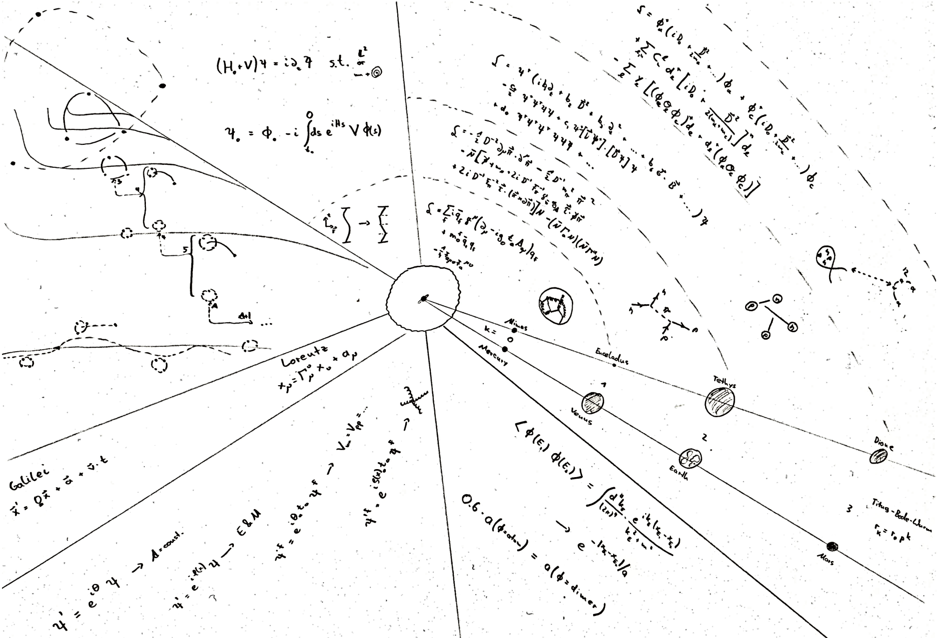
Universality in strongly-interacting systems: from QCD to atoms

ECT* 9-13 June (2025) org.: D. Blume, R. Briceño, J. Kirscher



Universality $\equiv$ microscopically differing systems realize common, *collective* behavior.

$\Rightarrow$ $\exists$ relativelly simple **root theory**

Degrees of freedom

- Fermions
- Bosons
- Anyons
- densities
- ...

Symmetries

- Lorentz & conformal
- Galilei & dilatation, special-conformal, U(1) global
- U(1) gauge
- ...

$$\hat{H}_{\text{internal}} + \hat{H}_{\text{background}}$$

Pathways to observables

- Path integral
- Schrödinger eq.
- Lippmann-Schwinger, AGS, FY
- S matrix $\Psi_{\text{out}}(E) = \hat{S}\Psi_{\text{in}}(E)$
- STM eq.
- ...

Observables

- spectral elements $|E\rangle$
- reactions between $|E\rangle$'s
- properties of sets $\{|E\rangle\}$
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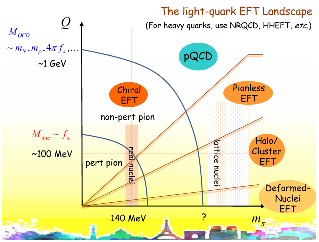

(U. van Kolck)

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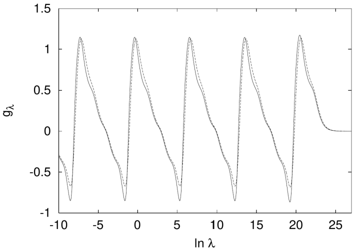

(S. Glazek)

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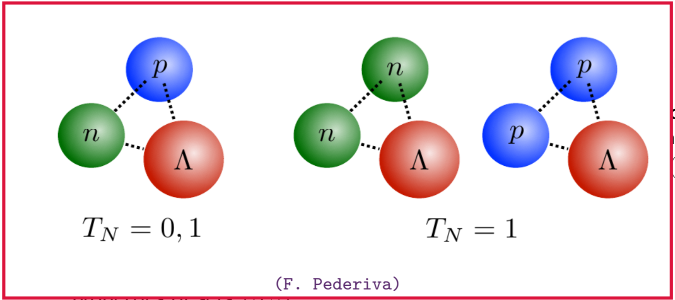

(F. Pederiva)

rvables

q.
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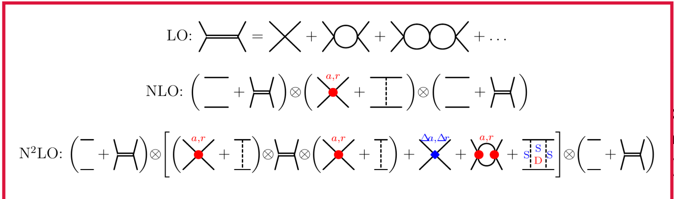

(H. Grießhammer)

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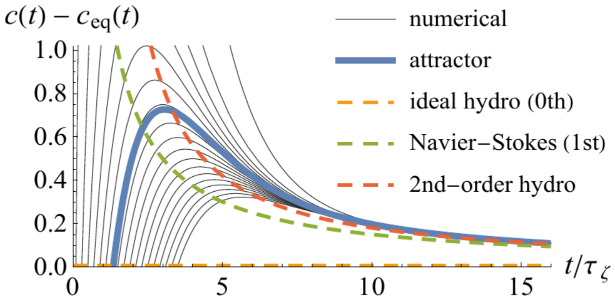

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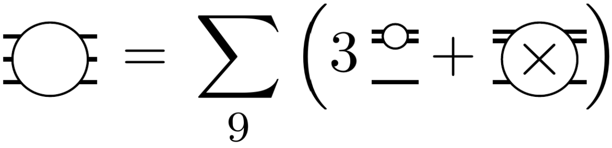

(A. Jakura)

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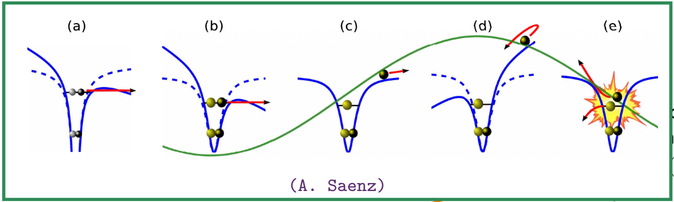

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are $O(1)$, but the matrix elements of the G operators are $O(N_c)$. Also, since baryons are composed of N_c quarks, the matrix elements of the quark number operator $\mathcal{N} = q^\dagger q$ scale as N_c. In summary, the following rules hold:

Isospin/spin scaling rules:

$$\langle N'|\frac{G^{ia}}{N_c}|N\rangle \sim 1\,, \qquad \langle N|\frac{\mathcal{N}}{N_c}|N\rangle \sim 1\,, \qquad \langle N'|\frac{S^i}{N_c}|N\rangle \sim N_c^{-1}\,, \qquad \langle N'|\frac{I^a}{N_c}|N\rangle \sim N_c^{-1}\,, \tag{6}$$

where the $|N\rangle$ are the nucleon states.

In the two-nucleon sector (a parallel argument holds for three and more nucleons) the objective is to compare a two-nucleon matrix element of an operator $\tilde{\mathcal{O}}$ involving the quark and gluon fields of QCD to a two-nucleon matrix

(M. Schindler)

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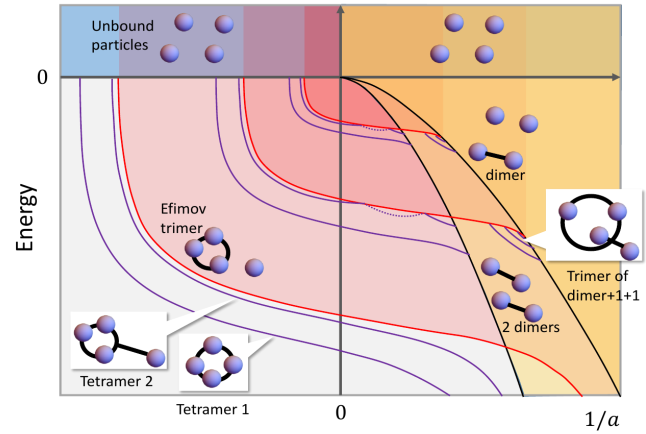

(A. Deltuva & P. Naidon, S. Endo)

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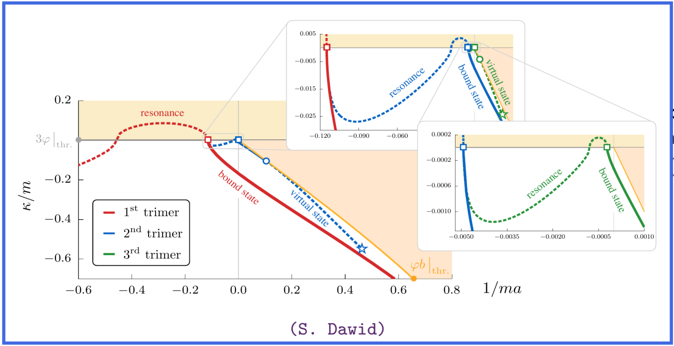

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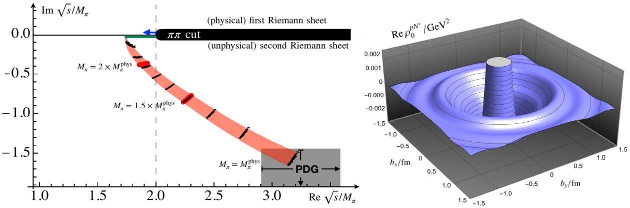

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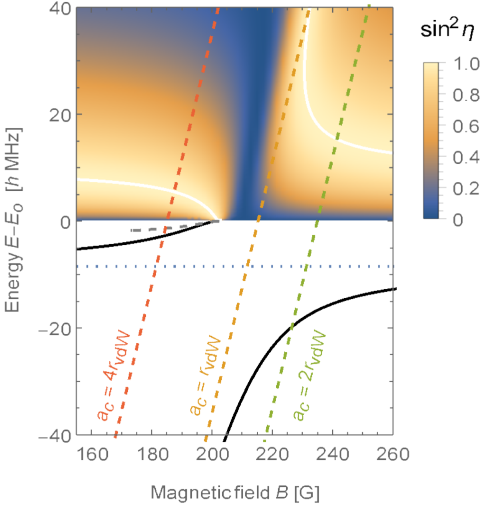

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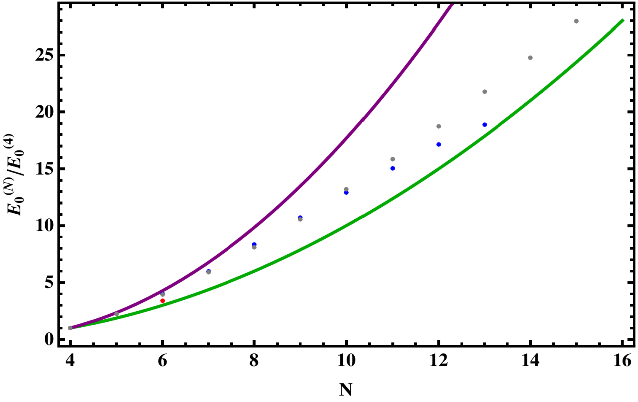

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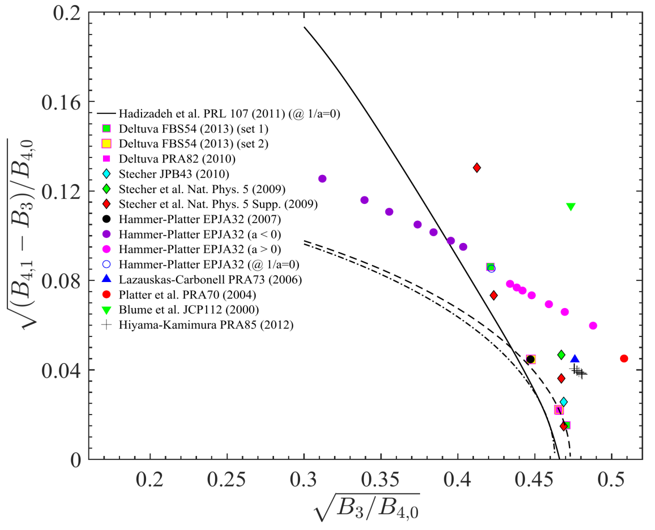

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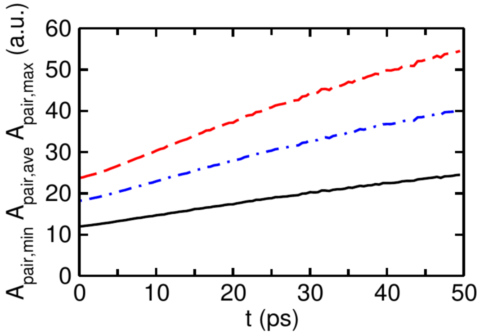

FIG. 4: Dynamics of internuclear distances. The black solid, blue dash-dotted, and red dashed lines show $A_{\text{pair,min}}$, $A_{\text{pair,ave}}$, and $A_{\text{pair,max}}$, respectively, as a function of time.

(Q. Guan)

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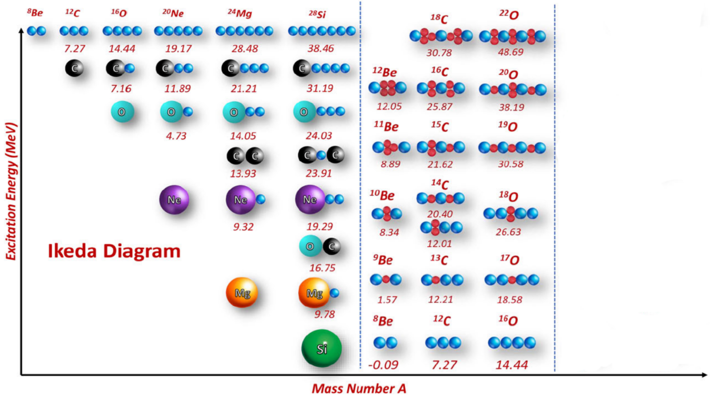

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$$\boxed{|\text{Your Problem}\rangle = \hat{\mathcal{O}}_{\lambda/\epsilon/\mu/\&c.}\,|\text{My Problem}\rangle}$$

$|$My Problem$\rangle$ =constituent-data parametrization of effective inter-cluster interactions

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cluster-relative motion

$$\sum_{j=1}^{n} \left[\mathfrak{D}_{ij}(E) + \tilde{\mathfrak{D}}_{ij}(E)\right] \left|\hat{\chi}_j\right\rangle = 0$$

$\hat{\phi}_i \prod_{m=1}^{n_F(i)} \delta\left(\xi_{i,m} - \xi_{i,m}{}'\right)$

$$\tilde{\mathfrak{D}}_{ij}\left(E;\xi_i{}',\xi_j{}''\right) = \sum\!\!\!\!\!\!\int_e \sum_{c,k=1}^{\tilde{n}} \left\langle \hat{\phi}_i \xi_i{}' \middle| (\mathcal{H} - E) \middle| \mathcal{A}\left\{\tilde{\hat{\phi}}_c \tilde{\hat{\chi}}_c(e)\right\}\right\rangle \cdot$$

$$\cdot \frac{1}{E - E(e)} \left\langle \tilde{\hat{\phi}}_k \tilde{\hat{\chi}}_k(e) \middle| (\mathcal{H} - E) \middle| \mathcal{A}\left\{\hat{\phi}_j \xi_j{}''\right\}\right\rangle$$

$$\mathfrak{D}_{ij}\left(E;\xi_i{}',\xi_j{}''\right) \equiv \left\langle \hat{\phi}_i \xi_i{}' \middle| (\mathcal{H} - e) \middle| \mathcal{A}\left\{\hat{\phi}_j \xi_j{}''\right\}\right\rangle$$

i, j specify fragmentation, e.g., $\hat{A}\Big\{ \underbrace{\phi_{i,1} \dots \phi_{i,n_F(i)}}_{=:\hat{\phi}_i} \hat{\chi}_i\left(\xi_{i,1}, \dots, \xi_{i,n_F(i)-1}\right)\Big\}$