UNIVERSALITY WITH EFFECTIVE FIELD THEORY

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cnrs

THE UNIVERSITY OF ARIZONA

Outline

- Structures and scales
- Two-body unitarity and scale invariance
- More bodies and discrete scale invariance
- Nucleons near unitarity
- Improved actions
- Conclusion

with

Contessi, Pavón

Contessi, Schäfer

Contessi, Schäfer, Gnech, Lovato

Structures and scales

"... were the world to be made between now and tomorrow
100 or 1,000 times larger or smaller than it is at present,
all its parts being enlarged or diminished proportionally,
everything would appear tomorrow exactly as now,
just as though nothing had been changed."

Nicolas Oresme
commentary on Aristotle's *De caelo et mundo*, 1377

stock

Here: $\hbar = 1, c = 1$

$$[m] = [E] = [p] = [r]^{-1} = [t]^{-1}$$

scale transformation

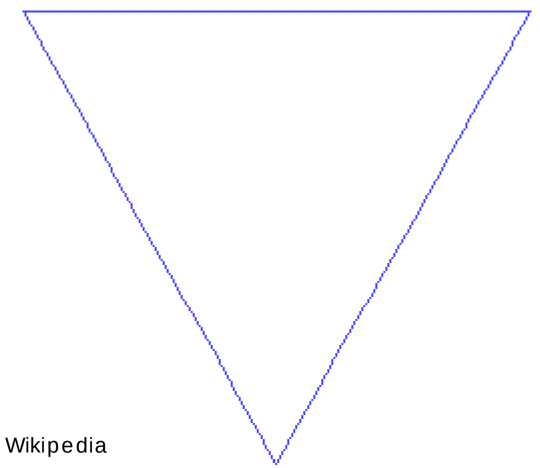
$$r \to \alpha r \quad \Leftrightarrow \quad p \to \alpha^{-1} p$$

$\alpha \geq 0$ (quantum) (nonrelativistic)

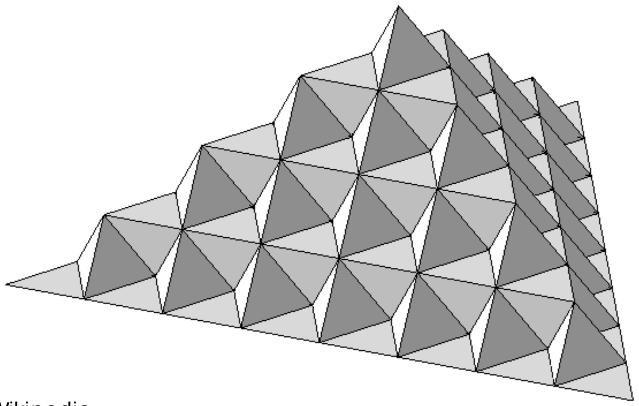
$$t/m \to \alpha^2 t/m \quad \Leftrightarrow \quad mE \to \alpha^{-2} mE$$

Simplest "complex" structures: one scale

von Koch 1904



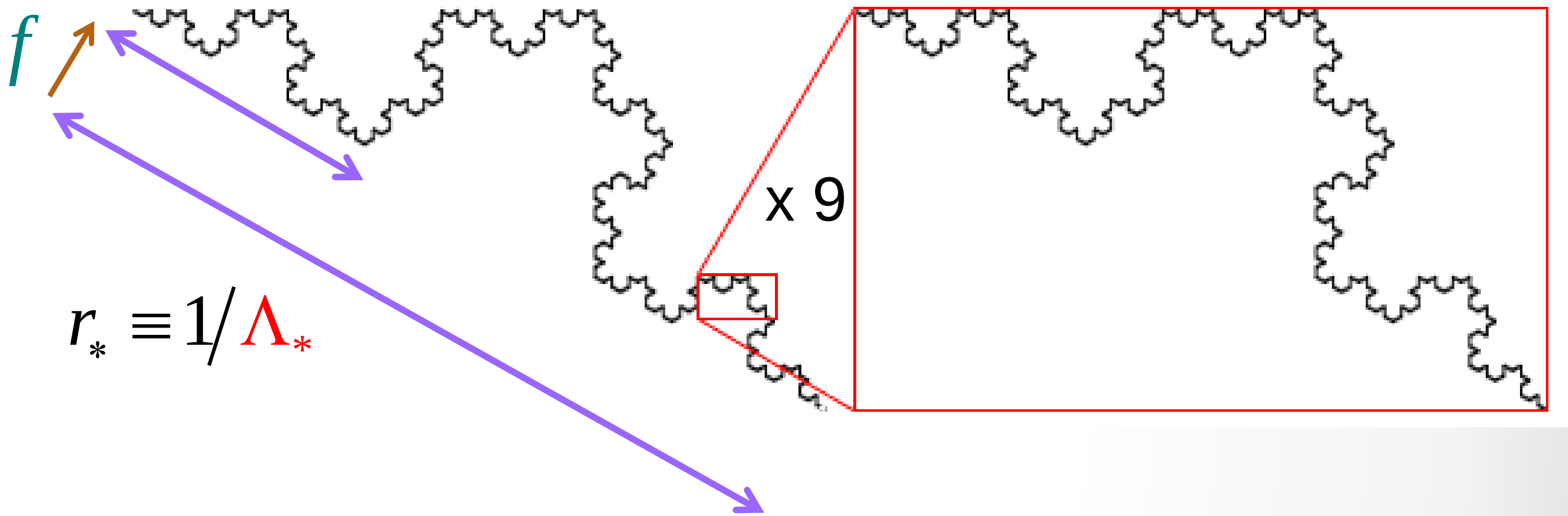
Wikipedia



Wikipedia

(Scaling) Fractal

yozh.org/2010/10/21/mset001/



$$r_* \equiv 1/\Lambda_*$$

$$r \to \alpha_n r = f^n r$$

$$\begin{cases} f \text{ real} \\ n \text{ integer} \end{cases}$$

Discrete scale invariance

romanesco or "fractal broccoli"

Nuclear physics: $\underbrace{\text{nucleons (proton or neutron) with spin } S=\tfrac{1}{2}\text{, nearly the same mass}}_{\text{four-component fermions}}$ $\quad m_N \simeq 940\ \mathrm{MeV}$

lightest exchanged particles: pions

$m_\pi \simeq 140\ \mathrm{MeV}$

$$V(r) = -\underbrace{\frac{g_{\pi N}^2}{m_N}\frac{e^{-m_\pi r}}{r^3}}_{\text{range } R \sim m_\pi^{-1} \simeq 1.4\ \mathrm{fm}} S_{12} + \dots$$

Yukawa '35

proton + neutron, S=1: deuteron

$B_2 \simeq 2.2\ \mathrm{MeV}$ $\Rightarrow$ $\dfrac{R}{a_2} \simeq \dfrac{\sqrt{m_N B_2}}{m_\pi} \approx \dfrac{1}{3}$

binding energy

scattering length ~ system size

Atomic physics: $\underbrace{\text{neutral atoms}}_{\text{fermions or bosons}}$, mass $m_{at} \approx A m_N$

lightest exchanged particles: two photons

$m_\gamma = 0$

$$V(r) = -\underbrace{\frac{l_{\mathrm{vdW}}^4}{4\pi m_{\mathrm{at}} r^6}}_{\text{“range” } R \sim l_{\mathrm{vdW}}} + \dots$$

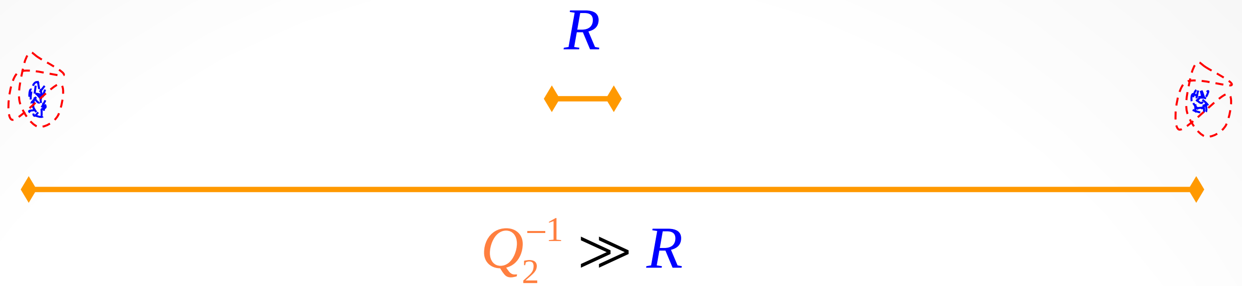
v.d. Waals 1873

two bosonic ^{4}He atoms, S=0: ^{4}He dimer

$B_2 \simeq 1.3\ \mathrm{mK}$

$l_{\mathrm{vdW}} \simeq 5.4\ \mathring{\mathrm{A}}$ $\Rightarrow$ $\dfrac{R}{a_2} \simeq \sqrt{m_N B_2}\, l_{\mathrm{vdW}} \approx \dfrac{1}{20}$

$Q_2 \equiv \sqrt{mB_2}$



multipole expansion of interactions in

$QR \equiv Q/M_{\rm hi}$

singular interactions regularization

e.g. $\delta_\Lambda(\vec r) = \left(\frac{\Lambda}{2\sqrt{\pi}}\right)^3 \exp\left(-\frac{\Lambda^2 r^2}{4}\right)$

$$V_2(\vec r) = \frac{4\pi}{m}\left[C_0(\Lambda)\, \delta_\Lambda^{(3)}(\vec r) + C_2(\Lambda)\, R^2\, \nabla^2 \delta_\Lambda^{(3)}(\vec r) + \ldots\right]$$

leading order

renormalization

next-to-leading order

Short-Range Effective Field Theory

vK '97'98
Bedaque + vK '97
Kaplan, Savage, Wise '98
...

cf. Bethe, Peierls '35

order by order

Bethe '49

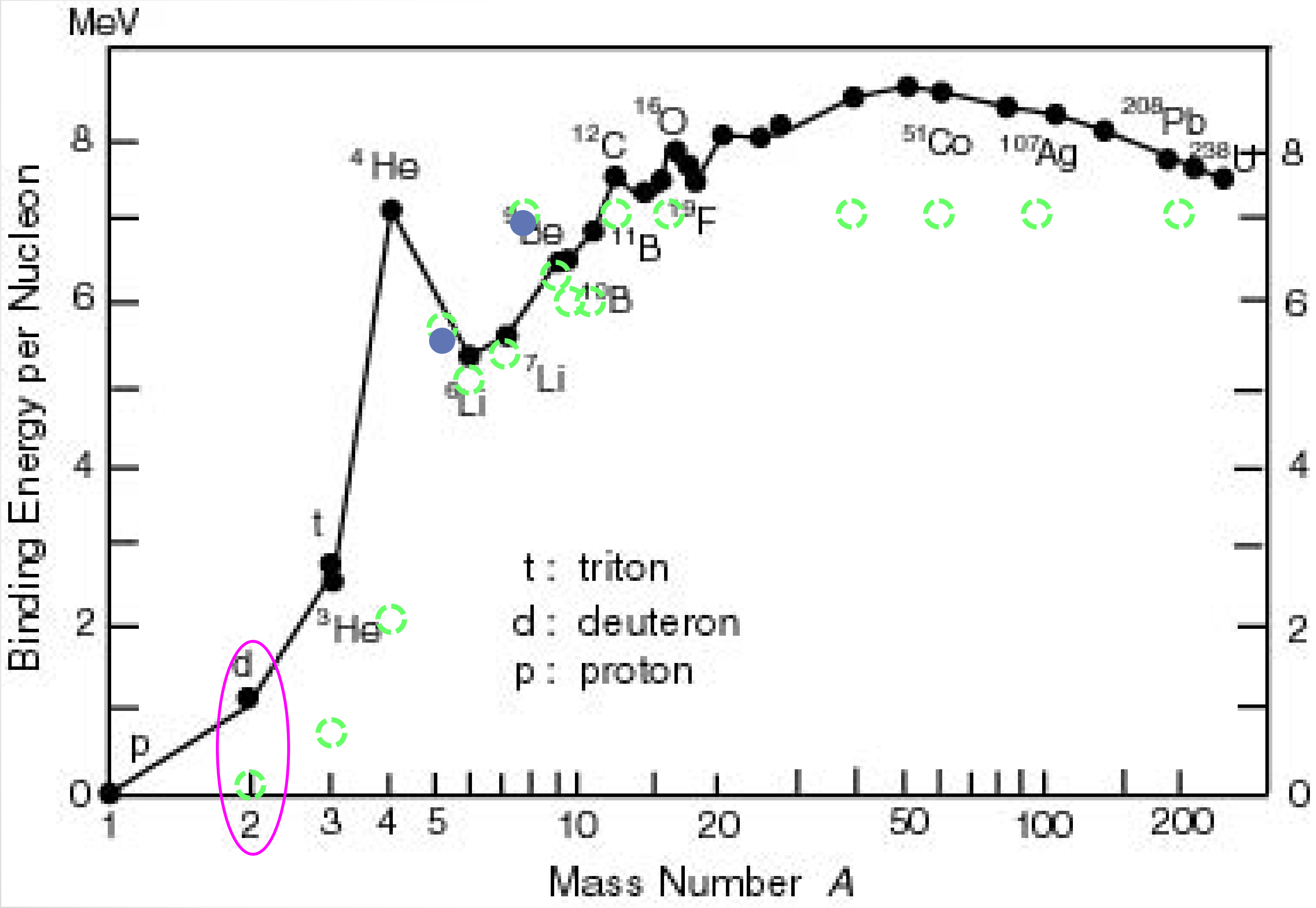
$$\Rightarrow\quad T_2(k \ll R^{-1}) = \frac{4\pi}{ikm}\left(1 + \frac{1}{ika_2} + \frac{ikr_2}{2} + \ldots\right)^{-1} \quad [+\, l > 0]$$

scattering length effective range

$k = \sqrt{mE}$ $\quad a_2^{-1} \approx \sqrt{mB_2}$ $\quad r_2 \sim \ldots \sim R$

typically

Two-Body Unitarity



Kenjiro Takada
https://ne.phys.kyushu-u.ac.jp/seminar/MicroWorld3_E/3Part3_E/3P34_E/nuclear_fusion_E.htm

$$A \geq 5$$

$$\left| \frac{B_A}{A} + \frac{E_{\text{th}}}{A} \right| \ll \frac{B_A}{A}$$

expect lots of clustering

$$\frac{B_2}{2} \ll \left(\frac{B_A}{A} \right)_{A \geq 3}$$

near unitarity limit:

$$\frac{B_2}{2} \to 0$$

near unitarity limit

LO

no scale!

(continuous) scale invariance

$$mE_2 = 0 \Leftrightarrow \alpha^{-2}\, mE_2 = 0$$

no two-body bound states...

subLOs

perturbative explicit breaking of scale invariance

additional expansion in $(Qa_2)^{-1}$

$$\Rightarrow \quad T_2(k \ll R^{-1}) = -\frac{4\pi}{ikm}\left[1 + (ika_2)^{-1} + \frac{ikr_2}{2} + \ldots\right]^{-1}$$

unitarity limit

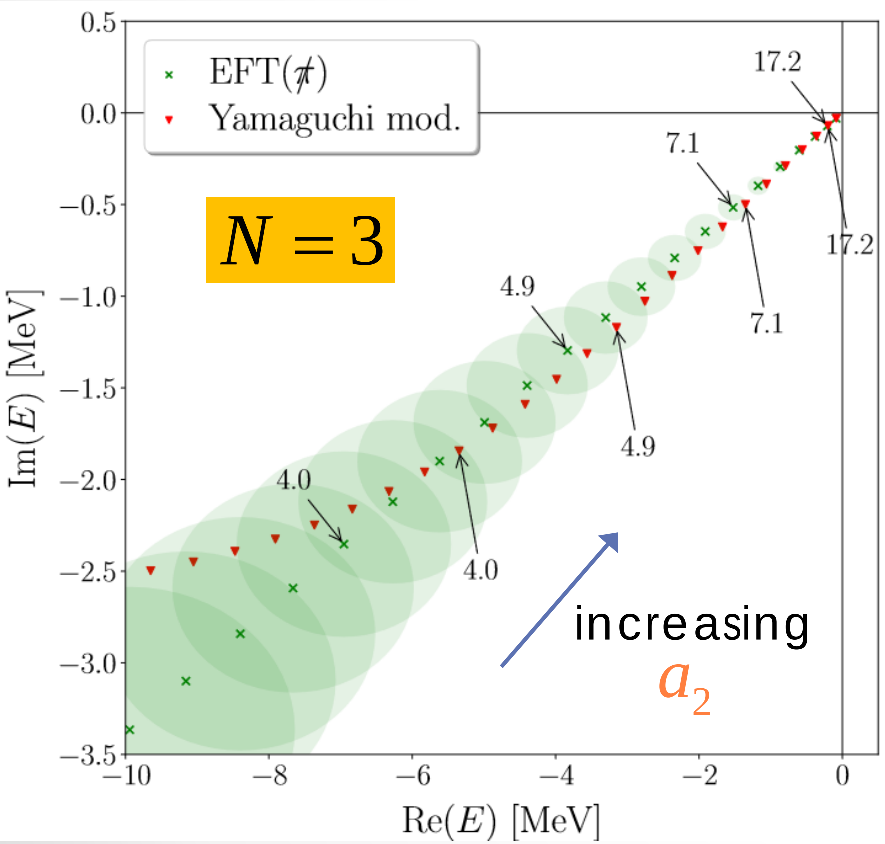
perturbative expansion in

$(Qa_2)^{-1}$ $\quad QR$

More bodies

Two-component fermions

e.g., neutrons



Dietz, Hammer, König, Schwenk, *Phys. Rev. C* **105** (2022) 064002

no isolated, finite, *low*-energy *S*-matrix poles

LO

cf. Oresme

unless
scale introduced by external interaction/trap

$$\left.\frac{E_N^{(0)}}{N}\right|_{N\to\infty} = \frac{3k_F^2}{10m}\left(\xi + \mathcal{O}\left(\frac{1}{k_F a_2}, k_F r_2\right)\right)$$

free-gas energy, uniform density

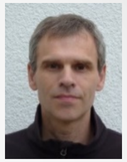
universal number

Bertsch '99

$$k_F = \left(3\pi^2\rho\right)^{1/3}$$

Other cases?

Bedaque, Hammer + vK ’99 ’00



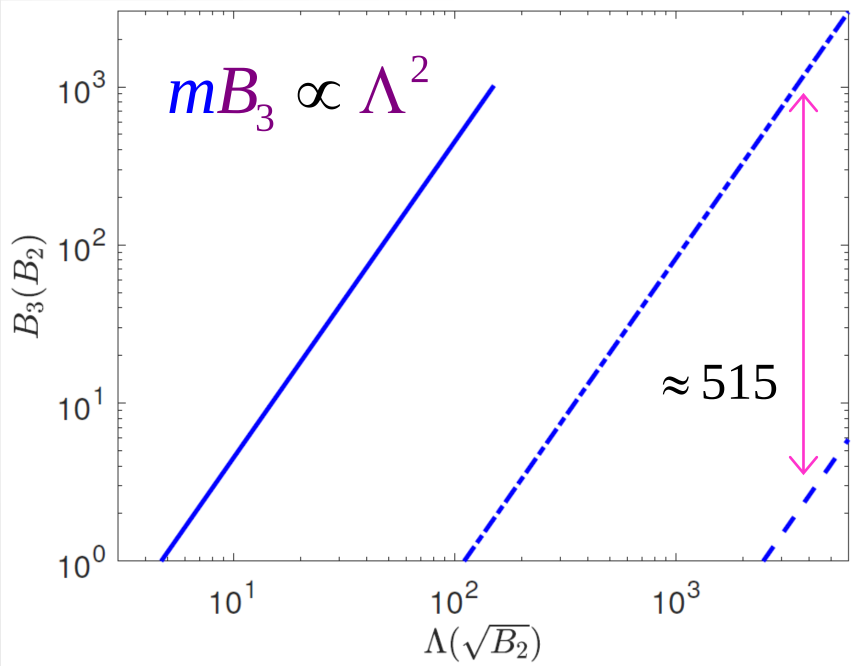
...

Bosons, more-component fermions

$A = 3$

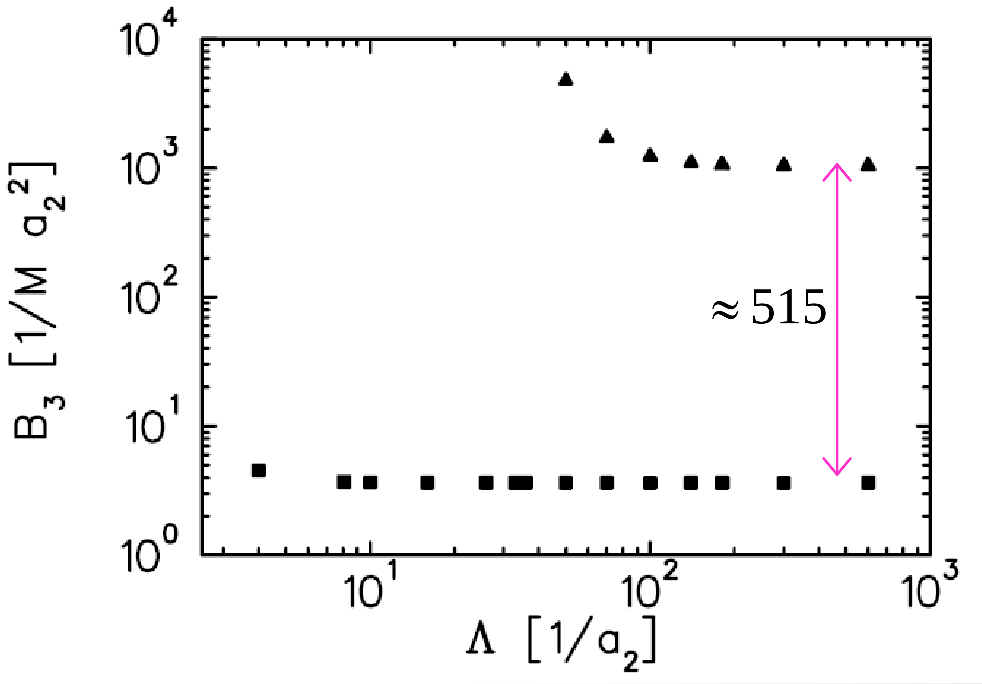
LO

Feng Wu ’22



“Thomas collapse”

Thomas ‘35



Bedaque, Hammer, vK, *Nucl. Phys. A* 676 (2000) 357

$$V_3(\vec{r}_1-\vec{r}_2,\vec{r}_2-\vec{r}_3)=\frac{(4\pi)^2}{m}D_0(\Lambda)\,\delta_\Lambda^{(3)}(\vec{r}_1-\vec{r}_2)\,\delta_\Lambda^{(3)}(\vec{r}_2-\vec{r}_3)$$

one parameter

determined by one three-body datum

Bedaque, Hammer + vK ’99 ’00 ...

$$H(\Lambda) \equiv \frac{\Lambda^2 D_0(\Lambda)}{m C_0^2(\Lambda)} \simeq \frac{\sin\left(s_0 \ln(\Lambda_*/\Lambda) - \arctan(s_0^{-1})\right)}{\sin\left(s_0 \ln(\Lambda_*/\Lambda) + \arctan(s_0^{-1})\right)}$$

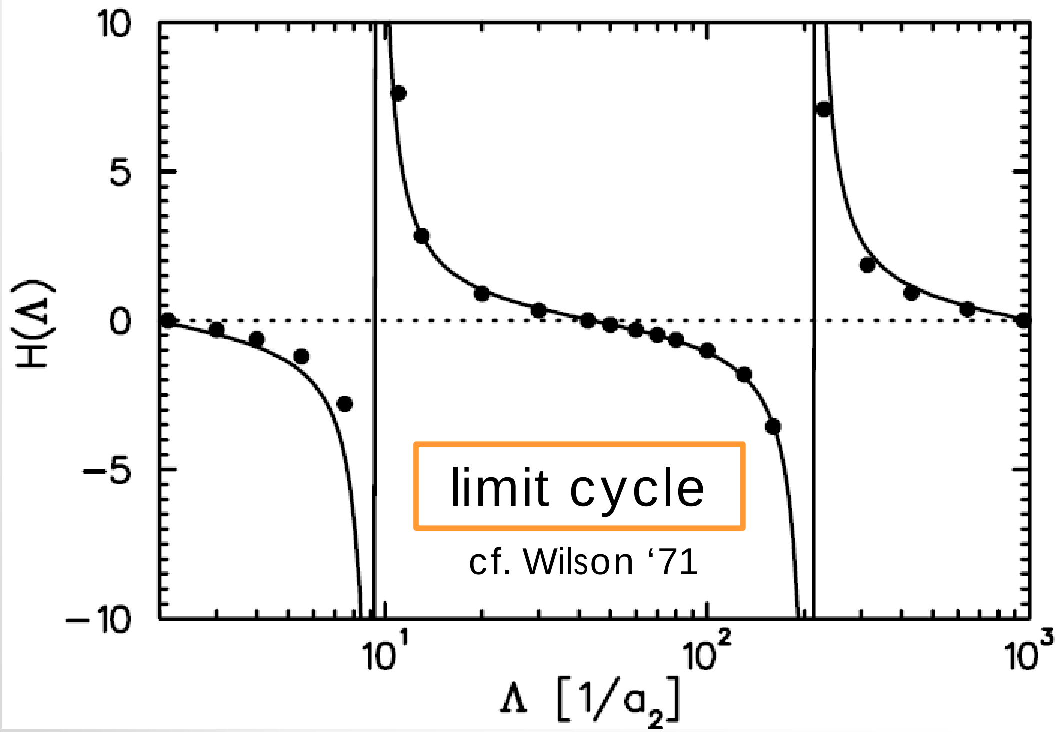
$$s_0 \simeq 1.00624$$

dimensionful parameter

anomalous breaking of (continuous) scale invariance

quantum phenomenon!

$$\xrightarrow{\quad f^{-1} \quad}$$



Bedaque, Hammer, vK, *Phys. Rev. Lett.* **82** (1999) 463

$$\Lambda \to \alpha_n^{-1}\Lambda = f^{-n}\Lambda$$

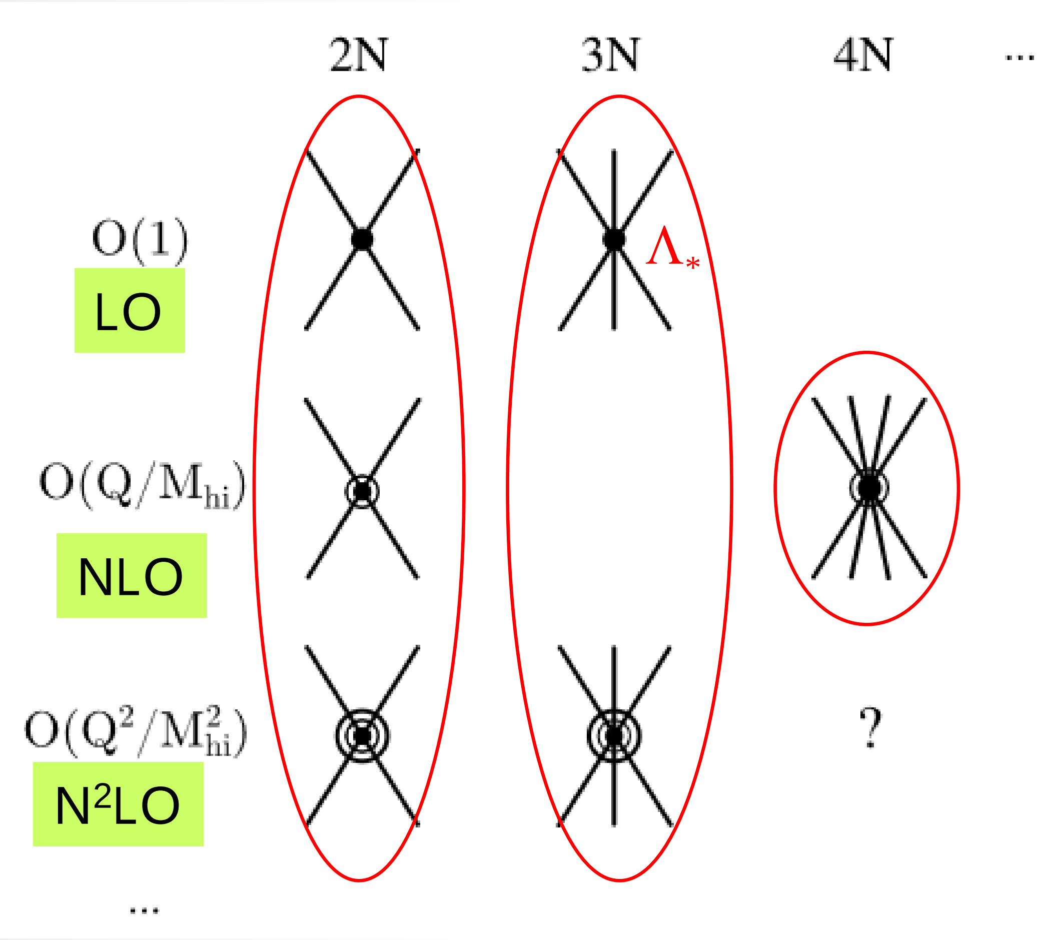
$$f^{-1} = \exp\left(\pi/s_0\right) \simeq 22.7$$

Discrete
scale invariance

STRUCTURE FROM A SINGLE SCALE

Potential

bosons



vK ’97’99
Kaplan, Savage,
Wise ’98
...

Bedaque,
Hammer,
vK ’99’00
...

Bazak, Kirscher,
König, Pavón,
Barnea, vK ’19
...

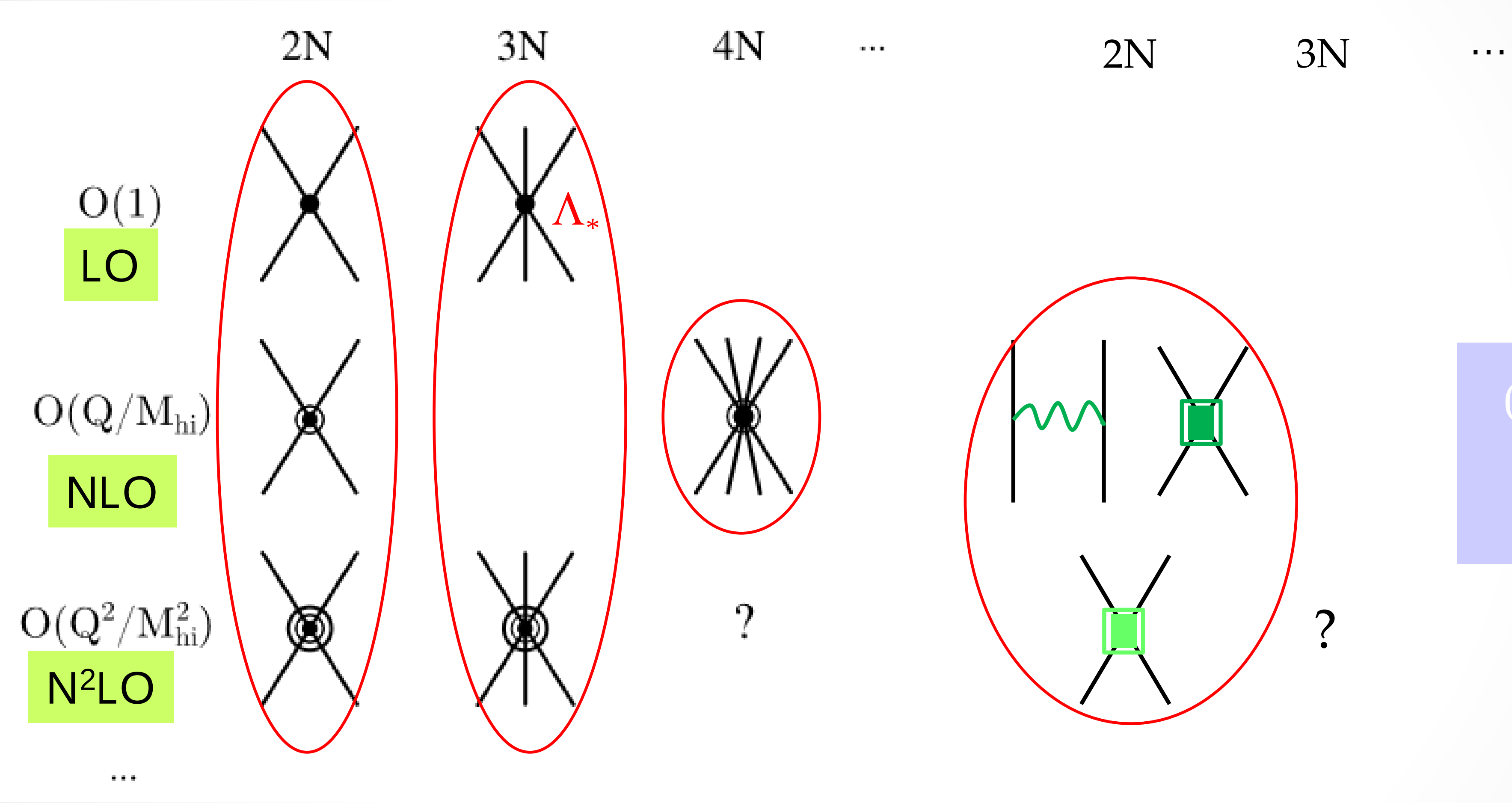
discrete scale invariance!

(distorted-wave) perturbation theory

Hammer, König, vK, *Rev. Mod. Phys.* **92** (2020) 025004

Potential

isospin-symmetric isospin-violating



vK ’97’99
Kaplan, Savage, Wise ’98
...

Bedaque, Hammer, vK ’99’00
...

Bazak, Kirscher, König, Pavón, Barnea, vK ’19
...

König, Griesshammer, Hammer, vK ’16
...

discrete scale invariance!

(distorted-wave) perturbation theory

Hammer, König, vK, *Rev. Mod. Phys.* **92** (2020) 025004

Two consequences

1) Towers of excited states

$$mB^{(0)}_{A,n} \to \alpha_l^{-2}\, mB^{(0)}_{A,n} = mB^{(0)}_{A,n+l}$$

$$\Rightarrow\; mB^{(0)}_{A,n}(\Lambda_*) = mB^{(0)}_{A,0}(\Lambda_*)\exp\left(-2n\pi/s_0\right)$$

ground state

fixes tower position

$A=3$ Efimov '70

$A=4$ Hammer, Platter '07

$A=5,6$ bosons von Stecher '10'11 Gattobigio, Kievsky, Viviani '11'12

Towers upon towers!

$A=3$ $A=4$

$n+1$ $m=1$

$m=0$

$\simeq 515$

$\simeq 22.7$

n $m=1$

$m=0$

Efimov state

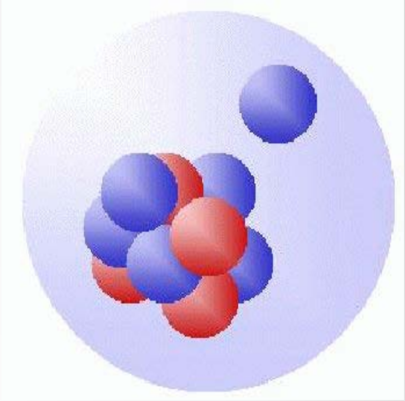
Efimov descendants

Wu, Frederico, Higa + vK '24

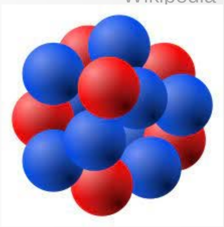
Halo EFT

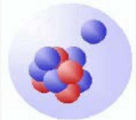
halo!

H. Maridi



Wikipedia





2) Ground-state correlations

single scale $\Rightarrow$ $$\frac{B^{(0)}_{A,0,0}(\Lambda_*)}{A} = \kappa_A \frac{B^{(0)}_{3,0}(\Lambda_*)}{3}$$

universal numbers

$$\begin{cases} \kappa_2 \equiv 0 \\ \kappa_3 \equiv 1 \\ \kappa_4 \simeq 3.5 \\ \kappa_{A\geq 5} \simeq ? \end{cases}$$

Hammer, Platter '07

von Stecher '10

...

Carlson, Gandolfi, Vitiello + vK '17

varying Λ_*

$A=4$

Tjon line

Platter, Hammer, Meißner '05

Tjon '75

Nakaichi, Akaishi, Tanaka, Lim '78

$A=5,6$

bosons

Generalized Tjon lines

Bazak, Eliyahu + vK '16

Nakaichi, Akaishi, Tanaka, Lim '79'80

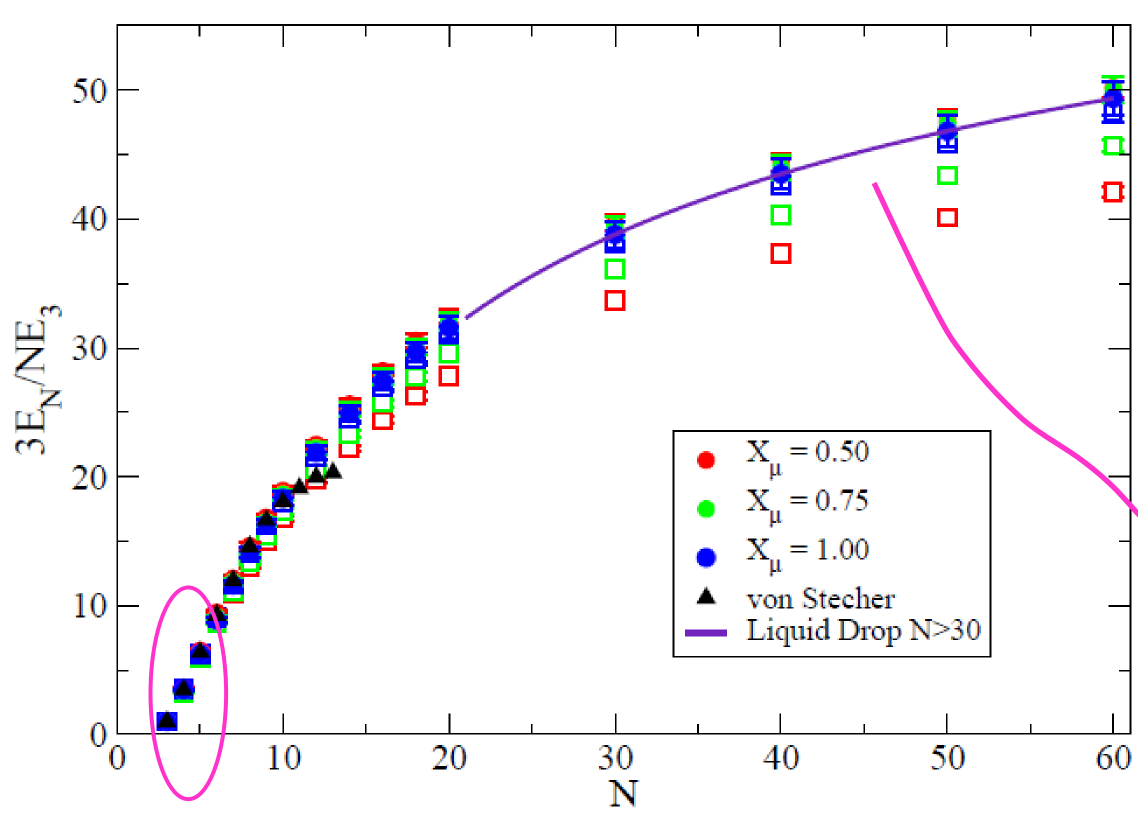
fixing Λ_*

$$\frac{B^{(0)}_{A,0,0}(B_{3,0})}{A} = \kappa_A \frac{B_{3,0}}{3}$$

Gandolfi, Carlson, Vitiello + vK ’17

unitary bosons

Carlson, Gandolfi, Vitiello, vK,
Phys. Rev. Lett. **119** (2017) 223002



increasing Λ

LO

Quantum Monte Carlo

saturation!

cf. Piatecki + Krauth ’14
Comparin + Krauth ‘16

$$\kappa_N \approx \frac{3}{N}(N-2)^2$$

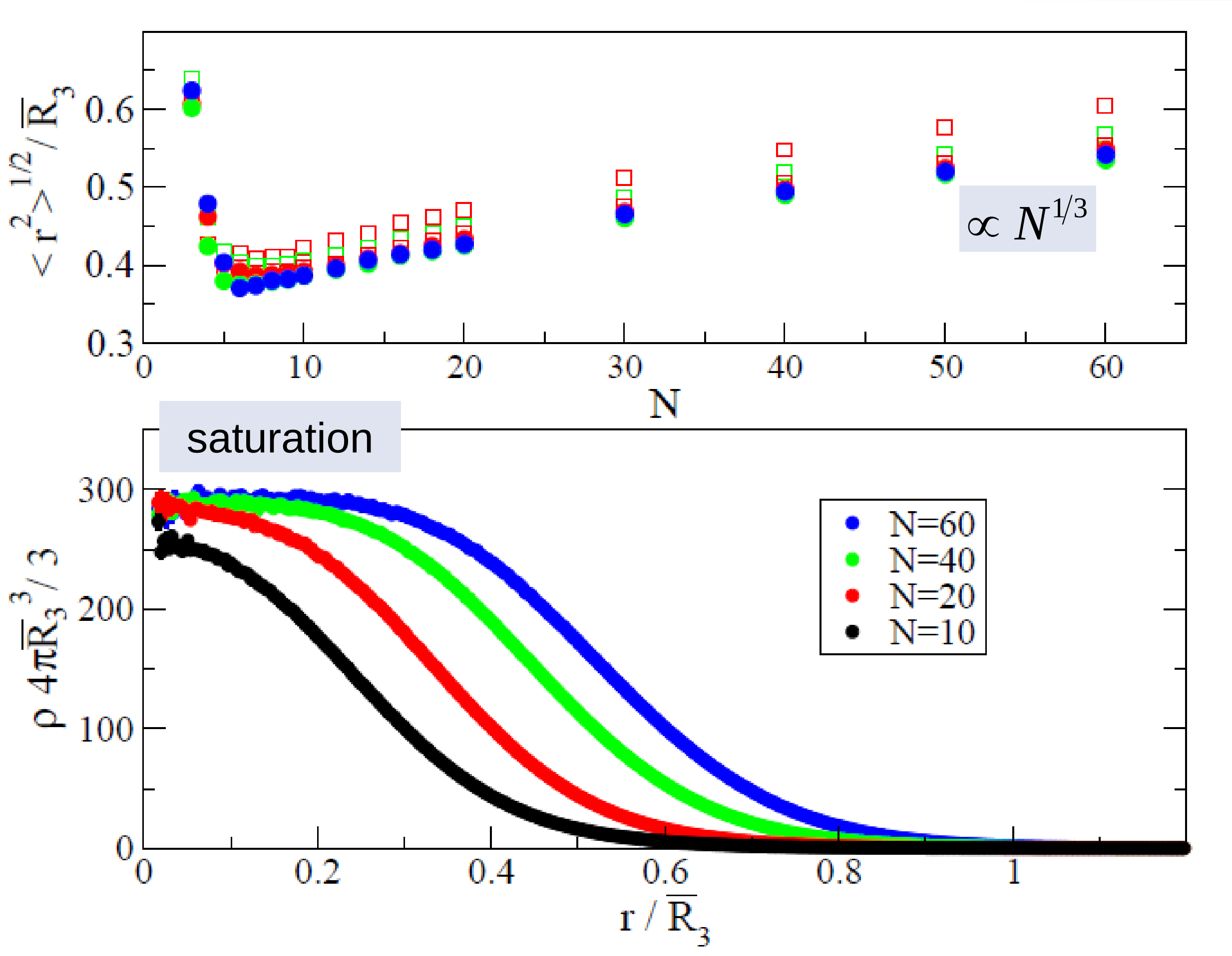
Bazak, Eliyahu + vK ‘16

$$\kappa_N = \kappa_\infty \left[1 - \eta_S N^{-1/3} + \mathcal{O}\left(N^{-2/3}\right)\right]$$

$$\kappa_\infty = 90 \pm 10 \qquad \eta_S = 1.7 \pm 0.3$$

Gandolfi, Carlson, Vitiello + vK ’17

A liquid indeed...



Carlson, Gandolfi, Vitiello, vK, *Phys. Rev. Lett.* **119** (2017) 223002

$$\overline{R}_3 \equiv \left(2mB_3\right)^{-1/2}$$

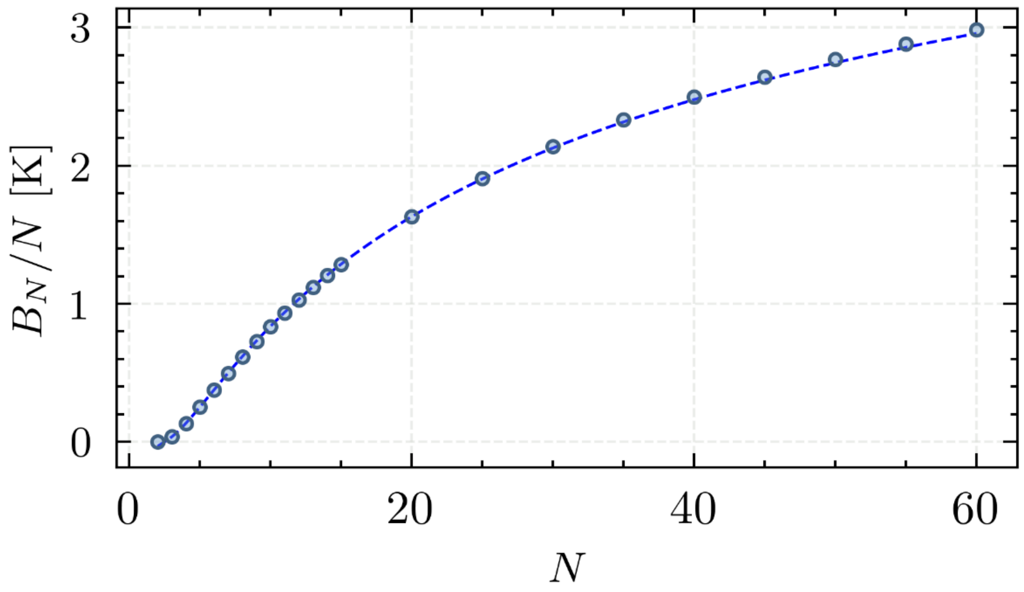
cf. ^{4}He

Pandharipande *et al.* '83
Madeira, Pederiva + vK, in progress

HFDHE2
phenomenological potential

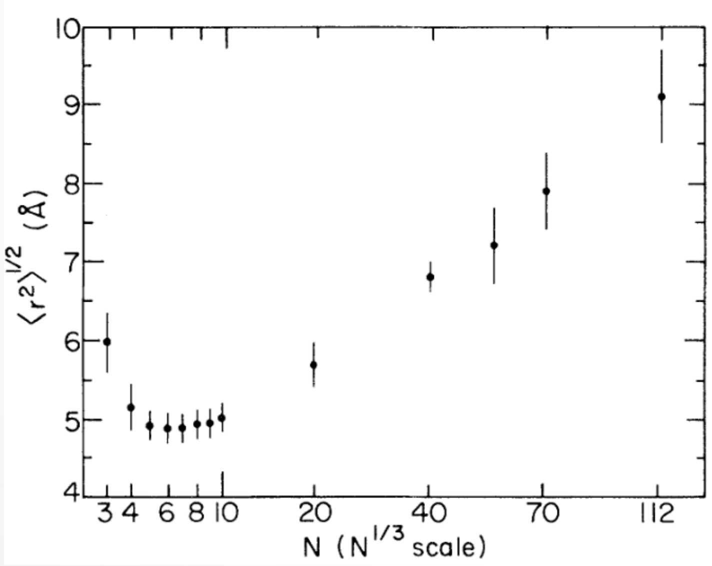
$$\frac{B_3(\Lambda_*)}{3} \simeq 0.013\ \text{K}$$

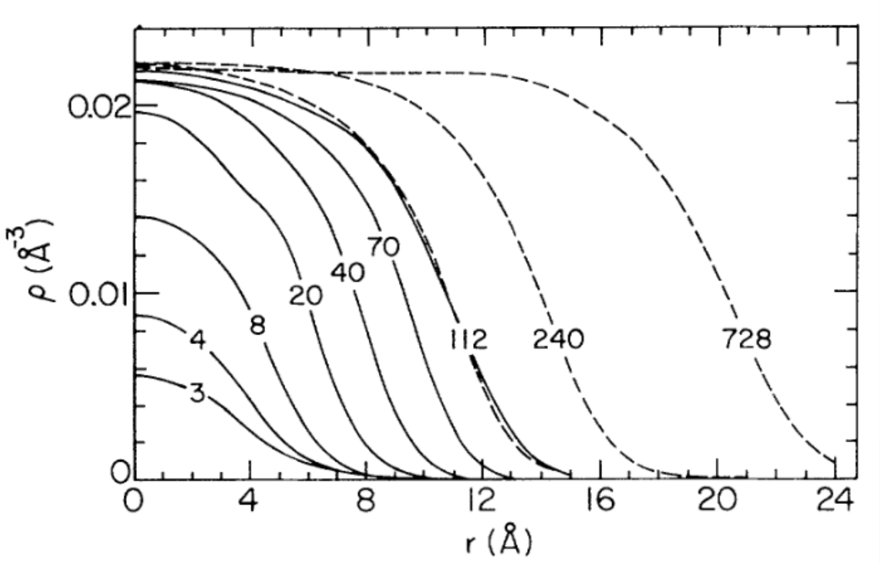
Lucas Madeira '25



$$\kappa_\infty \simeq 182$$

$$\eta_S \simeq 2.7$$





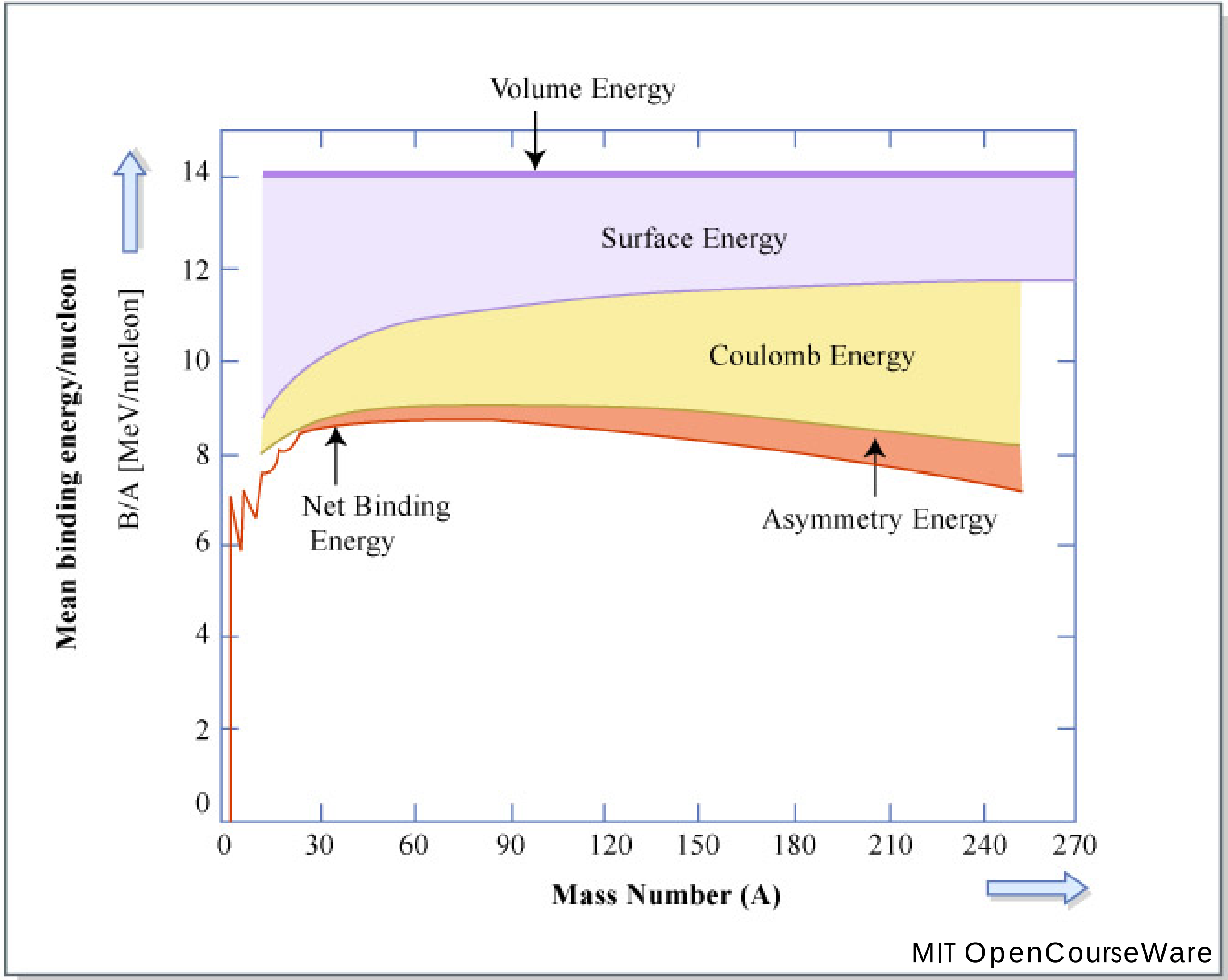
Pandharipande, Zabolitzky, Pieper, Wiringa, Helmbrecht , *Phys. Rev. Lett.* **50** (1983) 1676

Semi-empirical mass formula

Von Weizsäcker '35

$$\kappa_A = \kappa_\infty \left[1 - \eta_S A^{-1/3} - \eta_C A^{-4/3} Z(Z-1) - \eta_A (1 - 2Z/A)^2 + \ldots \right]$$

$$\frac{B_3(\Lambda_*)}{3} \simeq 2.8\ \mathrm{MeV}$$

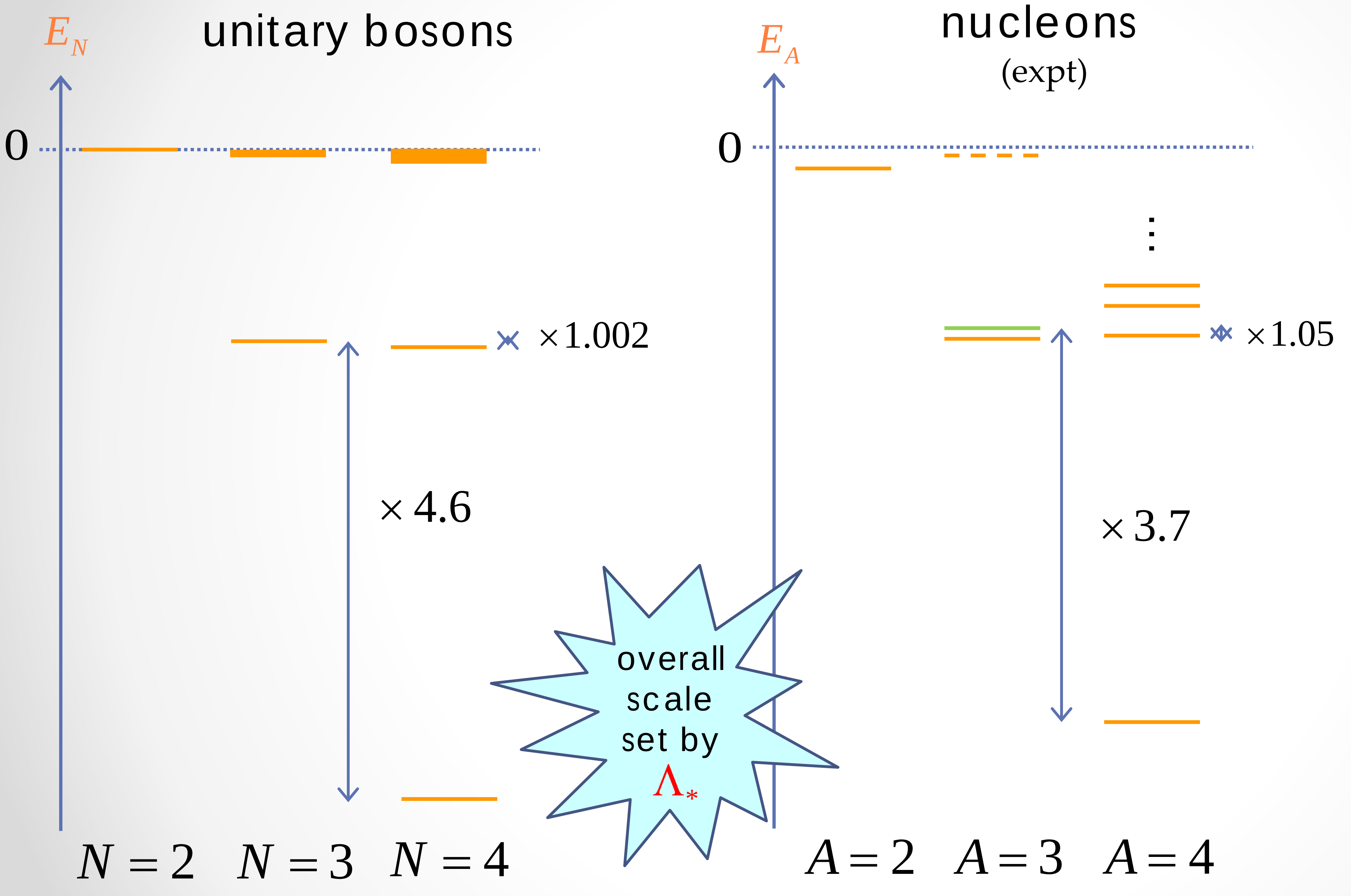


$\kappa_\infty \approx 5.6$

$\eta_S \approx 1.2$

$\eta_C \approx 0.05$

$\eta_A \approx 1.5$



$$\frac{3|B_2|}{2|B_3|} \simeq 0.4 < 1$$

subleading

virtual state in neutron-deuteron scattering

Rupak, Vaghani, Higa + vK ’18

^{3}He-^{3}H isospin splitting obtained at NLO

König, Grießhammer, Hammer + vK ’16

correction obtained at NLO

König, Grießhammer, Hammer + vK ’17

Nucleons around unitarity

LO

discrete scale invariance

Multiple towers of excited states?

Ground states

single scale $\Rightarrow$ $$\frac{B_A^{(0)}(\Lambda_*)}{A} = \kappa_A \frac{B_3(\Lambda_*)}{3}$$

$$\begin{cases} \kappa_2 \equiv 0 \\ \kappa_3 \equiv 1 \\ \kappa_4 \simeq 3.5 \\ \kappa_{A\geq 5} \simeq ? \end{cases}$$

= bosons

grows slower than bosons?

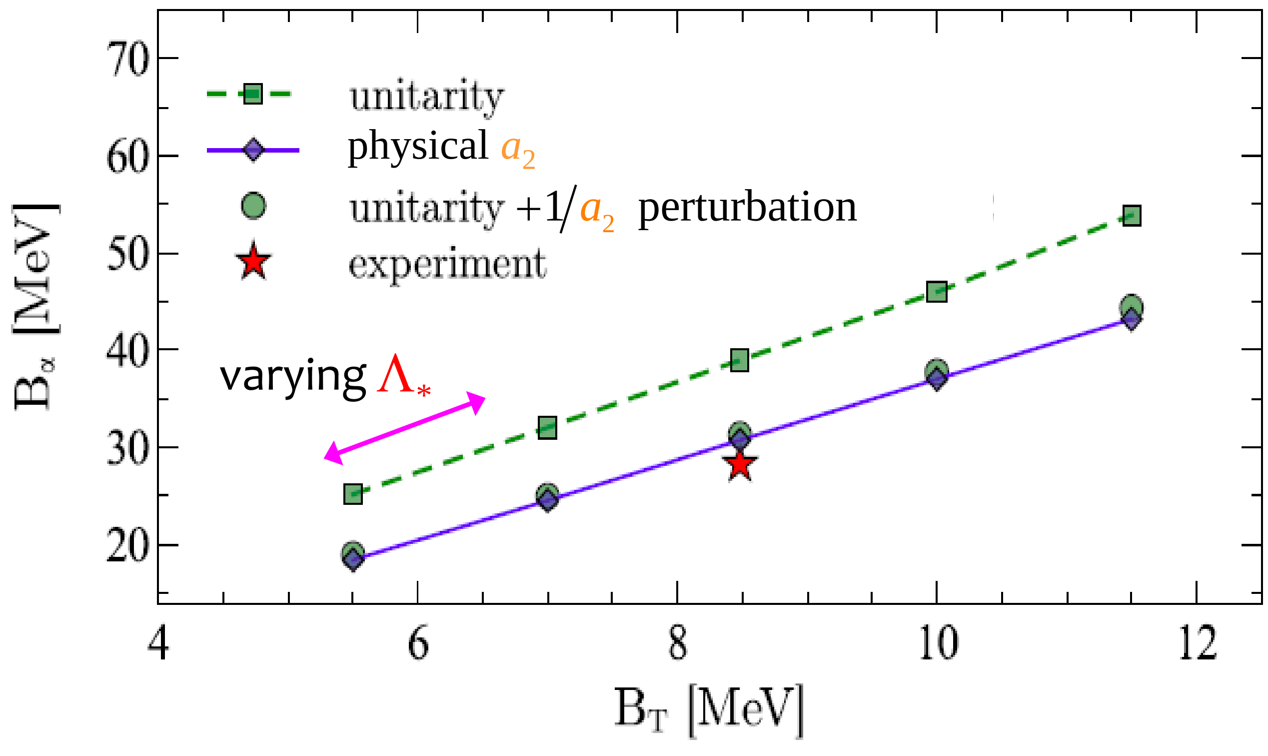
nuclear structure from a single parameter?

König, Grießhammer, Hammer + vK '17

$A = 4$

Tjon line

König, Grießhammer, Hammer, vK, *Phys. Rev. Lett.* **118** (2017) 202501



LO

incomplete NLO

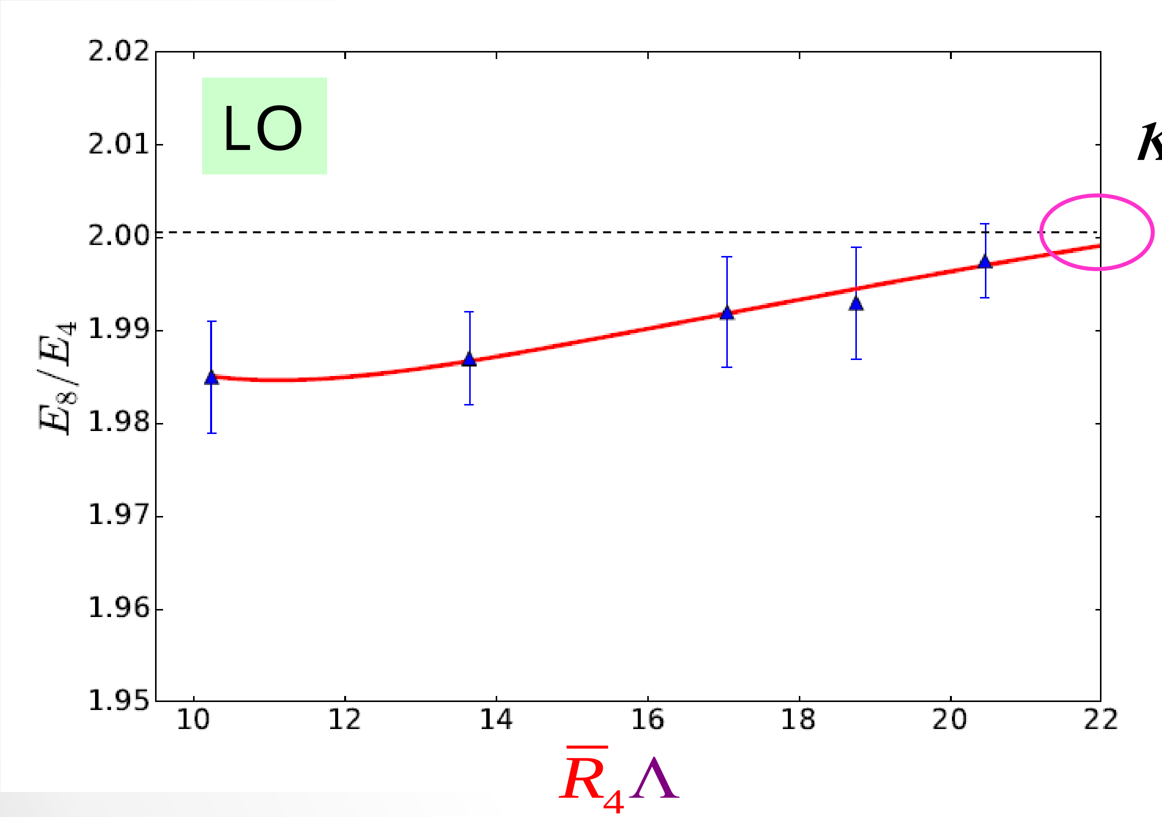
perturbatively close to unitarity limit!

Similar for ^{4}He atoms   Wu, Lin, König + vK, in progress

see Feng Wu's talk

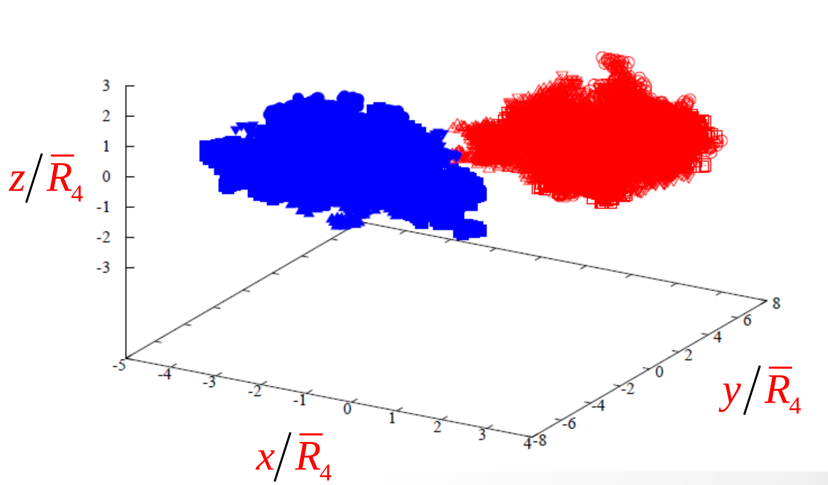
Dawkins, Carlson, vK + Gezerlis ’20

$A = 8$



$$\kappa_8 \simeq \kappa_4\ ?$$

$$\bar{R}_4 \equiv \left(2mB_4\right)^{-1/2}$$



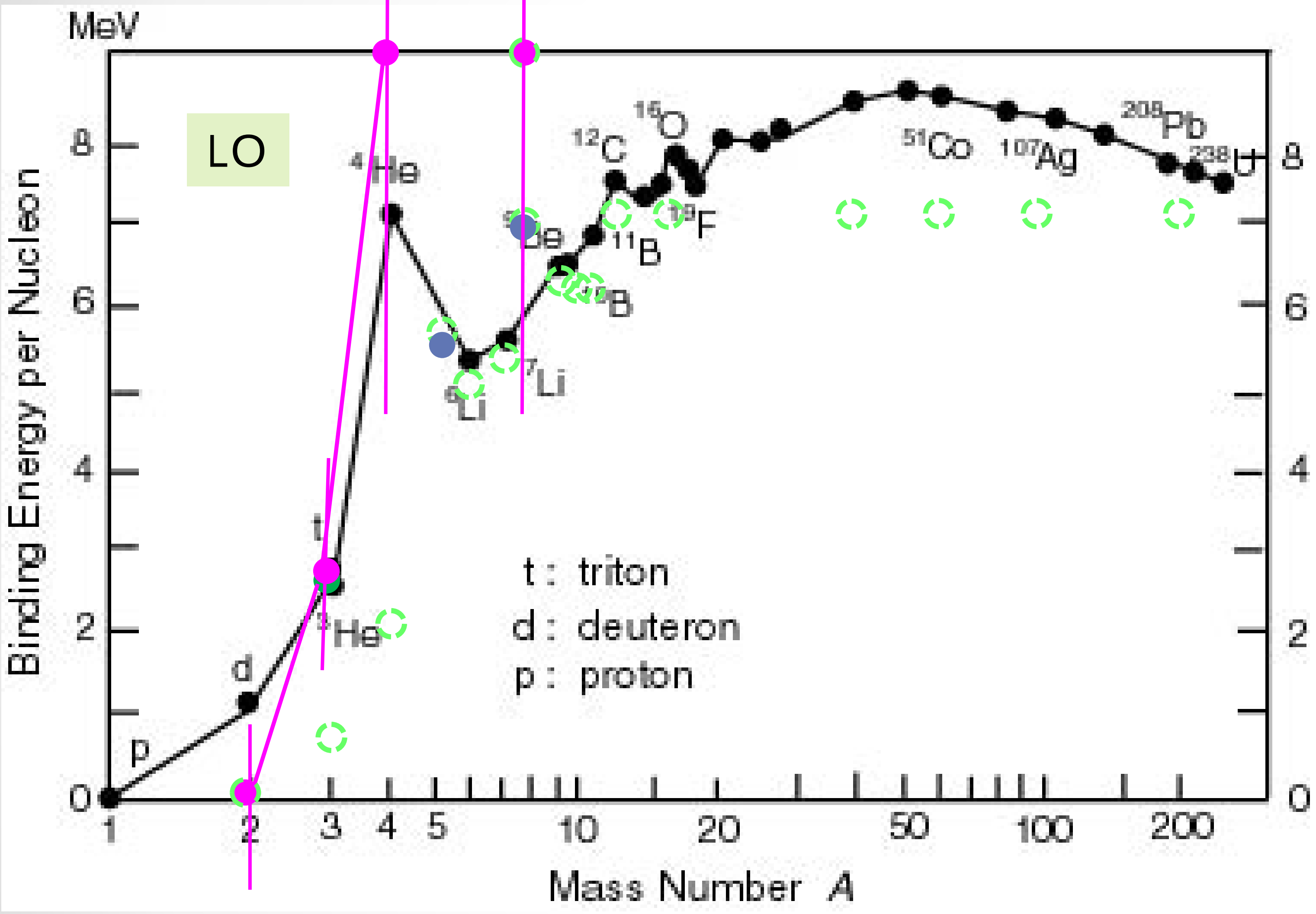
consistent with ^{8}Be

Dawkins, Carlson, vK, Gezerlis, *Phys. Rev. Lett.* **124** (2020) 143402

Dawkins, Carlson, vK, Gezerlis ‘20
Schäfer, Contessi, Kirscher, Mareš ’20
Contessi, Schäfer, Kirscher, Lazauskas, Carbonell ‘23

renormalization + near two-body unitarity: clustering of multicomponent fermions

Instability beyond alpha particle at LO



https://ne.phys.kyushu-u.ac.jp/seminar/MicroWorld3_E/3Part3_E/3P34_E/nuclear_fusion_E.htm

Practical problem: how to do (distorted-wave) perturbation theory?

Contessi, Schäfer, vK ’23
Contessi, Pavón, vK ’24
Contessi, Schäfer, Gnech, Lovato, vK ‘25

Improved Actions

Solution: add to LO *some* subLO corrections, while maintaining renormalization

- no new *physical* parameter at LO
- effect no larger than NLO → removed perturbatively at NLO

$$V^{(0)}(\vec{r};\Lambda)=C_0^{(0)}(\Lambda)\sum_{ij}\delta_\Lambda(\vec{r}_{ij})+D_0^{(0)}(\Lambda)\sum_{ijk}\delta_\Lambda(\vec{r}_{ij})\delta_\Lambda(\vec{r}_{ik})$$

(neglecting spin-isospin factors)

$$\Rightarrow \quad \tilde{V}^{(0)}(\vec{r};\Lambda)=V^{(0)}(\vec{r};\Lambda)+\Delta V(\vec{r};\Lambda)$$

within $\mathcal{O}\left(Q/M_{\text{hi}}\right)$

$$\Delta V(\vec{r};\Lambda)=\Delta V^{(1)}(\vec{r};\Lambda)+\ldots$$

$$V^{(1)}(\vec{r};\Lambda)=\ldots \quad \Rightarrow \quad \tilde{V}^{(1)}(\vec{r};\Lambda)=V^{(1)}(\vec{r};\Lambda)-\Delta V^{(1)}(\vec{r};\Lambda)$$

within $\mathcal{O}\left(Q^2/M_{\text{hi}}^2\right)$

etc.

Example 1: change of renormalization condition

fit to unitarity limit

$$C_0^{(0)}(\Lambda) = C_0^{(0)}\left(a_2^{(0)-1} = \sqrt{mB_2^{(0)}} = 0; \Lambda\right)$$

fit to two-body scattering length

$$\Delta V(\vec{r};\Lambda, a_2^{-1}, 0) = \left[C_0^{(0)}(a_2^{-1};\Lambda) - C_0^{(0)}(\Lambda)\right]\sum_{ij}\delta_\Lambda(\vec{r}_{ij})$$

infrared resummation of the expansion in $\left(Qa_2\right)^{-1}$

fit to A-body binding energy

$$\Delta V(\vec{r};\Lambda, \sqrt{mB_A}, 0) = \left[C_0^{(0)}(\sqrt{mB_A};\Lambda) - C_0^{(0)}(\Lambda)\right]\sum_{ij}\delta_\Lambda(\vec{r}_{ij})$$

$A > 4$

difficult to discern a potential failure of the expansion as density increases

cf. $\mathrm{NNLO_{sat}}$ chiral pot   Ekström *et al.* '15

^{4}He atomic clusters

$A \leq 6$

LO — Bazak, Eliyahu + vK ‘16

NLO — Bazak, Kirscher, König, Pavón, Barnea + vK ‘18

$A \geq 2$

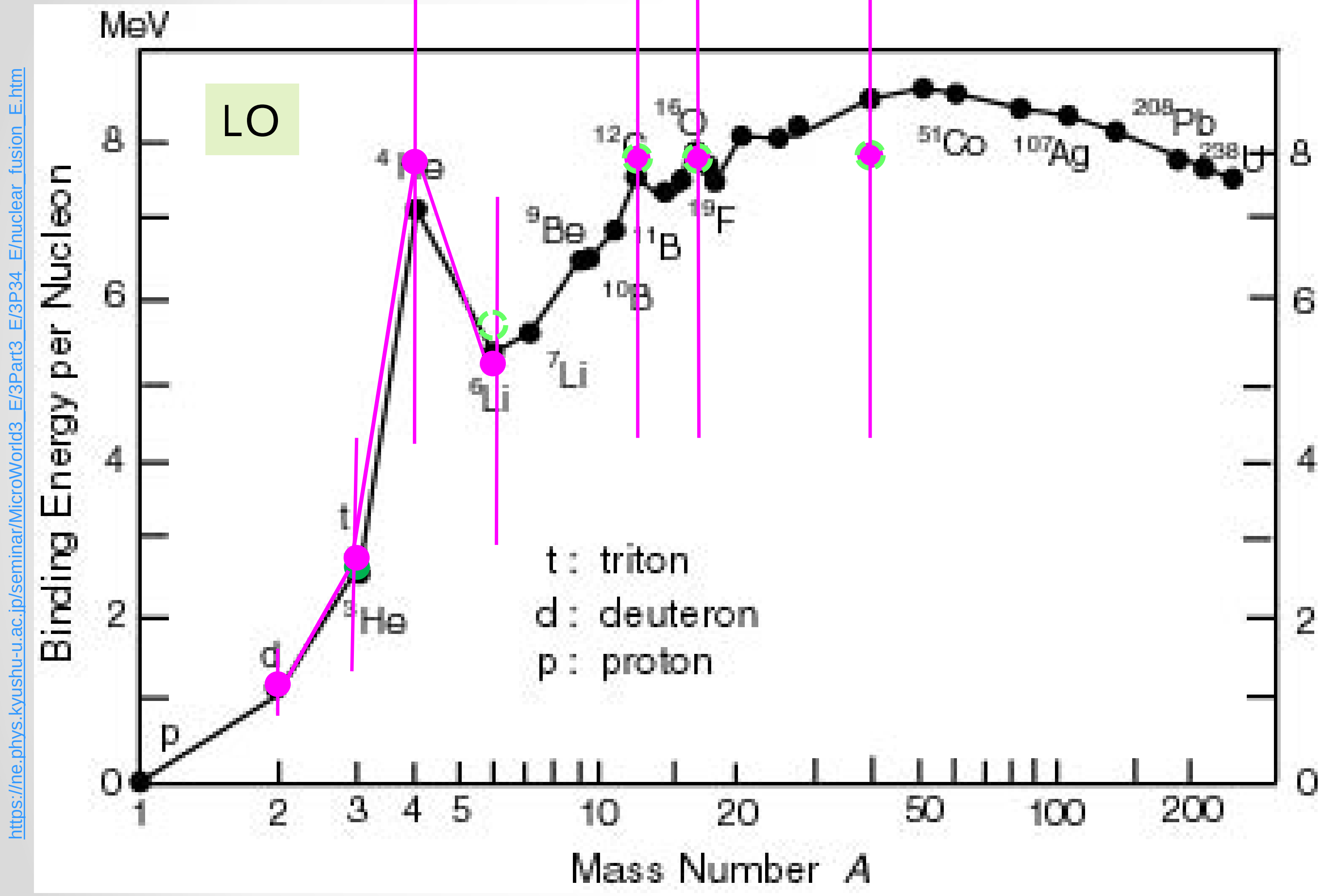
LO

NLO

Madeira, Pederiva + vK, in progress

see
Madeira’s talk

Instability beyond alpha particle at LO



^{6}Li — Stetcu, Barrett, vK '07; Contessi, Schäfer, Gnech, Lovato, vK '25

but n-α scattering good — Bagnarol, Schäfer, Bazak, Barnea '23

^{12}C — Contessi, Schäfer, Gnech, Lovato, vK '25

^{16}O — Contessi, Lovato, Pederiva, Roggero, Kirscher, vK '17; Bansal, Binder, Ekström, Hagen, Jansen, Papenbrock '18; Contessi, Schäfer, Gnech, Lovato, vK '25

^{40}Ca — Bansal, Binder, Ekström, Hagen, Jansen, Papenbrock '18

renormalization + near two-body unitarity: clustering of multicomponent fermions

Example 2: partial resummation of the two-body effective range

Contessi, Pavón, vK '24

(introduce dimer field)

$$\Delta V(\vec{r};\Lambda,a_2^{-1},x\,r_2/2,0)=\left[C_0^{(0)}(\sqrt{mE};\Lambda,a_2^{-1},x\,r_2/2)-C_0^{(0)}(\Lambda)\right]\sum_{ij}\delta_\Lambda(\vec{r}_{ij})$$

cf. $x=1$ Phillips, Rupak, Savage '00

$A=2$

Toy model

$$T_2(k)=-\frac{4\pi}{ikm}\left[1+\frac{1}{ika_2}+\frac{ikr_2}{2}-P_2\left(\frac{ikr_2}{2}\right)^3+P_3\left(\frac{ikr_2}{2}\right)^5\right]^{-1}$$

$k=\sqrt{mE}$

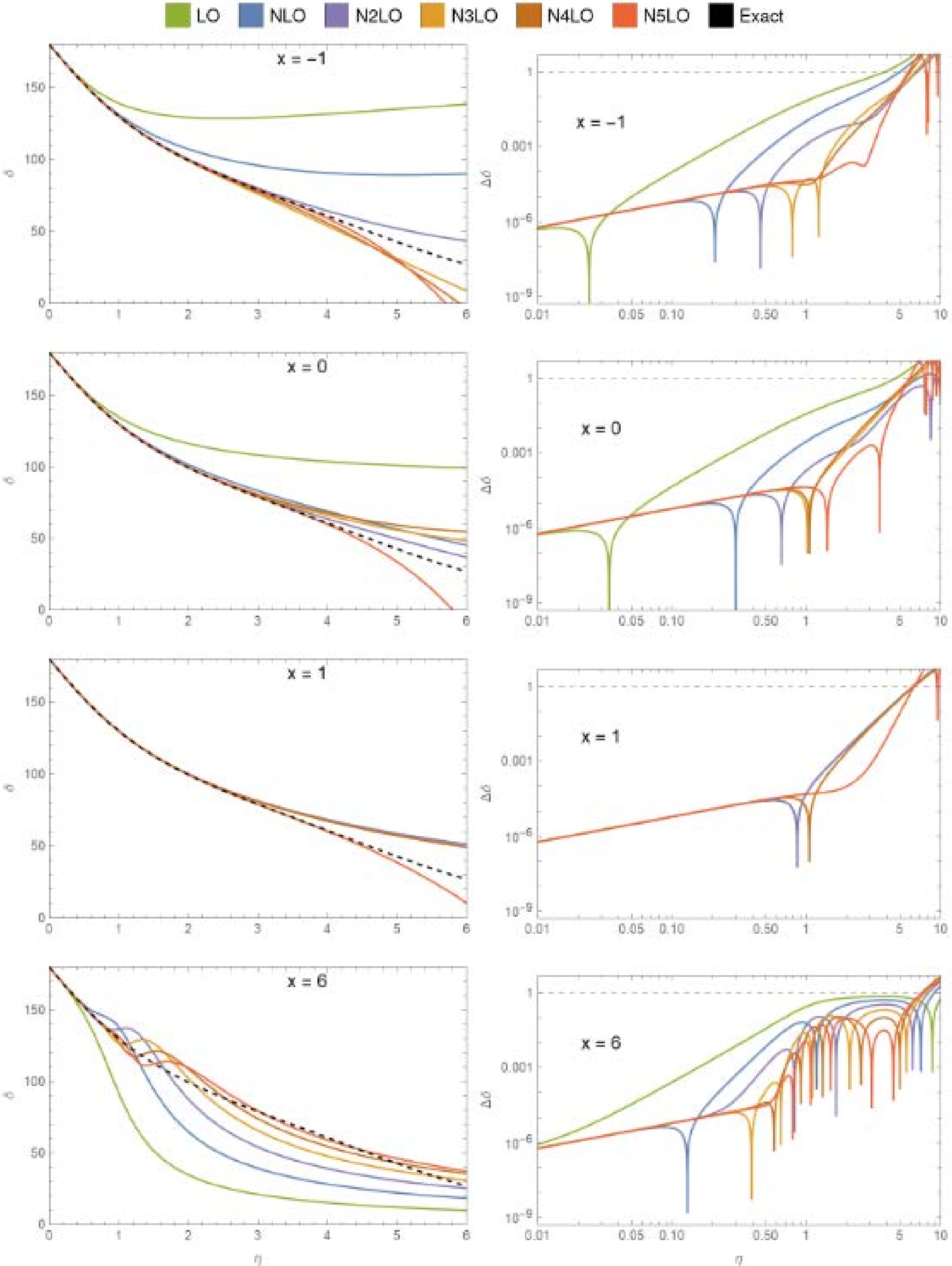
exact with ${}^3S_1$ np parameters

De Swart, Terheggen, Stoks '95

$m_\pi\frac{r_2}{2}\simeq 1.64$ $\alpha\equiv\frac{2a_2}{r_2}\simeq 6.2$ *etc.*

Contessi, Pavón, vK, *Phys. Lett. B* **856** (2024) 138903

phase shift



relative error in phase shift

eff range "anti" resummation

larger lower-order errors
but same breakdown scale

"standard" Short-Range EFT

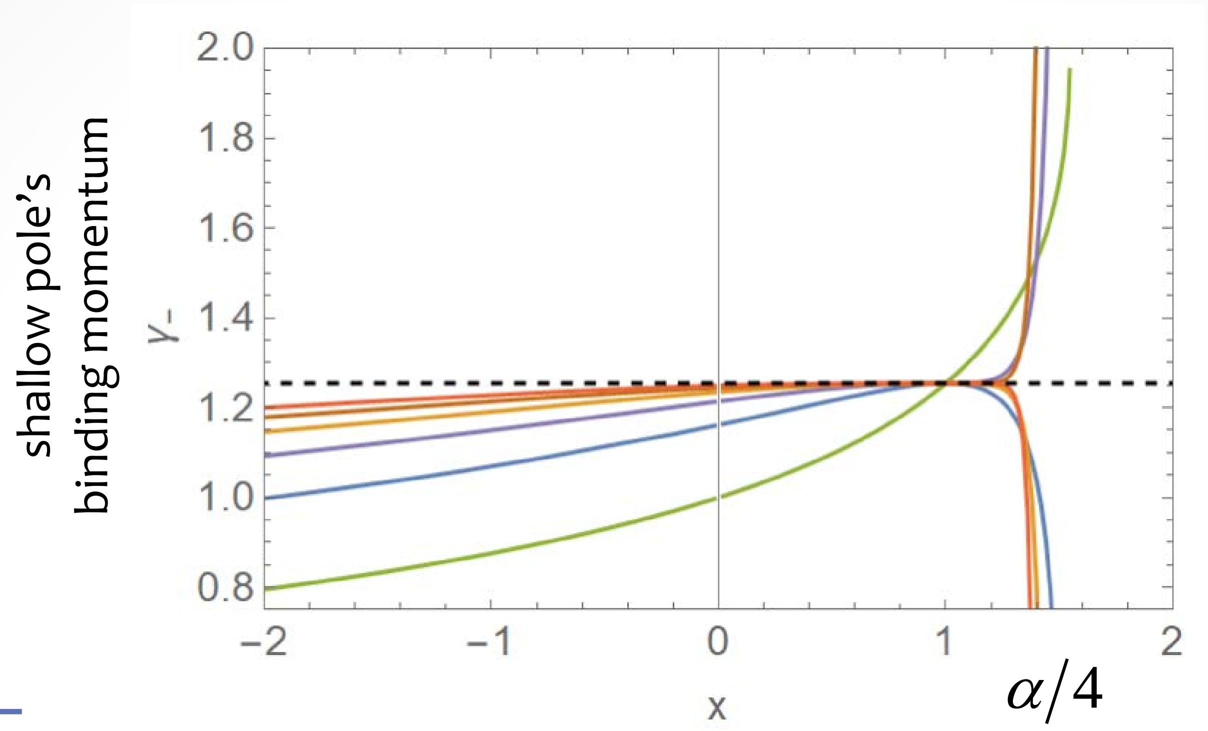
$$M_{\rm hi}\,\frac{r_2}{2} \simeq 1$$ as expected …

full eff range resummation

smaller lower-order errors
but same breakdown scale

near breakdown of
perturbation theory

$$\eta = k a_2$$



Contessi, Pavón Valderrama, vK,
Phys. Lett. B **856** (2024) 138903

two physical poles

one physical pole

γ_+: unphysical virtual state

poles coalesce

$\gamma_+ = \gamma_-$

(not surprising)
moral

- convergence radius and *a priori* error estimates do not change
- central values for various orders within *a priori* error estimates
- central values of lower orders closer to exact for $0 \lesssim x \lesssim 1$

BUT difficult to apply to more bodies

Example 3: partial resummation of a chain of ERE parameters

$$\Delta V(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3) = \sum_{ij}\left[\tilde{C}(\tilde{R}_2^{-1})\,\delta_{\tilde{R}_2^{-1}}(\vec{r}_{ij}) - C_0^{(0)}(\Lambda)\,\delta_\Lambda(\vec{r}_{ij})\right]$$

$$+\sum_{ijk}\left[\tilde{D}(\tilde{R}_2^{-1},\tilde{R}_3^{-1})\,\delta_{\tilde{R}_3^{-1}}(\vec{r}_{ij})\delta_{\tilde{R}_3^{-1}}(\vec{r}_{ik}) - D_0^{(0)}(\Lambda)\delta_\Lambda(\vec{r}_{ij})\delta_\Lambda(\vec{r}_{ik})\right]$$

"fake ranges"

Contessi, Schäfer, vK ‘23

Atomic Clusters

Two improvements

$$\tilde{R}_2 \equiv \tilde{R},\ \tilde{R}_3 = \Lambda^{-1} \Rightarrow \tilde{V}_I^{(0)}(\vec{r};\Lambda,\tilde{R}) = V^{(0)}(\vec{r};\Lambda) + \Delta V(\vec{r};\Lambda,\tilde{R},\Lambda^{-1})$$

$$\tilde{R}_2 = \tilde{R}_3 \equiv \tilde{R} \Rightarrow \tilde{V}_{II}^{(0)}(\vec{r};\tilde{R}) = V^{(0)}(\vec{r};\Lambda) + \Delta V(\vec{r};\Lambda,\tilde{R},\tilde{R})$$

cf. $\Delta V(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3) = 0$ Bazak, Eliyahu, vK ’16

^{4}He

$$M_{\text{hi}} \sim l_{vdW}^{-1} \simeq \left(5\,\mathring{A}\right)^{-1}$$

$$\frac{Q_3}{M_{\text{hi}}} \sim 0.4$$

parameters fitted to

LO two-body scattering length
trimer ground-state energy

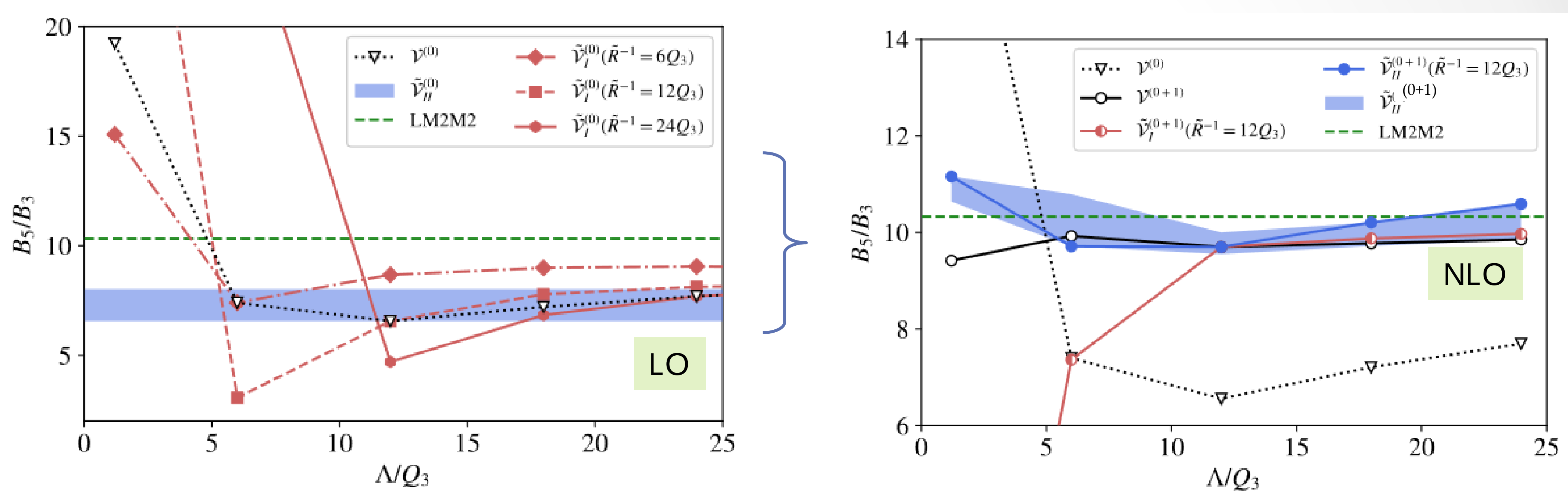
NLO two-body effective range
tetramer ground-state energy

from LM2M2 phenomenological potential

Janzen, Aziz ’95
Kolganova *et al.* ’04
Hiyama, Kamimura ’12
...

Example

$A = 5$

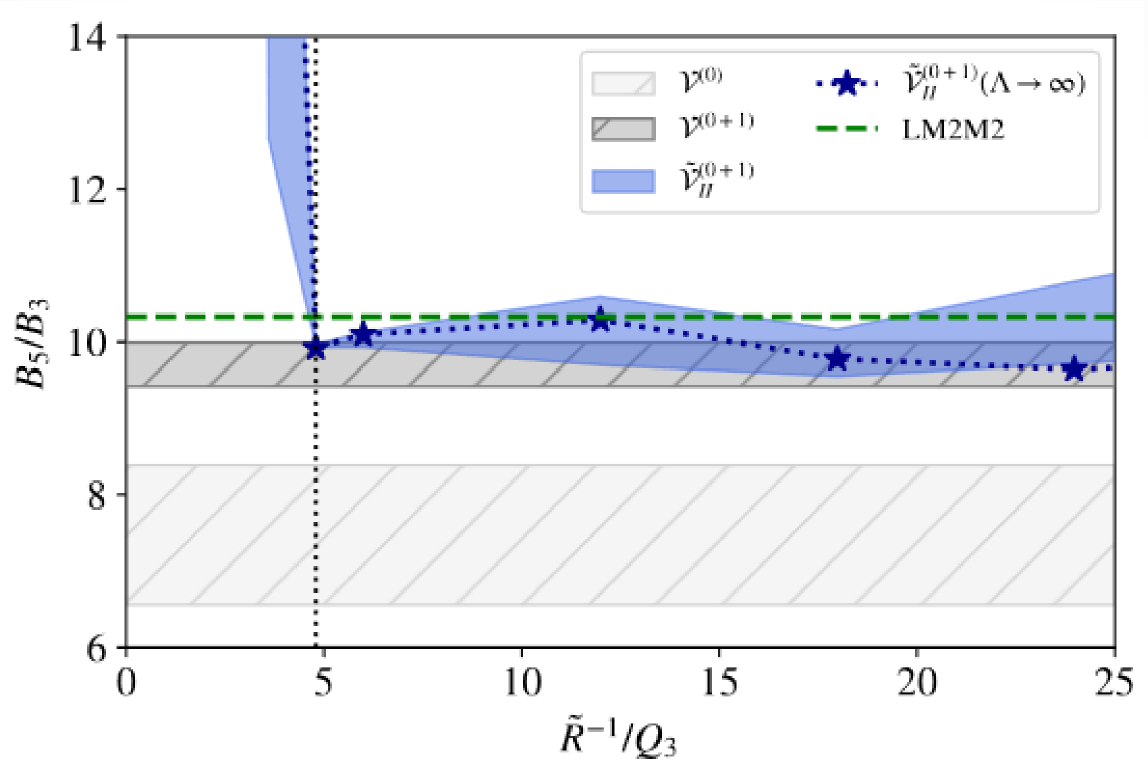


Contessi, Schäfer, vK, *Phys. Rev. A* **109** (2024) 022814

central values at various improved orders:

- slightly better than unimproved values
- differ from unimproved results no more than change in order

Contessi, Schäfer, vK,
Phys. Rev. A **109** (2024) 022814



NLO

zero range

- improvement at NLO nearly independent of fake range for a large range of values
- "wall" at $\tilde{R}_c^{-1} \simeq 2 l_{vdW}^{-1}$

two-body effective range at improved LO ~ LM2M2 effective range

Contessi, Schäfer, Gnech, Lovato, vK ‘25


Light-Medium Nuclei

Can we find a range of fake ranges that give stability at LO?

(two two-body S-wave channels)

one possible improvement

$$\tilde{R}_{2s} = xR_{2s} \qquad \tilde{R}_{2t} = xR_{2t}$$

fitted to corresponding effective ranges

$$\tilde{R}_{3} = xR_{3}$$

fitted for maximal improvement of alpha binding energy

$$\Rightarrow \quad \tilde{V}^{(0)}(\vec{r};x) = V^{(0)}(\vec{r};\Lambda) + \Delta V(\vec{r};\Lambda, xR_{2s}, xR_{2t}, xR_{3})$$

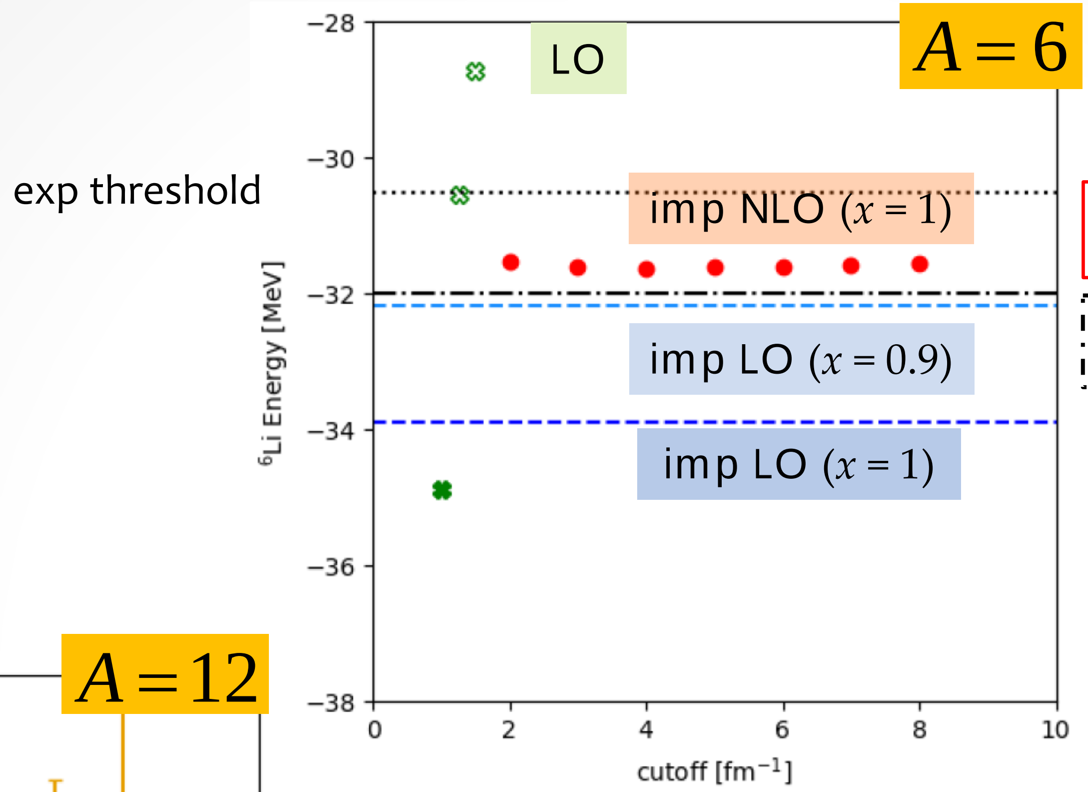
residual cutoff dependence at NLO

$$E_A^{(1)}(\Lambda) = E_A^{(1)}\left(1 + \frac{q_A^{(1)}}{\Lambda}\right)$$

uncertainty from variation for $\Lambda \geq 2\ \mathrm{fm}^{-1}$

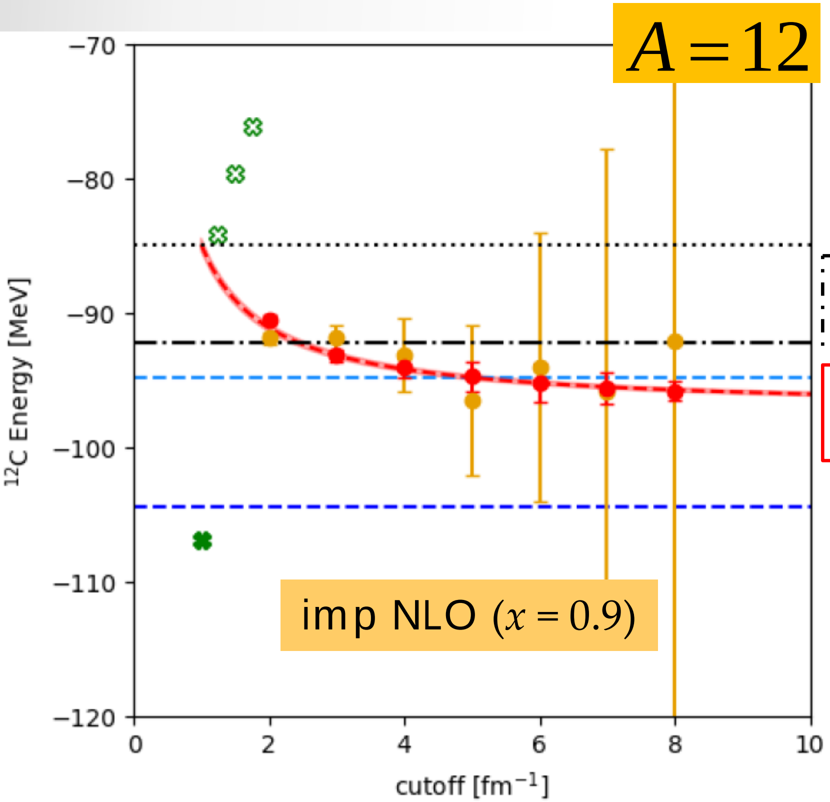
~ uncertainty from $\left(\dfrac{Q}{M_{\mathrm{hi}}}\right)^2 \sim \dfrac{1}{4}$

Contessi, Schäfer, Gnech, Lovato, vK, arXiv:2505.09299



$$E_6^{(1)} = -(31.57 \pm 0.02 \pm 0.3)\ \mathrm{MeV}$$

$$E_6^{(\mathrm{exp})} = -31.994\ \mathrm{MeV}$$

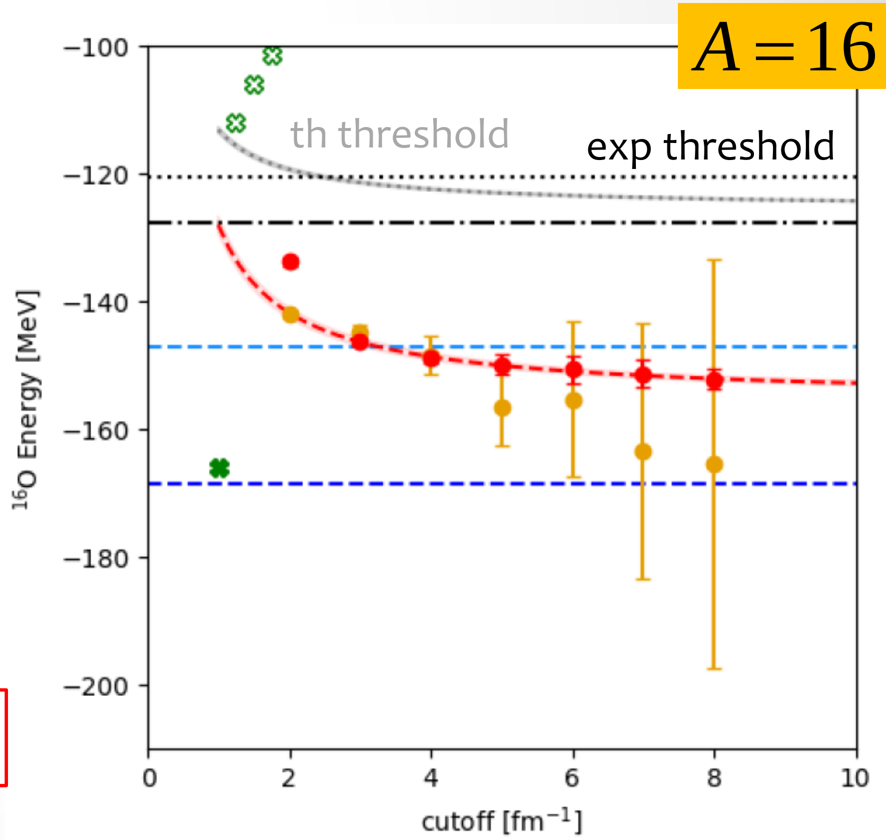


$$E_{12}^{(\mathrm{exp})} = -92.162\ \mathrm{MeV}$$

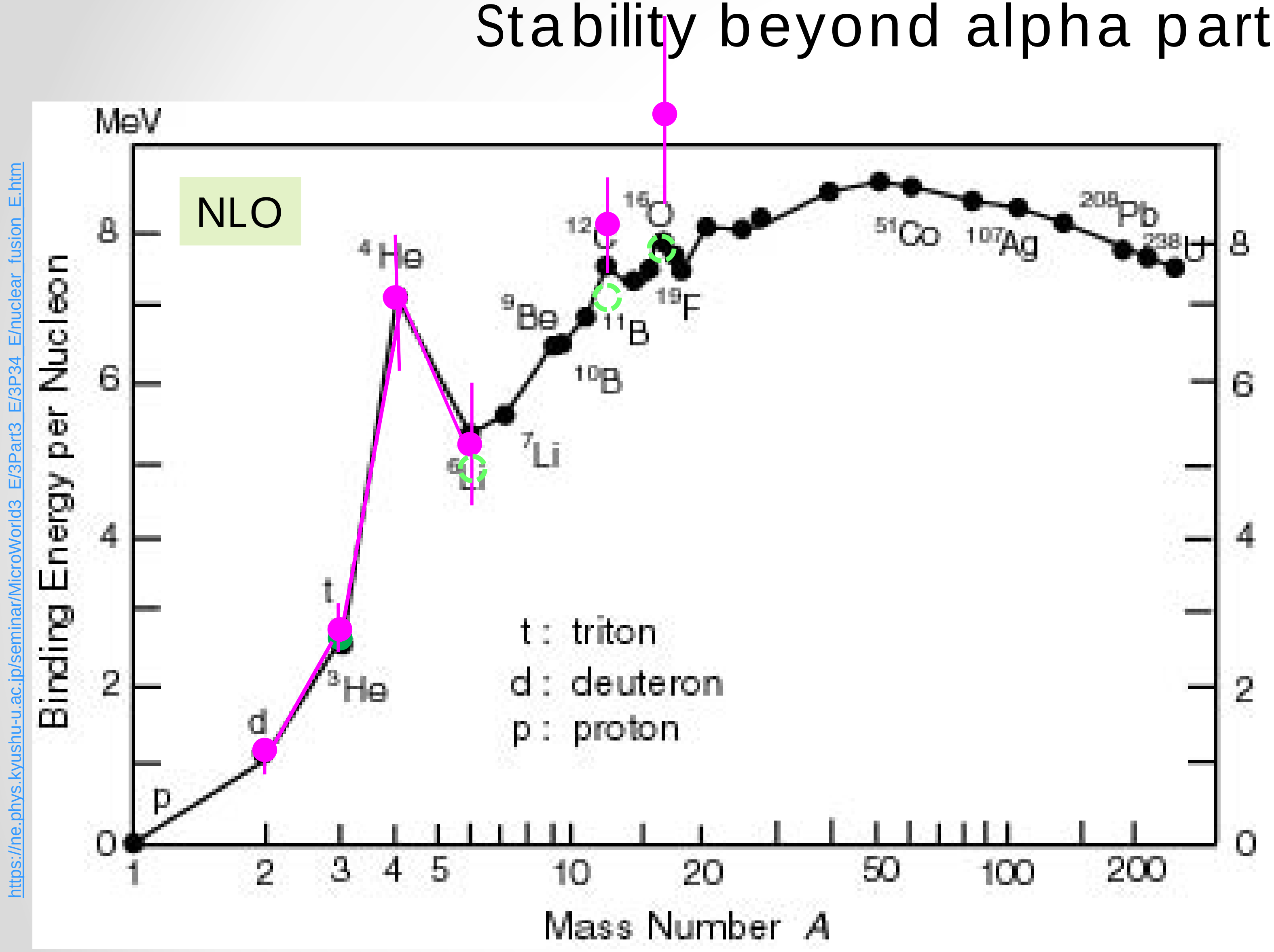
$$E_{12}^{(1)} = -(97.3 \pm 0.1 \pm 5)\ \mathrm{MeV}$$

$$E_{16}^{(\mathrm{exp})} = -127.619\ \mathrm{MeV}$$

$$E_{16}^{(1)} = -(155.6 \pm 0.3 \pm 20)\ \mathrm{MeV}$$



Stability beyond alpha particle at NLO



^{6}Li — Contessi, Schäfer, Gnech, Lovato, vK '25

^{12}C — Contessi, Schäfer, Gnech, Lovato, vK '25

^{16}O — Contessi, Schäfer, Gnech, Lovato, vK '25

renormalization + near two-body unitarity: clustering of multicomponent fermions

- cf. order-by-order weakening of Wigner bound

  Beck, Bazak, Barnea ‘20

  -- here more general

- cf. finite-range pionless potentials

  Kievsky *et al.* ’20
  Recchia *et al.* ’22
  …
  Bub *et al.* ‘24

  -- here preserving power counting

  (# parameters, perturbative corrections, breakdown scale, …)

BUT

- independence of improvement: restricted range of *x*, other forms?
- growth with *A* overestimated?

Conclusion

Boson and multi-component fermion systems near unitarity depend on essentially *one* (three-body!) parameter Λ_*

Bosons saturate and form a **quantum liquid**

Multi-component fermions tend to **cluster**

Improvement is sufficient to provide **stability** at LO and allow for perturbative corrections

Light-medium binding energies are within ~15% of experiment at NLO

Need: more observables, heavier nuclei, higher orders