Gravitational form factors from Continuum Schwinger methods

Universidad de Huelva
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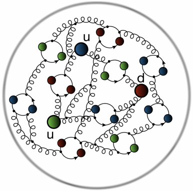
Zhao-Qian Yao (UHU&UPO)

The complex structure of strong interactions in Euclidean and Minkowski space
ECT*, Trento, 28.5.2025

Nucleon and QCD

Proton



- Proton(p): $u + u + d$; Neutron(n): $d + d + u$.
- infinitely many gluons and sea-quark.
- These quarks and gluons are bound together by the strong interactions, described by quantum chromodynamics (QCD).
- QCD's Running Coupling is getting large as energy scales decrease.

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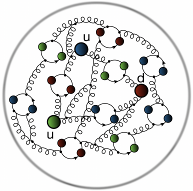

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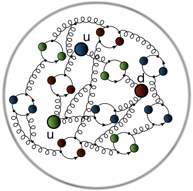

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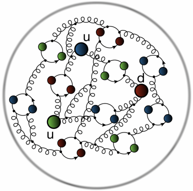

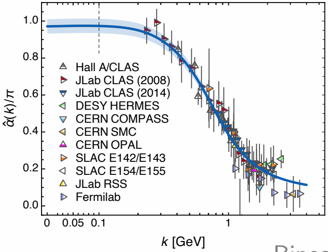

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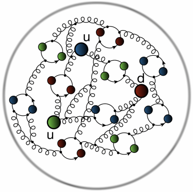

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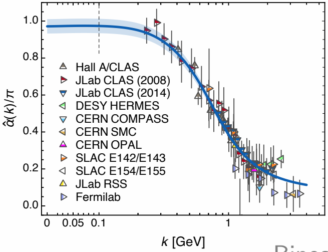

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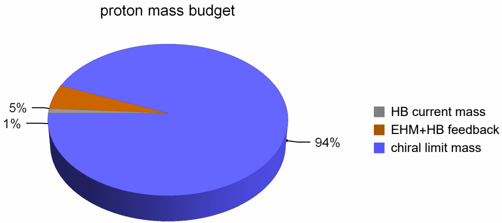

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Dyson-Schwinger Equations (DSEs) are a system of infinitely coupled equations for Schwinger functions.

DSEs provide a widely used approach to hadron structure. Roberts:1994dr

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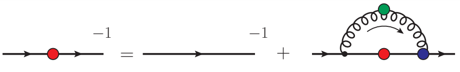
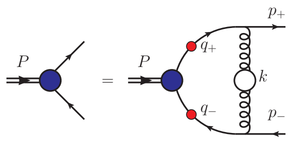


- Bethe-Salpeter Equations: DSEs for two quark bound state: $\Gamma(q,P)$



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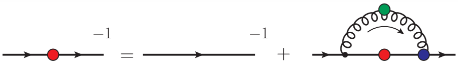

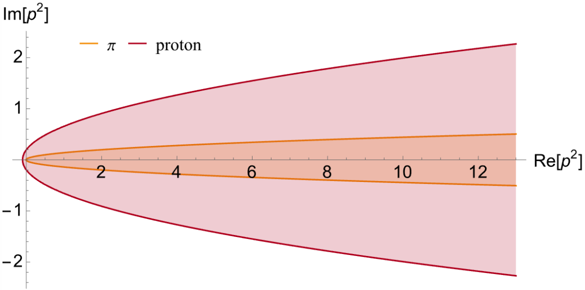

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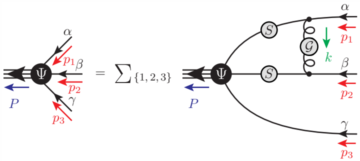
DSEs: Faddeev Equation - three quarks



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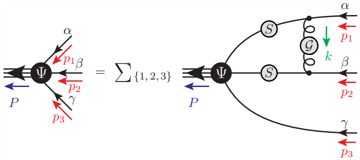

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Faddeev Amplitude:

$$\Psi_{ABCD}(p,q,P) = \left(\sum_{\rho=0}^{1} \psi^{\rho}_{\alpha\beta\gamma\mathcal{I}}(p,q,P) \otimes \mathrm{F}^{\rho}_{abcd} \right) \otimes \frac{\epsilon_{rst}}{\sqrt{6}},$$

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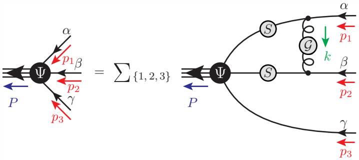

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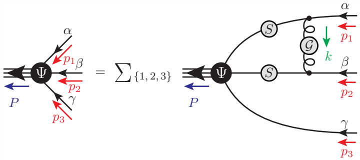

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- $\psi^{\rho}_{\alpha\beta\gamma\mathcal{I}}(p,q,P)$ is the **spin-momentum Faddeev amplitude**.

$$\psi^{\rho}_{\alpha\beta\gamma\mathcal{I}}(p,q,P) = \sum_{i}^{64} f^{\rho}_{i}\ \mathrm{X}_{i,\alpha\beta\gamma\mathcal{I}}(p,q,P)$$

Nucleon gravitational form factors

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[7] Technische Universität Dresden, 01062 Dresden, Germany

Received: 2 April 2025 / Accepted: 4 April 2025

Abstract A symmetry-preserving analysis of strong interaction quantum field equations is used to complete a unified treatment of pion, kaon, and nucleon electromagnetic and gravitational form factors. Findings include a demonstration that the pion near-core pressure is roughly twice that in the proton, so both are significantly greater than that of a neutron star; parton species separations of the nucleon's three gravitational form factors, in which, *inter alia*, the glue-to-quark ratio for each form factor is seen to take the same constant value, independent of momentum transfer; and a determination of proton radii orderings, with the mechanical (normal

These statements are a succinct expression of an emergent hadron mass (EHM) paradigm, developed via insightful use of continuum Schwinger function methods (CSMs) – see Ref. [6] and citations thereof. The three pillars of EHM are appearance of a gluon mass scale [7], infrared saturation and cessation of running in QCD's effective charge [8], and dynamical chiral symmetry breaking expressed in a nonzero chiral-limit running quark mass [9,10]. This paradigm is drawing support from results obtained using realistic simulations of lattice-regularised QCD (lQCD) [1–5] and being/will be tested by comparisons between EHM-based predictions

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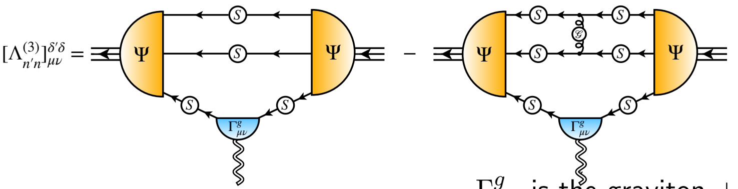
Nucleon Gravitational Form Factors

Matrix element energy-momentum tensor(EMT):



$\Gamma^g_{\mu\nu}$ is the graviton + quark vertex.

Xu:2023izo

- $A(Q^2)$: mass distribution form factor; $A(Q^2=0)=1$
- $J(Q^2)$: spin distribution form factor; $J(Q^2=0)=1/2$
- $D(Q^2)$: pressure distribution form factor. $D(Q^2=0)=?$

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\begin{aligned}
&m_N \Lambda^{Ng}_{\mu\nu}(Q) = \\
&-\Lambda_+(p_f)[K_\mu K_\nu A(Q^2) \\
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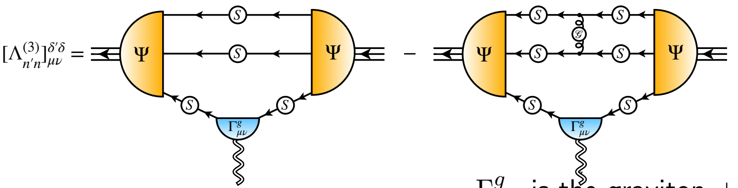

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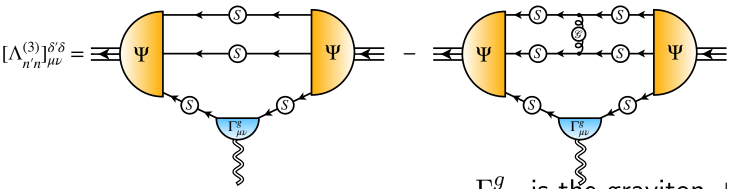

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Last unknown global property of nucleon

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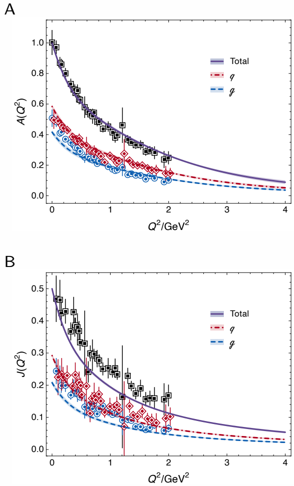
Gravitational Form Factors $A(Q^2)\&J(Q^2)$

- Our prediction for $A(Q^2)\&J(Q^2)$ compare with lQCD's results from Hackett:2023rif
- Symmetry-preserving character of CSM analysis is evident in values of $A(Q^2=0)=1$ and $J(Q^2=0)=1/2$.
- The total GFFs are scale independent. However these GFFs for quarks and gluons are scale dependent.
- GFFs species decomposition
  We arrive at the $\xi := \xi_2 = 2$ GeV.



Gravitational Form Factors $D(Q^2)$

- Nucleon pressure Form Factors $D(Q^2)$
- Nucleon “D-term”:
  Our prediction: $D(0) = -3.11(1)$
  lQCD's results:
  z-expansion: $-3.87(97)$; Dipole: $-3.35(58)$
  D:
  Cao:2024zlf;
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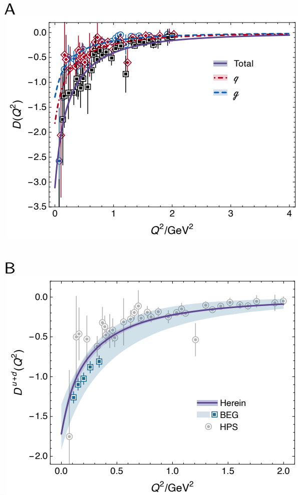
- Light quarks alone $D^{u+d}(0;\zeta_2) = -1.73(5)$
  Inference from available DVCS data yields
  $D^{u+d}(0;\zeta_2) = -1.63(29)$ Burkert:2018bqq



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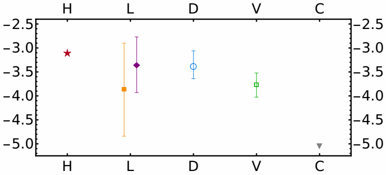

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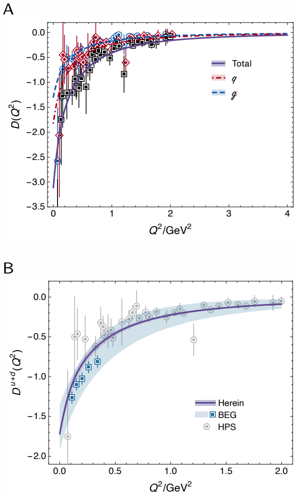

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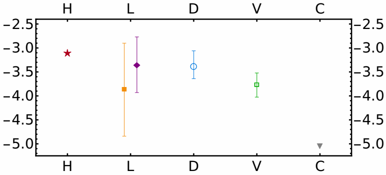

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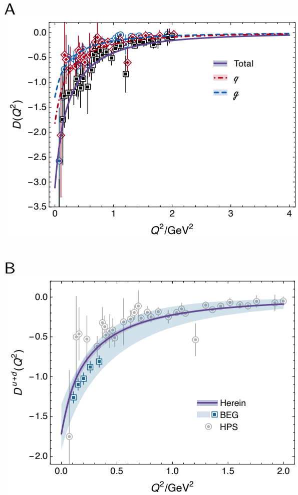

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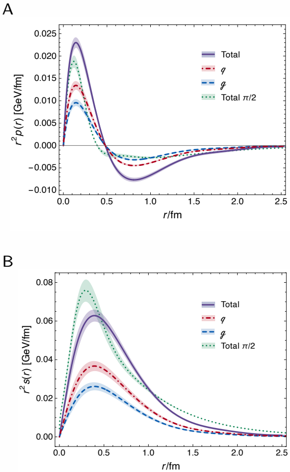
Gravitational Form Factors Pressure & Shear-force

- The pressure density positive peak is near $0.15\,\mathrm{fm}$ and changes sign near $0.5\,\mathrm{fm}$. The shear density peak is near $0.4\,\mathrm{fm}$.
- Pion (green) peak values are roughly twice those in the proton
- nucleon mass and mechanical radii: $r_{mass} = 0.81(5) r_{ch} > r_{mech} = 0.72(2) r_{ch}$
- Species decompostion(ζ_2)



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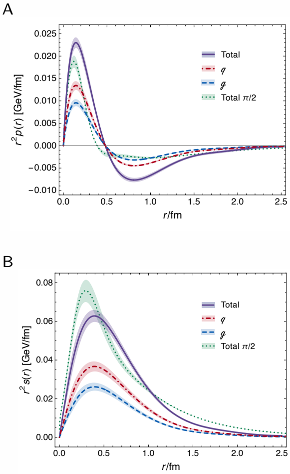

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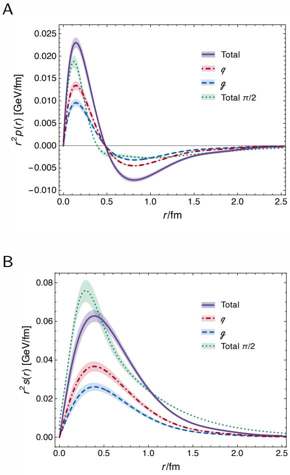

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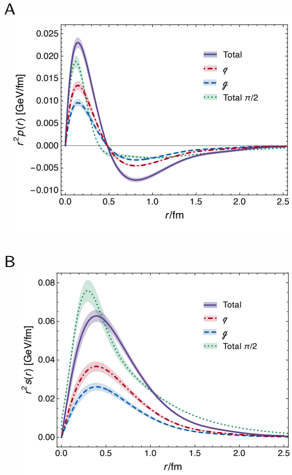
  - $r^q_{mass} = 0.62(4) r_{ch} > r^g_{mass} = 0.52(3) r_{ch}$
  - $r^q_{mech} = 0.55(2) r_{ch} > r^g_{mech} = 0.47(2) r_{ch}$



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- **Proton radius**
  $r_A = 0.621(06)\ \text{fm} < r_{mass} = 0.718(51)\ \text{fm} < r_{ch} = 0.887(3)\ \text{fm}$

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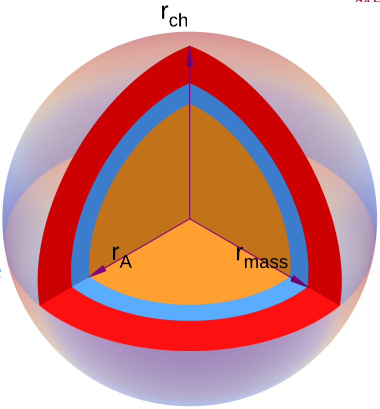
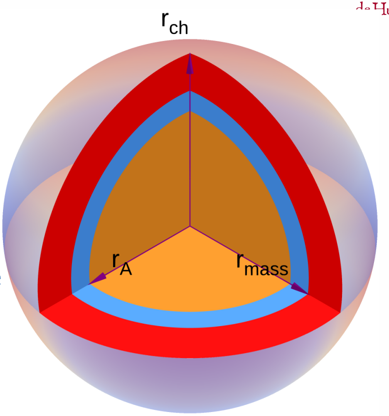
  $r_A = 0.621(06)\,\text{fm} < r_{mass} = 0.718(51)\,\text{fm}$



Summary

- **Gravitational Form Factors**
  - $A(Q^2)\&J(Q^2)$
  - Nucleon "D-term": $D(0) = -3.11(1)$
    Light quarks: $D^{u+d}(0;\zeta_2) = -1.73(5)$
  - Mass and mechanical radii:
    $r_{mass} = 0.81(5)r_{ch} > r_{mech} = 0.72(2)r_c$
  - Pressure & Shear-force
- **Proton radius**
  $r_A = 0.621(06)\,\text{fm} < r_{mass} = 0.718(51)\,\text{fm}$



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Summary

- **Gravitational Form Factors**
  - $A(Q^2) \& J(Q^2)$
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    Light quarks:
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    $r_{mass} = 0.81(5) r_{ch} > r_{mech} = 0.72(2) r_c$
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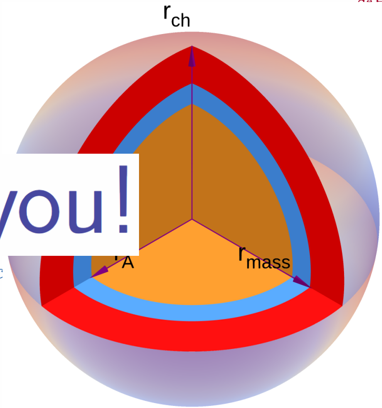
Thank you!



Supplementary Material

- The key element is the quark+quark scattering kernel, for which the RL truncation is obtained by writing:

$$\mathcal{K}_{tu}^{rs}(k) = \mathcal{G}_{\mu\nu}(k)[i\gamma_\mu \frac{\lambda^a}{2}]_{ts}[i\gamma_\nu \frac{\lambda^a}{2}]_{ur} \,, \tag{1a}$$

$$\mathcal{G}_{\mu\nu}(k) = \tilde{\mathcal{G}}(y) T_{\mu\nu}(k) \,, \tag{1b}$$

$k^2 T_{\mu\nu}(k) = k^2 \delta_{\mu\nu} - k_\mu k_\nu$, $y = k^2$.r, s, t, u represent colour, spinor, and flavour matrix indices (as necessary).

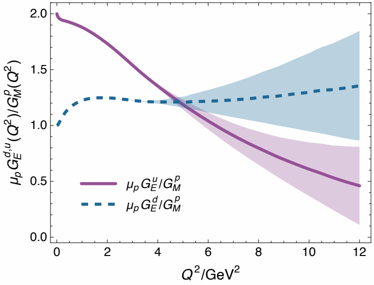
Supplementary Material

$$\tilde{\mathcal{G}}(y) = \frac{8\pi^2}{\omega^4} D e^{-y/\omega^2} + \frac{8\pi^2 \gamma_m \mathcal{F}(y)}{\ln\left[\tau + (1 + y/\Lambda_{\rm QCD}^2)^2\right]}, \tag{2}$$

where $\gamma_m = 12/25$, $\Lambda_{\rm QCD} = 0.234\,\text{GeV}$, $\tau = \text{e}^2 - 1$, and
$\mathcal{F}(y) = \{1 - \exp(-y/\Lambda_I^2)\}/y$, $\Lambda_I = 1\,\text{GeV}$.
We employ a mass-independent (chiral-limit) momentum-subtraction renormalisation scheme.
Contemporary studies employ $\omega = 0.8\,\text{GeV}$. With $\omega D = 0.8\,\text{GeV}^3$ and renormalisation point invariant quark current mass $\hat{m}_u = \hat{m}_d = 6.04\,\text{MeV}$, which corresponds to a one-loop mass at $\zeta = 2\,\text{GeV}$ of $4.19\,\text{MeV}$, the following predictions are obtained:

$$m_\pi = 0.14\,\text{GeV};$$
$$m_N = 0.94\,\text{GeV};$$
$$f_\pi = 0.094\,\text{GeV}.$$

Electromagnetic form factors $\mu_N G_E^N(Q^2)/G_M^N(Q^2)$



- In isospin symmetry, the flavour separation of the charge and magnetic form factors ($e_u = 2/3$, $e_d = -1/3$):

$$G_E^p = e_u G_E^u + e_d G_E^d\,, \quad G_E^n = e_u G_E^d + e_d G_E^u\,.$$

- G_E^p possesses a zero because G_E^u/G_M^p falls with increasing Q^2 whereas G_E^d/G_M^p is positive and approximately constant.
- G_E^n does not exhibit a zero because $e_u > 0$, G_E^d/G_M^p is large and positive, and $|e_d G_E^u|$ is always less than $e_u G_E^d$.

Flavor amplitudes

state	$F_{\mathcal{M}_\mathcal{A}}$	$F_{\mathcal{M}_\mathcal{S}}$
p	$\frac{1}{\sqrt{2}}(udu - duu)$	$\frac{1}{\sqrt{6}}(2uud - udu - duu)$
n	$\frac{1}{\sqrt{2}}(udd - dud)$	$\frac{1}{\sqrt{6}}(udd + dud - 2ddu)$

Table: Baryon octet flavor amplitudes; we define $\lambda_1\lambda_2\lambda_3 := \lambda_1 \otimes \lambda_2 \otimes \lambda_3$, and, $u^\dagger := (1\ 0)$, $d^\dagger := (0\ 1)$.