Stabilisation of three-body resonances to bound states in a continuum

Pascal Naidon, RIKEN

pascal.naidon@riken.jp

Lucas Happ, RIKEN

lucas.happ@riken.jp

Universality in strongly-interacting systems: from QCD to atoms
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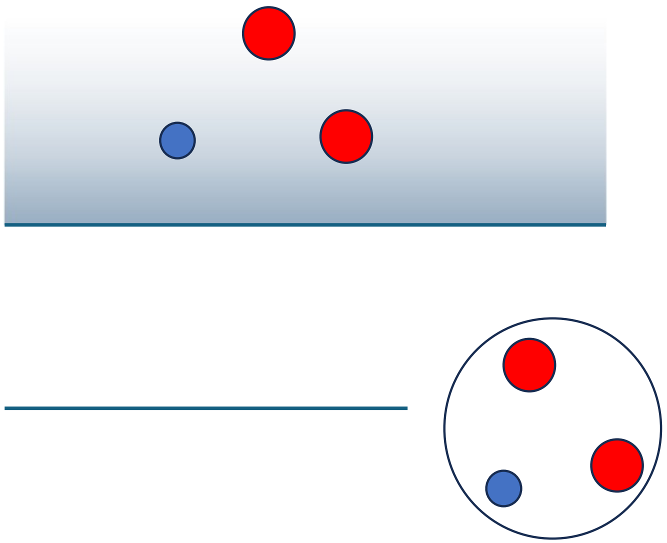
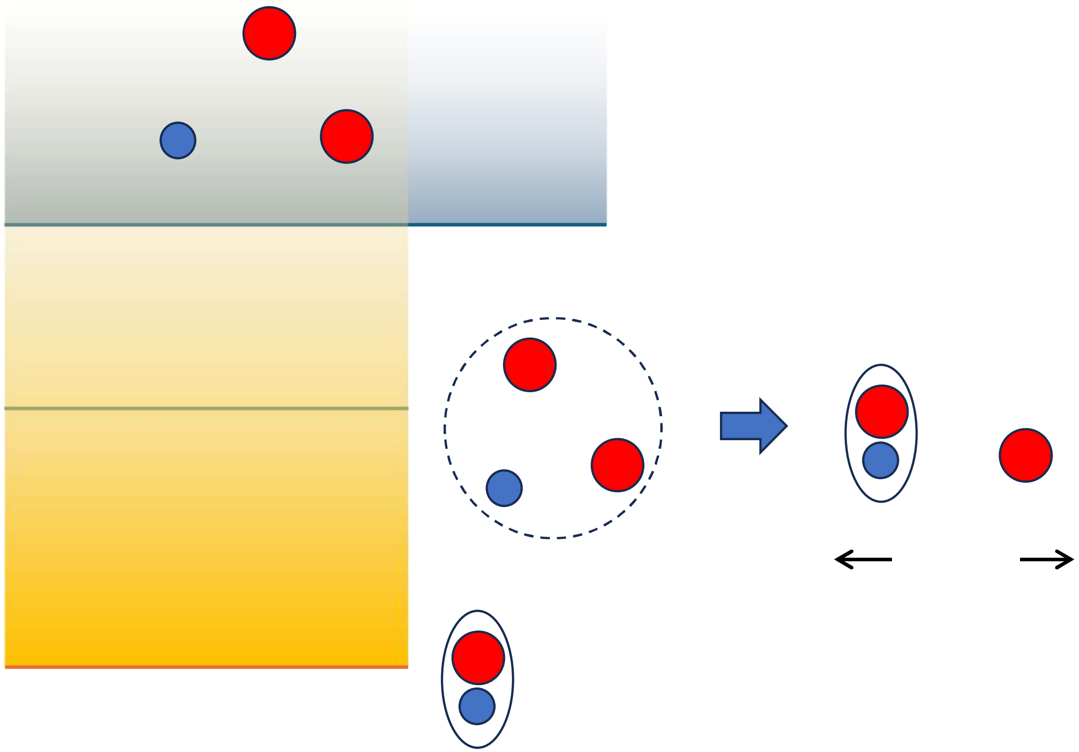
Introduction

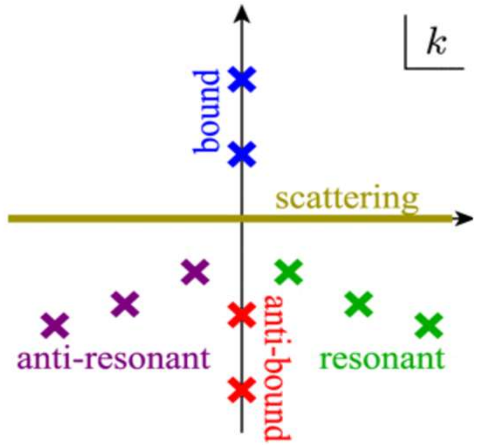
Few-body bound states



Introduction

Few-body resonances





Hatano et al., *J. Math. Phys.* **55**, 122106 (2014)

Complex energy

$$E = E_R - \frac{i}{2}\Gamma$$

Lifetime

$$\tau = \frac{\hbar}{\Gamma} = \frac{-\hbar}{2\, Im(E)}$$

Introduction

Few-body resonances are ubiquitous

- **Nuclear physics**

PHYSICAL REVIEW C **102**, 054303 (2020)

Resonant states of $^{9}_{\Lambda}$Be with $\alpha + \alpha + \Lambda$ three-body cluster model

Qian Wu,[1,*] Yasuro Funaki,[2,3,†] Emiko Hiyama,[4,3,‡] and Hongshi Zong[1,5,6,7,§]

Efimov states in excited nuclear halos

Shimpei Endo[1,*] and Junki Tanaka[2,3,†]

- **Ultracold atoms**

Evidence for the association of triatomic molecules in ultracold ^{23}Na^{40}K + ^{40}K mixtures

Huan Yang, Xin-Yao Wang, Zhen Su, Jin Cao, De-Chao Zhang, Jun Rui, Bo Zhao, Chun-Li Bai & Jian-Wei Pan

Nature 602, 229–233 (2022) | Cite this article

PHYSICAL REVIEW LETTERS **128**, 020401 (2022)

Bose-Einstein Condensation of Efimovian Triples in the Unitary Bose Gas

S. Musolino,[1,*] H. Kurkjian,[2] M. Van Regemortel,[3] M. Wouters,[4] S. J. J. M. F. Kokkelmans,[1] and V. E. Colussi[5,†]

Reshaped three-body interactions and the observation of an Efimov state in the continuum

Yaakov Yudkin[1], Roy Elbaz[1], José P. D'Incao[2,3], Paul S. Julienne[4] & Lev Khaykovich[1]

Control of reactive collisions by quantum interference

HYUNGMOK SON, JULIANA J. PARK, YU-KUN LU, ALAN O. JAMISON, TIJS KARMAN, AND WOLFGANG KETTERLE Authors Info & Affiliations

SCIENCE · 3 Mar 2022 · Vol 375, Issue 6584 · pp. 1006-1010 · DOI: 10.1126/science.abl7257

- **Excitons in semiconductors, …**

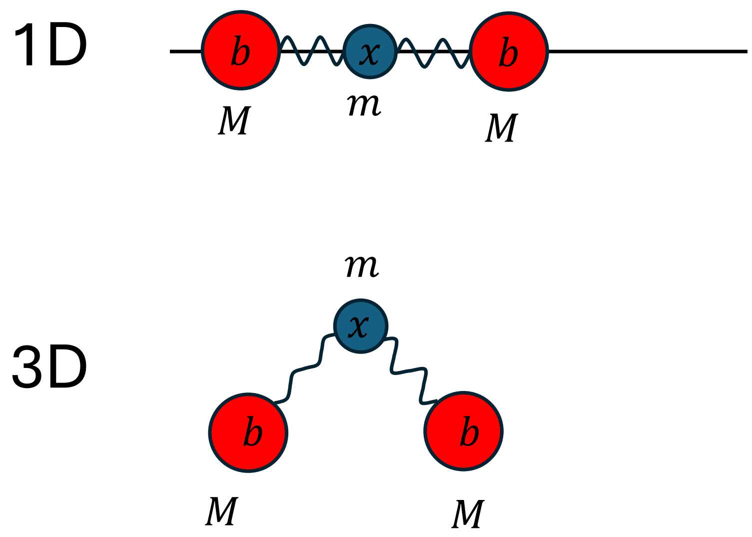
Linewidths and energy shifts of electron-impurity resonant states in quantum wells with infinite barriers

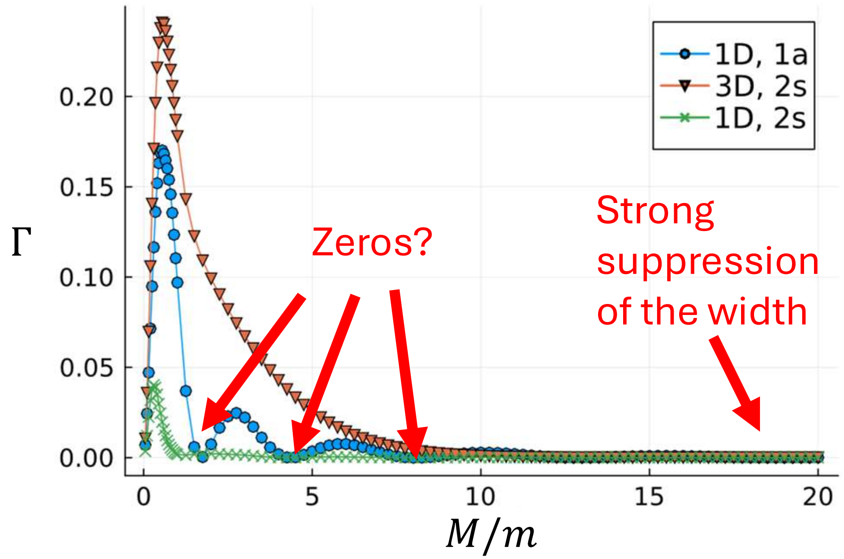
Pavel A. Belov[*]

Introduction

Mass ratio dependence of resonance width

Mass Ratio Dependence of Three-Body Resonance Lifetimes in 1D and 3D
Happp, Naidon, Hiyama, *Few-Body Systems* **65**, 38 (2024)





Many questions:

- Why such a strong effect of the mass ratio?
- Effect also for other parameters?
- What is the underlying mechanism?
- Is the width really vanishing?

Outline

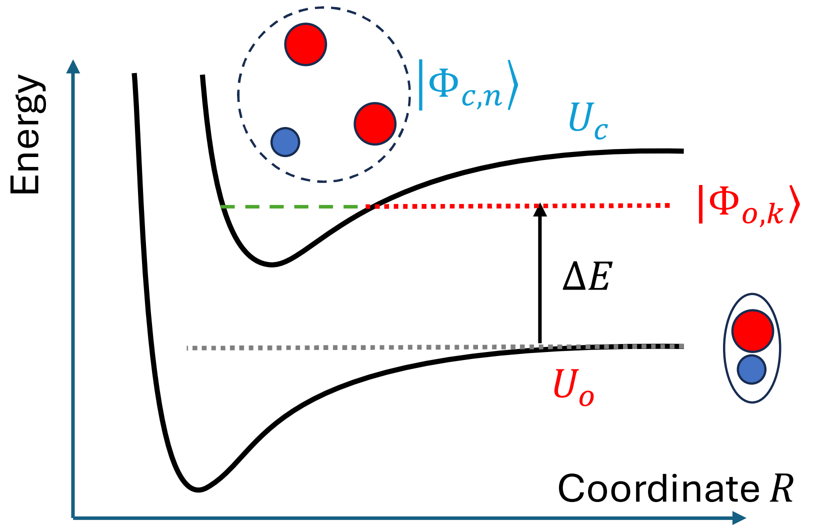
1. Two-channel description of few-body resonances
2. Bound state in a continuum (BIC)
3. Example in 1D
4. Example in 3D: the Efimov scenario
5. Conclusion

1. Two-channel description

Few-body system via
Feshbach-like two channel model

$$H = \begin{pmatrix} H_{o,o} & H_{o,c} \\ H_{c,o} & H_{c,c} \end{pmatrix}$$

$$H_{i,i} = T + U_i, \qquad i \in \{o, c\}$$



For instance,
Born-Oppenheimer
or
Hyper-spherical
representation

Bare (uncoupled) states:

$$(H_{o,o} - E_{o,m})|\Phi_{o,m}\rangle = 0$$

$$(H_{c,c} - E_{c,n})|\Phi_{c,n}\rangle = 0$$

Isolated resonance theory:

$$\boxed{\Gamma = 2\pi \left| \langle \Phi_{o,k} | H_{o,c} | \Phi_{c,n} \rangle \right|^2}$$

Chin et al., *Rev. Mod. Phys.* **82**, 1225 (2010)
Naidon, Pricoupenko, *PRA* **100**, 042710 (2019)

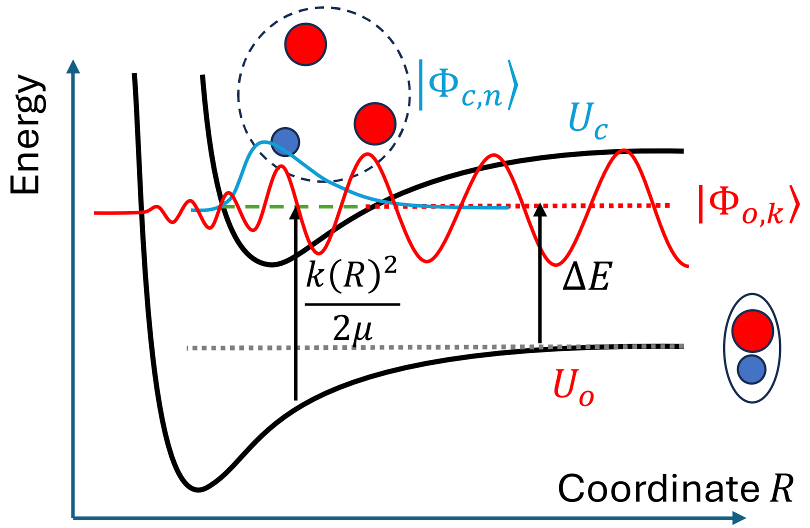
- Finite width Γ due to coupling between closed and open channel

1. Two-channel description

WKB approximation of the continuum state:

$$\Phi_{o,k}(R) \propto \frac{1}{\sqrt{k(r)}} \cos\left(\alpha + \int d^d R' k(R')\right)$$

With $k(R) \equiv \sqrt{2\mu[\Delta E - U_o(R)]}$



For instance, Born-Oppenheimer or Hyper-spherical representation

Key observations:

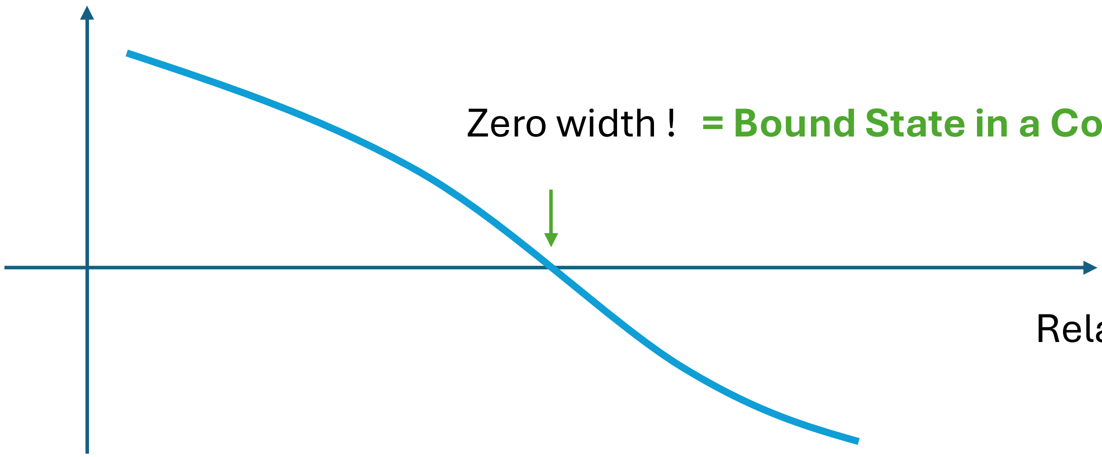
- Overlap with oscillating $\Phi_{o,k}(r)$ **can** lead to cancellation
- Wavelength of oscillations mainly governed by $k(R)$
- General mechanism for any dimension

Isolated resonance theory:

$$\boxed{\Gamma = 2\pi \left|\langle \Phi_{o,k} | H_{o,c} | \Phi_{c,n} \rangle\right|^2}$$

Chin et al., *Rev. Mod. Phys*. **82**, 1225 (2010)
Naidon, Pricoupenko, *PRA* **100**, 042710 (2019)

1. Two-channel description

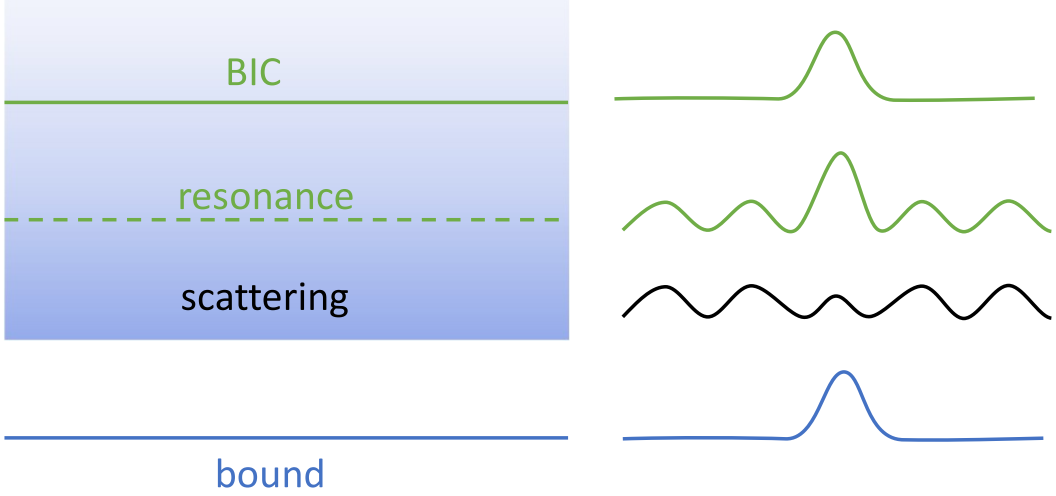


$$k \equiv \sqrt{2\mu[\Delta E - U_o(R)]}$$

Any parameter that changes the relative momentum:

- Mass ratios $\beta = M/m$
- Energy gap ΔE
- ...

2. Bound state in a continuum



Über merkwürdige diskrete Eigenwerte

J. von Neumann & E. P. Wigner

von Neumann and Wigner, *Phys. Z.* **30**, 465 (1929)

REVIEWS

Bound states in the continuum

Chia Wei Hsu[1]*, Bo Zhen[2,3]*, A. Douglas Stone[1], John D. Joannopoulos[2] and Marin Soljačić[2]

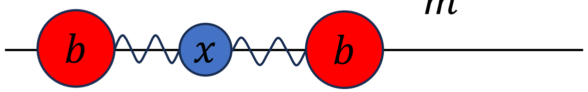
Hsu et al., *Nature Rev. M.* **1**, 16048 (2016)

Various mechanisms
- Engineered potentials
- Selection rules/protection from symmetries or separability
- Interferences of two resonances
- **Continuous tuning of system parameters**
- ...

3. Example in 1D

1D

$$\frac{M}{m} = \beta$$



Gaussian interspecies interaction

$$V_{ij}(r) = v_0 e^{-(r_{ij}/r_0)^2}$$

Born-Oppenheimer approximation:

$$H_0 \phi_C(\mathrm{Z};\mathrm{z}) = \boldsymbol{U_C(z)}\, \phi_C(Z;z), \qquad \langle \phi_C | \phi_{C'} \rangle = \delta_{C,C'}$$

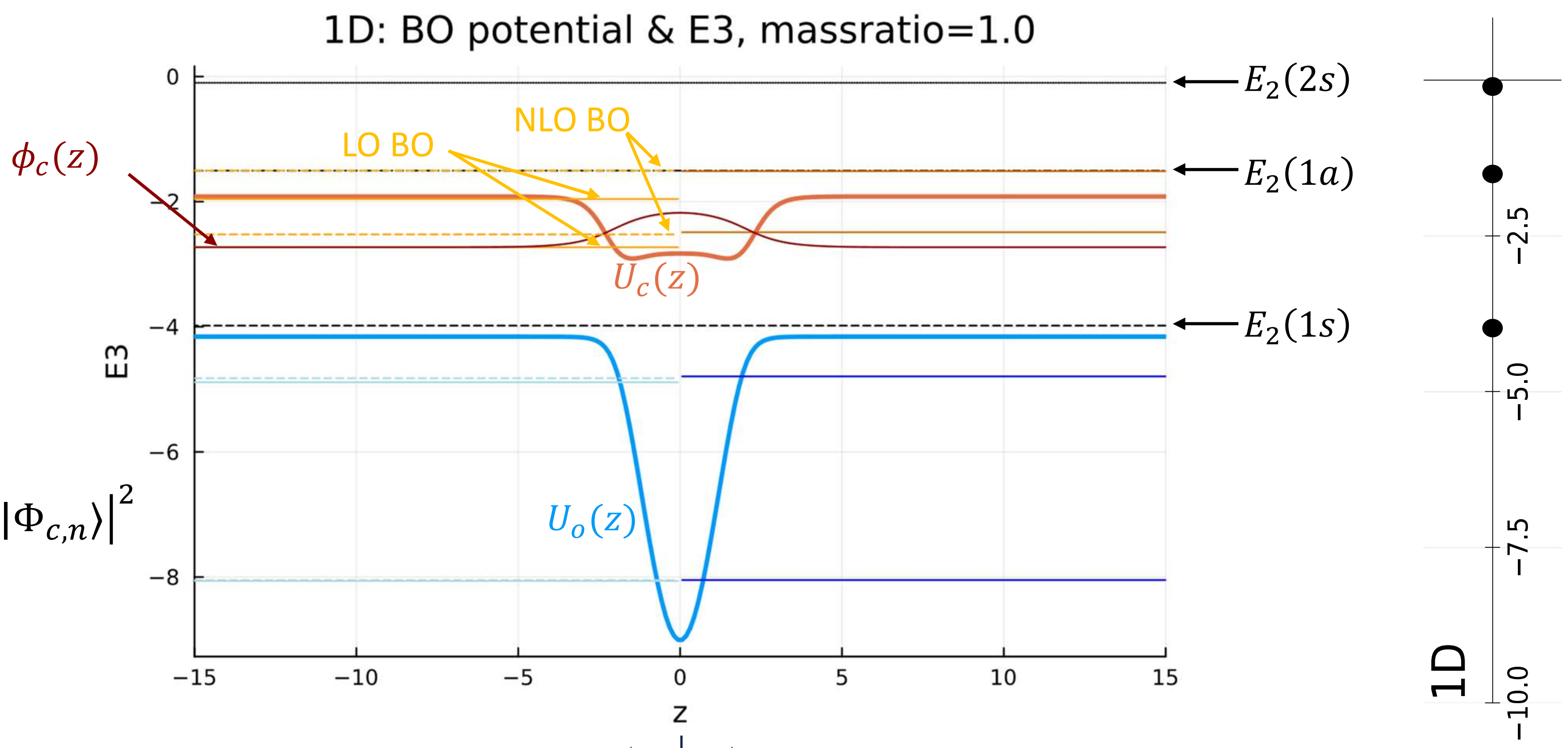
$$H_0 = -\frac{\hbar^2}{2\mu_{bb,x}} \frac{\partial^2}{\partial Z^2} + V(Z_+) + V(Z_-)$$

$$H_{C,C'} = \delta_{C,C'} \underbrace{\left[-\frac{\hbar^2}{2\mu_{bb}} \frac{\partial^2}{\partial z^2} + \boldsymbol{U_C(z)} \right]}_{H^{(0)}_{C,C'}} - \frac{\hbar^2}{2\mu_{bb}} \underbrace{\left[2\langle \phi_C | \frac{\partial}{\partial z} | \phi_{C'} \rangle \frac{\partial}{\partial z} + \langle \phi_C | \left(\frac{\partial^2}{\partial z^2} | \phi_{C'} \rangle \right) \right]}_{H^{(1)}_{C,C'}} = \begin{pmatrix} H^{(0)}_{oo} + H^{(1)}_{oo} & H^{(1)}_{oc} \\ H^{(1)}_{co} & H^{(0)}_{cc} + H^{(1)}_{cc} \end{pmatrix}$$

Lowest-order BO (0):
$H_{o,c} = H_{c,o} = 0$
→ No coupling; resonances are stable bare bound states

Next-order BO (1):
$H_{o,c} \neq 0$
→ coupling induces finite width $\Gamma \neq 0$
→ also: diagonal correction to resonance positions

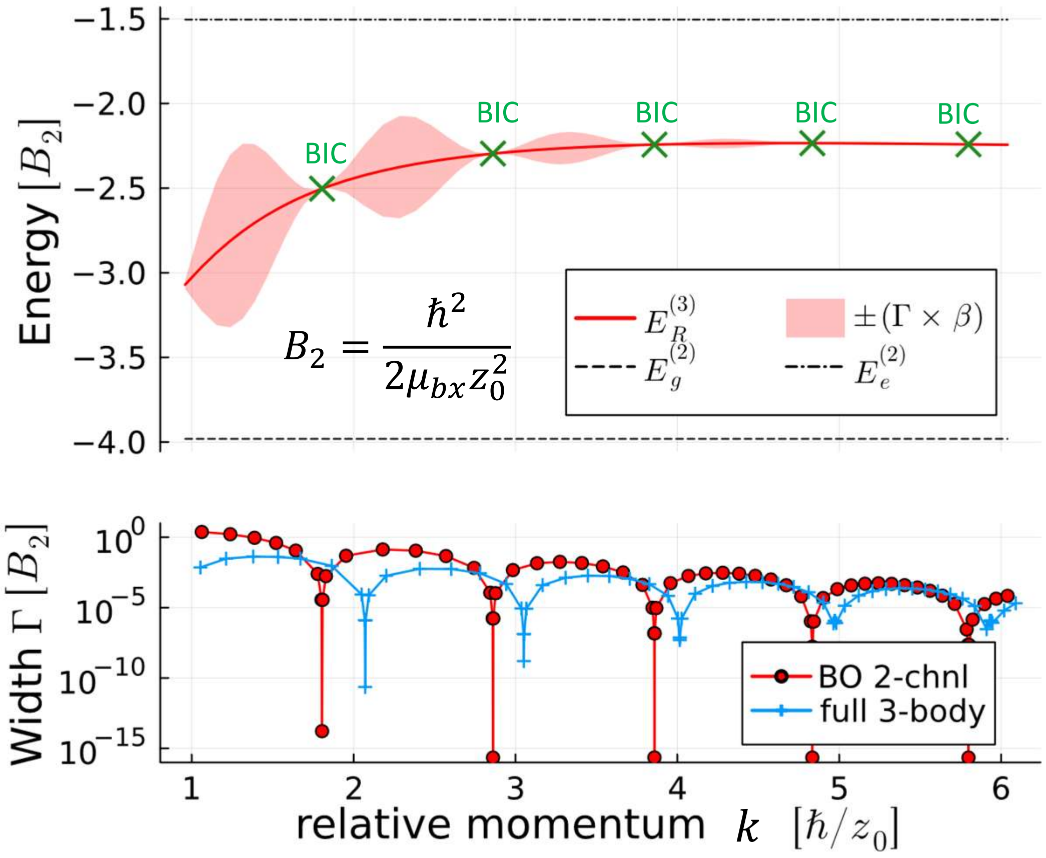
3. Example in 1D : Born-Oppenheimer



$$\Gamma = 2\pi \left| \langle \Phi_{o,k} | H_{o,c} | \Phi_{c,n} \rangle \right|^2$$

Born-Oppenheimer ⟷ full 3-body calculation

3. Example in 1D : Results

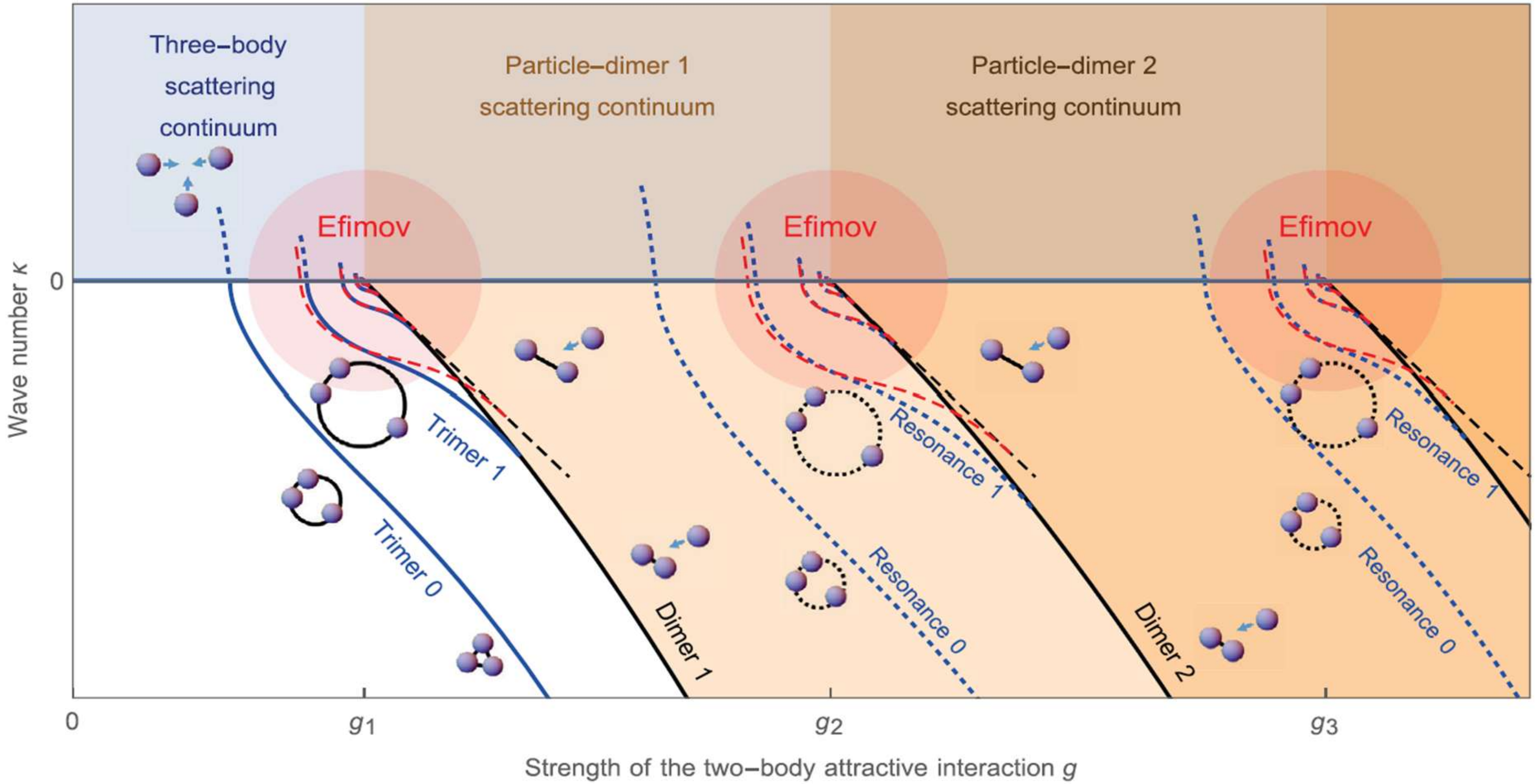


- Damped-oscillatory behaviour
- configurations with zero width: stabilised resonances (BIC)
- Good agreement between 2-channel BO and full 3-body calculation

Here: variation of k via the mass ratio $\frac{1}{20} \leq \beta \leq 20$ $\quad$ $k \propto \sqrt{1+\beta}$

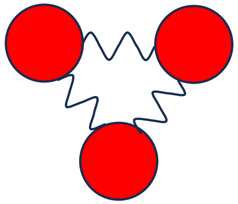
4. Example in 3D : the Efimov scenario

Efimov states are usually resonances



4. Example in 3D : the Efimov scenario

3D



$\beta = 1$

Three identical bosons in 3D

Two-body interactions:

Two-channel contact model:

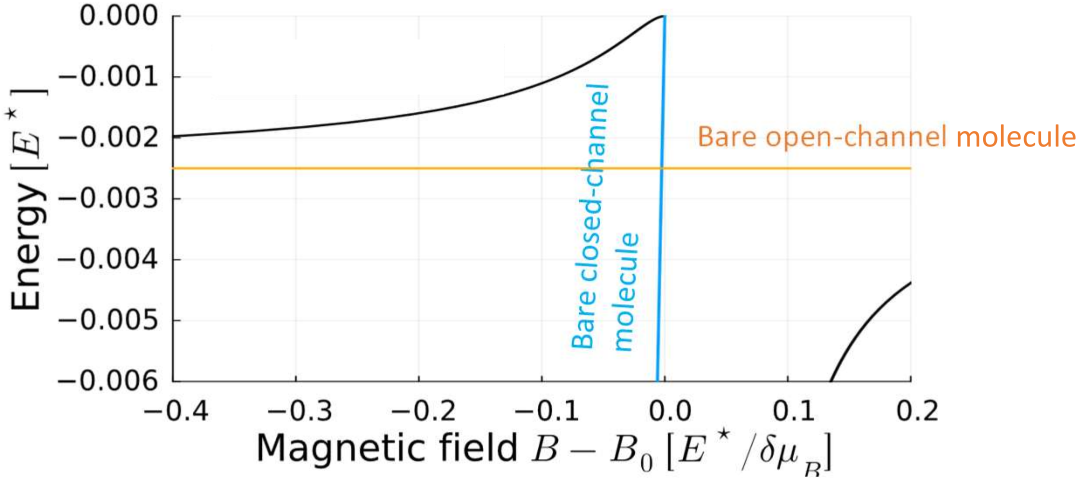
a_{bg} : Background scattering length

$R^{\star}$: Strength of the resonance

e_b: Shifted energy of the bare bound state

B : external magnetic field

Two-body solution



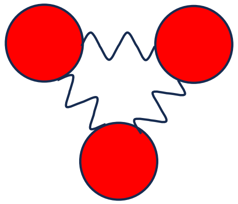
Momentum-dependent scattering length

$$a(k) = a_{\mathrm{bg}} - \frac{1}{R^\star(e_b - k^2)} \qquad e_b = \delta\mu_B(\mathrm{B} - \mathrm{B}_0)$$

Simple model of ultracold atoms close to a Feshbach resonance

4. Example in 3D : the Efimov scenario

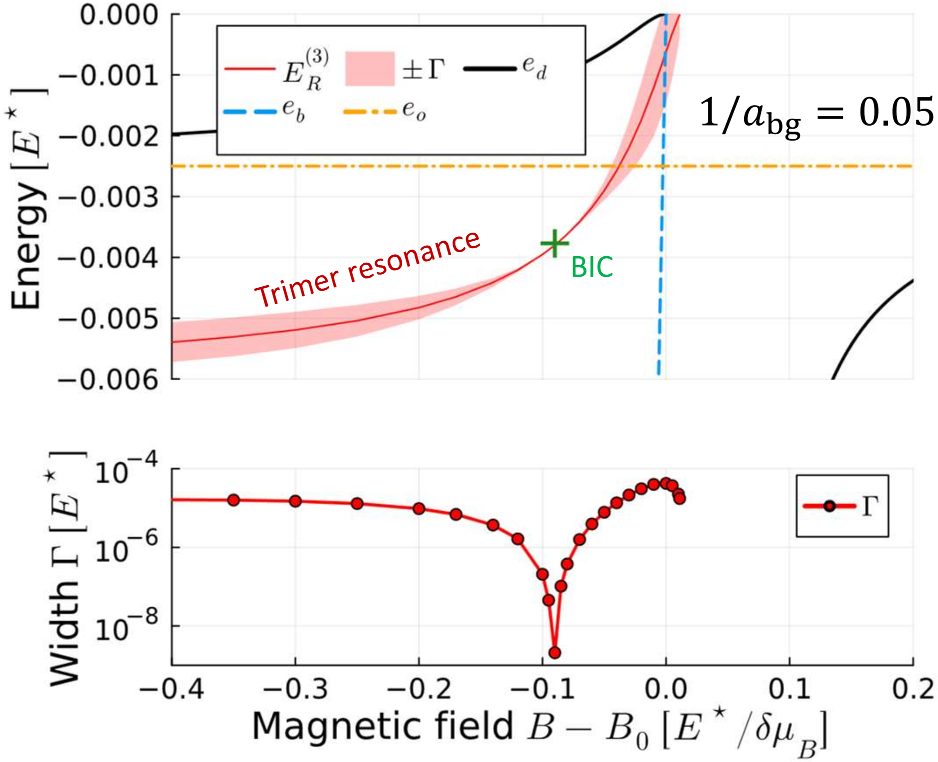
3D



$\beta = 1$

Three identical bosons in 3D

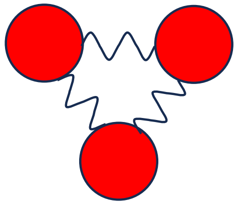
Three-body solution via STM equation



➢ Three-body BIC in Efimov scenario via tuning of an external magnetic field

4. Example in 3D : the Efimov scenario

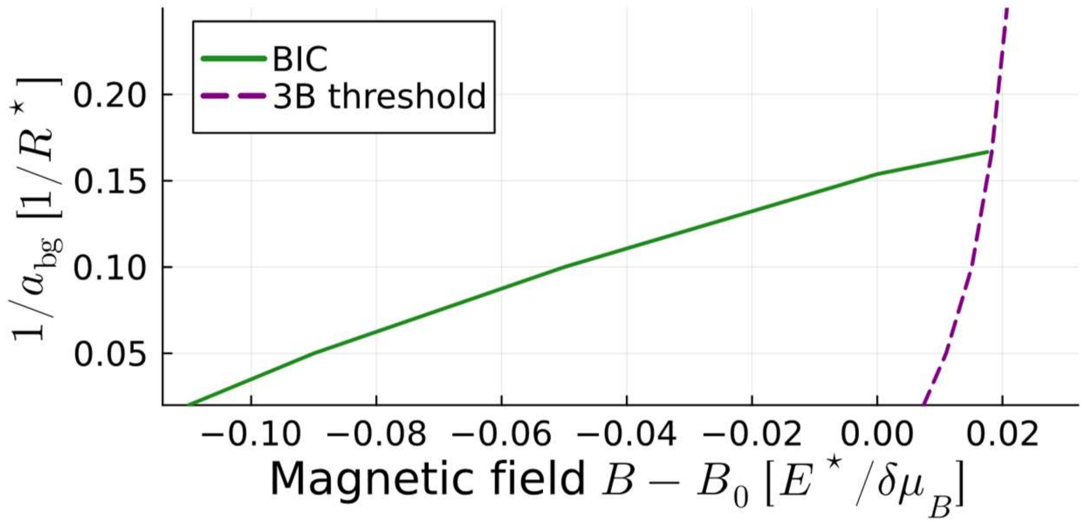
3D



$\beta = 1$

Three identical bosons in 3D

Several parameters can be tuned to reach a BIC



5. Conclusion

Summary:

- Two-channel picture explains BIC in few-body systems
- Stabilisation of resonances by continuously tuning of system parameters
- General phenomenon in different dimensions
- BIC in the Efimov scenario via tuning of an external magnetic field

Outlook:

- Universality?
- Other systems and tuneable parameters?

arxiv:2503.02037
pascal.naidon@riken.jp
lucas.happ@riken.jp