Talk at the workshop *Complexity of strong interactions and QCD*, ECT*, Trento, Italy, May 30, 2025

Breaking through the complexity of QCD in the Minkowski space-time

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This contribution concerns the complexity of the Hamiltonian formulation of QCD in the Minkowski space-time. The talk includes suggestions for overcoming the issues using the front form of dynamics and eigenvalue equations for describing bound states of the quanta of quark and gluon fields, and a method for computing effective Hamiltonians in quantum field theory, called the renormalization group procedure for effective particles. The latter is meant to provide a mathematical relationship between the complex parton-model picture of hadrons, viewed as relativistic many-body systems, with their simple classification as mostly made of just two or three constituent quarks. An outline of the steps of the computational scheme for deriving the quark and gluon structure of hadrons is provided as a summary.

For references, please see
K.Serafin, M.Gomez-Rocha, J.More, S.D. Glazek,
Dynamics of heavy quarks in the Fock space,
Phys. Rev. D, 109, 016017 (2024).

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Hamiltonian of QCD → front form of dynamics

why

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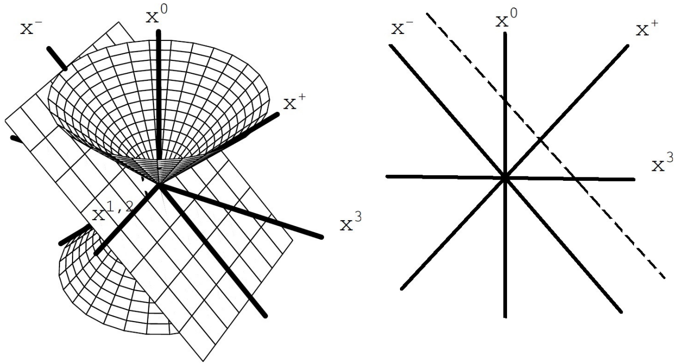

M. Wieckowski, S-matrix calculus using effective particles in the Fock space, hep-th/0511148

$x^{\pm} = x^0 \pm x^3$, $x^{\perp} = (x^1, x^2)$

$p^+ = E + p^3 > 0$

$px = \frac{1}{2} p^- x^+ + \frac{1}{2} p^+ x^- - p^{\perp} x^{\perp}$

energy $E = p^0 \to p^- = (p^{\perp 2} + m^2)/p^+$

boosting bound states, hadrons in the CMS, parton model in the IMF

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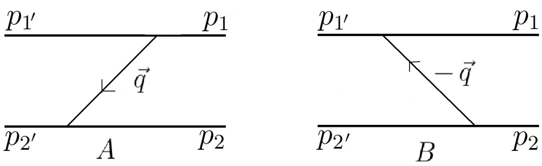

Instant form of dynamics $\quad E_q = \sqrt{m^2 + \vec{q}^{\,2}}$

$$A = \frac{1}{2E_q(E_1 + E_2 - E_{1'} - E_2 - E_q)} = \frac{1}{2E_q(E_1 - E_{1'} - E_q)} = \frac{1}{2E_q(q^0 - E_q)}$$

$$B = \frac{1}{2E_q(E_1 + E_2 - E_1 - E_{2'} - E_q)} = \frac{1}{2E_q(E_2 - E_{2'} - E_q)} = \frac{1}{2E_q(-q^0 - E_q)}$$

$$A + B = \frac{1}{2E_q(q^0 - E_q)} + \frac{1}{2E_q(-q^0 - E_q)} = \frac{1}{2E_q}\frac{-2E_q}{-q^{0\,2} + E_q^2} = \frac{1}{q^2 - m^2}$$

Front form of dynamics $\quad p^- = (m^2 + p^{\perp\,2})/p^+$

$$C = \frac{1}{q^+(p_1^- + p_2^- - p_{1'}^- - p_2^- - q^-)} = \frac{1}{q^+[p_1^- - p_{1'}^- - (q^{\perp 2} - m^2)/q^+]} = \frac{1}{q^+q^- - q^{\perp 2} - m^2} = \frac{1}{q^2 - m^2}$$

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Quantum fields

$A^+ = 0 \qquad n_f$

.

$$\hat{\psi} = \sum_{c=1}^{3} \sum_{\sigma=1}^{2} \int [p] \left[u_{p\sigma} \chi_c \hat{b}_{p\sigma c} e^{-ipx} + v_{p\sigma} \chi_c \hat{d}^\dagger_{p\sigma c} e^{ipx} \right]_{x^+=0}$$

$$\hat{A}^\mu = \sum_{c=1}^{8} \sum_{\sigma=1}^{2} \int [p] \left[\varepsilon^\mu_{p\sigma} T^c \hat{a}_{p\sigma c} e^{-ipx} + \varepsilon^{\mu *}_{p\sigma} T^c \hat{a}^\dagger_{p\sigma c} e^{ipx} \right]_{x^+=0}$$

$[p] = \int dp^+ d^2 p^\perp / [2p^+ (2\pi)^3]$ with $p^+ \geq 0$ Minkowski 3-dim p^+, p^1, p^2

$$\left\{ \hat{b}_{p\sigma c}, \hat{b}^\dagger_{p'\sigma' c'} \right\} = \left\{ \hat{d}_{p\sigma c}, \hat{d}^\dagger_{p'\sigma' c'} \right\} = \left[\hat{a}_{p\sigma c}, \hat{a}^\dagger_{p'\sigma' c'} \right] = 2p^+ (2\pi)^3 \delta^3(p - p') \delta_{\sigma\sigma'} \delta_{cc'}$$

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Hamiltonian $H \equiv P^-$ $A^+ = 0$

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$$\mathcal{L}_{\rm QCD} = \bar\psi \, (i \not\partial - g \not A - m) \, \psi - \frac{1}{2} \ {\rm Tr} \ G_{\mu\nu} G^{\mu\nu}$$

$$P^- \ = \ H_{\rm QCD} = \int d^2x^\perp dx^- \ \left[\frac{1}{2} \, \mathcal{T}^{+-} = \mathcal{H}_{\rm QCD} \right]$$

$$\begin{aligned}
\mathcal{H}_{\rm QCD} = \ & \bar\psi \frac{\gamma^+(-\partial^{\perp 2} + m^2)}{2i\partial^+} \psi + \ \frac{1}{2} \, A^a_\mu \partial^{\perp 2} A^{a\mu} - (J^{a\mu}_\psi + J^{a\mu}_A) A^a_\mu \\
& + \ \frac{1}{2} \, g^2 \bar\psi \not A \frac{\gamma^+}{i\partial^+} \not A \psi - \ \frac{1}{4} \, g^2 [A_\mu, A_\nu]^a [A^\mu, A^\nu]^a \\
& - \ \frac{1}{2} \, (J^{a+}_\psi + J^{a+}_A) \frac{1}{\partial^{+2}} (J^{a+}_A + J^{a+}_\psi)
\end{aligned}$$

$$J^{a\mu}_\psi = -g \bar\psi T^a \gamma^\mu \psi \qquad J^{a\mu}_A = ig[\partial^\mu A_\nu, A^\nu]^a$$

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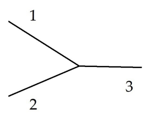
Divergent interactions

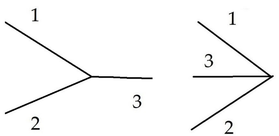
.

$$\mathcal{H}_{3g} = -J^{a\mu}_{Af} A^a_{f\mu} = -ig[\partial^\mu A_\nu, A^\nu]^a A^a_{f\mu} \quad \sim \quad A^3$$

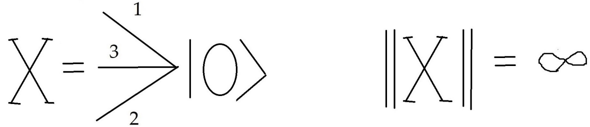
$$H_{3g} = \int dx^- d^2 x^\perp \, \mathcal{H}_{3g} = \sum_{\text{discrete}} \int [123] \, a_1^\dagger a_2^\dagger a_3 \; g \, Y_{123} \, \tilde{\delta}_{12.3} + h.c.$$

$$\tilde{\delta}_{12.3} = 2(2\pi)^3 \delta^3(p_1 + p_2 - p_3) \ , \quad Y_{123} = i f^{c_1 c_2 c_3} \left(\varepsilon_1^* \varepsilon_2^* \, \varepsilon_3 k_{12} - \varepsilon_1^* \varepsilon_3 \, \varepsilon_2^* k_{12}/x_2 - \varepsilon_2^* \varepsilon_3 \, \varepsilon_1^* k_{12}/x_1 \right)$$





Greatest threat: Vacuum



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Vacuum in the front form of dynamics

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$$H_I \sim \int_{\text{front}} d^3x\, A^3 \sim \left[\sum_p \left(a_p e^{-ipx} + a_p^\dagger e^{ipx}\right)\right]^3 \to a_1^\dagger a_2^\dagger a_3^\dagger \; \delta(1+2+3)$$

$$H_I|0\rangle = |123\rangle \quad \text{BUT} \quad (p_1+p_2+p_3)^+ = 0$$

Cutoff $p^+ \gtrsim \epsilon^+ > 0$

No terms like $a^{\dagger 3}$ in Hamiltonian with $\epsilon^+ \to 0$

$$\boxed{H|0\rangle = 0}$$

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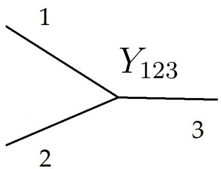
Regularization of interaction terms



$$Y_{123} \;\rightarrow\; f_\Delta(123)\; Y_{123}$$

RG $\rightarrow$ $\quad f_\Delta = e^{-(P^-_{12}-p^-_3)^2/\Delta^2} \qquad p^- \;=\; (p^{\perp 2}+m^2)/p^+ < \Delta \sim \epsilon^+$

$\boxed{\text{quarks} \quad m \qquad \text{gluons} \quad m_g \rightarrow 0}$

UV cutoff Δ implies $p^+ > (p^{\perp 2}+m_g^2)/\Delta$ and excludes $p^+ \lesssim \epsilon^+$

Two different kinds of variables, $p^+ > 0$ and $p^\perp$, no manifest rotation symmetry:

$\boxed{\text{The price of trivial FF vacuum } |0\rangle \text{ is complex FF renormalization problem.}}$

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Change of Wilsonian renormalization principle to SRG principle

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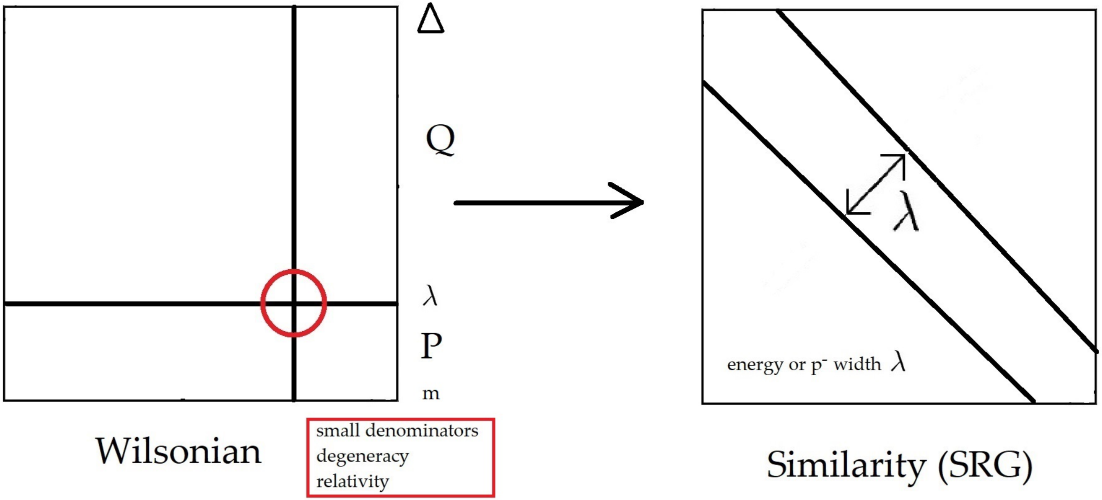
RG evolution changes the width λ from above Δ to about m



Another change: SRG → RGPEP

SRG Hamiltonian matrices → RGPEP Hamiltonian operators

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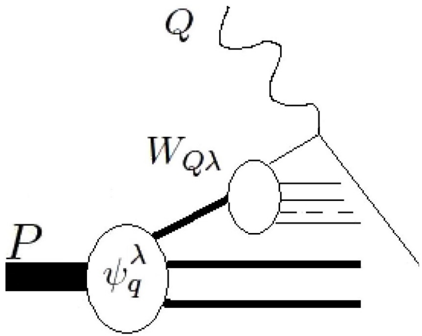
SRG $\to$ RGPEP: basis in the space of states $\to$ basis in the space of operators

.

$$a \to a_\lambda \qquad\qquad a^\dagger \to a_\lambda^\dagger$$

$$a_\lambda = \mathcal{U}_\lambda^\dagger \, a \, \mathcal{U}_\lambda \qquad\qquad a_\lambda^\dagger = \mathcal{U}_\lambda^\dagger \, a^\dagger \, \mathcal{U}_\lambda \qquad\qquad \mathcal{U}_\lambda = ?$$

Motivation is physical



$$H_\lambda(a_\lambda, a_\lambda^\dagger) = H(a, a^\dagger) \ , \quad a_Q = W_{Q\lambda} a_\lambda W_{Q\lambda}^\dagger \ , \quad W_{Q\lambda} = \mathcal{U}_Q^\dagger \mathcal{U}_\lambda$$

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Hamiltonian width λ and the RGPEP generator

$t = 1/\lambda^2 \qquad H_\lambda = H_f + H_{I\lambda} \qquad \mathcal{H}_\lambda = \mathcal{U}_\lambda^\dagger \, H_{\rm can} \, \mathcal{U}_\lambda \qquad \frac{d}{dt}\mathcal{H}_\lambda = \left[\mathcal{U}_\lambda'^\dagger \mathcal{U}_\lambda, \mathcal{H}\right]$

$t = 0 \quad \leftrightarrow \quad \lambda = \infty \qquad t \sim 1/m^2 \quad \leftrightarrow \quad \lambda \sim m$

initial condition $\mathcal{H}_{t=0} = \mathcal{H}_{\lambda=\infty} = H^\Delta_{\rm can} + CT_\Delta$

$\rightarrow$ cluster property, no disconnected effective interaction terms

SG&KW 1993-1998 $\mathcal{U}_\lambda'^\dagger \mathcal{U}_\lambda = \mathcal{F}[\mathcal{H}_\lambda]$	**for low orders of PT**	F.Wegner 1994 $\mathcal{U}_\lambda'^\dagger \mathcal{U}_\lambda = [\mathcal{H}_f, \mathcal{H}_{I\lambda}]$

PT refers to computation of the Hamiltonian operator $\mathcal{H}_\lambda$ itself, not states or observables.

$$\frac{d}{dt}\mathcal{H}_\lambda = [[\mathcal{H}_f, \mathcal{H}_{I\lambda}], \mathcal{H}_f + \mathcal{H}_{I\lambda}]$$

homogeneous solution $\mathcal{H}_{I\lambda \; mn} = \langle m|\mathcal{H}_{I\lambda}|n\rangle \; = \; e^{-(P_m^- - P_n^-)^2/\lambda^2} \, H_{I{\rm can}\, mn} \; = \; f_{\lambda\, mn} \, H_{I{\rm can}\, mn}$

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$g \to g_\lambda$ asymptotic freedom in $H_{\mathrm{QCD}\,\lambda}$ vertex form factors of width λ

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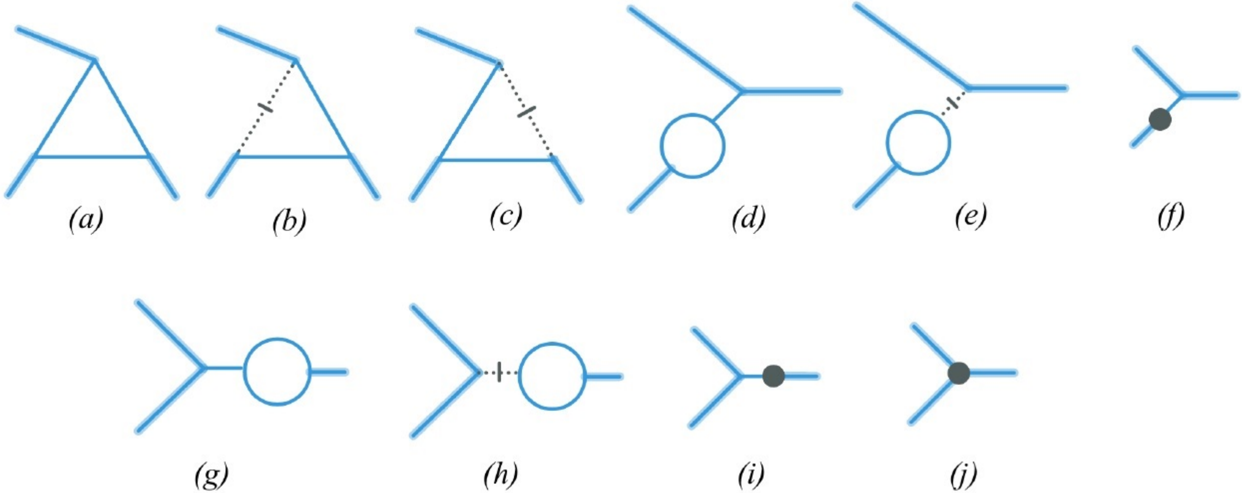

J.J.GalvezViruet, M.Gomez-Rocha, Phys. Rev. D108, 096001 (2023)

$$\boxed{gY \to g_\lambda f_\lambda Y}$$

Width λ leads to the nonperturbative window eigenvalue problems in AF models (theories).

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Window eigenvalue problem for hadrons

altered Wegner

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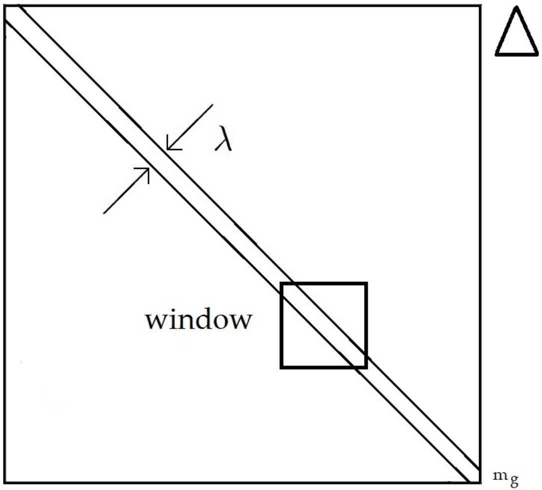

PT in $g_\lambda \ll 1$ for $\lambda > \lambda_0 \gtrsim m \gg \Lambda_{QCD}$

window middle eigenvalue matches whole matrix eigenvalue

the window eigenstate matches the whole matrix eigenstate

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Bound states in theories with asymptotic freedom (logarithmically altered generator)

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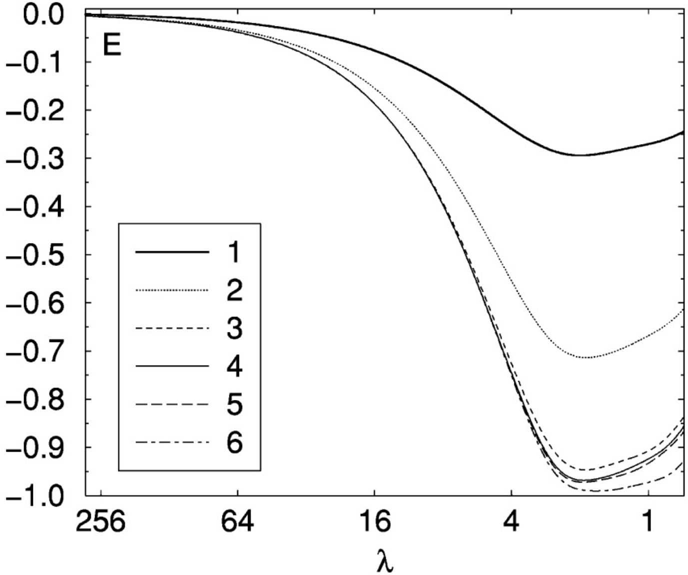

S.D. Glazek, J.Mlynik, Optimization of perturbative similarity renormalization group for Hamiltonians with asymptotic freedom and bound states, Phys. Rev. D 67, 045001 (2003).

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QCD windows differ from QED windows

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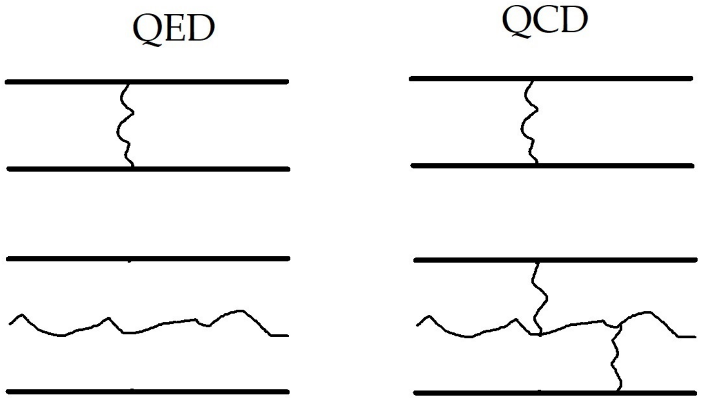

Gluons constantly (strongly) interact with quarks and gluons,

unlike photons with electrons and photons.

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blocking of gluons using $m_G \gg m_g$ and infrared confining effect (for $m_g \to 0$)

.

$$|\text{quarkonium}\rangle = |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + |Q\bar{Q}gg\rangle + |Q\bar{Q}ggg\rangle + ...$$

$$\to |Q\bar{Q}\rangle + |Q\bar{Q}G\rangle \qquad\qquad \mathbf{m_G \gg \Lambda}_{\text{QCD}}$$

$$H^{(2)}_{\text{QCD}\,\lambda}|\text{quarkonium}\rangle = \frac{P^{\perp 2} + M^2}{P^+}|\text{quarkonium}\rangle$$

$\boxed{\textbf{self-interaction} \sim \ln(\lambda/m_g)\ \textbf{canceled by gluon echange}\quad \text{‘t Hooft 1+1, Bloch-Nordsieck}}$

$$k^+ = \frac{1}{2}\,(1 + k^z/E_k) \qquad E_k = \sqrt{m^2 + \vec{k}^2} \qquad M = 2m + B \qquad m \gg \Lambda_{\text{QCD}} \qquad f_\lambda \to \text{NR appr.}$$

$$\frac{\vec{k}^{\,2}}{m}\,\psi(\vec{k}\,) + \int_q V_{\text{C}}(\vec{q}\,)\psi(\vec{k} - \vec{q}\,) + \int_q V_{\text{RG}}(\vec{q}\,)[\psi(\vec{k} - \vec{q}\,) - \psi(\vec{k}\,)] = B\psi(\vec{k}\,)$$

$$V_{\text{C}} = -\frac{4}{3}\,\frac{4\pi\alpha}{\vec{q}^{\,2}}\,(1 + \text{BF})$$

$$V_{\text{RG}} = \frac{4}{3}\,4\pi\alpha\,\left(\frac{1}{\vec{q}^{\,2}} - \frac{1}{q_z^2}\right)\,\frac{m_G^2}{m_G^2 + \vec{q}^{\,2}}\,\exp\left[-2\left(\frac{m\vec{q}^{\,2}}{\lambda^2 q_z}\right)^2\right]$$

Taylor expansion in $\vec{q}$ for $\psi(\vec{k} - \vec{q}\,) - \psi(\vec{k}\,)$

$Q\bar{Q}$ interaction and rotation symmetry

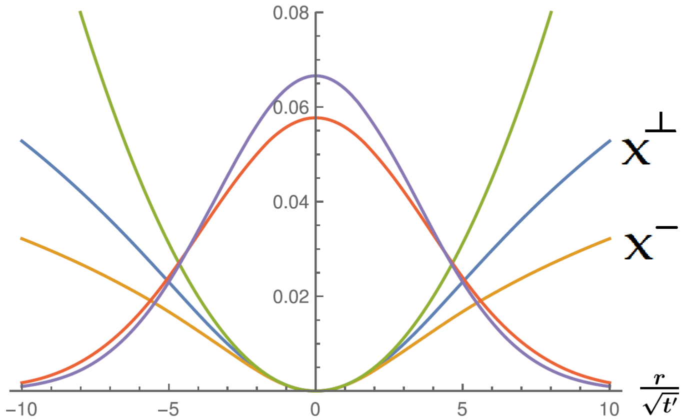
$\psi(\vec{k}-\vec{q}) - \psi(\vec{k}\,)$ Taylor in $\vec{q}$

.

$$V_{\rm RG} = -\,\frac{m\omega^2}{4}\,\Delta_{\vec{k}} \quad \underset{\rm F.T.}{\longleftrightarrow} \quad \tilde{V}_{\rm RG} \;=\; \frac{m\omega^2}{4}\,\vec{r}^{\,2}$$

$$\omega = \sqrt{\frac{4}{3}\,\frac{\alpha_\lambda}{\pi}}\,\left(\frac{\pi}{1152}\right)^{1/4}\frac{\lambda^2}{m^2}\,\lambda$$

the fourth-root factor is ~ 0.228520 extrapolation to $\alpha_\lambda/\pi \sim 0.1$ $m,\lambda \to \omega \sim \Lambda_{\rm QCD}$



K. Serafin ho $\to \ln r$, $-$, $\perp$, $\equiv$ coupling coherence, excited states $\to$ gluons, ω enough for strings

Example of an analytic RGPEP result

valley (m_Q, λ)

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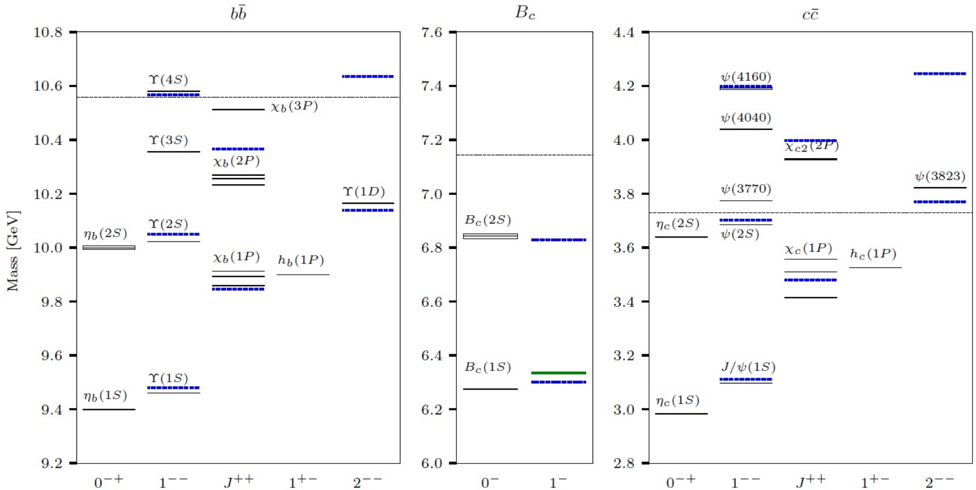

method	$\Omega_{ccc}(3/2+)$	$\Omega_{ccc}(3/2-)$
RGPEP [1]	**4797** (40,100) MeV	**5103** (40,100) MeV
Lattice [2]	**4793** (5)(7) MeV	**5094** (12)(13) MeV

[1] K.Serafin,M.Gómez-Rocha, J.More, S.D. Glazek,
Approximate Hamiltonian for baryons in heavy-flavor QCD, Eur. Phys. J. C 78, 964 (2018)

[2] N.S.Dhindsa, D.Chakraborty, A.Radhakrishnan, N.Mathur, M.Padmanath,
Precise study of triply charmed baryons (Ω_{ccc}), arXiv:2411.12729[hep-lat]

8 steps of the RGPEP

quantum Hamiltonian insights concerning QCD, SM and other theories

1. $\mathcal{L} \to \mathcal{H}_{\rm FF} \to H_{\rm FF}$
2. Regularization, ϵ^+, $\Delta \to \infty$, $m_g \to 0$ vacuum, UV, IR
3. Computation of H_λ order-by-order in weak-coupling expansion in $g \to 0$ $\Lambda_{\rm QCD} \ll m$
4. Counterterms $m^2_{\rm kinetic} \to m^2_\lambda$, $g \to g_\lambda$, $m_{\rm vertex} \to m_{\lambda\,\rm vertex}$, $h(x)$, . . .
5. Computation of windows AF $\to$ expansion in powers of g_λ
6. Extrapolation to relativistic g_λ
7. Diagonalization of windows, adjustment of masses to λ [**replace** m_G **with interactions**] g^4
8. Computation of observables NP from QCD

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QCD → Nuclear Physics

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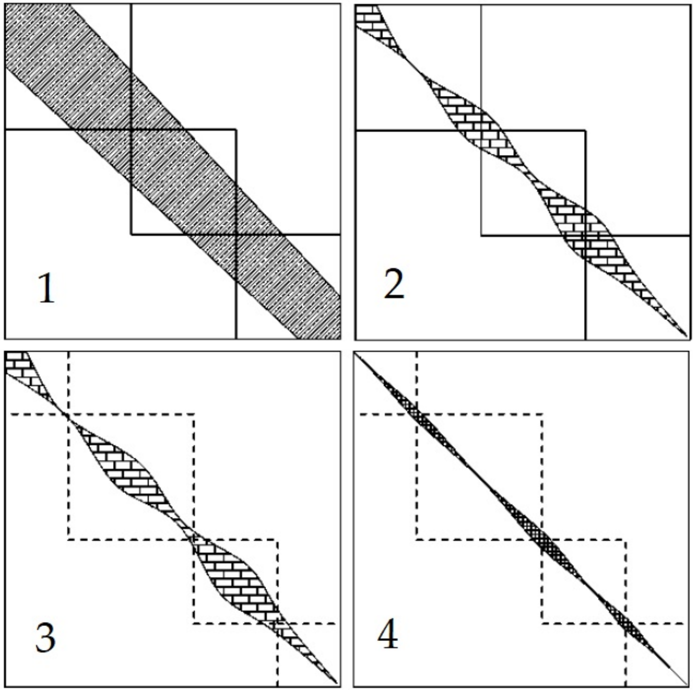

Large-momentum convergence of Hamiltonian bound-state dynamics of effective fermions in quantum field theory

SDG and Marek Więckowski, Phys. Rev D 66, 016001 (2002)

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