

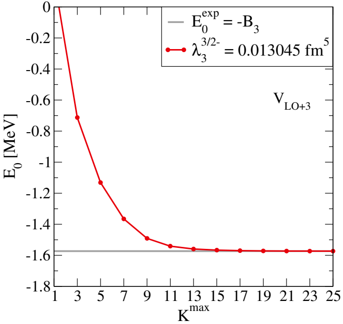
a Fourier Transform of the basis $f_m(Q)$ originally defined in momentum space is needed [Viviani, *et al.* FBS **39** (2006)]

$$g_{m,K}(\rho) = (-i)^K \int_0^\infty dQ \, \frac{Q^{3N-1}}{(Q\rho)^{\frac{3N}{2}-1}} \, J_{K+\frac{3N-3}{2}+\frac{1}{2}}(Q\rho) \, f_m(Q) \qquad N = A - 1$$

$\gamma + {}^9\text{Be} \to \alpha + \alpha + n$ $\quad |0\rangle : J^\pi = 3/2^- \to |f\rangle : J^\pi = 1/2^+$

^{9}Be ground state (LO)

$N_{\text{Lag}} = 30$



- EFT potential

$$\mathcal{V}_{LO+3} = \mathcal{V}_S^{\alpha\alpha} + \mathcal{V}_{P_{3/2}}^{\alpha n} + \mathcal{V}_3(300, \lambda_3^{3/2^-})$$

$\downarrow$

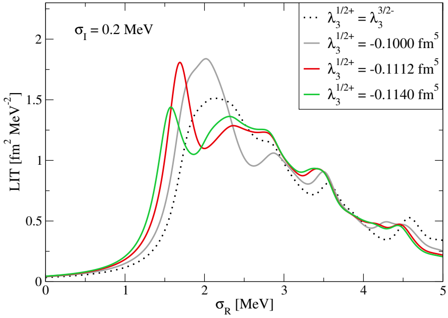
The LEC $\lambda_3^{3/2^-}$ is fixed to reproduce the experimental binding energy $B_3 = -1.573$ MeV

- HH Basis $\sim f_m(Q)\, \mathcal{Y}_{\{K\}}(\Omega_Q)$

  with $m = 1, \ldots, N_{\text{Lag}}$, $K = 1, \ldots, K^{\text{max}}$

$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n$ $\qquad |0\rangle : J^\pi = 3/2^- \rightarrow |f\rangle : J^\pi = 1/2^+$

$1/2^+$ LIT (LO)



- EFT potential ($\mathcal{V}_3$ is state-dependent)

$$\mathcal{V}_{LO+3} = \mathcal{V}_S^{\alpha\alpha} + \mathcal{V}_{P_{3/2}}^{\alpha n} + \mathcal{V}_3(300, \lambda_3^{1/2^+})$$

$\downarrow$

The LEC $\lambda_3^{1/2^+}$ is fixed to locate the resonance peak at the experimental energy

- Dipole operator

$$\mathcal{L}(\sigma_R) = \sum_L \frac{|\langle \Psi_L | \hat{d}_\lambda | \Psi_0 \rangle|^2}{(E_L - E_0 - \sigma_R)^2 + \sigma_I^2}$$

$\downarrow$

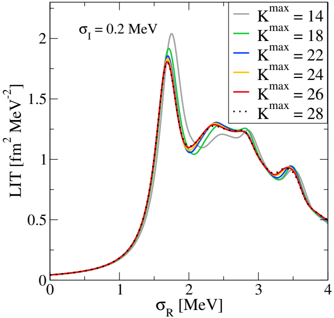
$\sigma_I \sim$ resolution

$\sigma_I \approx$ width of the resonance

$\gamma + {}^9\text{Be} \to \alpha + \alpha + n$ $\quad$ $|0\rangle : J^\pi = 3/2^- \to |f\rangle : J^\pi = 1/2^+$

$1/2^+$ LIT (LO)

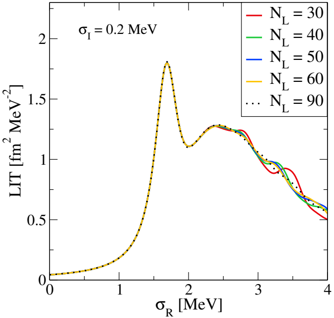
$N_{\text{Lag}} = 30$



← convergence: $K^{\text{max}} = 26$

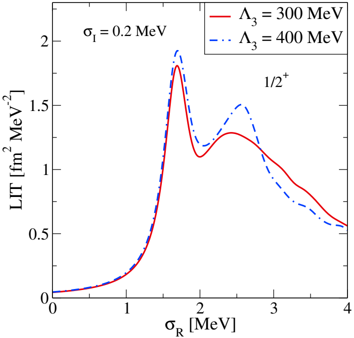
convergence: $N_{\text{Lag}} > 60$ →
(to avoid oscillations
in the tail region)

$K^{\text{max}} = 26$



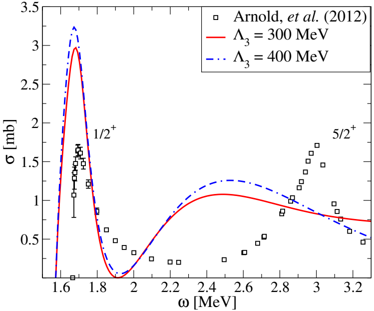
$\gamma + {}^{9}\text{Be} \to \alpha + \alpha + n$ $\qquad |0\rangle : J^\pi = 3/2^- \to |f\rangle : J^\pi = 1/2^+$

$1/2^+$ LIT and cross section (LO)



← *slight* dependence on the variation of the cutoff Λ_3

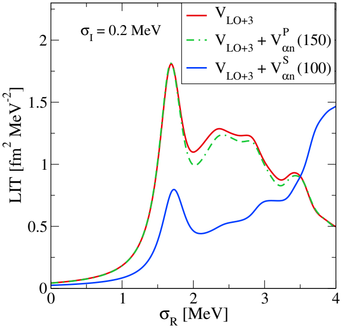
$\mathcal{L}(\sigma_R) \to \mathcal{R}(\omega) \to \sigma(\omega)$



Inclusion of other α–n partial waves ➠

$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n$ $\quad |0\rangle : J^\pi = 3/2^- \rightarrow |f\rangle : J^\pi = 1/2^+$

$1/2^+$ LIT (LO + $P_{1/2}$, LO + $S_{1/2}$)



- $\mathcal{V}^{\alpha n}_{P_{1/2}} \rightarrow$ almost no effect
- $\mathcal{V}^{\alpha n}_{S_{1/2}} \rightarrow$ peak lower in height! $\rightarrow$ **LO effect**

Choice: cutoffs $\Lambda^{\alpha n}_{S_{1/2}} > 100$ MeV

$\Rightarrow$ to "project out" the αn deep bound state we add a **projection potential**

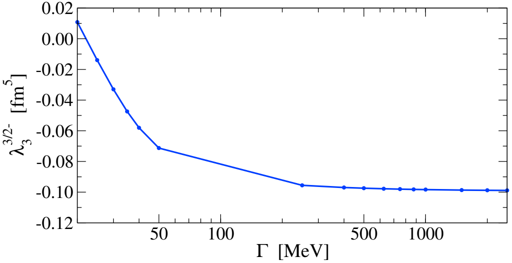
$$\mathcal{V}_{PR}(p, p') = \psi_{S_{1/2}}(p) \, \frac{\Gamma}{4\pi} \, \psi_{S_{1/2}}(p')$$

Theoretically: $\Gamma \rightarrow \infty$

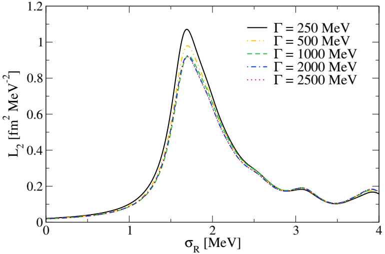
In practice: Γ-independence of the three-body results ➠

$$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^\pi = 3/2^- \rightarrow |f\rangle : J^\pi = 1/2^+$$

➠ Γ-independence of the three-body results: CHECKS



- ^{9}Be ground-state energy



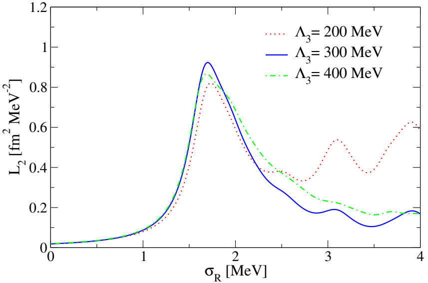
- calculation of the $1/2^+$ LIT

$\Gamma \gtrsim 10^3$ MeV ⇒ Γ is no more a "free parameter"

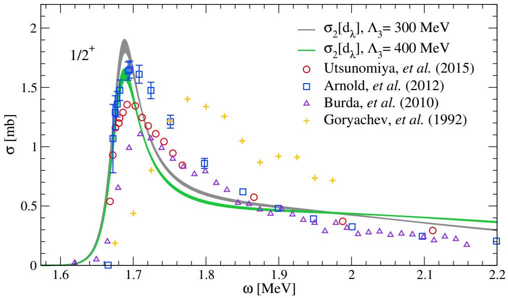
$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \rightarrow |f\rangle : J^{\pi} = 1/2^{+}$$

Final $1/2^+$ LIT and cross section

[YC *et al.*, arXiv:2506.05040]



slight dependence on the 3-body cutoff Λ_3

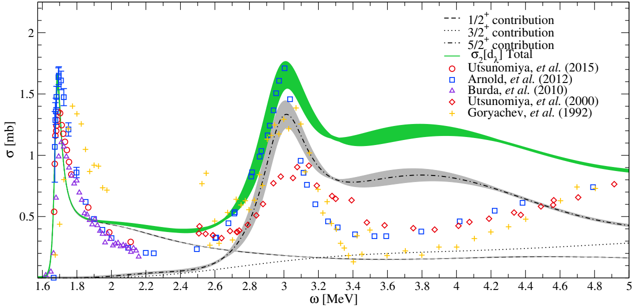


bands: *error* due to the inversion procedure $\mathcal{L}(\sigma_R) \rightarrow \mathcal{R}(\omega)$

$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n$ $\quad$ $|0\rangle : J^\pi = 3/2^- \rightarrow |f\rangle : J^\pi = 1/2^+, 5/2^+, 3/2^+$

Total photodisintegration cross section

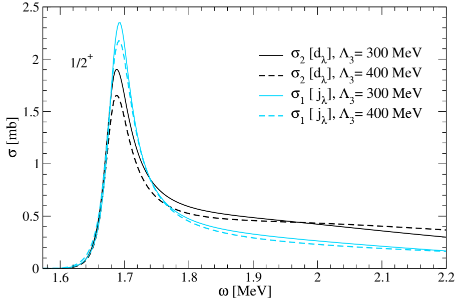
[arXiv:2506.05040]



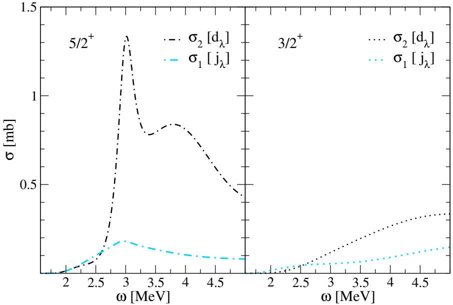
- $\Lambda_3 = 400$ MeV
- $1/2^+$
  error due to the inversion procedure
- $5/2^+$
  error from the model-space convergence
- $3/2^+$
  broad resonance
- inclusion of the $P_{3/2}$ shape parameter
  $\rightarrow$ improvement at higher energies

$$\gamma + {}^9\text{Be} \to \alpha + \alpha + n \qquad |0\rangle : J^\pi = 3/2^- \;\to\; |f\rangle : J^\pi = 1/2^+, 5/2^+, 3/2^+$$

$1/2^+, 5/2^+, 3/2^+$ cross section: $j_\lambda - d_\lambda$ comparison



⇒ *non-negligible* contribution of the many-body currents

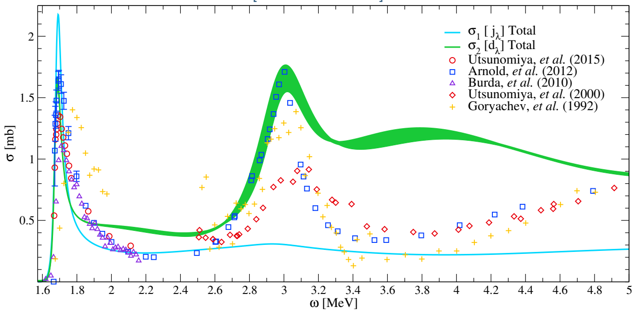


⇒ *dominant* effect of the many-body currents

$$\gamma + {}^{9}\text{Be} \rightarrow \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \rightarrow |f\rangle : J^{\pi} = 1/2^{+}, 5/2^{+}, 3/2^{+}$$

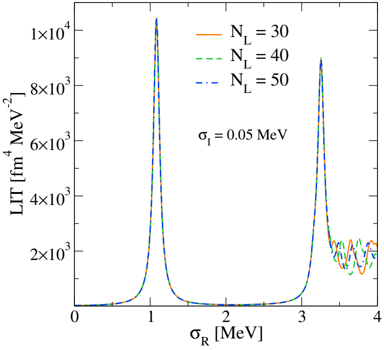
Total photodisintegration cross section: $j_\lambda - d_\lambda$ comparison

[arXiv:2506.05040]



$$\gamma + {}^{12}\mathrm{C} \to \alpha + \alpha + \alpha \qquad |0\rangle : J_n^\pi = 2_1^+ \text{ (bound state)} \to |f\rangle : J_n^\pi = 0_2^+ \text{ (Hoyle state)}$$

0^+ LIT (LO)



- Quadrupole operator
- $\mathcal{V}_{LO+3} = \mathcal{V}_{S_0}^{\alpha\alpha} + \mathcal{V}_{\mathbf{3}}(\Lambda_3, \lambda_3)$

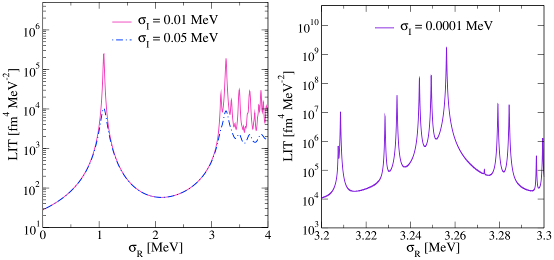
  $\lambda_3^{2^+}$ chosen to reproduce $E^{\exp}(2_1^+) = -2.875$ MeV,

  $\lambda_3^{0^+}$ chosen to locate the resonance peak at the experimental energy ≈ 3.25 MeV
- 0_1^+ (ground state) and 0_2^+ are **NOT** simultaneously reproduced

  $E(0_1^+) = -1.792 \text{ MeV} \neq E^{\exp}(0_1^+) = -7.275 \text{ MeV}$

$\gamma + {}^{12}\mathrm{C} \rightarrow \alpha + \alpha + \alpha$ $\quad |0\rangle : J_n^\pi = 2_1^+$ (bound state) $\rightarrow |f\rangle : J_n^\pi = 0_2^+$ (Hoyle state)

0^+ LIT (LO)



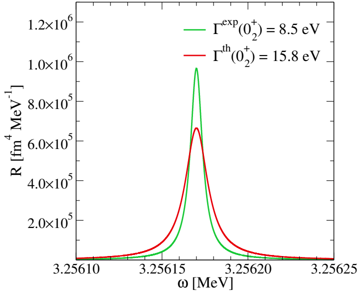
Construction of the response function:

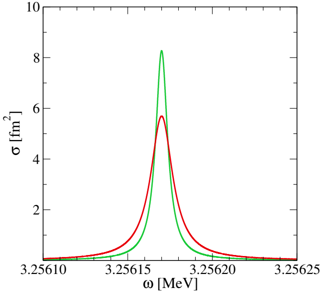
- we take the highest peak as a single "LIT state"
- we impose the experimental width of the resonance
  ($\Gamma^{\exp}(0_2^+) \sim$ eV, very narrow!)
- we impose a width Γ^{th} from Suno, *et al.* PRC **94** (2016)

$\gamma + {}^{12}\mathrm{C} \rightarrow \alpha + \alpha + \alpha$ $\quad |0\rangle : J_n^\pi = 2_1^+$ (bound state) $\rightarrow |f\rangle : J_n^\pi = 0_2^+$ (Hoyle state)

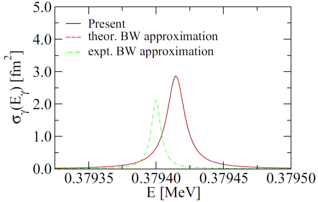
0^+ cross section (LO)

[Γ^{th} from Suno, *et al.* (2016)]





[Suno, *et al.* PRC **94**(2016)]



Comparison

$\Rightarrow$ factor ~ 4 or ~ 2

Conclusions and outlook

Photodisintegration reactions of three-body cluster nuclei *at low energy*

- potentials from Cluster EFT
- LIT method: cross sections
- study of the effect of the many-body currents

^{9}Be [arXiv:2506.05040]

- LO cross section: overestimation of the data
- LO+$S_{1/2}$ cross section: agreement with the data
- non-vanishing effect of the many-body currents, dominant in the $5/2^+$ channel

☞ Calculation of the magnetic transition $M1$

^{12}C [YC PhD Thesis (2024)]

- LO calculation (early stage!): overestimation of the data $\longrightarrow$ How the calculation can be improved? (*D*-wave potential)

☞ Improvement in the convergence

- Effect of the many-body currents in ^{12}C nucleus
- Four-body calculations
- *NN* interaction from EFT (^{10}Be,...)
- Final goal: $\gamma + {}^{16}\mathrm{O} \to \alpha + \alpha + \alpha + \alpha$ $\longrightarrow$ Is this EFT approach still valid?

thank you for your attention