Hydrodynamic attractors- Fluid Dynamic beyond equilibrium

Toshali Mitra

INSTITUT FÜR THEORETISCHE PHYSIK

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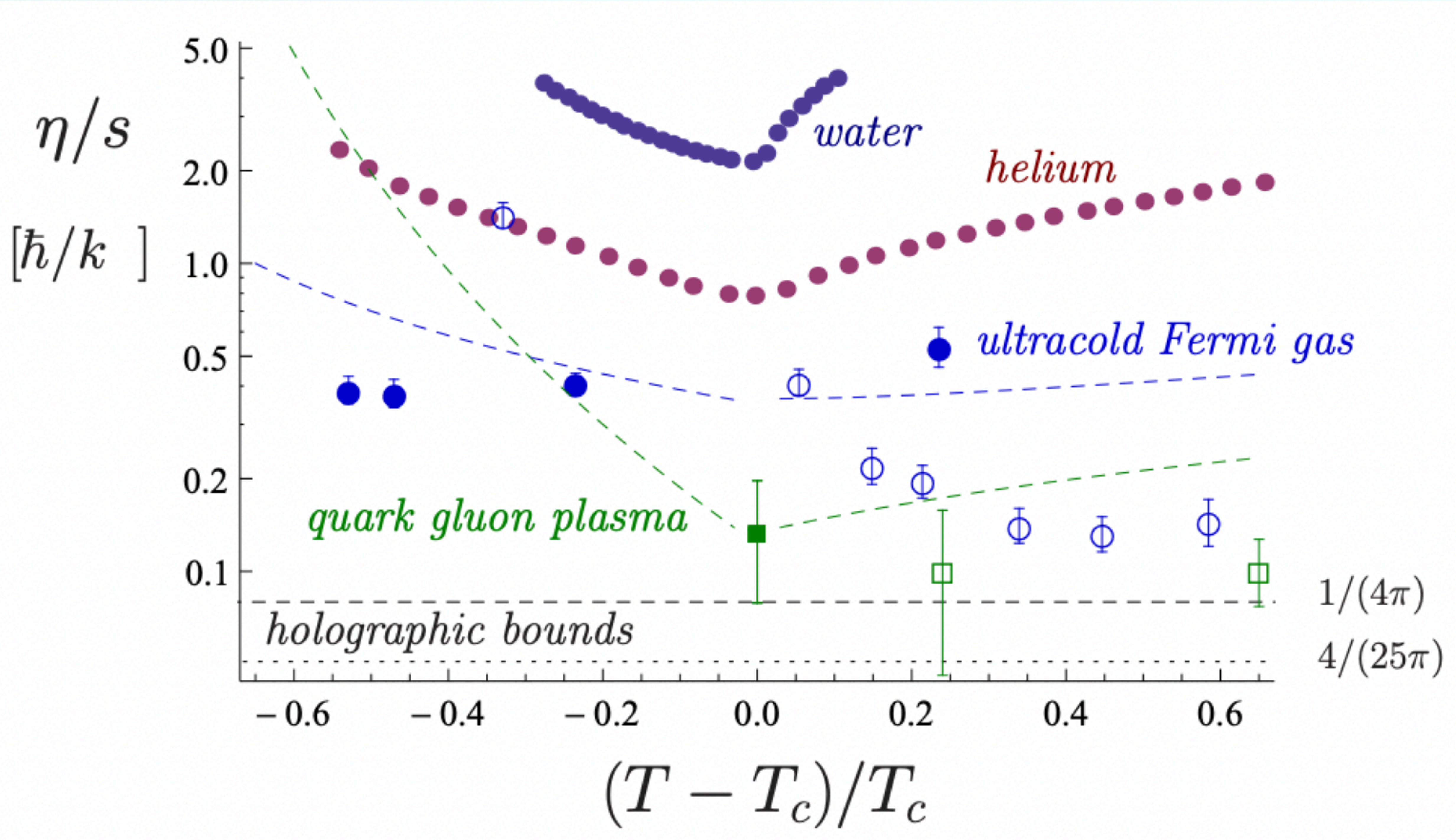
Universality in strongly-interacting system from QCD to Atoms

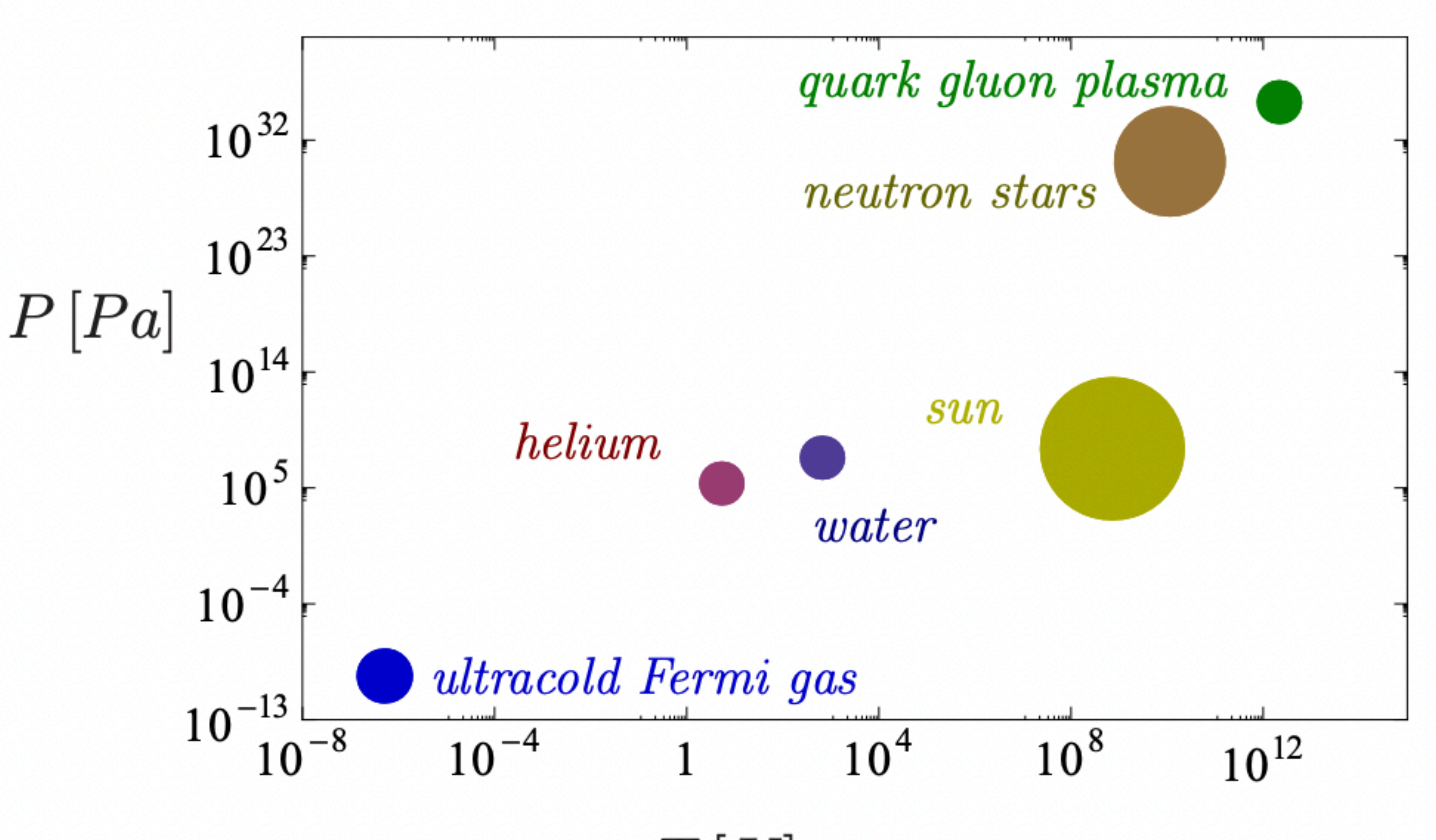
Strongly interacting quantum systems

- Strongly correlated quantum fluids.
  - Quark-Gluon Plasma
  - Ultracold atomic Fermi gas

- Similar Hydro flow
- Low ratio of η/s

- Described by holographic duality





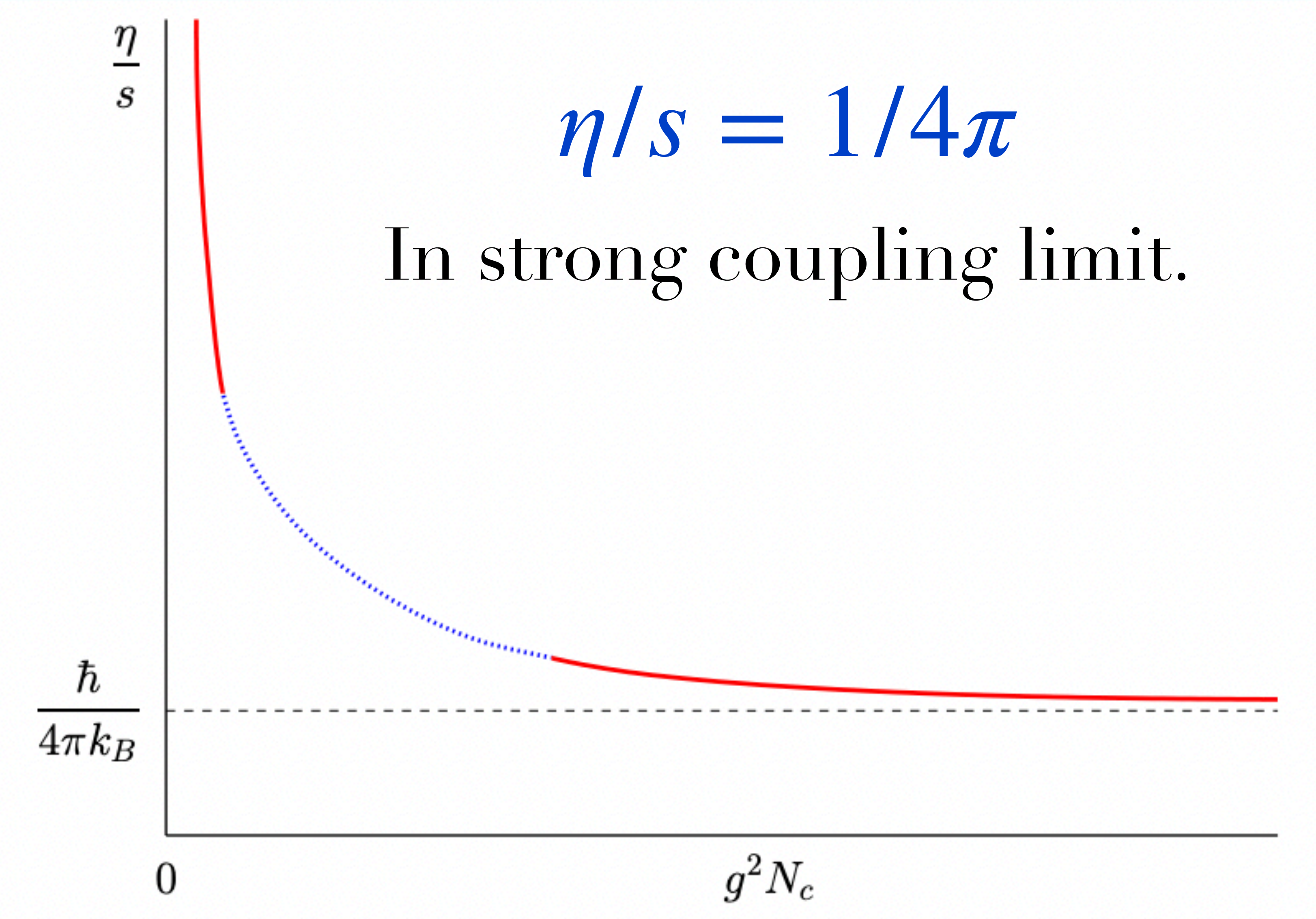
Thomas et al (2012)

Thomas et al (2012)

Holographic duality

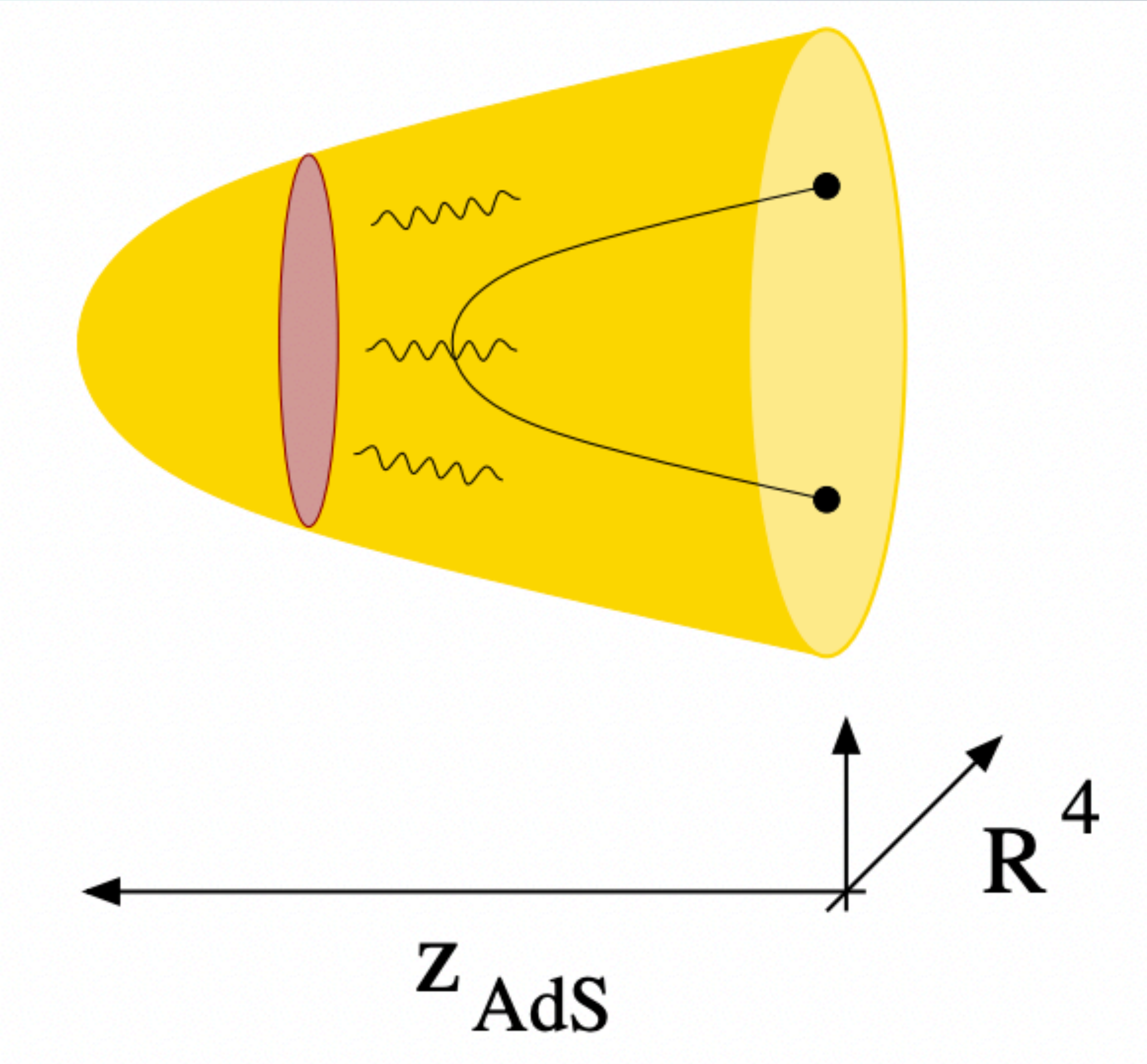
- Strong-weak duality

Strongly coupled field theory in R^4	⟷	Weakly coupled Gravity on AdS_5 black hole
CFT temperature	⟷	Hawking temperature.
CFT entropy and shear viscosity	⟷	Area of event horizon.



Thomas Schäfer Lecture Schleching 2025

AdS/CFT



- AdS/CFT correspondence predicts a lower bound

$$\eta/s \geq 1/4\pi$$

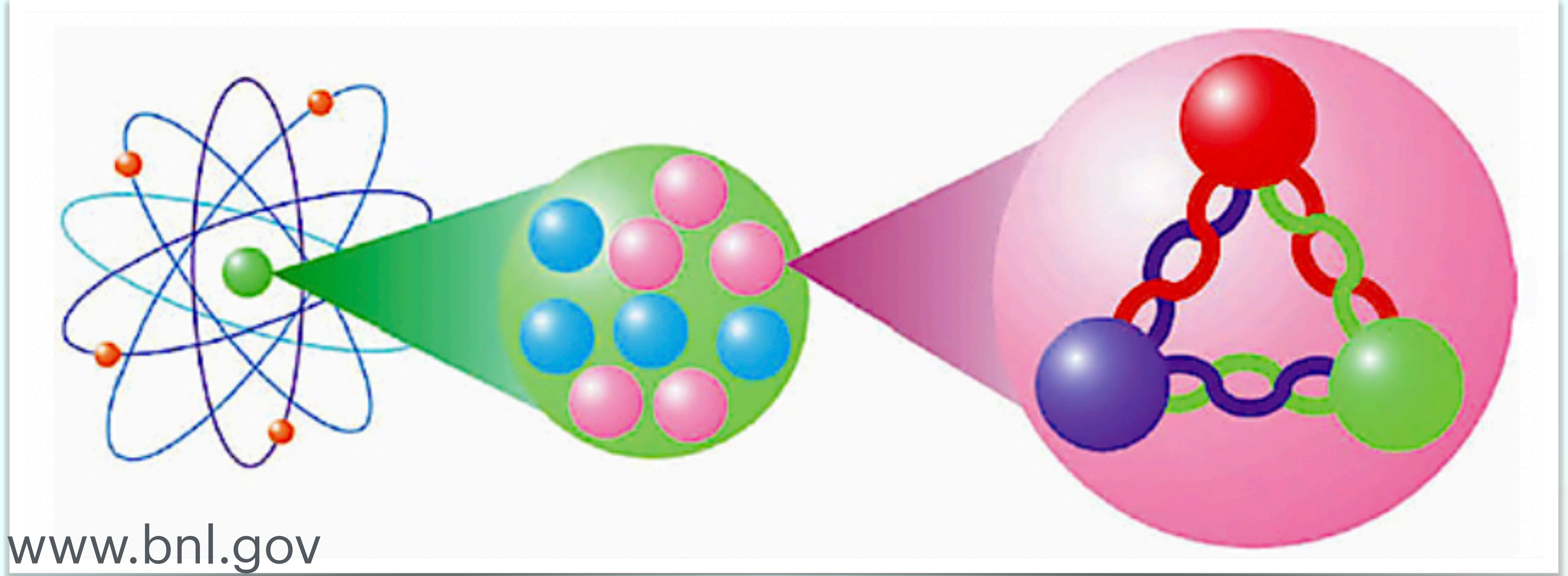
Kovtun, Son, Starinets 2005

Outline

- Strongly interacting system and hydro behaviour
- Hydrodynamics as universal effective theory
- Hydrodynamics Far from equilibrium in heavy ion collision
- Attractors in cold atoms
- Attractors and spontaneous symmetry breaking
- Summary and outlook

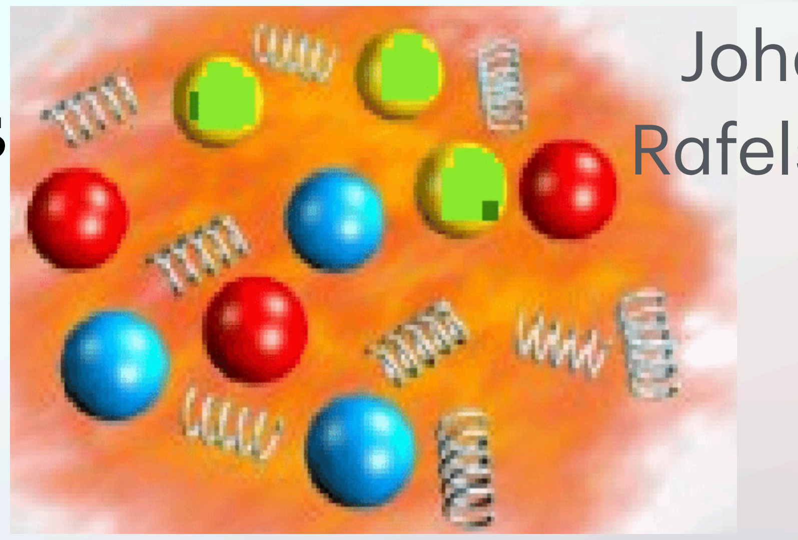
Quark-Gluon Plasma

- Fundamental particles of QCD
  - Quarks - Proton and Neutron
  - Gluons- Mediator of strong interaction.



www.bnl.gov

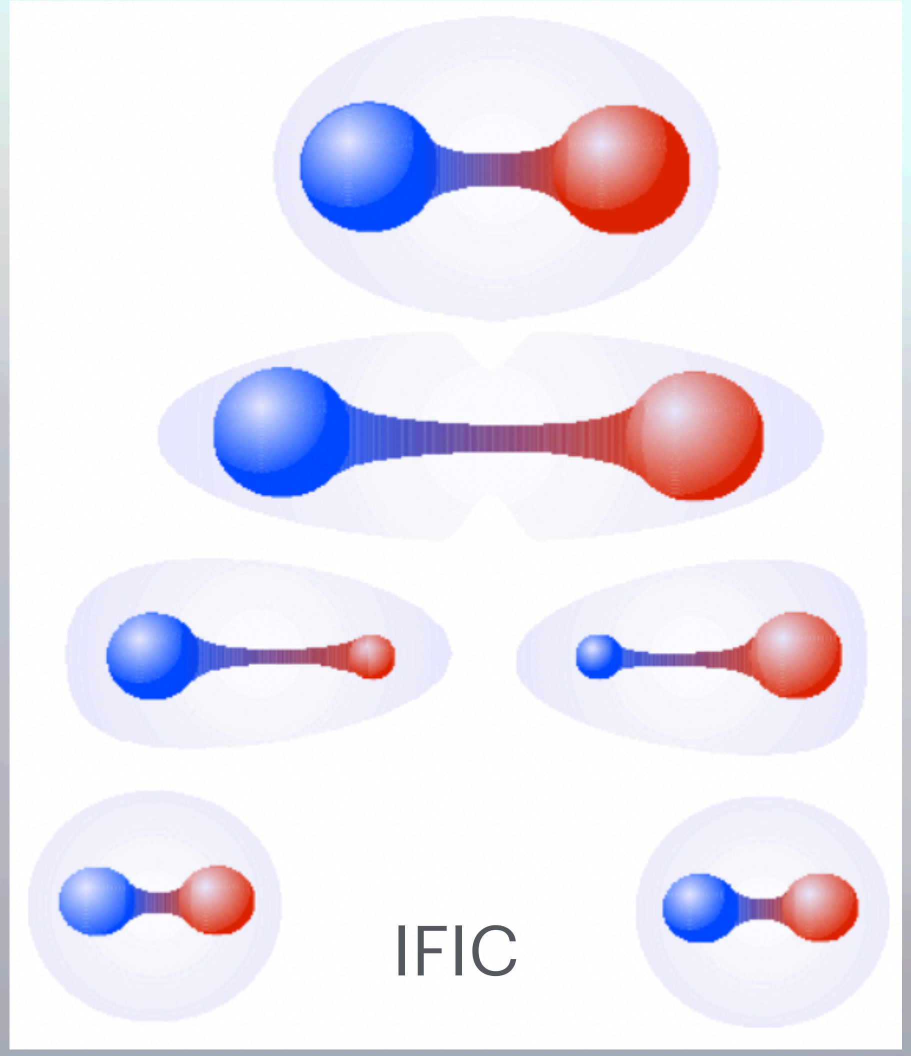
Deconfined quarks and gluons

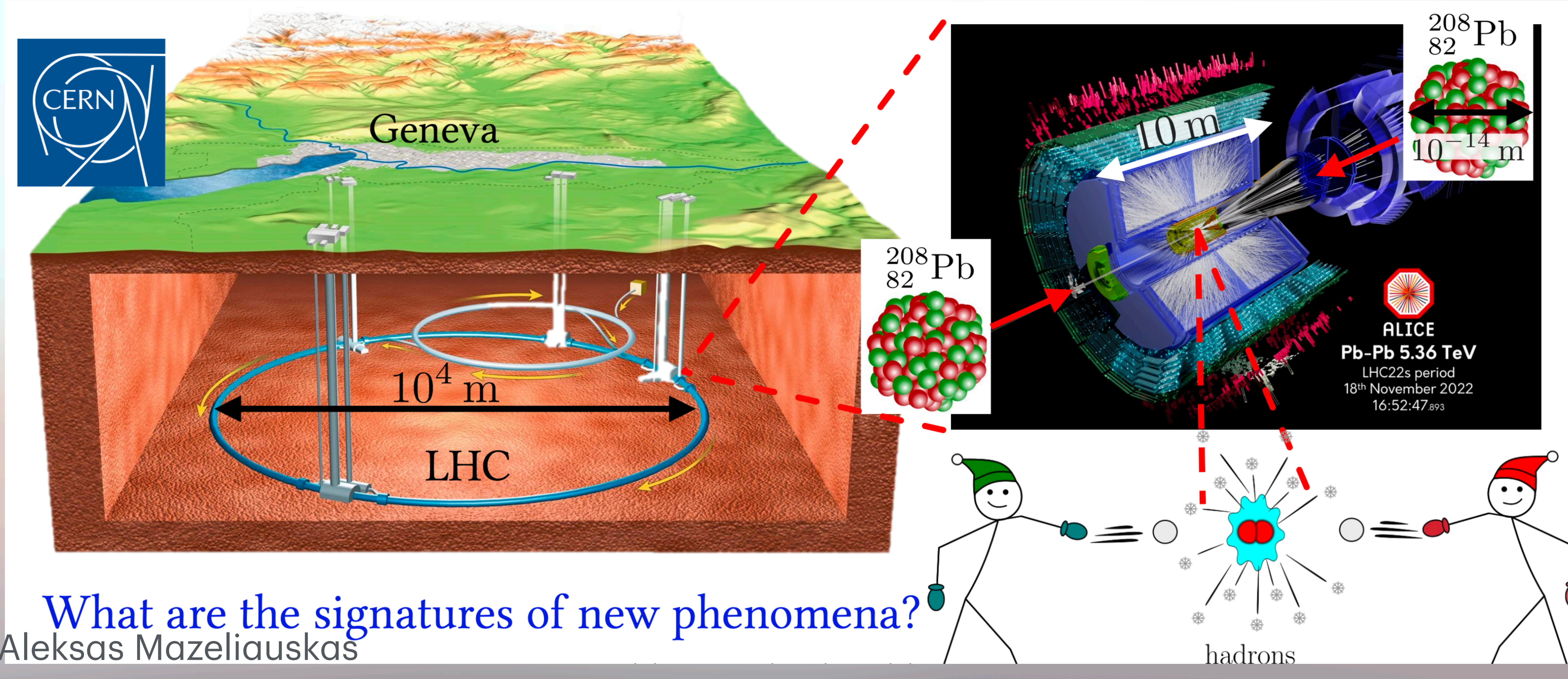


Johann Rafelski'15

- RHIC - 2000 and 2010 Energy Frontier at LHC

Confinement

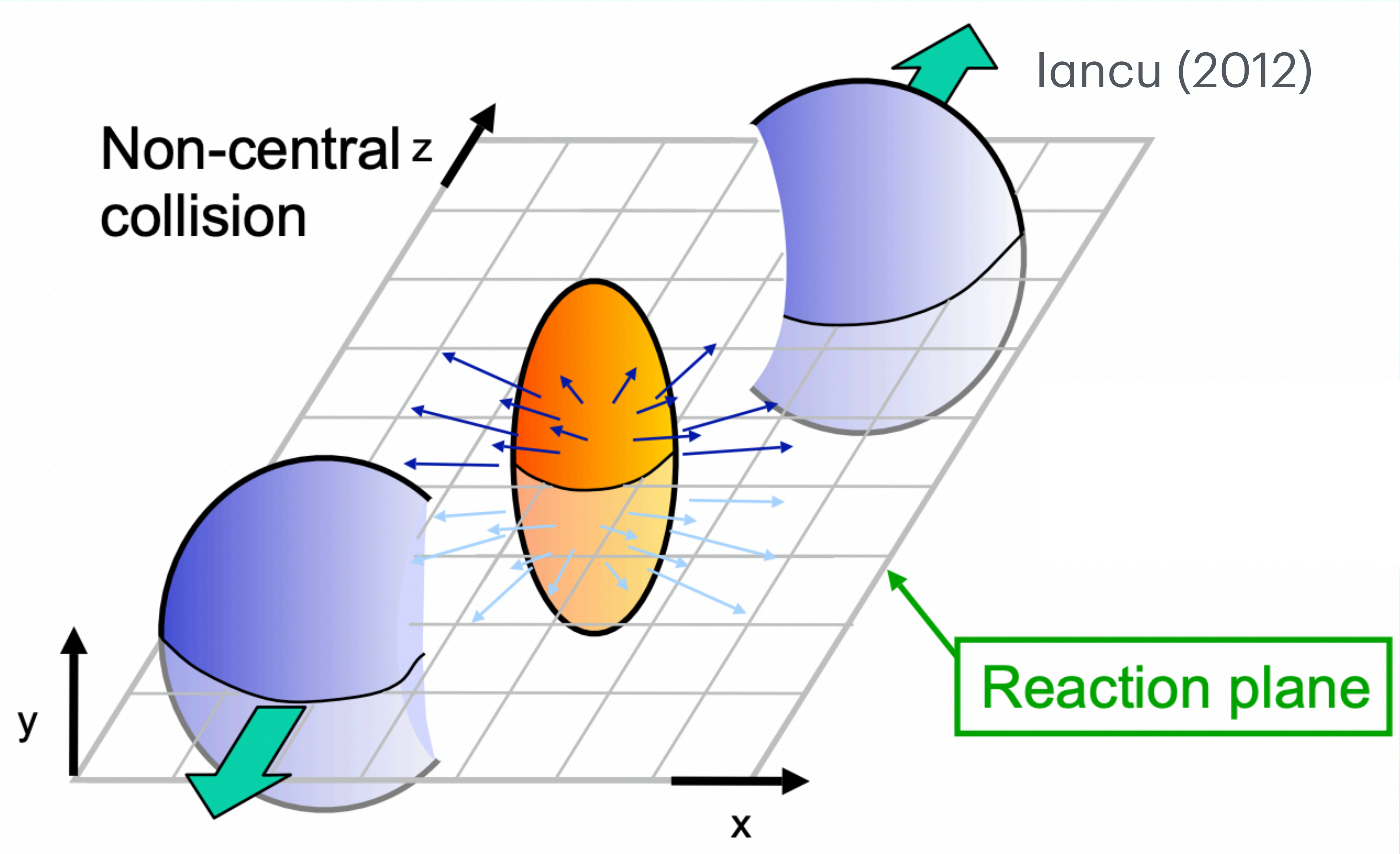




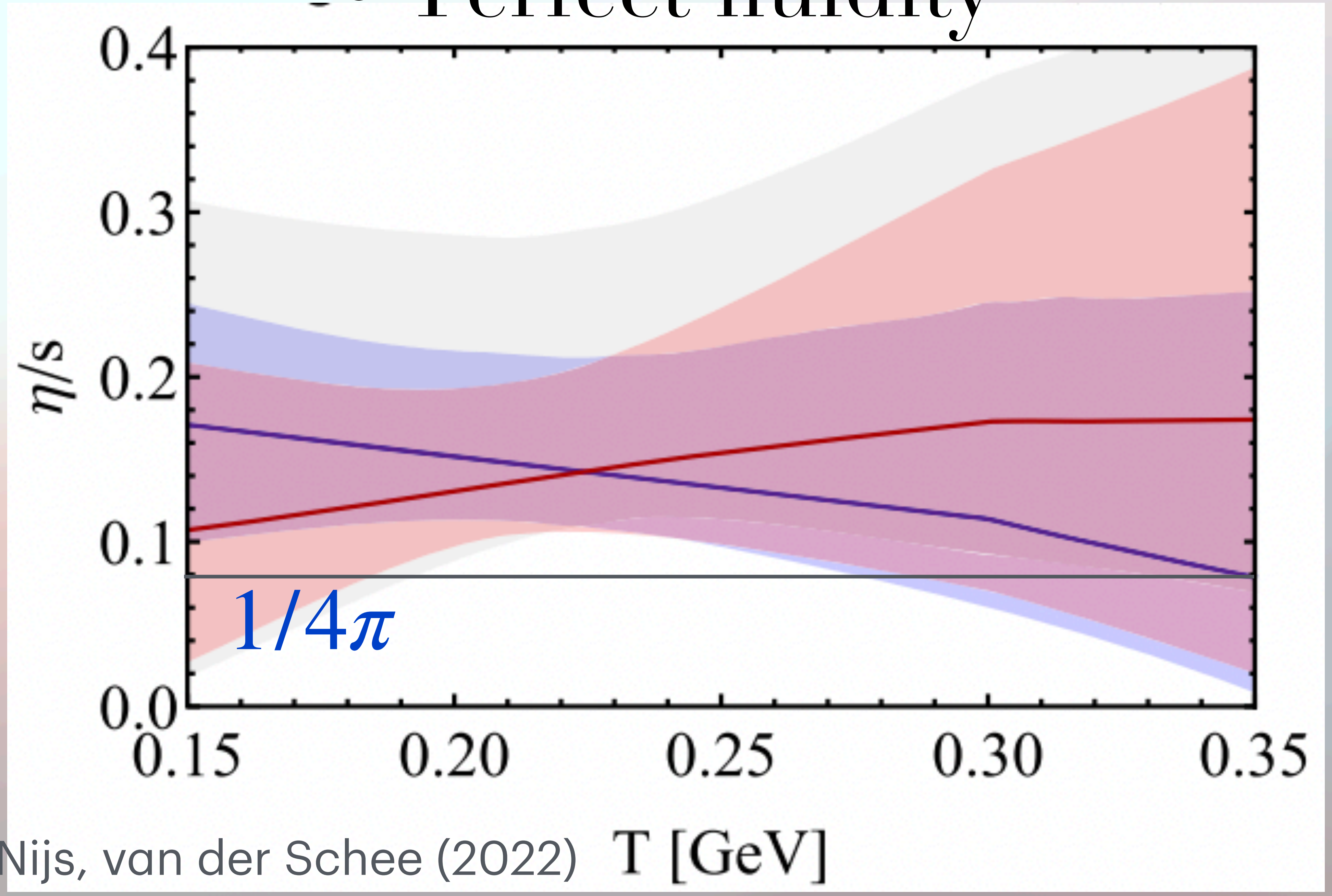
Aleksas Mazeliauskas

Emergence of hydrodynamic

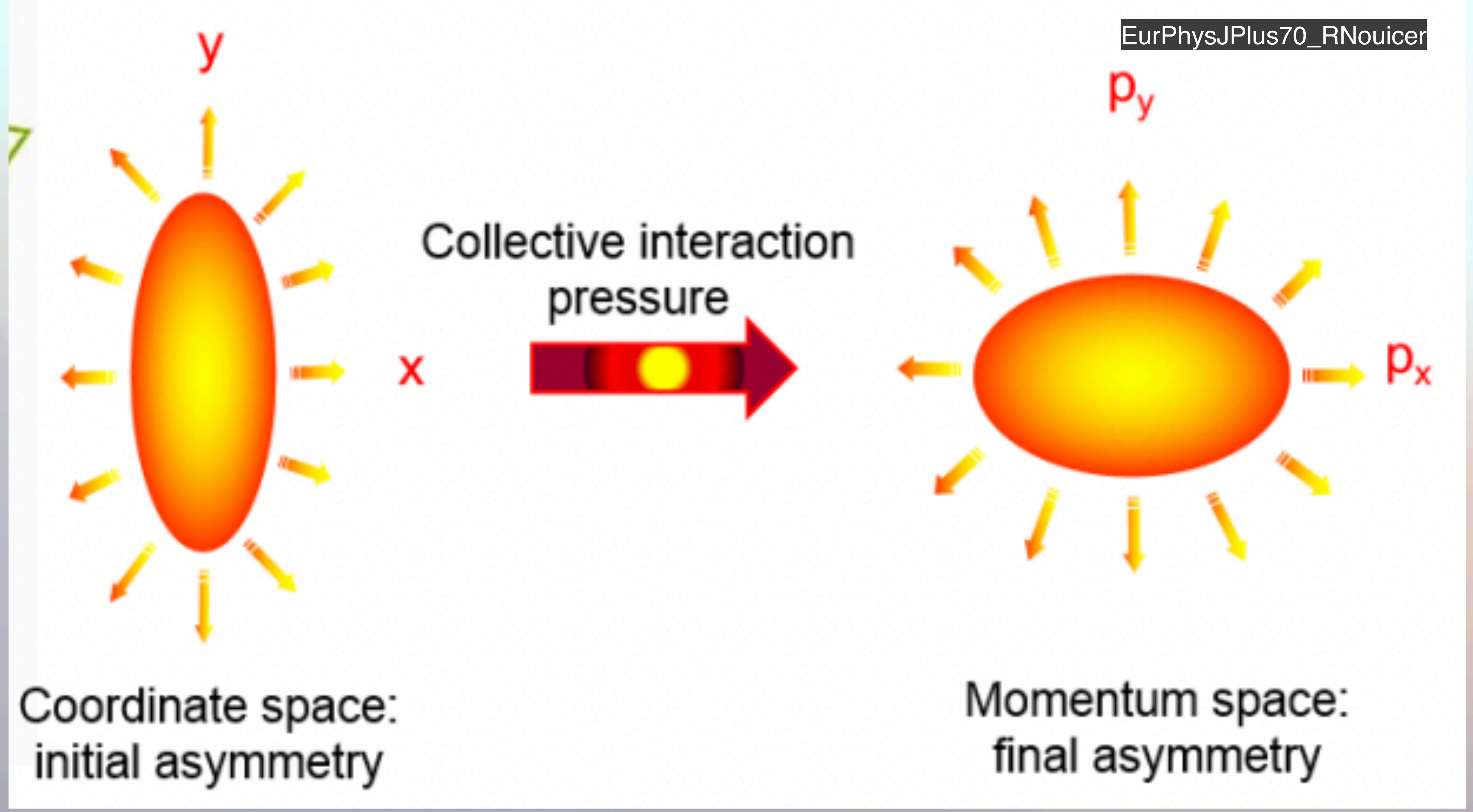
- QGP-
  - Perfect fluid like behaviour
  - Strong interaction
- Effective theory hydrodynamic



- Perfect fluidity



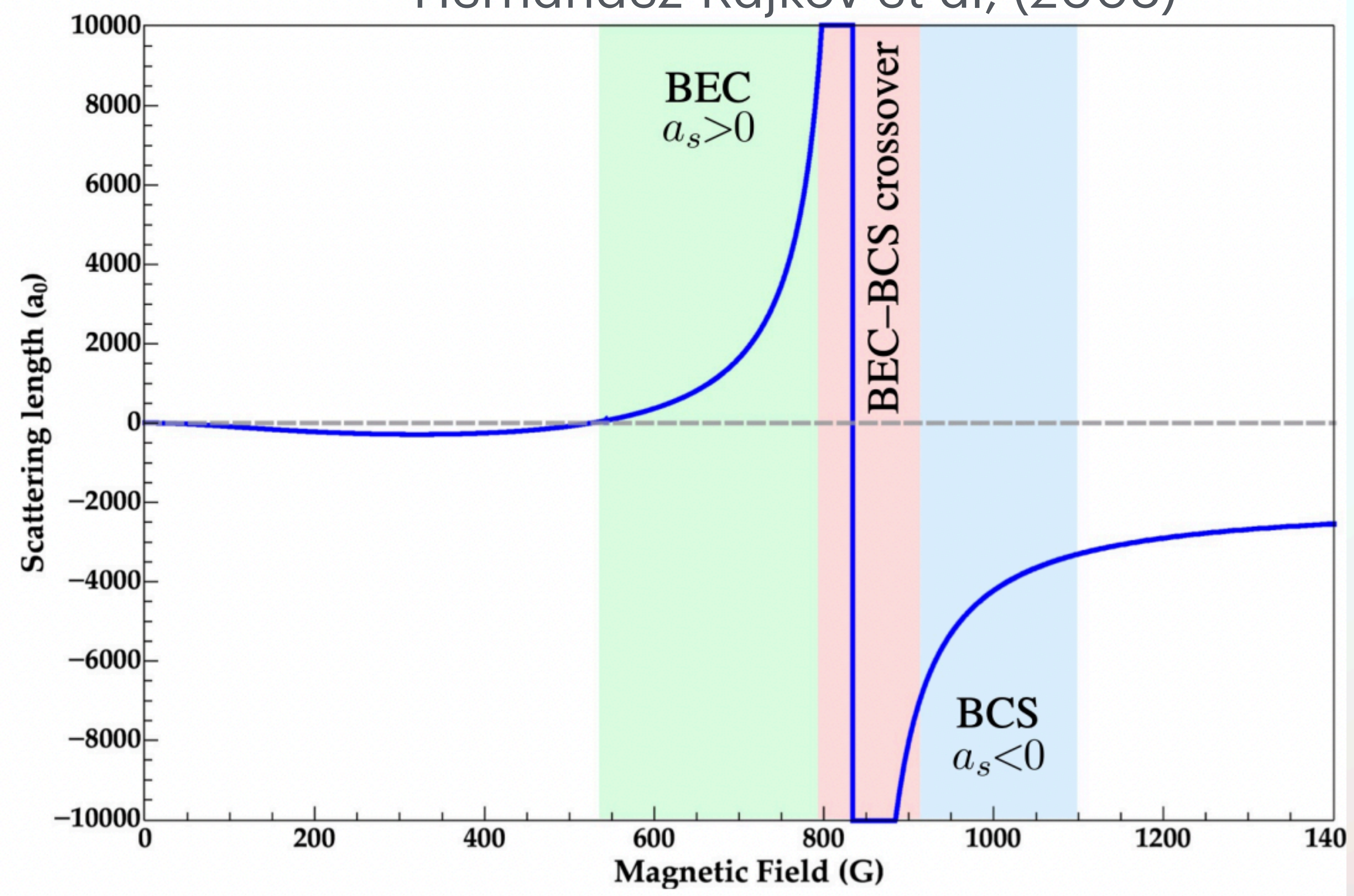
Nijs, van der Schee (2022)



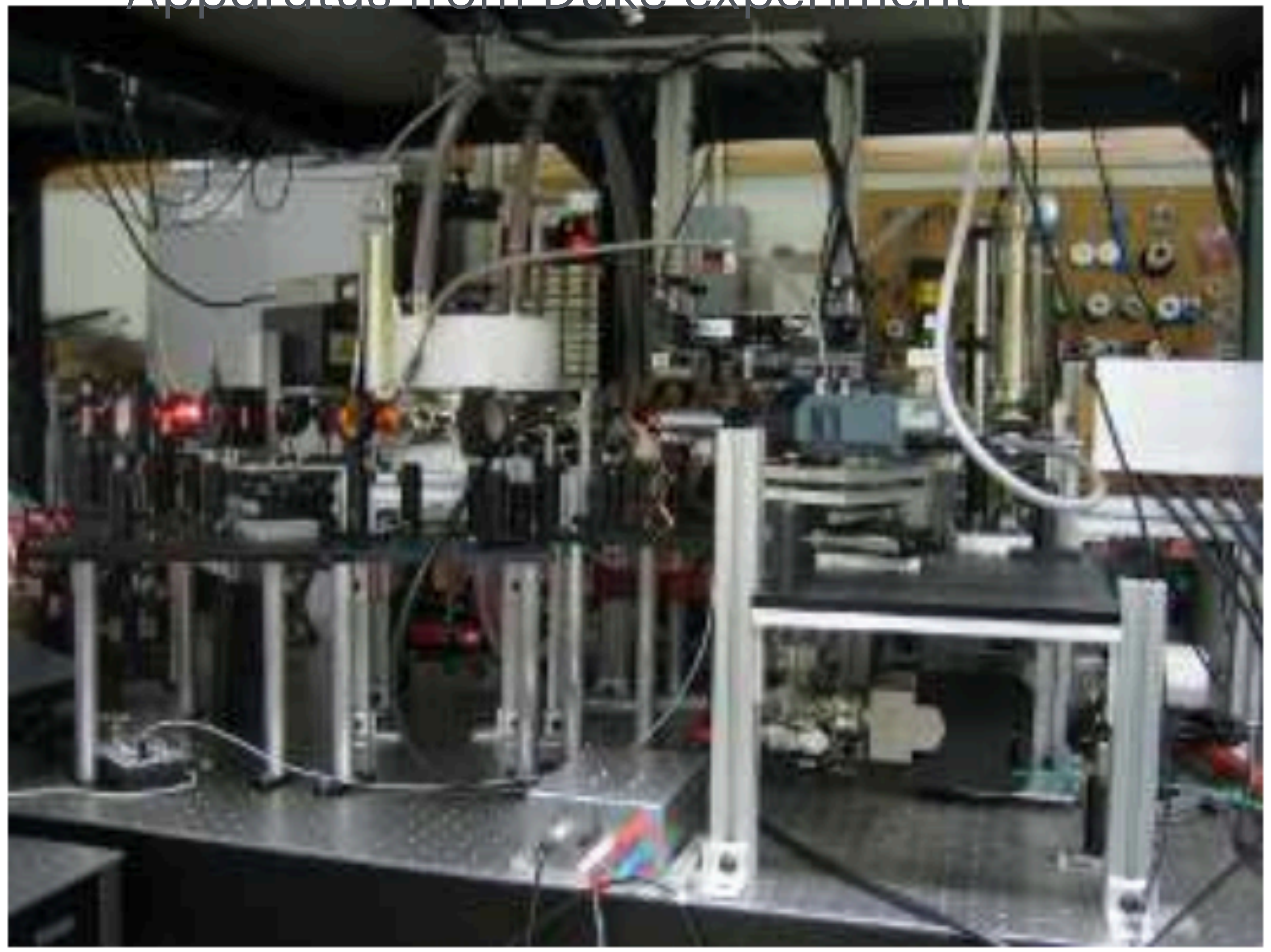
Ultracold atoms

- Table-top experiment
  - Clouds of dilute fermions
  - Deformed by harmonic trapping fields
  - Tunable scattering length $a(G)$

Hernández-Rajkov et al, (2008)

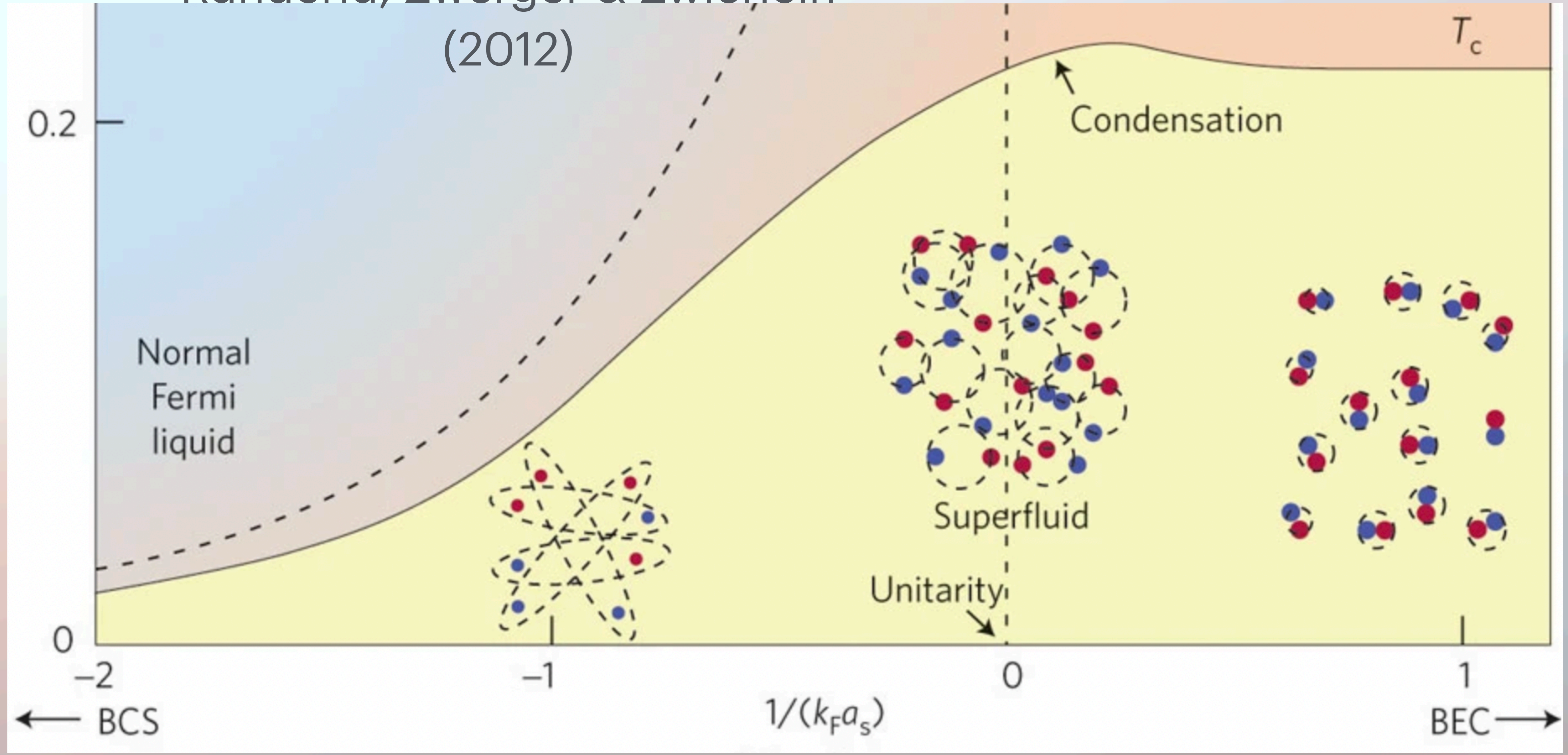


Apparatus from Duke experiment



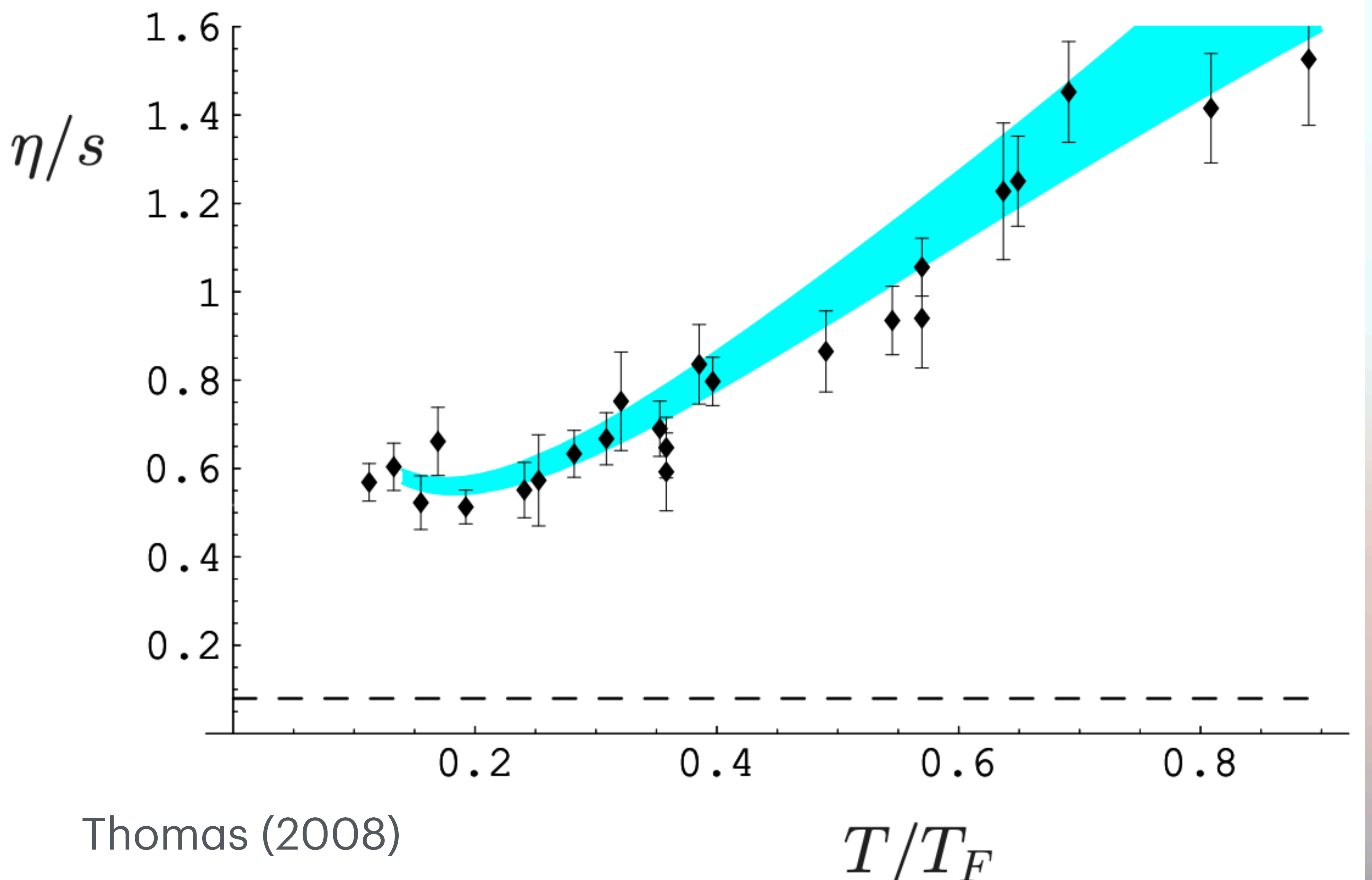
Thomas et al (2012)

Randeria, Zwerger & Zwierlein (2012)



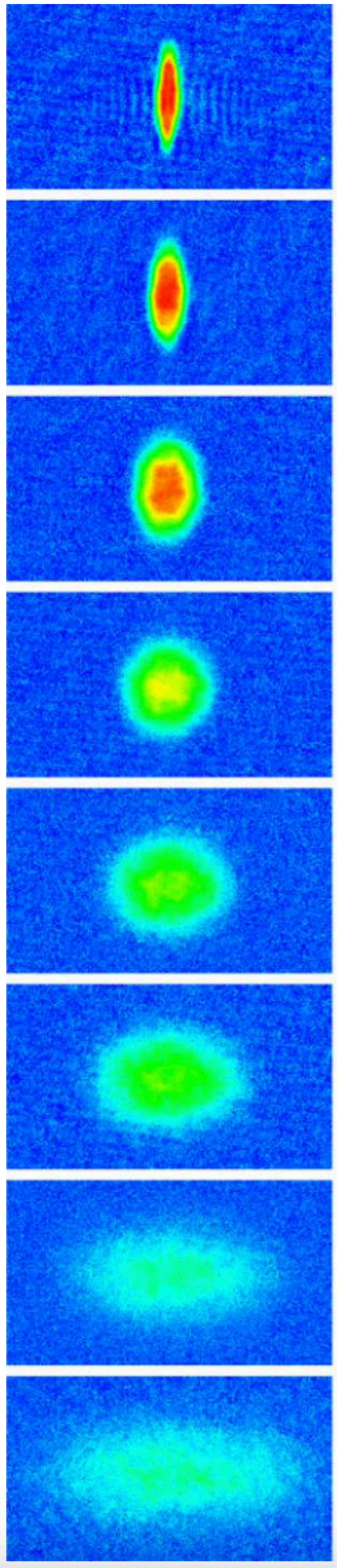
Fluidity in cold atoms

- Elliptic flow
  - Released from Cigar-shaped optical trap.
  - Large pressure gradient.
  - Low ratio of η/s in non-relativistic case.

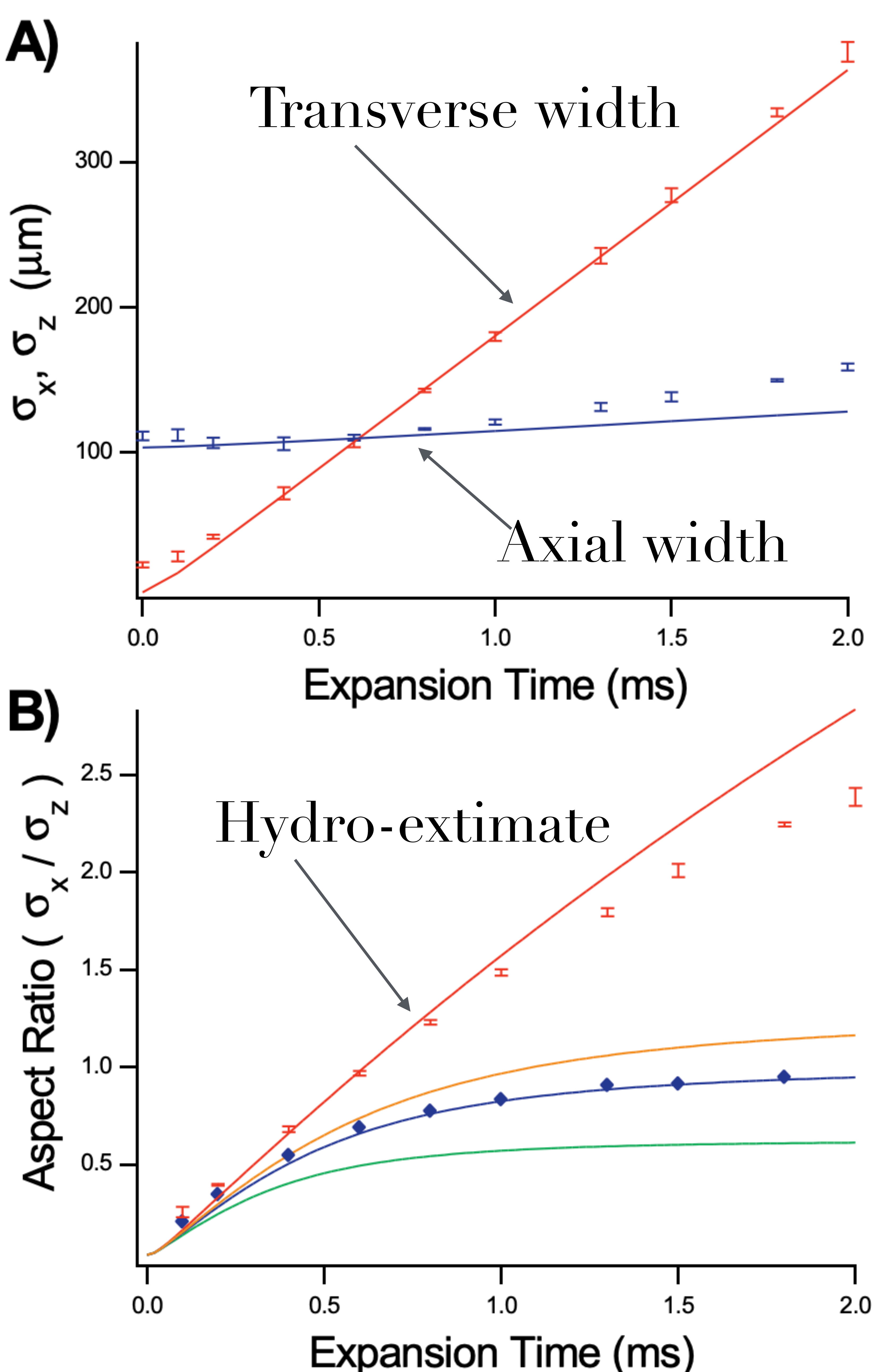


Thomas (2008)

O' Hara et al (2002) ^6Li



O' Hara et al (2002) ^6Li

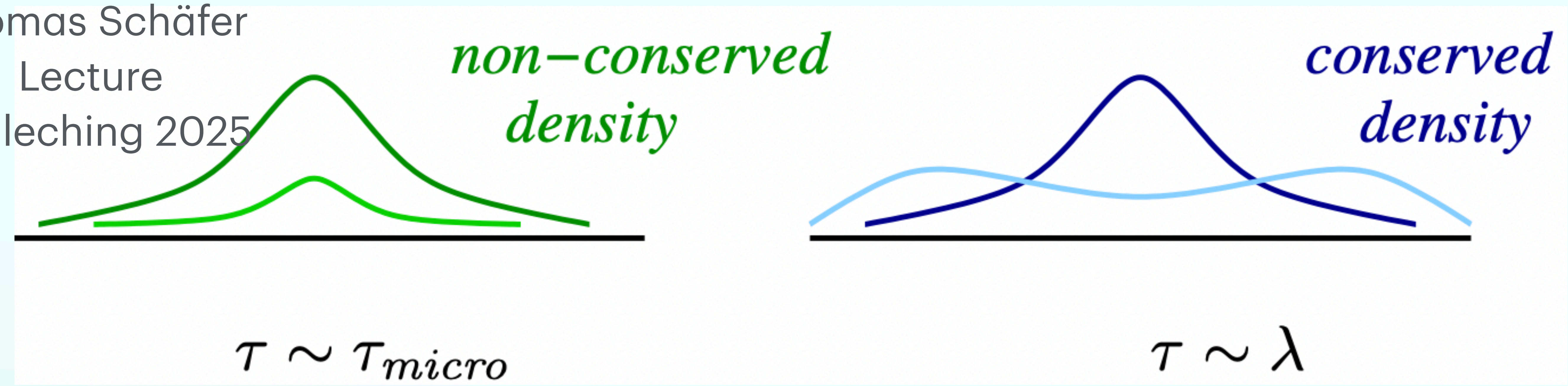


Hydrodynamics

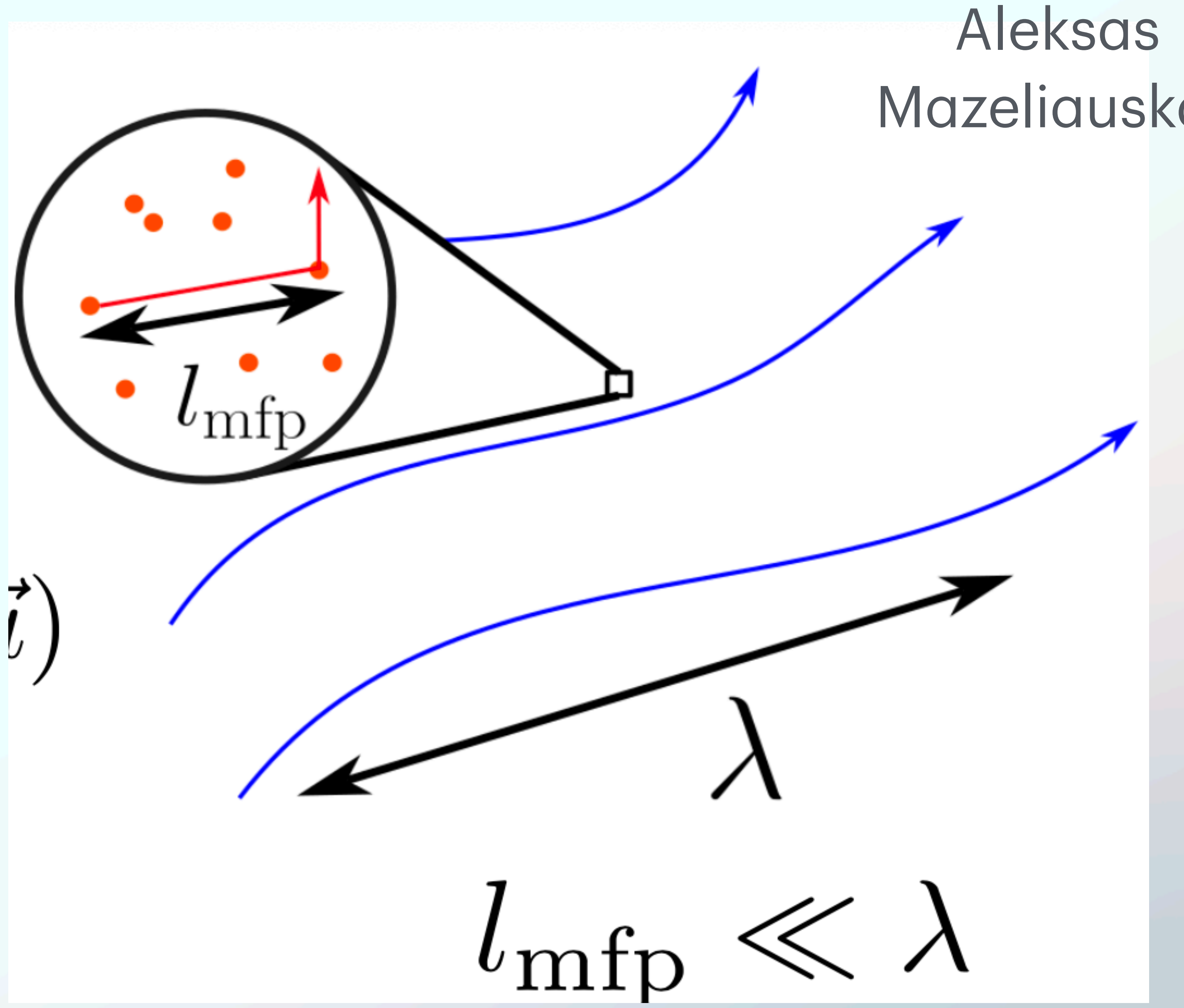
Hydrodynamics

- Effective description of many-body physics in the long wavelength limit.

Thomas Schäfer Lecture Schleching 2025

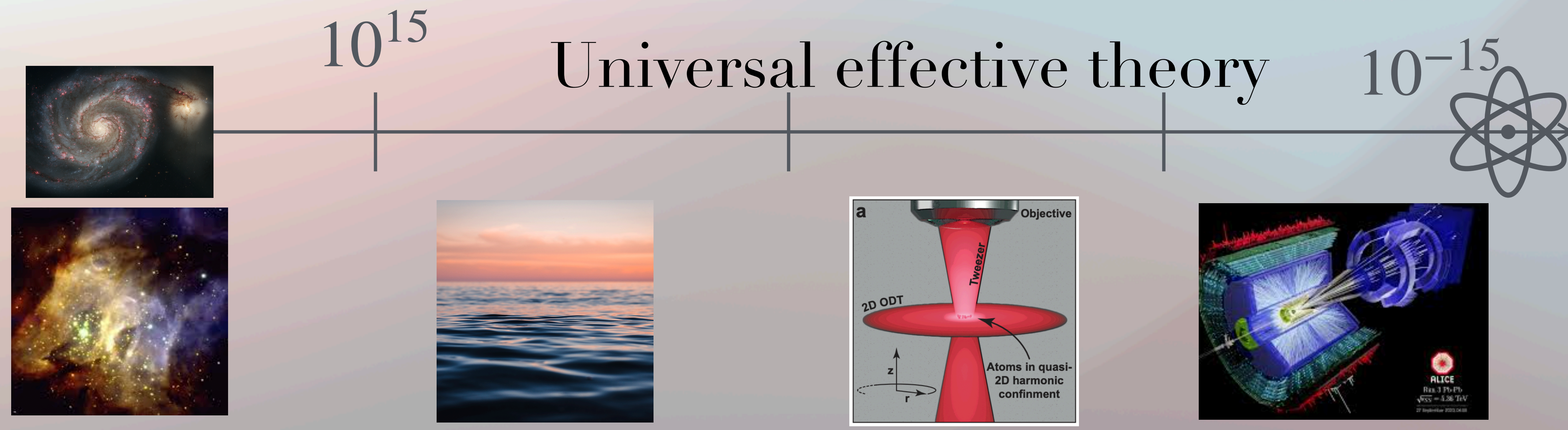


Aleksas Mazeliauskas



- Knudsen number: $Kn = \frac{l_{mfp}}{\lambda}$ $Kn << 1$

- Applicable to non-equilibrium systems with large gradient expansion.
  - Regime of applicability?



Relativistic Hydrodynamics

- Macroscopic quantities of interest: $\nabla_\mu T^{\mu\nu} = 0$

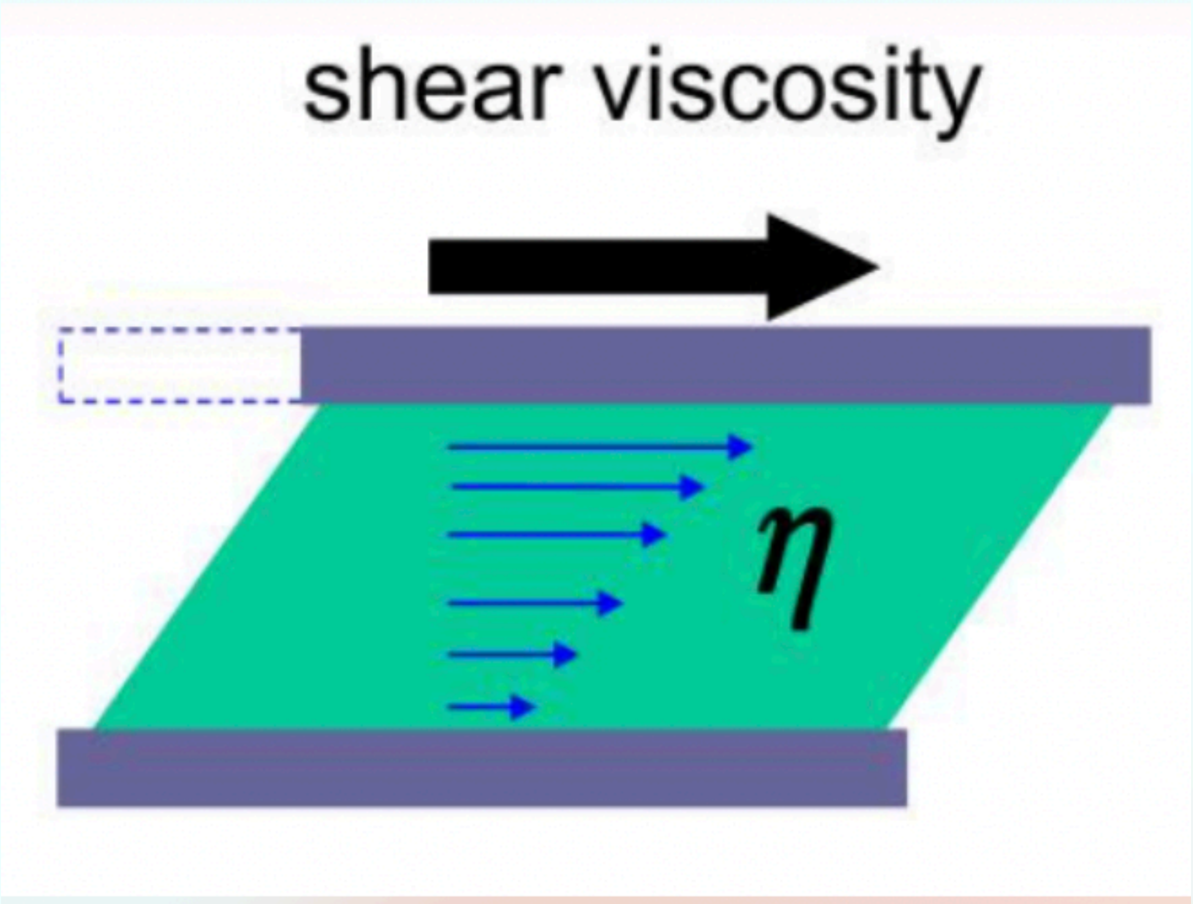
- Constitutive relation: $T^{\mu\nu} = \underbrace{(\varepsilon + P)u^\mu u^\nu + Pg^{\mu\nu}}_{Perfect\ \ fluid} + \Pi^{\mu\nu}$

$$P = P(\varepsilon)$$

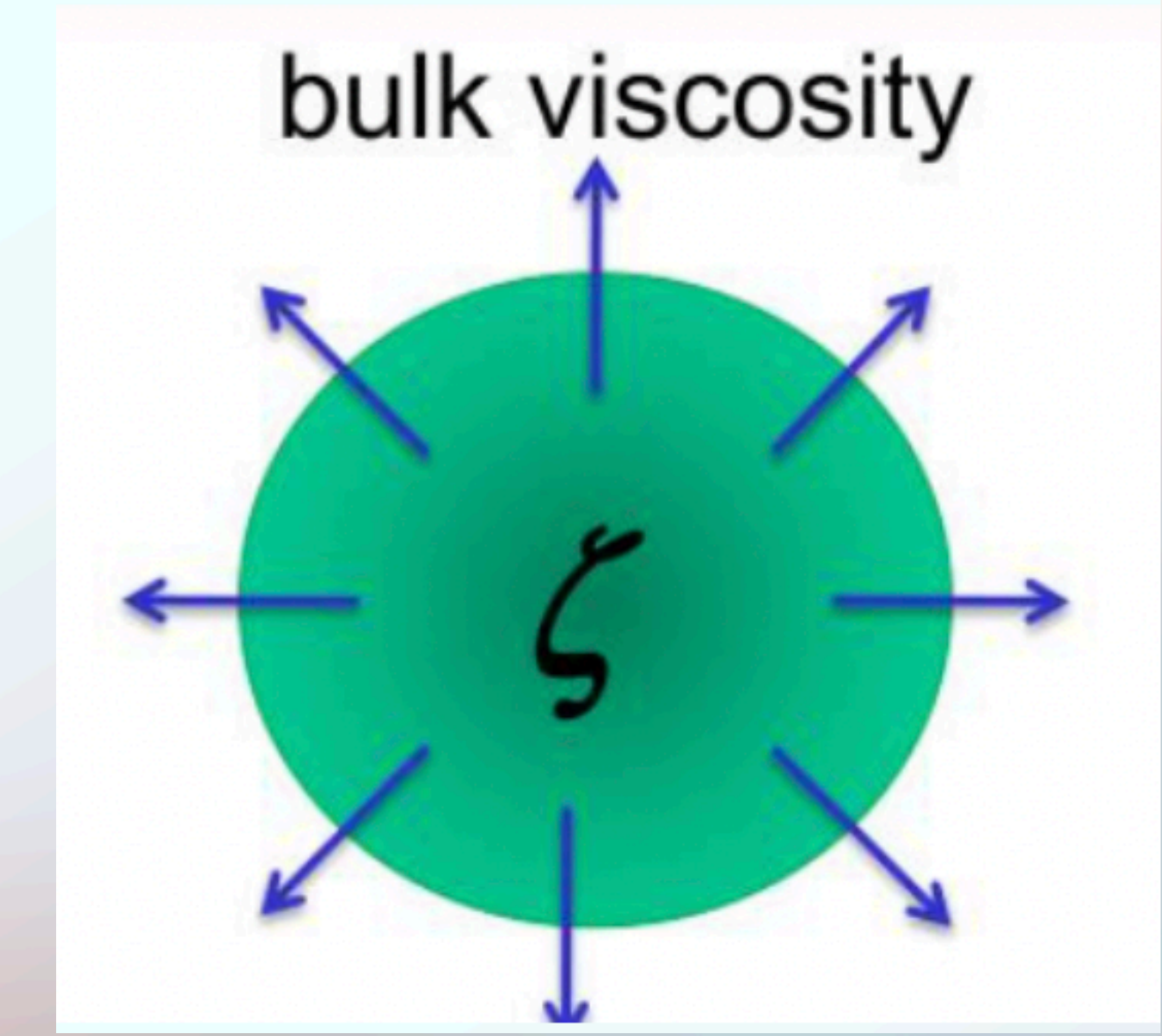
- Constitutive relation for dissipative correction:

$$\Pi^{\mu\nu} = \underbrace{-\eta\sigma^{\mu\nu} - \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha}_{1st\ order\ correction} + higher\ order\ correction$$

$$\sigma_{\mu\nu} \propto \nabla_\mu u_\nu$$



Song and Heinz (QM 2009)



How to describe Hydro beyond equilibrium?

Hydrodynamic Attractors!!

Attractors in Heavy-ion collision

Attractors in ultracold atoms

Attractors in Heavy-ion collision

Stages of heavy-ion collisions

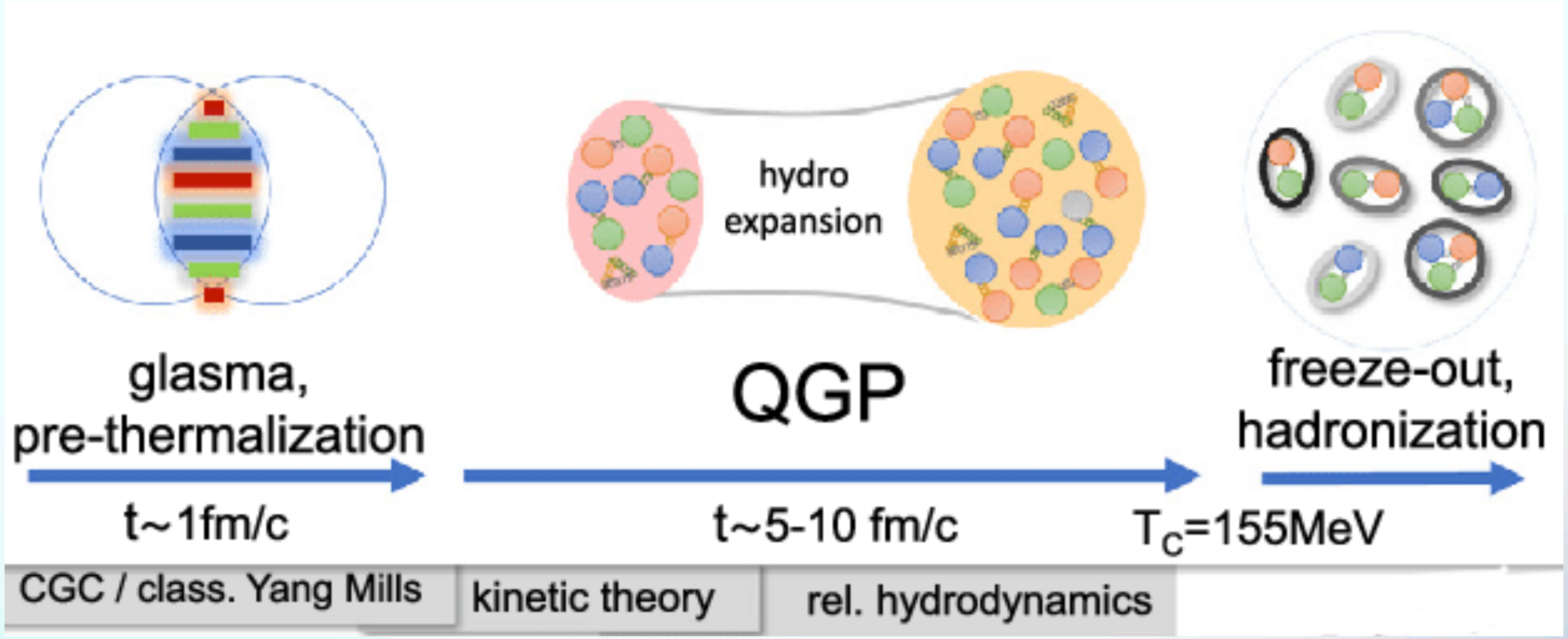


Figure taken from Heavy quarkonium in extreme conditions by Alexander Rothkopf.

Incoming nuclei ⟶ Pre-thermalisation ⟶ QGP ⟶ Hadronisation ⟶ Freeze-out

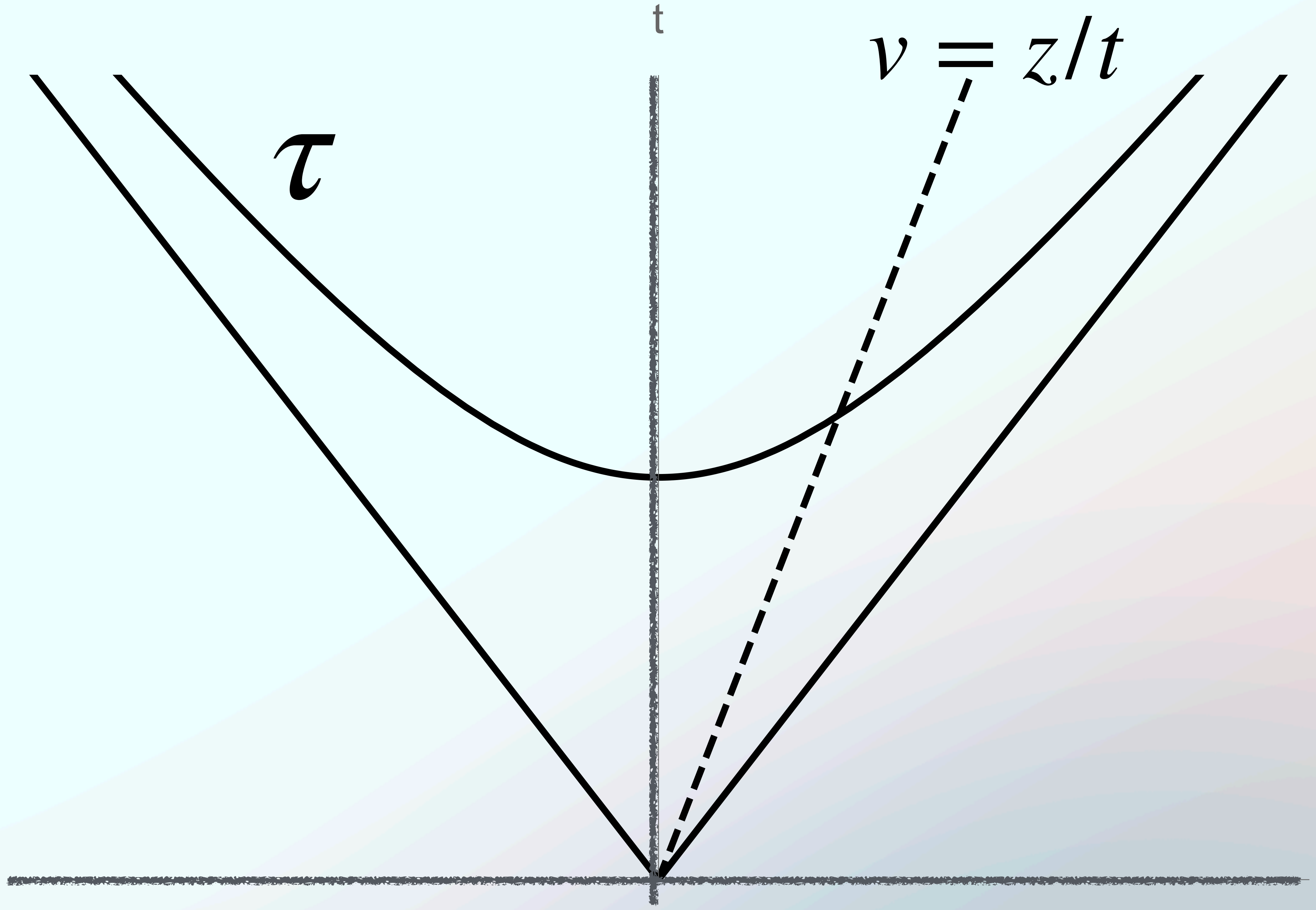
Bjorken Flow: Longitudinal expansion

- $1 - D$ model to describe nuclei-nuclei collisions.
  - x, y independent
  - Boost invariant
- Proper time: $\tau = \sqrt{t^2 - z^2}$
- Energy-momentum tensor:

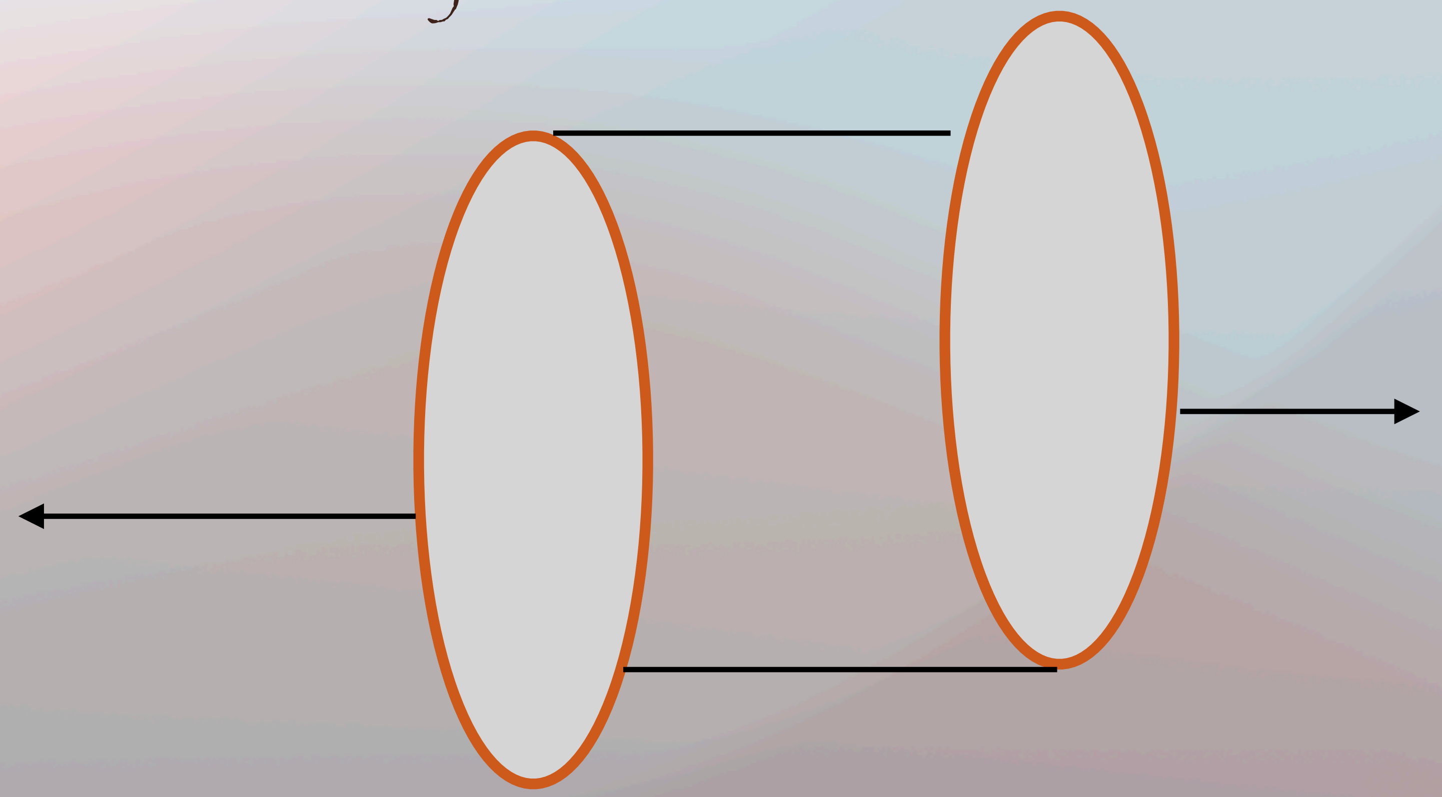
$$T^{\mu}_{\nu} = diag(-\varepsilon(\tau), p_T(\tau), p_T(\tau), p_L(\tau)),$$

$$p_T = p - \pi_T \qquad p_L = p - \pi_L$$

$$\pi_{\mu\nu} = diag(0,\ \pi_T,\ \pi_T,\ \pi_L) \qquad \pi_{\mu\nu} = -\eta\sigma_{\mu\nu}$$



Bjorken Flow



Gradient expansion

- For a conformal fluid: $p = \varepsilon/3$

- Evolution of energy density:

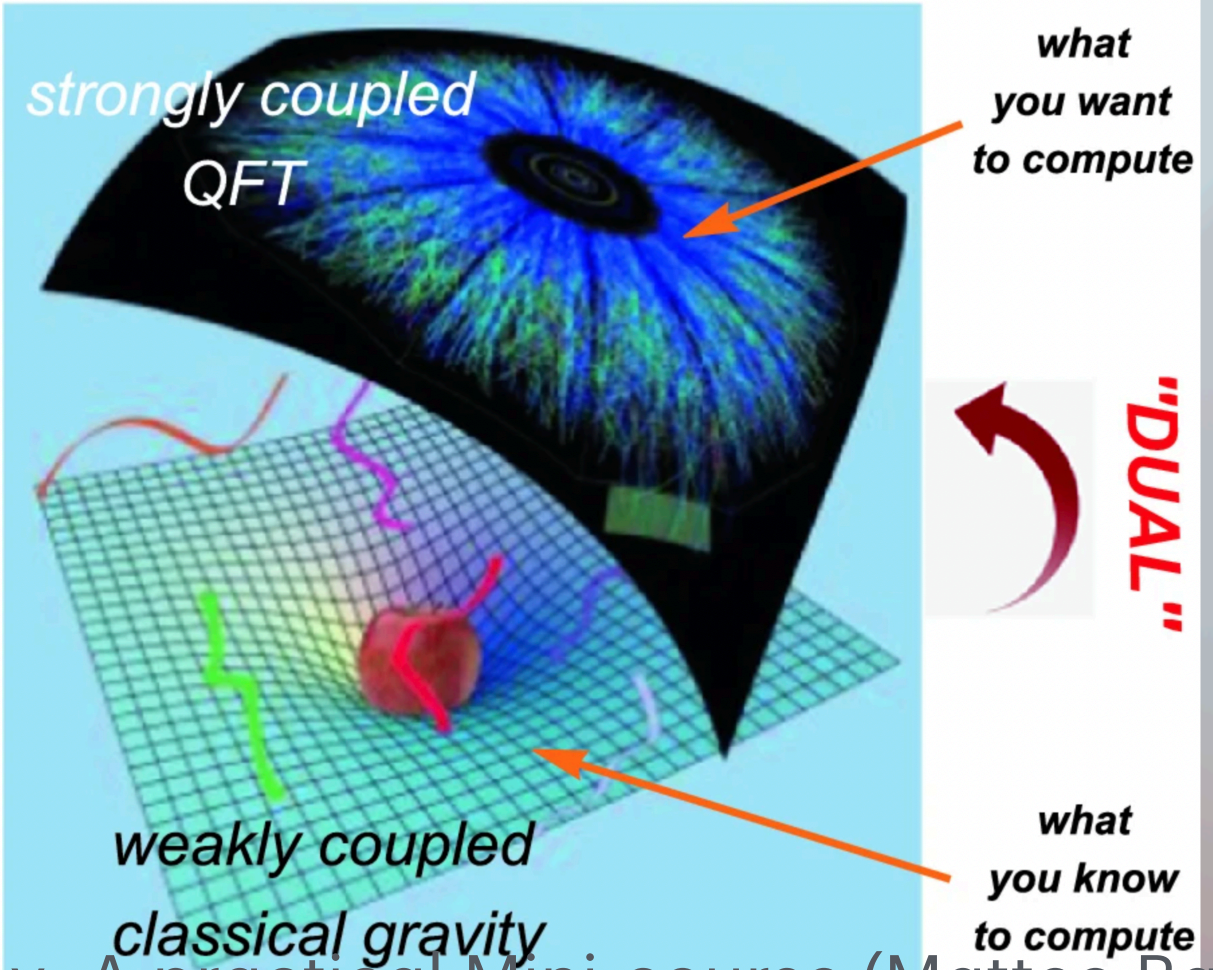
$$\nabla_\mu T^{\mu\nu} = 0 \Rightarrow \partial_\tau \varepsilon = -\frac{1}{\tau}\left(\varepsilon + p_L\right)$$

- Ideal hydro: $p_L = p$

$$\varepsilon = \tau^{-4/3}$$

- First order hydrodynamics: $p_L = p - \frac{4}{3}\eta\left(\frac{1}{\tau}\right)$ ($\frac{1}{\tau}$: $\nabla_\mu u^\mu$)

$$\varepsilon = \tau^{-4/3} + \cdots$$



- Gradient expansion: $\varepsilon(\tau) \approx \epsilon_0\left(\frac{\tau_0}{\tau}\right)^{4/3} + 6C_\eta\, \pi\, \epsilon_0^{4/3}\frac{\tau_0}{\tau^2} + \ldots\ldots$

Heller and Janik 2007.

Phenomenological model: MIS theory

- MIS formulation: $\pi^{\mu\nu}$ independent dynamical variable,

$$\tau_\pi u^\alpha \nabla_\alpha \pi^{\mu\nu} + \pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$$

Relaxation time

$$C_\eta \varepsilon^{3/4}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\tau_\pi \partial_\tau \pi_L + \pi_L\left(1 + \frac{4}{3}\tau_\pi\right) + \frac{4}{3}\frac{\eta}{\tau} = 0 \qquad \tau\partial_\tau \varepsilon + \frac{4}{3}\varepsilon - \pi_L = 0$$

$$C_\pi T^{-1}$$

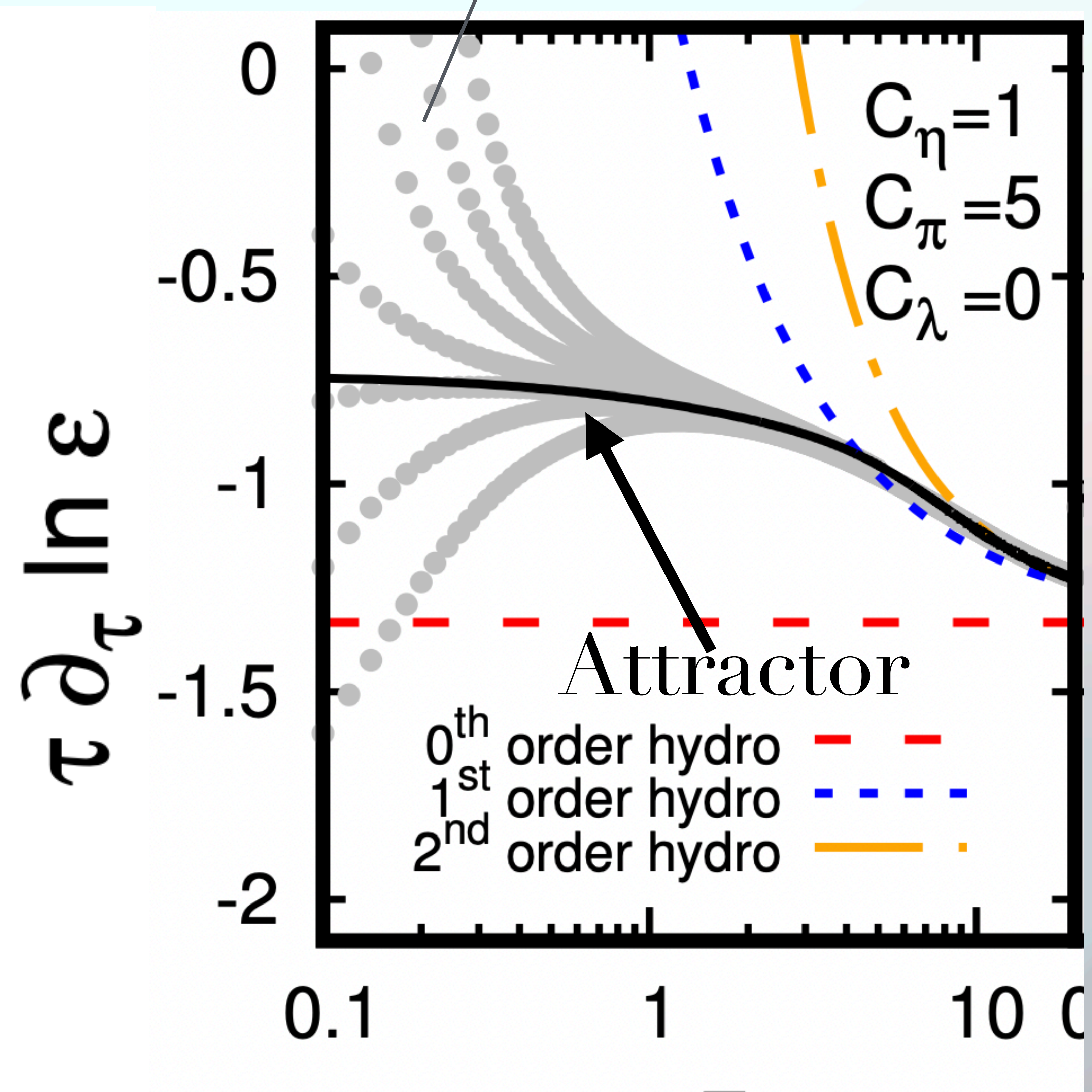
Factorially divergent series

- Energy density: $\tau\partial_\tau \ln\varepsilon = -\frac{4}{3} + \frac{16C_\eta}{9\tau T} + \frac{C_\eta C_\pi}{27\tau^2 T^2} + \cdots$

Resummed series

$$\tau\partial_\tau \ln\varepsilon = (\tau\partial_\tau \ln\varepsilon)_{attract} + (\tau\partial_\tau \ln\varepsilon)_{non-hydro}$$

$$\delta f = e^{\frac{-3\tau T}{2C_\pi}}$$



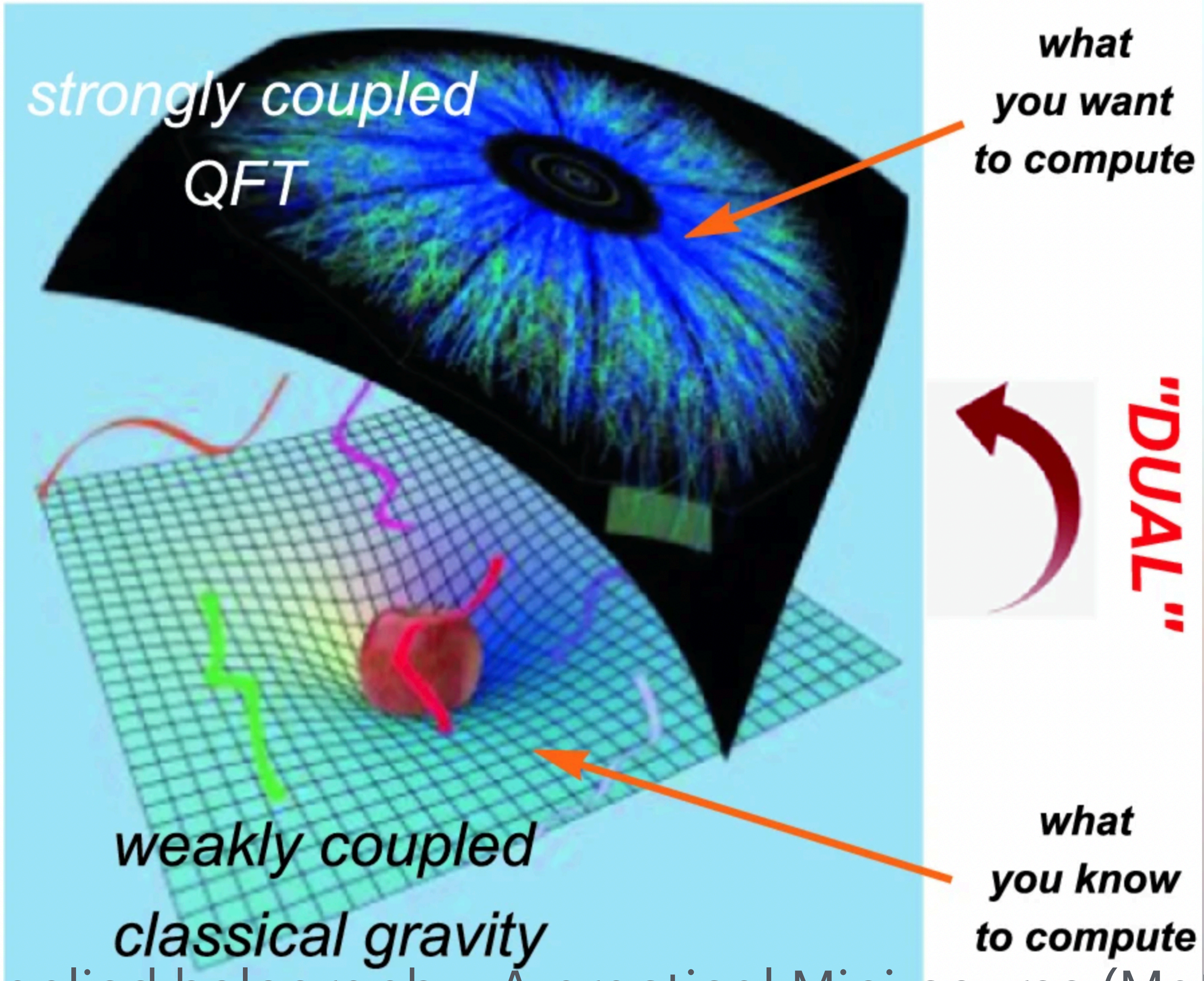
Romatschke (2017)

Kinetic theory and holographic attractors

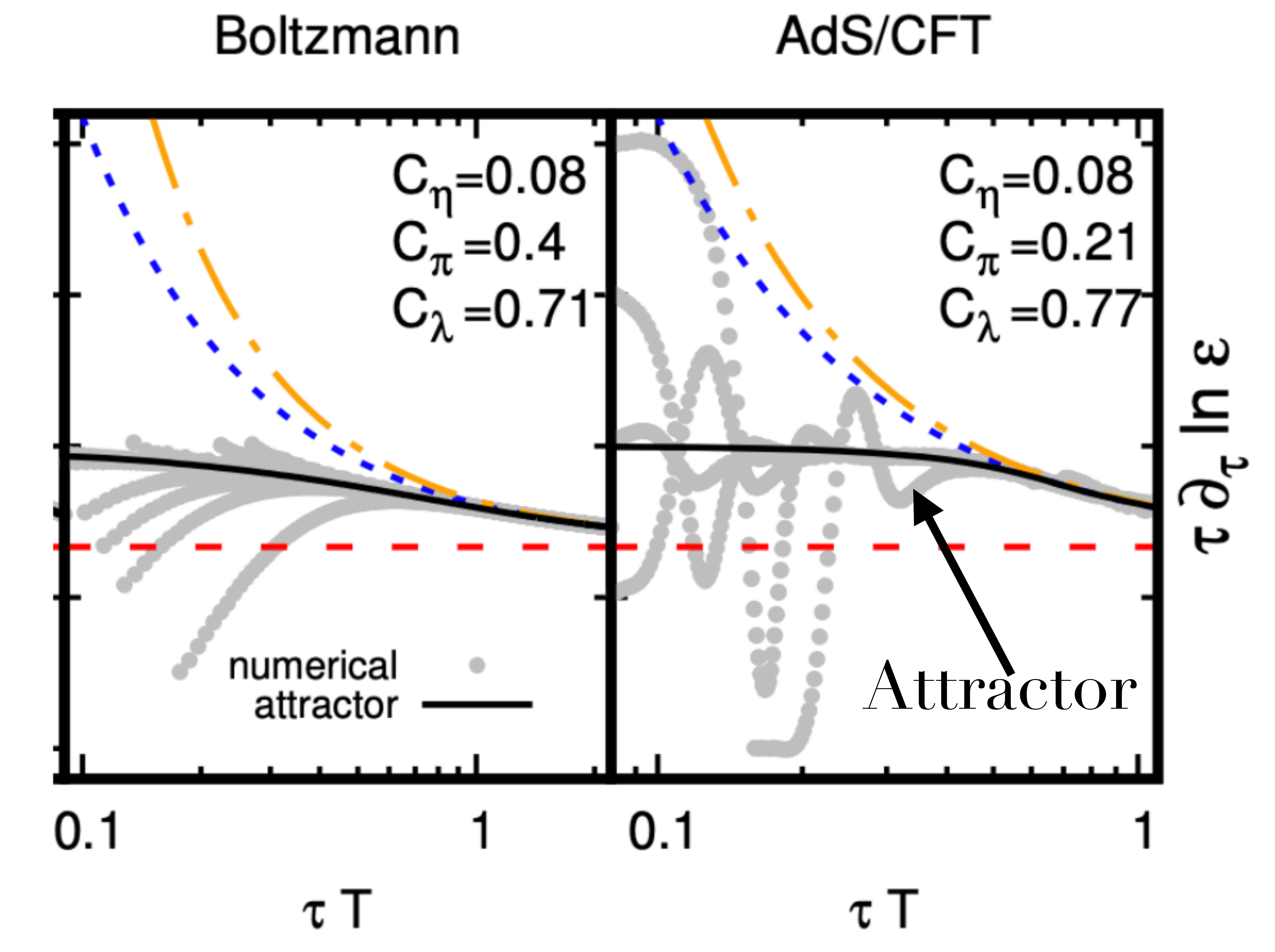
- Kinetic theory- Quasi-particle description

$$\partial_\tau f - \frac{p_z}{\tau}\partial_{p_z} f = C[f] \qquad \frac{\eta}{s} = \frac{5}{T}\tau_{rel}$$

- Holography - Strong-weak duality



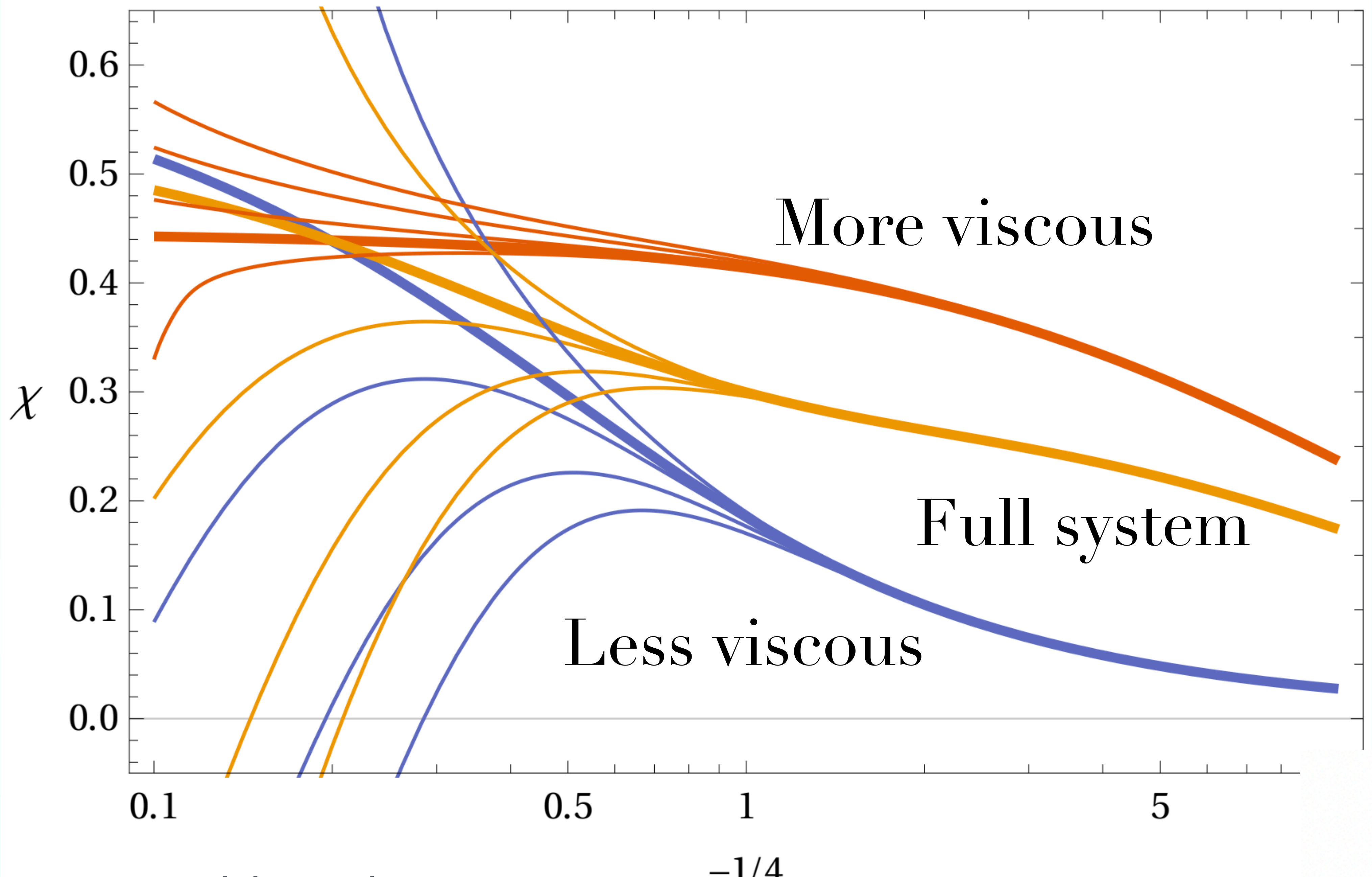
$\eta/s = 1/4\pi$



Romatschke (2017)

Attractors in other models

Hybrid viscous fluid model

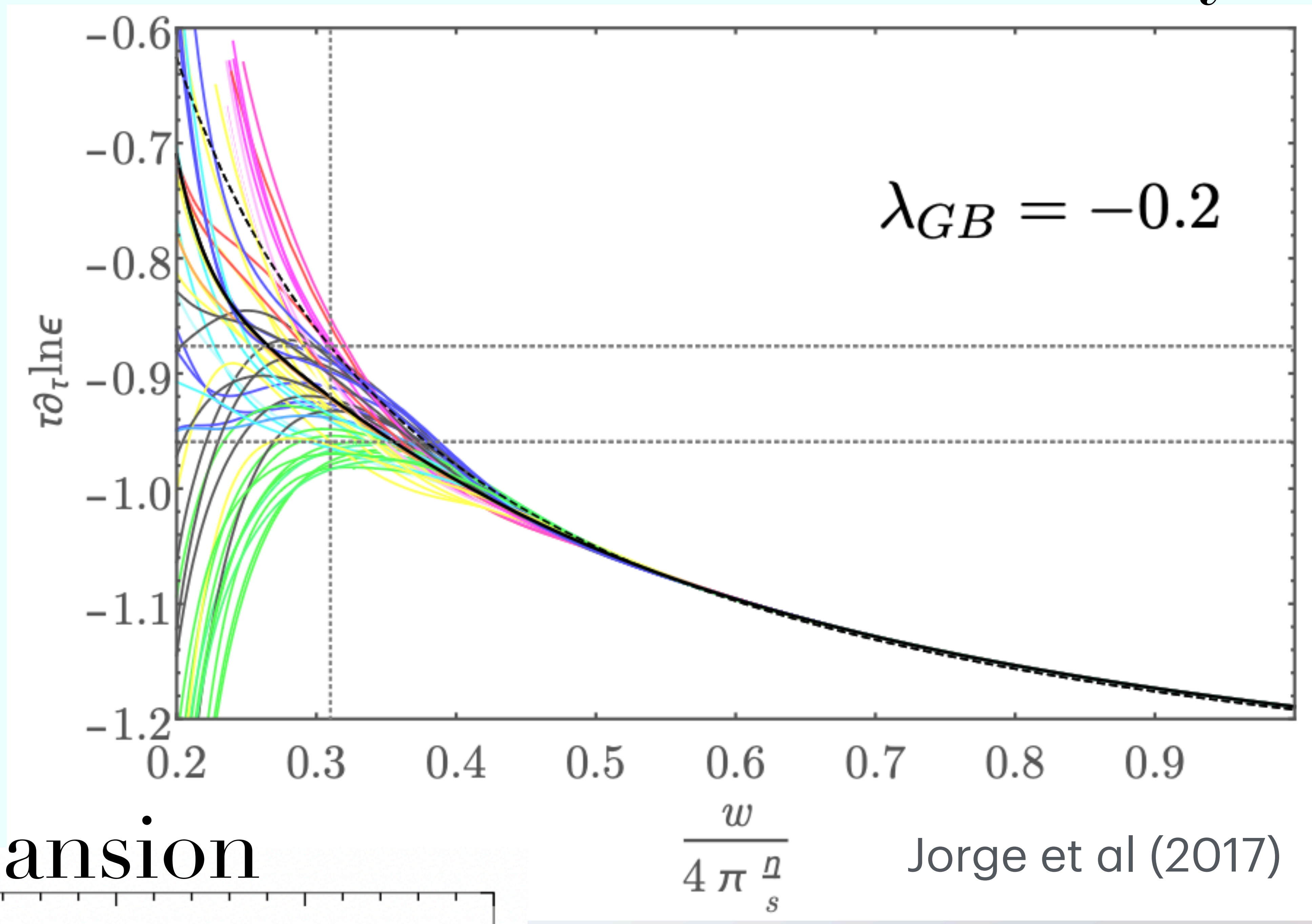


Mitra et al (2022)

Time →

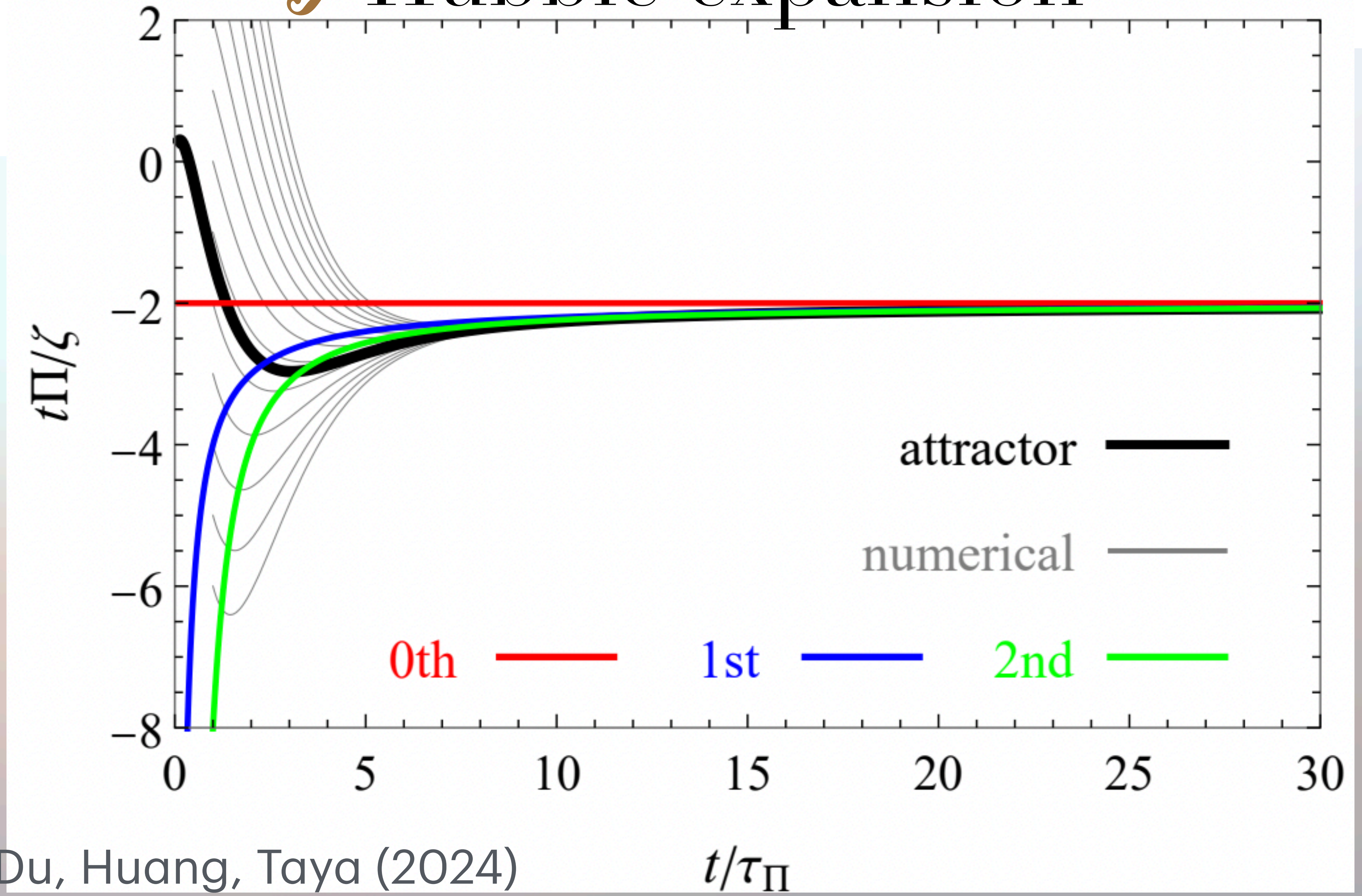
↑ Anisotropy

Gauss-Bonnet Gravity



Jorge et al (2017)

Hubble expansion



Du, Huang, Taya (2024)

Hydrodynamic Attractors: Universal Phenomena

- Hydrodynamic attractor: *Evolution of non-equilibrium system for arbitrary initial conditions to the same late-time behaviour characteristic by hydrodynamic degrees of freedom.*

- Heavy-ion collision: *Attractor has been observed in an expanding boost-invariant background.*

Implications:

- Hydrodynamic attractor in heavy- ion collision.

  - Hydro far-from-equilibrium. Soloviev, review on hydrodynamic attractors (2021)
  - Phenomenological implication. Mazeliauskas et al(2019)
  - Rich mathematical structure. Heller and Spalinski (2015)

- Can attractor be observed experimentally?

Attractors in ultra cold atom

Ultracold atom

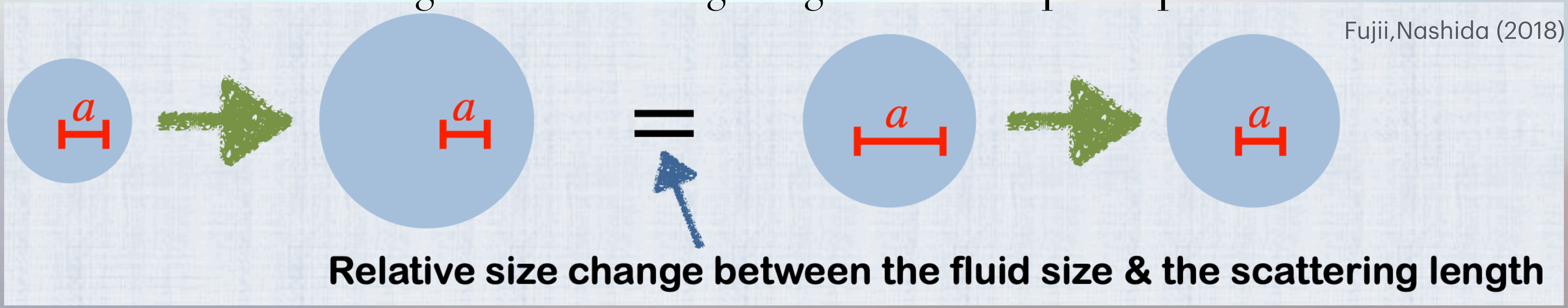
Fujii and Enss (2024)

- Hydrodynamic relaxation model
  - Contact interaction
    - Scattering length a.
    - Tunable by magnetic field.
  - Fluid remains uniform and rest.
  - Isotropic expansion. $\zeta \neq 0$

- Unitarity limit: $\frac{1}{a} \to 0$
  - $\eta/s \sim 0.5$ and $\zeta/s \sim \frac{1}{a^2} \to 0$

$a^2\zeta \to constant$

Change in Scattering length ≡ Isotropic expansion



Attractor in ultra cold atoms

- Bulk viscosity

$$-3\zeta \log a^{-1}(t)$$

$$\Pi_{NS} \equiv \frac{1}{3}T^i_i - p_{eq} = -\zeta \vec{\nabla} . \vec{v}$$

- MIS formulation

$$\tau_\zeta d_t \Pi(t) + \Pi(t) = \Pi_{NS}$$

- Monotonic drive: Traditional hydro attractor

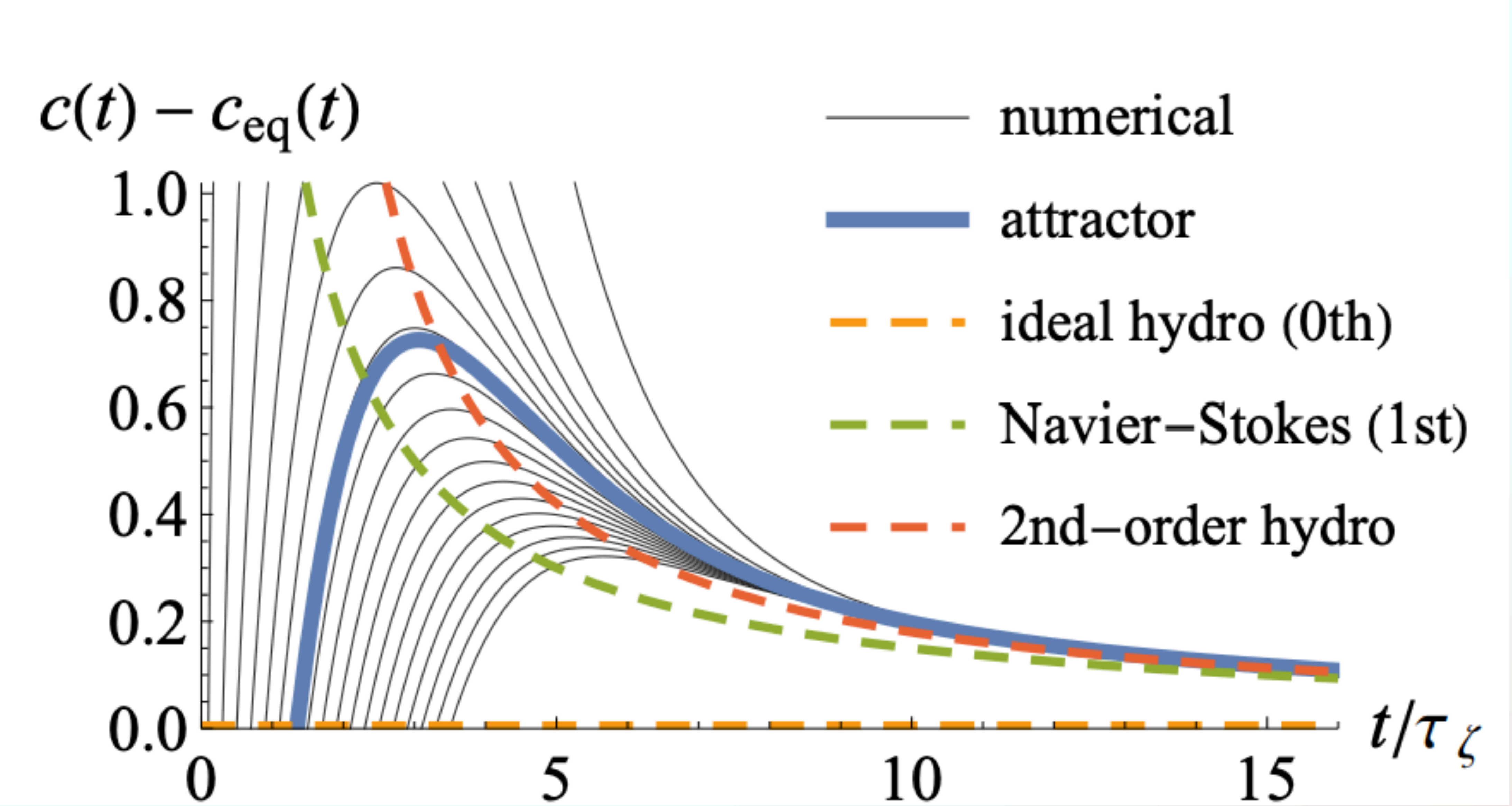
$$a(t)^{-1} \propto t^{-k}$$

- Periodic drive: New cyclic attractor

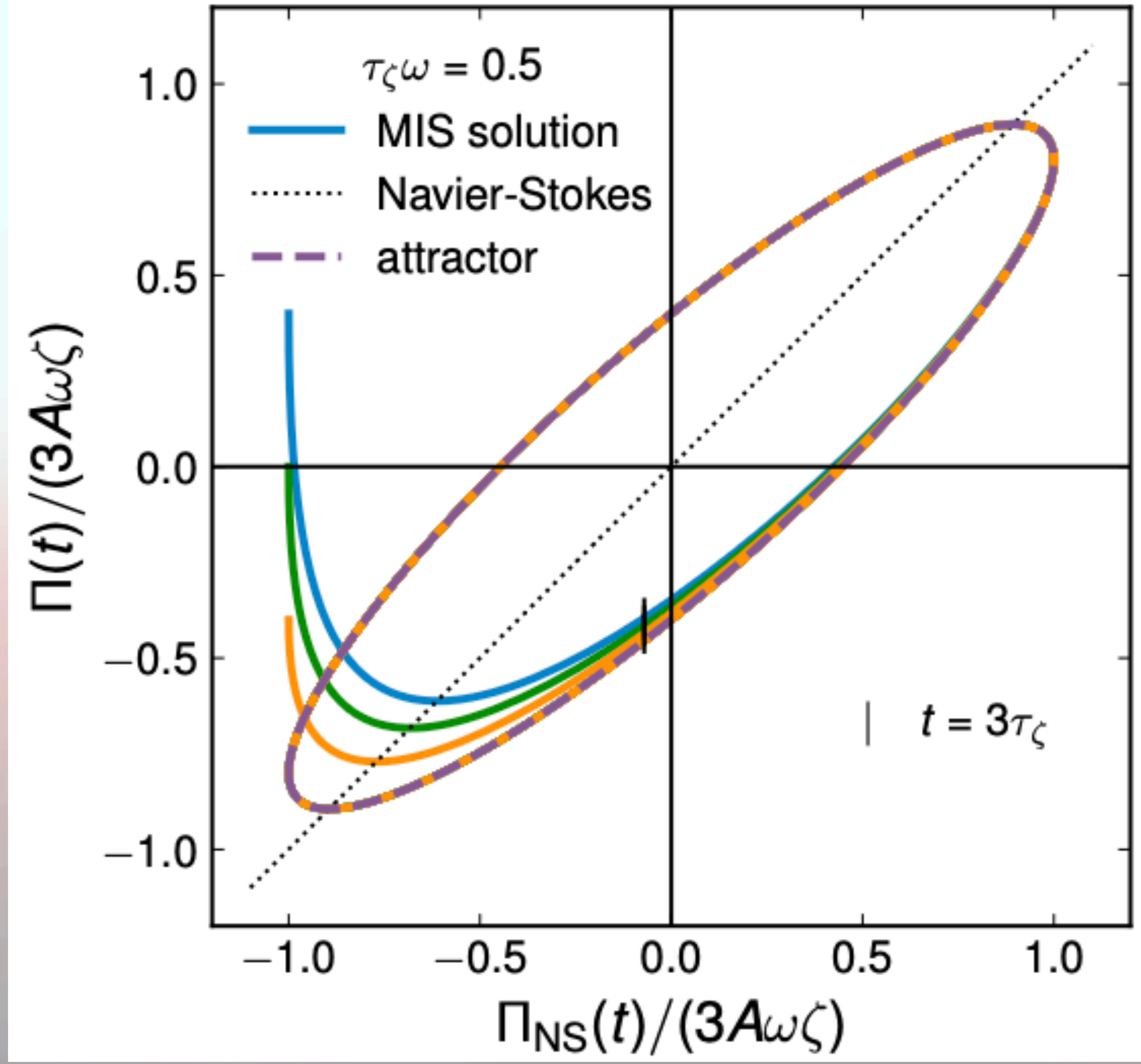
Enss, Mazeliauskas (2025)

$$a(t)^{-1} \propto (1 + A\cos\omega t)$$

$\Pi \sim$



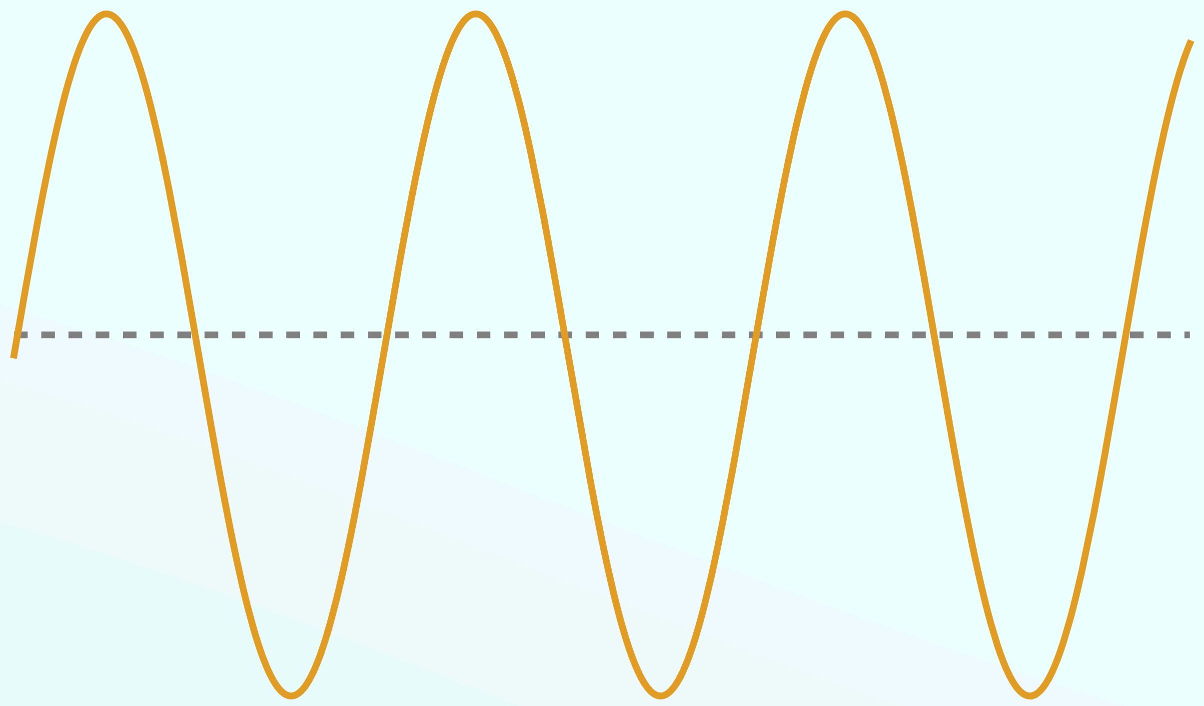
Enss, Fujii (2024)



Anisotropic Periodic oscillation - ongoing work

Mitra, Mazaliauskas and Enss

- Periodic oscillation in $z-$ direction.

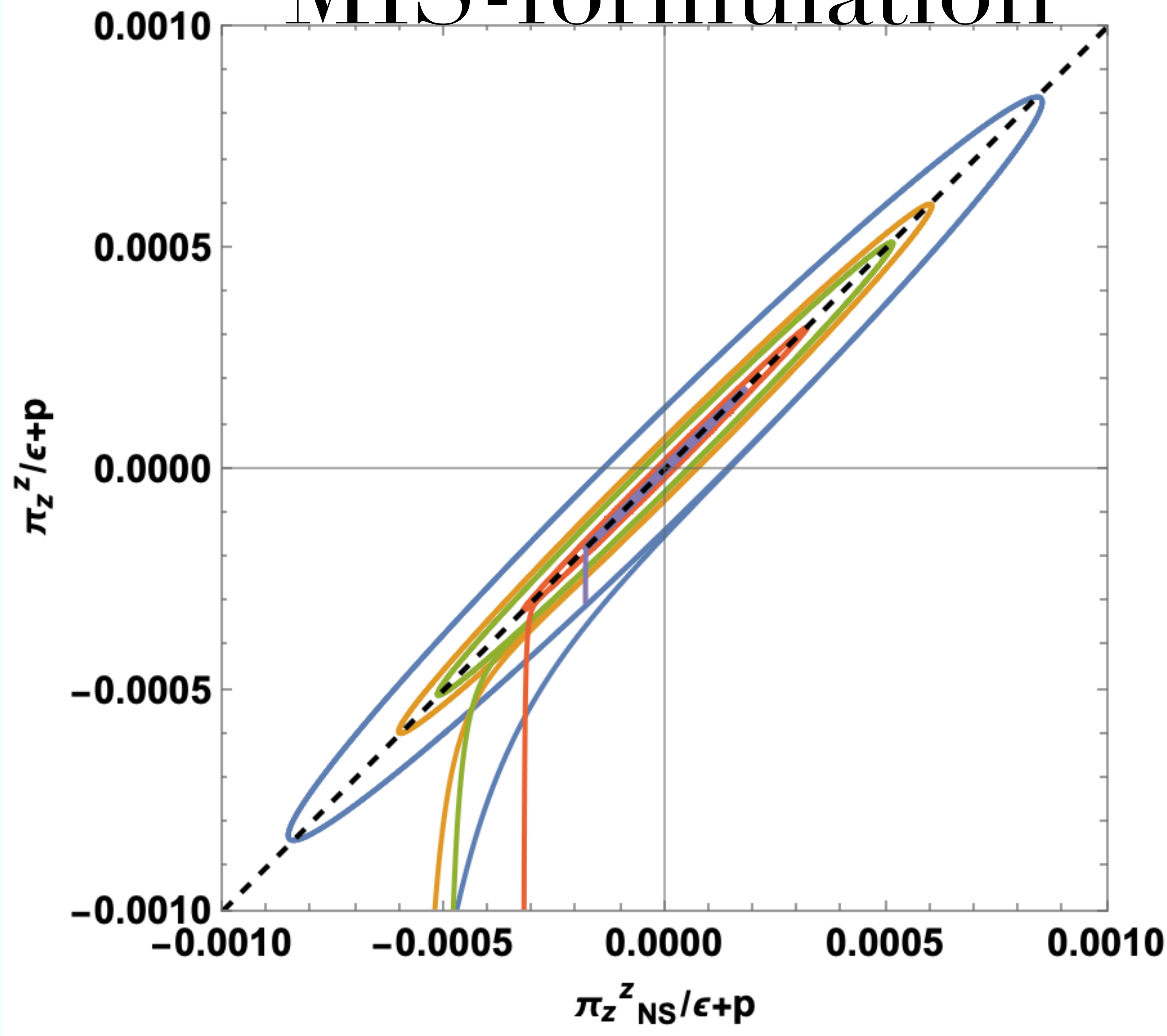


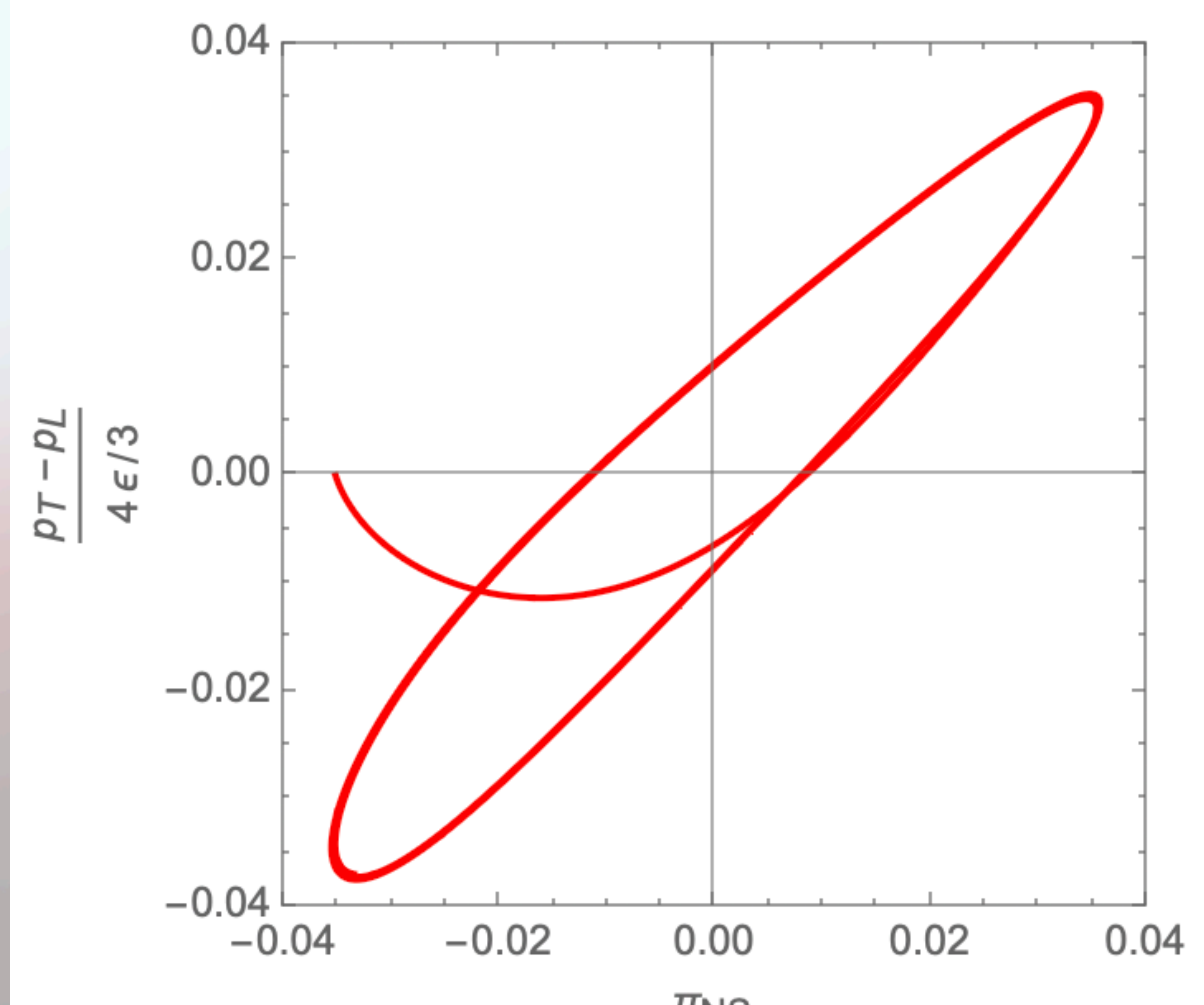
$$1+\beta\sin(\omega t)$$

$$\pi_{NS} = -\frac{4\ \beta C_\eta \omega \cos(t\omega)}{3\ \sqrt[4]{\epsilon(t)}(1+\beta\sin(t\omega))}$$

- Interpretation of oscillatory attractor in HIC??

MIS-formulation





Holography

Attractors and spontaneous symmetry breaking

Buza, Mitra, Soloviev (2024)

Attractor meets symmetry breaking

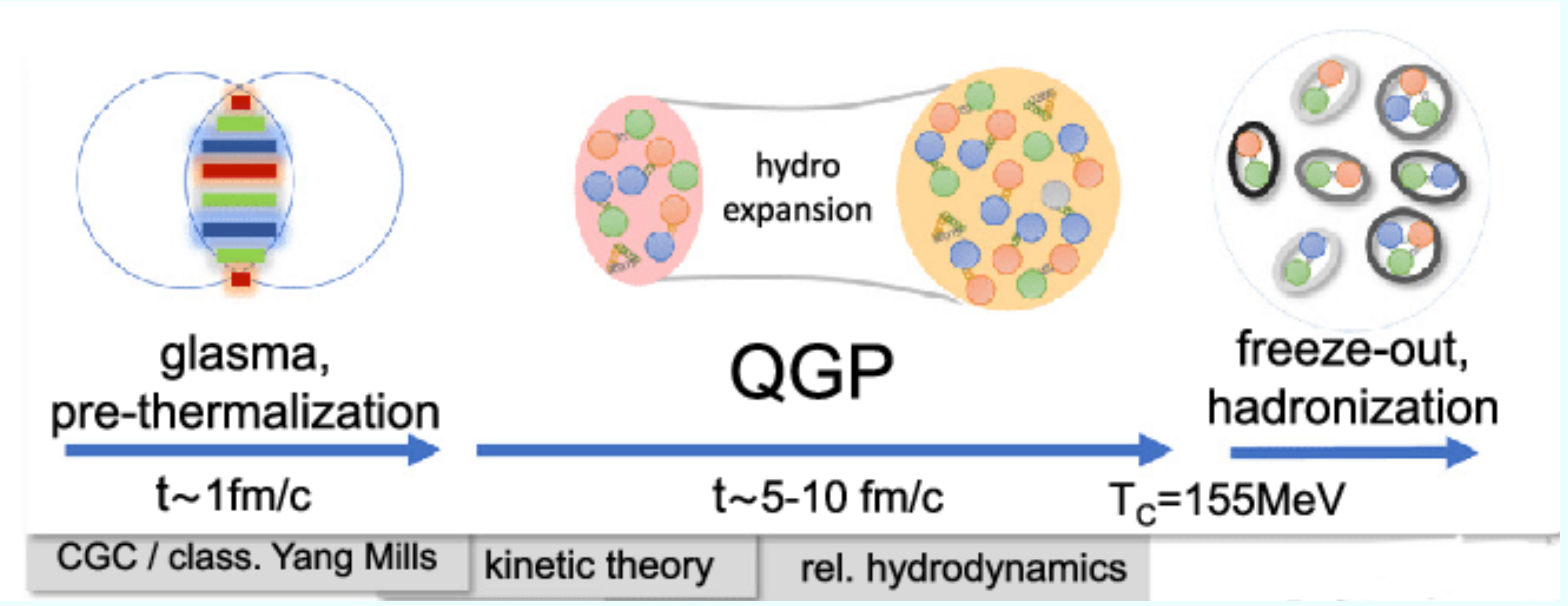
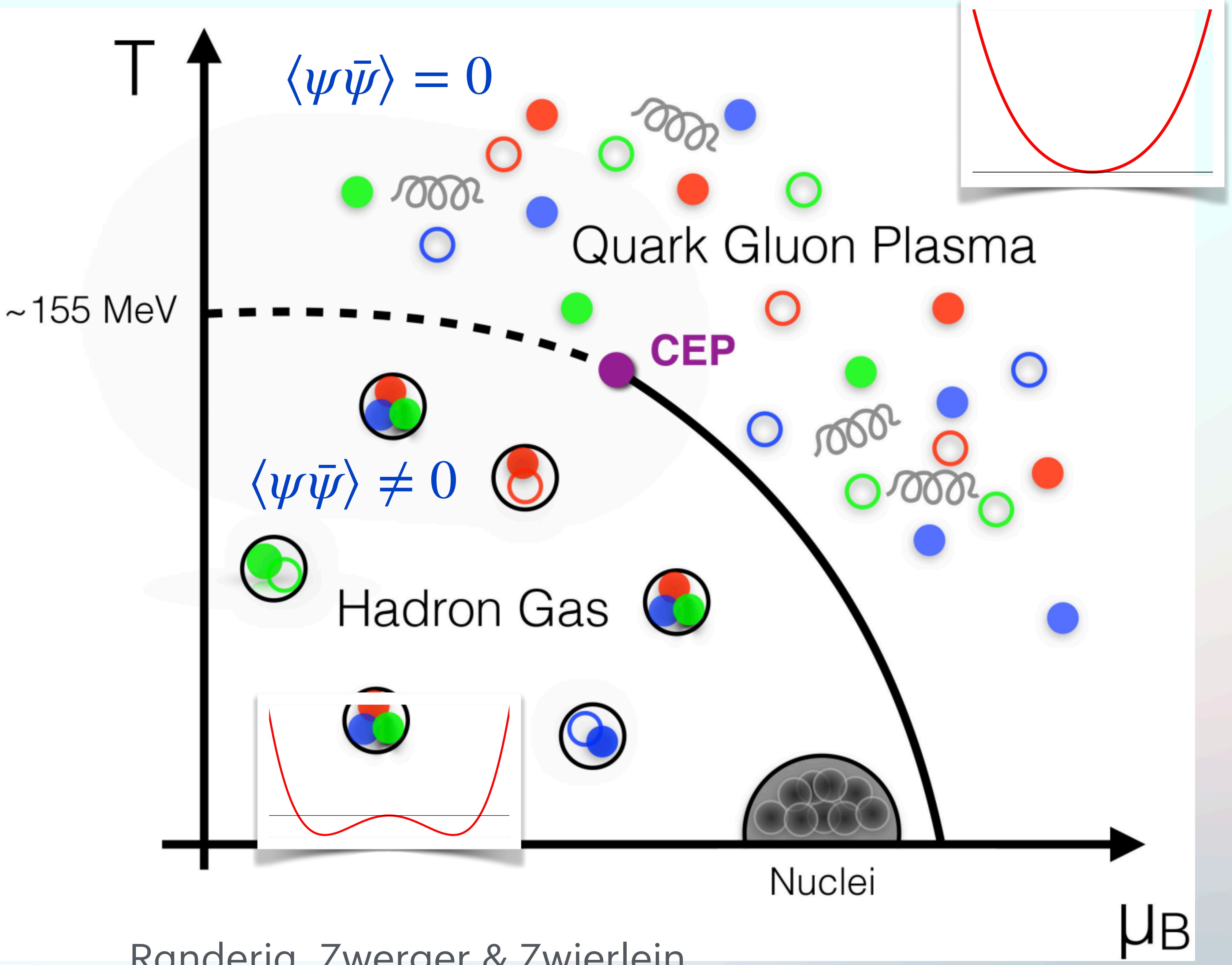
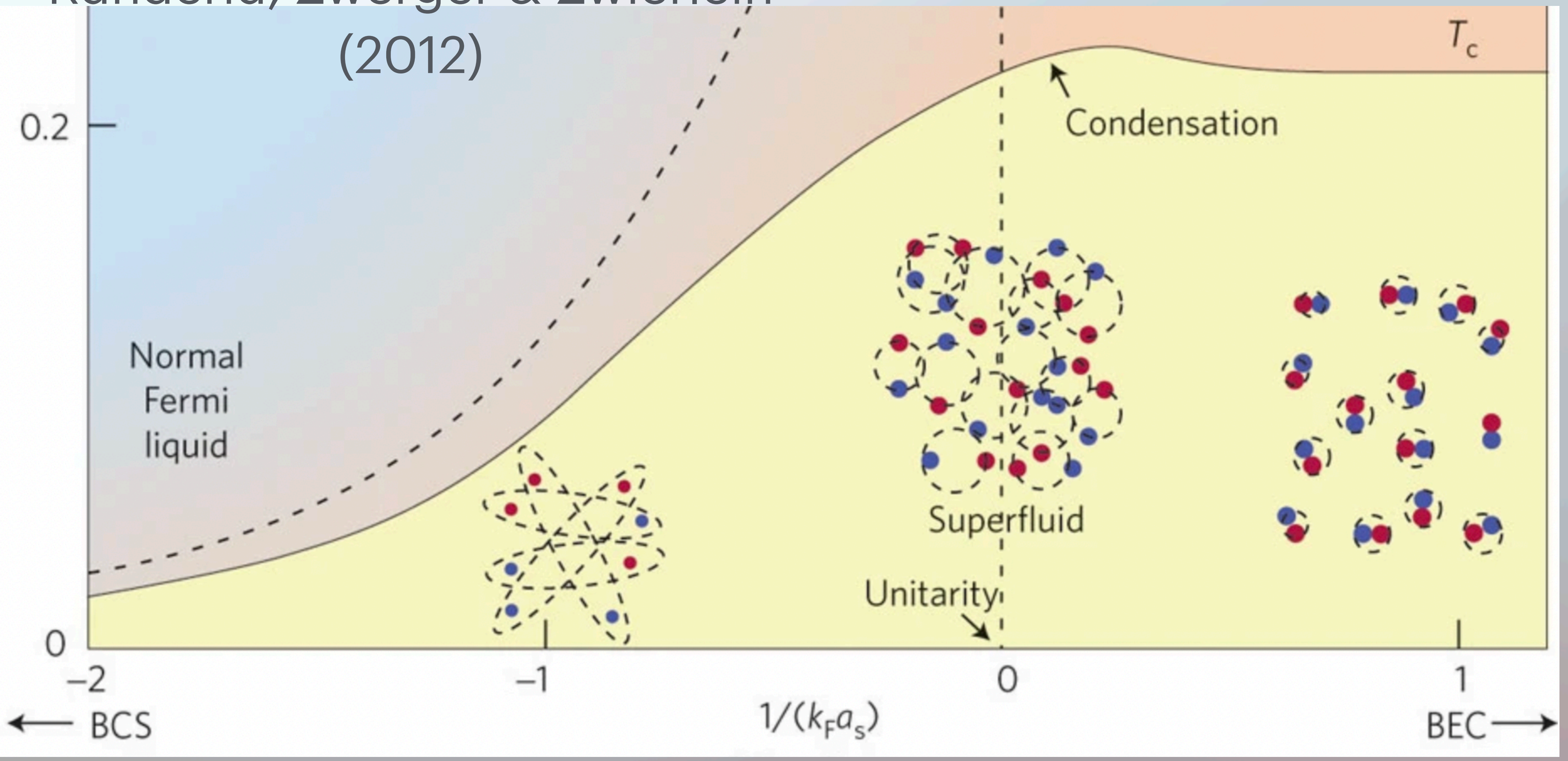


Figure taken from Heavy quarkonium in extreme conditions by Alexander Rothkopf.



Randeria, Zwerger & Zwierlein (2012)



- Transition from symmetric phase to broken phase?

Hydrodynamic + Condensate

σ : condensate and ψ : phase, constant phase

- Couple a viscous fluid with a scalar field $\Sigma = \sigma e^{i\psi}$ of mass $m \sim (T - T_c)$

- Energy-momentum tensor:

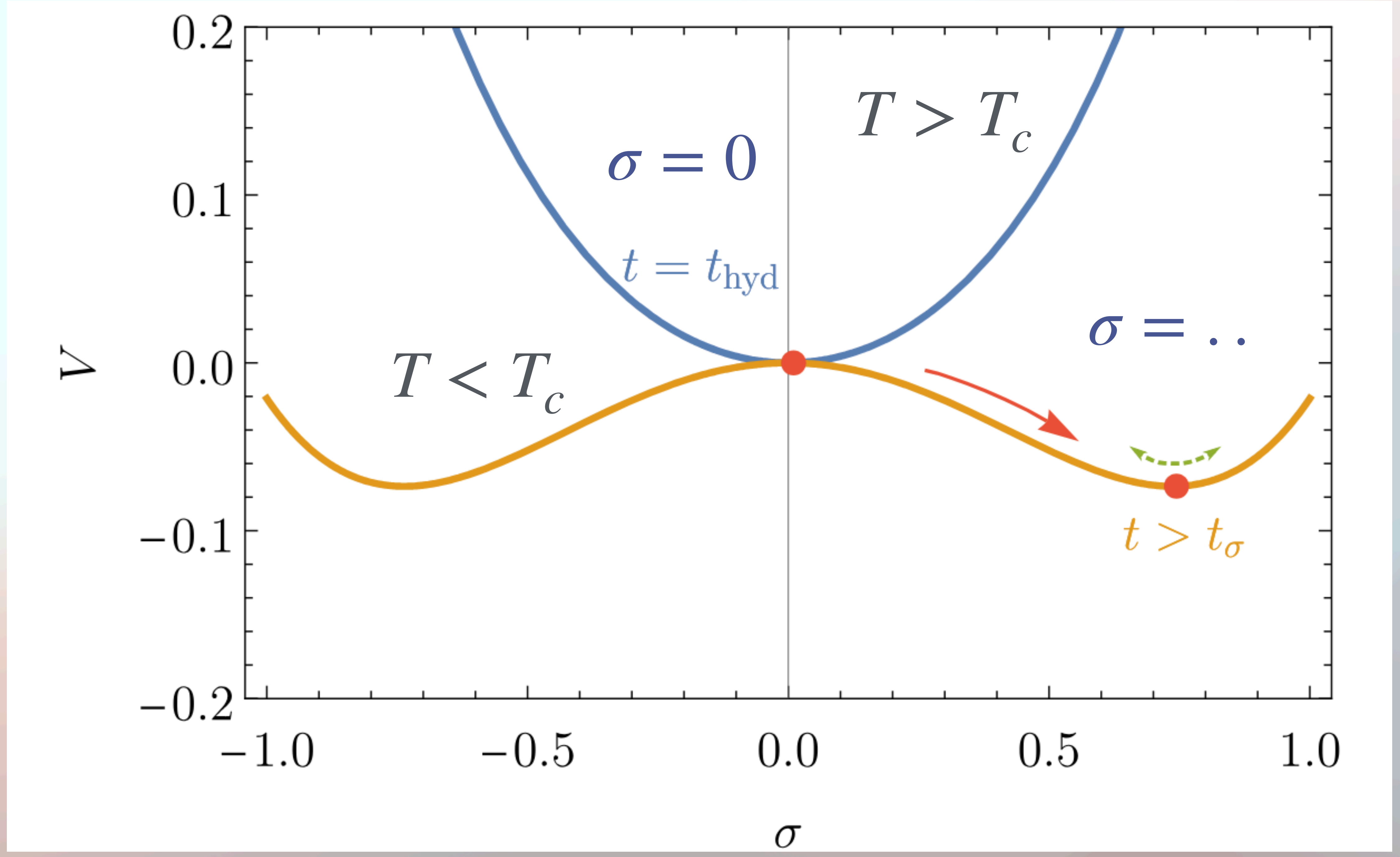
$$T^{\mu\nu} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{scalar}$$

$$T^{\mu\nu}_{ideal} + \Pi^{\mu\nu}$$

- Scalar field equation

$$(Eom)_{scalar} - u \, . \, \nabla \sigma = 0$$

$$V = \frac{1}{2} m_0 (T - T_c) \sigma^2 + \frac{1}{4} \lambda \sigma^4$$



Potential as a function of temperature and condensate. Figure taken from Buza, Mitra, Soloviev (2024)

Modified fluid equations

- Conservation of energy-momentum:

$$\tau\partial_\tau \log T - \frac{1}{3}(\chi - 1) = m_0 \frac{\sigma^2 + 2\tau\sigma\sigma'}{8T^3} + \frac{\tau}{4T^3} C_{\kappa_1} \sigma'^2$$

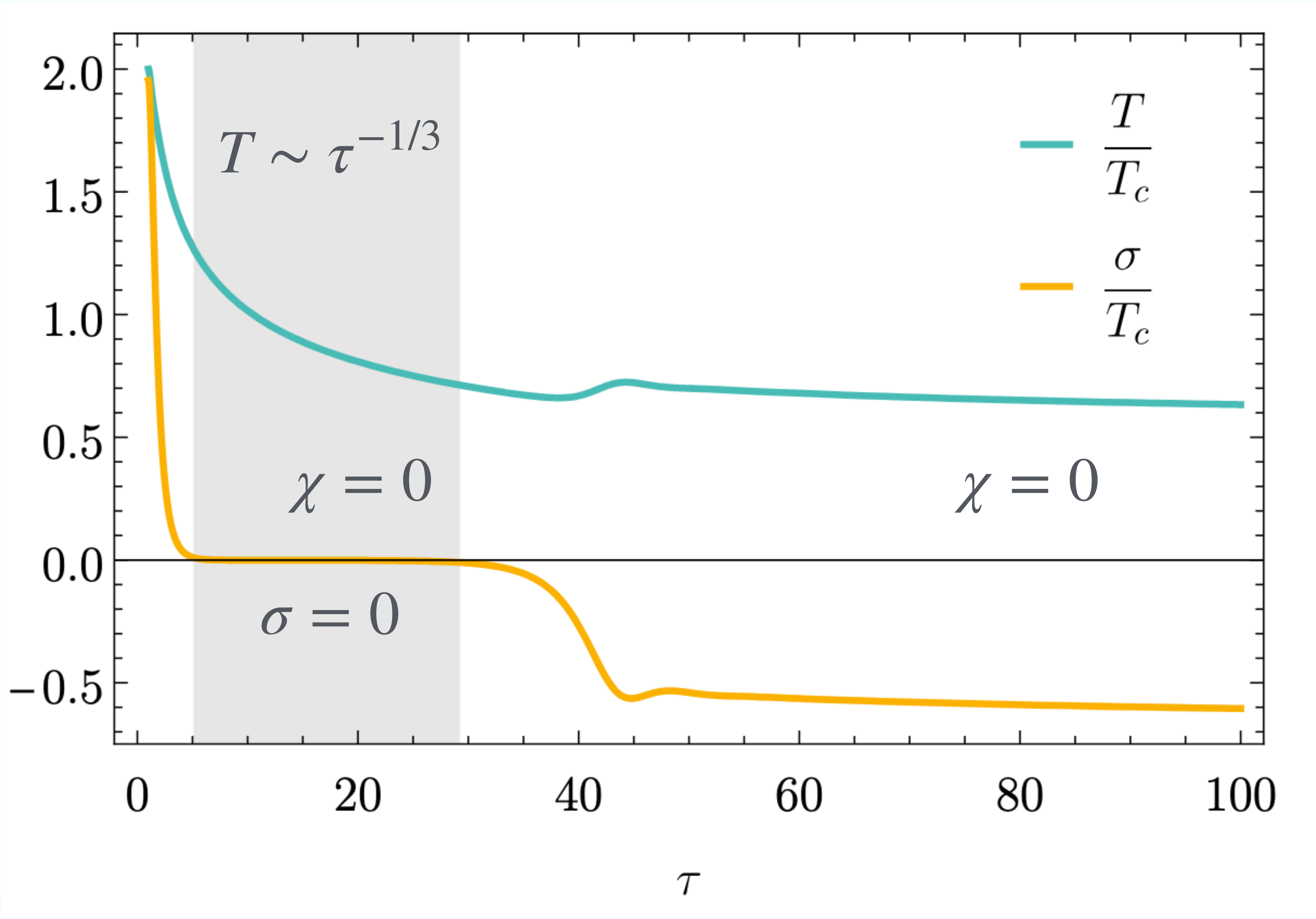
- Scalar field equation:

$$\sigma'' + \frac{\sigma'}{\tau} + \lambda\sigma^3 + m_0(T - T_c)\sigma + C_{\kappa_1} T\sigma' = 0$$

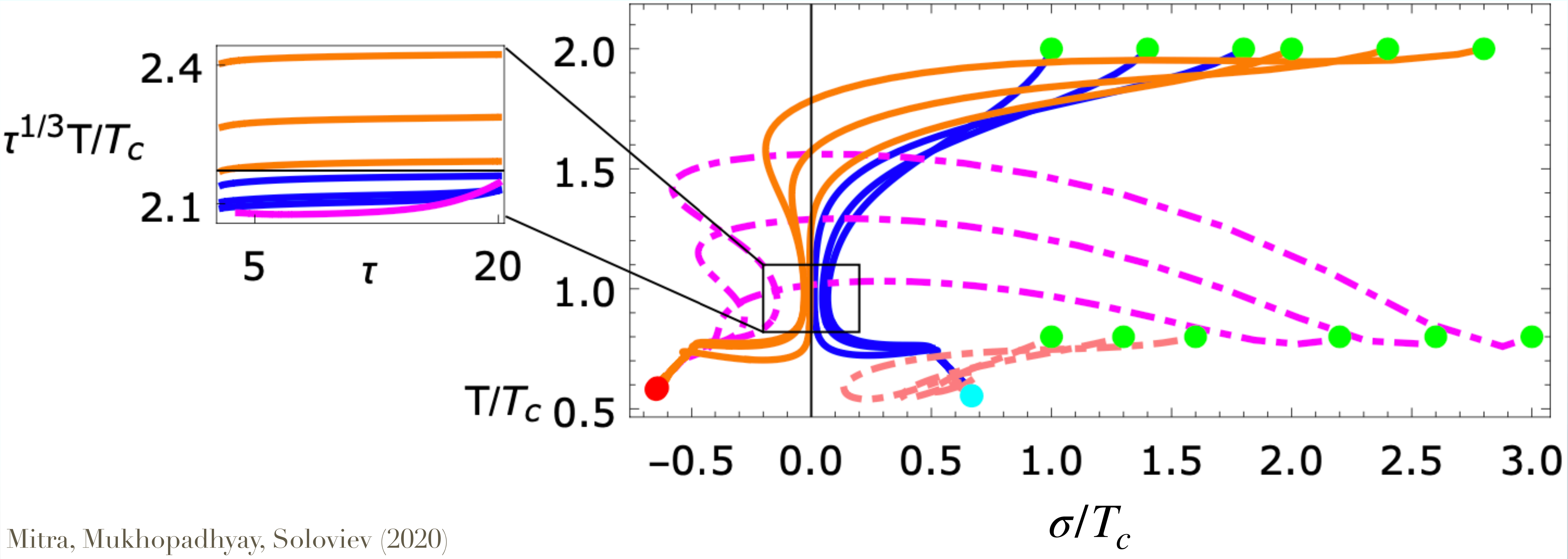
- MIS equation:

$$\tau\chi' + \frac{4}{3}\left(\chi - \frac{C_\eta}{C_\pi}\right) + 4\chi\frac{\tau T'}{T} + \frac{\tau}{C_\pi}\chi T = 0.$$

$T_0 > T_c$



Buza, Mitra, Soloviev (2024)



Evolution of the system shown on $T-\sigma$ phase space

End of attractor

- Transition from hydro-like behaviour to the symmetry breaking fixed points.

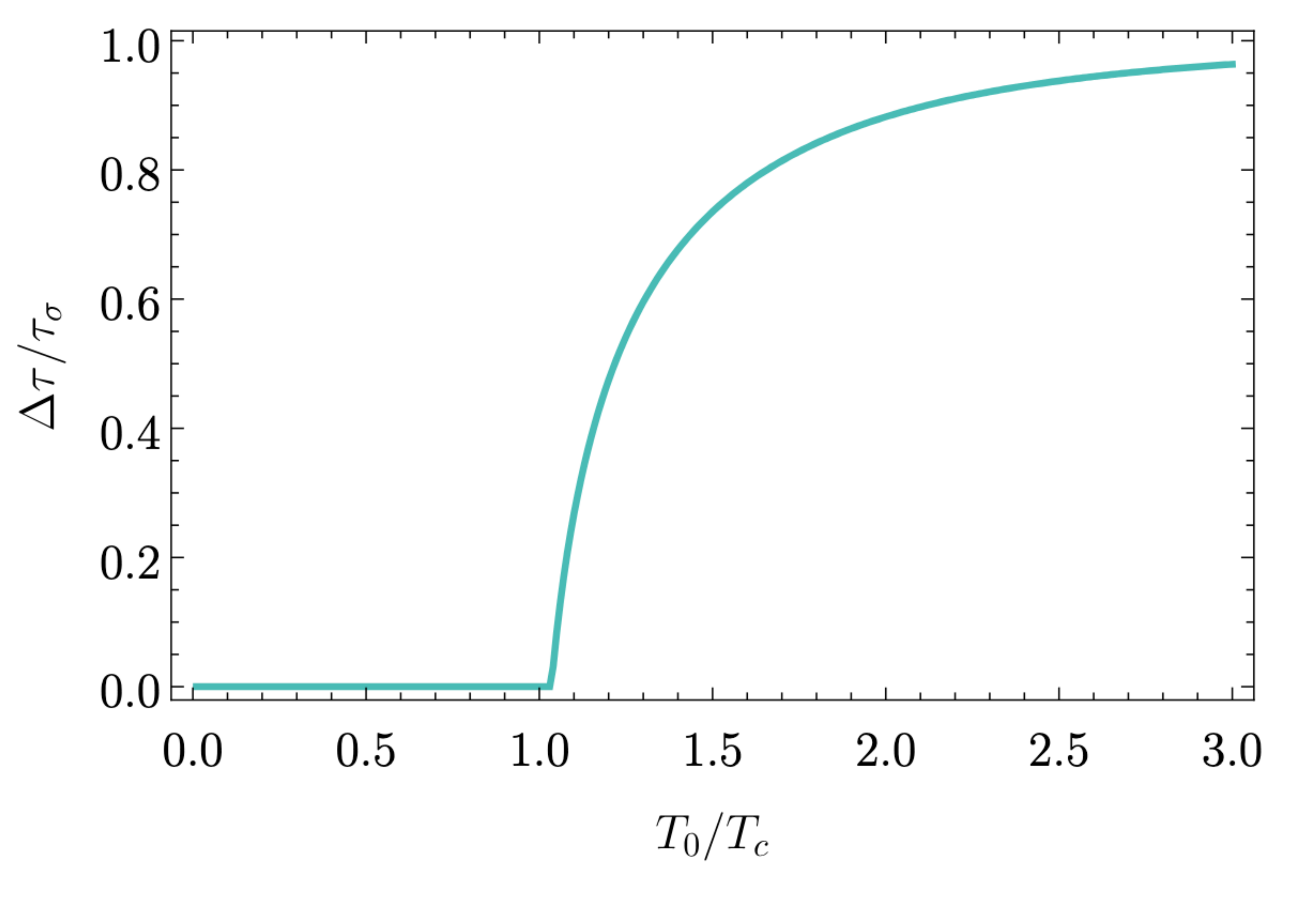
- Attractor time: Timescale from $\tau_{hyd}(\sigma = 0, \chi = 0)$ to $\tau_{\sigma}(\chi = 0, \sigma \neq 0)$.

- For Bjorken flow,

$$\frac{\Delta\tau}{\tau_0} = \frac{\tau_\sigma - \tau_{hyd}}{\tau_0} \equiv |\sigma(\tau)| \leq 10^{-2} \sim 13.2$$

- Freeze-out time

$$\tau/\tau_0 \sim 10$$



Buza, Mitra, Soloviev (2024)

Summary and outlook

- Hydrodynamic attractor - Hydro beyond equilibrium.
- Heavy-ion: Attractors observed in monotonically expanding system.
- Cold atoms: Cyclic attractor.
- Attractors + Condensate evolves the system out of hydro.

- Experimental observation of attractors in cold atoms?
- Cyclic attractor in HIC
- Attractor in a periodically driven system + condensate?
- Attractors close to superfluid phase transition in cold atoms?

Thank you

Plan to experimentally realise attractor in cold atoms.

Fujii and Enss (2024)

The attractor behavior can be probed in current cold-atom experiments, e.g., in a two-component Fermi gas of ^{40}K atoms with Fermi energy $E_F \sim h \times 20\,\text{kHz}$ at $T/T_F = 0.25$. The effect is expected to be large near unitarity, where the bulk viscous relaxation time is predicted as $\tau_\zeta \sim 0.7\hbar/k_B T \sim 0.15\,\text{ms}$ [35]. Let us consider the power-law protocol with $\alpha = 1/2$ and $1/k_F\tilde{a} = 0.5$. As starting times of the ramp, one can choose $t_k/\tau_\zeta = 0.5, 1, 2$; the maximum rate of change of $1/k_F a$ is then $\sim 4\,\text{kHz}$, well within experimental capabilities near the narrow Feshbach resonance at $202\,\text{G}$. The attractor is expected to occur in the time window $t \sim (5\text{–}10) \times \tau_\zeta \sim 0.75\text{–}1.5\,\text{ms}$. It will be visible in the time-dependent contact, which has already been measured with $0.1\,\text{ms}$ time resolution [55, 56].

Stages of heavy-ion collisions

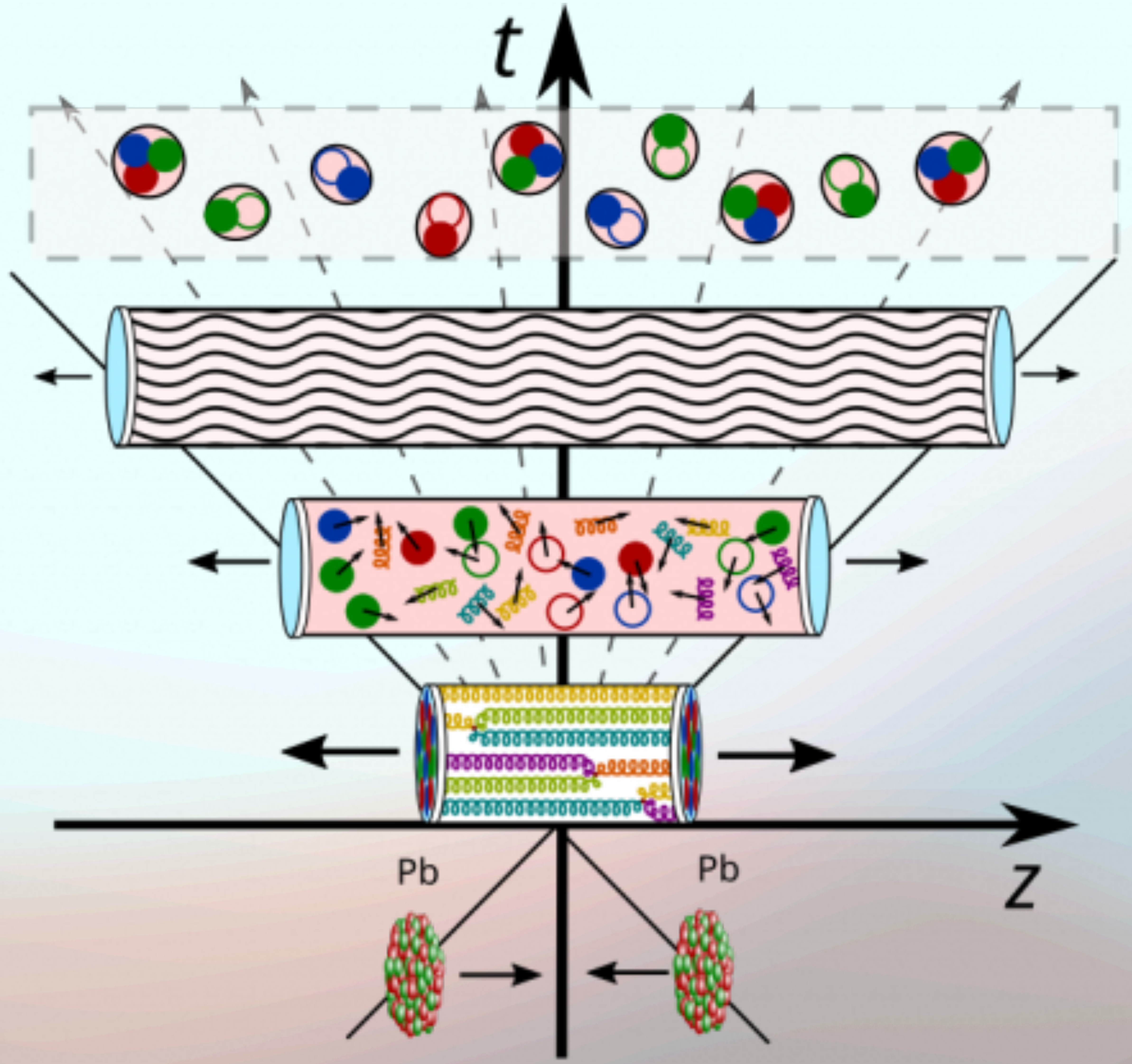
Hadronisation: $t > 10 fm/c$

Fluid expansion: $t \sim 1 - 10 fm/c$

QGP equilibration $t \sim 1 fm/c$

Deposition of initial energy

Incoming nuclei



Mazeliauskas et al (2021)