

The complex structure of strong interactions in Euclidean and Minkowski space

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Outline

Euclidean vs Minkowski

Minkowski-space quantities

The complex structure of confinement

(Non)perturbative methods

- ▶ Formal field theory
- ▶ Lattice
- ▶ Dyson–Schwinger equations
- ▶ Functional renormalisation group
- ▶ Gribov–Zwanziger approach
- ▶ Analytic / massive perturbation theory

Most nonperturbative calculations are in euclidean space
— but we live (?) in Minkowski space!

Euclidean vs Minkowski

Direct Minkowski calculations?

- ▶ Spectral or Nakanishi representation [Dudal, Mon]
- ▶ Real-time lattice (Complex Langevin, Lefschetz thimble/flow)
- ▶ Hamiltonian formalism [Glazek, Fri]

Euclidean vs Minkowski

Direct Minkowski calculations?

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Numerical analytical continuation

[Salas-Bernandez, Thu; Jay, Fri]

- ▶ Linear methods (Backus–Gilbert, HTL)
- ▶ Bayesian methods (MEM, BR, Gaussian processes)
- ▶ Padé, continued fractions (Schlessinger)
- ▶ Nevanlinna–Pick

Complex complications

- ▶ Obstructions to analytical continuation from singularity structure? [van Egmond, Tue]
- ▶ Complicated contour deformations [Huber, Ferreira, Thu]
- ▶ How do topological features translate? [Krein, Mon; Hayashi, Fri]

Bound states, decays, scattering

[Thursday talks]

- ▶ **Stable state** = pole on **timelike** real axis
- ▶ **Resonance** = pole off real axis
- ▶ **Multiparticle threshold** = branch cut on timelike real axis

Stable states only can be extracted **directly** on lattice from exponential decay in euclidean time

Momentum-space BSEs require sols in **complex momentum space**

Minkowski-space quantities

- ▶ parton distribution functions [Wed talks; Glazek, Fri]
- ▶ fragmentation functions [Roberts, Wed]
- ▶ scattering, decay [Lo, Tue; Thomas, Salas-Bernárdez, Thu]
- ▶ transport, thermal broadening [Lowdon, Tue]
- ▶ equilibration, dynamical critical phenomena

Confinement?

How to define confinement of coloured entities?

- ▶ Wilson/Polyakov loop [Cohen, Fri]
— only valid in ∞ heavy quark limit
- ▶ Failure of cluster decomposition, dressing fields
[Peruzzo, Mon]
- ▶ Non-positive norm states, BRST quartet mechanism
- ▶ Absence of asymptotic states

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- ▶ Does absence of phase transition prevent definition?
[Reinosa, Tue]
- ▶ Can we distinguish between confinement and BEH?

Analytical structure

Proposal:

confinement = absence of asymptotic states
with quark/gluon quantum numbers
= absence of poles on real timelike axis

- ▶ prospects for determining analytical structure? [Comitini, Tue]
- ▶ lessons from unstable particles, decays? [Lo, Tue]
- ▶ Confining models in Euclidean and Minkowski
[Palhares, Pereira, Krein, Mon]

Deconfinement?

How to define deconfined matter?

- ▶ Does absence of phase transition prevent definition?
[Reinosa, Tue]
- ▶ Changes in analytic structure for confined particles?
[Comitini, Tue]
- ▶ Can we distinguish between confinement and BEH?
- ▶ No asymptotic states exist in any thermal medium
→ no poles on real axis

THANK YOU!