Limit cycles and related renormalization group behaviors

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Some basic examples suggest that the renormalization-group limit cycles and even more complex behaviors may be commonplace for quantum Hamiltonians requiring renormalization, unlike in the case of classical systems, where fixed points are commonly considered. The behaviors are identified using the renormalization group transformations, which allows one to draw qualitative conclusions concerning complex systems before finding solutions. Also, simple systems can behave in complex ways.

For some references, please see:

K.Serafin, M.Gomez-Rocha, J.More, S.D.Glazek, *Dynamics of heavy quarks in the Fock space*
Phys. Rev. D, 109, 016017 (2024)

M.Girgus, S.D.Glazek, *Spiral flow of quantum quartic oscillator with energy cutoff*
Nucl. Phys. B 1010, 116776 (2025)

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Outline

1-5 in detail, 6-9 brief

.

1. RG limit cycle — 1971
2. Wilsonian RGT for Hamiltonians (Gaussian elimination) — transformation
3. $-g/r^2$, limit cycle and bound states in familiar example — discrete scaling
4. g, ih, asymptotic freedom, limit cycle, chaos, a numerical puzzle — tuning
5. ϕ^4, hidden complexity of quantum dynamics (surprise) — cutoff on # of quanta
6. RGT $\rightarrow$ RGPEP for Hamiltonians in QFT — similarity transformation
7. Brief look at RGPEP: hadrons, analytic Ω_{ccc}, QCD $\rightarrow$ NP — oscillator and $m_g \rightarrow 0$
8. BSM suggestion — asymptotic freedom in a cycle
9. Conclusion

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1. Limit cycle closed trajectory

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Prediction of limit cycles in K.G.Wilson, PRD 3, 1818 (1971): Eqs. (1.6), (1.7), or (3.54), (3.55) in Sec. H

$$\frac{dg_1}{dt} = \psi(g_1, g_2)$$
$$\frac{dg_2}{dt} = \phi(g_1, g_2) \qquad t \sim \ln(\Lambda/\Lambda_0)$$

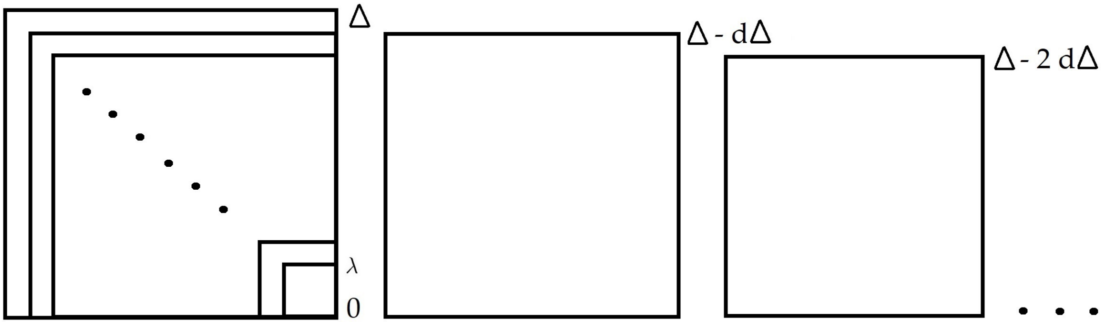
Example

$$\begin{bmatrix} \psi(g_1, g_2) \\ \phi(g_1, g_2) \end{bmatrix} = \begin{bmatrix} -g_2 \\ g_1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

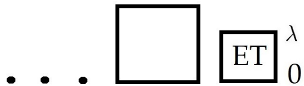
$$\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{bmatrix} \begin{bmatrix} g_{10} \\ g_{20} \end{bmatrix}$$

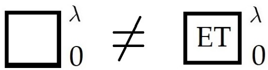
at least 2 couplings

2. Wilsonian RGT for Hamiltonians



Wilsonian RGT for H ~ Gaussian elimination





3. Schroedinger's particle in potential $-g/r^2$

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S.M.Dawid, R.Gonsior, J.Kwapisz, K.Serafin, M.Tobolski, Phys. Lett. B 777, 260 (2018)

$$H = \frac{\vec{p}^{\,2}}{2m} - \frac{g}{\vec{r}^{\,2}} \quad , \quad H\tilde{\phi} = E\,\tilde{\phi}$$

$$\frac{p^2}{2m}\phi(\vec{p}) - \frac{g}{4\pi}\int d^3q \; \frac{\phi(\vec{q}\,)}{|\vec{p} - \vec{q}\,|} = E\,\phi(\vec{p}\,)$$

$l = 0$

$$p^2\phi_0(p) + \int_0^\infty dq \; q^2 \; V_0(p,q) \; \phi_0(q) = 2mE\,\phi_0(p)$$

$$V_0(p,q) = -\alpha \left[\frac{\theta(p-q)}{p} + \frac{\theta(q-p)}{q}\right] \quad , \quad \alpha = 2mg$$

regularization: $\int_0^\infty \to \int_0^\Delta$

computing ET by reducing the cutoff

.

$$\Delta \;\rightarrow\; (\Delta - d\Delta) \;\rightarrow\; (\Delta - 2d\Delta) \;\rightarrow\; . \;\; . \;\; . \;\; \Lambda \rightarrow (\Lambda - d\Lambda) \;\rightarrow . \;\; . \;\; .$$

$$\int_0^{\Delta} dp\, X(p) = \int_0^{\Delta - d\Delta} dp\, X(p) + d\Delta\, X(\Delta)$$

$$p^2 \phi_0(p) + \int_0^{\Delta - d\Delta} dq\; q^2\; V_0(p,q)\; \phi_0(q) + d\Delta\, \Delta^2\, V_0(p,\Delta)\, \phi_0(\Delta) = 2mE\, \phi_0(p)$$

$$\phi_0(\Delta) = \frac{-1}{\Delta^2 - 2mE} \int_0^{\Delta - d\Delta} dq\; q^2\; V_0(\Delta, q)\, \phi_0(q)$$

$$p^2\; \phi_0(p) + \int_0^{\Delta - d\Delta} dq\; q^2 \left[V_0(p,q) - d\Delta\, V_0(p,\Delta) V_0(\Delta, q) \right]\; \phi_0(q) = 2mE\, \phi_0(p)$$

$$V_0(p,\Delta) \,=\, V_0(\Delta,q) \,=\, \frac{-\alpha}{\Delta} \qquad \text{independent of p and q}$$

$$V_0(p,q) \to V_0(p,q) + \gamma_\Lambda \qquad\qquad \lim_{\Lambda\to\infty} \gamma_\Lambda \stackrel{?}{=} 0$$

$$p^2\ \phi_0(p) + \int_0^{\Lambda-d\Lambda} dq\ q^2 \left[V_0(p,q) + \gamma_\Lambda - d\Lambda \left(-\frac{\alpha}{\Lambda} + \gamma_\Lambda \right)^2 \right]\ \phi_0(q) = 2mE\,\phi_0(p)$$

$$\gamma_{\Lambda-d\Lambda} = \gamma_\Lambda - d\Lambda\ \left[-\frac{\alpha}{\Lambda} + \gamma_\Lambda \right]^2$$

the Ricatti equation

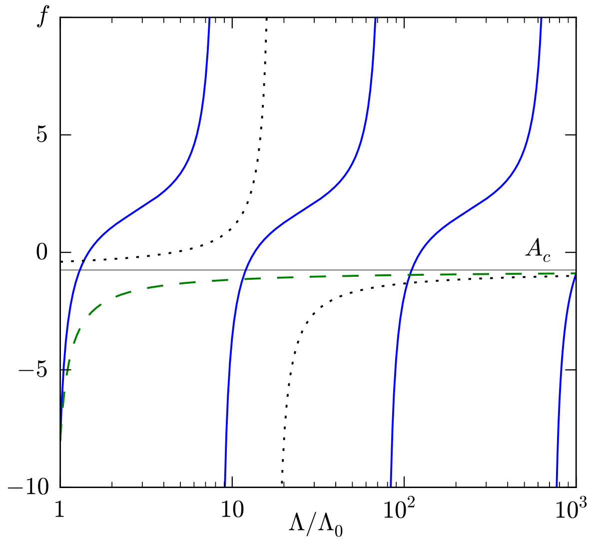
$$\frac{d\gamma}{d\Lambda} = \left[\gamma - \frac{\alpha}{\Lambda} \right]^2 \qquad \gamma = f/\Lambda \qquad \Lambda\,\frac{\partial f}{\partial \Lambda} = \beta(f)$$

$$A = \alpha - 1/2,\ \ B = \sqrt{\alpha - 1/4} \qquad \beta(f) = (f-A)^2 + B^2$$

real $B \neq 0$, or $\sqrt{\alpha} > 1/2$ $\qquad$ B imaginary $\to$ 2 fixed points

$$f = B\tan\left(\varphi + B\ln\frac{\Lambda}{\Lambda_0}\right) + A \qquad \varphi = \arctan\left(\frac{f_0 - A}{B}\right)$$

.



$l = 0 \Rightarrow f$ limit cycle (solid blue line) for a single coupling constant

$l = 1 \Rightarrow f$ approaches a constant in a logarithmic fashion (dashed line)

$g = 9/(8m)$ $\quad f_0 = f(\Lambda_0) = -8.0$ $\quad B$ real

discrete scaling with cutoff $p = x\Lambda \qquad q = y\Lambda \qquad E \ll \Lambda$

.

$$p^2\phi_0(p) + \int_0^\Lambda dq\ q^2\ \left[V_0(p,q) + f/\Lambda\right]\ \phi_0(q) = 2mE\,\phi_0(p)$$

$$x^2\phi(x) + \int_0^1 dy\ y^2\ \left[-\alpha\left(\frac{\theta_{x-y}}{x} + \frac{\theta_{y-x}}{x}\right) + f\right]\ \phi(y) = \frac{2mE}{\Lambda^2}\,\phi(x)$$

$$\mathcal{E}(\alpha, f) = 2mE/\Lambda^2$$

$$f = B\tan\left(\varphi + B\ln\frac{\Lambda}{\Lambda_0}\right) + A \qquad B\ln\frac{\Lambda}{\Lambda_0} = \pi \qquad \Lambda^2 = \left(e^{-2\pi/B}\right)^n\Lambda_0^2$$

a bound state with E $\quad\rightarrow\quad$ **bound states with** $E_n = e^{-2\pi n/B}\,E$

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SG and KW, PRD47, 4657 (1993), PRB69, 094304 (2004)

4. g, ih, AF, LC, chaos, a numerical puzzle

tuning to cycle

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$$H_{mn}(g) = E_m\,\delta_{mn} - g\sqrt{E_m E_n} \qquad E_n \;=\; b^n \qquad \text{e.g., } b = e > 1$$

regularization $\quad -M \le m, n \le N \qquad \Delta = b^N$

Gaussian elimination $\quad H^{(N)}\psi = E\psi \qquad \Delta \to \Lambda = b^n$

$$H^{(N)} \to H^{(N-1)} \to H^{(N-2)} \to \,.\;.\;.\; H^{(n)} \qquad 1 \gg E/\Lambda \sim 0$$

.

$$g \to g_N \to g_{N-1} \to g_{N-2} \to \,.\;.\;.\; g_n$$

$$g_{n-1} = g_n + \frac{g_n^2}{1 - g_n - E/E_n} \to g_{n-1} \;=\; g_n + \frac{g_n^2}{1-g_n} \;=\; \frac{g_n}{1-g_n}$$

$$\frac{1}{g_{n-1}} = \frac{1}{g_n} - 1 \qquad \frac{1}{g_n} \;=\; \frac{1}{g_N} - (N-n) \qquad g_N \;=\; \frac{g_n}{1 + g_n(N-n)}$$

$$N - n_0 = \ln(\Delta/\Lambda_0) \qquad g_0 = g_{n_0} = \frac{1}{\ln(\Lambda_0/\mu)}$$

$$\boxed{g_\Delta = \frac{g_0}{1 + g_0 \ln(\Delta/\Lambda_0)} = \frac{1}{\ln(\Delta/\mu)} \qquad \text{AF}}$$

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SG and KW, PRD47, 4657 (1993), PRB69, 094304 (2004)

Hamiltonian: real symmetric → complex Hermitian QM

$$H_{mn}(g,h) = \sqrt{E_m E_n}\ \left[\delta_{mn} - g - ih\,\mathrm{sign}(m-n)\right]$$

$$g_{n-1} = g_n + \frac{g_n^2}{1-g_n-E/E_n} \quad \to \quad g_{n-1} = g_n + \frac{g_n^2+h^2}{1-g_n-E/E_n}$$

$$g_{n-1} = \frac{g_n}{1-g_n} \quad \to \quad \tilde g_{n-1} = \frac{\tilde g_n + h}{1-\tilde g_n\, h} \qquad \tilde g_n = g_n/h$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\ \tan\beta} \qquad \arctan\tilde g_{n-1} = \arctan\tilde g_n + \arctan h$$

$$g_n = h\tan\left[\arctan g_N + (N-n)\arctan h\right]$$

$$h = \tan(\pi/p) \qquad \begin{cases} p \ \text{integer} & \to \ \text{limitcycle} \\ p \ \text{rational} & \to \ \text{convoluted limit cycle} \\ p \ \text{irrational} & \to \ \text{chaos} \end{cases}$$

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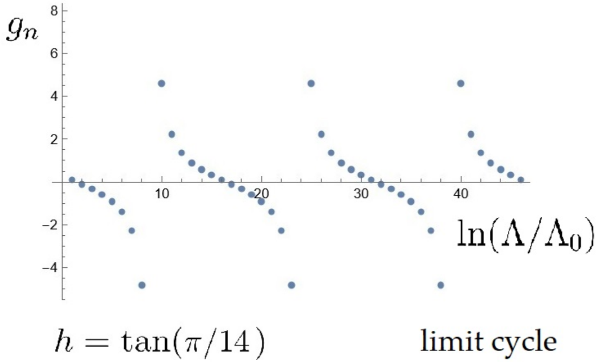

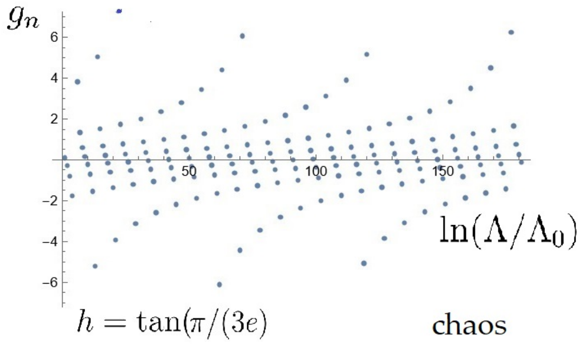

both g and ih contribute to the logarithmic divergence ultimate QM log divergence

set $(g, 0)$ is of measure 0 on a complex plain (g, ih)

$(g, 0) \quad \overset{RGT}{\rightarrow} \quad (g, 0) \qquad\qquad (0, h) \quad \overset{RGT}{\rightarrow} \quad (g, h)$

continuum QM is more likely cyclic than fixed philosophy of ultimate truth?

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universality and tuning to the limit cycle natural $p > 2$

$g_n = g_n(E)$ $\quad g_n,\, g_{n-1},\, .\ .\ .\ g_{n-p+1}$ $\quad E \neq 0 \to g_{n-p} \neq g_n$

$$g_n(E)/h = f_n(x)|_{x=x_n} \qquad x_n = E/E_n = E/b^n \qquad x_{n-p} = r\ x_n \qquad r\ =\ b^p$$

fixed-point analysis modulo cycle

$$g_{n-p}(0) = g_n(0) \qquad g_{n-p}\ =\ h \tan\left[\arctan(g_n/h) + e_1 E + e_2 E^2 + e_3 E^3 + O(E^4)\right]$$

$$f^*_k(x) = g^*_k + c^*_{k1} x + c^*_{k2} x^2 + c^*_{k3} x^3 + O(x^4) \qquad k = 1, 2, ..., p$$

$f^*(rx) = R[f^*(x)]$ $\quad R$ computed in p steps of a cycle

$$R[f^* + df] = f + L\, df \qquad L\, df\ =\ w\, df$$

1 marginal operator $w = 1 = r^{\lambda_0}$, $\lambda_0 = 0$ — adjust H_{NN}, H_{NN-1}, H_{N-1N-1}

irrelevant operators $w_l = r^{-l}$, $\lambda_l = -l$ — eliminate first 2 irrelevant

anomalous dimensions the same on lc — accelerated approach to LC

1 bound state per cycle (jump of tan)

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$b = 2,\ p = 3,\ b^p = 8,\ N = 17,\ M = -51,\ g_N = -7$

E	Ratio	w_3^{-1}
0.115359460521		
0.326274994387	2.828333219599	
0.922875684170	2.828521033011	
2.610199955097	2.828333219599	
7.383005473358	2.828521033011	
20.881599640790	2.828333219600	
59.064043787554	2.828521033043	
167.052797170395	2.828333220314	
472.512353132001	2.828521049247	
1336.422560688180	2.828333591344	
3780.110931254790	2.828529719903	
10692.223889183900	2.828547649430	
30306.851247105530	2.834475929536	
91327.456629399810	3.013426102394	
1824274.370928830000	19.975092247797	
−3.321031582839		
−26.568252662770	8.000000000017	~400
−212.546021531960	8.000000008649	511
−1700.369094987040	8.000004341325	502
−13606.149375382800	8.001879953885	433
−111890.939577163000	8.223556605927	119

SG, KW, PRB69, 094304 (2004)

rapid approach to ratio 8 for bound states

puzzle for scattering states:

$b^{p/(p-1)} = \sqrt{8} \sim 2.828\,427\,124\,746\,19$

$$2.828\,333\,219\,599 + 2.828\,521\,033\,011 = 2 \times 2.828\,427\,126\,305$$
$$2.828\,333\,219\,599 \times 2.828\,521\,033\,011 \sim 7.999\,999\,999\,999\,491$$

$8^3 = 512$

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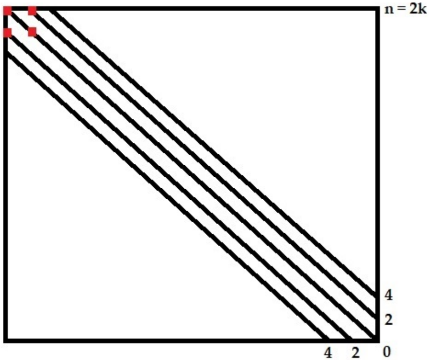
5. ϕ^4, hidden complexity of quantum dynamics (surprise) cutoff on # of quanta

$$H = -\frac{d^2}{d\phi^2} + A\,\phi^2 + B\,\phi^4 \qquad A, B > 0 \qquad \phi \sim a + a^\dagger \qquad [a, a^\dagger] = 1$$

$$H/(\hbar\omega) = a^\dagger a + g(a^\dagger + a)^4 \qquad a^\dagger a\;|k\rangle = (k!)^{-1/2} a^{\dagger k}|0\rangle = k|k\rangle \qquad H^\infty_{k,l} = \langle k|H|l\rangle$$

cutoff $k, l \le N$ $\qquad N \to N-1 \to N-2 \to ...n \qquad H^n_R = \lim_{N\to\infty} H^N_n$



$$H^N_{n;n,n} = n + \xi_1\left(H^N_{n,n} - n\right)$$
$$H^N_{n;n-2,n-2} = n - 2 + \xi_2\left[H^N_{n-2,n-2} - (n-2)\right]$$
$$H^N_{n;n,n-2} = \xi_3 H^N_{n,n-2}$$
$$H^N_{n;n-2,n} = \xi_3 H^N_{n-2,n} \qquad \vec{\xi} = [\xi_1, \xi_2, \xi_3]$$

$$\boxed{\vec{\xi}_{n-2} = RGT[\vec{\xi}_n]}$$

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M.Girgus, S.D.Glazek, *Spiral flow of quantum quartic oscillator with energy cutoff*
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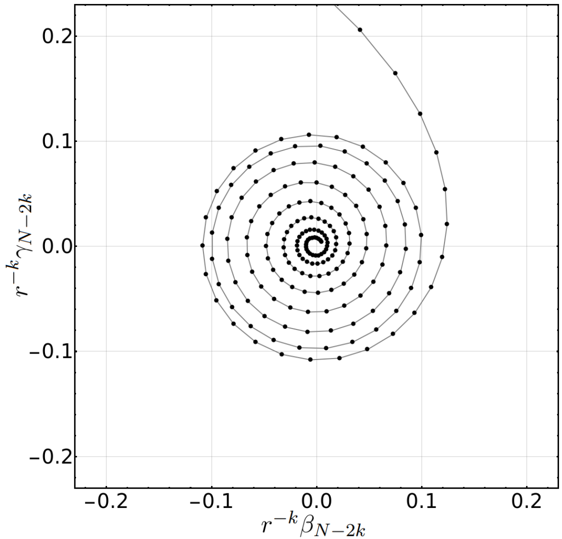

$\vec{\xi}^* = (1/6, 5/6, 1/2)$, $t \sim \pi/p \sim \pi$, $p^2 = \sqrt{a + a^2/4} - a/2$, $a = 1/(4gN)$

$r = (1-p)/(1+p)$, $\lambda = 1$, $\lambda_\pm = e^{\pm i\nu}$, $\nu = 2 \arcsin p$

SSB $A < 0$ (6 dim $\vec{\xi}$), 2 coupled oscillators (10 dim $\vec{\xi}$), $\phi \stackrel{?}{=} \phi_0$, vacuum in QFT, Higgs

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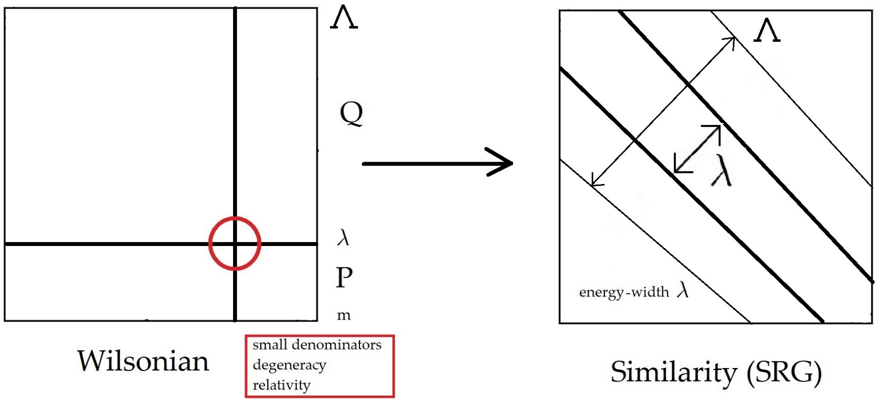
6. Need for change of RGT in QFT

similarity transformation

Cutoff on energy → cutoff on change of energy

SDG, K.G. Wilson, *Renormalization of Hamiltonians*, PRD48, 5863 (1993)



Another change: SRG → RGPEP

refs in SDG, APPB43, 1843 (2012)

SRG: Hamiltonian matrices → RGPEP: Hamiltonian operators

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7. Look at RGPEP: hadrons, analytic Ω_{ccc}, QCD $\to$ NP

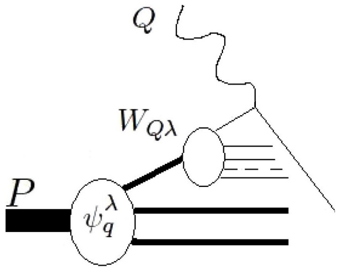
oscillator, $m_g \to 0$

basis in virtual Fock space $\to$ basis in virtual Fock-space operators

$$a \to a_\lambda \qquad a^\dagger \to a_\lambda^\dagger$$

$$a_\lambda = \mathcal{U}_\lambda^\dagger \, a \, \mathcal{U}_\lambda \qquad a_\lambda^\dagger = \mathcal{U}_\lambda^\dagger \, a^\dagger \, \mathcal{U}_\lambda \qquad \mathcal{U}_\lambda = ?$$

Motivation is physical



$$H_\lambda(a_\lambda, a_\lambda^\dagger) = H(a, a^\dagger) \; , \quad a_Q = W_{Q\lambda} a_\lambda W_{Q\lambda}^\dagger \; , \quad W_{Q\lambda} = \mathcal{U}_Q^\dagger \mathcal{U}_\lambda$$

Hamiltonian width λ and the RGPEP generator

$H_\lambda = H_f + H_{I\lambda}$ $\qquad \mathcal{H}_\lambda = \mathcal{U}_\lambda^\dagger H_{\rm can} \mathcal{U}_\lambda$ $\qquad \frac{d}{d\lambda^p}\mathcal{H}_\lambda = \left[\mathcal{U}_\lambda'^\dagger \mathcal{U}_\lambda, \mathcal{H}\right]$

initial condition $\mathcal{H}_{\lambda=\infty} = H_{\rm can}^\Delta + CT_\Delta$

$\to$ cluster property, no disconnected effective interaction terms

For low orders of PT follow F.Wegner 1994 $\mathcal{U}_\lambda'^\dagger \mathcal{U}_\lambda = [\mathcal{H}_f, \mathcal{H}_{I\lambda}]$

PT refers to computation of $\mathcal{H}_\lambda$, not observables

$$\frac{d}{dt}\mathcal{H}_\lambda = [[\mathcal{H}_f, \mathcal{H}_{I\lambda}], \mathcal{H}_f + \mathcal{H}_{I\lambda}]$$

homogeneous solution:

$$\mathcal{H}_{I\lambda\ mn} = \langle m|\mathcal{H}_{I\lambda}|n\rangle \ = \ e^{-(E_m - E_n)^2/\lambda^2} \, H_{I{\rm can}\, mn} \ = \ f_{\lambda\, mn} \, H_{I{\rm can}\, mn}$$

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Window eigenvalue problem for hadrons

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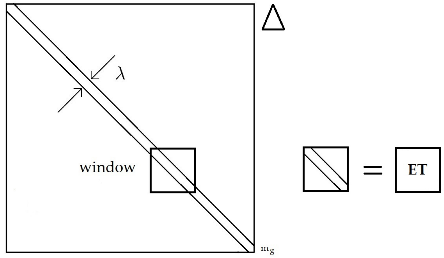

PT in $g_\lambda \ll 1$ $V_c \sim e^2$

window middle eigenvalue matches whole matrix eigenvalue

the window eigenstate matches the whole matrix eigenstate

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QCD windows differ from QED windows

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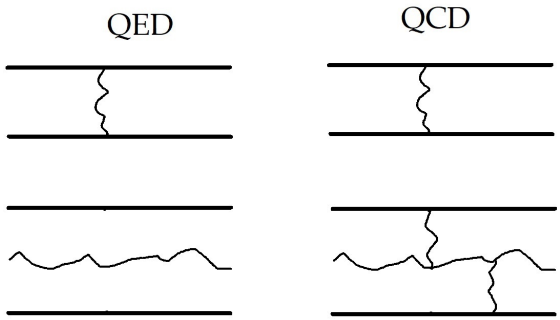

Gluons constantly (strongly) interact with quarks and gluons,

unlike photons with electrons and photons.

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blocking of gluons using $m_G \gg m_g$ and infrared confining effect

.

$$|\text{quarkonium}\rangle = |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + |Q\bar{Q}gg\rangle + |Q\bar{Q}ggg\rangle + ...$$

$$\rightarrow |Q\bar{Q}\rangle + |Q\bar{Q}G\rangle \qquad m_G \gg \Lambda_{\text{QCD}}$$

$$H^{(2)}_{\text{QCD}\,\lambda}|\text{quarkonium}\rangle = M|\text{quarkonium}\rangle$$

self-interaction $\sim \ln(\lambda/m_g)$ **canceled by gluon echange** ‘t Hooft 1+1, Bloch-Nordsieck

$$M = 2m + B \qquad m \gg \Lambda_{\text{QCD}} \qquad f_\lambda \to \text{NR appr.}$$

$$\frac{\vec{k}^{\,2}}{m}\,\psi(\vec{k}\,) + \int_q V_{\text{C}}(\vec{q}\,)\psi(\vec{k}-\vec{q}\,) + \int_q V_{\text{RG}}(\vec{q}\,)[\psi(\vec{k}-\vec{q}\,) - \psi(\vec{k}\,)] = B\psi(\vec{k}\,)$$

$$V_{\text{C}} = -\frac{4}{3}\,\frac{4\pi\alpha}{\vec{q}^{\,2}}\,(1+\text{BF})$$

$$V_{\text{RG}} = \frac{4}{3}\,4\pi\alpha\,\left(\frac{1}{\vec{q}^{\,2}} - \frac{1}{q_z^2}\right)\,\frac{m_G^2}{m_G^2+\vec{q}^{\,2}}\,\exp\left[-2\left(\frac{m\vec{q}^{\,2}}{\lambda^2 q_z}\right)^2\right]$$

Taylor expansion in $\vec{q}$ for $\psi(\vec{k}-\vec{q}\,) - \psi(\vec{k}\,)$

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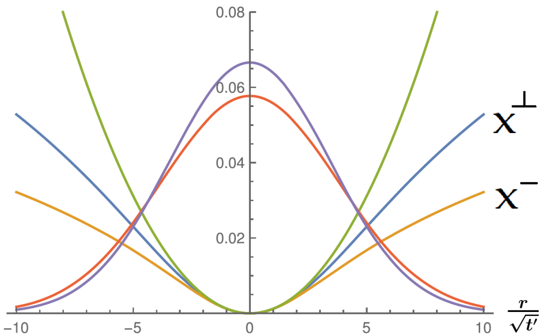
$Q\bar{Q}$ **effective interaction** (QCD+m_G) $\psi(\vec{k}-\vec{q}) - \psi(\vec{k}\,)$ Taylor series

.

$$V_{\rm RG} = -\frac{m\omega^2}{4}\,\Delta_{\vec{k}} \quad \underset{\text{F.T.}}{\longleftrightarrow} \quad \tilde{V}_{\rm RG} = \frac{m\omega^2}{4}\,\vec{r}^{\,2}$$

$$\omega = \sqrt{\frac{4}{3}\,\frac{\alpha_\lambda}{\pi}}\,\left(\frac{\pi}{1152}\right)^{1/4}\frac{\lambda^2}{m^2}\,\lambda$$

the fourth-root factor is ~ 0.228520 extrapolation to $\alpha_\lambda/\pi \sim 0.1$



K. Serafin ho $\to \ln r$, $+$, $\perp$, excited states $\to$ gluons, oscillator enough for strings

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Example of an analytic RGPEP result

valley (m_Q, λ)

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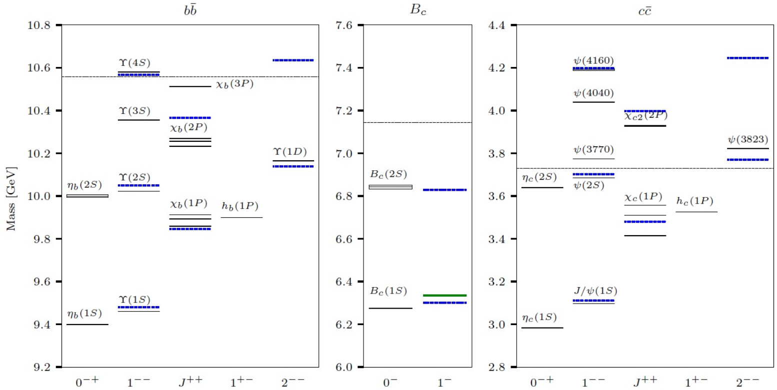

method	$\Omega_{ccc}(3/2+)$	$\Omega_{ccc}(3/2-)$
RGPEP [1]	**4797** (40,100) MeV	**5103** (40,100) MeV
Lattice [2]	**4793** (5)(7) MeV	**5094** (12)(13) MeV

[1] K.Serafin,M.Gómez-Rocha, J.More, S.D. Glazek,
Approximate Hamiltonian for baryons in heavy-flavor QCD, Eur. Phys. J. C 78, 964 (2018)

[2] N.S.Dhindsa, D.Chakraborty, A.Radhakrishnan, N.Mathur, M.Padmanath,
Precise study of triply charmed baryons (Ω_{ccc}), arXiv:2411.12729[hep-lat]

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QCD → Nuclear Physics

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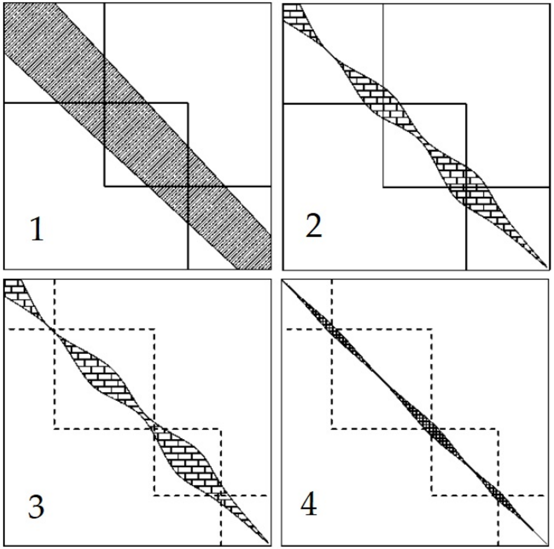

SDG, Marek Więckowski, *Large-momentum convergence of Hamiltonian bound-state dynamics of effective fermions*
Phys. Rev D 66, 016001 (2002)

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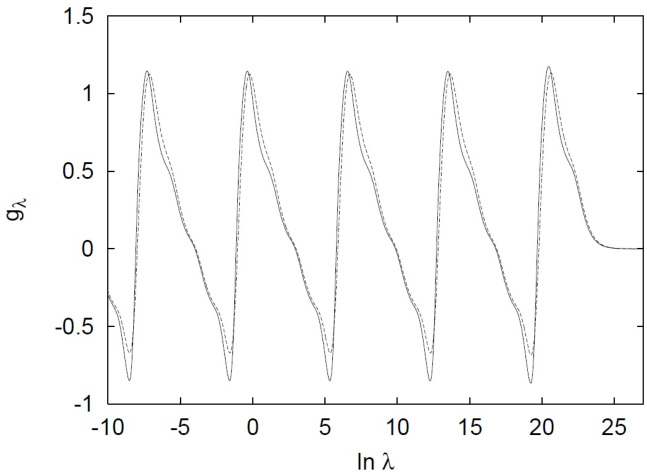
8. SM suggestion

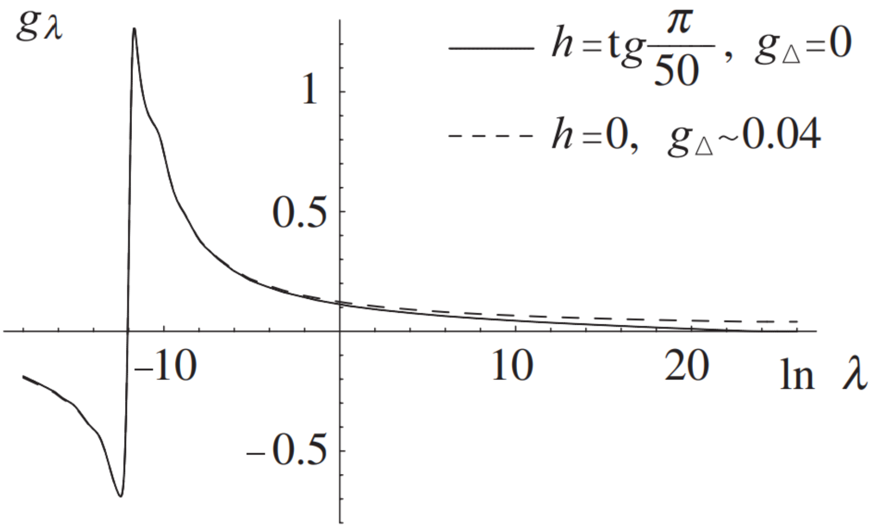
"creative"

.

$$H_{mn}(g,h) = \sqrt{E_m E_n}\ [\delta_{mn} - g - ih\,\mathrm{sign}(m-n)]$$

solution to the Wegner equation for the matrix of H_λ





$b = 4,\ N = 16,\ M = -25,\ E = -7.644479\ 10^{-6}$ $\qquad N \to 66,\ E' = 4^{50} E$

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9. Conclusion

renormalization of Hamiltonians → revival of Hamiltonian QM

points of interest

space of interacting quanta (vacuum excitation)

H as energy operator (form of dynamics, constituent quarks and partons)

bound states (special relativity of extended objects)

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