S-MATRIX APPROACH TO THERMODYNAMICS OF HADRONS

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University of Wroclaw

ECT* WORKSHOP
THE COMPLEX STRUCTURE OF STRONG INTERACTIONS IN EUCLIDEAN AND MINKOWSKI SPACE
05.2025, TRENTO

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969).

R. Venugopalan and M. Prakash,
Nucl. Phys. A546, 718 (1992).

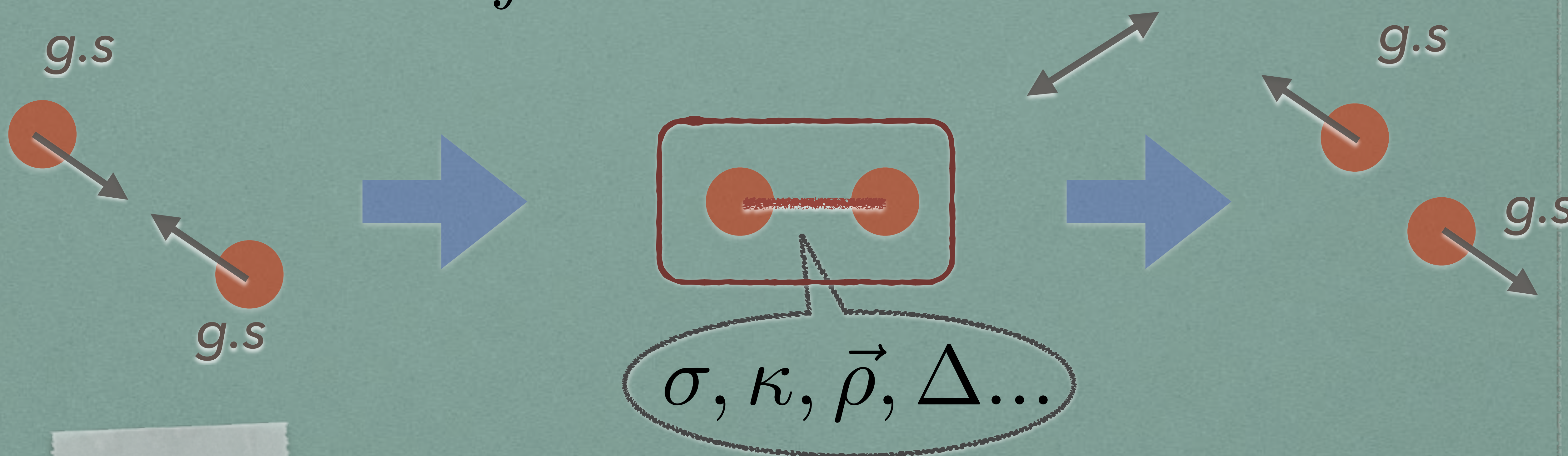
(study notes)

PML, EPJC **77** no.8 533 (2017)

PML PRD **102**, 034038 (2020)

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE\, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr}\, (\delta_E)\,.$$



+ repulsions

PWA
X
S-matrix thermo.

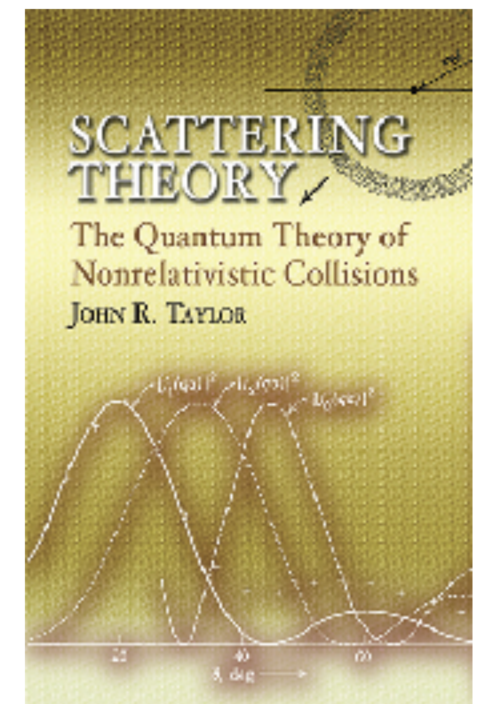
$$\delta \longrightarrow \mathcal{Q}(M) \equiv \frac{1}{2}\, \mathrm{Im}\, (\mathrm{tr} \ln \mathrm{S})$$

Thermodynamics & Scattering

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

lousy derivation

$$\operatorname{tr} e^{-\beta \hat{H}} \to \int \frac{dE}{2\pi} e^{-\beta E} \operatorname{tr} \underline{2\pi \delta(E - \hat{H})}$$



$$-2\, \mathrm{Im} \frac{1}{E - \hat{H} + i\delta} = \frac{1}{i} \frac{\partial}{\partial E} \ln \left({G^*}^{-1} G \right)$$

subtract the free part $$-\frac{1}{i} \frac{\partial}{\partial E} \ln \left({G_0^*}^{-1} G_0 \right)$$

$$S_E = G_0^* {G^*}^{-1} G G_0^{-1} \to I - 2\pi i\, \delta(E - \hat{H}_0) \times T_E$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

- Separation of thermo & dynamics, unlike imaginary time FTFT
- Shift of burden: S-matrix elements
- Model Independence w scattering data
- Consistent way to include resonances + non-resonant effects
- Poles, Roots & Riemann sheets; Coupled-channel effects
- In-medium effects

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr}\left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

QM *N-body* *QFT*

$$\operatorname{tr}\{...\} \Longleftrightarrow \int d^3q \, \langle q| \ldots |q\rangle \longrightarrow \int (d\,k)\, \langle k_1 k_2| \ \ldots \ |k_1 k_2\rangle$$

Fock Space Expansion

$$\int (dk)\,(\cdots) \rightarrow \int \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\,(\cdots)$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

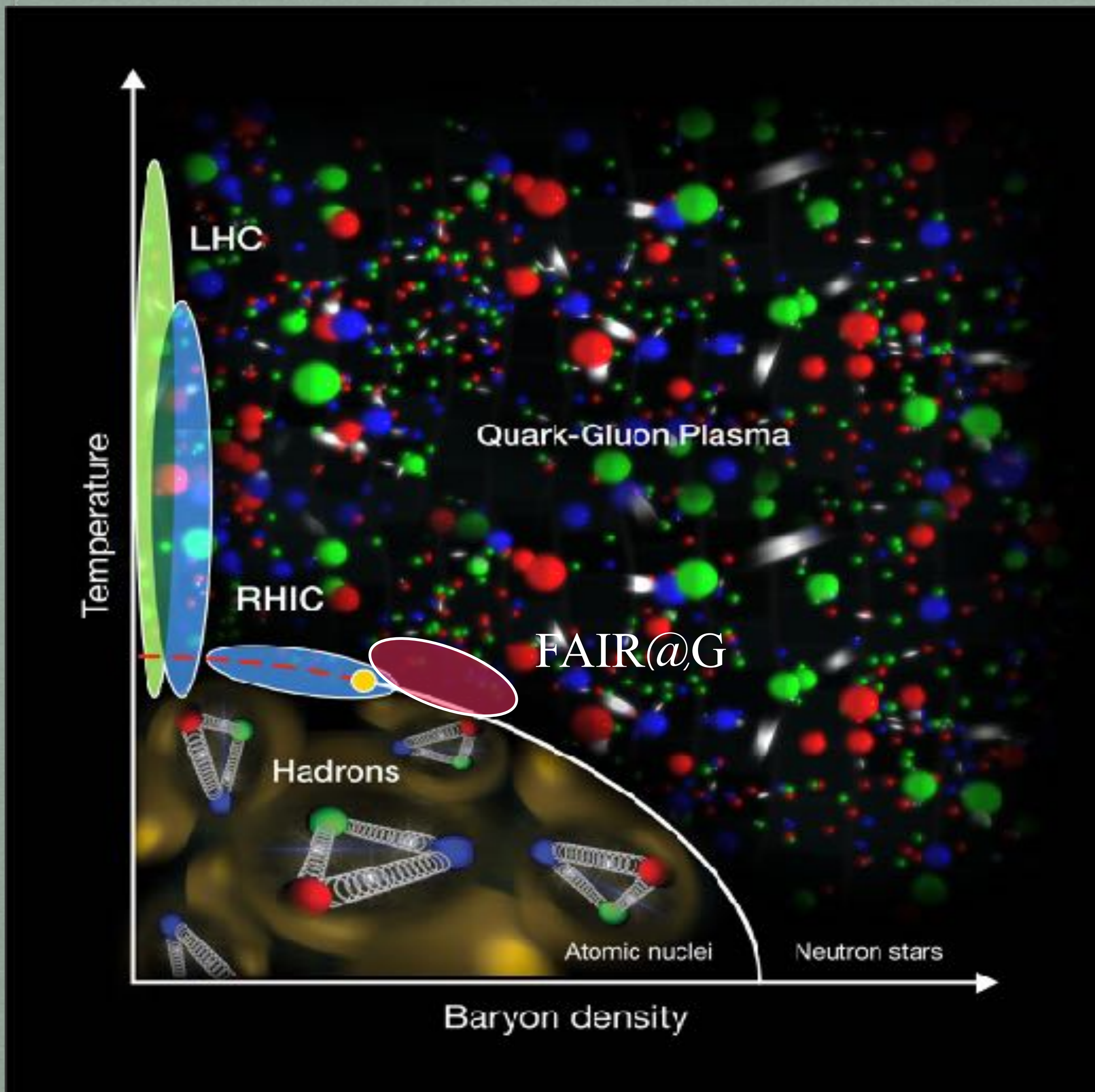
completeness assumption: via asymptotic states

$$b_{\pi\pi}\xi_\pi^2 + b_{\pi K}\xi_\pi\xi_K + b_{\pi N}\xi_\pi\xi_N + b_{\pi\eta}\xi_\pi\xi_\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \ldots$$

$$b_{\pi N} = 2 \times b_{\pi N}^{I=1/2} + 4 \times b_{\pi N}^{I=3/2}$$

orbital L: S, P, D, F, etc..

S-MATRIX FORMULATION OF STATISTICAL MECHANICS



dynamical

$$\ldots \mathrm{r} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

...pleteness assumption:

... asymptotic states

$$\ldots_{\pi\eta}\xi_\pi\xi_\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \ldots$$

$$\ldots 4 \times b_{\pi N}^{I=3/2}$$

orbital L:

S, P, D, F, etc..

S-M... OF
STA...

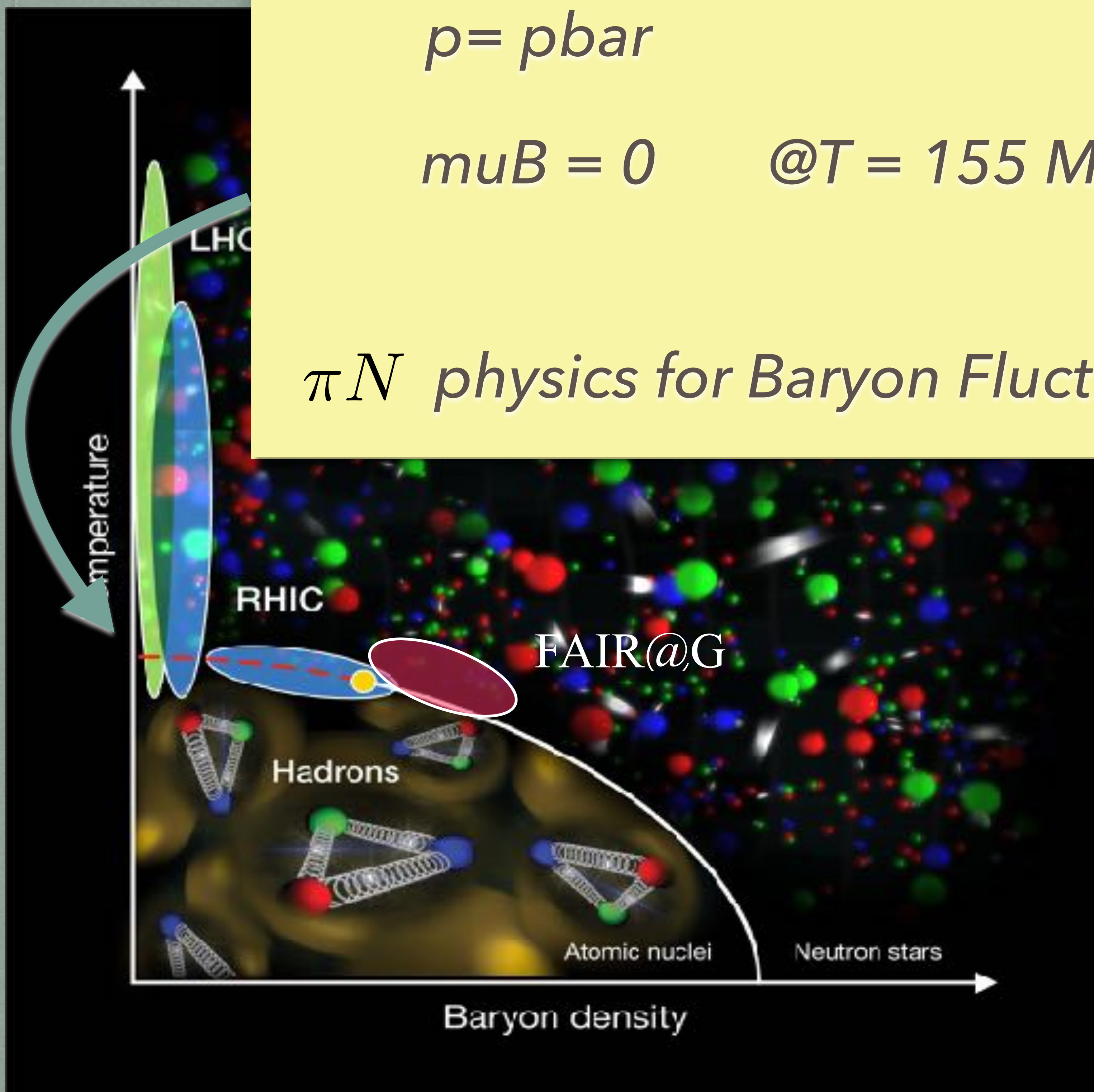
LHC conditions

$$\langle \pi \rangle / \langle p \rangle \approx 15$$

p= pbar

muB = 0 @T = 155 MeV

πN *physics for Baryon Fluctns*



...ical

$$\ldots S_E \Big\}_c$$

...leteness assumption:

...asymptotic states

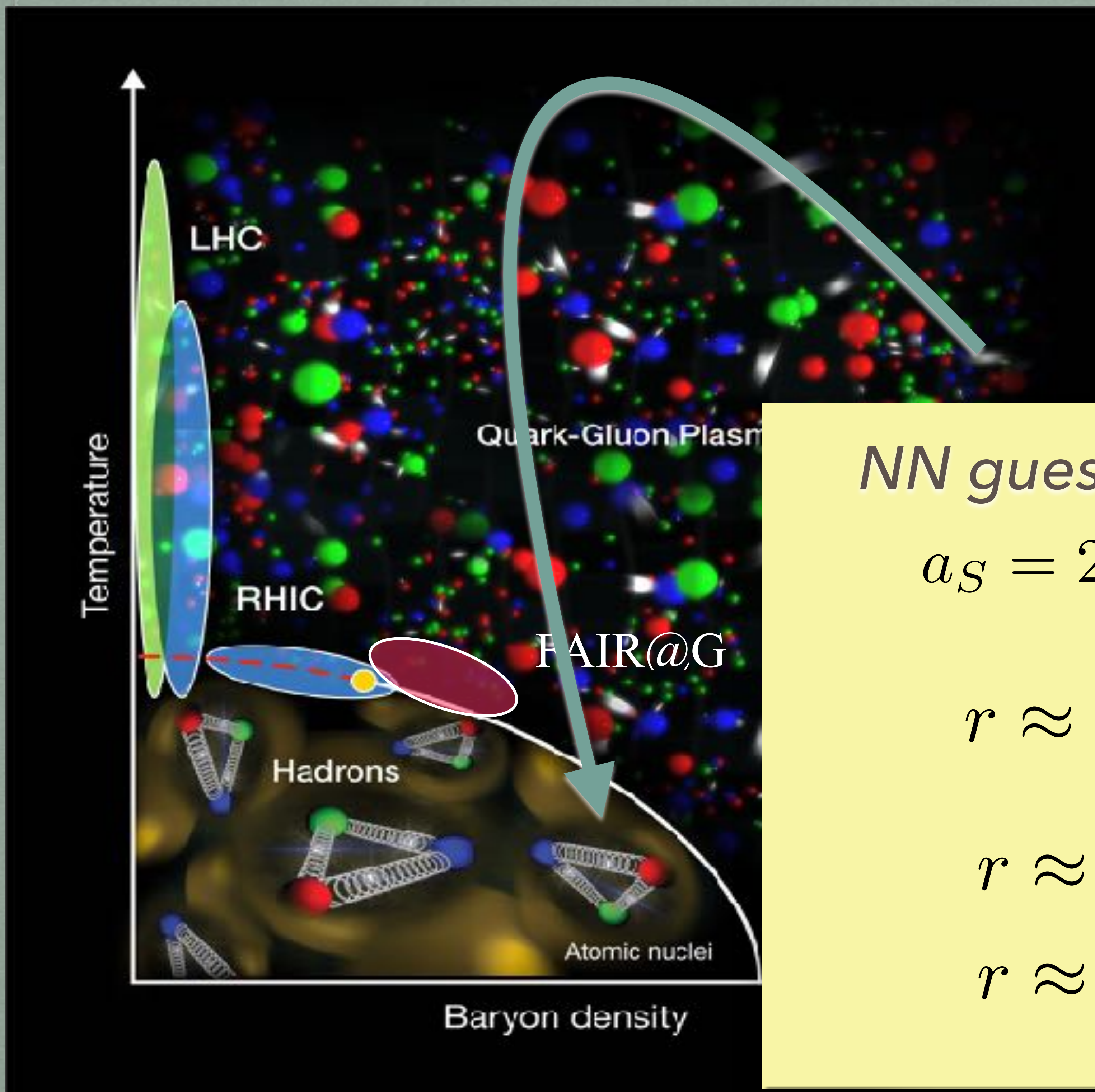
$$\ldots_{\pi\eta}\xi_\pi\xi_\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \ldots$$

$$\ldots 4 \times b_{\pi N}^{I=3/2}$$

orbital L:

S, P, D, F, etc..

S-MATRIX FORMULATION OF STATISTICAL MECHANICS



dynamical

$$\mathrm{r}\left\{ S_E^{-1}\frac{\partial}{\partial E} S_E \right\}_c$$

NN guesstimate

$a_S = 20\, fm$

$r \approx 0.0727$ $\quad LHC$

$r \approx 0.36$ $\quad HADES$

$r \approx 1.92$ $\quad T = 60\, MeV$

$\mu_B = 700, 800\, MeV$

HOW TO RELATE PHASE SHIFTS TO THERMODYNAMICS?

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

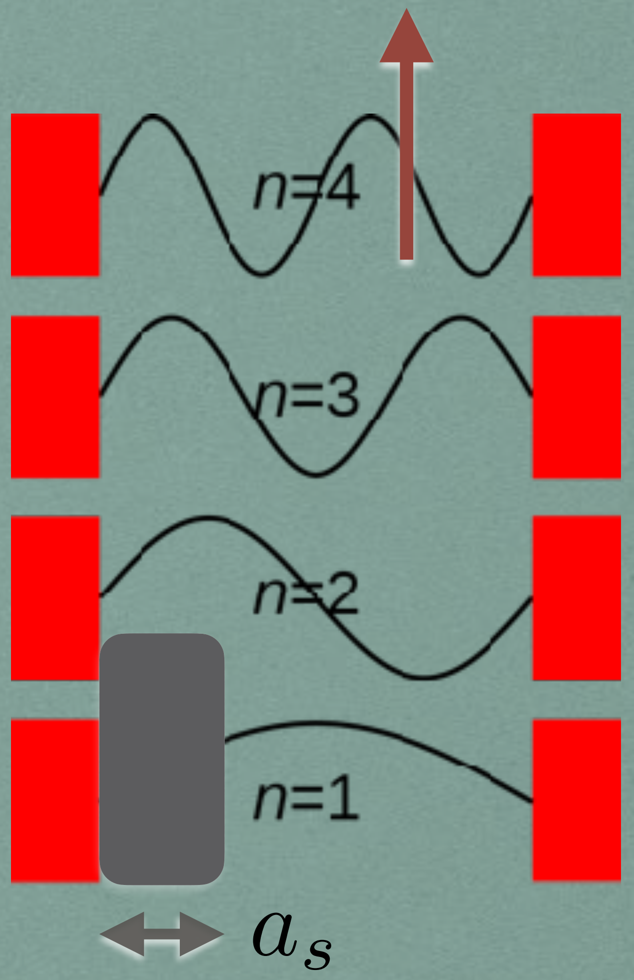
single channel, elastic $\frac{1}{\pi} \frac{d}{dE} \delta$

N-body &
Coupled-Channel problem
multi (coupled) channel $\frac{1}{\pi} \frac{d}{dE} \mathcal{Q}$

$$\mathcal{Q} = \frac{1}{2} \mathrm{ImTr} \, \ln S$$
$$= \sum_{\text{channels}} \lambda_i$$

REPULSION AND RESONANCES

PHASE SHIFT AND DENSITY OF STATES



particle in a box

$$\psi \sim \sin(k^{(0)}x) \qquad k^{(0)} = \frac{n\pi}{L}$$

in the presence of a scattering potential

$$\psi \sim \sin(kx + \delta(k))$$

density of states

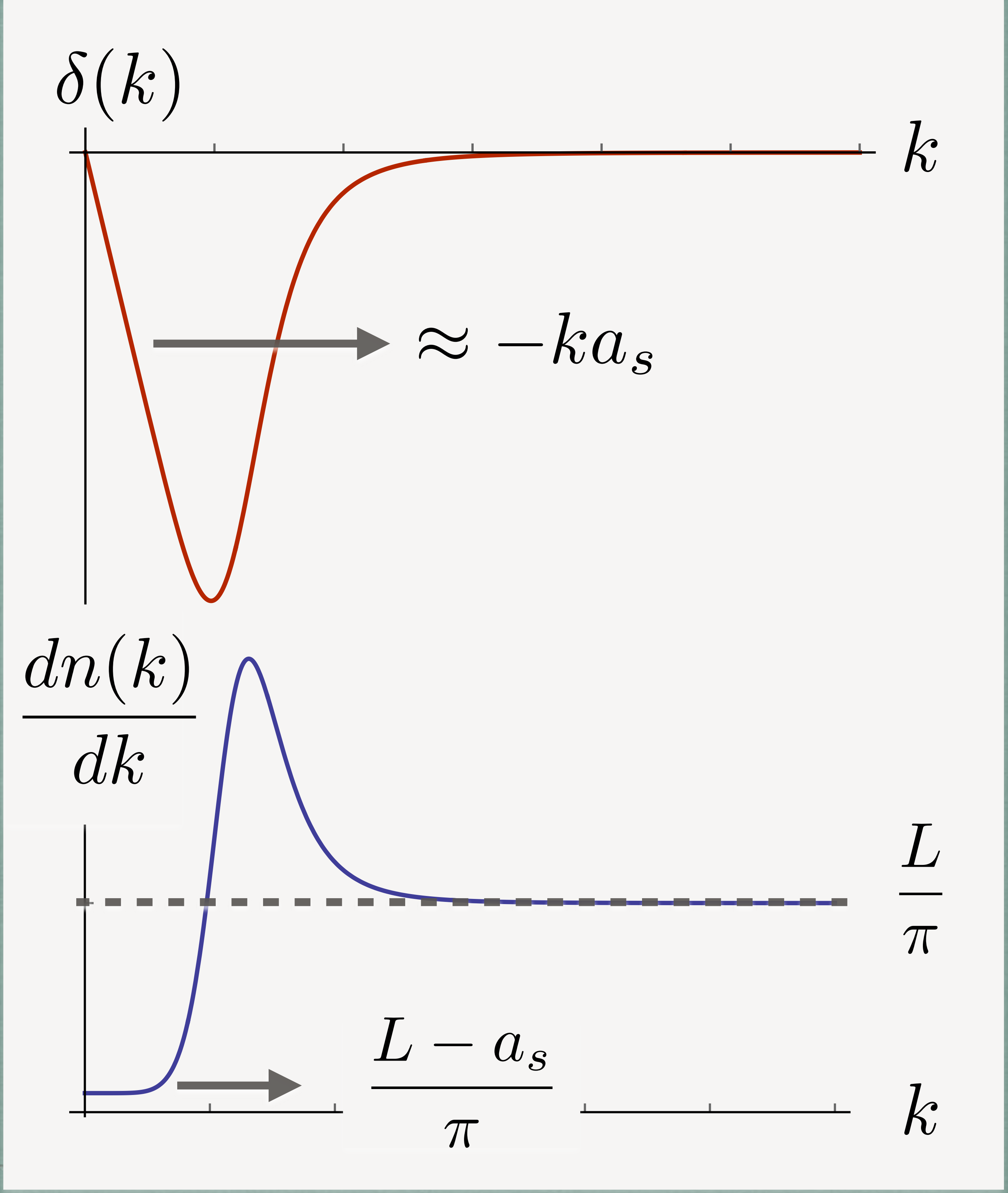
$$kL + \delta(k) = n\pi \quad \Rightarrow \quad \frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

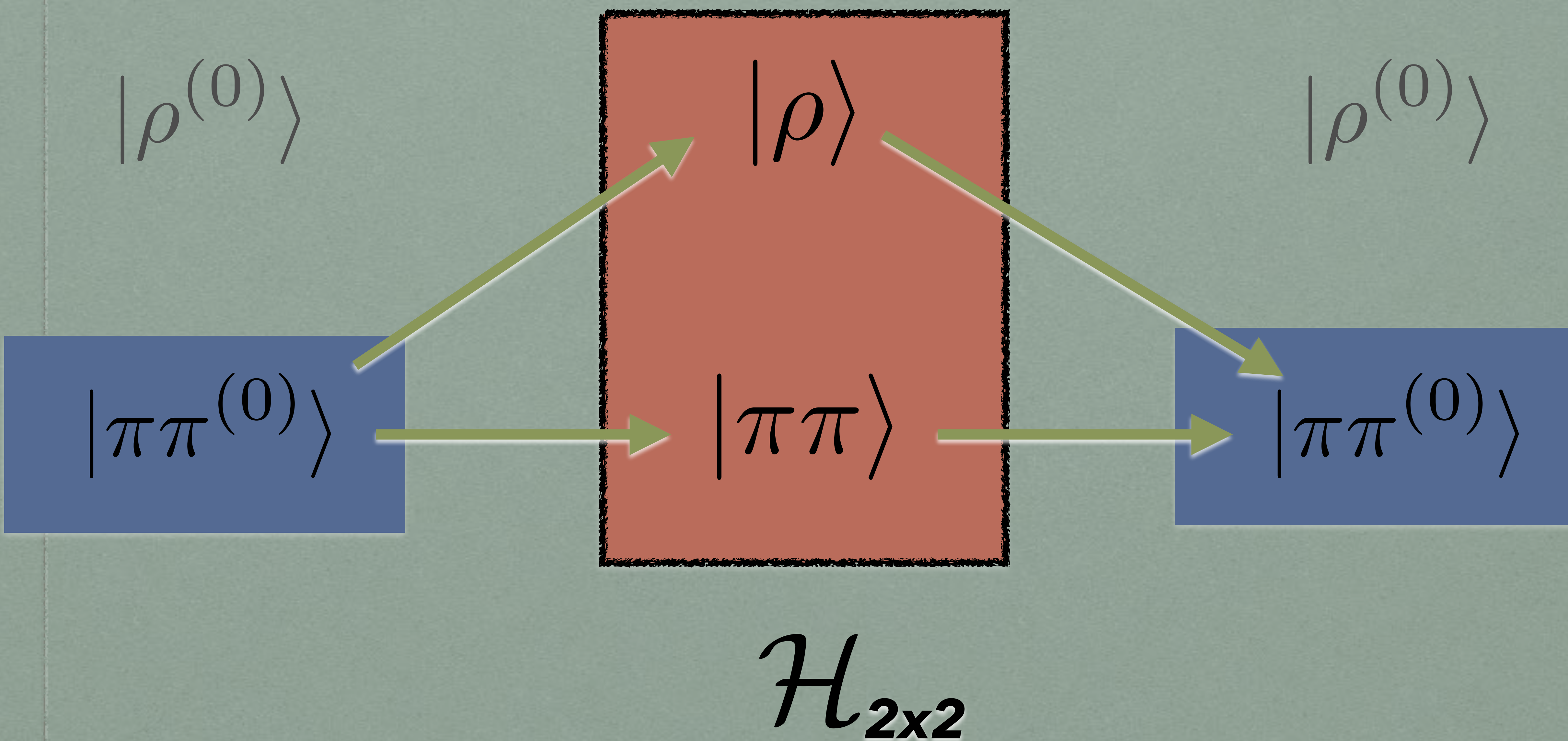
change in d.o.s. due to int.

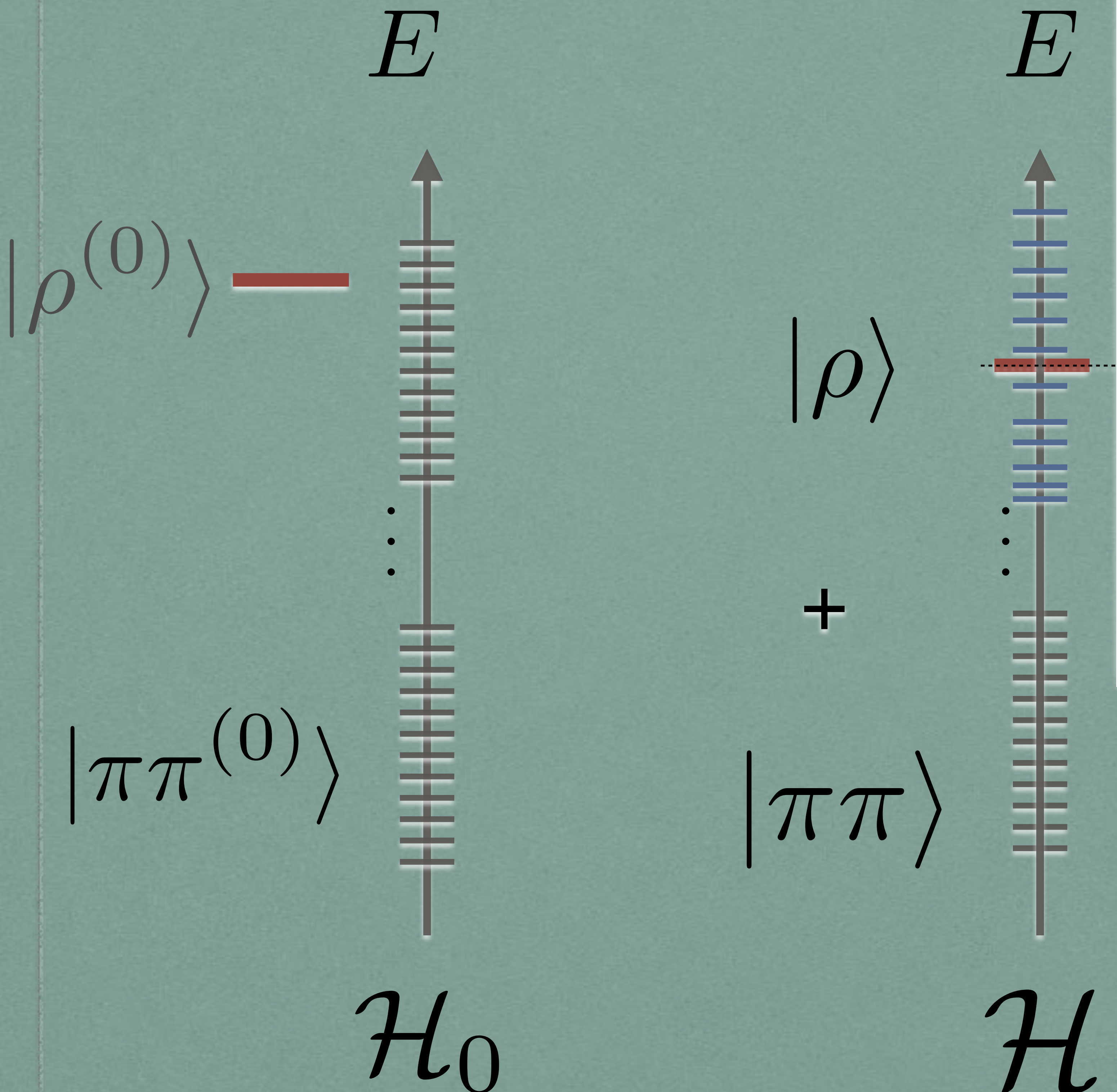
Effect of repulsive interaction: pushing states from low k to high k

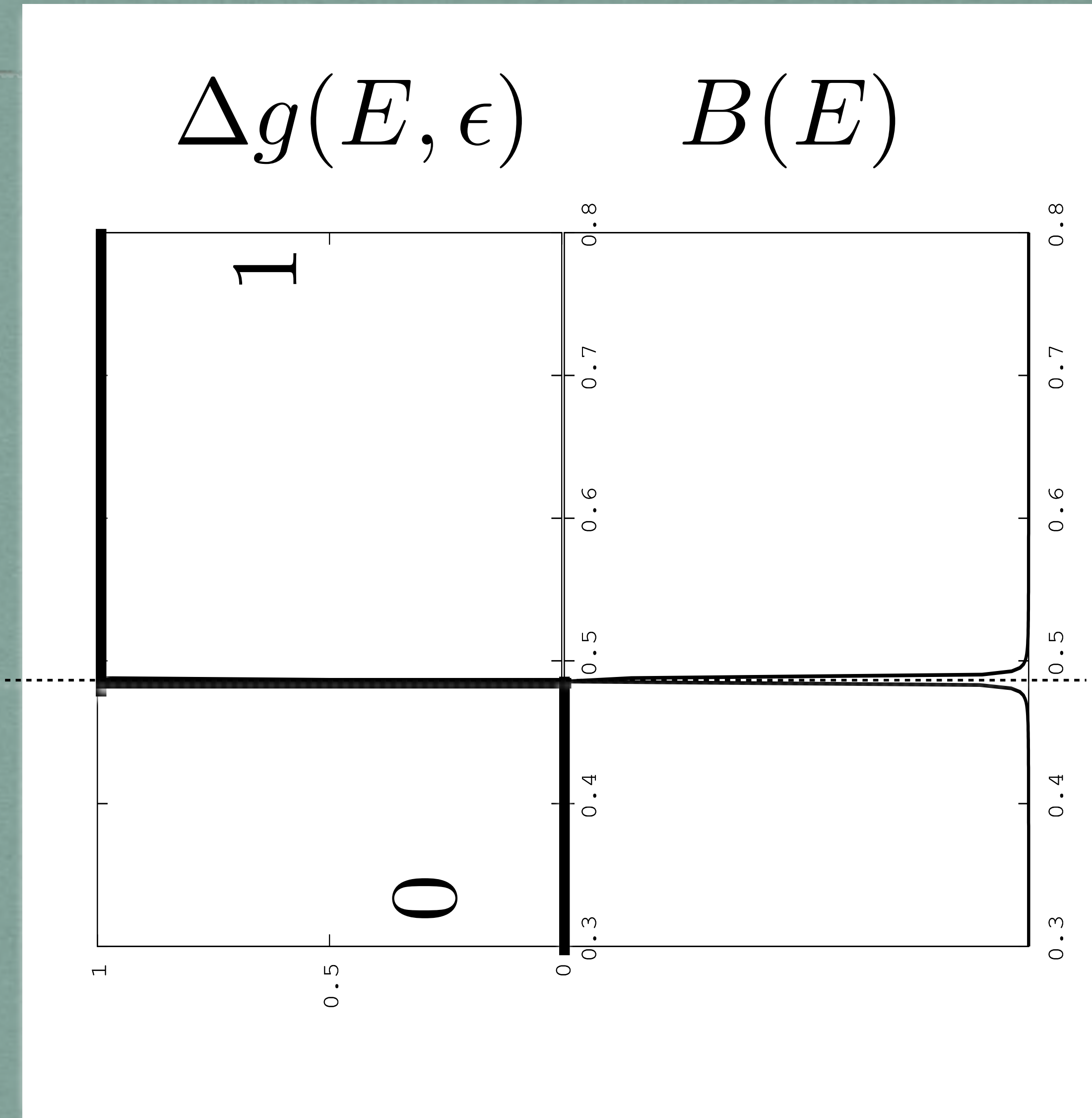


phase shift and d.o.s. (schematics)

SCATTERING THEORY VS HAMILTONIAN (LEE MODEL)



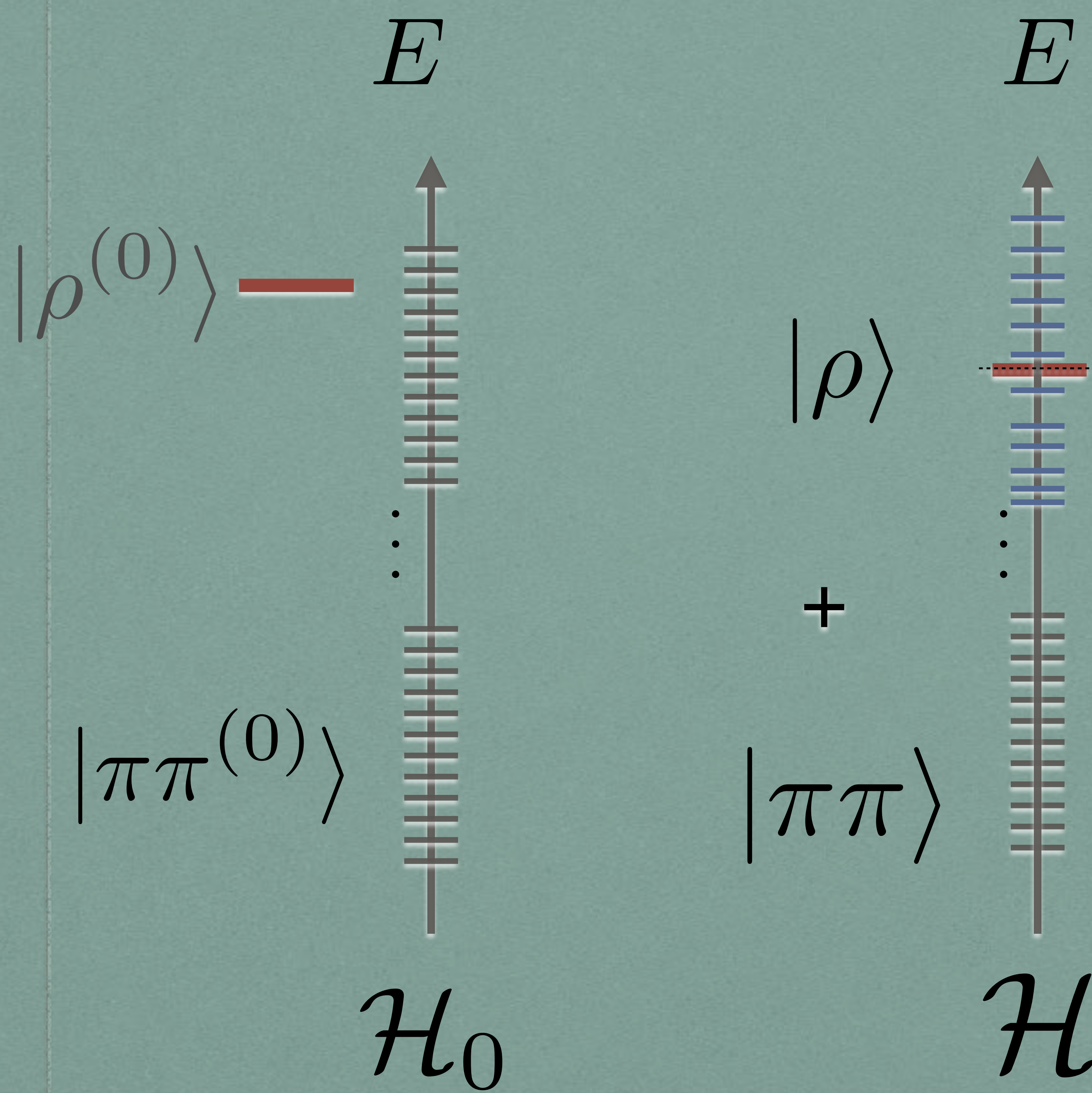


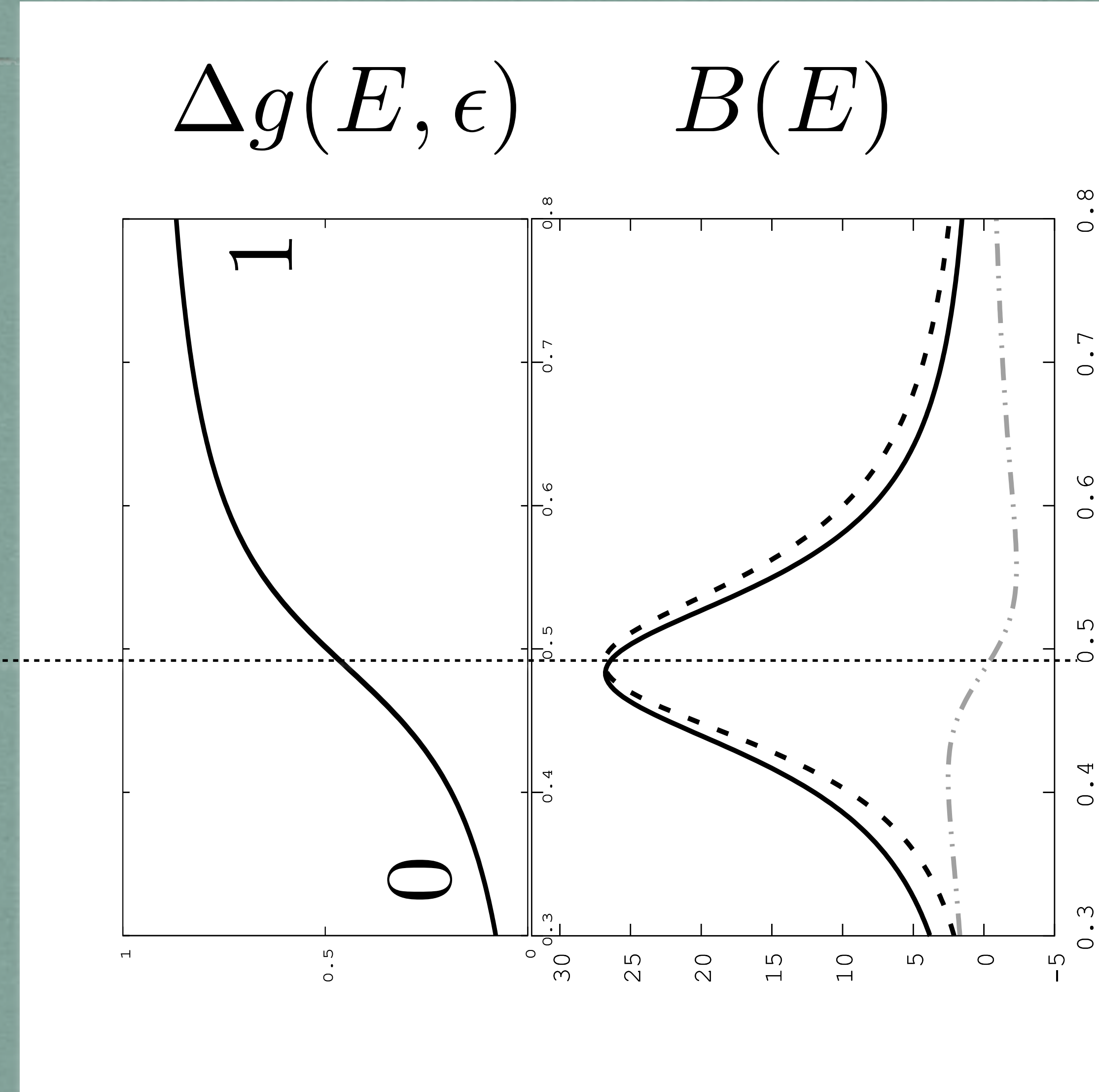


$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E,\epsilon)$$

PML, Francesco Giocosa EPJC **79** (2019)





$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E,\epsilon)$$

$$= A_\rho + \Delta A_{\pi\pi}$$

$\mathrm{Tr}\, e^{-\beta \mathcal{H}_0}$ *vs* $\mathrm{Tr}\, e^{-\beta \mathcal{H}}$

PML, Francesco Giocosa EPJC **79** (2019)

W. Weinhold & B. Friman, Phys. Lett. B 433, 236 (1998).

PHYSICS OF B

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

physical interpretation:

$$B = 2\,\frac{\partial}{\partial E}\delta$$

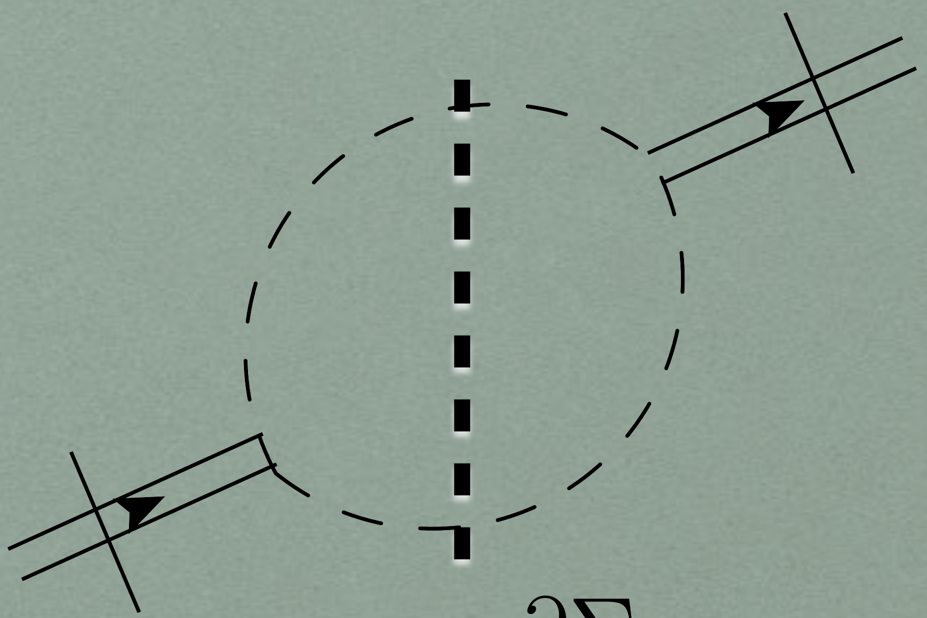
$$= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1}$$

$$= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E} G_\rho]$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

$$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$$ pipi -> pipi

contribution from correlated pi pi pair



$$\frac{\partial \Sigma_\rho}{\partial E}$$

PHYSICS OF B

to rho or not to rho?
that's OUT of the question!

resonance's picture:

$$B(E) = A_\rho(E) + \Delta A_{\pi\pi}$$

rho

scattering picture:

$$B_1 = \frac{\partial}{\partial E}\,\mathrm{Tr}\,\hat{t}_{\mathrm{re}}$$

$$B_2 = \frac{1}{2}\,\mathrm{Im}\,\mathrm{Tr}\,\hat{t}^\dagger\,\overleftrightarrow{\partial_E}\,\hat{t}$$

pipi -> pipi

$\delta = -\mathrm{Im}\,\mathrm{T}$

$$B = 2\,\frac{\partial}{\partial E}\delta$$

$= -2\,\mathrm{Im}\,\frac{\partial}{\partial E}$

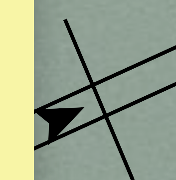
$= -2\,\mathrm{Im}[G$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

on:

n

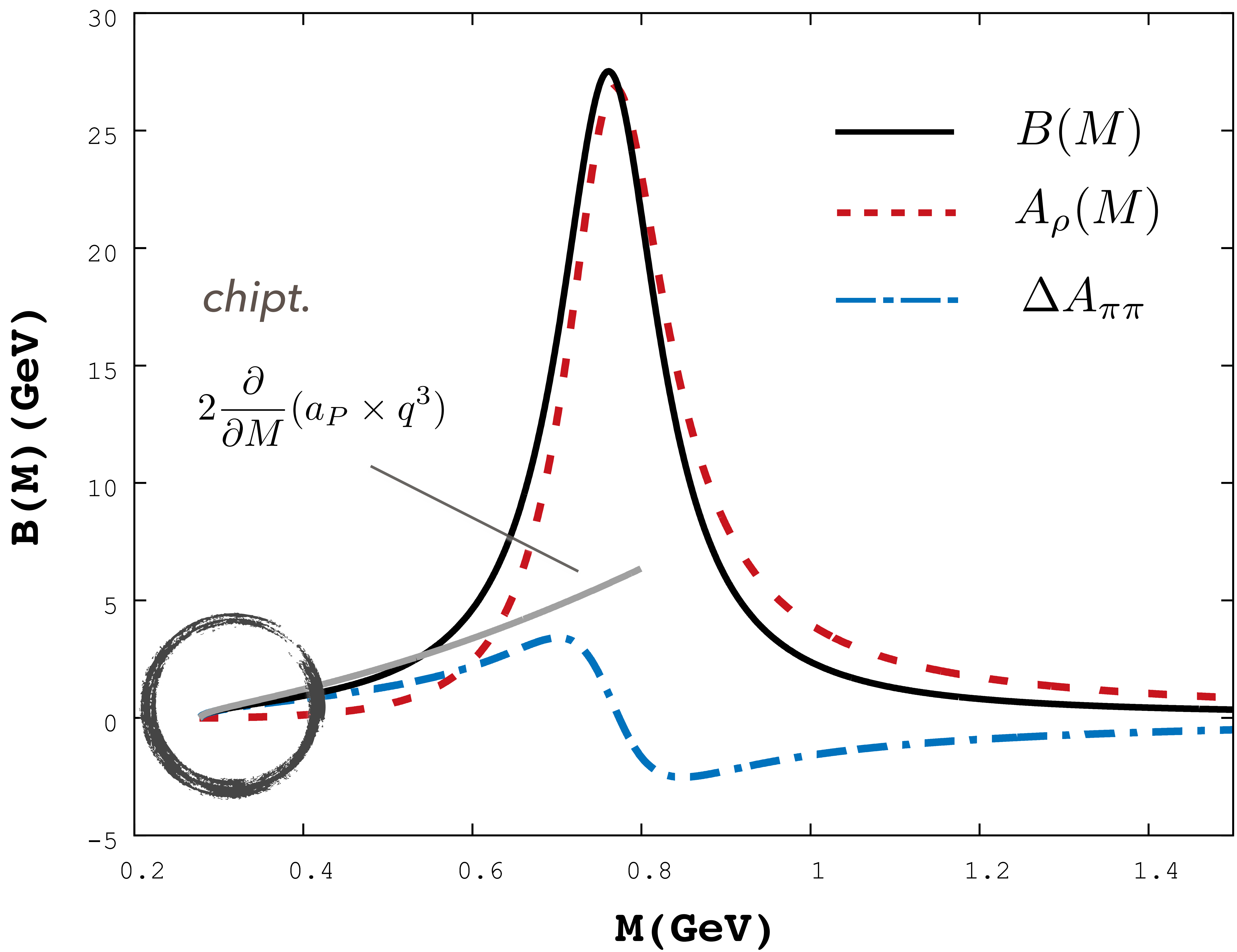
air



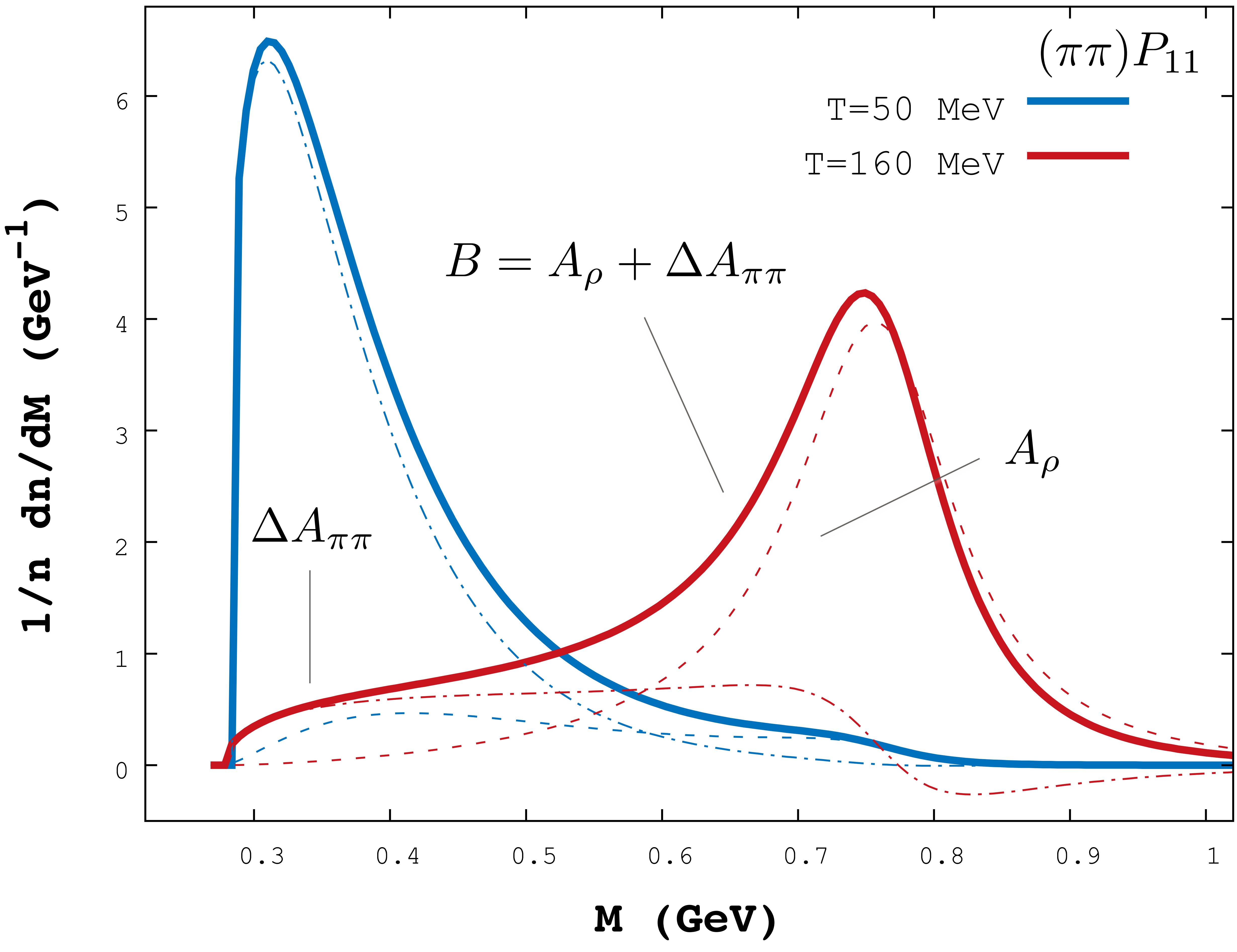
$$\frac{\partial \Sigma_\rho}{\partial E}$$

$$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$$

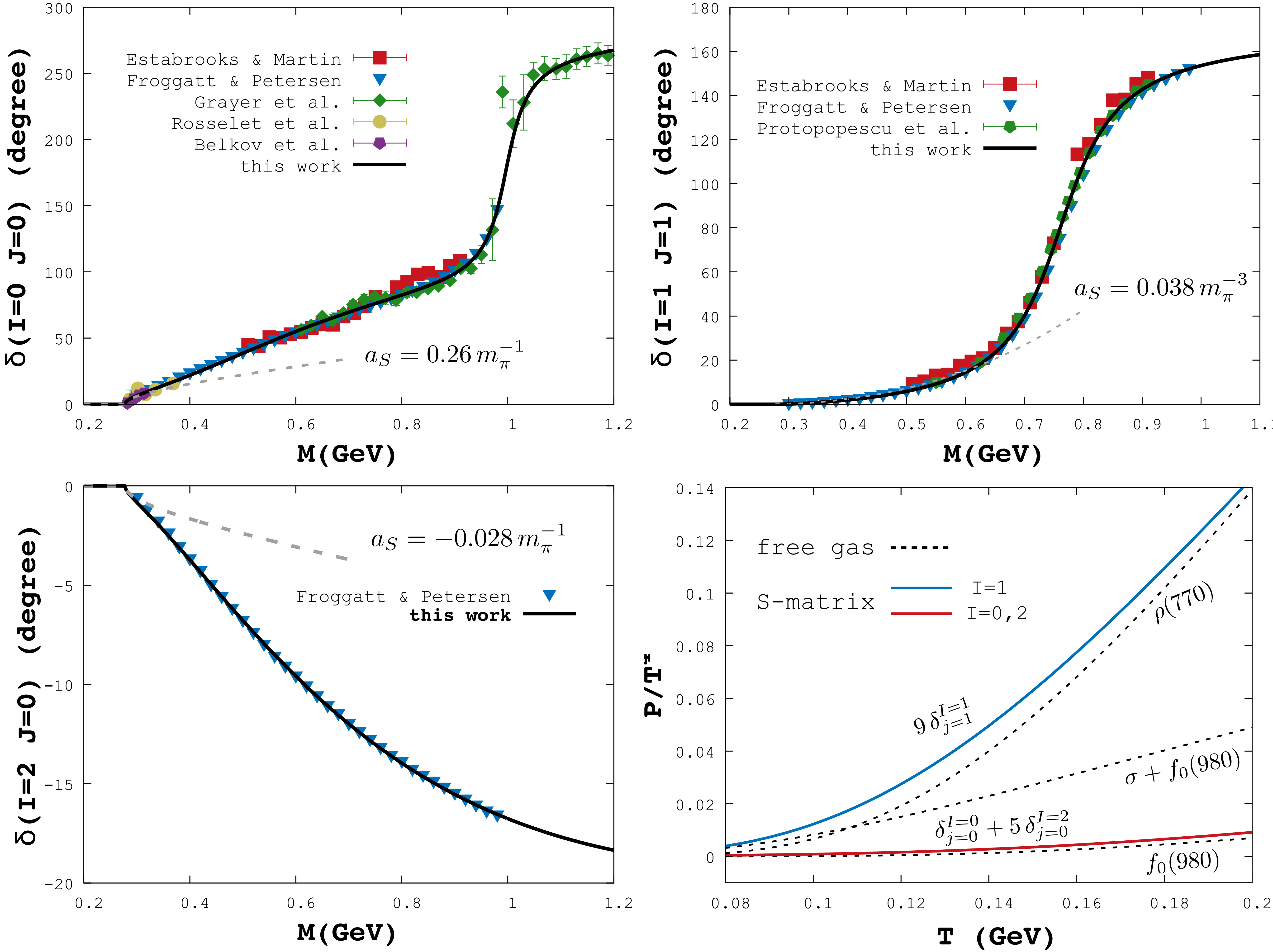
pipi -> pipi



B(M) (GeV)
M(GeV)
$B(M)$
$A_\rho(M)$
$\Delta A_{\pi\pi}$
chipt.
$2\frac{\partial}{\partial M}(a_P \times q^3)$
30
25
20
15
10
5
0
-5
0.2
0.4
0.6
0.8
1
1.2
1.4



$(\pi\pi)P_{11}$
T=50 MeV
T=160 MeV
$B = A_\rho + \Delta A_{\pi\pi}$
A_ρ
$\Delta A_{\pi\pi}$
1/n dn/dM (GeV^{-1})
M (GeV)
0
1
2
3
4
5
6
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



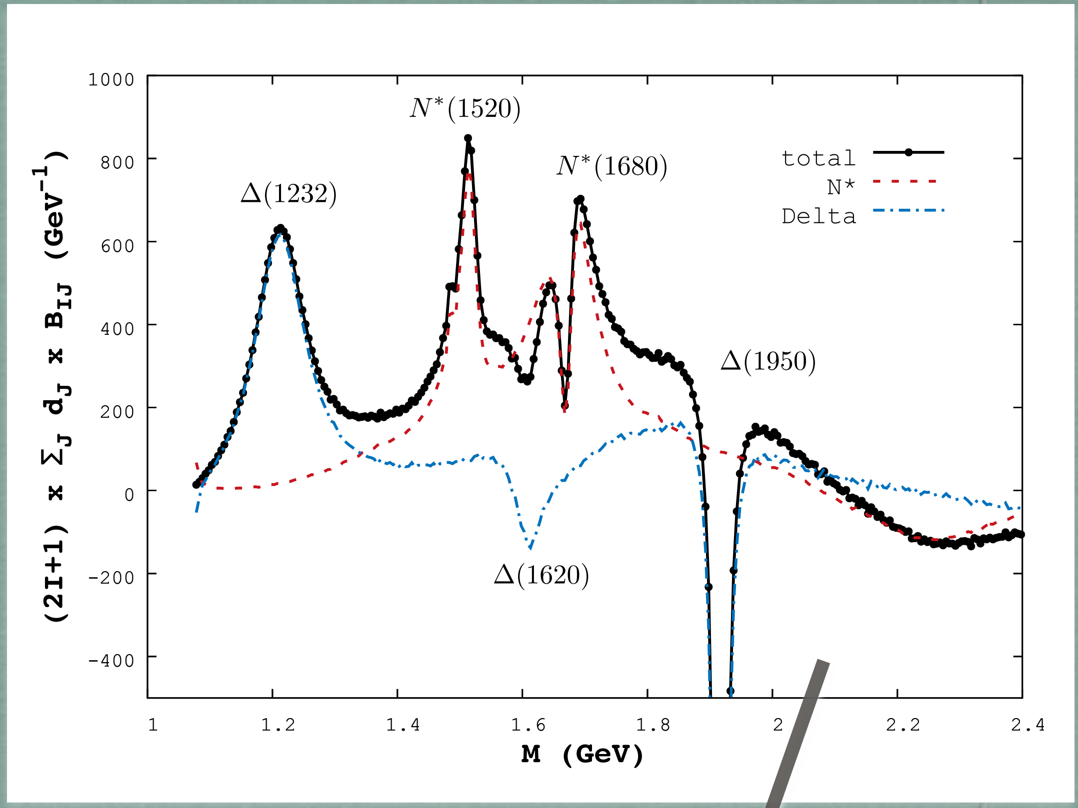
δ(I=0 J=0) (degree)
M(GeV)
Estabrooks & Martin
Froggatt & Petersen
Grayer et al.
Rosselet et al.
Belkov et al.
this work
$a_S = 0.26\, m_\pi^{-1}$
δ(I=1 J=1) (degree)
M(GeV)
Estabrooks & Martin
Froggatt & Petersen
Protopopescu et al.
this work
$a_S = 0.038\, m_\pi^{-3}$
δ(I=2 J=0) (degree)
M(GeV)
$a_S = -0.028\, m_\pi^{-1}$
Froggatt & Petersen
this work
P/T⁴
T (GeV)
free gas
S-matrix
I=1
I=0,2
$\rho(770)$
$9\,\delta_{j=1}^{I=1}$
$\sigma + f_0(980)$
$\delta_{j=0}^{I=0} + 5\,\delta_{j=0}^{I=2}$
$f_0(980)$

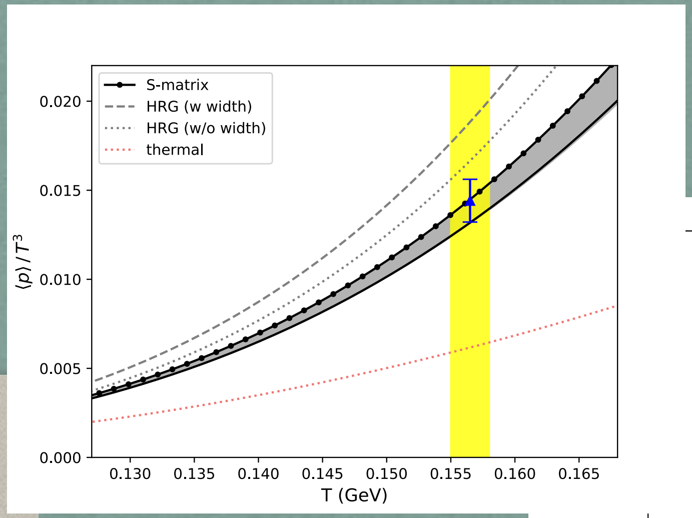
THE S-MATRIX PROGRAM

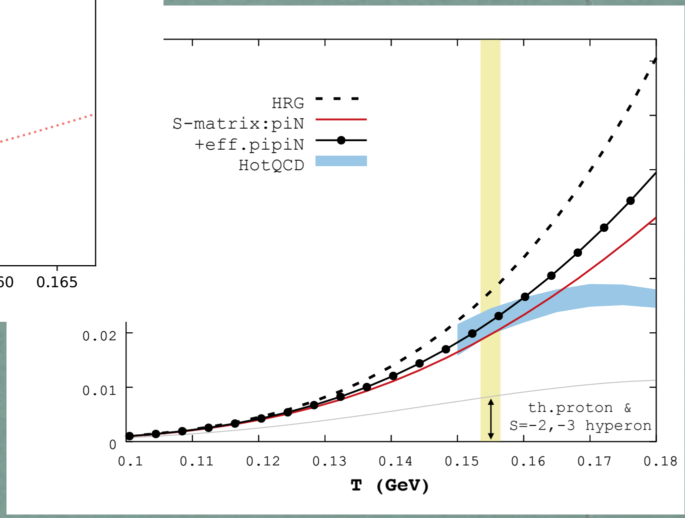
step 1: build a model for S-matrix

step 2: adjust model parameters to match scattering experiments

broad resonances & thresholds
coupled channel effects
energy dependent branchings

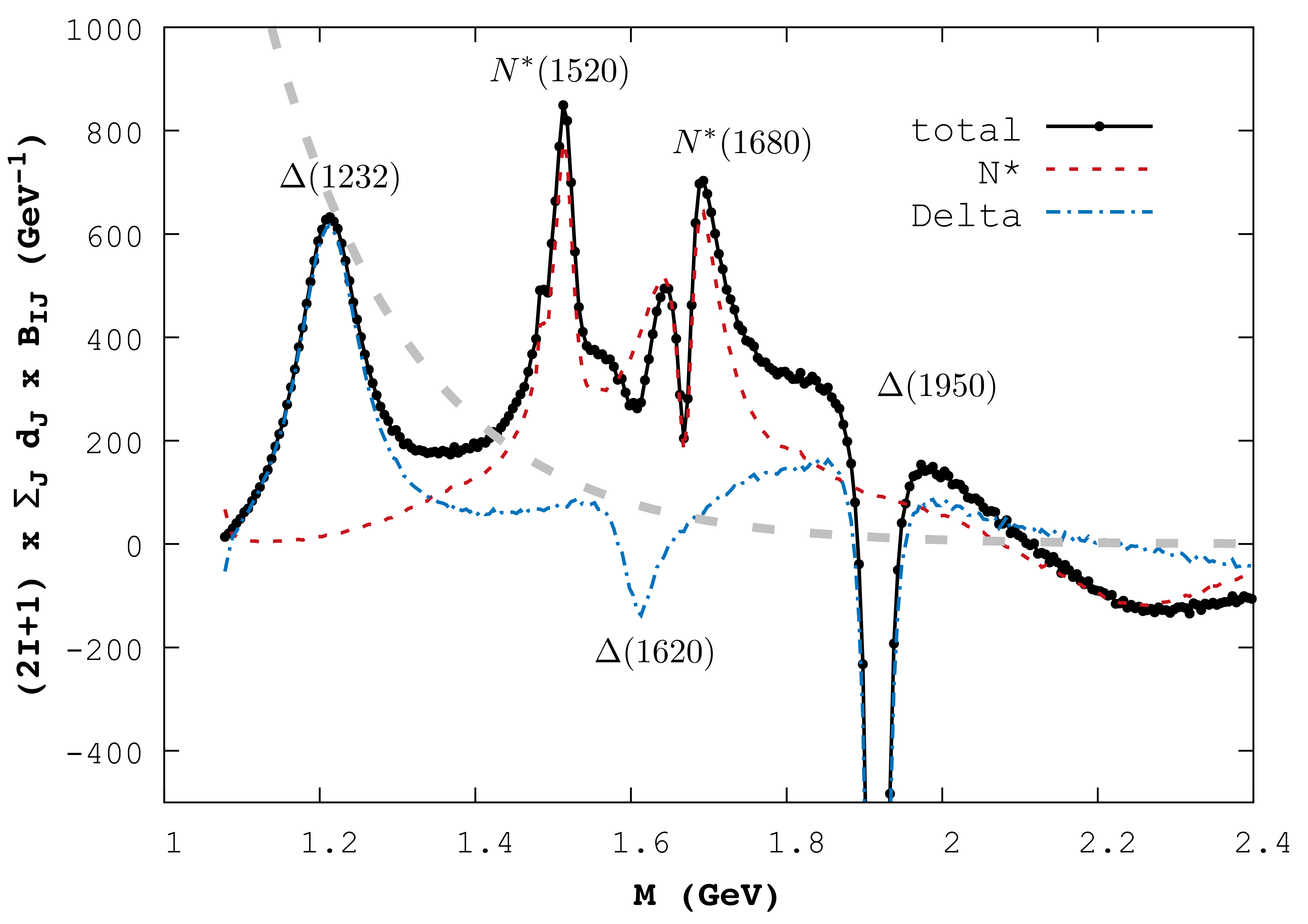




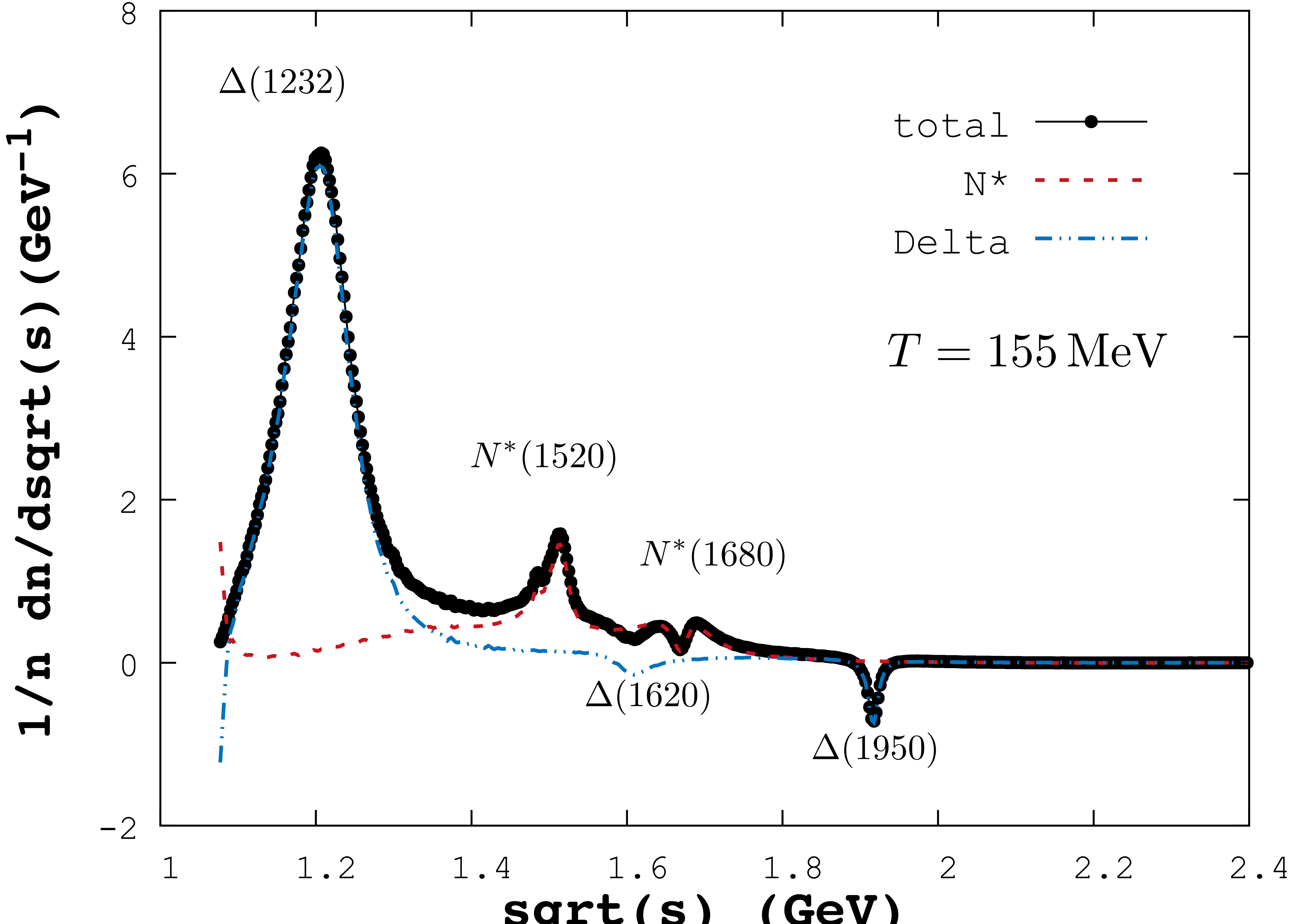


step 3: compute thermodynamics

HICs & LQCD



1000
800
600
400
200
0
-200
-400
1
1.2
1.4
1.6
1.8
2
2.2
2.4
(2I+1) x Σ_J d_J x B_{IJ} (GeV^{-1})
M (GeV)
$\Delta(1232)$
$N^*(1520)$
$N^*(1680)$
$\Delta(1950)$
$\Delta(1620)$
total
N*
Delta



Δ(1232)
total
N*
Delta
T = 155 MeV
N*(1520)
N*(1680)
Δ(1620)
Δ(1950)
8
6
4
2
0
-2
1
1.2
1.4
1.6
1.8
2
2.2
2.4
1/n dn/dsqrt(s) (GeV⁻¹)
sqrt(s) (GeV)

S = -1 HYPERONS COUPLED CHANNEL SYSTEM

JPAC, PRD **93**, 034029 (2016)

C. Fernandez-Ramirez, PML, and P. Petreczky, PRC **98**, 044910 (2018)

J. Cleymans, PML, K. Redlich, and N. Sharma PRC **103**, 014904 (2021)

JPAC, PRD **93**, 034029 (2016)

PHASE SHIFT FROM PWA

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Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

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Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)
Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

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Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.
--------------------------------------------------------------------------
```

- 1 $\to \bar{K}N$,
- 2 $\to \pi\Sigma$,
- 3 $\to \pi\Lambda$,
- 4 $\to \eta\Lambda$,
- 5 $\to \eta\Sigma$,
- 6 $\to \bar{K}_1 N$,
- 7 $\to \left[\bar{K}_3 N\right]_-$,
- 8 $\to \left[\bar{K}_3 N\right]_+$,
- 9 $\to [\pi\Sigma^*]_-$,
- 10 $\to [\pi\Sigma^*]_+$,
- 11 $\to \left[\bar{K}\Delta\right]_-$,
- 12 $\to \left[\bar{K}\Delta\right]_+$,
- 13 $\to [\pi\Lambda(1520)]_-$,
- 14 $\to [\pi\Lambda(1520)]_+$,
- 15 $\to \pi\pi\Lambda$,
- 16 $\to \pi\pi\Sigma$.

elastic scatterings (elementary)

Effective elementarity

quasi elastic scatterings

unitarity background

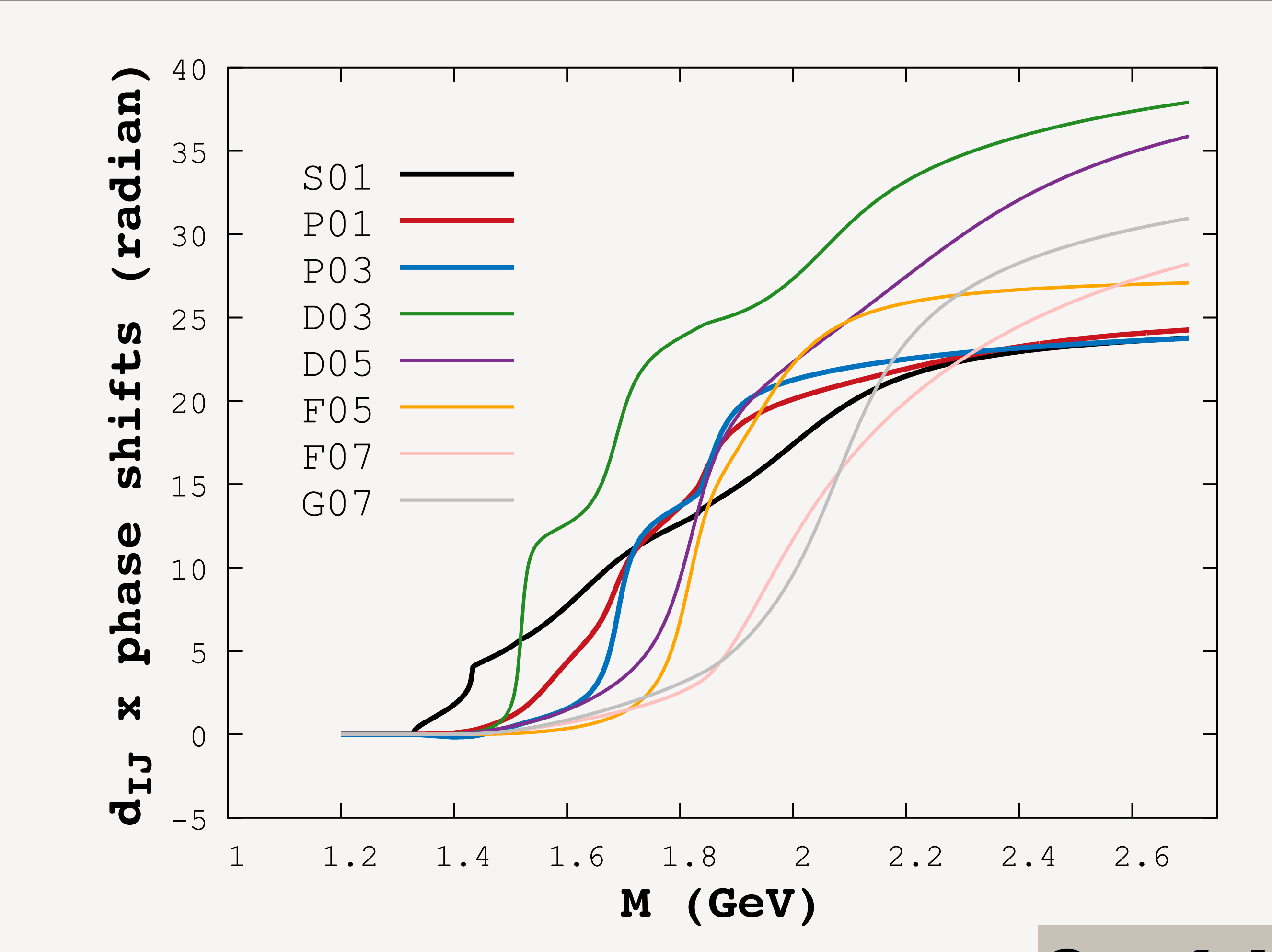
STRANGENESS CONTENT IN A HADRON GAS

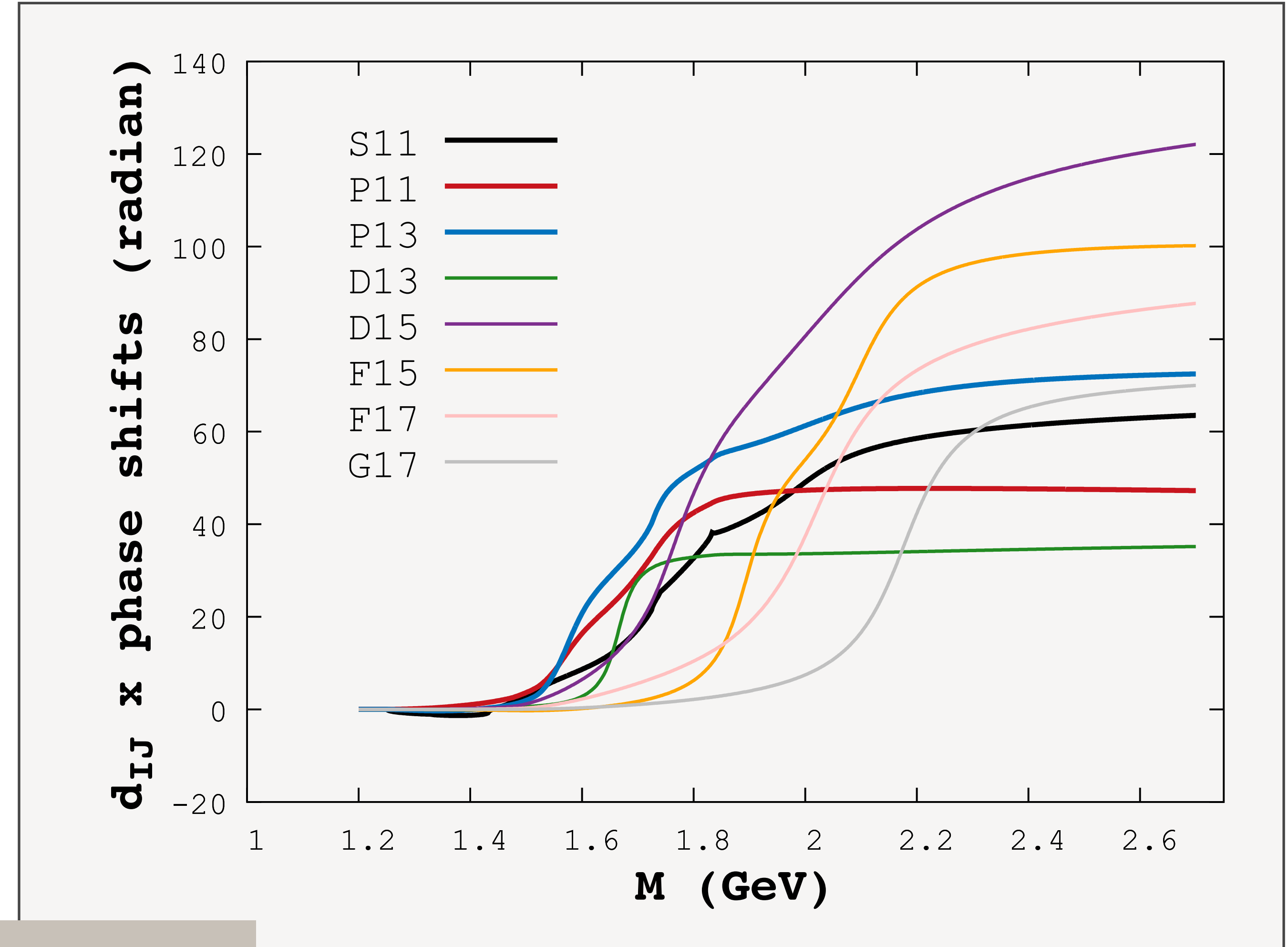
- K-N system requires a coupled channel analysis

$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \ldots$ *16 basis states*

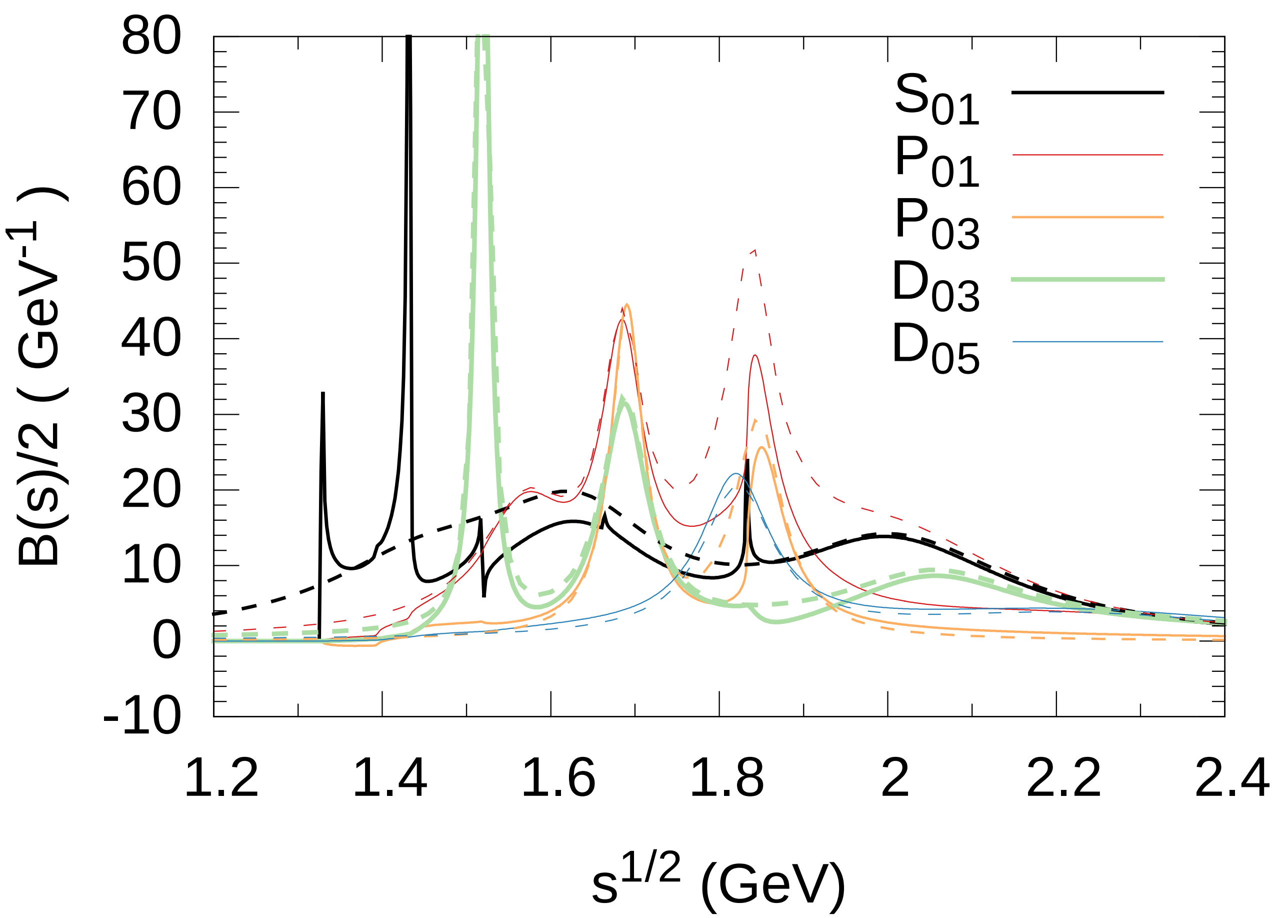
$$\begin{aligned} \mathcal{Q}(M) &\equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\,\mathrm{S}) \\ &= \frac{1}{2}\,\mathrm{Im}\,(\ln\,\det\,[\mathrm{S}]) \\ &= \delta_{\bar{K}N} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \ldots \end{aligned}$$

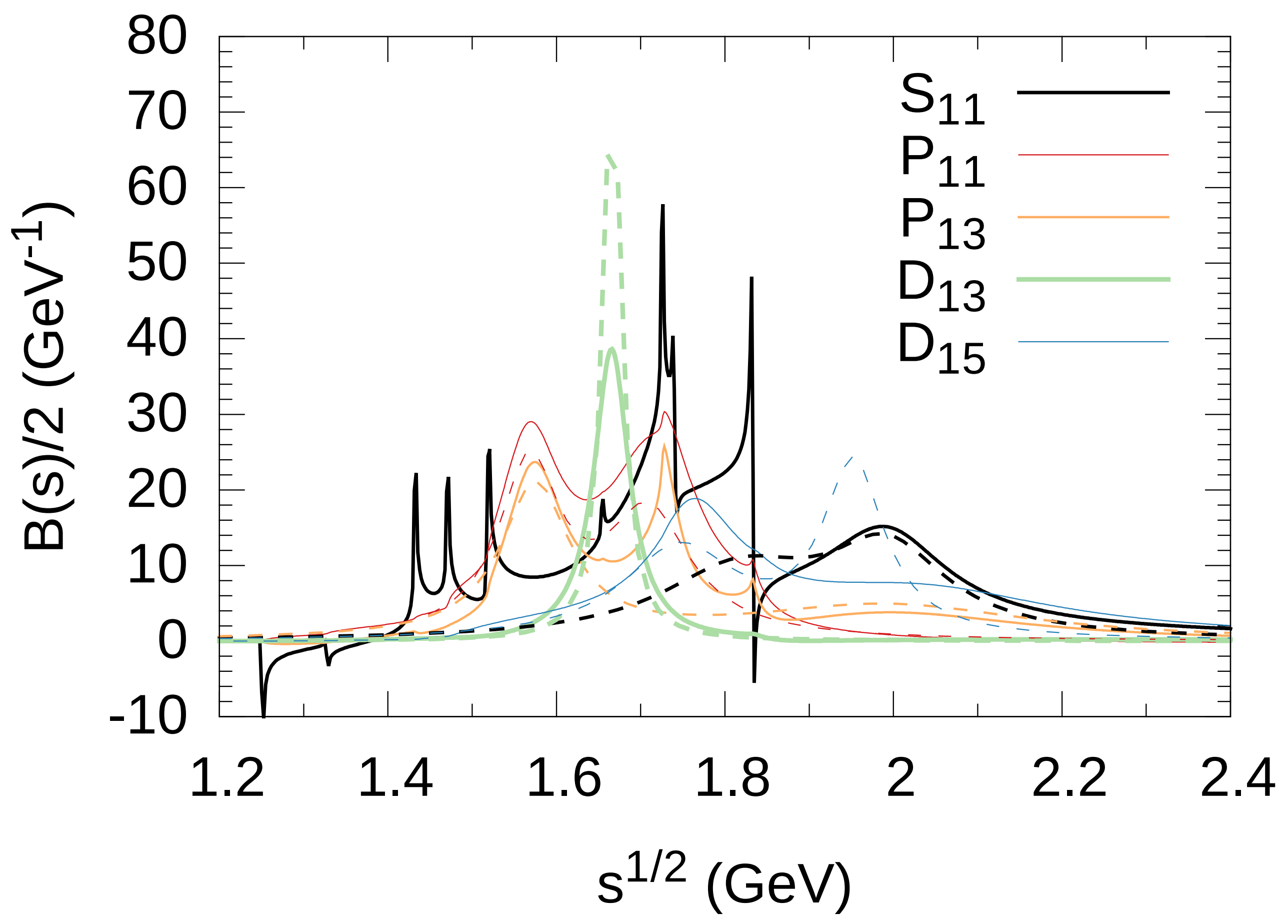
Compute det S for each sqrt(s) for each channel isospin conserving


d_{IJ} x phase shifts (radian)
M (GeV)
S01
P01
P03
D03
D05
F05
F07
G07

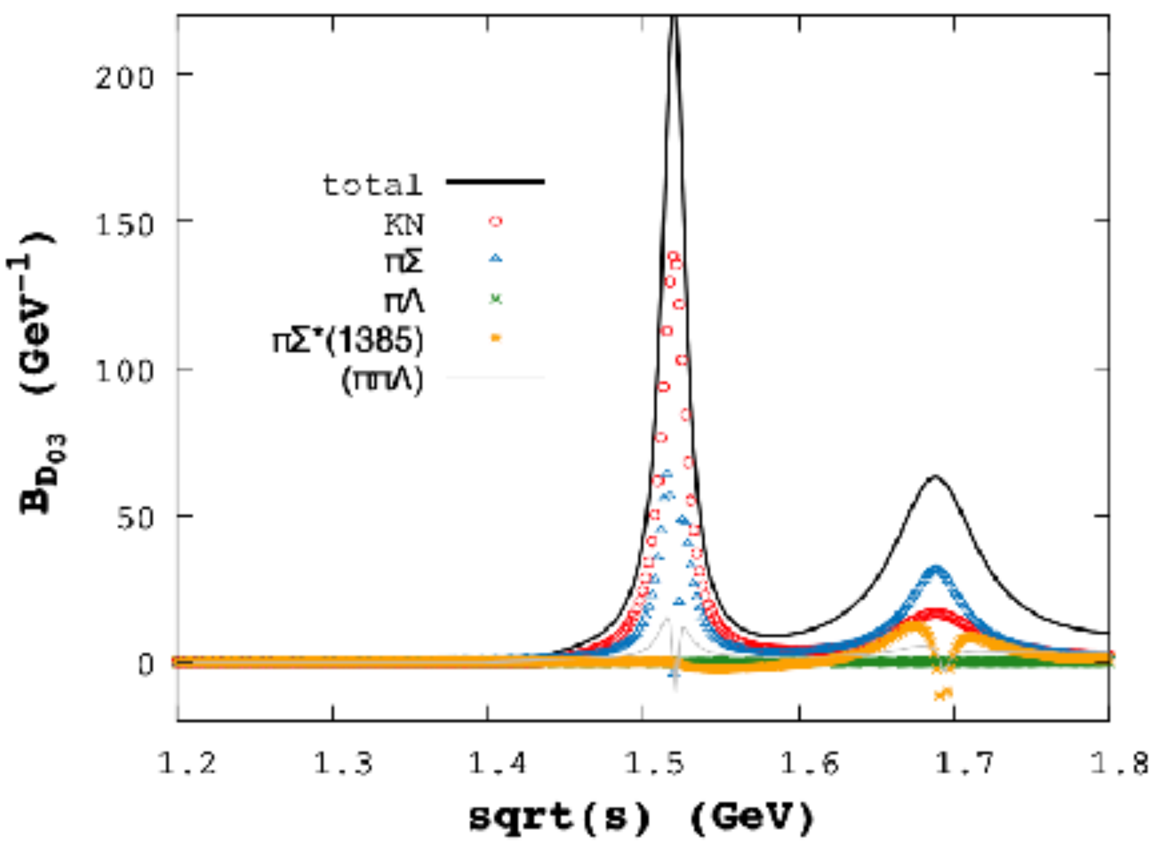

d_{IJ} x phase shifts (radian)
M (GeV)
S11
P11
P13
D13
D15
F15
F17
G17

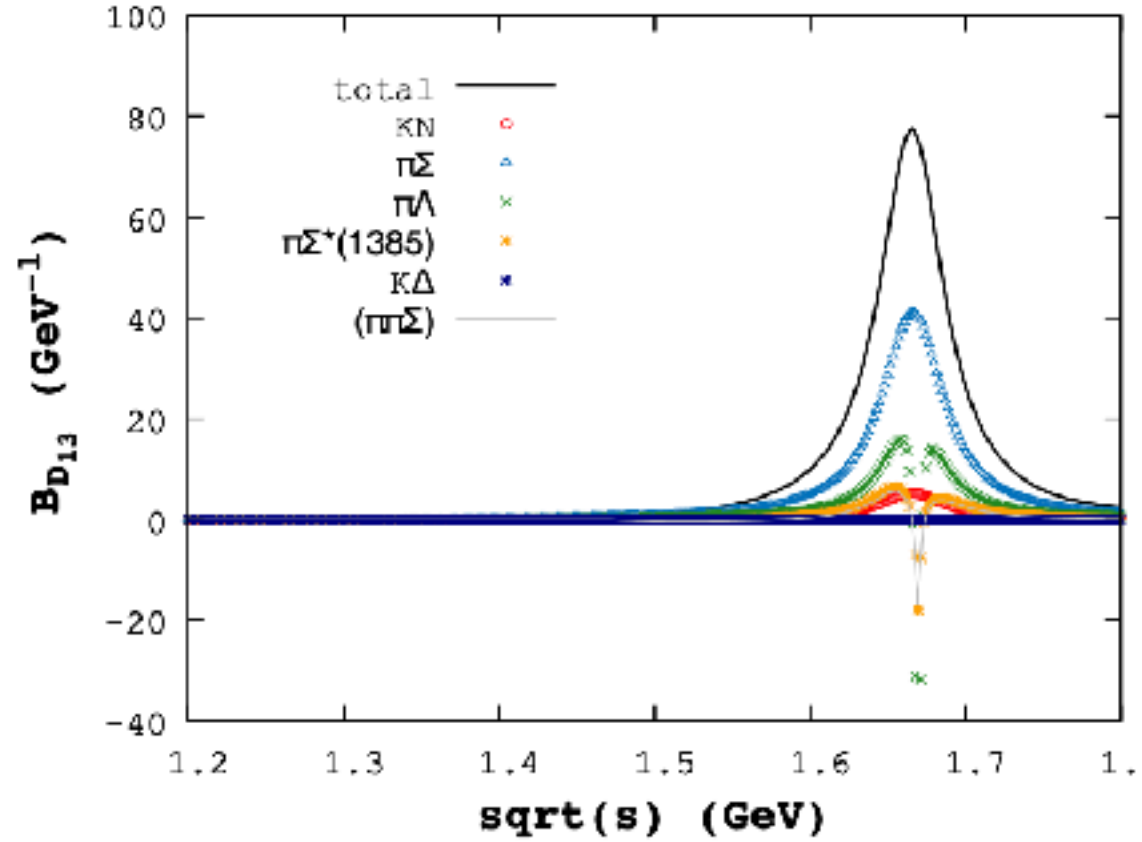
S=-1 Hyperon


B(s)/2 (GeV^{-1})
$s^{1/2}$ (GeV)
S_{01}
P_{01}
P_{03}
D_{03}
D_{05}


B(s)/2 (GeV^{-1})
$s^{1/2}$ (GeV)
S_{11}
P_{11}
P_{13}
D_{13}
D_{15}

1520, 1690	1670





Λ(1520) 3/2⁻ $I(J^P) = 0(\frac{3}{2}^-)$

Mass $m = 1519.5 \pm 1.0$ MeV [d]
Full width $\Gamma = 15.6 \pm 1.0$ MeV [d]
$p_{beam} = 0.39$ GeV/c $4\pi\lambda\!\!\!^-{}^2 = 82.8$ mb

Λ(1520) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	45 ± 1%	243
$\Sigma\pi$	42 ± 1%	268
$\Lambda\pi\pi$	10 ± 1%	259
$\Sigma\pi\pi$	0.9 ± 0.1%	169
$\Lambda\gamma$	0.85 ± 0.15%	350

Λ(1690) 3/2⁻ $I(J^P) = 0(\frac{3}{2}^-)$

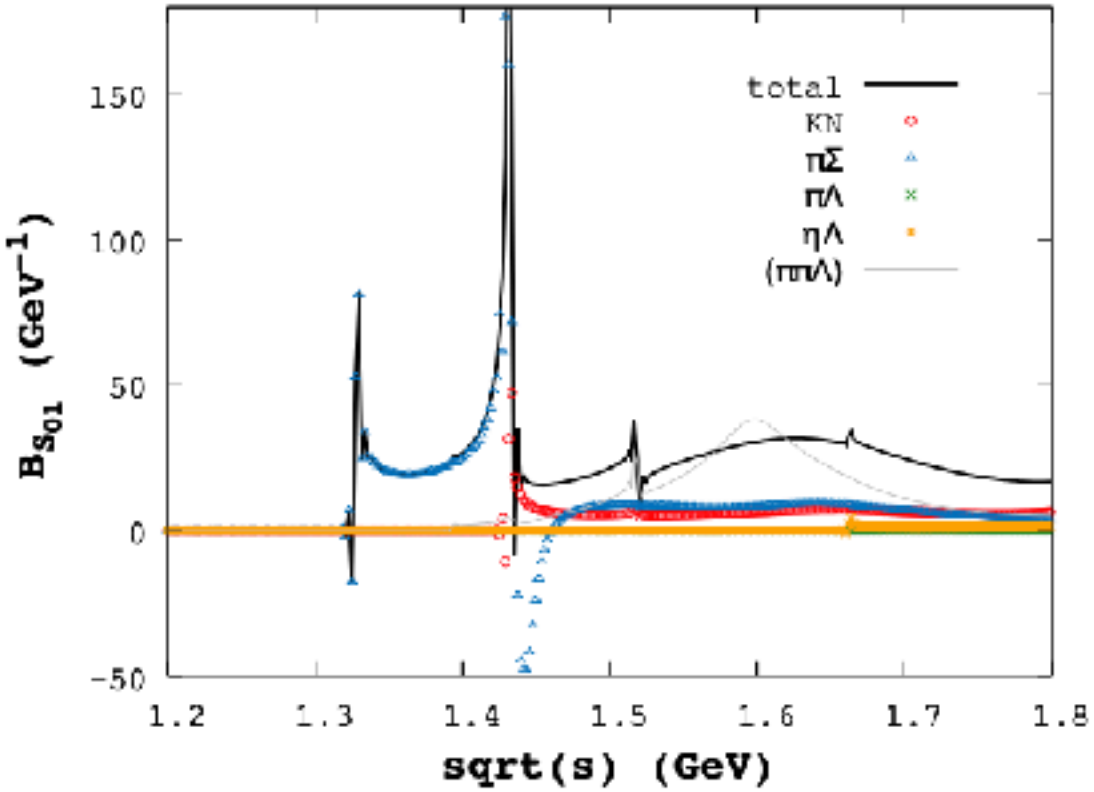
Mass m = 1685 to 1695 (≈ 1690) MeV
Full width Γ = 50 to 70 (≈ 60) MeV
$p_{beam} = 0.78$ GeV/c $4\pi\lambda\!\!\!^-{}^2 = 26.1$ mb

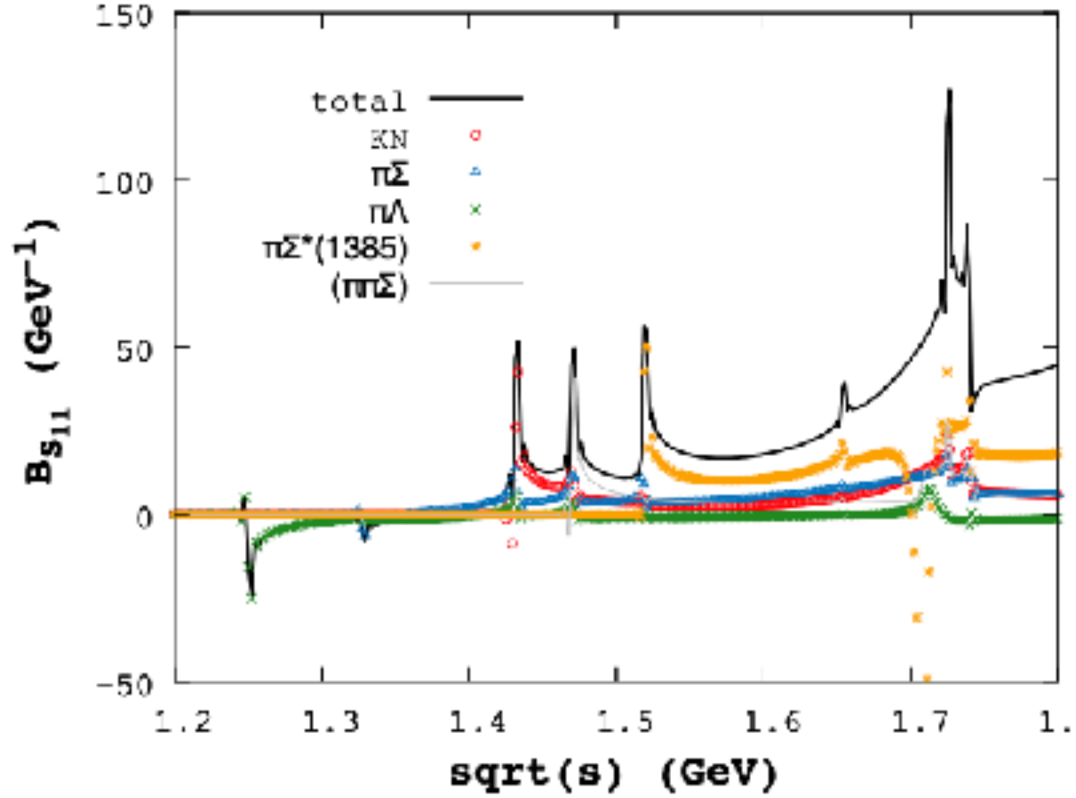
Λ(1690) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	20–30 %	433
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	∼ 25 %	419
$\Sigma\pi\pi$	∼ 20 %	358

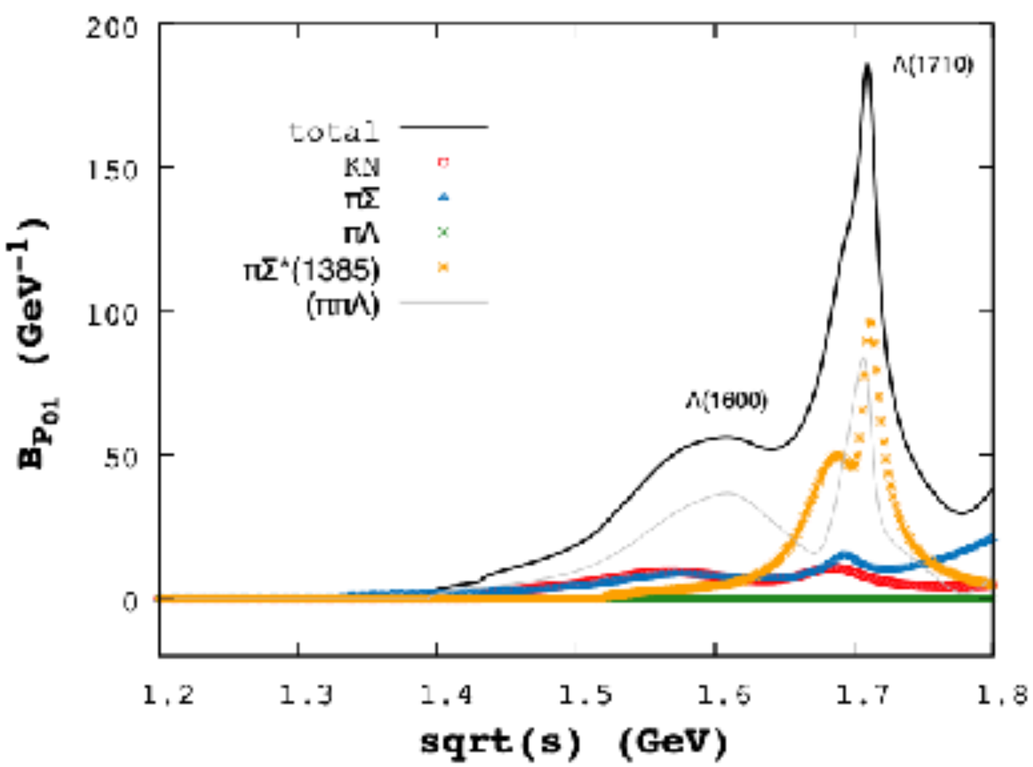
Σ(1670) 3/2⁻ $I(J^P) = 1(\frac{3}{2}^-)$

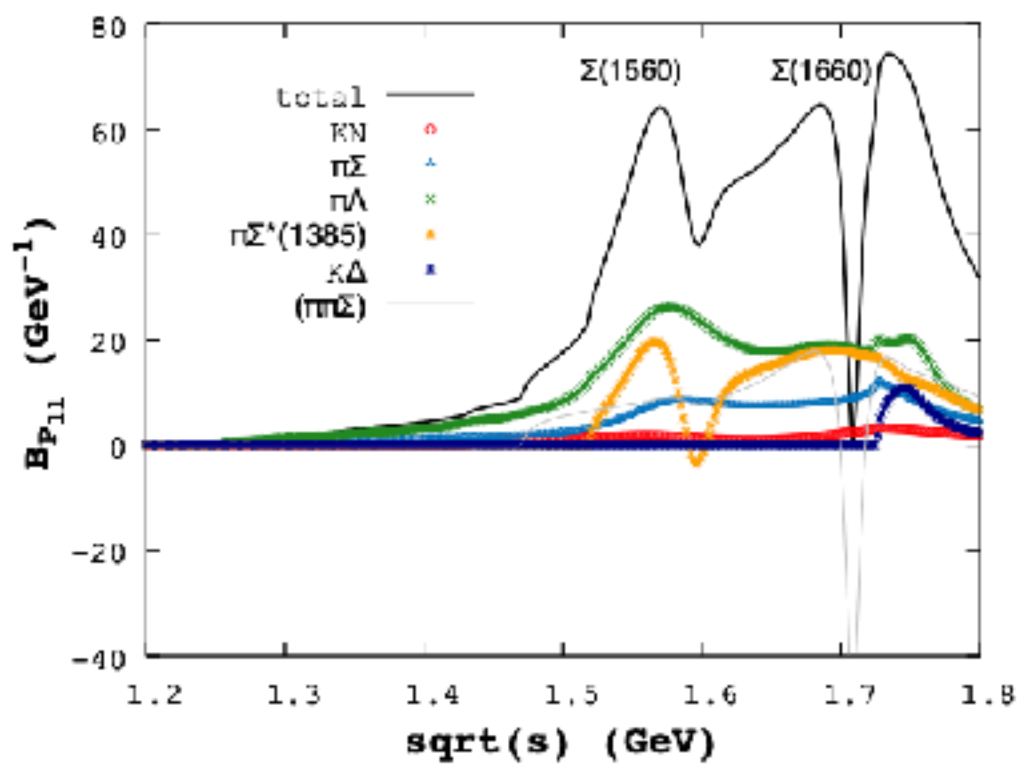
Mass m = 1665 to 1685 (≈ 1670) MeV
Full width Γ = 40 to 80 (≈ 60) MeV
$p_{beam} = 0.74$ GeV/c $4\pi\lambda\!\!\!^-{}^2 = 28.5$ mb

Σ(1670) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394







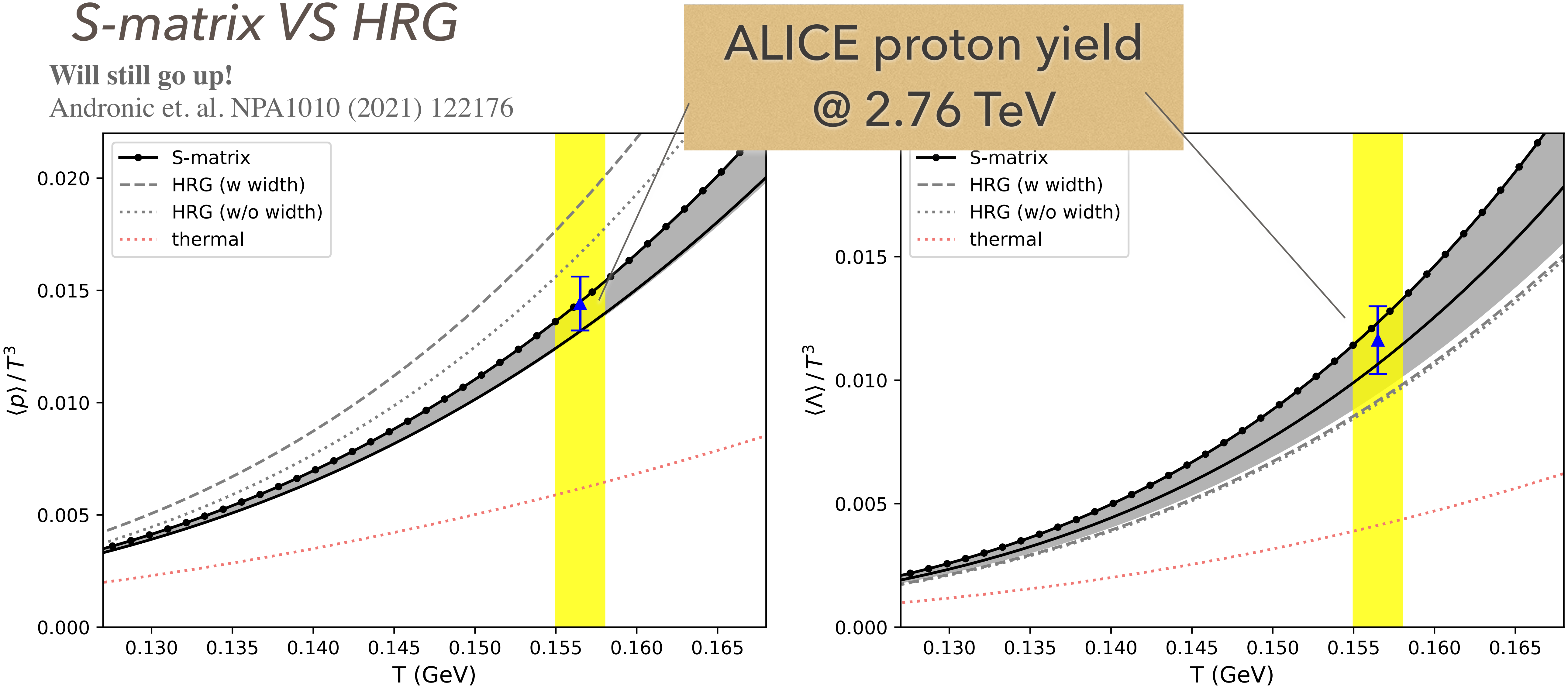


branching ratio?

S-matrix VS HRG

Will still go up!

Andronic et. al. NPA1010 (2021) 122176



piN phase shifts

pipiN BGs

hyperons

consistent treatment of res and non-res. int.

Coupled-Channel model:

$$\bar{k}N, \pi\Lambda, \pi\Sigma, \ldots$$

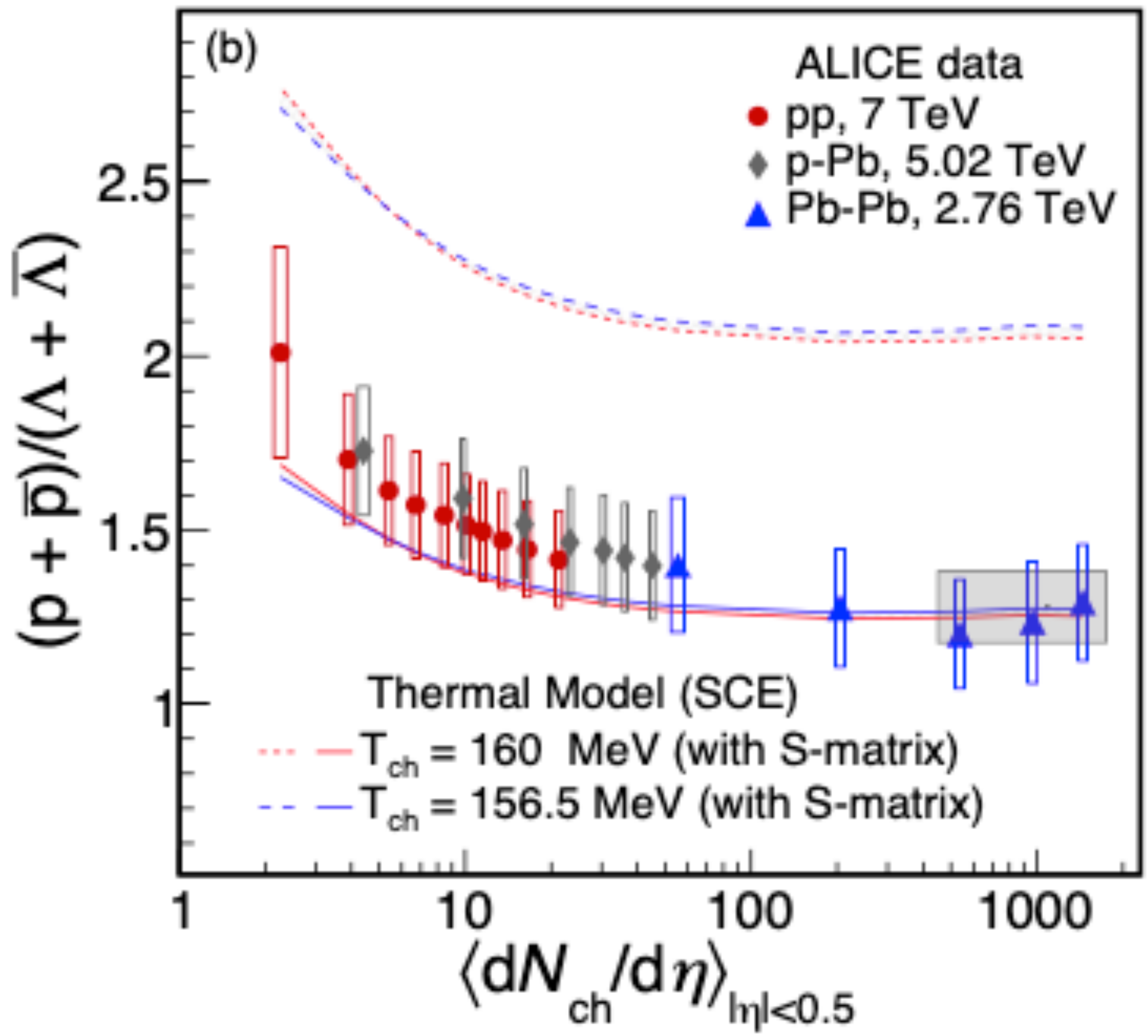
extra hyperon states

beyond PDG

unitarity BGs

less protons

more lambdas

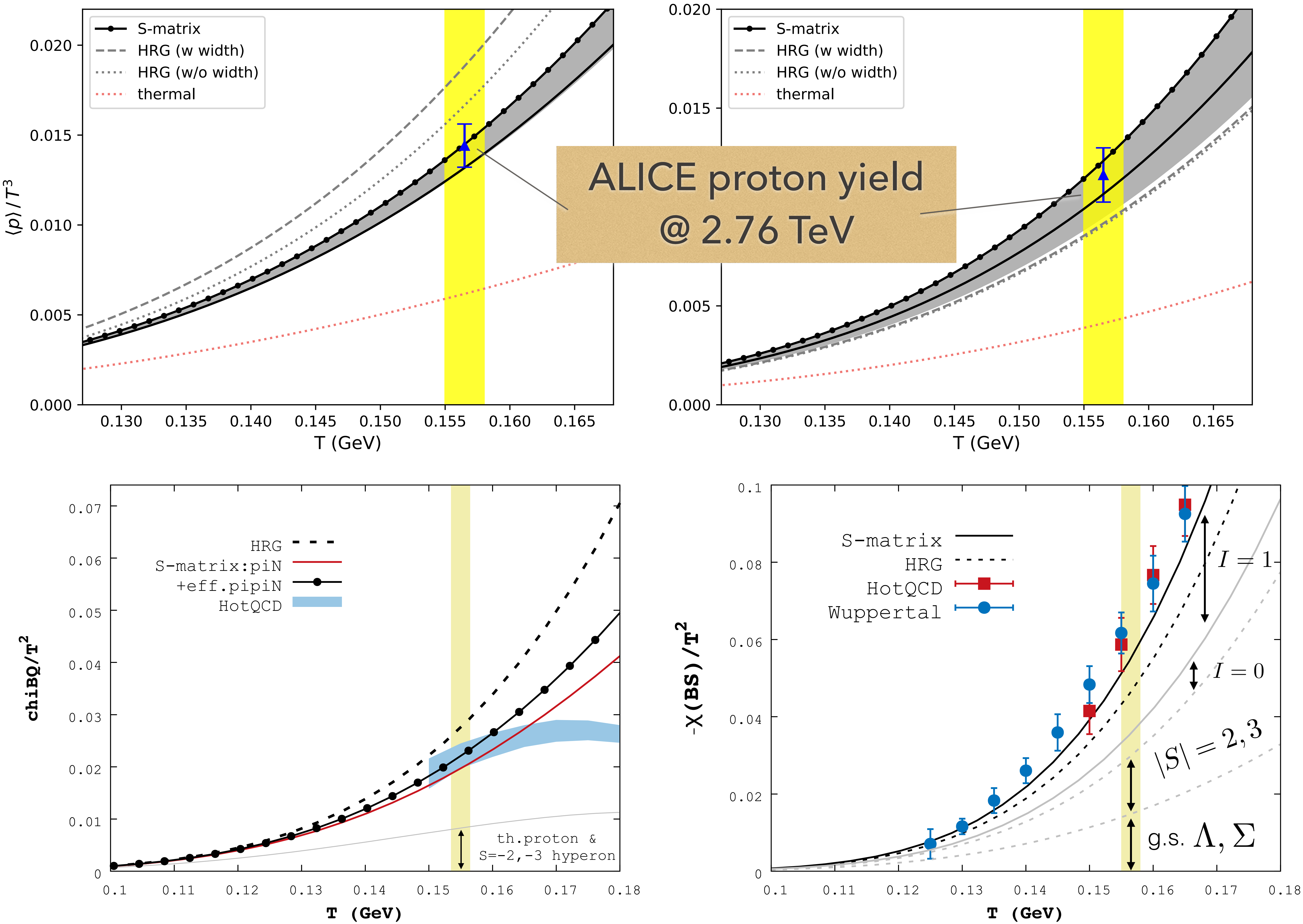


hrg

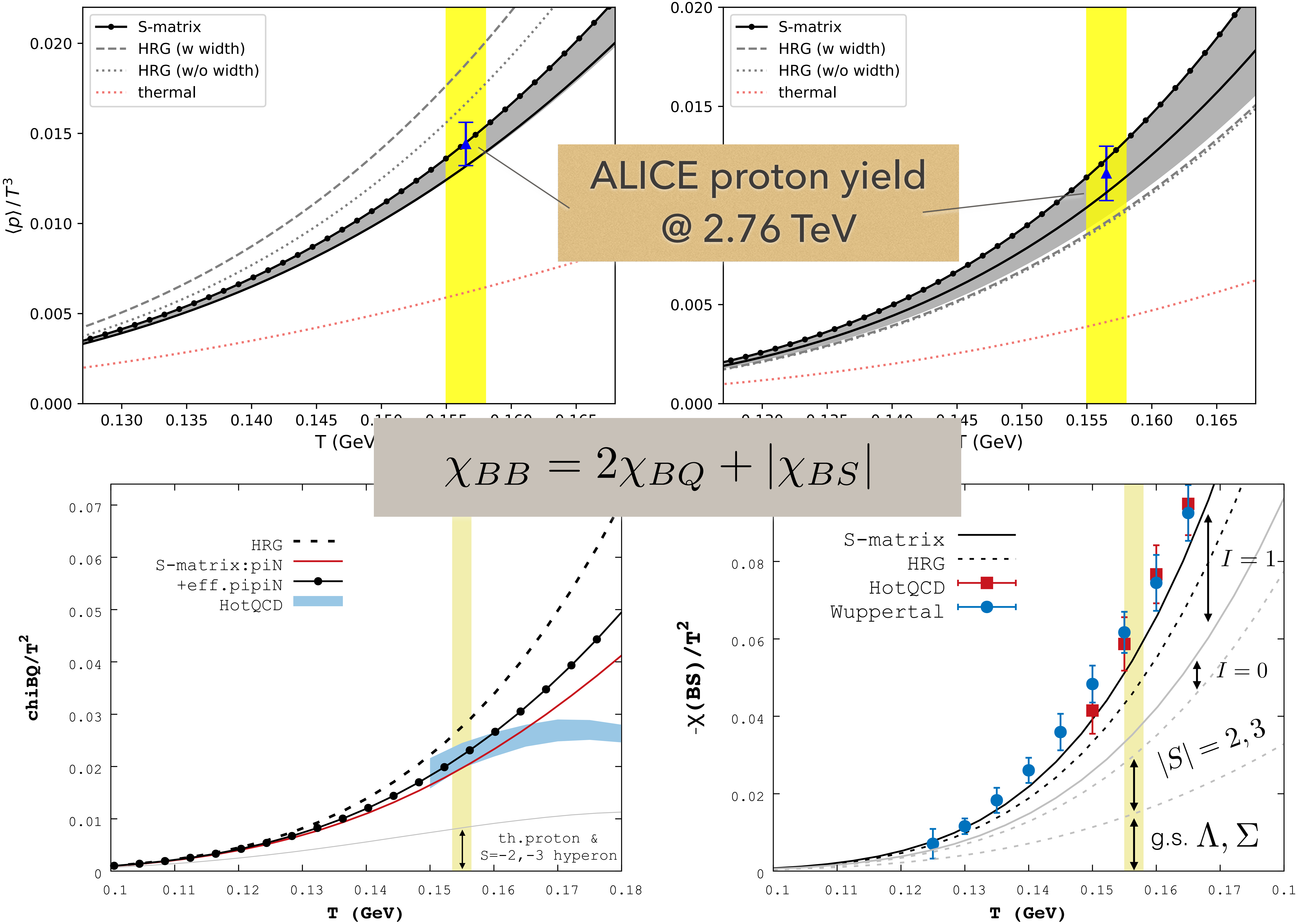
smat

Phys. Rev. C 103, 014904 (2021).

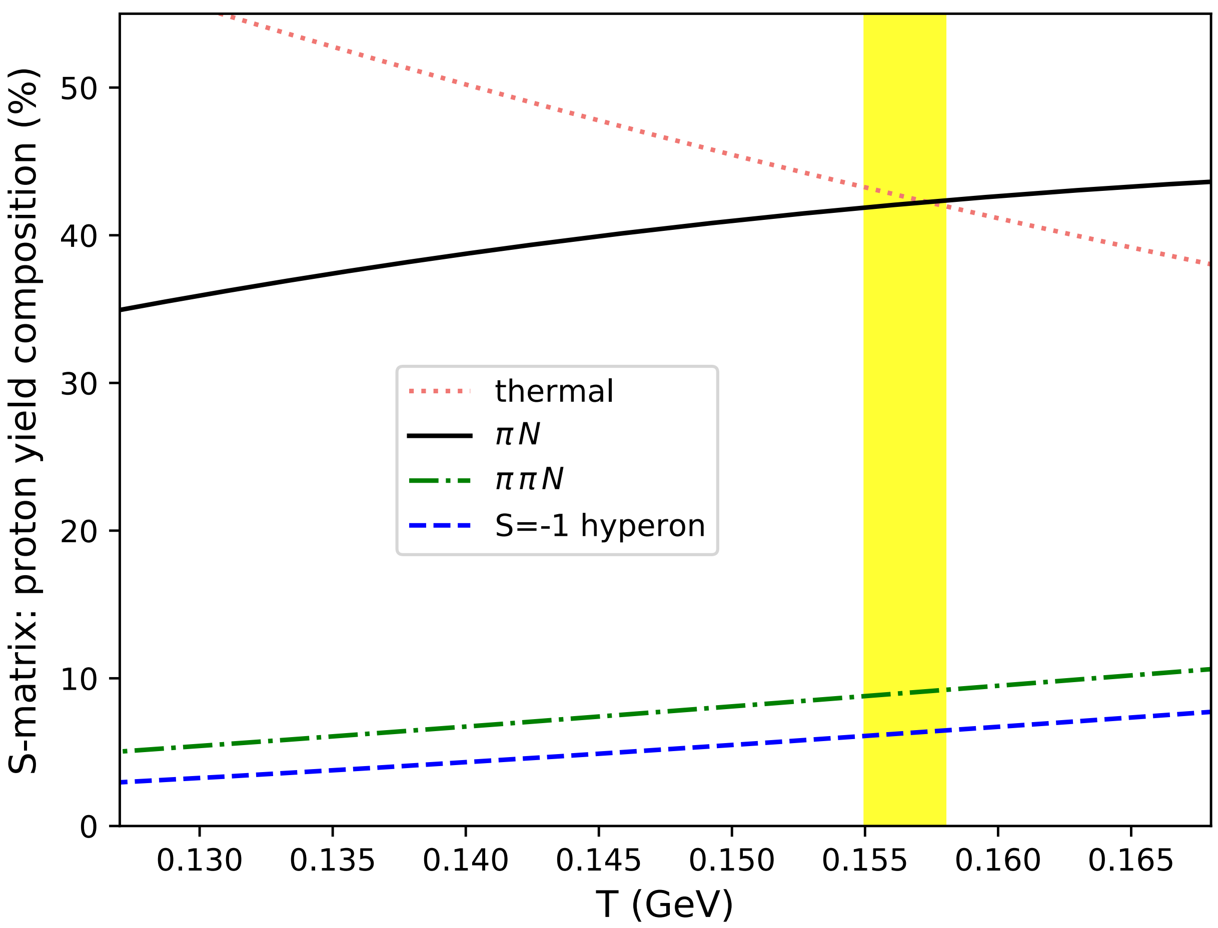
Phys. Lett. B 792, 304 (2019).

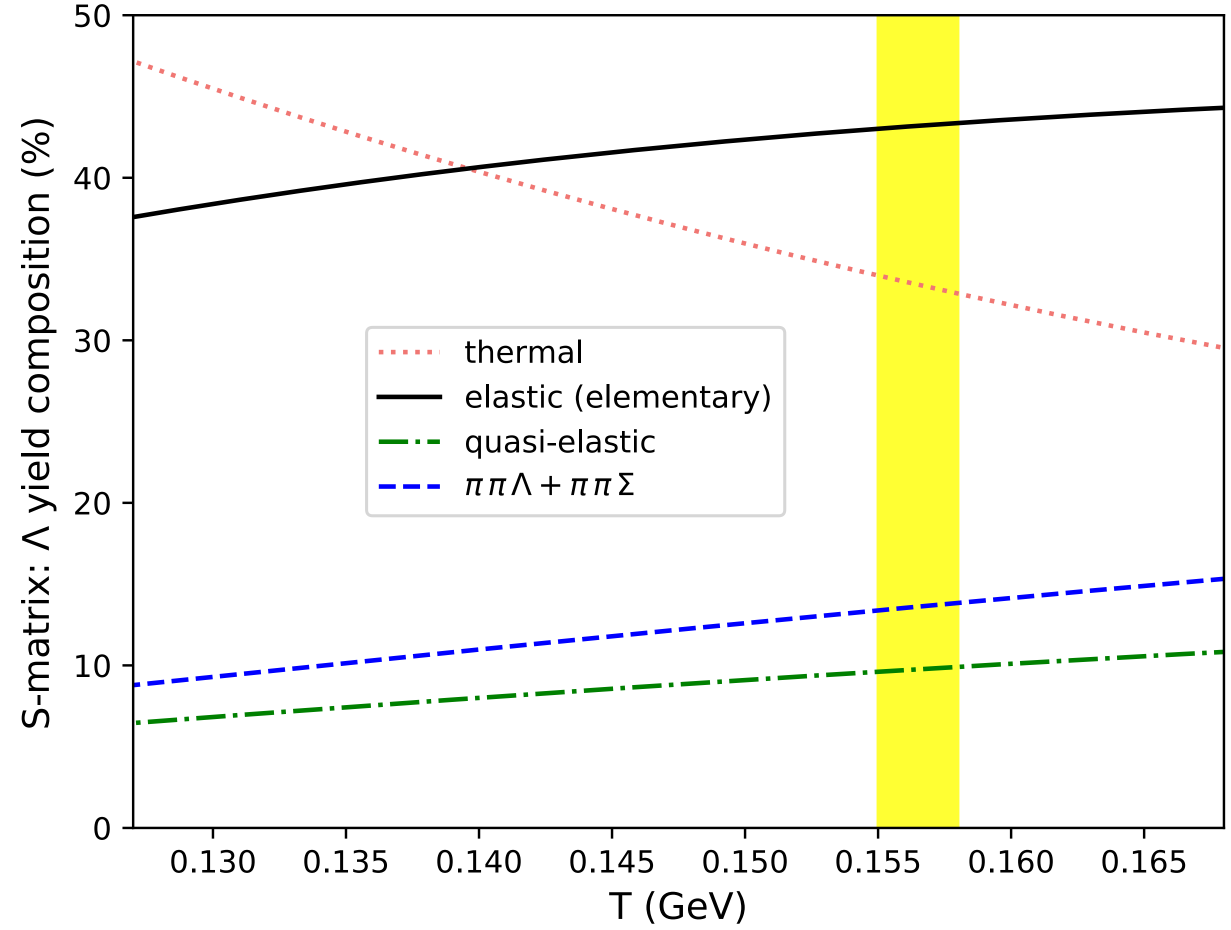


S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩ / T^3
T (GeV)
ALICE proton yield
@ 2.76 TeV
HRG
S-matrix:piN
+eff.pipiN
HotQCD
chiBQ/T^2
th.proton &
S=-2,-3 hyperon
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
-χ(BS)/T^2
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ
T (GeV)



$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$





SAID GWU

JPAC

`piN phase shifts`
`pipiN BGs`
`hyperons`

Coupled-Channel system:

$\bar{k}N, \pi\Lambda, \pi\Sigma, ...$

extra hyperon states

beyond PDG

unitarity BGs

consistent treatment of res and non-res. int.

COUPLED-CHANNEL DYNAMICS

PML PRD **102,** 034038 (2020)

DYNAMICAL GENERATION OF BS / RESONANCES

- dynamical generation of bound states / resonances:
  f(980) close to K Kbar threshold
  f(500) dynamically generated

- coupling of open channels: pipi, kkbar with a $|q\bar{q}\rangle$ state

Locher, Markushin, Zheng, EPJC 4 (1998)
Kaminski, Lesniak, Loiseau, EPJC 9 (1999)

what you give $\neq$ *what you get* *1 in 5 out!*

$$\frac{1}{E-\mathcal{H}_0} = \begin{matrix} |\pi\pi\rangle \\ |K\bar{K}\rangle \\ |R^0\rangle \; (|q\bar{q}\rangle) \end{matrix} \begin{bmatrix} \Pi_{\pi\pi}(E) & & \\ & \Pi_{K\bar{K}}(E) & \\ & & \frac{1}{E-m^0_{res}} \end{bmatrix}$$

$$V_{int} = \begin{bmatrix} g_{\pi\pi} & g_{\pi K} & g_{\pi R} \\ g_{\pi K} & g_{KK} & g_{KR} \\ g_{\pi R} & g_{KR} & \end{bmatrix}$$

$$G = G_0 + G_0 V_{int} G$$

TESTING THE ROBUSTNESS

$$\mathcal{Q}(E) = \frac{1}{2}\mathrm{Im}\mathrm{Tr}\{\ln S_E\}$$

Getting Effective DOS on REAL Energy

effective DOS

$$B = 2\frac{d}{dE}\mathcal{Q}$$

what is being counted?

can it handle dynamically generated states?

TESTING THE ROBUSTNESS

$$\mathcal{Q}(E) = \frac{1}{2}\mathrm{Im}\mathrm{Tr}\{\ln S_E\}$$

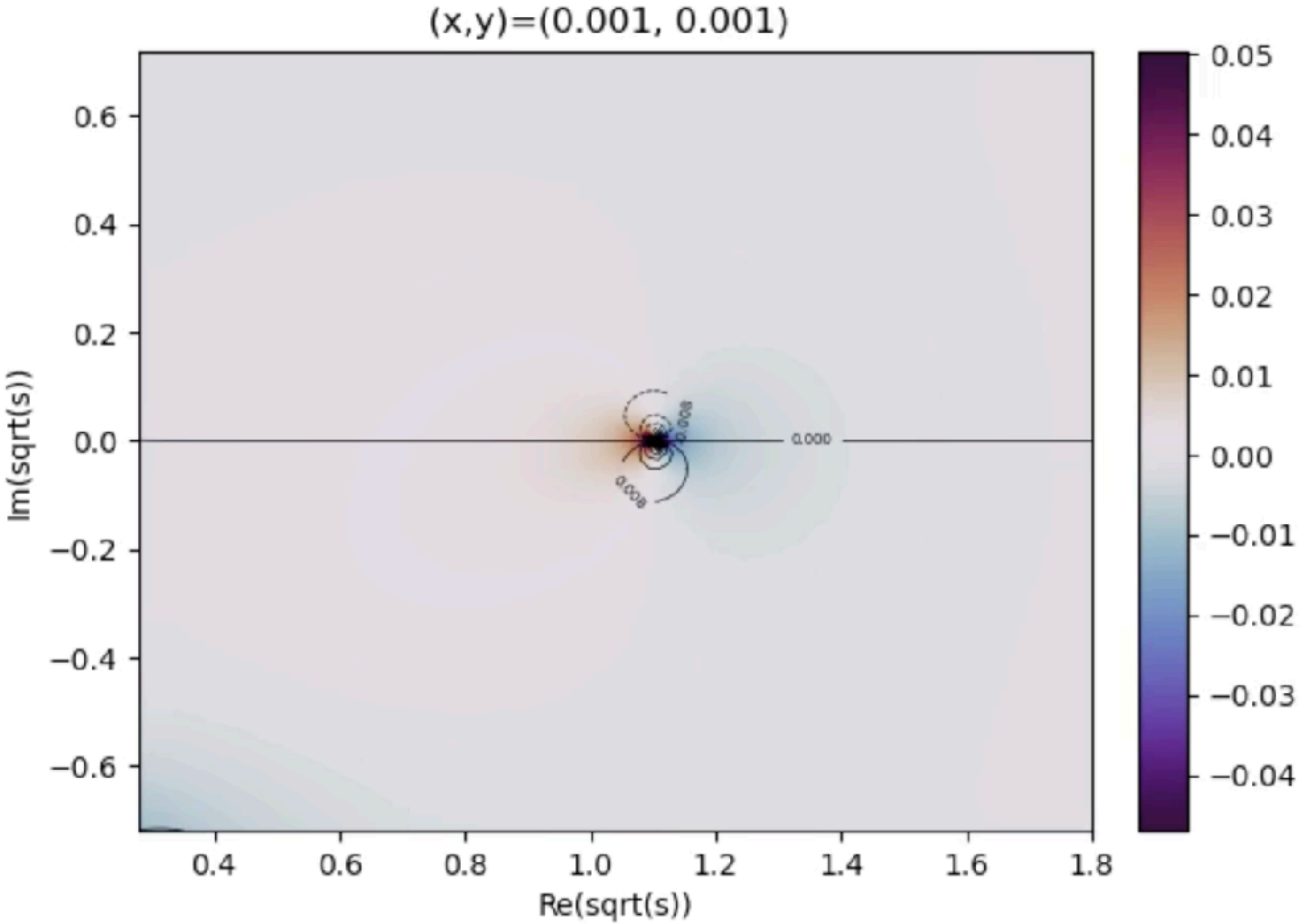
Getting
Effective DOS
on
REAL Energy

effective DOS

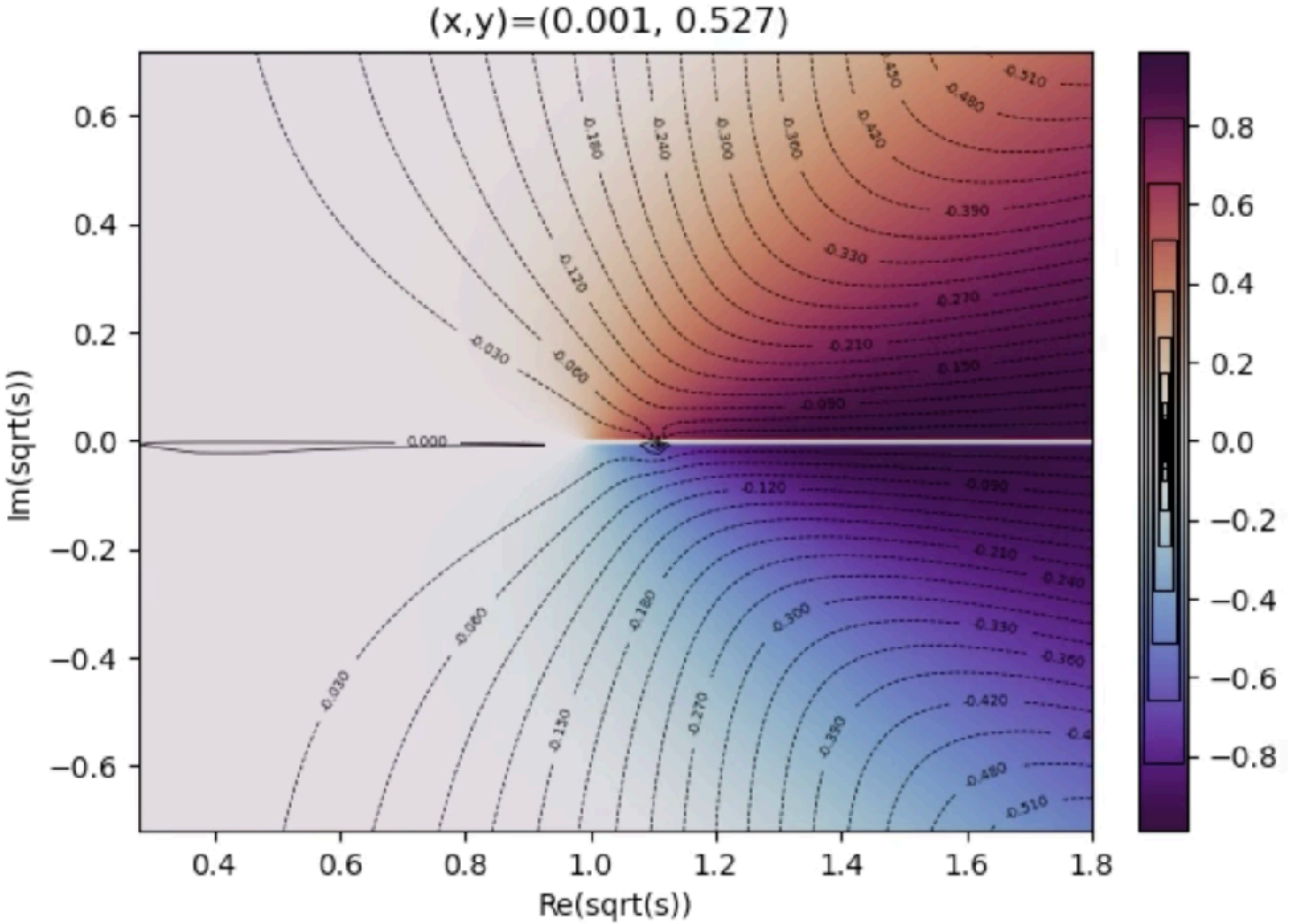
$$B = 2\frac{d}{dE}\mathcal{Q}$$

what is being counted?

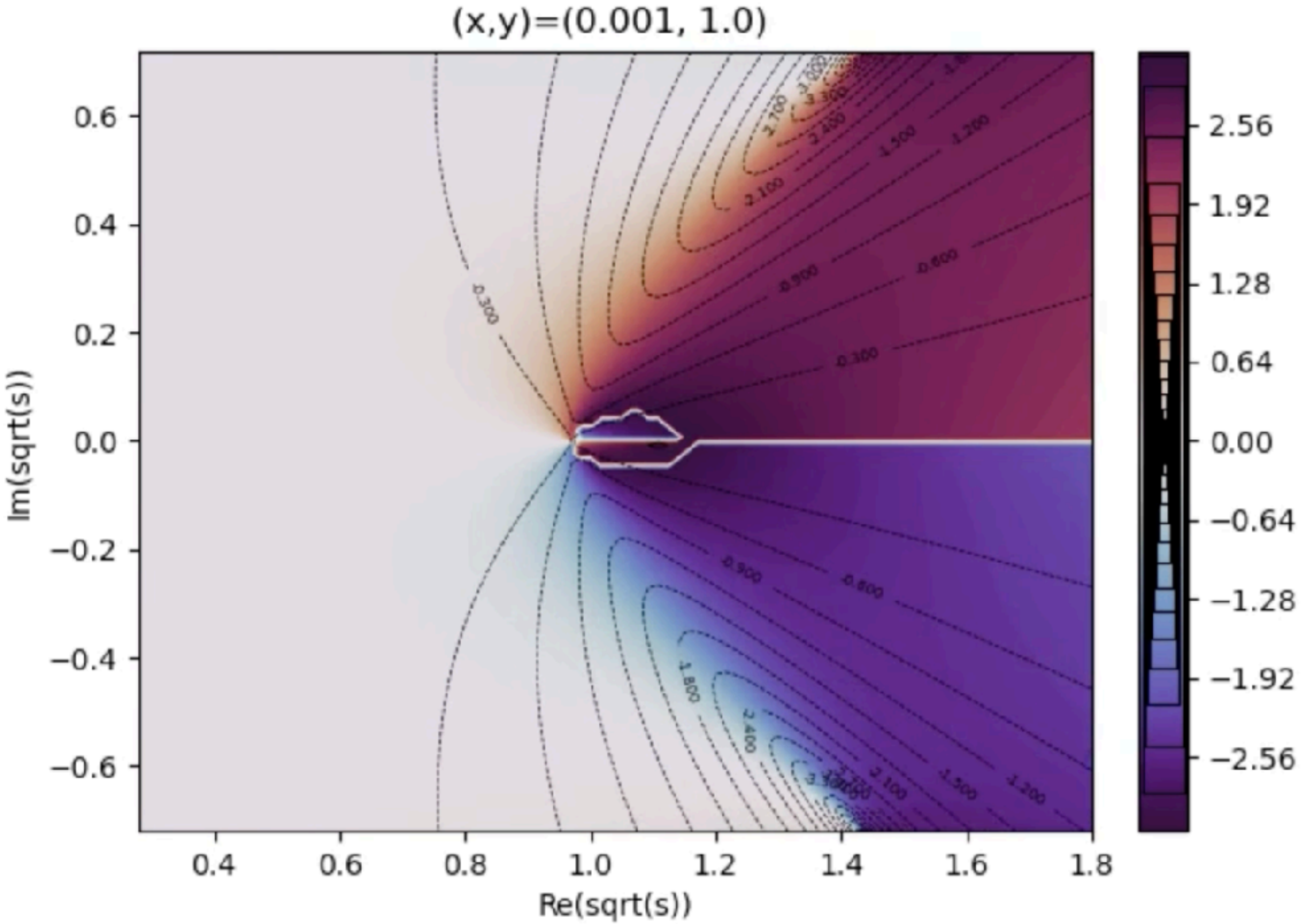
can it handle dynamically generated st



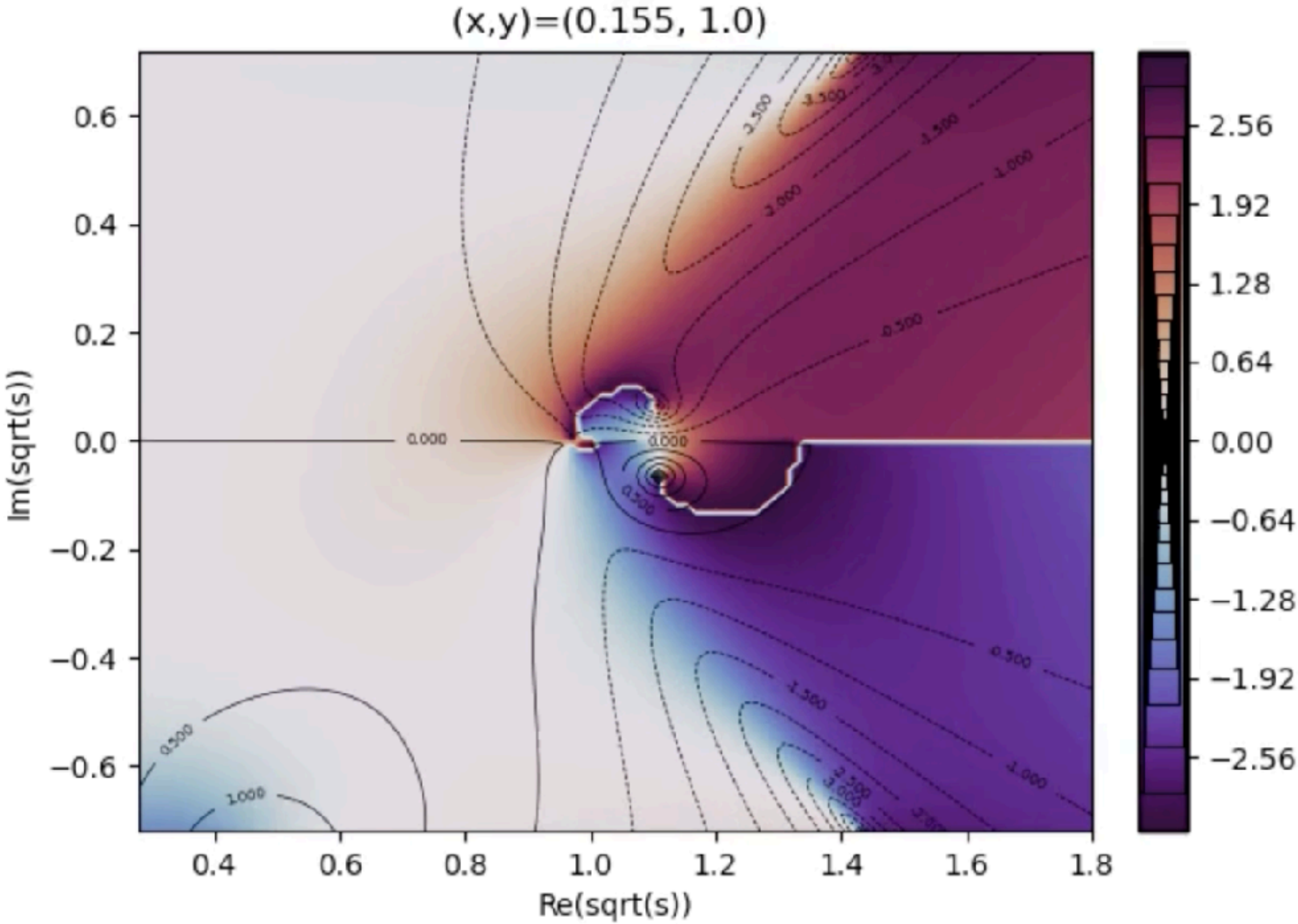
(x,y)=(0.001, 0.001)
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Im(sqrt(s))
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Re(sqrt(s))
0.000
0.05
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04

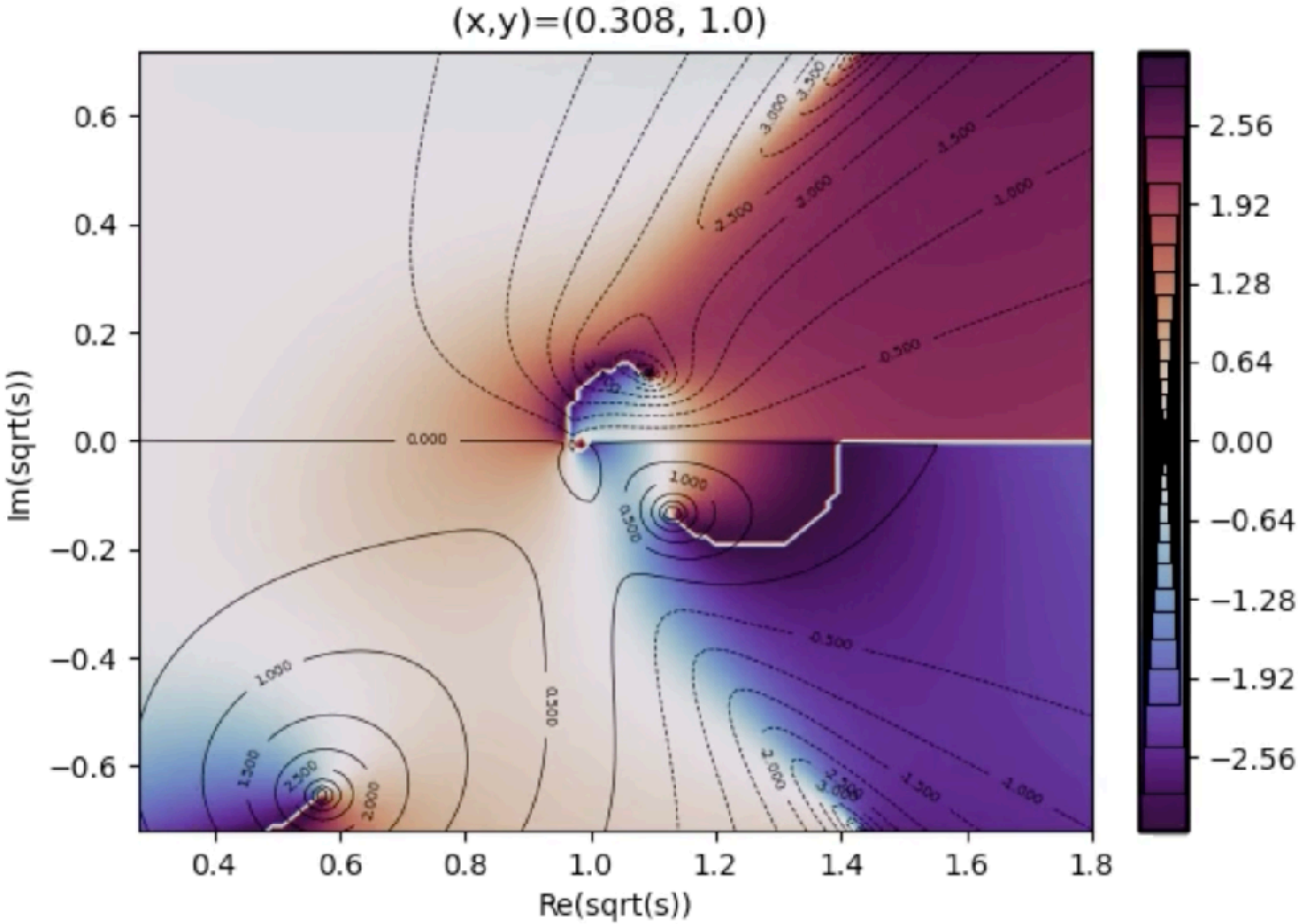


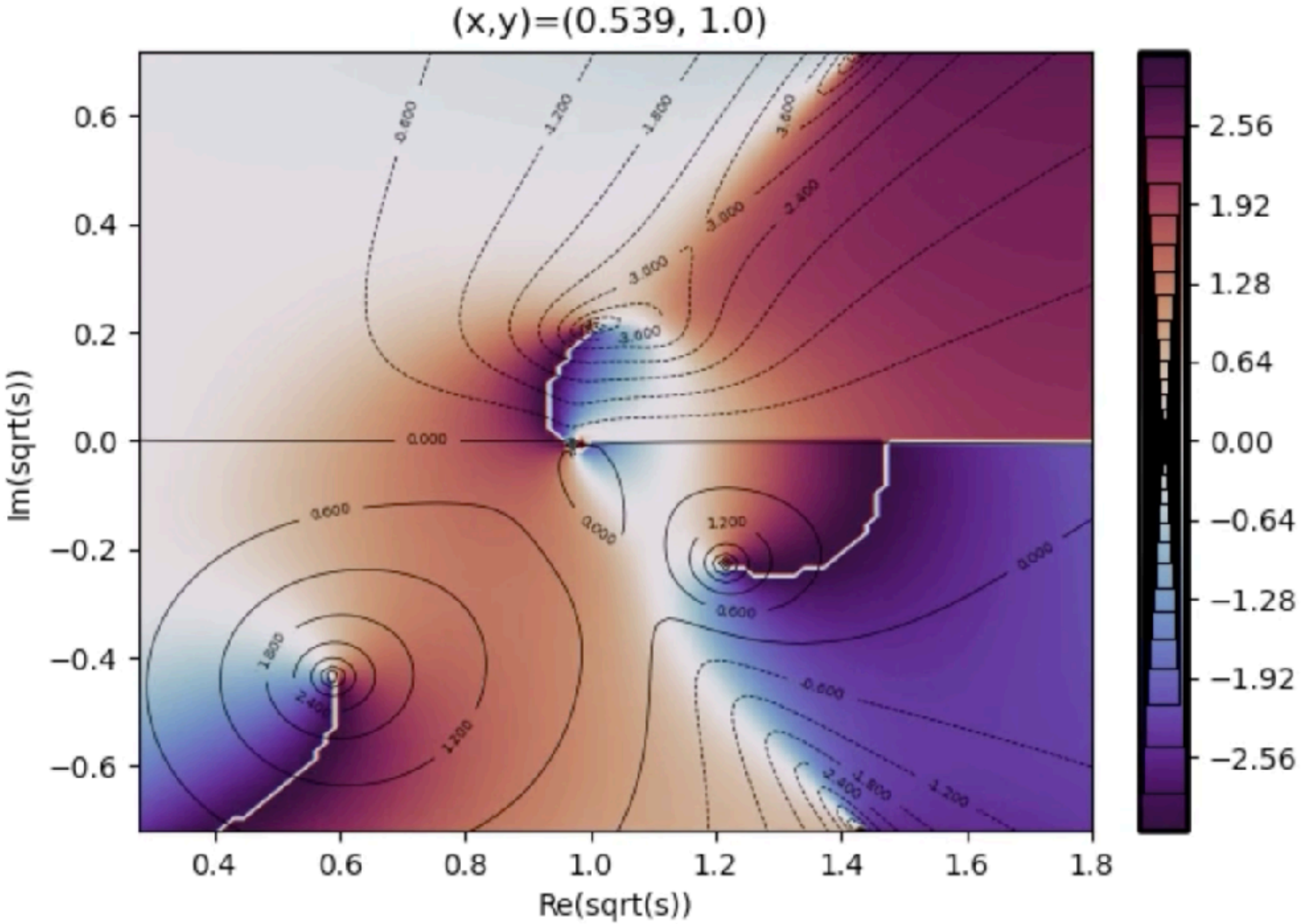
(x,y)=(0.001, 0.527)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8



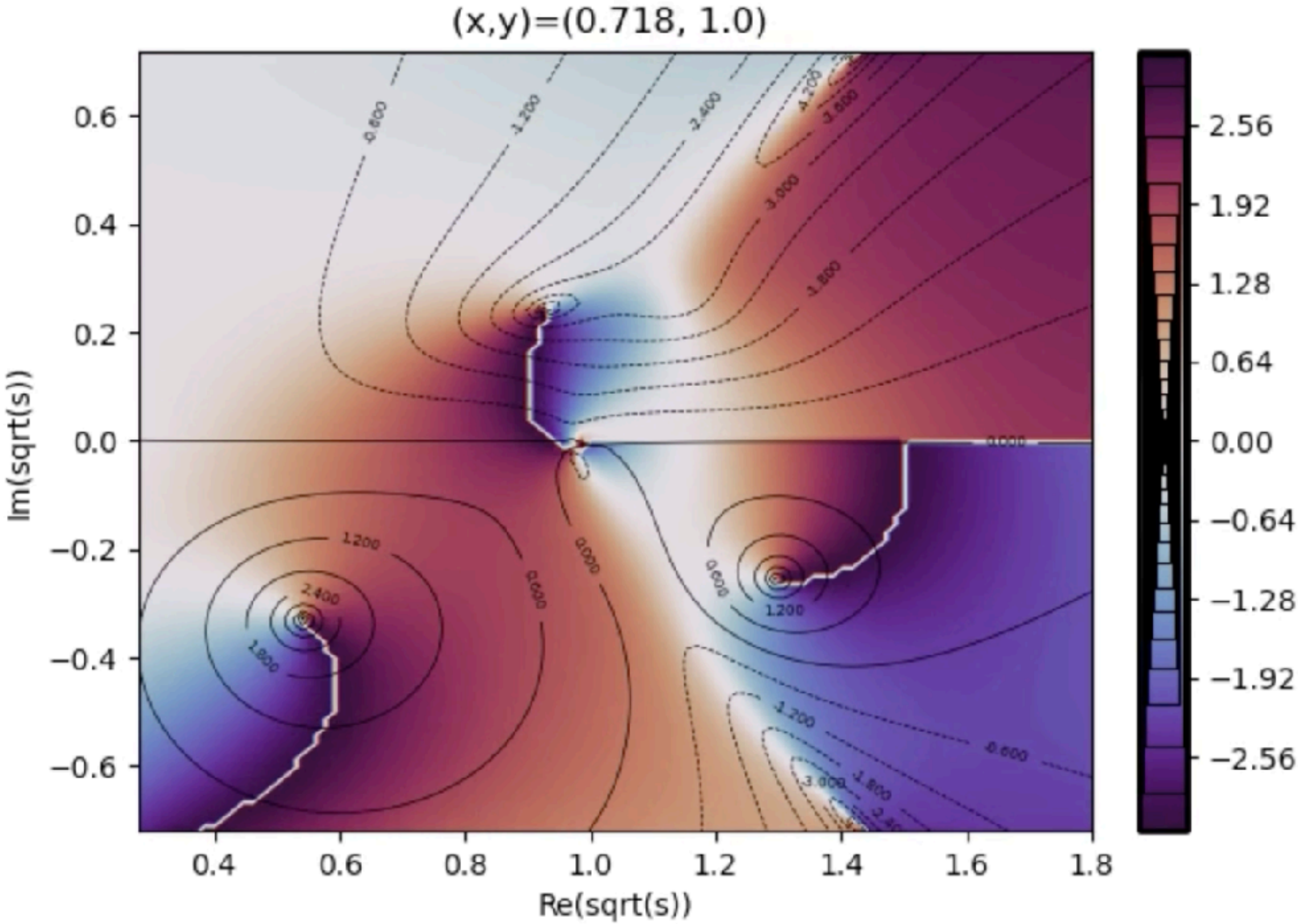
(x,y)=(0.001, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56

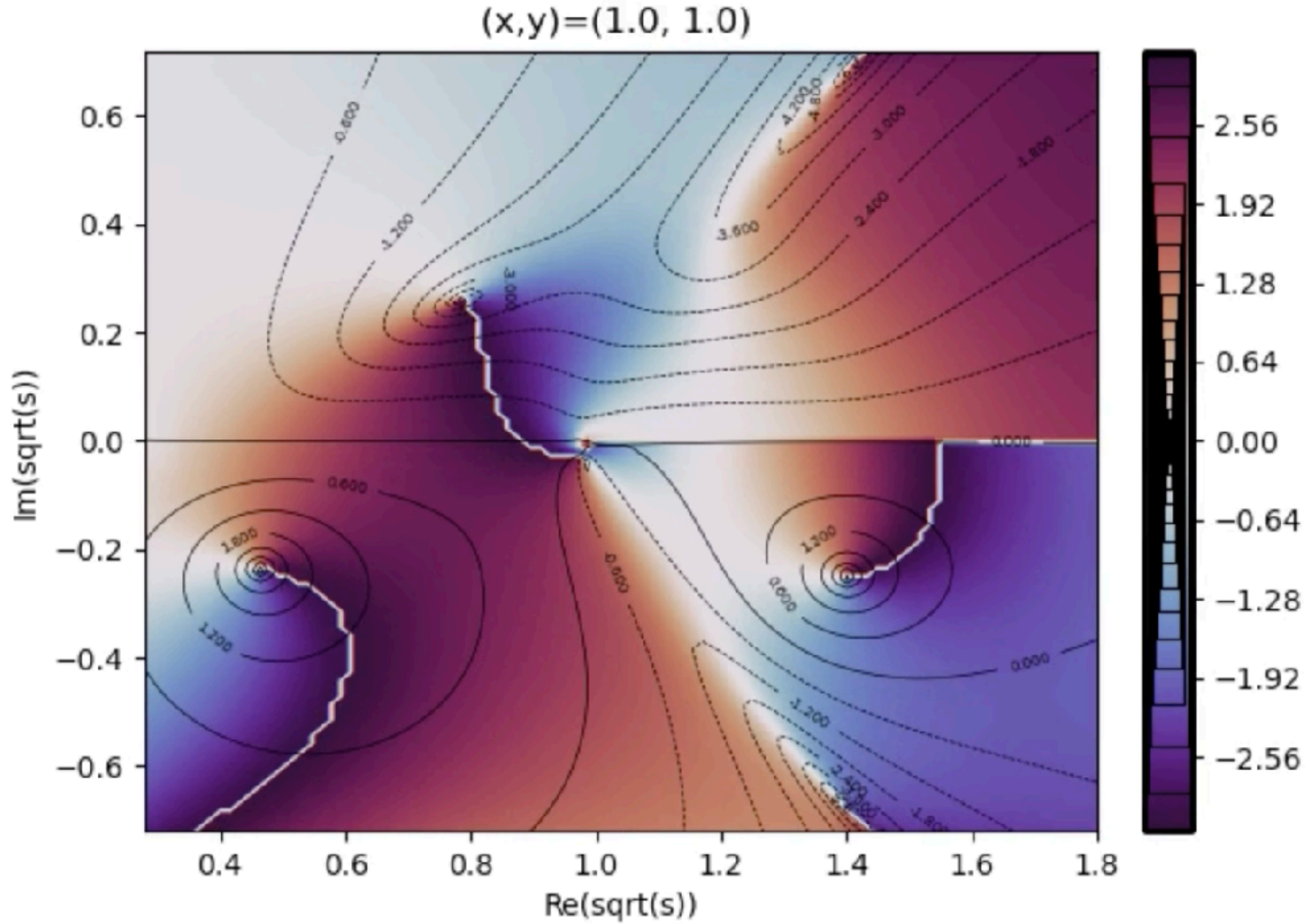

(x,y)=(0.155, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56

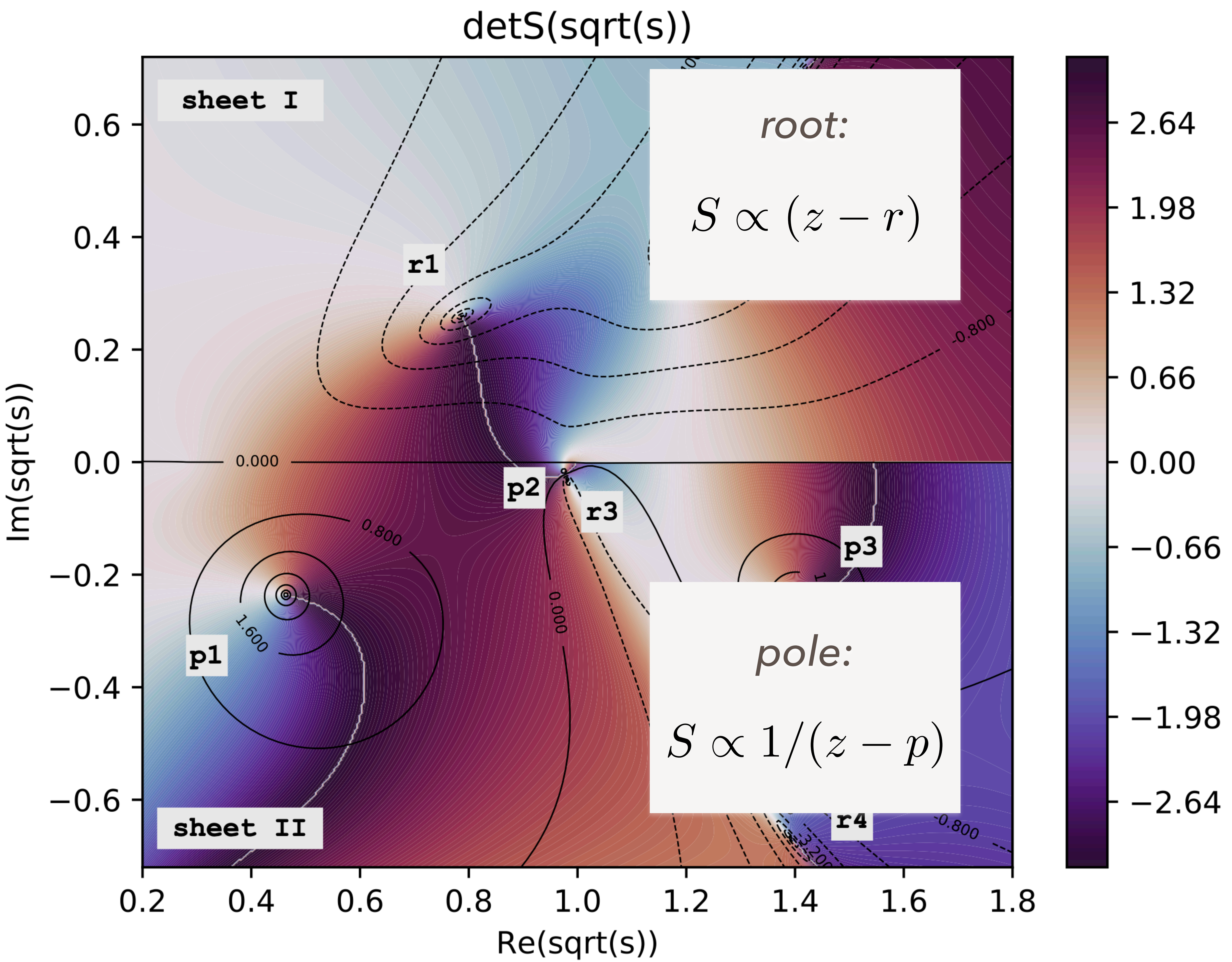

(x,y)=(0.308, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56



(x,y)=(0.539, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56

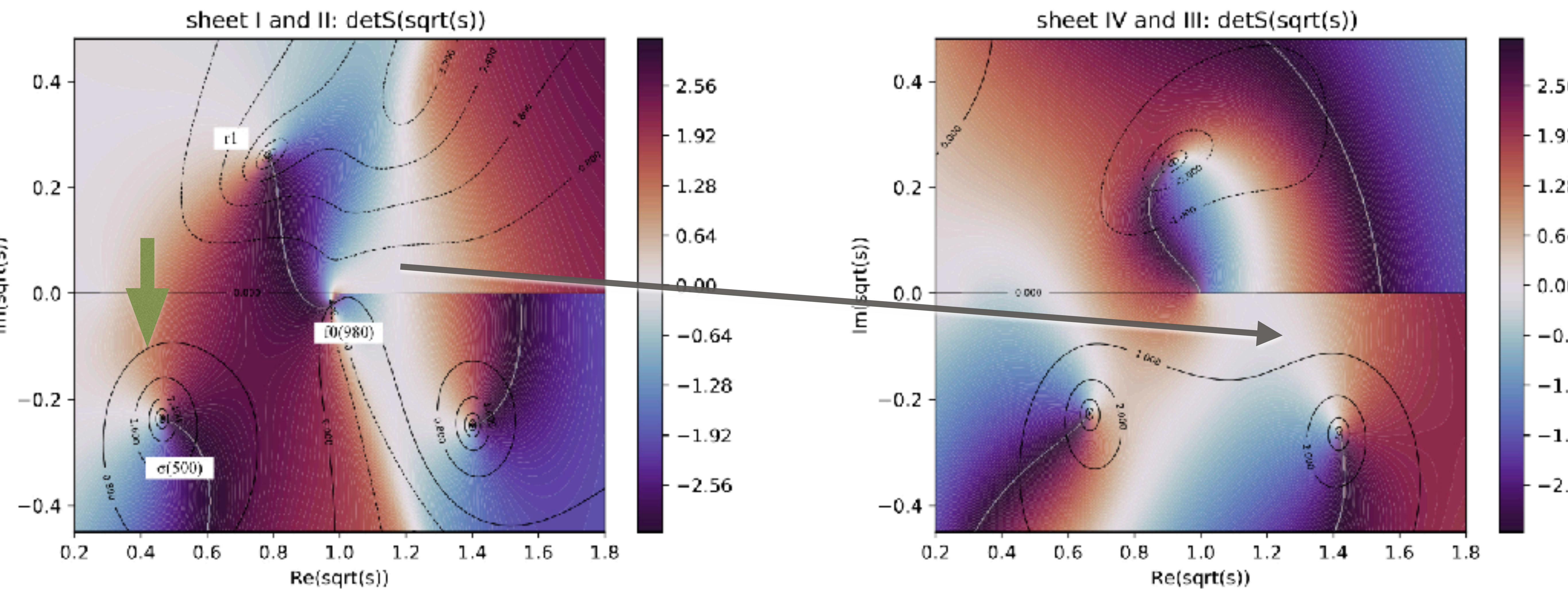

(x,y)=(0.718, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56
1.200
2.400
1.800
0.600
0.000
-1.200
-0.600
-1.800
-3.000
3.000
-2.400
4.200
3.600
1.800


(x,y)=(1.0, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56

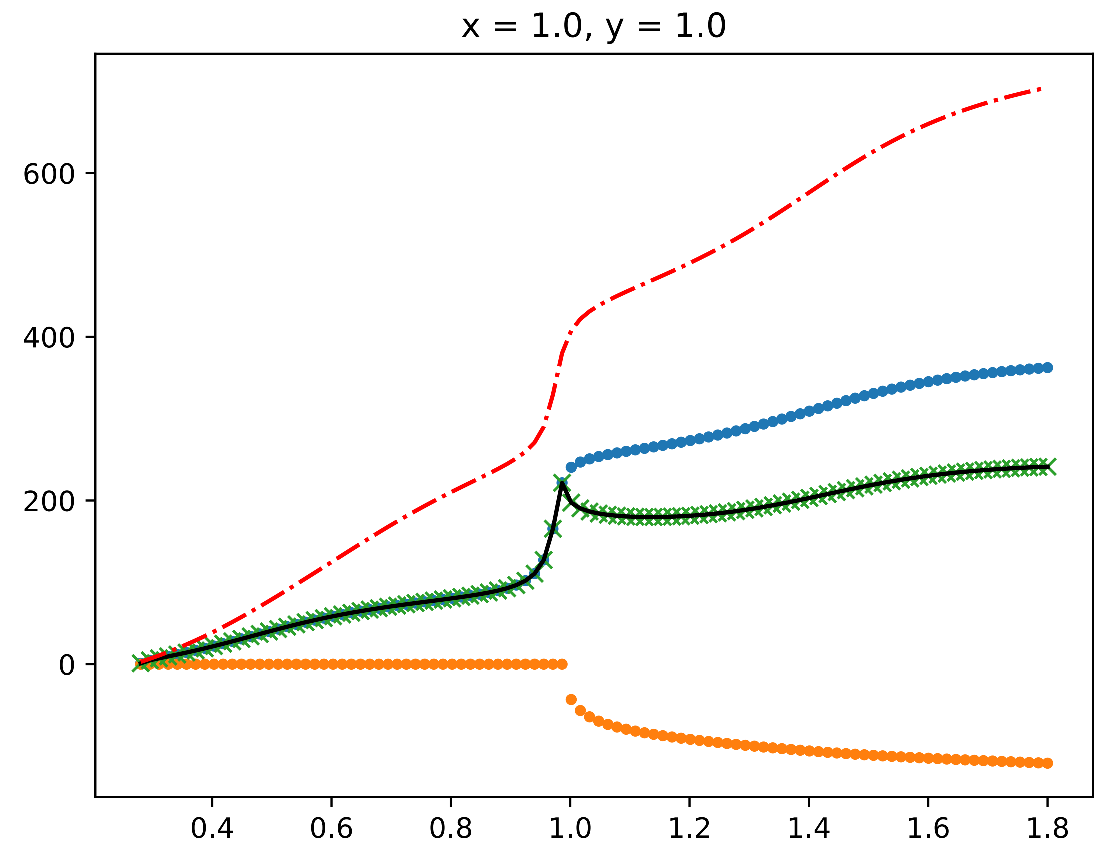


detS(sqrt(s))
sheet I
sheet II
root:
$S \propto (z - r)$
pole:
$S \propto 1/(z - p)$
r1
r3
r4
p1
p2
p3
0.000
0.800
1.600
-0.800
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.64
1.98
1.32
0.66
0.00
−0.66
−1.32
−1.98
−2.64

Continuity of Phase Change

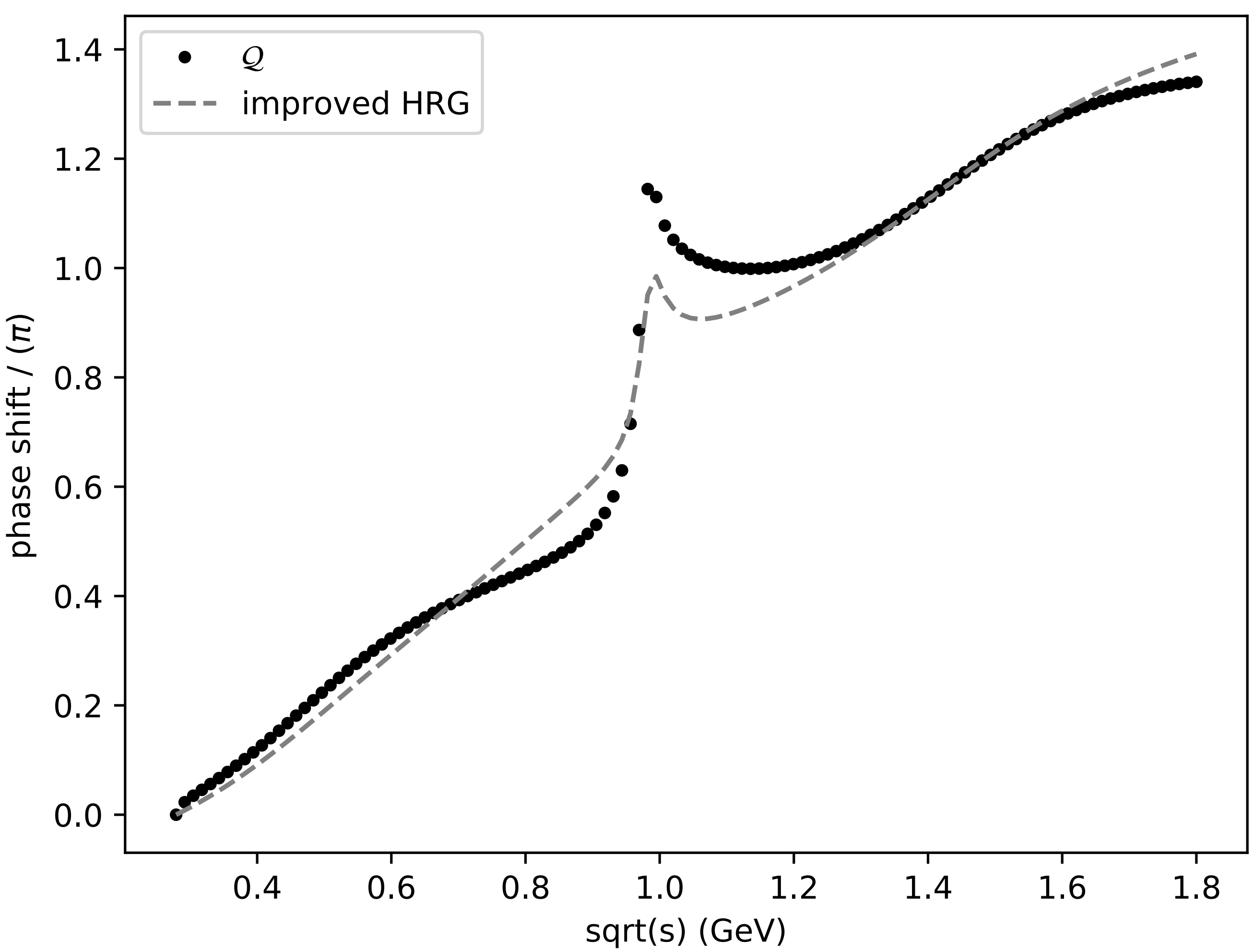


f. [54, 55]



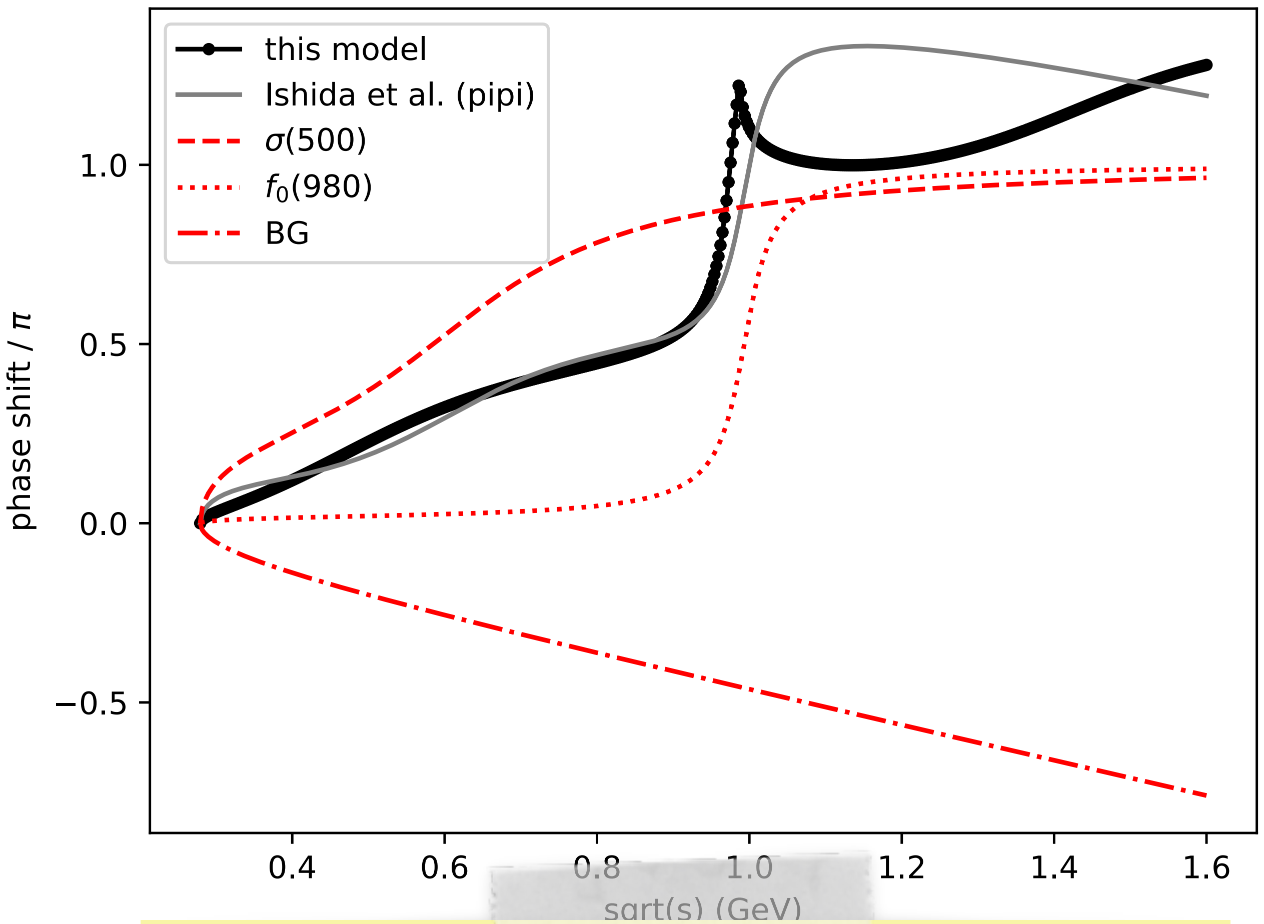
	Re $\sqrt{s}$	Im $\sqrt{s}$	sheet
p1	0.4637	-0.2357	II
p2	0.975	-0.0164	II
p3	1.401	-0.249	II
p4	0.6654	-0.2263	III
p5	1.4176	-0.2640	III
r1	0.787	+0.259	I
r2	1.410	+0.691	I
r3	0.981	-0.032	II
r4	1.393	-0.669	II
r5	0.918	+0.248	IV

II. Location of resonance poles (p_i) and roots (r_i) d in the model.



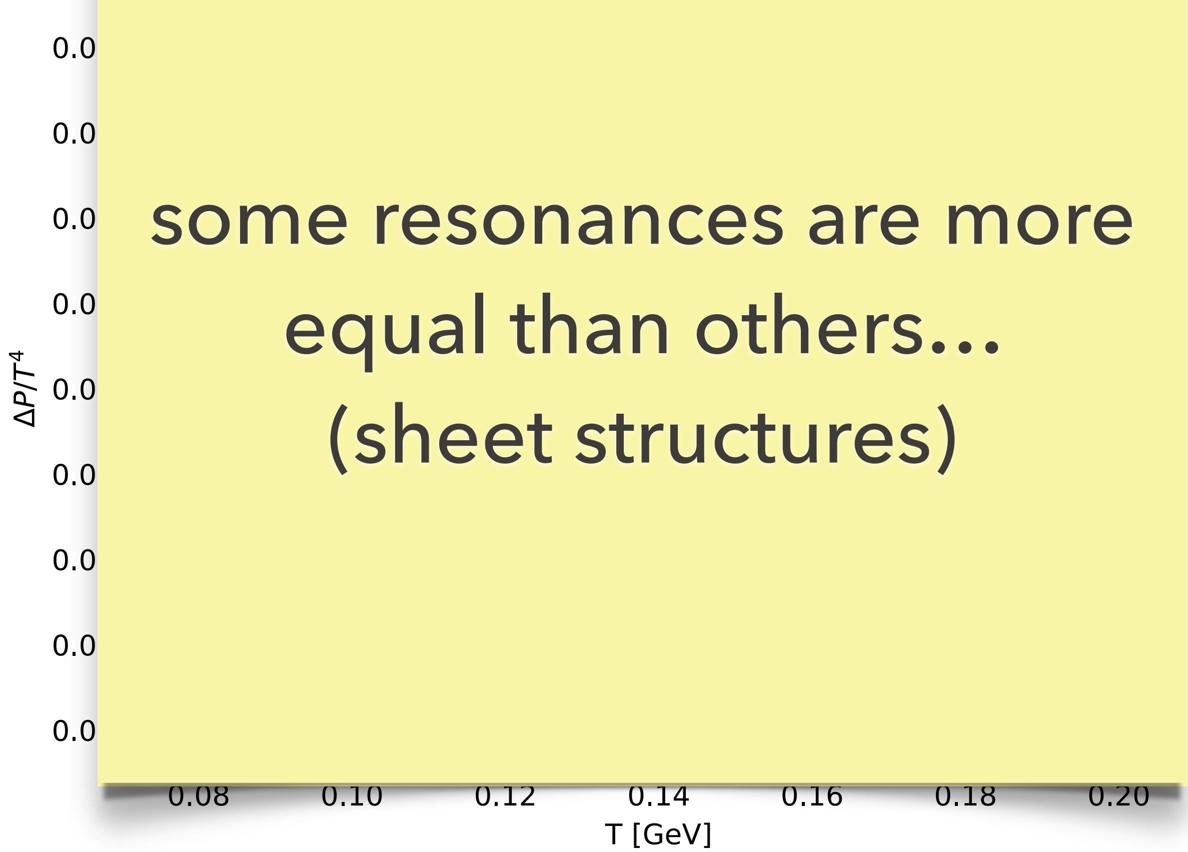
repulsive corrections in HRG-like scheme: via roots

	$\mathrm{Re}\sqrt{s}$	$\mathrm{Im}\sqrt{s}$	sheet

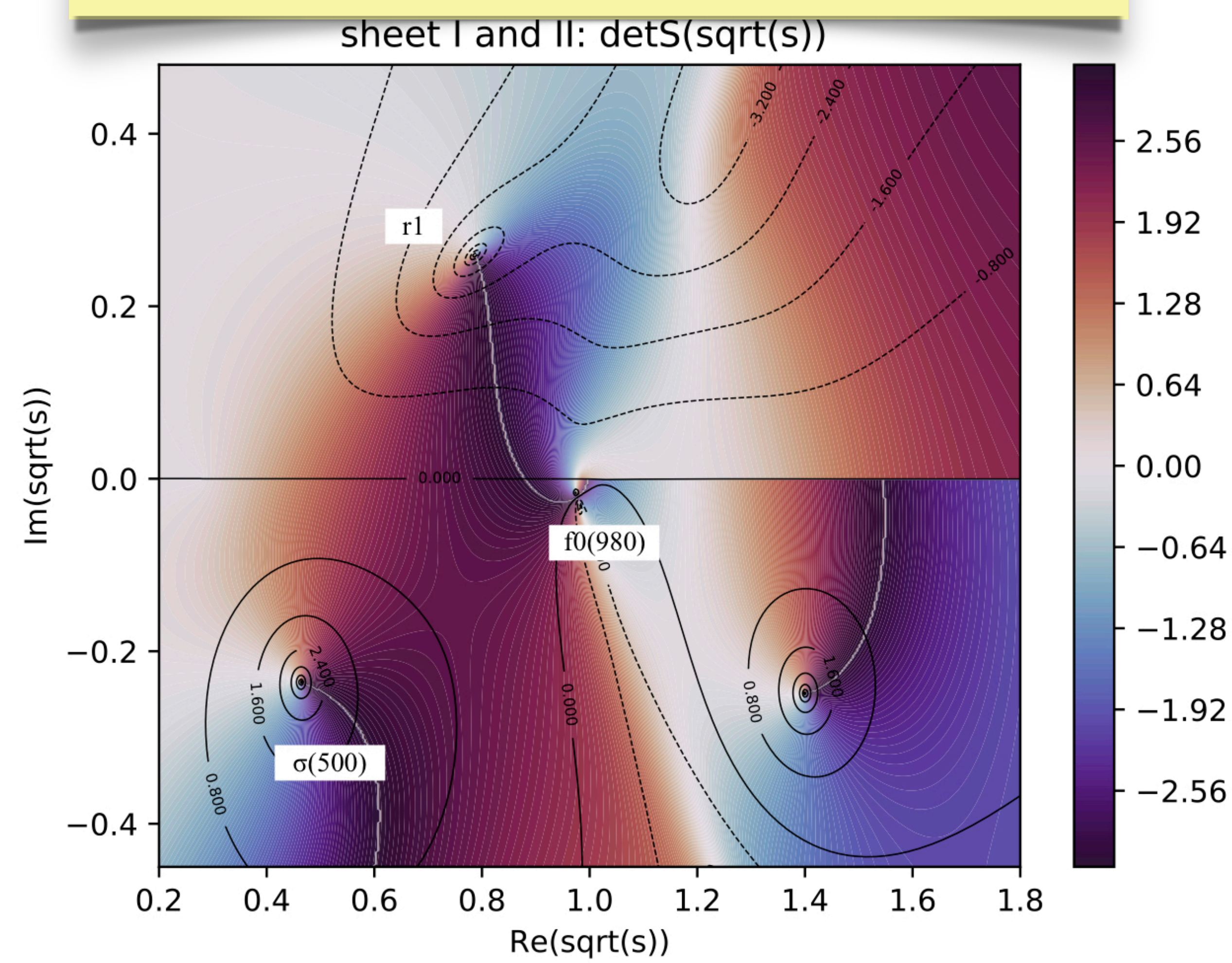


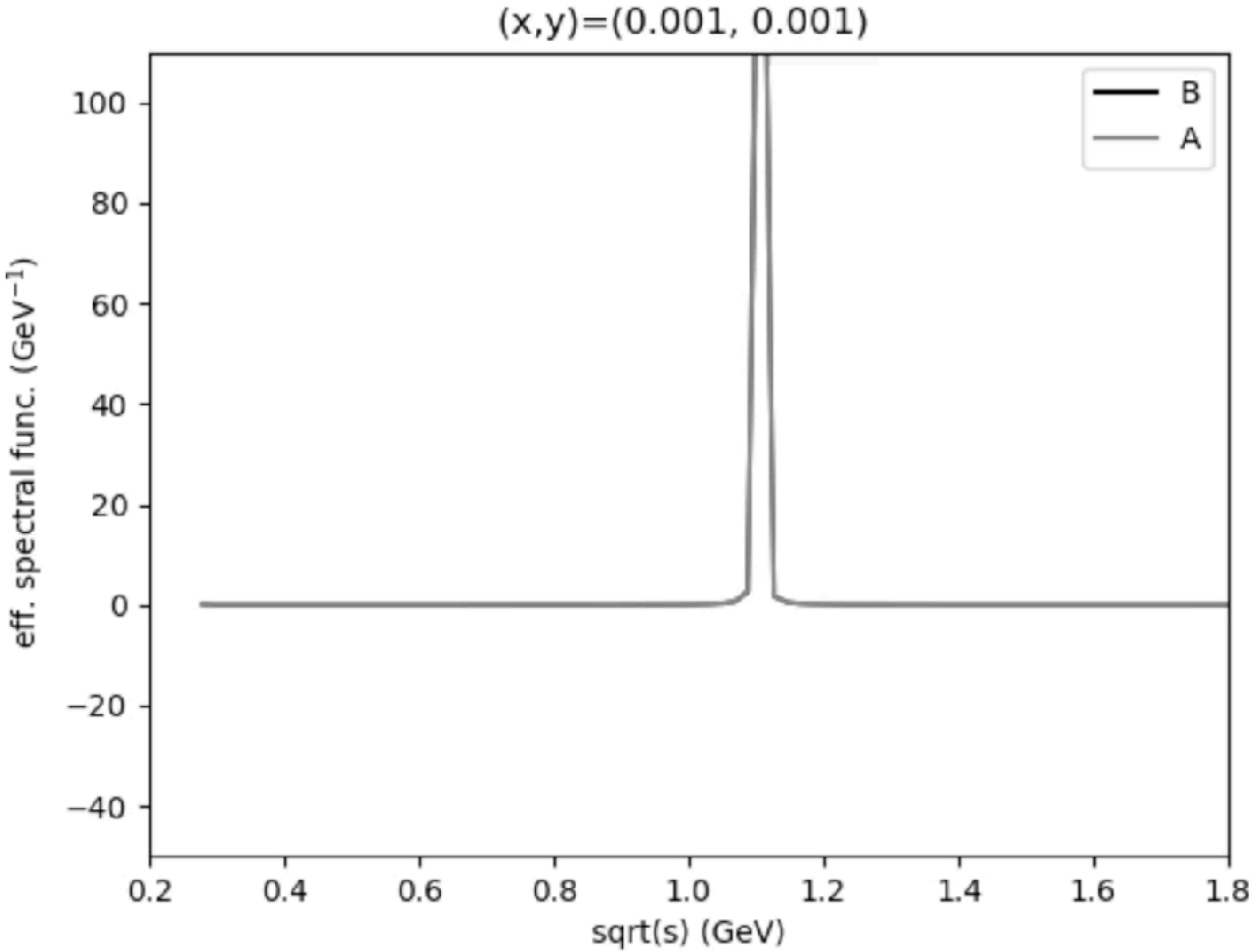
non-Briet-Wigner
=> poles and roots distribution

subtractive corrections

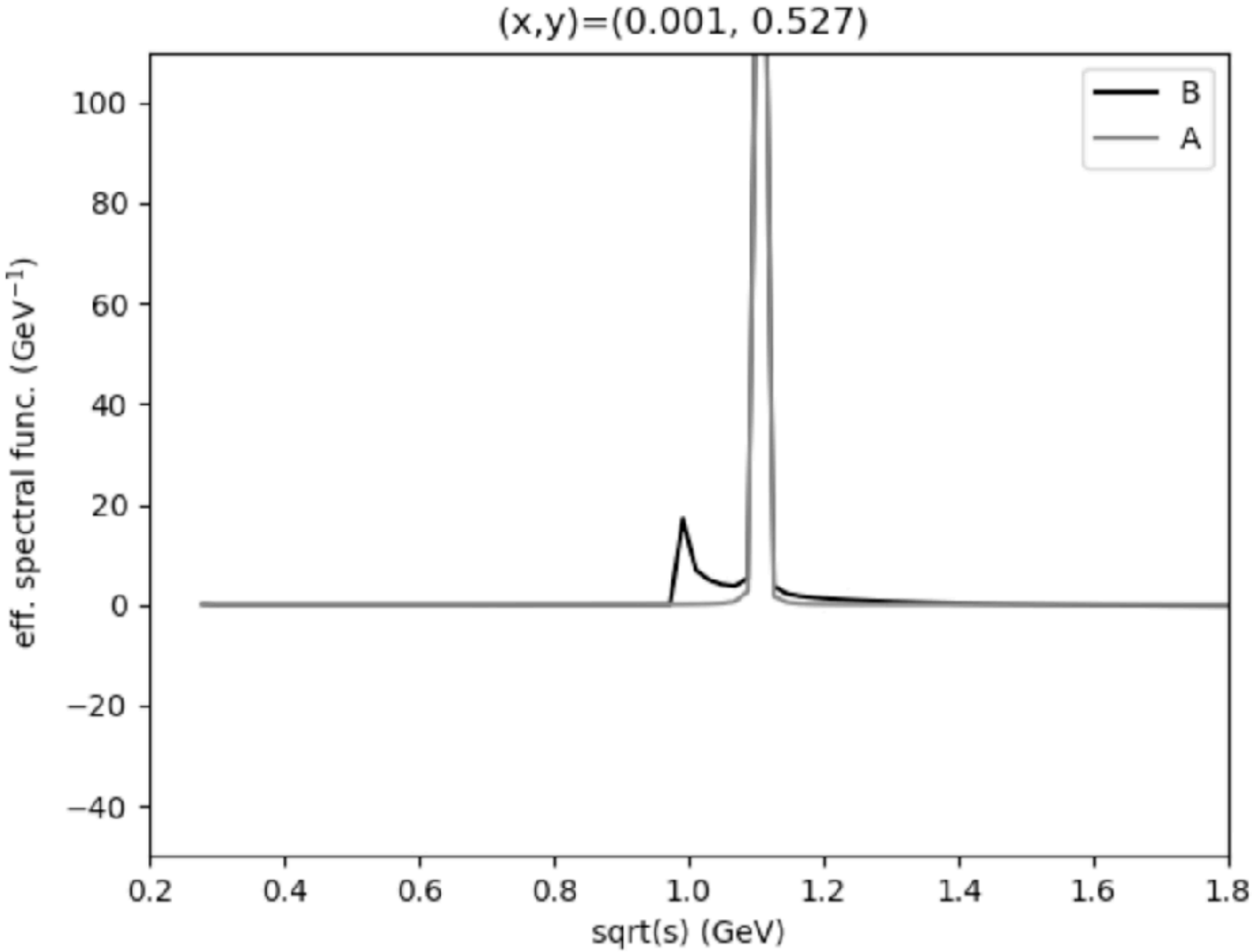


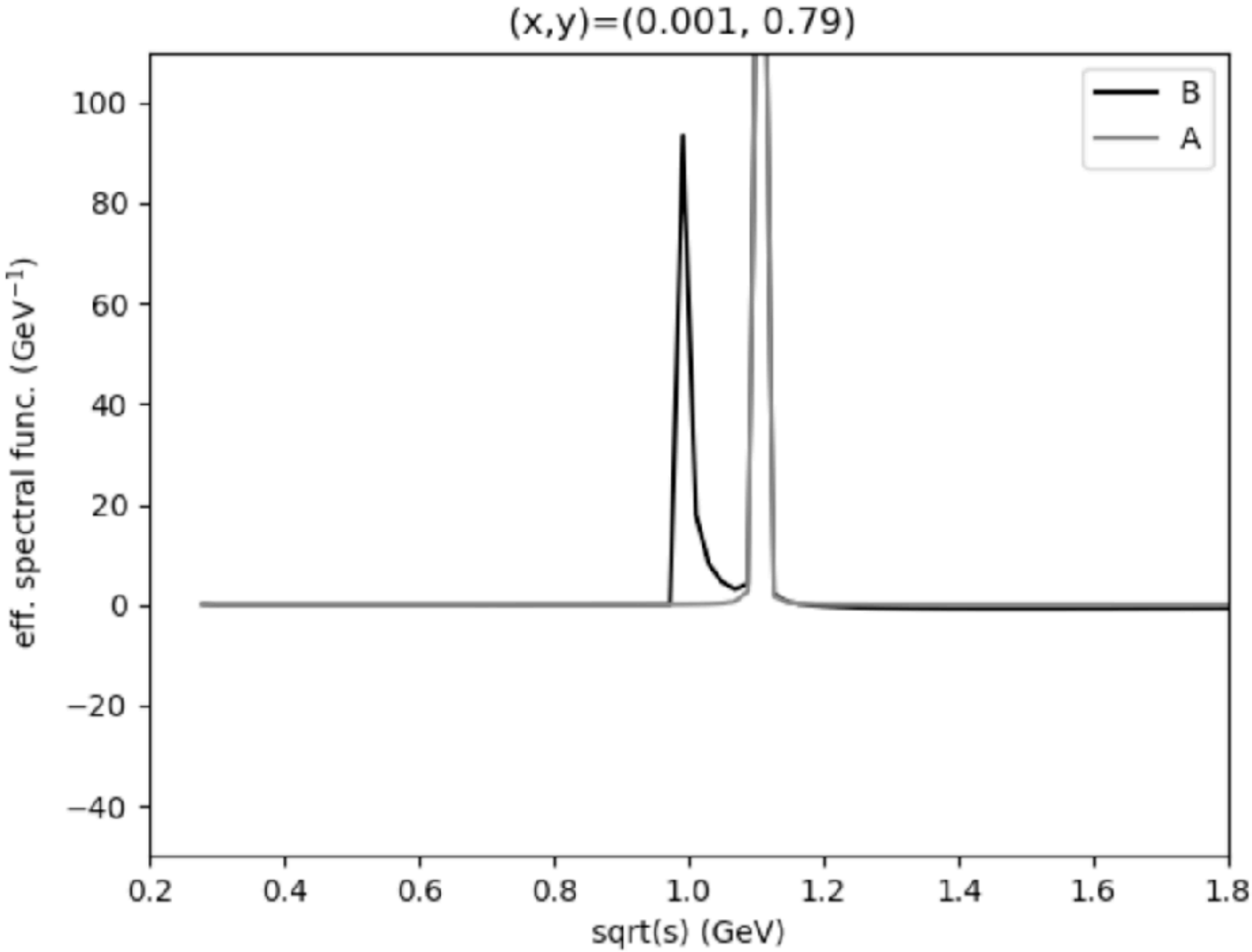
some resonances are more equal than others…
(sheet structures)

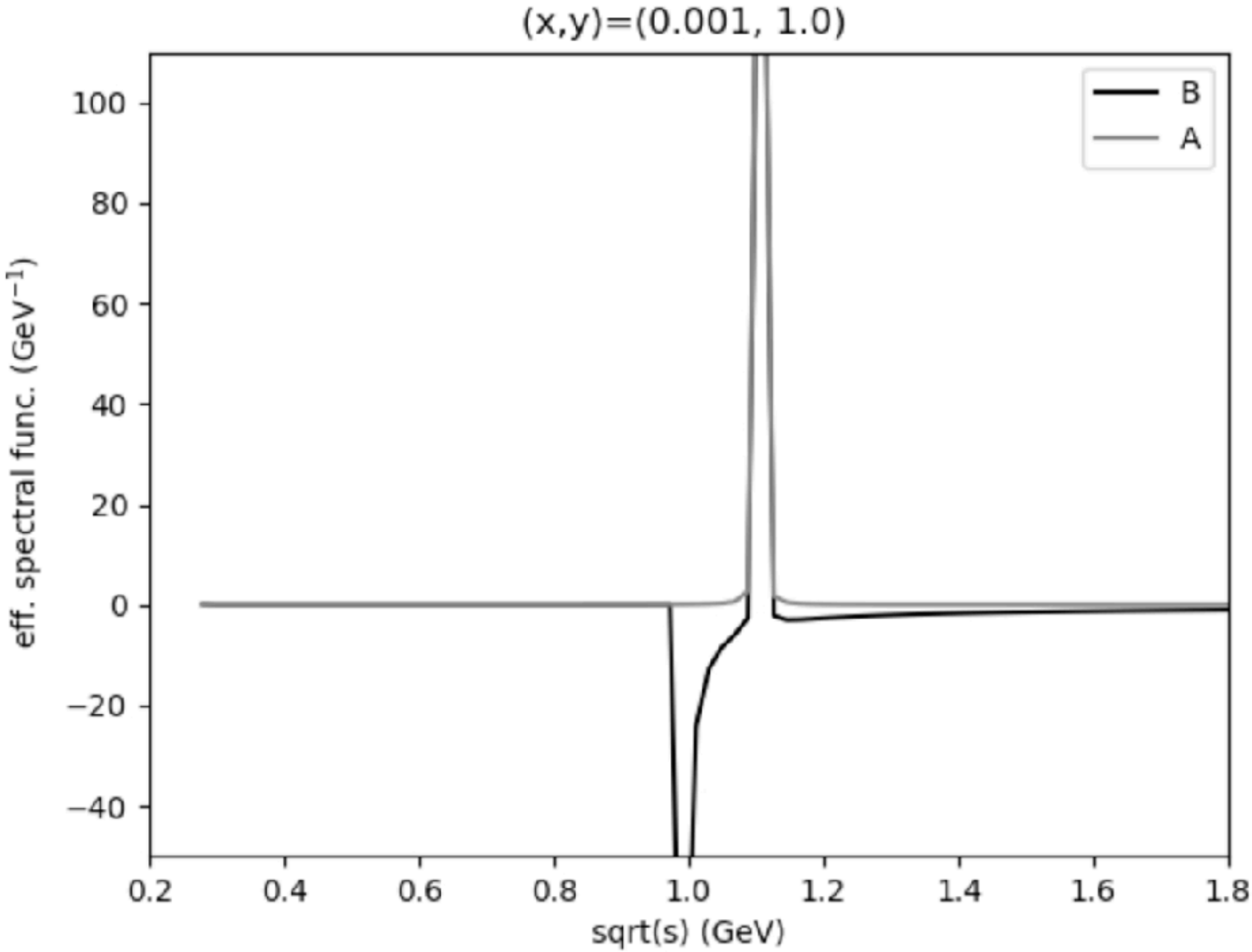


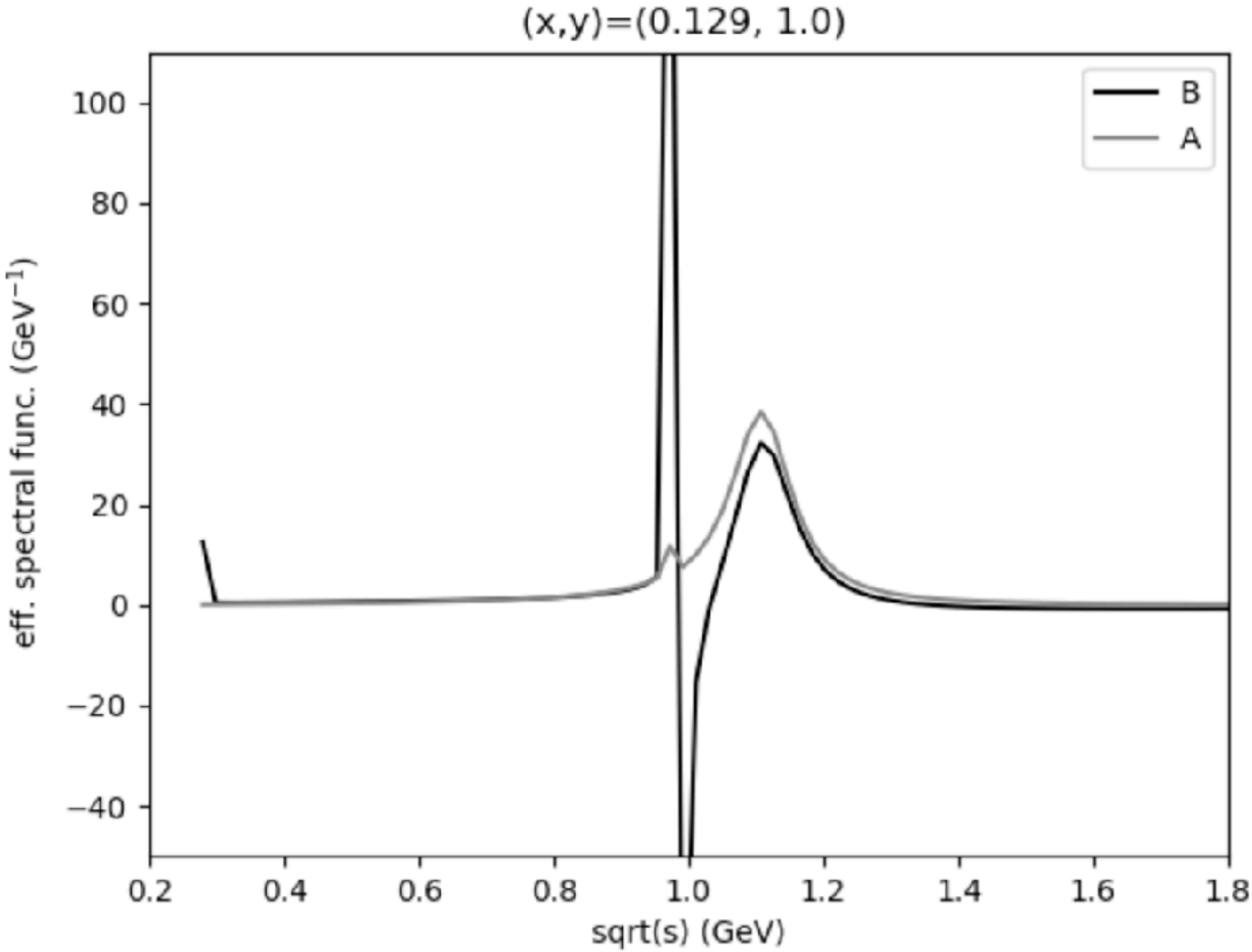


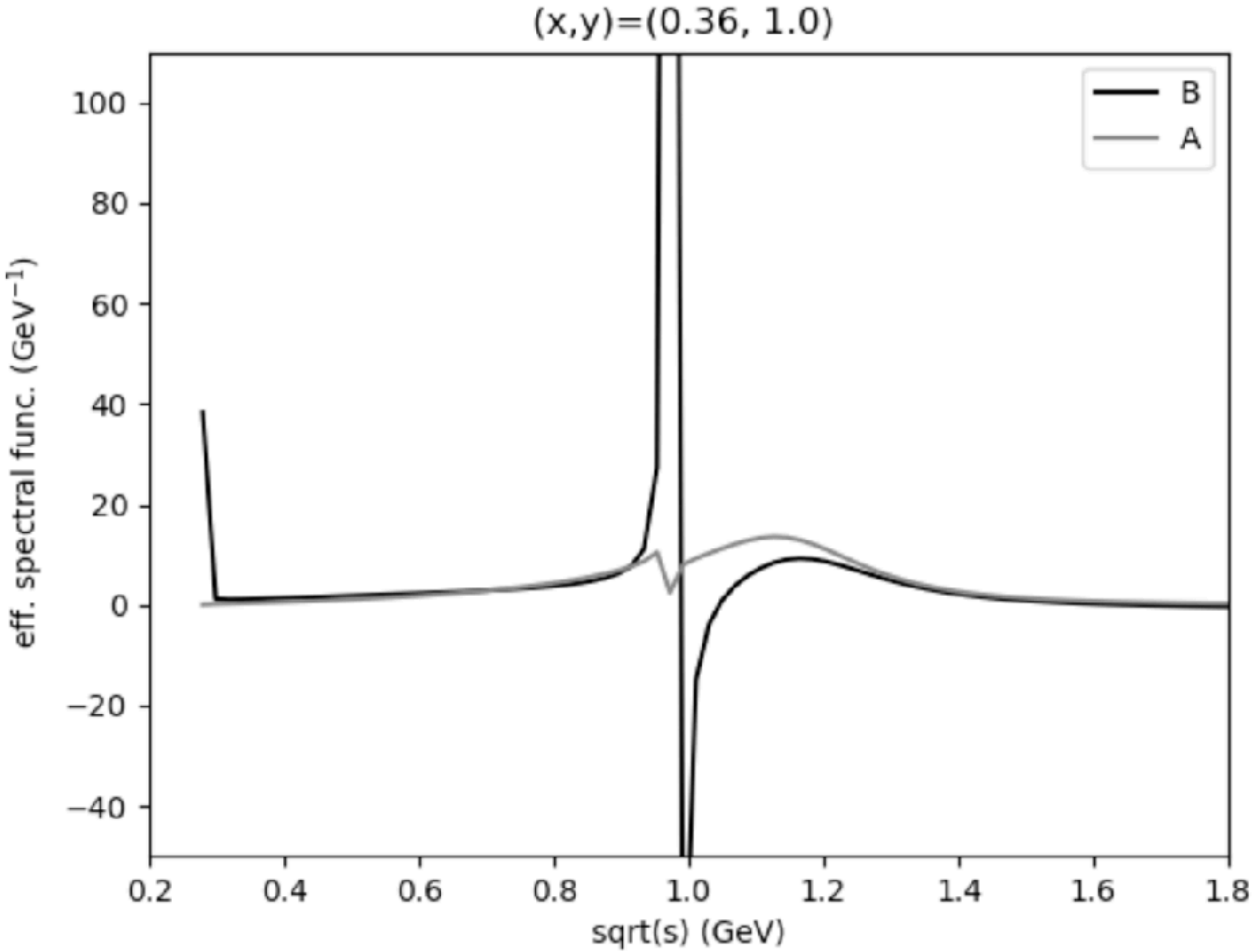
(x,y)=(0.001, 0.001)
B
A
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)
eff. spectral func. (GeV^{-1})

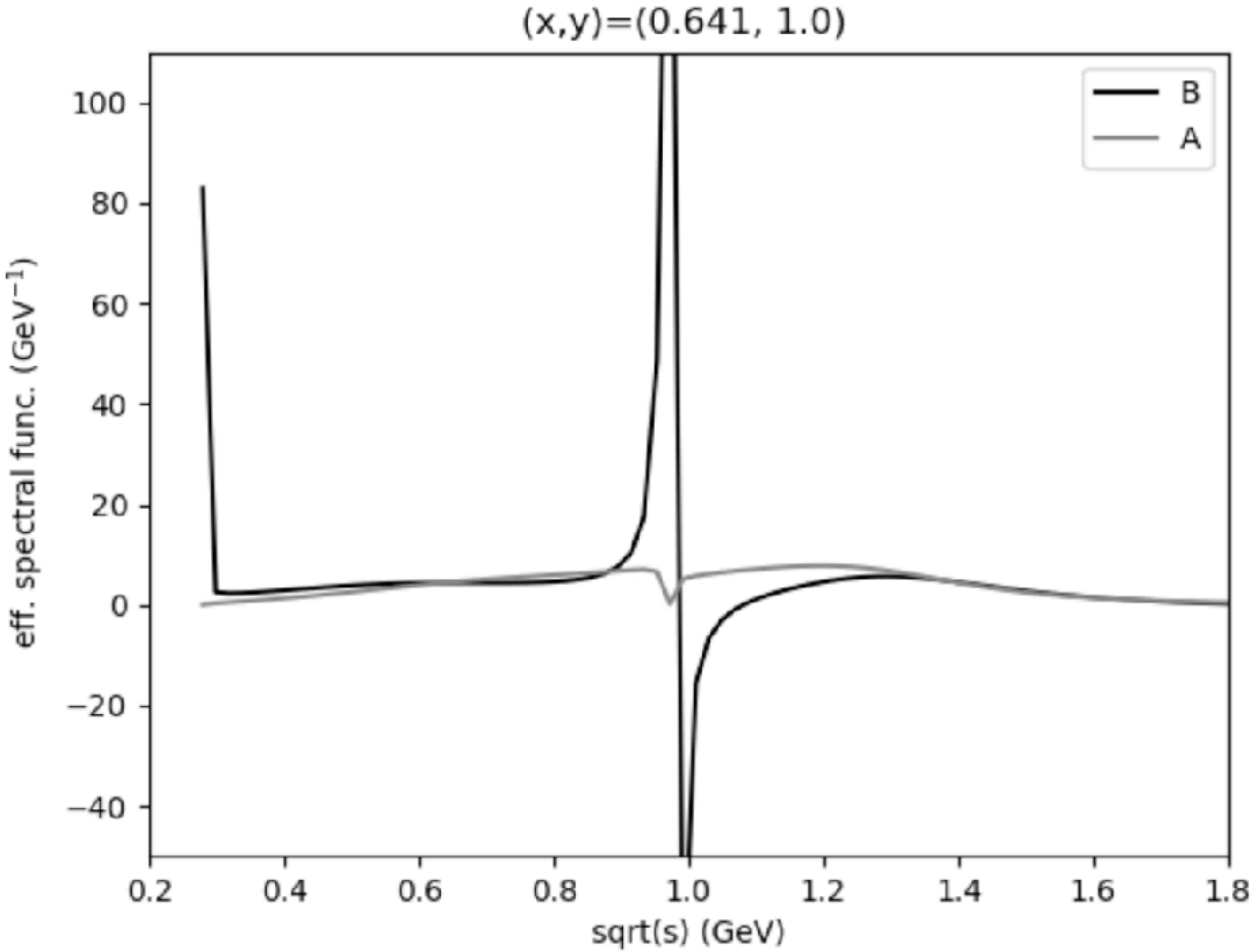

(x,y)=(0.001, 0.527)
B
A
eff. spectral func. (GeV^{-1})
sqrt(s) (GeV)
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

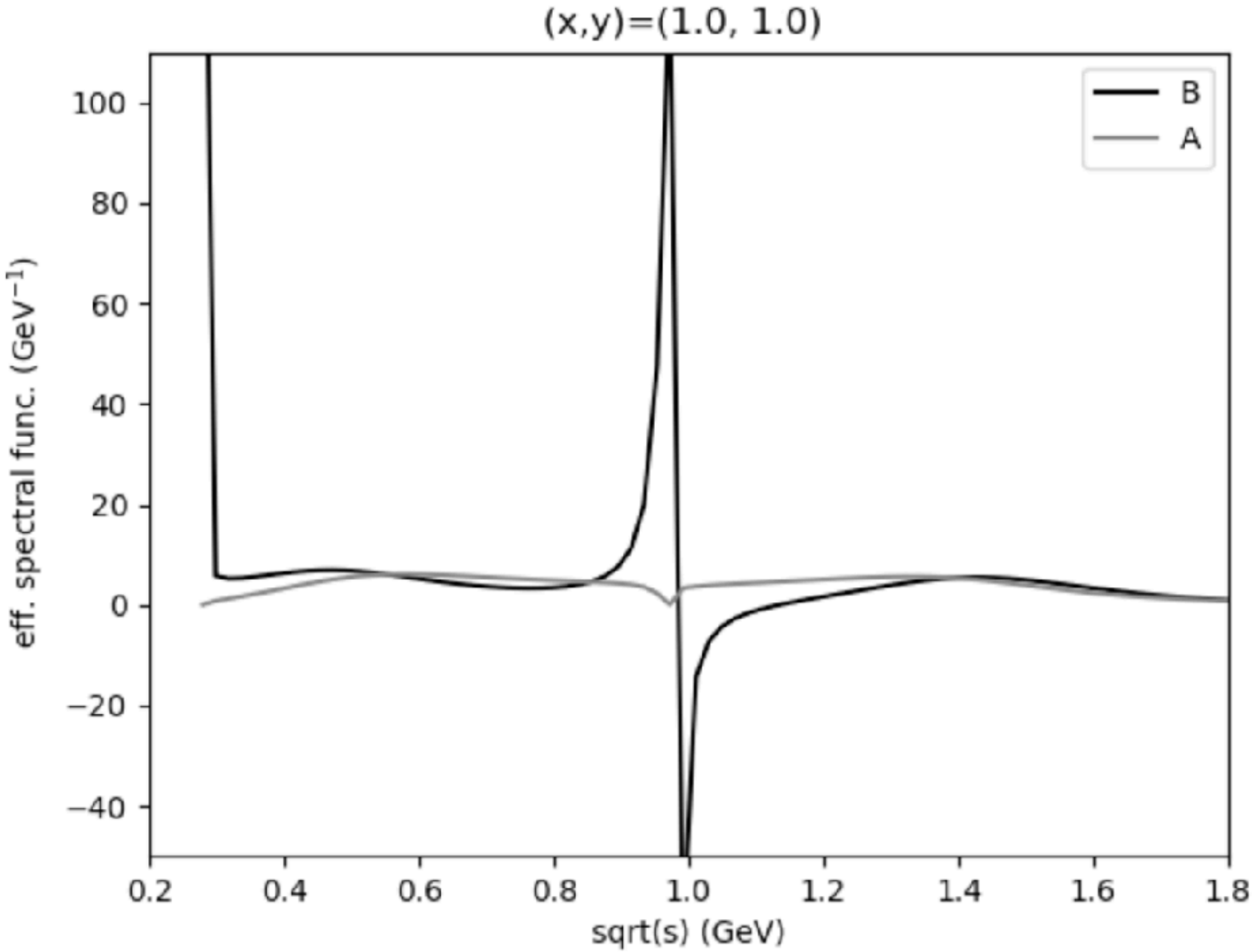

(x,y)=(0.001, 0.79)
B
A
eff. spectral func. (GeV^{-1})
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)


(x,y)=(0.001, 1.0)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)


(x,y)=(0.129, 1.0)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)


(x,y)=(0.36, 1.0)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)

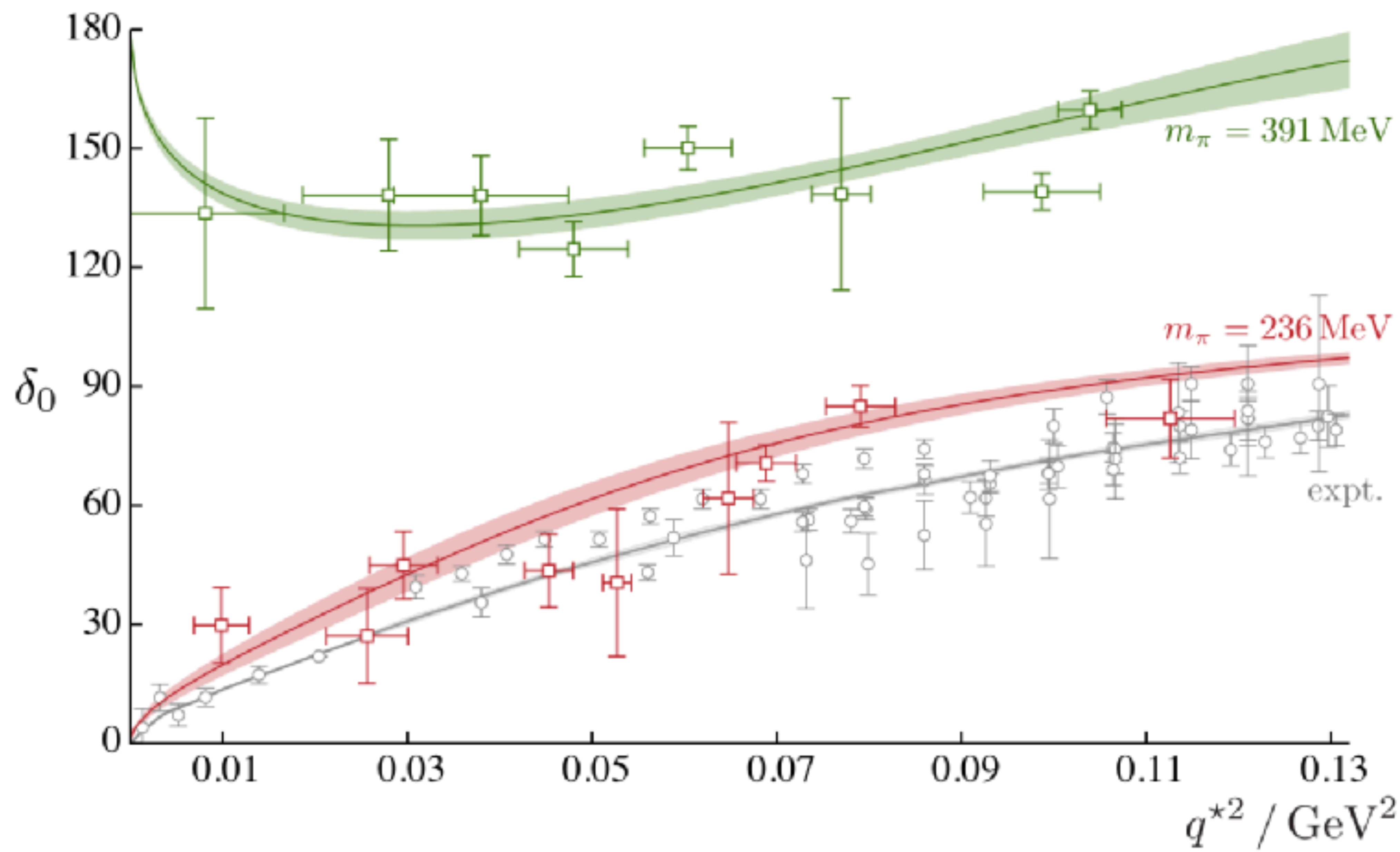

(x,y)=(0.641, 1.0)
B
A
eff. spectral func. (GeV^{-1})
sqrt(s) (GeV)
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8



(x,y)=(1.0, 1.0)
B
A
eff. spectral func. (GeV^{-1})
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)

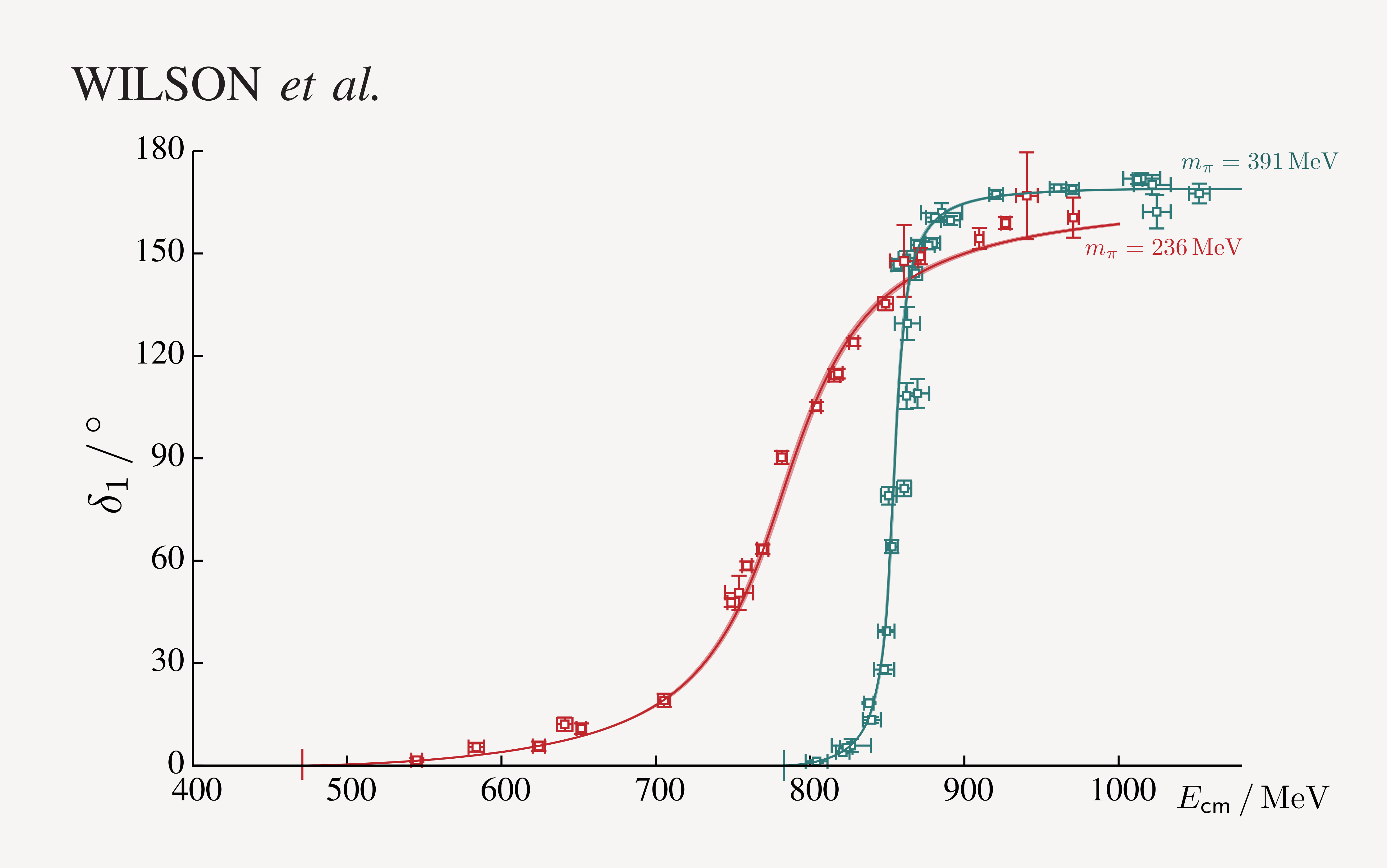
LATTICE COMPUTATIONS ON PHASE SHIFT

deuteron physics?



R. A. Briceno, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

LATTICE COMPUTATIONS ON PHASE SHIFT



NO SERIOUS MESON SPECTROSCOPY
WITHOUT SCATTERING*

GEORGE RUPP

CFIF, Instituto Superior Técnico, Universidade de Lisboa, 1049-001, Portugal

EEF VAN BEVEREN

CFC, Departamento de Física, Universidade de Coimbra, 3004-516, Portugal

SUSANA COITO

Institute of Modern Physics, CAS, Lanzhou 730000, China

(Received January 25, 2015)

CONTINUUM

$$Z = \sum_{\alpha=B,M} \langle\alpha|e^{-\beta H}|\alpha\rangle$$

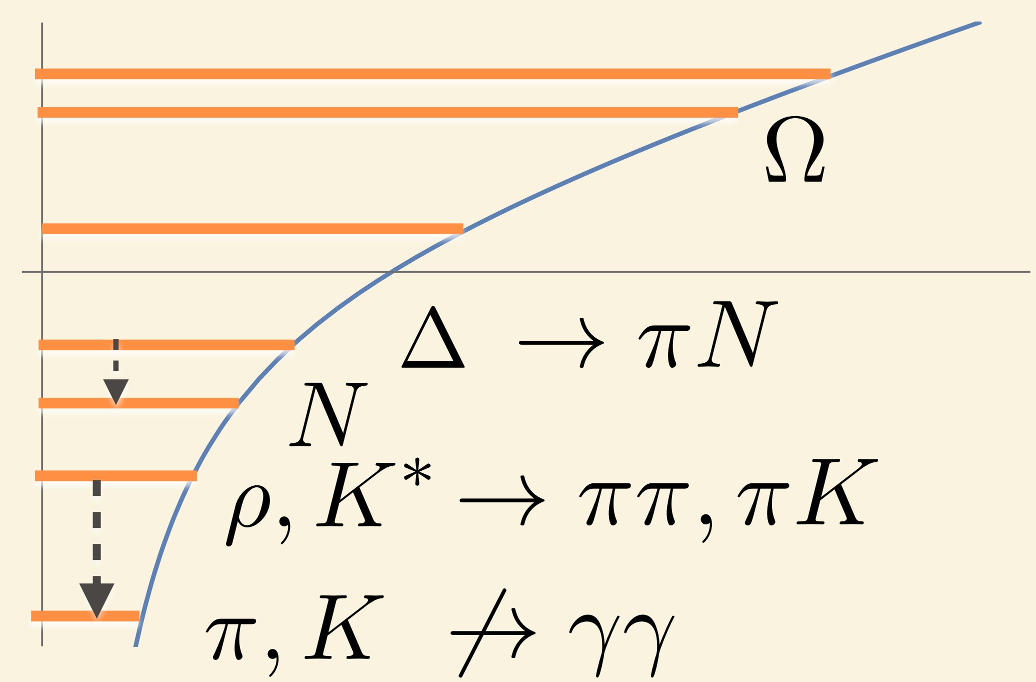
meson loops effects:

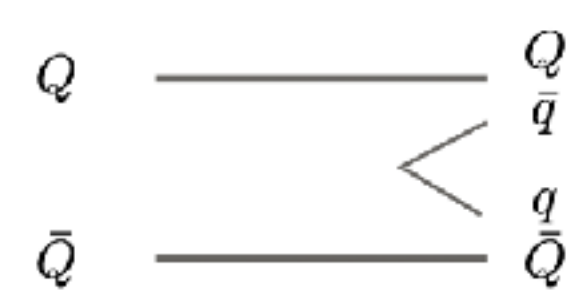
shift in hadron masses

Quark Model States:

mixing with continuum

QCD spectrum





indecently optimistic...

VOLUME 34, NUMBER 3 JULY, 1962

S-Matrix Theory of Strong Interactions without Elementary Particles*†

GEOFFREY F. CHEW
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1. INTRODUCTION

IN this paper I present an indecently optimistic view of strong interaction theory. My belief is that a major breakthrough has occurred and that within a relatively short period we are going to achieve a depth of understanding of strong interactions that a few years ago I, at least, did not expect to see within my lifetime. I know that few of you will be convinced by the arguments given here, but I would be masking my feelings if I were to employ a conventionally cautious attitude in this talk. I am bursting with excitement, as are a number of other theorists in this game.

tell me that this is a fetish, that field theory is an equally suitable language, but to me the basic strong-interaction concepts, simple and beautiful in a pure *S*-matrix approach, are weird, if not impossible, for field theory. It must be said, nevertheless, that my own awareness of these concepts was largely achieved through close collaboration with three great experts in field theory, M. L. Goldberger, Francis Low, and Stanley Mandelstam. Each of them has played a major role in the development of the strong interaction theory that I describe,[1] even though the language of my description may be repugnant to them. Murray Gell-Mann, also, although he has not actu-

PARTICLES AS S–MATRIX POLES; HADRON DEMOCRACY *

satisfy unitarity. There is no "reason" for any others. Similarly, as Feynman and Heisenberg have both emphasized, there is no reason why some particles should be on a different footing from others. The elementary particle concept is unnecessary, at least for baryons and mesons.

The second assumption may turn out to be closely related to the first, perhaps even a consequence, but

but some are more equal than the others?

Geoffrey F. Chew

Chew at his California home on July 2014

Born	June 5, 1924 Washington, D.C., United States
Died	April 12, 2019 (aged 94) Berkeley, California, United States
Nationality	American
Alma mater	University of Chicago
Known for	S-matrix theory, bootstrap theory, strong interactions, Chew–Frautschi plot
Awards	Hughes Prize (1962) Lawrence Prize (1969) Majorana Prize (2008)
Scientific career	
Fields	Theoretical physics
Institutions	University of Illinois UC Berkeley
Doctoral advisor	Enrico Fermi
Doctoral students	David Gross John H. Schwarz John R. Taylor

THE END (1)

IN-MEDIUM EFFECTS

VACUUM PHYSICS?

Quantum statistical mechanics of gases in terms of dynamical filling fractions and scattering amplitudes

André LeClair

Newman Laboratory, Cornell University, Ithaca, NY, USA

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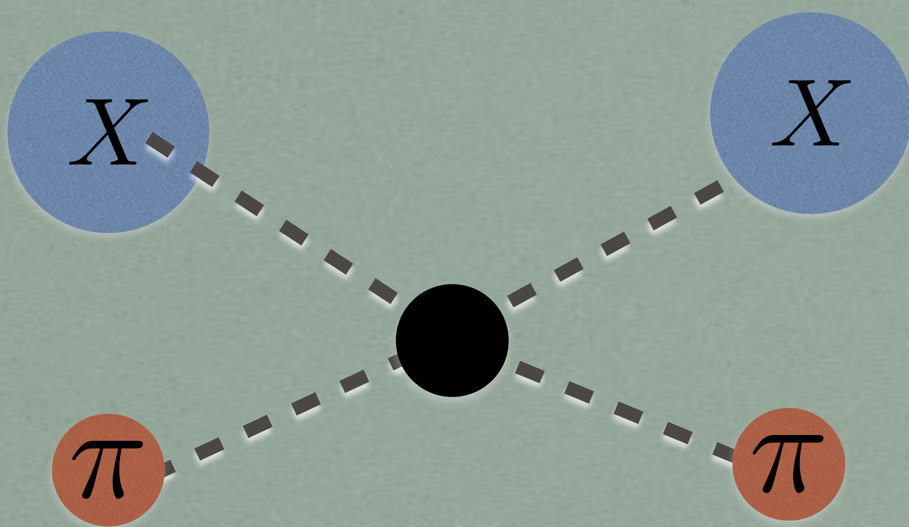
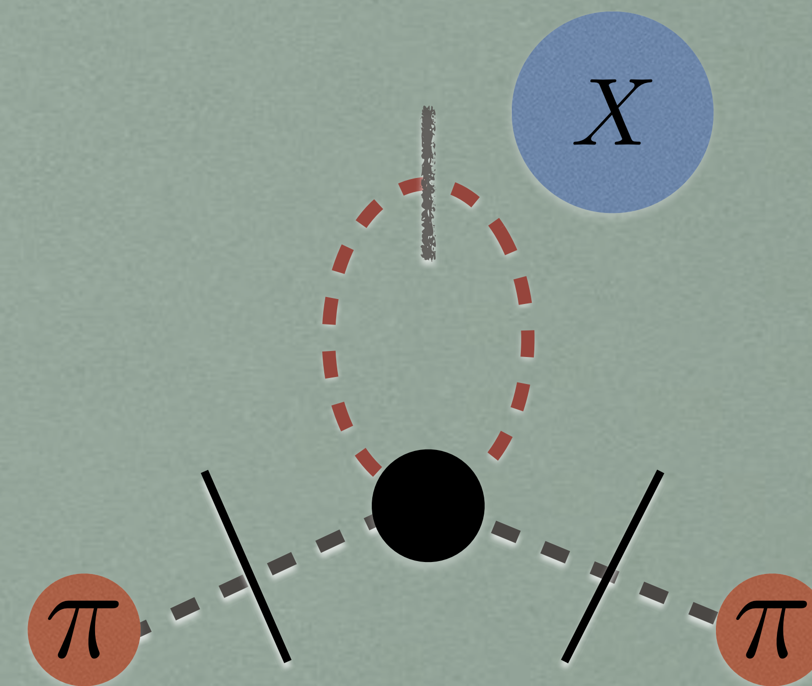
It helps to realize that at least in principle it is possible to decouple the zero temperature dynamics and the quantum statistical sums. The argument is simple: the computation of the partition function $Z = \mathrm{Tr}(e^{-\beta H})$ is in principle possible from the complete knowledge of the zero temperature eigenstates of the Hamiltonian H. In practice this is rather difficult and one resorts to perturbative methods such as the Matsubara method, which unfortunately entangles the zero temperature dynamics from the quantum statistical mechanics. However,

IN-MEDIUM EFFECTS FROM S-MATRIX





$$\Delta P = N_{\mathrm{th}}^{A}\, N_{\mathrm{th}}^{B} \times \frac{4\pi f}{2m_{\mathrm{red}}}.$$

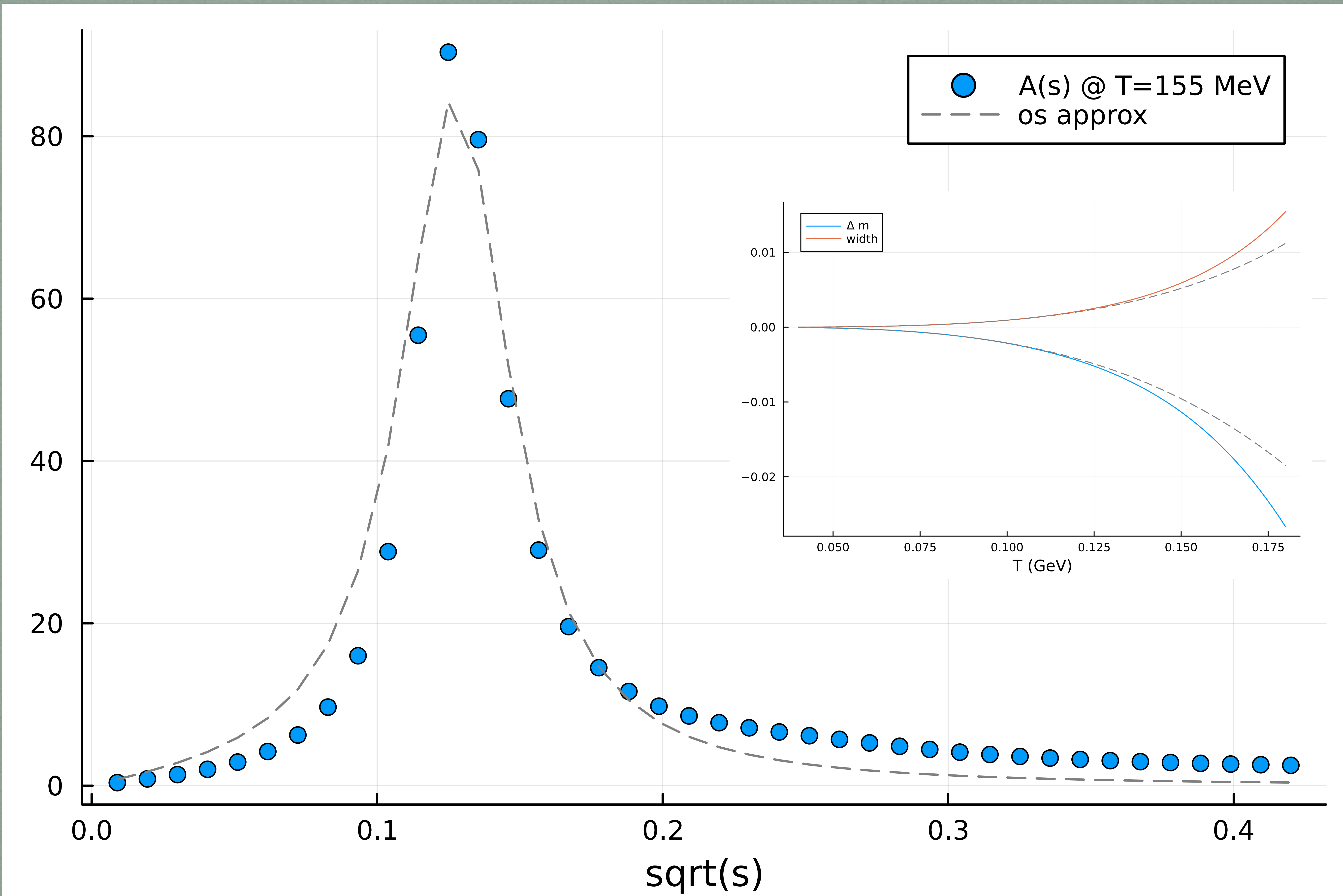
$$\Sigma_A(E_A) = \int \frac{d^3k_B}{(2\pi)^3}\frac{1}{2E_B}\, n_{\mathrm{th}}(E_B)\, T(AB \to AB).$$

$$\Delta m_A = \frac{1}{2E_A}\, \mathrm{Re}\, \Sigma_A(p)$$

$$\approx N_{\mathrm{th}}^{B} \times \frac{-4\pi f}{2m_{\mathrm{red}}}.$$

A. Schenk NPB 363(1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)



A. Schenk NPB 363(1991)

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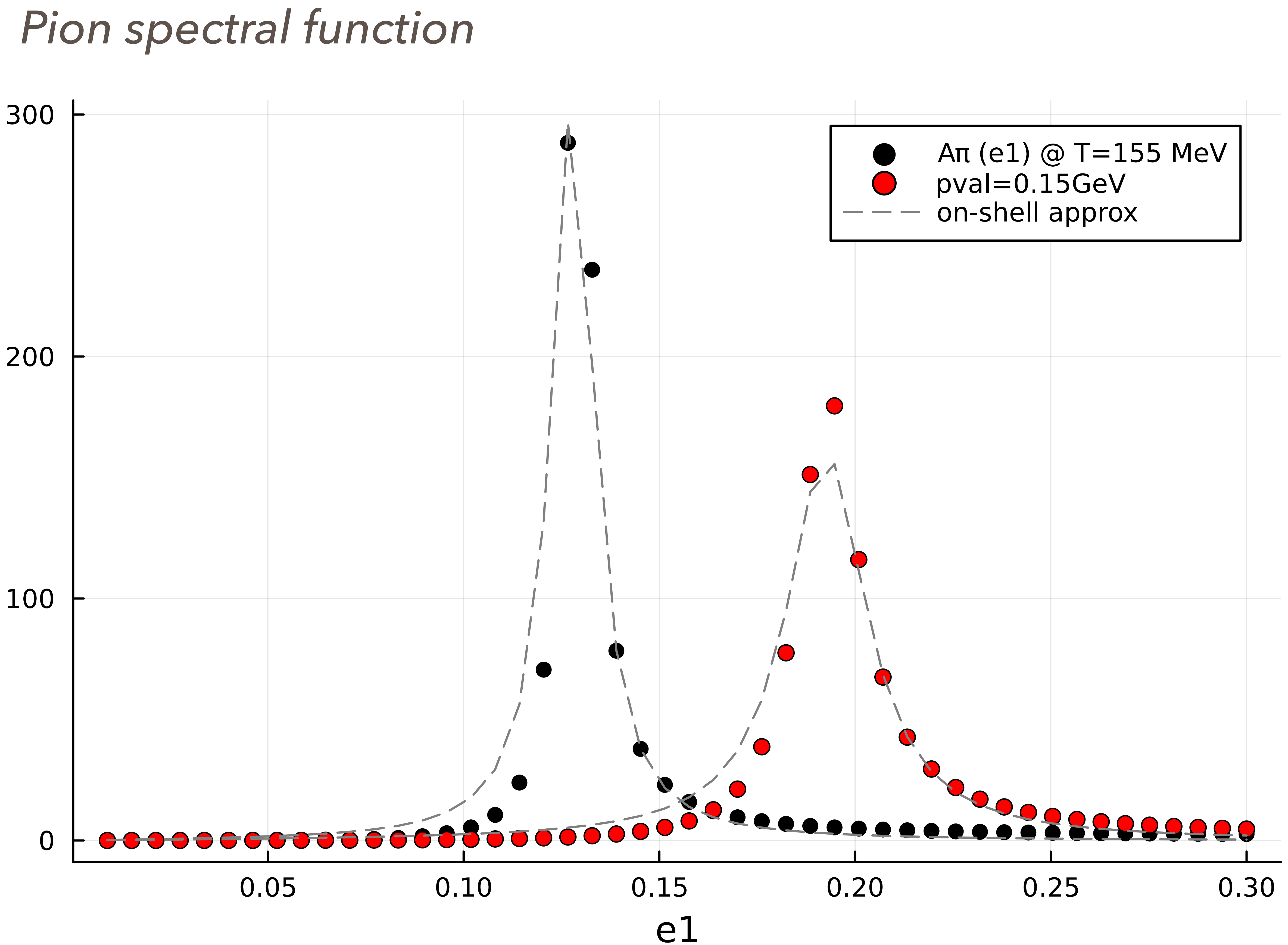
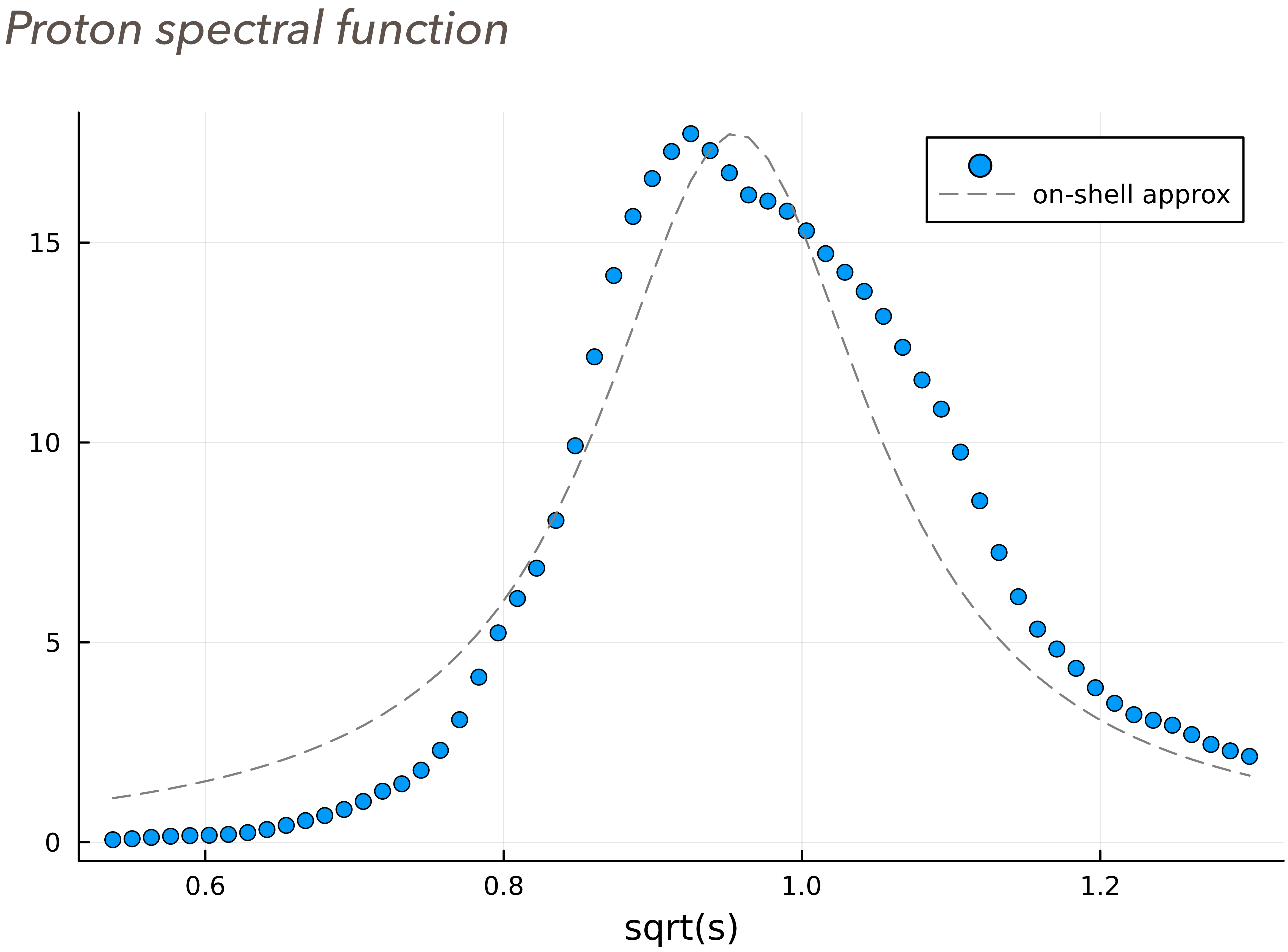
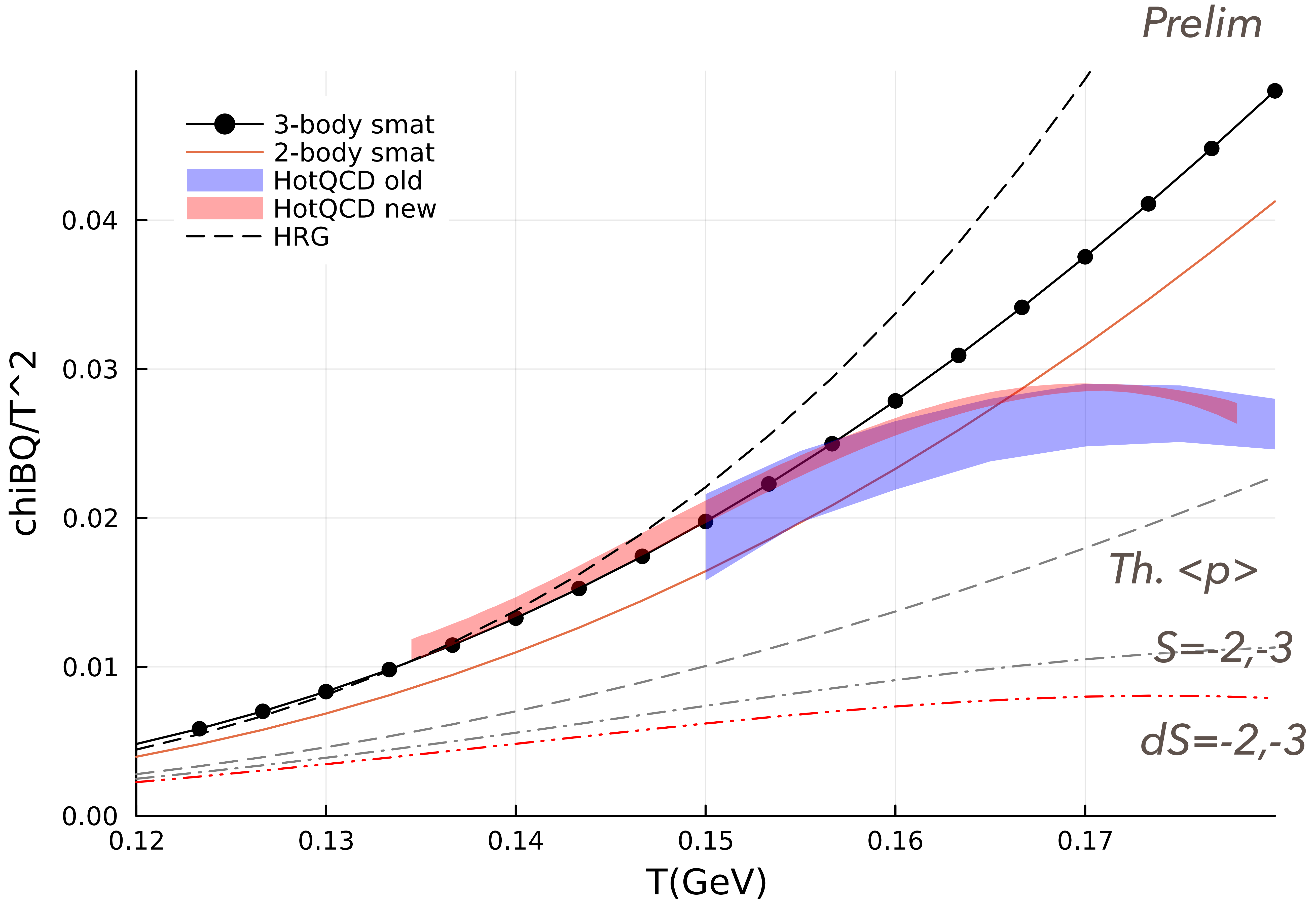


Pion spectral function
300
200
100
0
0.05
0.10
0.15
0.20
0.25
0.30
e1
Aπ (e1) @ T=155 MeV
pval=0.15GeV
on-shell approx

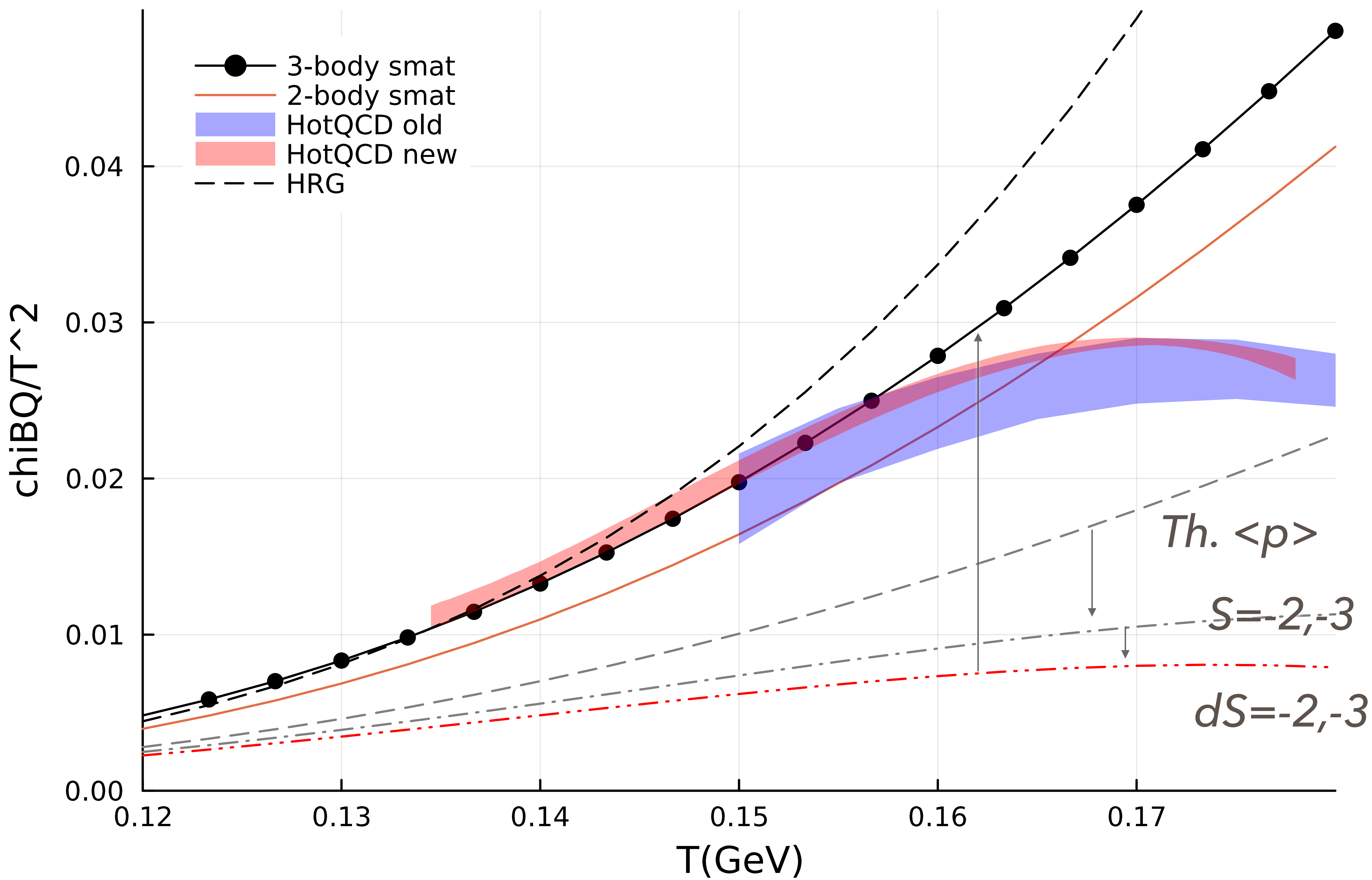

Proton spectral function
on-shell approx
0
5
10
15
0.6
0.8
1.0
1.2
sqrt(s)

A SMALL UPDATE



Prelim
3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
0.04
0.03
0.02
0.01
0.00
chiBQ/T^2
0.12
0.13
0.14
0.15
0.16
0.17
T(GeV)
Th. <p>
S=-2,-3
dS=-2,-3

Prelim



3-body smat
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HotQCD old
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S=-2,-3
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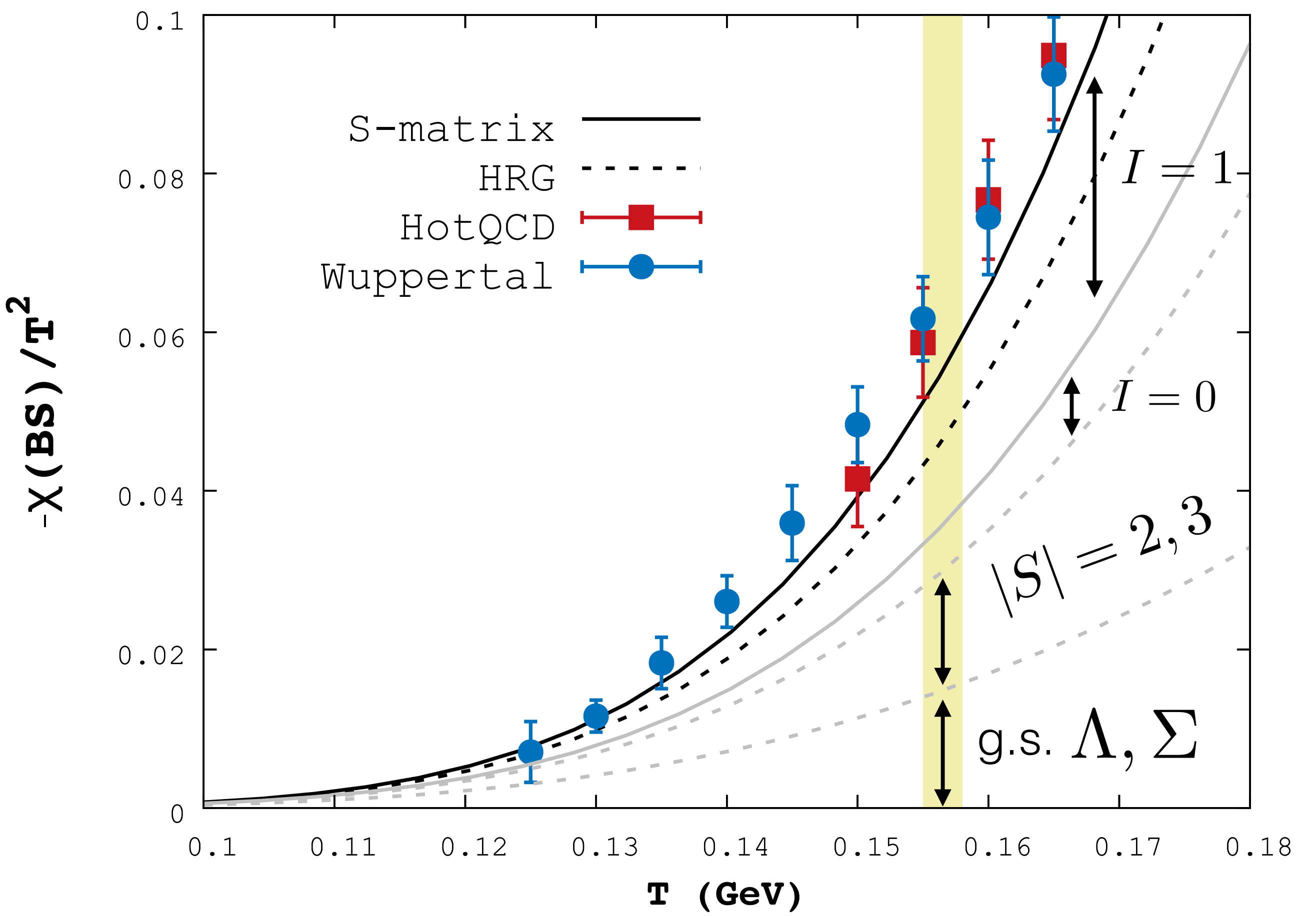
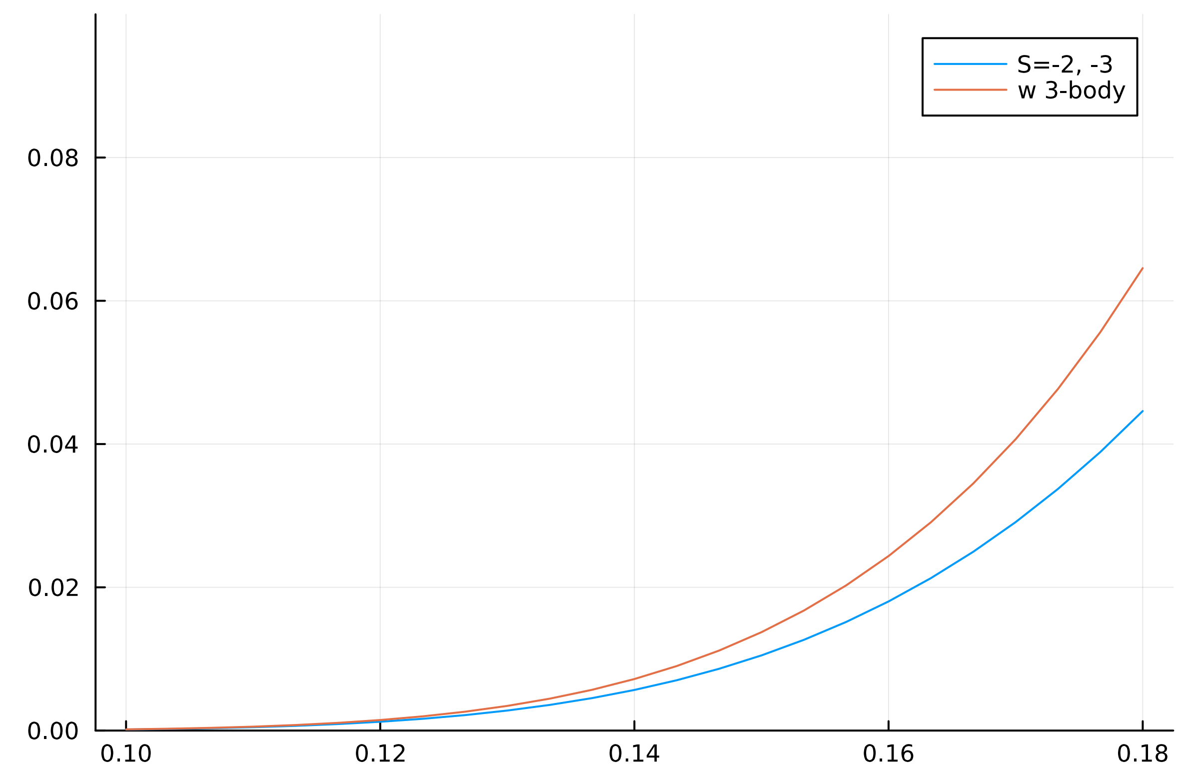


S-matrix
HRG
HotQCD
Wuppertal
$I = 1$
$I = 0$
$|S| = 2, 3$
g.s. Λ, Σ
-χ(BS)/T²
T (GeV)
0
0.02
0.04
0.06
0.08
0.1
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18



S=-2, -3
w 3-body
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18

THANK YOU!

THE END (2)