Center vortices and gluon mass generation

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The Complex Structure of Strong Interaction is Euclidean and Miskowski Space

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Outline

1. Motivation: role of center vortices in lattice QCD
2. Model: features observed in simulations
3. The model vacuum state, degrees of freedom
4. Results: area law (known), gluon mass (here)
5. Conclusions & Perspectives

Work with D. Rosa Jr, L.E. Oxman and B.R. Soares

Important gluon configurations

Center vortices seem to be responsible for most of the nonpertubative long-range physics*.

Lattice QCD, removing them:

- Pure Yang-Mills: no flux tubes
- Dynamical quarks:
  - no hadron mass generation
  - spectral positivity violation

Difficult to model these configurations

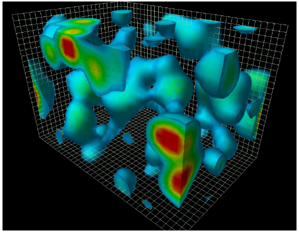
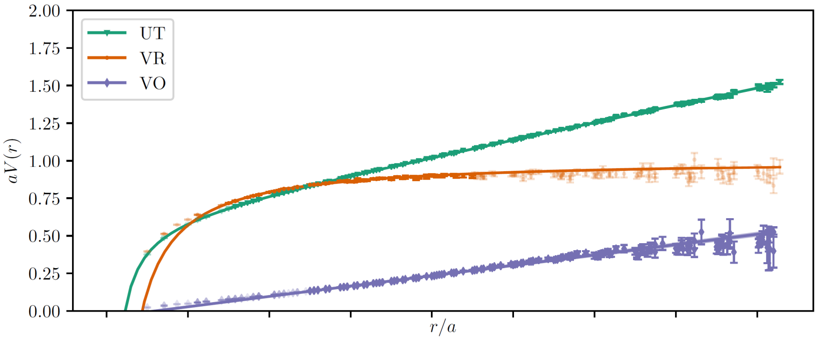


Fig. from D. Leinweber et al.

*'t Hooft, Greensite, Faber, Langfeld, Olejník, Leinweber, Oxman, Reinhardt, Rosa Jr., ...

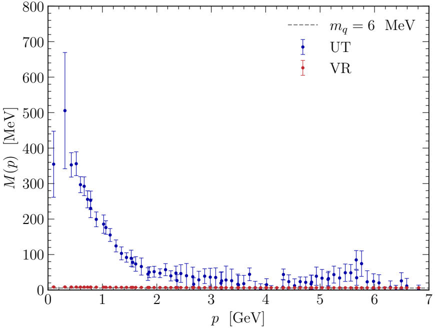
No vortices, no flux tubes*

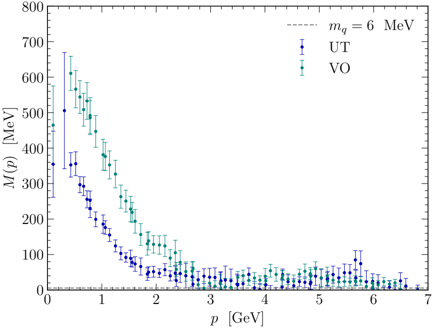
Pure Yang-Mills



J. C. Biddle, W. Kamleh, D. B. Leinweber, PRD 106, 054505

No vortices, no mass generation*

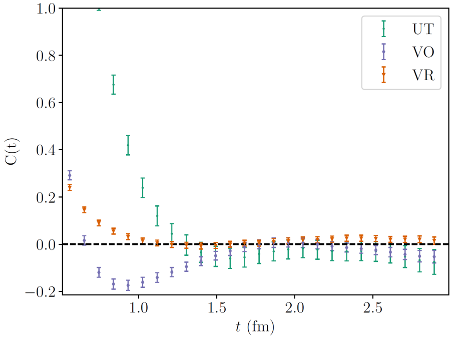




*W. Kamleh, D. B. Leinweber, A. Virgili, PRD 110, L051502 (2024)

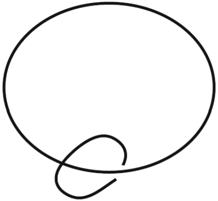
Positivity violation (gluon prop.)*

$$C(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp_0 \int_0^{\infty} d\sigma^2 \, \frac{\rho(\sigma^2)}{p_0^2 + m^2} \, e^{-ip_0 t} = \int_0^{\infty} d\sigma \, e^{-\sigma t} \, \rho(\sigma^2)$$



*J. C. Biddle, W. Kamleh, D. B. Leinweber, PRD 107, 094507(2023)

Center vortices



Wilson loop

- $W(C) = \mathrm{Tr}\, \mathcal{P} \exp\left(i \oint_C A^i dx^i\right)$
- Center vortex: $W(C) \to z\, W(C),\ z \in Z(N)$
- SU(2): $W(C) \to -W(C)$

Vortex : $A_\theta \sim \dfrac{1}{r}\, \sigma_3,$ thin vortex

Generic curve: $\partial^j F_{ji}(x) = j_i(x)$

$$j_i(x) = \sigma_3\, 2\pi \int_C d\sigma\, \frac{dx_i(\sigma)}{d\sigma}\, \delta^{(3)}(x - x(\sigma))$$

The Model*

— Formulated in Minkowski space
— Based on the Hamiltonian formalism
— Analytical calculation done in $3+1$ spacetime dimensions
— Correlation functions computed with a vacuum functional
— Features and degrees of freedom observed in lattice simulations:

- Abelian dominance, large vortices (percolating)
- Mixed ensemble of oriented and nonoriented (monopoles) thin center vortices
- Monopoles: flux orientation change
- Open oriented lines matching at common end points (monopole-antimonopole)

— Describes Wilson loop area law
— **Here: gluon mass generation**
— Validity: (deep) infrared

*D. Rosa Jr., H. Reinhardt, L.E. Oxman, PRD 106, 114021 (2022)

Vacuum functional*

— Example: magnetic field of an oriented vortex + monopole: $\boldsymbol{B}^q = \boldsymbol{\nabla} \times \boldsymbol{a}^q(\gamma) - \boldsymbol{\nabla}\zeta^q(\gamma)$

$$\boldsymbol{a}^q(\gamma) = \frac{\boldsymbol{\nabla} \times \boldsymbol{j}^q(\gamma)}{-\nabla^2} \quad \text{and} \quad \zeta_i^q = \frac{\boldsymbol{\nabla} \cdot \boldsymbol{b}^q(\gamma)}{-\nabla^2} \quad \text{with} \quad \boldsymbol{b}^q(\gamma) = \boldsymbol{j}^q(\gamma)$$

$$j_i^q(\gamma) = \mathcal{C}^q \int_\gamma ds \, \frac{d\bar{x}_i(s)}{ds} \delta(x - \bar{x}(s)) \quad \text{where} \quad q = 1, \cdots, N-1 \;\; \leftarrow SU(N)$$

— Vacuum state:

Written as functional of the sum of all field configurations centered at the thin vortex fields $(a^q(\gamma), \zeta^q(\gamma))$, weighted by the probability amplitude ψ_γ of occurrence of a given configuration $\{\gamma\}$

*D. Rosa Jr., H. Reinhardt, L.E. Oxman, PRD 106, 114021 (2022)

Vacuum functional (cont'd)

The probability amplitude ψ_γ

— $\psi_\gamma = \psi_\gamma(\kappa, \mu, \lambda_0, \xi_0, \vartheta_0)$

- $1/\kappa$: stiffness, μ: tension λ_0: contact interaction strength $\leftarrow$ of vortex lines
- ξ_0: prob. ampl. of an $N-$matching point
- ϑ_0: prob. ampl. of a monopole

— Prob. ampl. due to intrinsic properties of vortices: $\psi_\gamma^{\text{intr.}} = e^{-S_0(\gamma)-S_{\text{int}}(\gamma)}$

- $S_0(\gamma) = \int_\gamma ds \, \left(\frac{1}{2\kappa}\dot{u}\cdot\dot{u} + \mu\right)$, $\quad x = x(s)$, $\quad \dot{x} = \frac{dx}{ds}$, $\quad \dot{u} = \frac{\dot{x}}{\sqrt{\dot{x}}}$
- $S_{\text{in}} = \frac{\lambda_0}{2}\int d^3x \, \rho^2(x)$, $\quad \rho(x) = \int_\gamma ds \, \delta(x - x(s))$

— Complete prob. ampl.: for V matching points and M monopoles

$$\boxed{\psi_\gamma = {\xi_0}^V \, \vartheta_0^M \, \psi_\gamma^{\text{intr.}}}$$

Vacuum functional (cont'd)

— Work in electric field representation (Fourier transform):

$$\widetilde{\Psi}_{\text{c.v.}}(E,\eta) = \int [DA][D\zeta]\, e^{i(E,A)+i(\eta,\zeta)}\ \Psi_{\text{c.v.}}(A,\zeta)$$

$$\Psi(A,\zeta) = \sum_{\{\gamma\}} \delta(A - a(\{\gamma\}))\, \delta(\zeta - (-\Delta)^{-1}\nabla b(\{\gamma\}))\ \psi_{\{\gamma\}}$$

— Sum/integration over configurations replaced by an effective field theory

$$\widetilde{\Psi}_{\text{c.v.}}(E,\eta) = \int [D\Phi^\dagger][D\Phi]\, e^{-\int d^3x \left[\mathrm{Tr}|\boldsymbol{D}(E,\eta)\Phi)|^2 + V(\Phi)\right]}$$

$$\boldsymbol{D}(E,\eta) = \boldsymbol{\nabla} - iC^q \left(\frac{\boldsymbol{\nabla}\times\boldsymbol{E}^q}{-\nabla^2} + \frac{\boldsymbol{\nabla}\eta^q}{-\nabla^2} \right)$$

$$V(\Phi) = \frac{\lambda}{2}\,\mathrm{Tr}(\Phi^\dagger\Phi - aI_N)^2 - \xi\,(\det\Phi + \det\Phi^\dagger) - \vartheta\,\mathrm{Tr}(\Phi^\dagger T_A \Phi T_A)$$

$$\lambda = 9\kappa^2\lambda_0, \quad a^2 = -\frac{\mu}{3\kappa\lambda_0} - \frac{\vartheta_0}{3\kappa\lambda_0}\frac{N-1}{N}, \quad \xi = (3\kappa)^{\frac{N}{2}}\xi_0, \quad \vartheta = 6\kappa N\vartheta_0\ .$$

Wilson loop - area law*

For $(\lambda, \kappa) > 0$ and $\mu < 0$: percolating vortices, large center vortices are favored

$\Rightarrow$ SSB driven by the first term in $V(\Phi)$: $\frac{\lambda}{2}\,\mathrm{Tr}(\Phi^\dagger\Phi - aI_N)^2$

String tension: $\langle W(C)\rangle \sim e^{-\sigma A}$ $\leftarrow$ σ determined by two parameters: v and ϑ:

$$\sigma = 2v^2\,\frac{N-1}{N}\int_{-\infty}^{+\infty} dz\,\left[\frac{\partial\theta(z)}{\partial z}\right]^2$$

$$\frac{\partial^2\theta(z)}{\partial z^2} - \frac{\vartheta}{2}\theta(z) = 0, \qquad \theta(-\infty) = 0, \quad \theta(+\infty) = 2\pi, \qquad C \text{ in } x - y \text{ plane}$$

$$2\lambda N(v^2 - a^2) - 2\xi N v^{N-2} - \vartheta\,\frac{N^2-1}{N} = 0, \quad v^2 = \langle(\Phi^\dagger\Phi)^2\rangle \sim \langle\rho^2\rangle \text{ (density center vortex lines)}$$

*D. Rosa Jr., H. Reinhardt, L.E. Oxman, PRD 106, 114021 (2022)

Gluon mass

Magnetic field correlation function: $\langle B_i^a(\boldsymbol{k}) B_j^b(\boldsymbol{k'})\rangle = \langle \Psi | B_i^a(\boldsymbol{k}) B_j^b(\boldsymbol{k'}) | \Psi \rangle$

- $\Phi(x) \simeq v + iT^q \theta^q(x)$
- Expand action up to quadratic terms in θ
- Compute exactly the correlation function
- Result:

$$\langle B_i^a(\boldsymbol{k}) B_j^b(\boldsymbol{k'})\rangle = \delta^{pq}\,(2\pi)^3 \delta(\boldsymbol{k'} - \boldsymbol{k})\, 8\pi^2 N v^2 \left(\delta_{ij} - \frac{k_i k_i}{\boldsymbol{k}^2 + \boldsymbol{m}_g^2}\right)$$

$$\boldsymbol{m_g^2} = \frac{\boldsymbol{\vartheta}^2}{2}$$

Value of m_g: determined by one of the parameters that determine the string tension

Significance of the result

Compute correlation function with a naive "Gribov" action:

$$\Psi(A) \sim e^{-\frac{1}{2}\int d^3x \int d^3y\ A^a(x)\, w(x-y) A^a(y)}$$

$$\omega(k) = \sqrt{\boldsymbol{k}^2 + m_G^4/\boldsymbol{k}^2}$$

- $B_i = \frac{1}{2}\epsilon_{ijk}F_{jk}, \qquad F_{ij} = \partial_i A_j - \partial_j A_i + ig[A_i, A_j]$
- $\rho^q = \boldsymbol{k} \cdot \boldsymbol{B}^q,\ \langle \rho^q(\boldsymbol{k})\rho^p(\boldsymbol{k}')\rangle \xrightarrow[k\to 0]{} \delta^{pq}\,\delta(\boldsymbol{k}+\boldsymbol{k}')\ \boldsymbol{k}^2$
- ρ^q correlation function vanishes in the infrared (as in center vortex model)

Relevance in QCD: string breaking

Lattice results - static sources*

– Energy stored: $E_\sigma = 1.1$ GeV/fm

– Breaking distance r_c:

- $m_\pi = 280$ MeV: $r_c = 1.2$ fm
- $m_K = 460$ MeV: $r_c = 1.3$ fm

Nonstatic sources:

$$m_\pi = 140 \text{ MeV} \to r_c = \frac{280 \text{ MeV}}{1.1 \text{GeV/fm}} < 1/3 \text{ rm}$$

Breaking occurs within a hadron

* J. Bulava, B. Hörz, F. Knechtli, V. Koch, G. Moir, C. Morningstar, M. Peardon, PLB 793, 493 (2019)

Conclusion & Perspectives

1. Continumm model for center vortices in YM theory:
   - Known: determine string tension
   - Here: generate gluon mass
2. Input: oriented and nonoriented (monopoles) flux lines
3. Enough parameters to accommodate phenomenological values
4. Next:
   - Go beyond quadratic action, positivity violation
   - Generalize to thick vortices
   - Couple quarks, string breaking
   - Dynamical chiral symmetry breaking

Thank you

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