The gluon propagator at finite temperature and density

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The complex structure of strong interactions in Euclidean and Minkowski space

ECT* Trento, May 26-30 2025

Myself

- PostDoc researcher @ University of Catania (IT), research grant "Analytical approaches to non-perturbative QCD and properties of the Quark-Gluon Plasma" within NGEU funding (PRIN2022, Project "Advanced probes of the Quark Gluon Plasma", p.c. 2022SM5YAS)
- Joint PhD @ University of Catania & KU Leuven in 2023 under the supervision of F. Siringo and D. Dudal
- Research interests: dynamical mass generation in QCD, gluon condensation, confinement and deconfinement, hamiltonian formulation of gauge theories, analytic structure of the gluon propagator, ...

This talk

- Topics
  - Behavior of the Landau-gauge gluon propagator in pure Yang-Mills theory and in full QCD at finite temperature and finite density
  - Sensitivity of the gluon propagator to the deconfinement transition and QCD phase diagram
  - Analytic structure of the gluon propagator at finite temperature and density (work in progress)
- Methods: mainly, the screened massive expansion of QCD

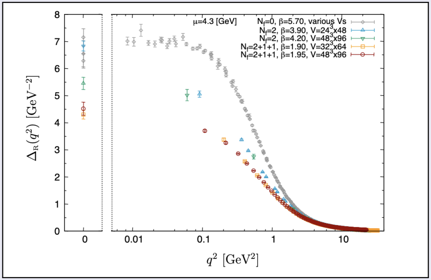
The screened massive expansion of pure Yang-Mills theory and full QCD

The screened massive expansion

F. Siringo, Nucl. Phys. B 907 (2016); F. Siringo and G. C., Phys. Rev. D 98 (2018)

Gluons in the infrared and perturbation theory (PT)

- Well-established (at least in this room): in linear covariant gauges (and beyond), the vacuum Euclidean gluon propagator acquires an infrared mass both in YM theory and in full QCD



Ayala, *et al.*, Phys. Rev. D86 (2012)

- IR QCD is not captured by ordinary PT *in general* (because of the Landau pole),
- but *specifically* ordinary PT is unable to describe mass generation: gluons remain massless to every finite order

The screened massive expansion

F. Siringo, Nucl. Phys. B 907 (2016); F. Siringo and G. C., Phys. Rev. D 98 (2018)

Gluons in the infrared and perturbation theory (PT)

- Yet, this does not imply that perturbative formulations of QCD are necessarily off-limits: a clear example is the Curci-Ferrari model and its many achievements
- Can we obtain a viable perturbation theory for QCD without changing the QCD action?
- The simplest way to obtain one is by adding a mass term for the gluons in its kinetic Lagrangian while subtracting the same term from the interaction Lagrangian to keep the total action unchanged $\Longrightarrow$ **screened massive expansion**

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_0^{\text{(std.)}} + \mathcal{L}_{\text{int}}^{\text{(std.)}} \to (\mathcal{L}_0^{\text{(std.)}} + \mathcal{L}_{m^2}) + (\mathcal{L}_{\text{int}}^{\text{(std.)}} - \mathcal{L}_{m^2})$$

The screened massive expansion

F. Siringo, Nucl. Phys. B 907 (2016); F. Siringo and G. C., Phys. Rev. D 98 (2018)

Set-up and Feynman rules

- Specifically, in the screened massive expansion we take as the order zero of QCD perturbation theory

$$S_0 = S_0^{\text{(std.)}} + \int \frac{d^d p}{(2\pi)^d}\, \frac{m^2}{2}\, A^a_\mu t^{\mu\nu}(p) A^a_\nu$$

where $S_0^{\text{(std.)}}$ is the (standard) kinetic term of the Faddeev-Popov gauge-fixed action in a linear covariant gauge. This yields a zero-order gluon propagator $\Delta^{ab}_{\mu\nu}(p)$

μ, a  ν, b

$$= \delta^{ab}\left[\frac{-i t_{\mu\nu}(p)}{p^2 - m^2} + \frac{-i\xi\, \ell_{\mu\nu}(p)}{p^2}\right]$$

with a transverse mass equal to m^2 (ext. parameter) and a longitudinal component which is exact

The screened massive expansion

F. Siringo, Nucl. Phys. B 907 (2016); F. Siringo and G. C., Phys. Rev. D 98 (2018)

Set-up and Feynman rules

- At the same time, we take the interaction to be

$$S_{\text{int}} = S_{\text{int}}^{\text{(std.)}} - \int \frac{d^d p}{(2\pi)^d}\, \frac{m^2}{2}\, A^a_\mu t^{\mu\nu}(p) A^a_\nu$$

- This keeps the action unchanged while adding a new two-gluon, non-perturbative vertex to the Feynman rules, the gluon "mass counterterm" ($\leftarrow$ unrelated to renormalization)

μ, a  $\nu, b \quad = \quad -im^2\, t^{\mu\nu}(p)\, \delta_{ab}$

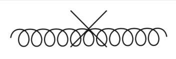
- The new vertex is non-perturbative in the sense that it is not proportional to any fixed power of the coupling g

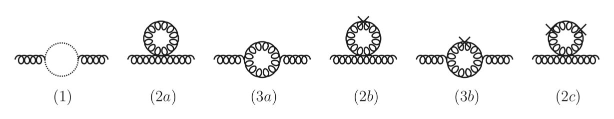
Set-up and Feynman rules

- Nonetheless, n-point functions are computed using the usual PT rules. E.g., for the 1-loop transverse gluon propagator

$$\Delta(p^2) = \frac{-i}{Z_A\, p^2 - m^2 - \Pi(p^2)}$$

one computes the diagrams





The pure Yang-Mills infrared gluon propagator at finite temperature

Thermal perturbation theory

- At finite temperature, one can use the Matsubara formalism to compute thermal averages of operators: define the theory in Euclidean space $(\tau, \mathbf{x})$ and integrate over $\tau \in [0, \beta = 1/T]$
- Because of Lorentz symmetry breaking, at $T \neq 0$ the Landau-gauge gluon propagator can be expressed in terms of two scalar functions separately depending on $\omega = \omega_n = 2\pi n T$ (Matsubara frequency) and on $|\mathbf{p}|$:

$$\Delta_{\mu\nu}(\omega, \mathbf{p}) = \Delta_T(\omega, |\mathbf{p}|)\, P^T_{\mu\nu}(\omega, \mathbf{p}) + \Delta_L(\omega, |\mathbf{p}|)\, P^L_{\mu\nu}(\omega, \mathbf{p})$$

where $P^{T,L}_{\mu\nu}(\omega, \mathbf{p})$ are (4-dimensionally transverse) 3-dimensionally transverse and longitudinal projectors

The pure Yang-Mills IR gluon propagator at finite T

F. Siringo and G.C., Phys. Rev. D 103 (2021); G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Screened massive expansion

- 1-loop Euclidean gluon propagator computed in the screened massive expansion:

$$p^2 \Delta_{T,L}(\omega, |\mathbf{p}|) = \frac{z_\pi}{\pi_{T,L}(\omega, |\mathbf{p}|; m^2, T) + \pi_0}\ , \qquad p^2 = \omega^2 + |\mathbf{p}|^2$$

- $\pi_{T,L}(\omega, |\mathbf{p}|; m^2, T)$ contain a vacuum and a thermal term,

$$\pi_{T,L}(\omega, |\mathbf{p}|; m^2, T) = \pi^{\text{(vac.)}}(p^2; m^2) + \pi^{\text{(therm.)}}_{T,L}(\omega, |\mathbf{p}|; m^2, T)$$

  with the first an analytic function and the second a 1D integral involving the Bose distribution

Screened massive expansion

$$\begin{aligned}
\Pi^{(a-c)}_{L,T}(p) &= \left[\frac{3p^4}{2m^4}-1\right] I^{00}_{L,T}(p) + \left[4+\frac{3p^4+8m^2p^2+4m^4}{2m^4}\right] I^{mm}_{L,T}(p) + \\
&- \left[\frac{3p^4+4m^2p^2+m^4}{m^4}\right] I^{m0}_{L,T}(p) + \frac{2p^2(p^2+2m^2)}{m^2}\, I^{mm}(p) + \\
&- \frac{2p^2(p^2+m^2)}{m^2}\, I^{0m}(p) - \left[\frac{2p^2+3m^2}{m^2}\right] J_m + \left[\frac{2p^2+m^2}{m^2}\right] J_0 + \\
&- \left[8m^2+\frac{(p^2+2m^2)^2}{m^2}\right] \partial I^{mm}_{L,T}(p) + \frac{(p^2+m^2)^2}{m^2}\, \partial I^{m0}_{L,T}(p) + \\
&- 2p^2(p^2+4m^2)\, \partial I^{mm}(p) + (p^2+m^2)^2\, \partial I^{m0}(p) + (p^2+3m^2)\, \partial J_m .
\end{aligned}$$

$$I^{\alpha\beta}(y,\omega) = \int_0^\infty \frac{x\mathrm{d}x}{8\pi^2 y} \left\{ \frac{n(\epsilon_{x,\alpha})}{\epsilon_{x,\alpha}}\ \mathrm{Re}\ L_\beta(\omega + i\epsilon_{x,\alpha};\, y, x) +\ \alpha \leftrightarrow \beta \right\}$$

$$L_\alpha(z;y,x) = \log \frac{z^2+\epsilon^2_{x+y,\alpha}}{z^2+\epsilon^2_{x-y,\alpha}}$$

Screened massive expansion

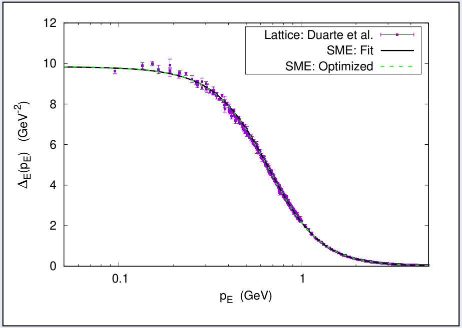
- 1-loop Euclidean gluon propagator computed in the screened massive expansion:

$$p^2\Delta_{T,L}(\omega,|\mathbf{p}|)=\frac{z_\pi}{\pi_{T,L}(\omega,|\mathbf{p}|;m^2,T)+\pi_0}\,,\qquad p^2=\omega^2+|\mathbf{p}|^2$$

- 3 free parameters: the gluon mass parameter m^2, an additive constant π_0 left over from field-strength renormalization and a multiplicative renormalization factor z_π
- The coupling α_s is reabsorbed into z_π and π_0. Choosing a renormalization scheme (e.g. MOM) fixes one of these

Screened massive expansion

- At $T = 0$ one can optimize π_0 based on principles of gauge invariance: $\pi_0 = -0.876$. m^2 is then the only free parameter. Excellent agreement is found with the IR lattice data[a]



[a] $m = 0.657$ GeV for Duarte, Oliveira and Silva, Phys. Rev. D 94 (2016)

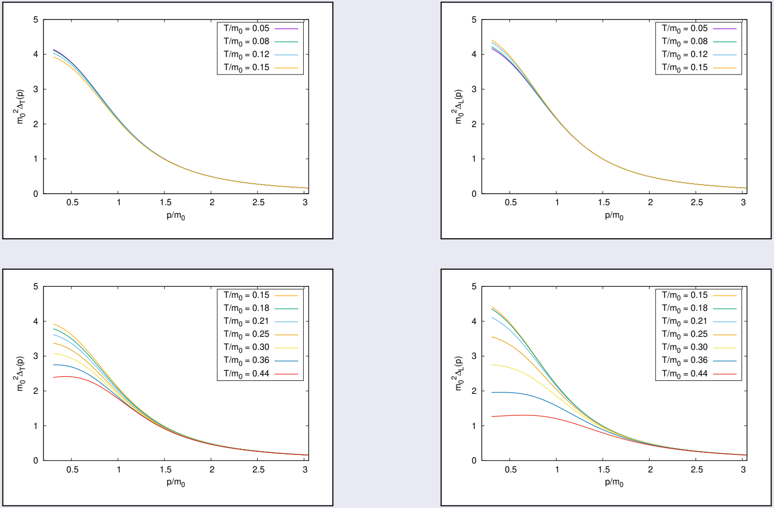
Screened massive expansion

- Both m^2 and π_0 can in principle depend on temperature
- At $T \geq 0$ we can adopt two approaches
  - use the (optimized) $T = 0$ parameters
  - fit temperature-dependent parameters from the lattice
- (Note: optimization is not excluded in principle at $T > 0$, but we never attempted it)

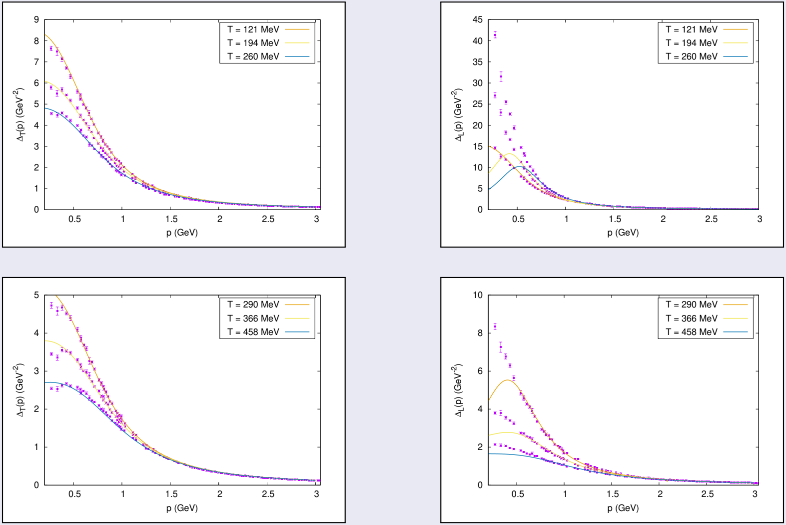
The pure Yang-Mills IR gluon propagator at finite T

F. Siringo and G.C., Phys. Rev. D 103 (2021); G.C. and F. Siringo, Phys. Rev. D 111 (2025)

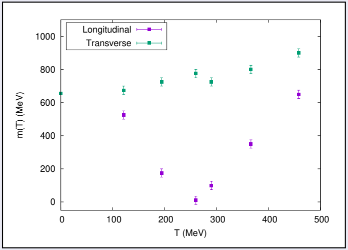
Screened massive expansion: parameters fixed from $T = 0$ $(\omega_n = 0)$

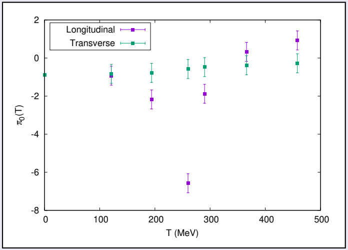


Screened massive expansion: parameters from the lattice ($\omega_n = 0$)



Screened massive expansion: parameters from the lattice ($\omega_n = 0$)





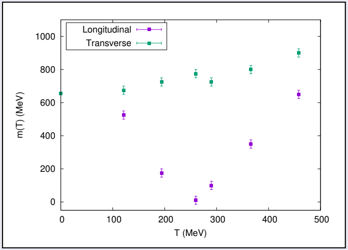
Silva, Oliveira, Bicudo, PRD 89 (2014)

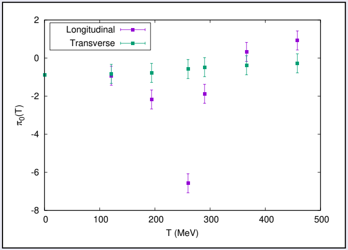
- The transverse fit is quite good up to $T = 458$ MeV, the longitudinal one isn't below $|\mathbf{p}| \approx 0.7$ GeV around $T \approx T_c$
- Need to fit the parameters separately for the two components
- Possible explanations: needs separate mass parameters $m^2_{T,L}(T)$ for the two components, Polyakov loop unaccounted for, etc.

The pure Yang-Mills IR gluon propagator at finite T

F. Siringo and G.C., Phys. Rev. D 103 (2021); G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Screened massive expansion: parameters from the lattice ($\omega_n = 0$)





Silva, Oliveira, Bicudo, PRD 89 (2014)

- The transverse mass parameter grows from 650 to 900 MeV from $T = 0$ to $T = 458$ MeV
- Consistent with a transverse gluon mass still influenced by the behavior of vacuum, enhanced by thermal effects
- We'll come back to this (hopefully) at the end

Screened massive expansion

To summarize:

- The qualitative behavior of both the components is already captured using fixed parameters
- The transverse propagator is in good quantitative agreement with the lattice when slowly increasing parameters are used
- The longitudinal propagator does not grow nearly as much as expected as $T \to T_c$, but it is still sensitive to the phase transition as observed on the lattice

The gauge sector at 1-loop seems to be enough under control for qualitative results to be obtained in full QCD

The full QCD infrared gluon propagator at finite temperature and density

Screened massive expansion: modeling IR quarks

- Obv., quark chiral symmetry breaking cannot be described by keeping the quark sector decoupled from the gluons'
- In the spirit of the screened massive expansion, what we would do in the quark sector is shift the expansion point of the quarks' perturbative series too:

$$\overline{\psi}(i\not{D} - m_q)\psi = \left[\overline{\psi}(i\not{\partial} - M_q)\psi\right] + \left[\overline{\psi}(g\not{A} + m_q - M_q)\psi\right]$$

- This strategy proved successfull in vacuum[a], but it is overshoot if the aim is to obtain qualitative results *in the gluon sector*

[a] See e.g. G.C., D. Rizzo, M. Battello and F. Siringo, PRD 104 (2021)

The full QCD IR gluon propagator at finite T and μ

G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Screened massive expansion: modeling IR quarks

- Instead, we employ the simplest possible model for IR quarks: we use an enhanced mass $M_q \approx 350 - 450$ MeV, as suggested by lattice, SDE, etc., and keep it constant at all momenta, temperatures and densities:

$$(\mathcal{L}_{\text{QCD}})_0 = (\mathcal{L}_{\text{YM}})_{0,\text{SME}} + \sum_q \overline{\psi}_q(\partial\!\!\!/ + M_q)\psi_q$$

  without further subtracting the quark mass

- We will discuss the validity of this approximation (and propose alternatives) esp. at large T and μ later

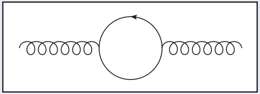
- We work with $N_f = 2 + 1$, $M_1 = 350$ MeV, $M_2 = 450$ MeV and *baryonic* chemical potential, i.e. $\mu_1 = \mu_2 = \mu := \mu_B/3$

Screened massive expansion: parameters

- 1-loop Euclidean gluon propagator, as before,

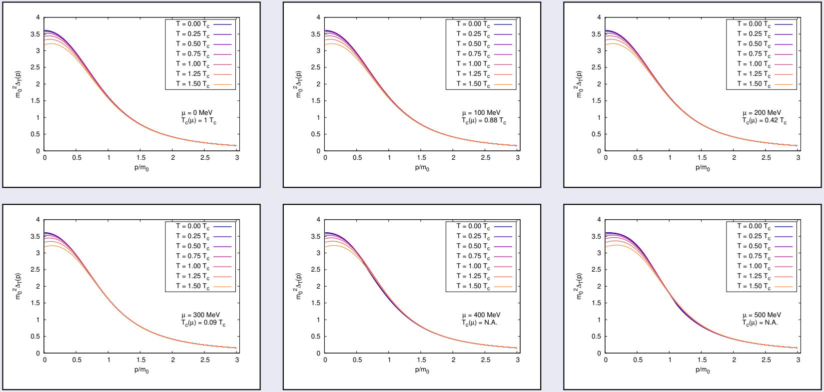
$$p^2\Delta_{T,L}(\omega,|\mathbf{p}|)=\frac{z_\pi}{\pi_{T,L}(\omega,|\mathbf{p}|;m^2,T,\mu)+\pi_0}$$

  where $\pi_{T,L}(\omega,|\mathbf{p}|;m^2,T,\mu)$ now also depends on chemical potential μ and contains the quark loop



- Same free parameters as pure YMT, but not enough data for a fit over the whole (T,μ) domain $\Longrightarrow$ use $T=0$ YMT parameters. Obj.: provide at least a qualitative picture

Screened massive expansion, results: transverse propagator

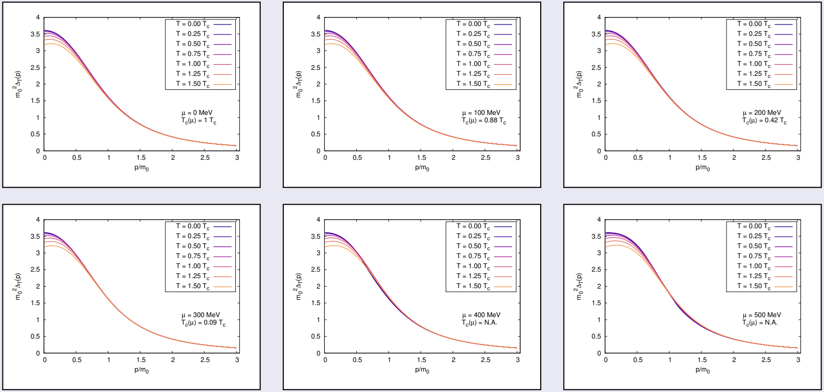


Depends only slightly on μ. Strictly decreases with T at $\mu = 0$

The full QCD IR gluon propagator at finite T and μ

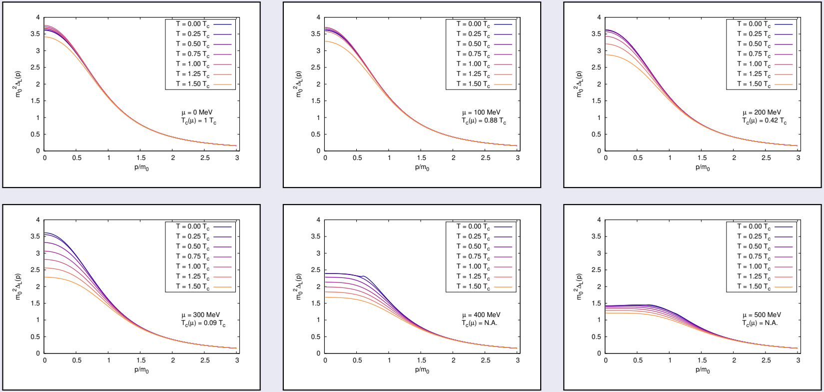
G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Screened massive expansion, results: transverse propagator



Expands to larger $|\mathbf{p}|$ with μ, slightly increasing @ interm. $|\mathbf{p}|$

Screened massive expansion, results: longitudinal propagator

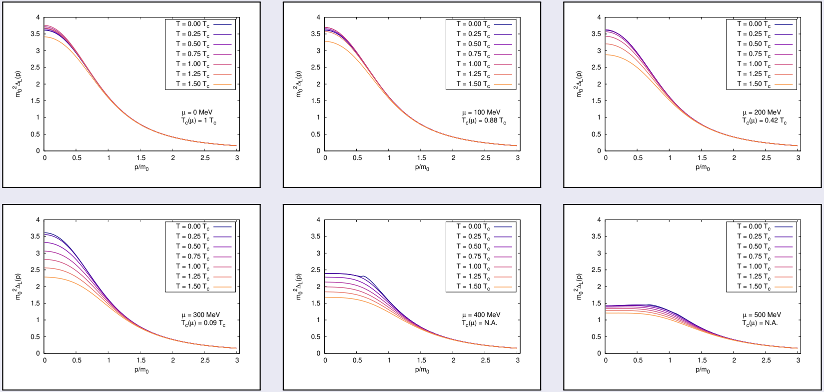


Marked dependence on μ, non-monotonic with T

The full QCD IR gluon propagator at finite T and μ

G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Screened massive expansion, results: longitudinal propagator

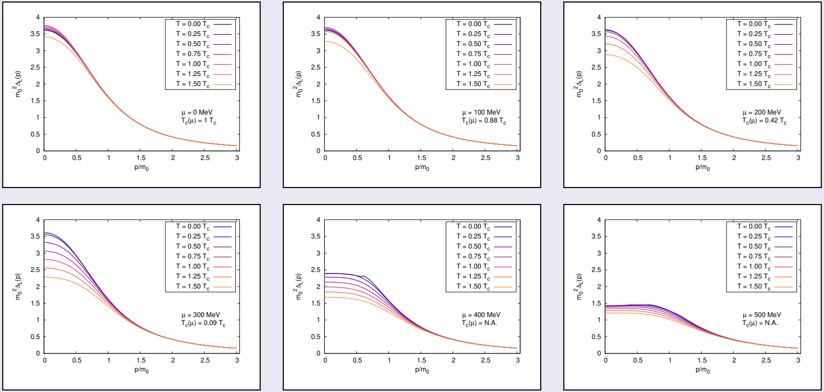


Same as pure YMT at low μ, decreasing with T for $\mu \gtrsim M_1$

The full QCD IR gluon propagator at finite T and μ

G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Screened massive expansion, results: longitudinal propagator

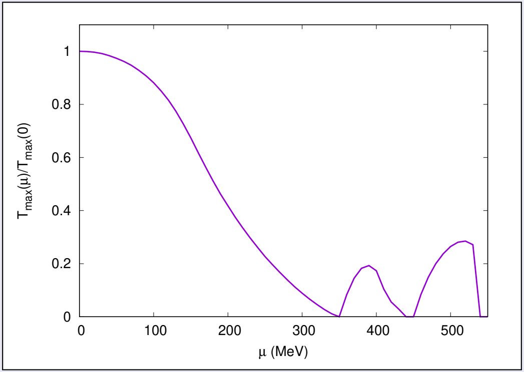


Crossing at low T, $|\mathbf{p}|$, and $\mu \gtrsim M_1, M_2$. 1-loop artefact? Feature?

Screened massive expansion, results: longitudinal propagator

- If we assume the longitudinal propagator in full QCD to be sensitive to the phase transition just like in pure YMT, we may draw a parallelism between the two and identify the turning point $T_{\text{max}}(\mu)$ with $T_c(\mu)$
- This should mostly be understood in a weak sense: we do not expect a sharp phase transition, but rather a crossover @ lower μ and M_q, so no sharp, “true” T_c
- Based on our pure YMT results, this can at most provide a *qualitative* estimate of the behavior of $T_c(\mu)$

Screened massive expansion, results: $T_{\text{max}}(\mu)$ @ $|\mathbf{p}| = 0$

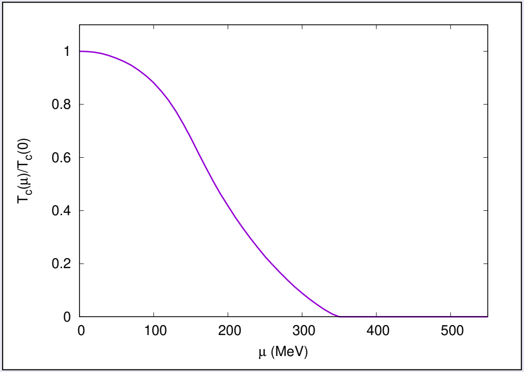


Humps on the right: low-T suppression at small $|\mathbf{p}|$ due to chem.pot. μ crossing mass thresholds $(M_1, M_2) = (350, 450)$ MeV

The full QCD IR gluon propagator at finite T and μ

G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Screened massive expansion, results: $T_{\text{max}}(\mu)$ @ $|\mathbf{p}| = 0$



Model-dependent: they disappear if (M_1, M_2) drop beyond the first portion of $T_{\text{max}}(\mu)$. Here $(M_1, M_2) = (125, 225)$ MeV

Screened massive expansion, results: $T_{\text{max}}(\mu)$ @ $|\mathbf{p}| = 0$

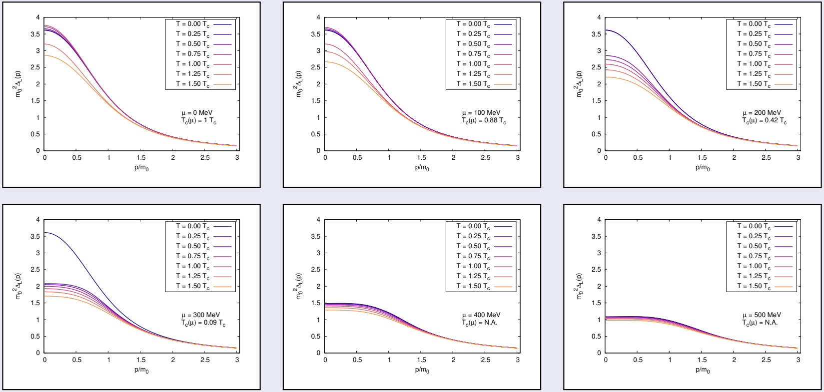
In more detail:

- Keeping (M_1, M_2) constant across all (T, μ) yields a long. propagator with a max wrt. T at fixed μ and $|\mathbf{p}| \implies T_{\text{max}}(\mu)$
- As μ increases, the (matter part of the) quark loops turns on at lower T's and makes $T_{\text{max}}(\mu)$ decrease until $T_{\text{max}}(\mu) = 0$ at $\mu = M_1$
- Then:
  - If we stick with the constant-mass model, $T_{\text{max}}(\mu)$ develops two humps
  - If we assume chiral symmetry restoration beyond $\mu = M_1$, we should decrease the masses $\implies$ the humps disappear

The full QCD IR gluon propagator at finite T and μ

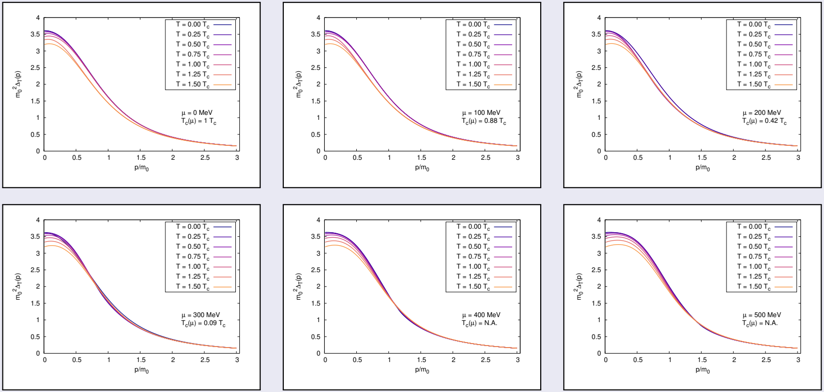
G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Screened massive expansion, results: long. prop. + sudden CS rest.



No crossing at low T. Increased separation between $T < T_c$ and $T > T_c$ curves because of mass jump

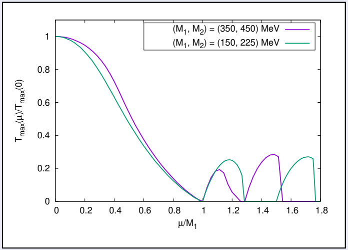
Screened massive expansion, results: trans. prop. w/ CS rest.

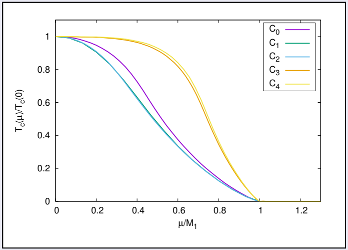


Here too, increased separation between $T < T_c$ and $T > T_c$ curves because of mass jump

Screened massive expansion, results: stability

Parameters arbitrary $\Longrightarrow$ test dependence of results





The qualitative features are parameter-independent as long as we use (T, μ)-independent parameters

Screened massive expansion, results: stability

Parameters arbitrary $\Longrightarrow$ test dependence of results

Configuration	M_1 (MeV)	M_2 (MeV)	m (MeV)
C_0	350	450	656
C_1	150	225	656
C_2	350	450	1000
C_3	1100	1300	656
C_4	350	225	200

The qualitative features are parameter-independent as long as we use (T, μ)-independent parameters

Screened massive expansion, fixed params results: summary

- At $\mu = 0$, the full QCD IR gluon propagator behaves as in pure Yang-Mills theory wrt. T: strictly decreasing transverse component, non-monotonic longitudinal component
- As μ increases, the (matter component of the) quark loops suppress the long. component at lower and lower T. Their influence on the transverse component is minor for $\mu \lessapprox m$
- The long. suppression eventually (at $\mu = M_1$) makes the long. propagator a decreasing function of T
- If the quark masses are kept constant, for $\mu > M_1, M_2$ the long. prop. is still slightly suppressed at low T and $|\mathbf{p}|$

Screened massive expansion, results: summary

- By comparison with pure Yang-Mills theory, one could interpret $T_{\text{max}}(\mu)$ as a transition temperature $T_c(\mu)$
- If the quark masses are decreased for $T > T_c(\mu)$, the long. propagator is decreasing with T for small $|\mathbf{p}|$ too. Additionally, the separation between $T < T_c(\mu)$ and $T > T_c(\mu)$ curves increases (realistically, faster suppression)
- The qualitative behavior described above does not depend on the free parameters of the expansion, as long as they are taken to be (T, μ)-independent

Analytic structure of the gluon propagator at finite temperature and density (in progress)

Analytic structure at finite T and μ

(G.C. and F. Siringo, in preparation)

Beyond Euclidean space

- The screened massive expansion is a modified perturbation theory $\Longrightarrow$ it provides analytic expressions up to a 1D integral at finite (T, μ) (Bose/Fermi distributions)
- Very easy to extend to the complex plane: replace $\omega \in 2\pi\mathbb{Z}$ (Matsubara freq.) with $\omega = p^4 \in \mathbb{C}$ in functions/integrands and if complex singularities are where expected that's the (unique) analytic continuation
- On indefinite-metric Hilbert spaces, propagators can only have two kinds of singularities:
  - a branch cut along $\omega \in i\mathbb{R}$ (Minkowski axis)
  - complex-conjugate singularities for $\omega \notin \mathbb{R}, i\mathbb{R}$ (e.g. c.c. poles)

Beyond Euclidean space

- Branch cut and c.c. singularities fully determine the propagator over the whole $\mathbb{C}$ via spectral representations. E.g., in the presence of simple c.c. poles $\{z_i(|\mathbf{p}|)\}$,

$$\Delta_{T,L}(z,|\mathbf{p}|)=\sum_i \frac{Z_i(|\mathbf{p}|)}{z-z_i(|\mathbf{p}|)}+\int_{-\infty}^{+\infty}\mathrm{d}\omega\ \frac{\rho_{T,L}(\omega,|\mathbf{p}|)}{\omega+iz}$$

where the spectral functions $\rho_{T,L}(\omega,|\mathbf{p}|)$ live on the branch cut,

$$\rho_{T,L}(\omega,|\mathbf{p}|)=-\frac{1}{\pi}\,\mathrm{Im}\left\{\Delta_{T,L}(i\omega+\epsilon,|\mathbf{p}|)\right\}$$

Analytic structure at finite T and μ

(G.C. and F. Siringo, in preparation)

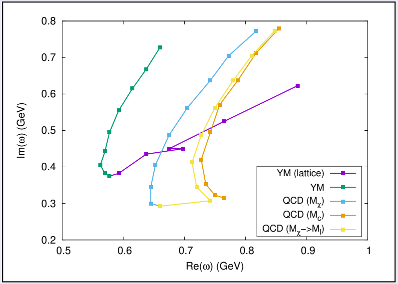
Screened massive expansion: objectives

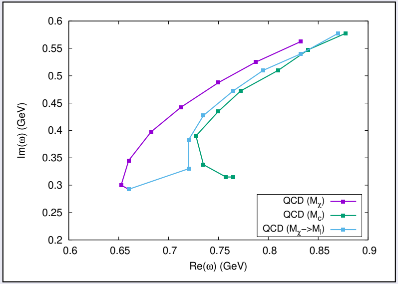
- For gluons in pure Yang-Mills theory we've already found:
  - c.c. poles for $T \geq 0$
  - positivity-violating spectral function for $T = 0$
- Compute the spectral functions for pure YMT at $T > 0$ and extend these results to full QCD at $(T, \mu) > 0$
- Explore whether the spectral functions have peaks, indicating e.g. quasi-particle-like behavior
- Start from $|\mathbf{p}| = 0$, where $\Delta_T = \Delta_L$ for all $(T, \mu) > 0$

Analytic structure at finite T and μ

(G.C. and F. Siringo, in preparation)

Preliminary results: poles



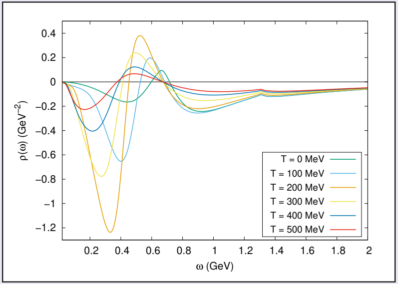


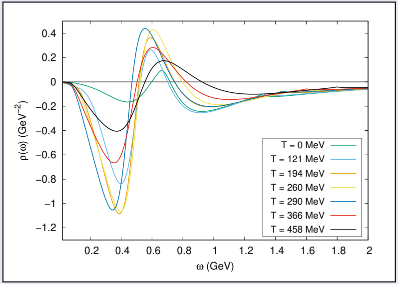
Left: $\mu = 0$, $T \in [0, 450]$ MeV. Right: $T = 0$, $\mu \in [0, 1000]$ MeV

Analytic structure at finite T and μ

(G.C. and F. Siringo, in preparation)

Preliminary results: YM spectral functions



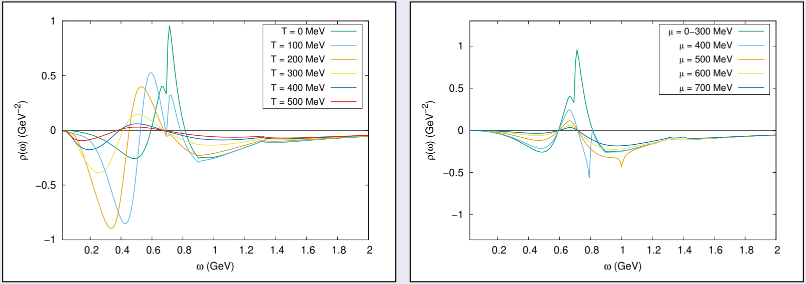


Left: fixed $T = 0$ parameters. Right: parameters from lattice (transverse prop., Silva, Oliveira, Bicudo PRD 89, 2014)

Analytic structure at finite T and μ

(G.C. and F. Siringo, in preparation)

Preliminary results: QCD spectral functions ($T = \mu = 0$ params.)



Left: $\mu = 0$. Right: $T = 0$. Constant $(M_1, M_2) = (350, 450)$ MeV.

Analytic structure at finite T and μ

(G.C. and F. Siringo, in preparation)

Preliminary results: QCD spectral functions ($T = \mu = 0$ params.)

- Behavior of c.c. poles in full QCD similar to pure YMT, except for a dependence on the quark masses at low T and μ
- Spectral functions still show positivity violation at $T, \mu \neq 0$
- No distinct peaks in spectral functions at $T, \mu > 0 \implies$ no strong indications of quasi-particle-like behavior in the deconfined phase, at least for (T, μ) accessible at colliders
- **HOWEVER**, this is in Landau gauge. What would happen in a “physical” gauges?

Analytic structure at finite T and μ

(G.C. and F. Siringo, in preparation)

Preliminary results: QCD spectral functions ($T = \mu = 0$ params.)

Work to do:

- Thorough exploration of parameter space
- Careful analysis of singularities near the Minkowski axis
- Go beyond $|\mathbf{p}| = 0$, where $\Delta_T \neq \Delta_L$ (future work?)

Thank you