Radiative corrections in the Refined Gribov-Zwanziger framework and its coupling to matter

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“The complex structure of strong interactions in Euclidean and Minkowski space”
ECT*

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1. Introduction

- **Yang-Mills theories/QCD become strongly coupled at large distances**
- **Computing observables in the IR requires sophisticated non-perturbative methods and/or effective models**
- **From a continuum quantum-field theoretic perspective, gauge-fixing is necessary in order to compute correlation functions**
- **Observables are encoded in gauge-invariant correlation functions [See Giovani's talk today!]**
- **Correlation functions of elementary fields are the seeds for the extraction of observables**

- **However, the underlying assumptions of the traditional gauge-fixing procedure (Faddeev-Popov) do not hold at strong coupling**
- **It does not select one representative for each gauge orbit (*gauge-fixing needs to be fixed!*)**
- **The existence of several configurations living on the same gauge orbit *and* satisfying the same gauge fixing condition is the (in)famous *Gribov problem***

1. Introduction

- **Taking into account the existence of such configurations might play an important role in the IR**
- **This has been (partially) addressed by Gribov and Zwanziger and, later on, refined by Dudal, Gracey, Sorella, Vandersickel and Verschelde *[(Refined) Gribov-Zwanziger (RGZ) setup]***
- **However, explicit computations of loop corrections within this scenario are still scarce and the issue how to include matter is debatable *[there is a fork in the road]***

- **This talk:**
  - A. **I will briefly review the RGZ framework**
  - B. **Present recent results on radiative corrections to two-point functions in RGZ**
  - C. **Debate on how matter can be consistently included**
  - D. **Present recent results on radiative corrections to the two-point functions of RGZ-matter systems**
  - E. **Collect concluding remarks**

Warning: This talk is totally Euclidean!

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No time!

2. Brief overview of the Gribov problem

Our starting point is the four-dimensional Euclidean Yang-Mills action and $SU(N)$ as gauge group

$$S_{\mathrm{YM}} = \frac{1}{4}\int \mathrm{d}^d x \; F^a_{\mu\nu} F^a_{\mu\nu}$$

$$a = 1, \dots, N^2 - 1$$

$$[T^a, T^b] = i f^{abc} T^c$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$$

coupling constant

structure constants

The action S_{YM} is invariant under

$$(A')^a_\mu T^a = U(A^a_\mu T^a) U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger$$

$$U = \exp\left(-ig\alpha^a T^a\right) \; \in \; SU(N)$$

For infinitesimal α^a,

$$A'^a_\mu \approx A^a_\mu - D^{ab}_\mu \alpha^b$$

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - g\, f^{abc} A^c_\mu$$

2. Brief overview of the Gribov problem

Formal Quantization

$$\mathcal{Z}_{\mathrm{YM}} = \int [\mathscr{D}A]\mathrm{e}^{-S_{\mathrm{YM}}[A]}$$

gauge Invariant

gauge Invariant

$$A \equiv \left\{ A^a_\mu(x)\,,\, x \in \mathbb{R}^d \right\}$$

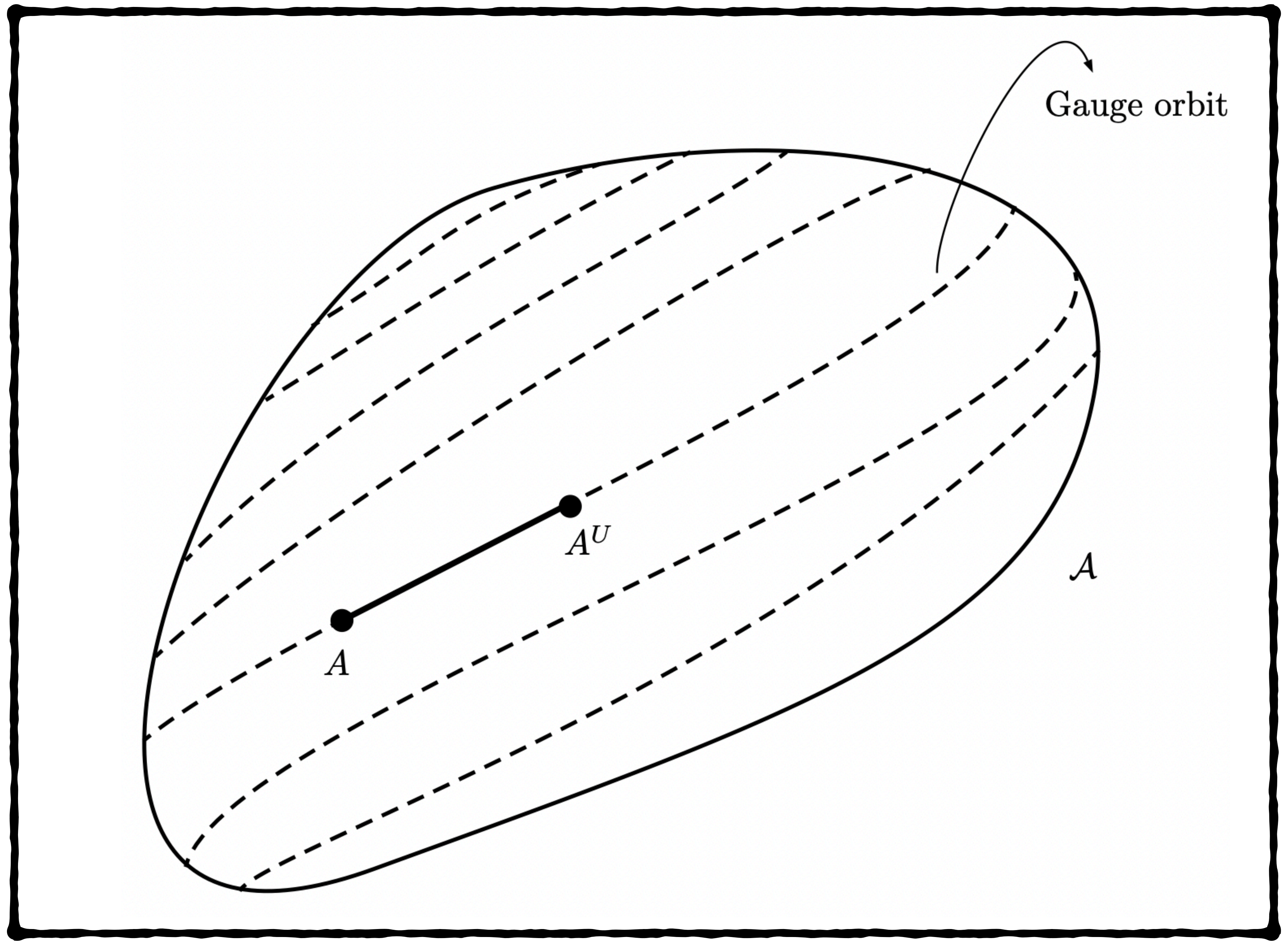
$$[\mathscr{D}A] = \prod_x \prod_\mu \prod_a \mathrm{d}A^a_\mu(x)$$

We should restrict the functional integration to the moduli space. Formally, it is defined by

$$\mathscr{A} = \{A\} \qquad \mathscr{G} = \{U\}$$

$$U = \left\{ U(x) \in SU(N)\,;\, x \in \mathbb{R}^d \right\}$$

$$\mathscr{C} = \frac{\mathscr{A}}{\mathscr{G}}$$



2. Brief overview of the Gribov problem

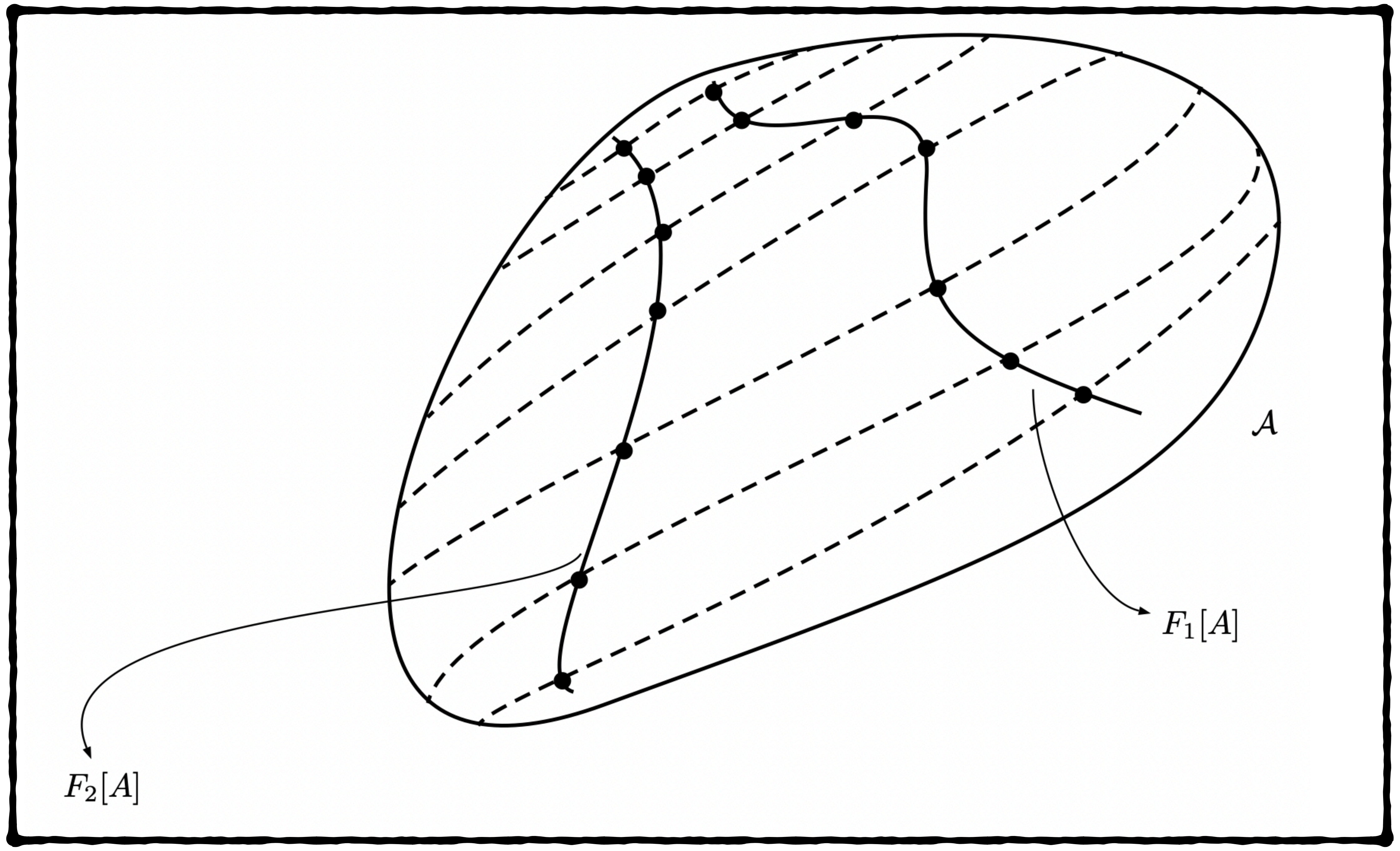
The Faddeev-Popov procedure aims at tracing global sections on the wild field space slicing each gauge orbit once

In practice, one chooses a condition that must be satisfied by the gauge field, namely,

$$F^a[A] = 0$$

Common choices are: $\partial_\mu A^a_\mu = 0$, $\partial_i A^a_i = 0$, $\partial_\mu A^a_\mu = \alpha b^a$,...

Ideally



2. Brief overview of the Gribov problem

Faddeev-Popov procedure: Systematics

Toy version:

$\{x_i\}$ **are zeros of** $f(x)$**. We** ***assume*** **that** $f'(x)$ **exists and does not vanish at** $\{x_i\}$

$$\delta(f(x)) = \sum_{i=1}^{n} \frac{\delta(x - x_i)}{|f'(x_i)|} \quad \Rightarrow \quad \int \mathrm{d}x\, \delta(f(x)) = \sum_{i=1}^{n} \frac{1}{|f'(x_i)|}$$

$$\boxed{\frac{1}{\sum_{i=1}^{n} \frac{1}{|f'(x_i)|}} \int \mathrm{d}x \delta(f(x)) = 1}$$

Let us ***assume*** **that** $f(x)$ **has just one root and its derivative is positive. Hence**

$$\boxed{f'(x_i) \int \mathrm{d}x\, \delta(f(x)) = 1}$$

2. Brief overview of the Gribov problem

Faddeev-Popov procedure: Systematics

“Real-life” version:

The Faddeev-Popov procedure is the functional generalization of the toy example retaining its key assumptions. The resulting gauge-fixed path integral is

$$\mathscr{Z}_{\mathrm{YM}} = \int [\mathscr{D}A] \det \left(\frac{\delta F[A^U]}{\delta \alpha^a} \bigg|_{\alpha=0} \right) \delta[F[A]] \, \mathrm{e}^{-S_{\mathrm{YM}}[A]}$$

The assumptions in the toy model version can be translated as: the gauge condition picks one representative *per* gauge orbit (one root) and the determinant is positive.

The expression above can be nicely written in terms of auxiliary fields that lift the determinant and delta-functional to the Boltzmann factor and generate a gauge-fixed Yang-Mills action

2. Brief overview of the Gribov problem

Faddeev-Popov procedure: Systematics

$$\mathcal{Z}_{\mathrm{YM}} = \int [\mathcal{D}A][\mathcal{D}\bar{c}][\mathcal{D}c][\mathcal{D}b]\; \mathrm{e}^{-S_{\mathrm{FP}}[A,\bar{c},c,b]}$$

$$S_{\mathrm{FP}}[A,\bar{c},c,b] = S_{\mathrm{YM}} + \int \mathrm{d}^d x\, \left(b^a F^a[A] + \bar{c}^a \mathcal{M}^{ab} c^b\right)$$

The $(\bar{c}^a, c^b)$ are the so-called Faddeev-Popov ghosts and b^a is the Nakanishi-Lautrup field.

The gauge-fixed action provides an incredibly efficient tool to perform computations in the UV

However…

$$\partial_\mu A^a_\mu = 0$$
$$\partial_\mu A'^a_\mu = 0$$

$$A'^a_\mu = A^a_\mu - D^{ab}_\mu \zeta^b$$

implies

$$-\partial_\mu D^{ab}_\mu \zeta^b = 0$$

2. Brief overview of the Gribov problem

$-\partial_\mu D_\mu^{ab}$ **is the Faddeev-Popov operator in the Landau gauge**

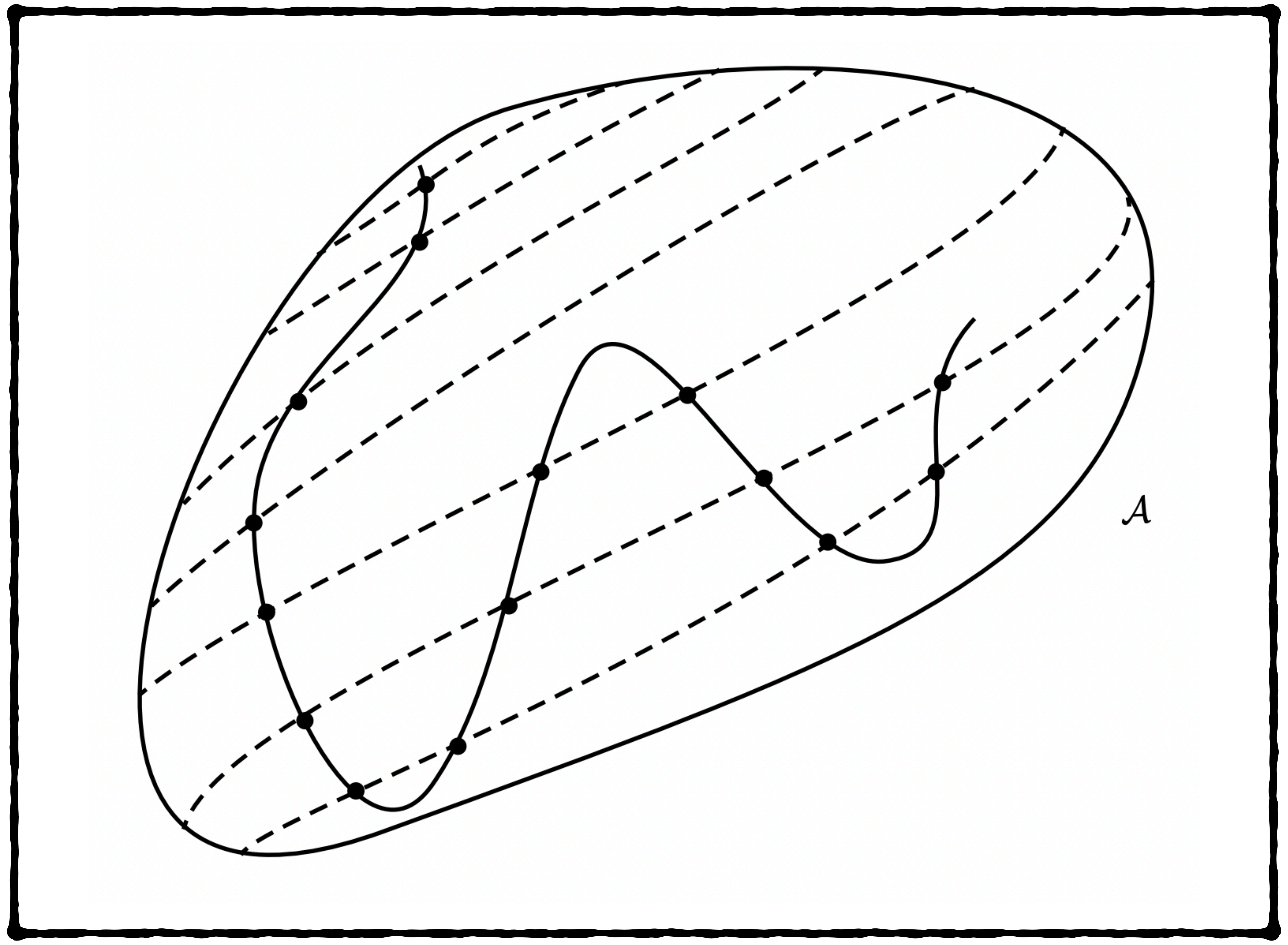
Existence of (infinitesimal) Gribov copies

$\Updownarrow$

Existence of normalizable zero modes of the FP operator

They exist!

[Gribov 1978; Singer 1978]



2. Brief overview of the Gribov problem

NPB 139 (1978) 1-19

$-\partial_\mu D_\mu^{ab}$ is

QUANTIZATION OF NON-ABELIAN GAUGE THEORIES

V.N. GRIBOV

Leningrad Nuclear Physics Institute, Gatchina, Leningrad 188350, USSR

Received 13 January 1978

It is shown that the fixing of the divergence of the potential in non-Abelian theories does not fix its gauge. The ambiguity in the definition of the potential leads to the fact that, when integrating over the fields in the functional integral, it is apparently enough for us to restrict ourselves to the potentials for which the Faddeev-Popov determinants are positive. This limitation on the integration range over the potentials cancels the infrared singularity of perturbation theory and results in a linear increase of the charge interaction at large distances.

[Gribov 1978; Singer 1978]

2. Brief overview of the Gribov problem

Perturbatively

$$\boxed{-\partial_\mu D^{ab}_\mu \zeta^b = -\,\partial^2 \zeta^a + g f^{abc} A^c_\mu \partial_\mu \zeta^b}$$

the first term dominates and the FP operator does not develop zero modes

This is in agreement with the success of perturbative computations using the FP procedure

It also reinforces that the Gribov problem becomes relevant at the strongly-coupled regime

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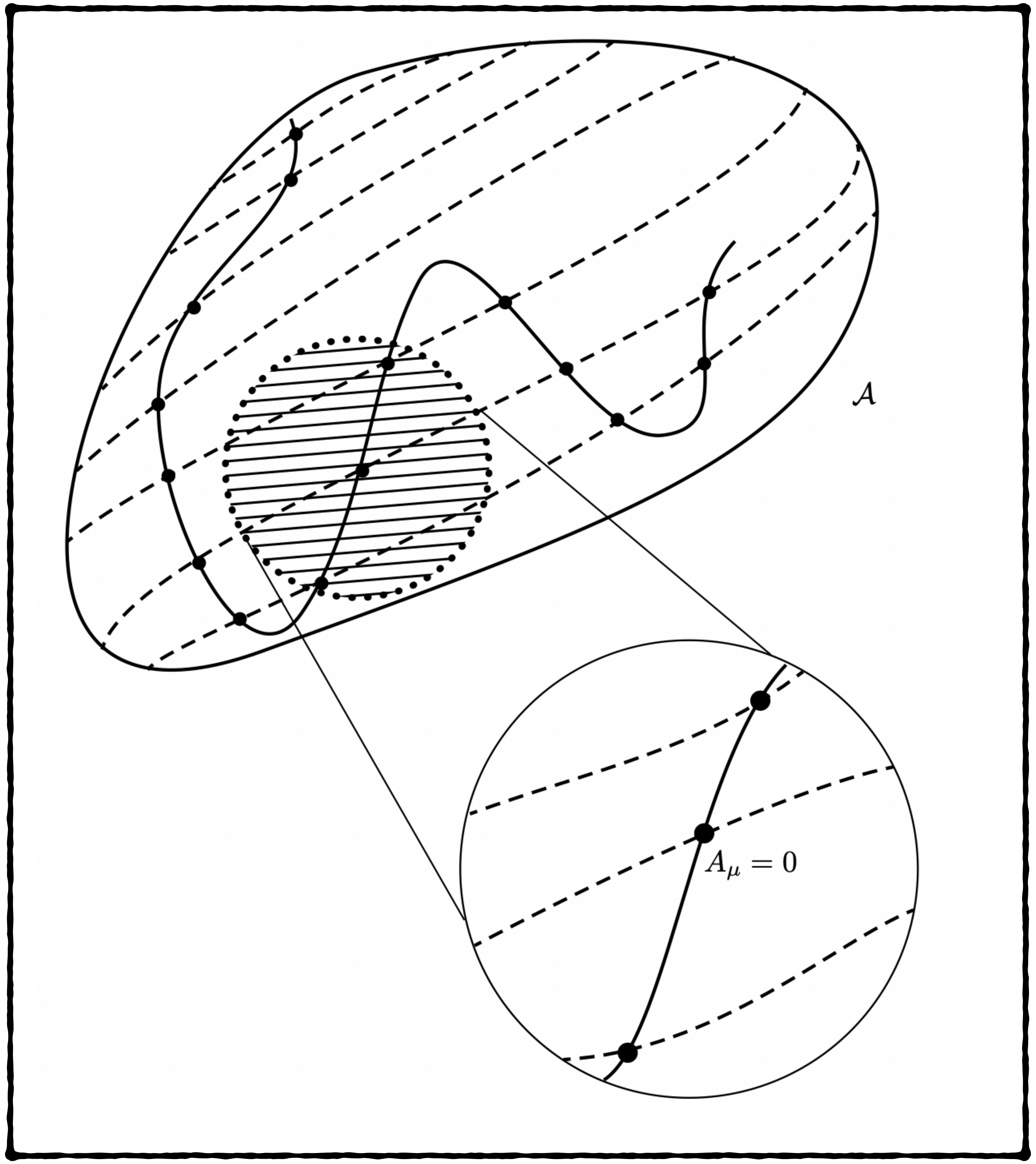

3. Don't cross the Horizon! (aka RGZ)

Infinitesimal copies are linked to normalizable zero modes of the FP operator

One way of dealing with them is to restrict the functional integral to a region where the FP operator does not develop zero modes (it is positive)

Remarkably, in the Landau gauge, such a region exists and it features highly non-trivial geometrical properties

This region is known as the Gribov region and it is bounded. Its boundary is known as the Gribov horizon.

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This region is known as the Gribov region and it is bounded. Its boundary is known as the Gribov horizon.

Moreover, all gauge orbits cross it *at least* once

[Dell’Antonio and Zwanziger 1991]

$$\mathscr{Z}_{\mathrm{GZ}} = \int_{\Omega} [\mathscr{D}A][\mathscr{D}\bar{c}][\mathscr{D}c][\mathscr{D}b]\ \mathrm{e}^{-S_{\mathrm{FP}}[A,\bar{c},c,b]}$$

The restriction to Ω can be worked out and it effectively engenders a non-local modification to the FP action

$$\mathscr{Z}_{\mathrm{GZ}} = \int [\mathscr{D}A][\mathscr{D}\bar{c}][\mathscr{D}c][\mathscr{D}b]\ \mathrm{e}^{-S_{\mathrm{GZ}}[A,\bar{c},c,b]+d\gamma^4(N^2-1)V}$$

$$S_{\mathrm{GZ}}[A,\bar{c},c,b] = S_{\mathrm{FP}}[A,\bar{c},c,b] - \gamma^4 H(A)$$

3. Don't cross the Horizon! (aka RGZ)

$$H(A) = g^2 \int_{x,y} f^{abc} A^b_\mu(x) \left[\mathscr{M}^{-1}\right]^{ad}(x,y) f^{dec} A^e_\mu(y)$$

Horizon Function

Non-local due to the inverse
of the FP operator

γ **is a mass-like parameter - Gribov parameter**

This parameter is not free but fixed by a gap equation

$$\langle H(A) \rangle = dV(N^2 - 1)$$

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Remarkably, the Horizon function can be localized

$$\Sigma_{\mathrm{GZ}}[A,\bar{c},c,b] = S_{\mathrm{FP}}[A,\bar{c},c,b] + S_H$$

$$S_H = \int_x \left(\bar{\varphi}^{ab}_\mu \mathcal{M}^{ac}(A) \varphi^{cb}_\mu - \bar{\omega}^{ab}_\mu \mathcal{M}^{ac}(A) \omega^{cb}_\mu \right) + ig\gamma^2 \int_x f^{abc} A^a_\mu \left(\varphi + \bar{\varphi}\right)^{bc}_\mu$$

[Zwanziger 1989]

$$\mathcal{Z}_{\mathrm{GZ}} = \int [\mathcal{D}\mu]_{\mathrm{GZ}}\, \mathrm{e}^{-\Sigma_{\mathrm{GZ}}[A,\bar{c},c,b] + d\gamma^4 (N^2-1)V}$$

$$[\mathcal{D}\mu]_{\mathrm{GZ}} = [\mathcal{D}A][\mathcal{D}\bar{c}][\mathcal{D}c][\mathcal{D}b][\mathcal{D}\bar{\varphi}][\mathcal{D}\varphi][\mathcal{D}\bar{\omega}][\mathcal{D}\omega]$$

3. Don't cross the Horizon! (aka RGZ)

The local action $\Sigma_{\rm GZ}$ is renormalizable and effectively eliminates infinitesimal Gribov copies

However, it has IR instabilities - Stabilizing (refining) condensates are introduced: Refined GZ

[Dudal, Gracey, Sorella, Vandersickel and Verschelde 2008]

$$S_{\rm RGZ} = S_{\rm GZ} + S_{\rm cond}$$

$$S_{\rm cond} = \int_x \left[\frac{m^2}{2} A^a_\mu A^a_\mu + M^2 \left(\bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu \right) \right]$$

The mass parameters m^2 and M^2 are not free but fixed by their own gap equations

[Dudal, Felix, Palhares, Rondeau, Sorella, Vandersickel, Vercauteren]

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The tree-level gluon propagator in the RGZ setup is

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \mathscr{D}_A(p^2)$$

$$\mathscr{D}_A(p^2) = \frac{p^2 + M^2}{(p^2 + m^2)(p^2 + M^2) + 2\gamma^4 g^2 N}$$

It is finite at vanishing momentum.
The ghost propagator is not enhanced in the IR: Massive/Decoupling behavior

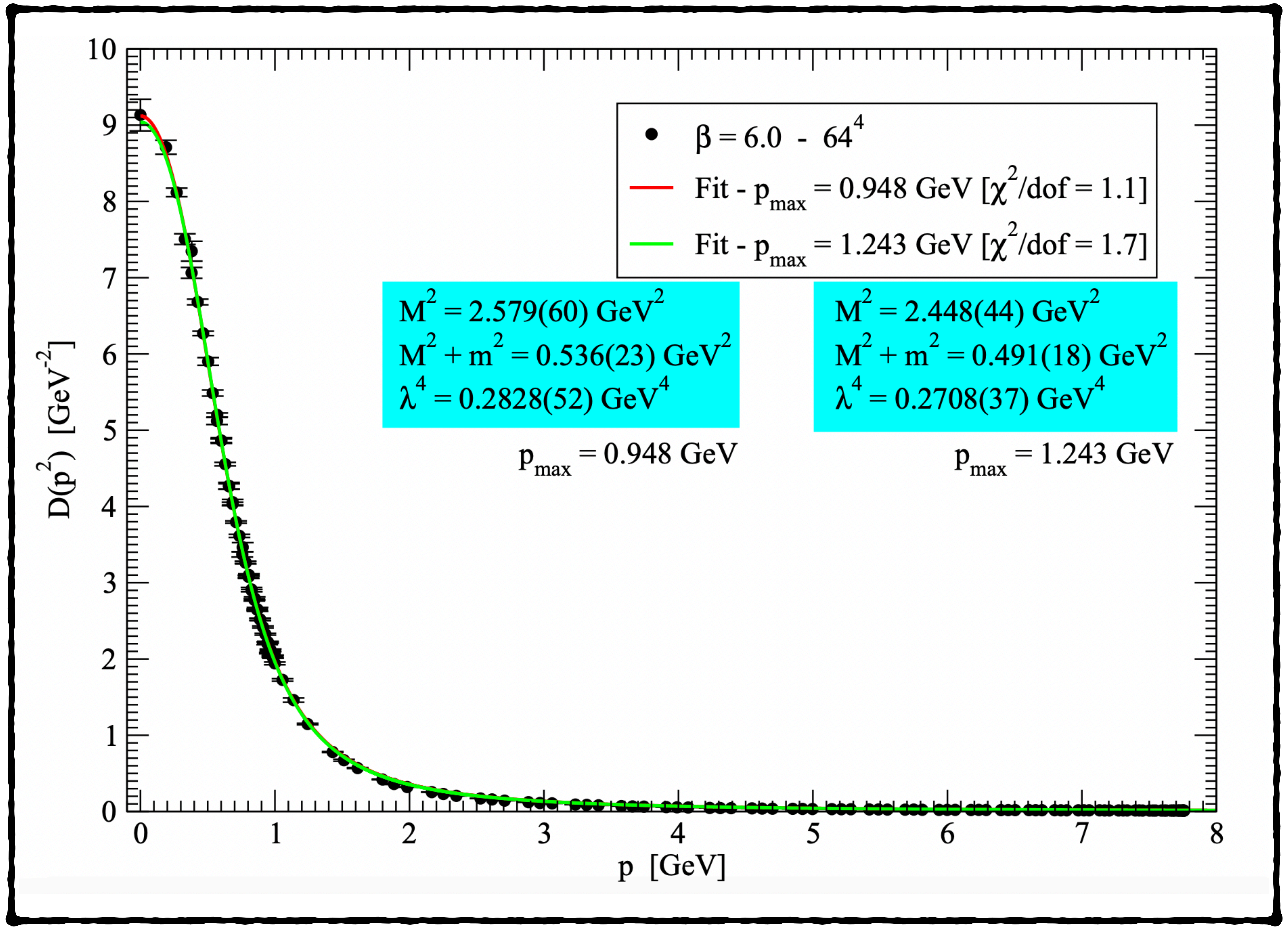
At vanishing condensate masses (GZ):
Scaling behavior

3. Don't cross the Horizon! (aka RGZ)

Solving the corresponding gap equations: Too difficult for quantitative precision

Strategy: Using the mass-like parameters to fit the tree-level propagator to lattice data

The functional form of the propagator fits very well lattice data in the IR



[Dudal, Oliveira and Vandersickel 2010]

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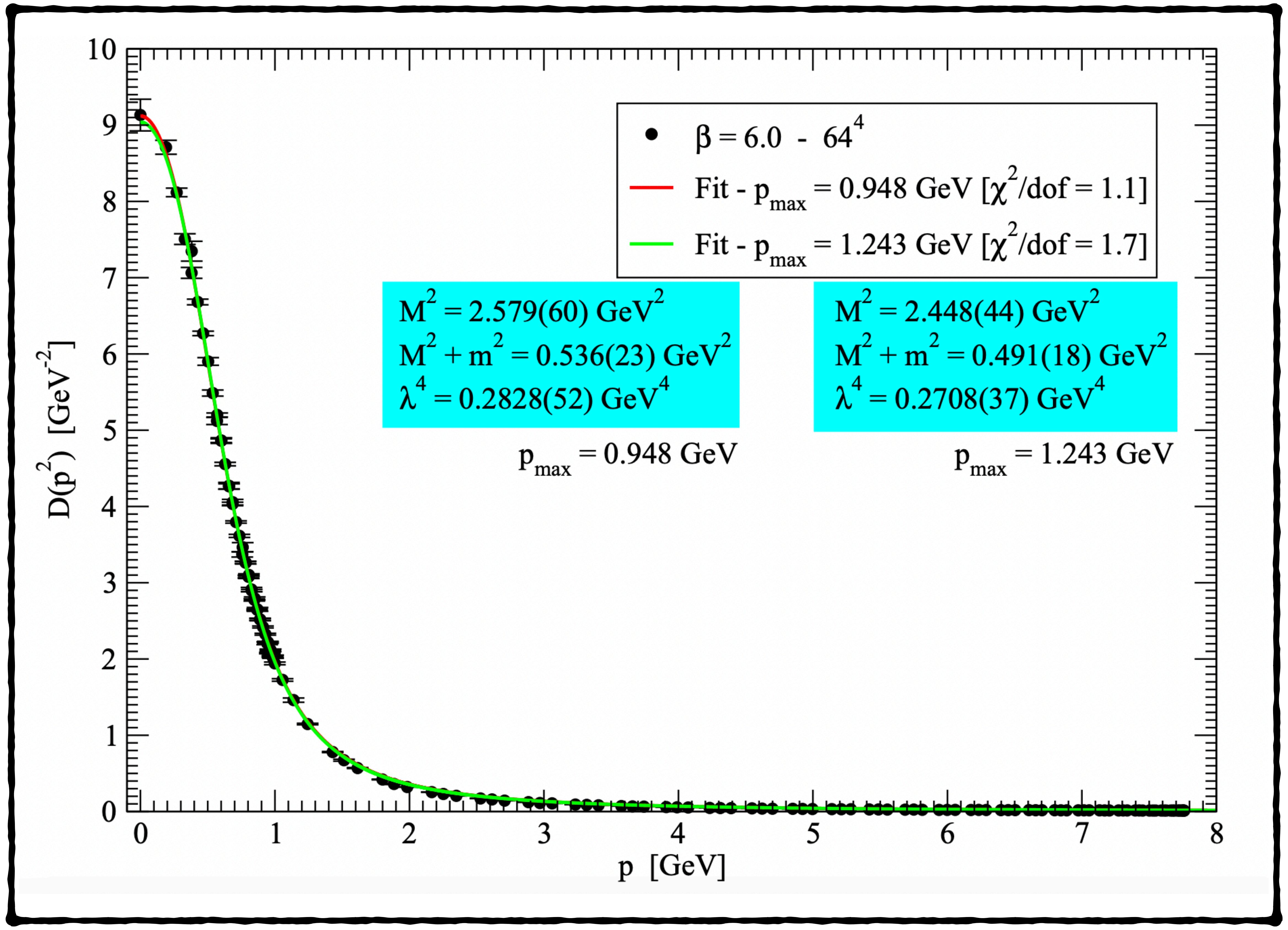
Non-perturbatively-informed tree-level propagator!



[Dudal, Oliveira and Vandersickel 2010]

3. Don’t cross the Horizon! (aka RGZ)

What happens if one-loop corrections are added to the tree-level expression?

This is not a trivial task due to the proliferation of diagrams arising from the extra fields in the RGZ

Inspired by the success of the Curci-Ferrari model, it is expected that one-loop corrections will not spoil the good agreement achieved at tree level

Recently, the one-loop computation was performed at fixed coupling and the result compared to lattice data

[de Brito and AP 2024]

[Lattice data: Cucchieri and Mendes; Duarte, Oliveira and Silva]

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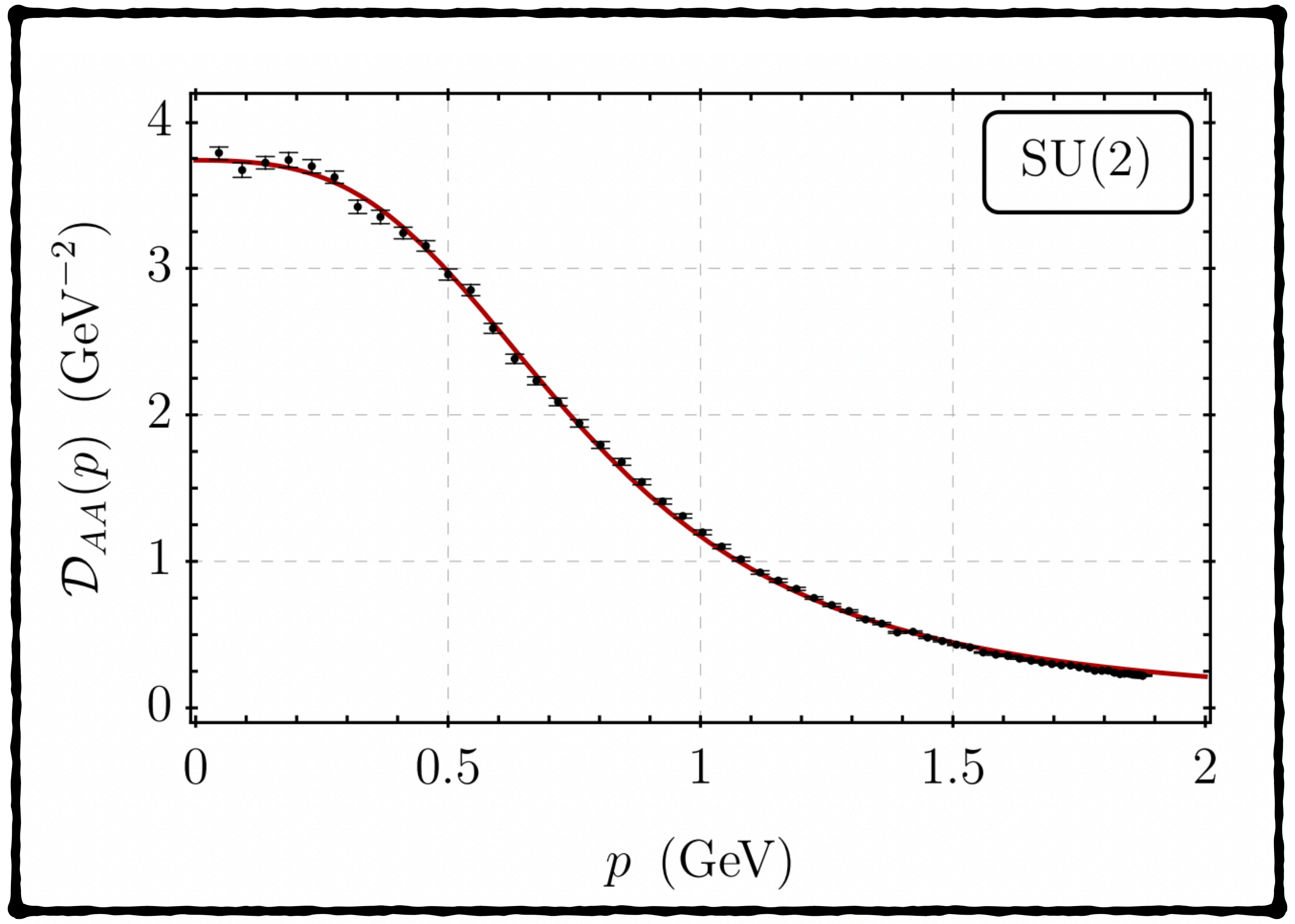

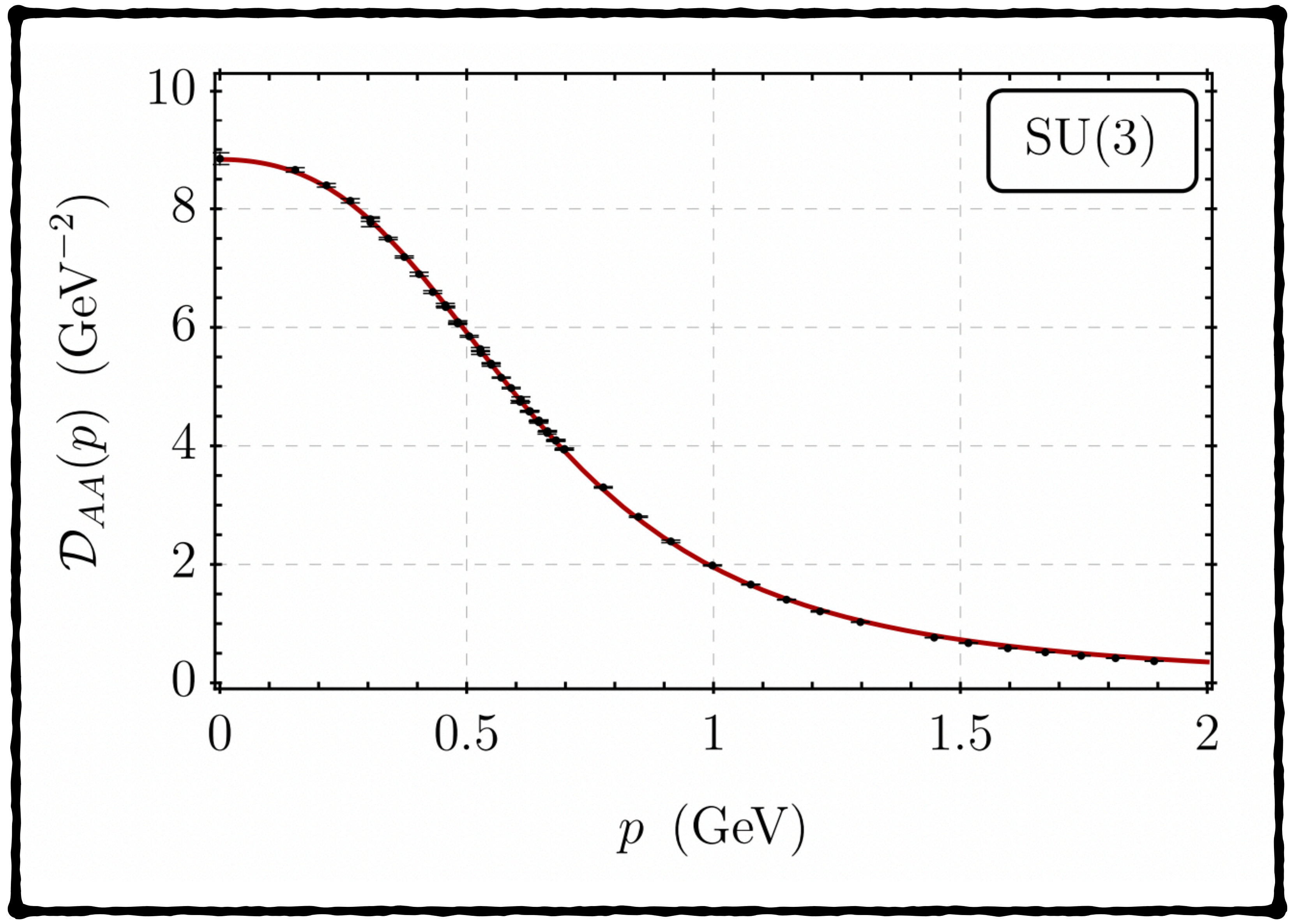

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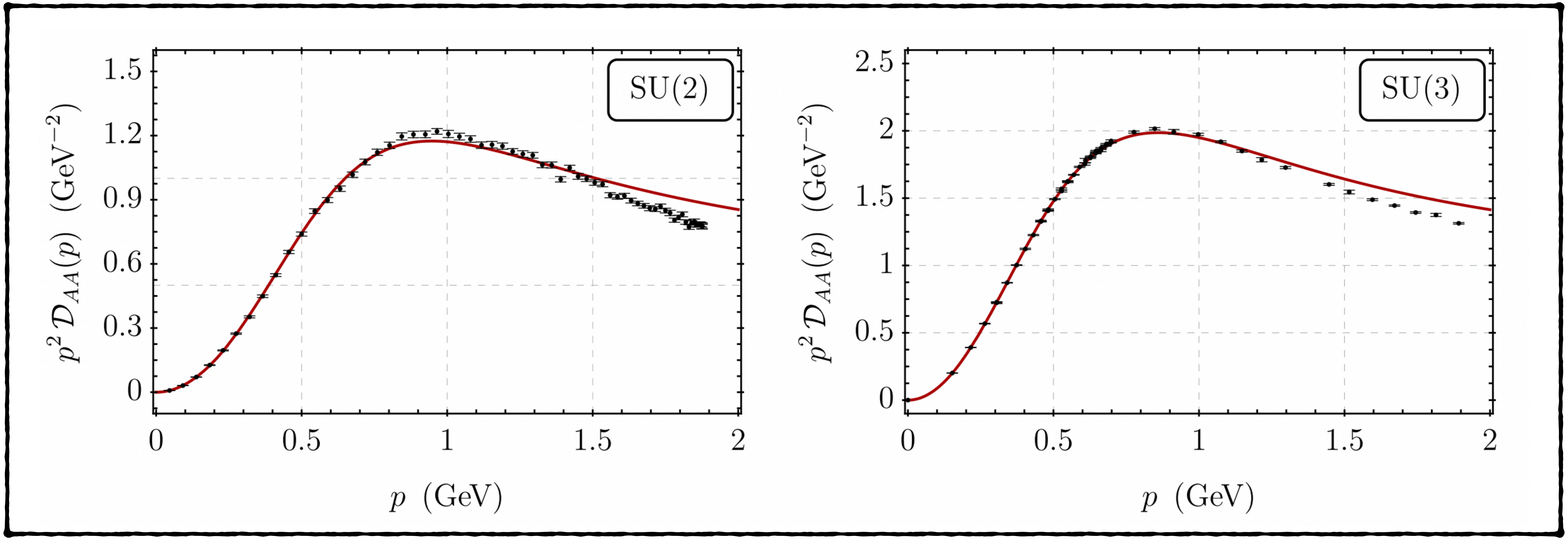

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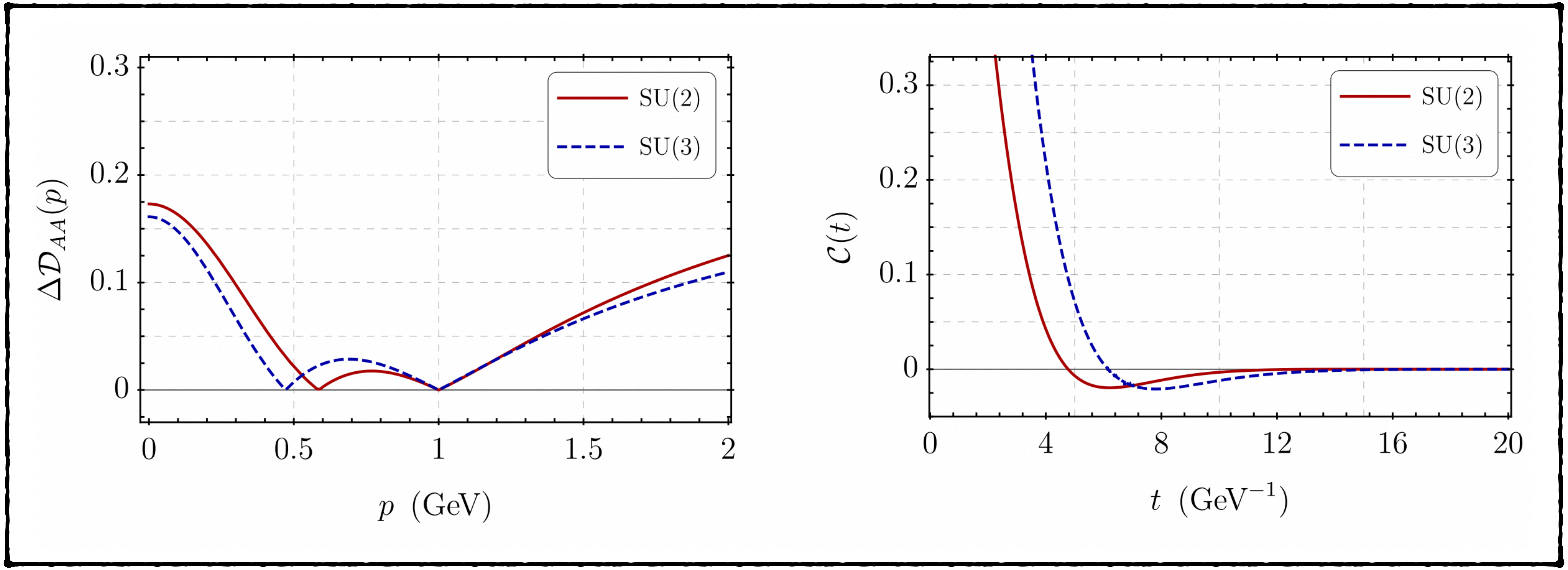


[de Brito and AP 2024] [Lattice data: Cucchieri and Mendes; Duarte, Oliveira and Silva]

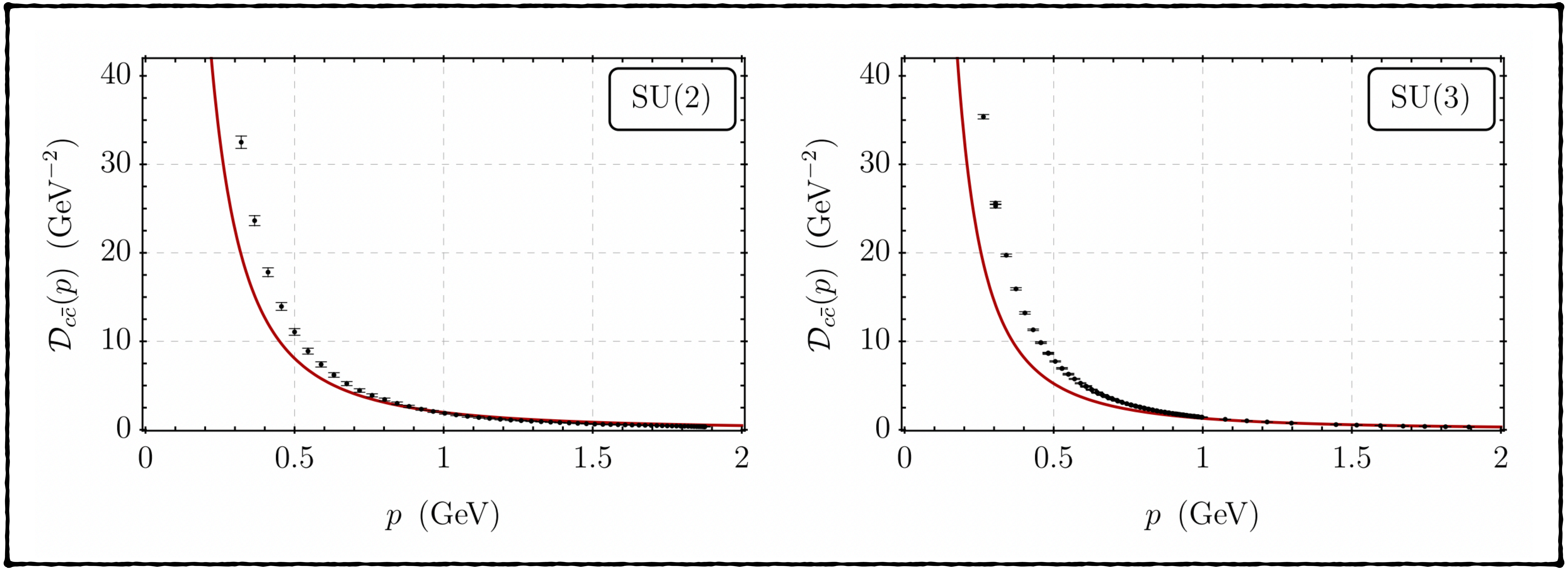
Complex conjugate poles

$$p_\pm^2|_{\mathrm{SU(2)}} \approx -0.184 \pm 0.667i\,\mathrm{GeV}^2$$

$$p_\pm^2|_{\mathrm{SU(3)}} \approx -0.404 \pm 0.332i\,\mathrm{GeV}^2$$

3. Don't cross the Horizon! (aka RGZ)

In our setup, the ghost propagator was a "prediction".
However, our procedure needs optimization *[Work in Progress]*



[de Brito and AP 2024] [Lattice data: Cucchieri and Mendes; Duarte, Oliveira and Silva]

3. Don’t cross the Horizon! (aka RGZ)

It’s not always rainbows and butterflies…

The RGZ action is not invariant under BRST transformations

$$sS_{\text{RGZ}} \neq 0$$

BRST is a powerful tool and particularly important for gauge-parameter dependence

The RGZ was formulated in the Landau gauge. What about linear covariant gauges with a free gauge parameter?

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A BRST-invariant reformulation of the RGZ framework was proposed

[Capri, Dudal, Fiorentini, Guimaraes, Justo, Mintz, Palhares, AP, Sobreiro, Sorella]

This is achieved by means of dressed gauge-invariant fields

[Lavelle, McMullan, Zwanziger,…]

$$\mathscr{A}^2_{\text{min}} = \min_U \text{Tr} \int_x A^U_\mu A^U_\mu$$

[See Giovani's talk this afternoon]

3. Don't cross the Horizon! (aka RGZ)

$$A_\mu^h = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\chi_\nu$$

$$\chi_\nu = A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + \mathcal{O}(A^3)$$

$sA_\mu^h = 0$

In the Landau gauge: $A_\mu^h \to A_\mu$

BRST-invariant reformulation of the Horizon function

$$H(A^h) = g^2 \int_{x,y} f^{abc} A_\mu^{h,b}(x) \left[\mathcal{M}^{-1}(A^h)\right]^{ad}(x,y) f^{dec} A_\mu^{h,e}(y)$$

The Gribov parameter couples to a BRST-closed term: it is *not* akin to a gauge parameter

3. Don't cross the Horizon! (aka RGZ)

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The Gribov parameter couples to a BRST-closed term: it is *not* akin to a gauge parameter

The same happens with the refining mass-like parameters

The gauge-invariant *dressed* field A^h_μ can be localized by means of a Stuckelberg-like field

The BRST-invariant RGZ action is local but non-polynomial. Yet, it is renormalizable

[Capri, Fiorentini, AP, Sorella 2017]

Correlation functions of gauge fields and FP ghosts are equivalent in the BRST-invariant and the standard RGZ frameworks

4. Going beyond pure YM-RGZ

The elimination of (infinitesimal) Gribov copies affects, directly, only the gauge sector

The RGZ introduces non-perturbative information to the tree-level gluon propagator

If matter is minimally coupled to the gauge fields, the tree-level matter propagator is the perturbative one

Should matter be introduced with a non-perturbative structure that mimics the gauge sector?

Or should the non-perturbative information be introduced via gauge loops?

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Or should the non-perturbative information be introduced via gauge loops?

A non-perturbative coupling was proposed

[Dudal, Guimaraes, Palhares, Sorella 2013]
[Capri, Guimaraes, Justo, Palhares, Sorella 2014]
[Capri, Fiorentini, AP, Sorella 2017]

For a given matter field $F^i(x)$:

$$\mathscr{H}_F = g^2 \int_{x,y} (T^a)^{ij} F^i(x) \left[\mathscr{M}^{-1}\right]^{ab}(x,y)(T^b)^{jk} F^k(y)$$

This corresponds to a Horizon-like term for the matter fields

This is what we shall call a *non-perturbative* coupling between matter and the RGZ

Mass-like parameters are needed and should be fixed by appropriate gap equations

4. Going beyond pure YM-RGZ

As a concrete example: Scalar fields in the adjoint representation

$$S^{\mathrm{nm}}_{\phi\mathrm{RGZ}} = S_{\mathrm{RGZ}} + S^{\mathrm{nm}}_{\phi}$$

$$S^{\mathrm{nm}}_{\phi} = \int_x \left[\frac{1}{2}(D^{ab}_{\mu}\phi^b)(D^{ac}_{\mu}\phi^c) + \frac{m^2_{\phi}}{2}\phi^a\phi^a + \frac{\lambda}{4!}(\phi^a\phi^a)^2\right]$$

$$+\int_x \left(\bar{\zeta}^{ab}\mathcal{M}^{ac}(A)\zeta^{cb} - \bar{\theta}^{ab}\mathcal{M}^{ac}(A)\theta^{cb}\right) + ig\sigma^2\int_x f^{abc}\phi^a(\bar{\zeta}+\zeta)^{bc}$$

$$+M^2_{\phi}\int_x \left(\bar{\zeta}^{ab}\zeta^{ab} - \bar{\theta}^{ab}\theta^{ab}\right)$$

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Standard minimally coupled scalar-field action

$$+\int_x \left(\bar{\zeta}^{ab}\mathscr{M}^{ac}(A)\zeta^{cb} - \bar{\theta}^{ab}\mathscr{M}^{ac}(A)\theta^{cb}\right) + ig\sigma^2\int_x f^{abc}\phi^a(\bar{\zeta}+\zeta)^{bc}$$

Localization of the Horizon-like function for the matter sector

$$+M^2_{\phi}\int_x \left(\bar{\zeta}^{ab}\zeta^{ab} - \bar{\theta}^{ab}\theta^{ab}\right)$$

Condensates of the auxiliary localizing fields

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As a concrete example: Scalar fields in the adjoint representation

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The tree-level scalar-field propagator is

$$\langle \phi^a(p)\phi^b(-p)\rangle = \delta^{ab}\frac{p^2 + M_\phi^2}{(p^2 + m_\phi^2)(p^2 + M_\phi^2) + 2Ng^2\sigma^4}$$

[Capri, Guimaraes, Justo, Palhares, Sorella 2014]

Using the mass-like parameters to fit lattice data, the tree-level expression provides a quite good description of the scalar-field propagator

4. Going beyond pure YM-RGZ

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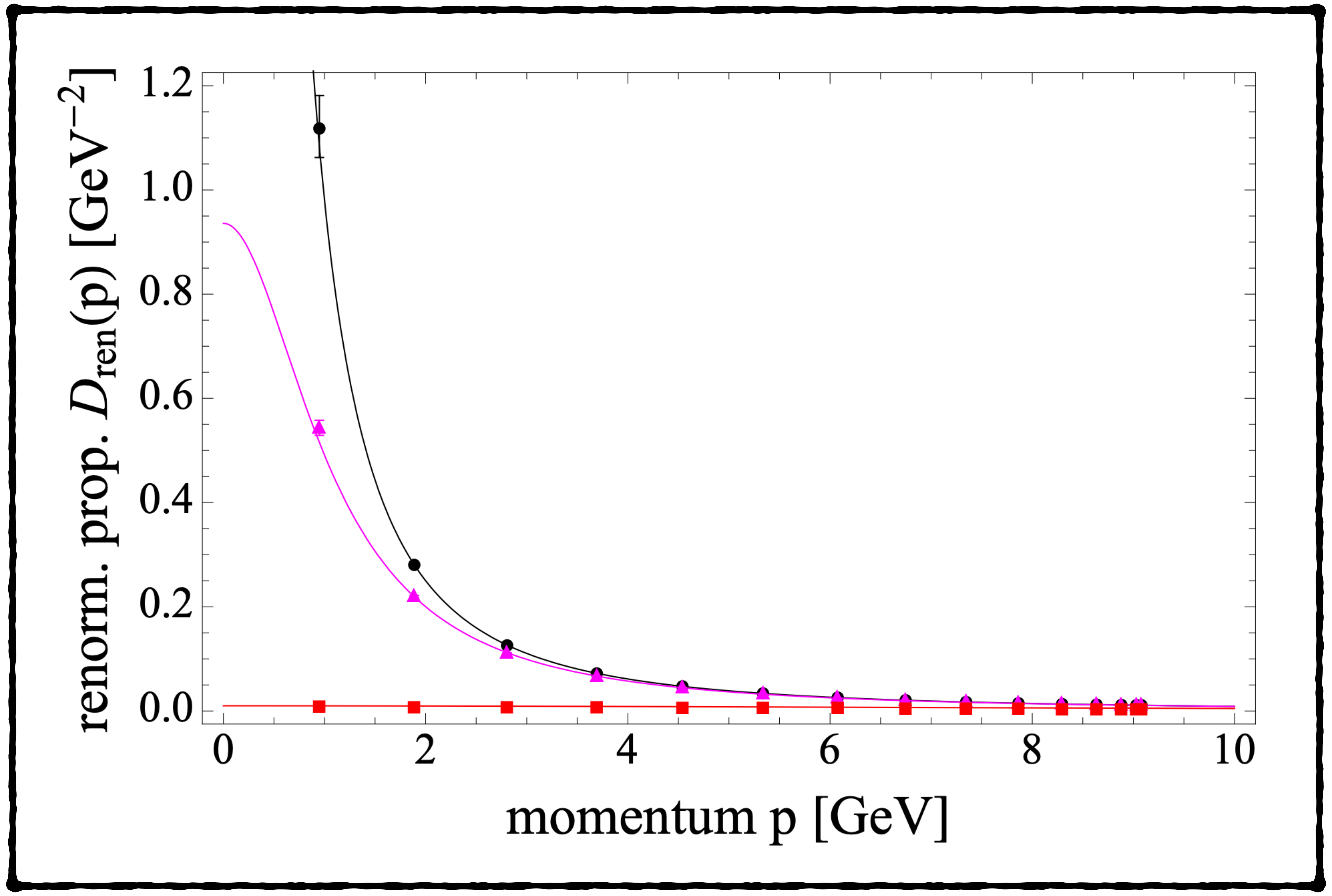

[Capri, Guimaraes, Justo, Palhares, Sorella 2014]

[Lattice data: Maas]

4. Going beyond pure YM-RGZ

Back to minimal coupling

$$S_{\phi\mathrm{RGZ}} = S_{\mathrm{RGZ}} + S_{\phi}$$

$$S_{\phi} = \int_x \left[\frac{1}{2}(D^{ab}_{\mu}\phi^b)(D^{ac}_{\mu}\phi^c) + \frac{m^2_{\phi}}{2}\phi^a\phi^a + \frac{\lambda}{4!}(\phi^a\phi^a)^2 \right]$$

Compute one-loop corrections to the scalar-field propagator arising from its coupling to the RGZ dynamics

[de Brito, de Fabritiis, AP 2023]

Use the values for the mass-like parameters of the RGZ from the tree-level fitting of the (quenched) gluon propagator

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The computation is performed at fixed coupling

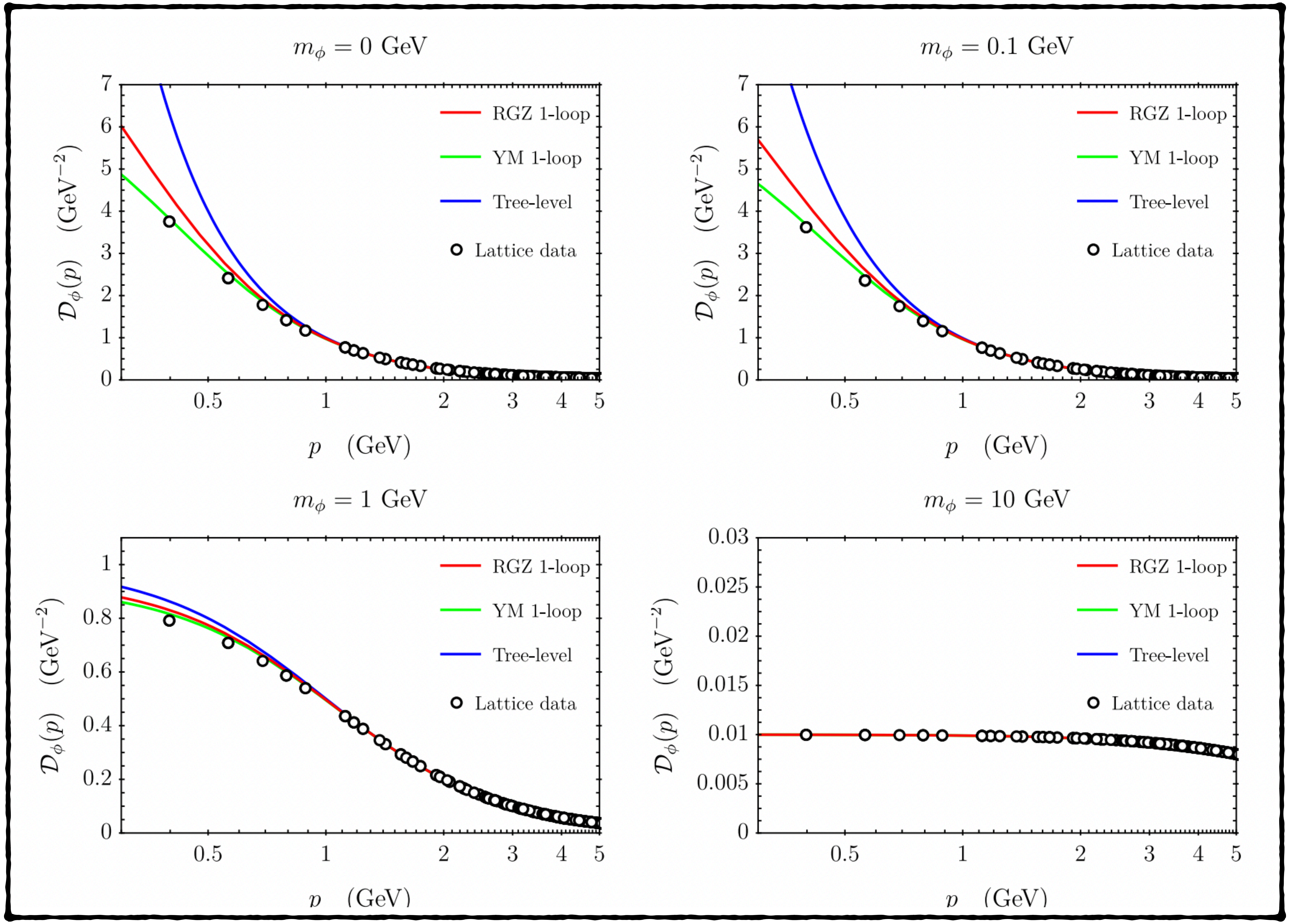
Despite the large set of Feynman rules and mixings in the RGZ, the scalar propagator *at one-loop* order is obtained directly from the inversion of the 1PI two-point function

$$\langle \phi^a(p)\phi^b(-p)\rangle = \delta^{ab}\mathscr{D}_\phi(p^2)$$

$$\mathscr{D}_\phi(p^2) = \left(\Gamma^{(2)}_{\phi\phi}(p^2)\right)^{-1}$$

4. Going beyond pure YM-RGZ

[de Brito, de Fabritiis, AP 2023] [Lattice data: Maas (scalar sector); Cucchieri and Mendes (quenched gluon propagator)]



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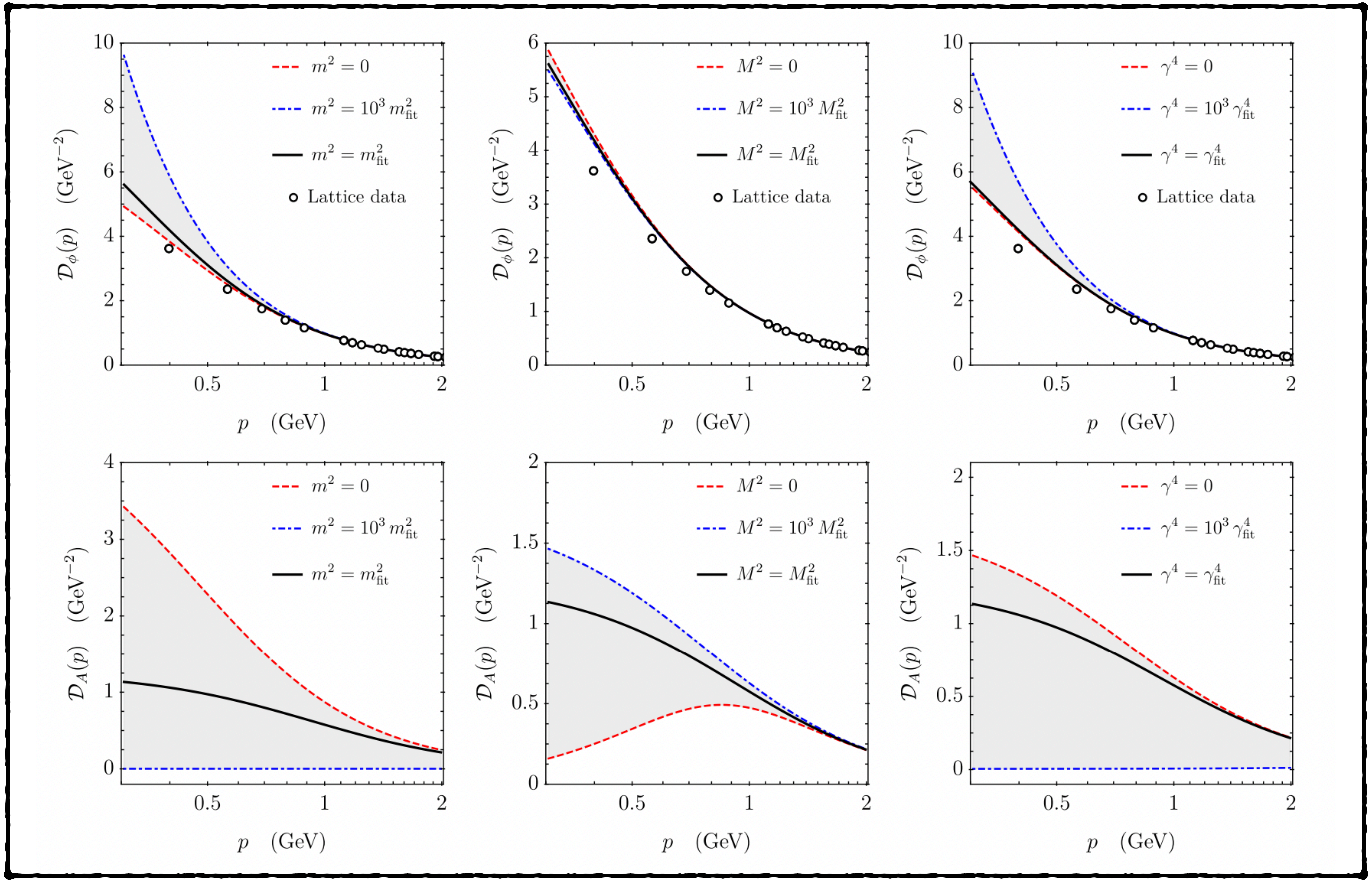

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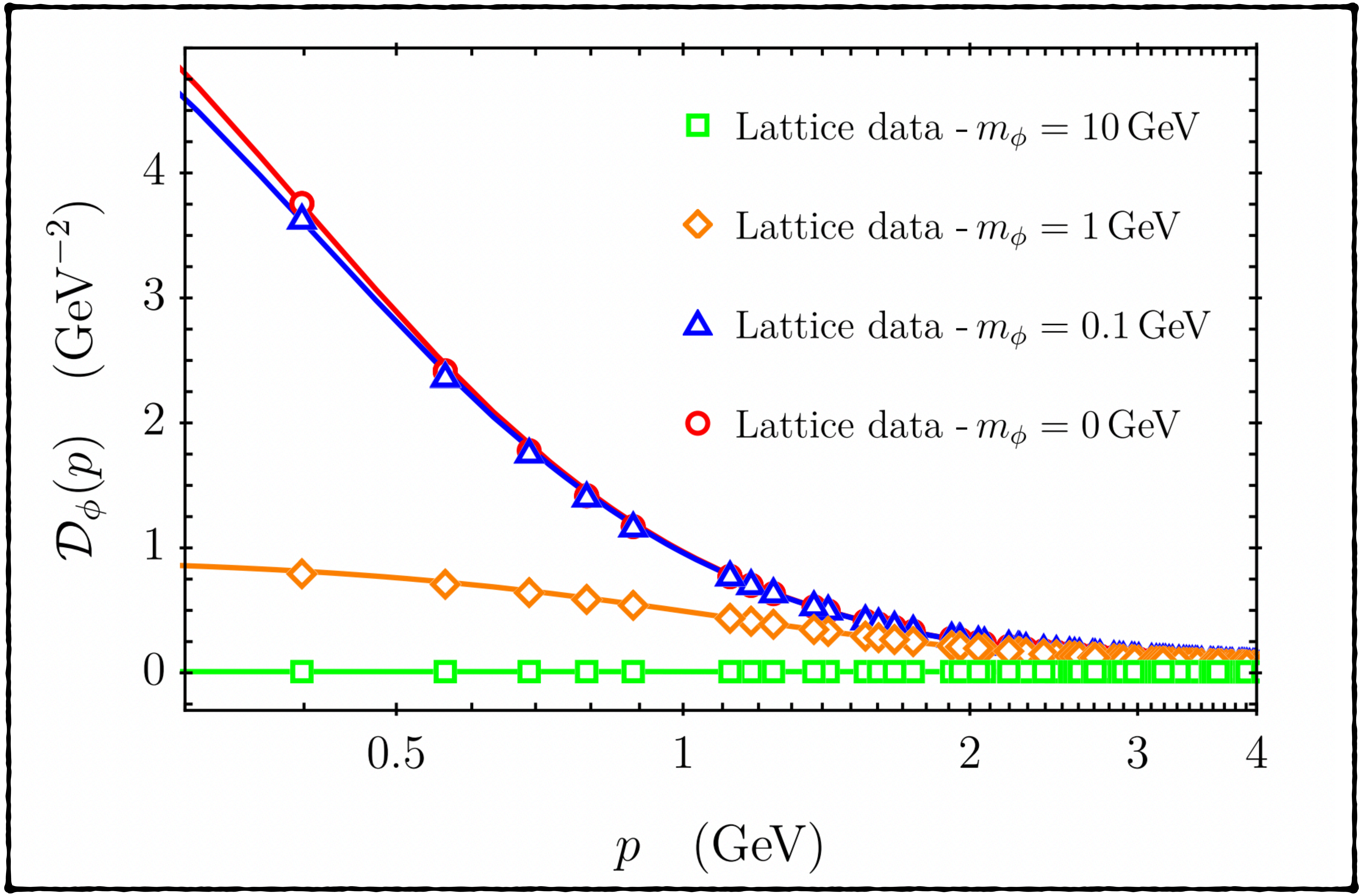
If we fit g as well, the results are considerably improved

Within our setup, we were not able to identify positivity violation conclusively



4. Going beyond pure YM-RGZ

For RGZ-Scalar systems:

The scalar-field propagator can be well described by minimal and non-perturbative matter coupling

The non-perturbative coupling has the advantage (or rather drawback) of introducing new parameters that are independent of those in the gluon propagator

Fitting the scalar-field propagator is a completely independent procedure

Further investigation within this scheme is necessary

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In the minimal scheme, the scalar-field propagator receives quantum corrections that depend on parameters of the original RGZ action

The scalar-field propagator can be "predicted" if the mass parameters are fixed by the gluon propagator

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The setup employed needs to be improved in several directions. First results are encouraging

[Work in Progress]

4. Going beyond pure YM-RGZ

What about quarks?

The non-perturbative matter coupling gives rise to

$$\boxed{S^{\rm nm}_{\psi\rm RGZ} = S_{\rm RGZ} + S^{\rm nm}_{\psi}}$$

$$S^{\rm nm}_{\psi} = \int_x \sum_{f=1}^{N_f} \left[\bar{\psi}^i_f \gamma_\mu D^{ij}_\mu \psi^j_f - m^f_\psi \bar{\psi}^i_f \psi^i_f\right]$$

$$+ \int_x \sum_{f=1}^{N_f} \left(\bar{\lambda}^{ai}_f \mathscr{M}^{ab}(A) \lambda^{bi}_f + \bar{\theta}^{ab}_f \mathscr{M}^{ac}(A) \theta^{cb}_f\right) + ig \int_x \sum_{f=1}^{N_f} M_f^{3/2} \left(\bar{\lambda}^{ai}_f (T^a)^{ij} \psi^j_f + \bar{\psi}^{ai}_f (T^a)^{ij} \lambda^j_f\right)$$

$$+ \int_x \sum_{f=1}^{N_f} \mu^2_{\psi,f} \left(\bar{\lambda}^{ai}_f \lambda^{ai}_f + \bar{\theta}^{ai}_f \theta^{ai}_f\right)$$

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Standard minimally coupled Dirac action

$$+\int_x \sum_{f=1}^{N_f} \left(\bar{\lambda}^{ai}_f \mathscr{M}^{ab}(A)\lambda^{bi}_f + \bar{\theta}^{ab}_f \mathscr{M}^{ac}(A)\theta^{cb}_f\right) + ig\int_x \sum_{f=1}^{N_f} M^{3/2}_f \left(\bar{\lambda}^{ai}_f (T^a)^{ij}\psi^j_f + \bar{\psi}^{ai}_f (T^a)^{ij}\lambda^j_f\right)$$

Localization of the Horizon-like function for the matter sector

$$+\int_x \sum_{f=1}^{N_f} \mu^2_{\psi,f}\left(\bar{\lambda}^{ai}_f \lambda^{ai}_f + \bar{\theta}^{ai}_f \theta^{ai}_f\right)$$

Condensates of the auxiliary localizing fields

4. Going beyond pure YM-RGZ

What about quarks?

The non-perturbative matter coupling gives rise to

$$S^{\rm nm}_{\psi{\rm RGZ}} = S_{\rm RGZ} + S^{\rm nm}_{\psi}$$

The tree-level quark propagator for degenerate $N_f = 2$ is

$$S(p) = Z(p)\frac{i\gamma_\mu p_\mu - A(p)}{p^2 + A^2(p)}$$

[Capri, Guimaraes, Justo, Palhares, Sorella 2014]

$$Z(p) = 1 \qquad A(p) = \frac{g^2 M^3}{p^2 + \mu_\psi^2} + m_\psi$$

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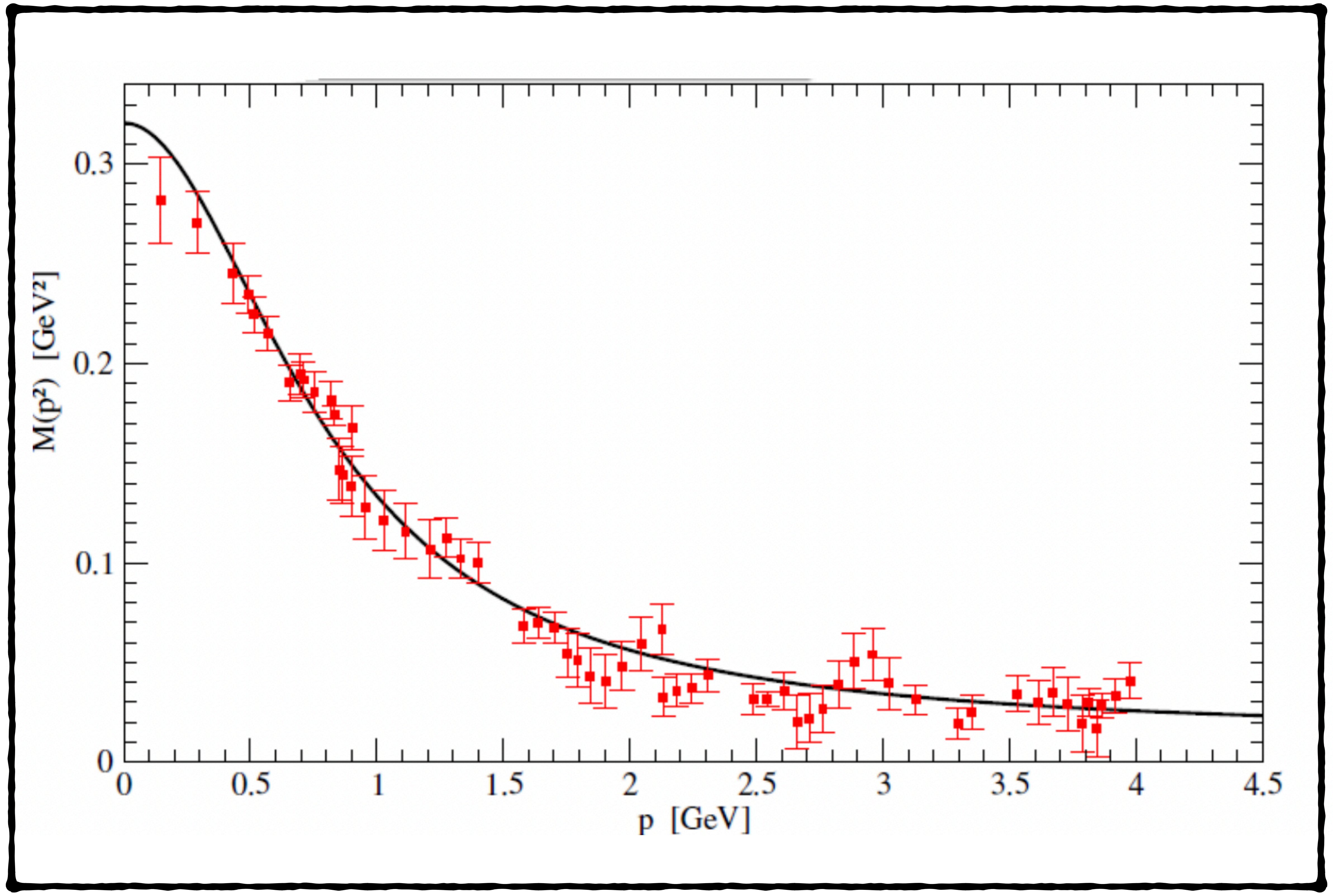

[Dudal, Guimaraes, Palhares, Sorella 2013]

[Capri, Guimaraes, Justo, Palhares, Sorella 2014]

Non-trivial momentum-dependence on the mass function already at tree-level

This mass function captures a non-trivial chiral limit.

4. Going beyond pure YM-RGZ

Minimally coupled quarks

$$S_{\psi\text{RGZ}} = S_{\text{RGZ}} + S_{\psi}$$

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Compute one-loop corrections to the quark propagator arising from its coupling to the RGZ dynamics

[de Brito, de Fabritiis, AP 2025]

Fit the mass-parameters as well as the coupling using the unquenched gluon propagator at one-loop order

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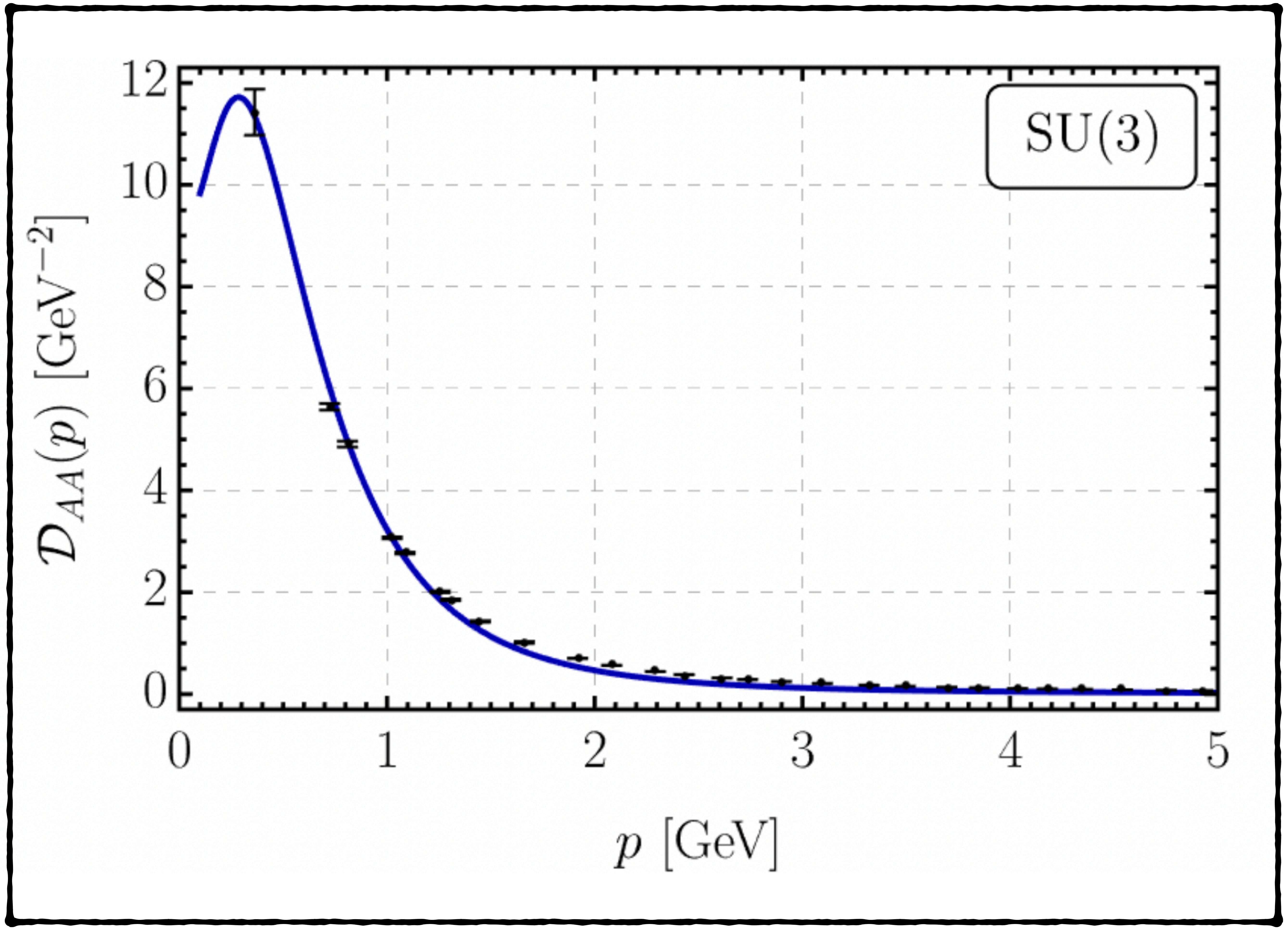
A toy-running for the coupling is used, namely,

$$g^2(p) = \frac{g_0^2}{1 + \frac{g_0^2}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\log\left(\frac{p^2+\Lambda^2}{\Lambda^2}\right)}$$

Λ stands for a IR cutoff to circumvent the perturbative Landau pole

4. Going beyond pure YM-RGZ

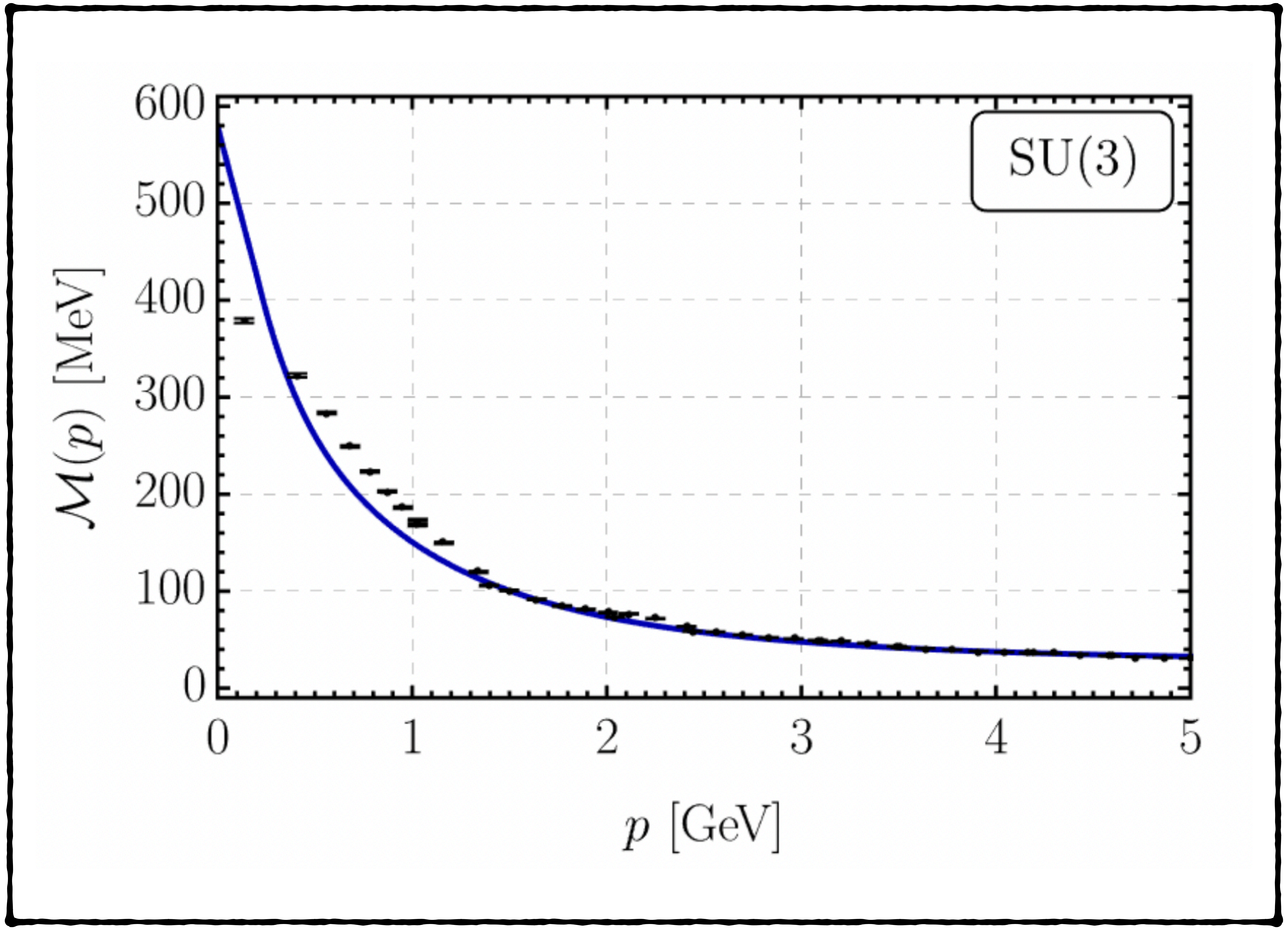


[de Brito, de Fabritiis, AP 2025]

[Lattice data: Sternbeck, Maltman, Muller-Preussker and von Smekal]

Unquenched gluon propagator at one-loop order from RGZ minimally coupled to quarks

4. Going beyond pure YM-RGZ



[de Brito, de Fabritiis, AP 2025]

[Lattice data: Oliveira, Silva, Skullerud and Sternbeck]

Quark mass function at one-loop order: The mass-parameters from the RGZ are fixed by the gluon propagator

$$m_\psi = 37\,\text{MeV} \quad M_\pi = 422\,\text{MeV}$$

No extra parameter is introduced. This is a "prediction".

In the present treatment, chiral-symmetry breaking effects are *not* captured by the one-loop (or any finite order) correction - One should perform such analysis away from the chiral limit

4. Going beyond pure YM-RGZ

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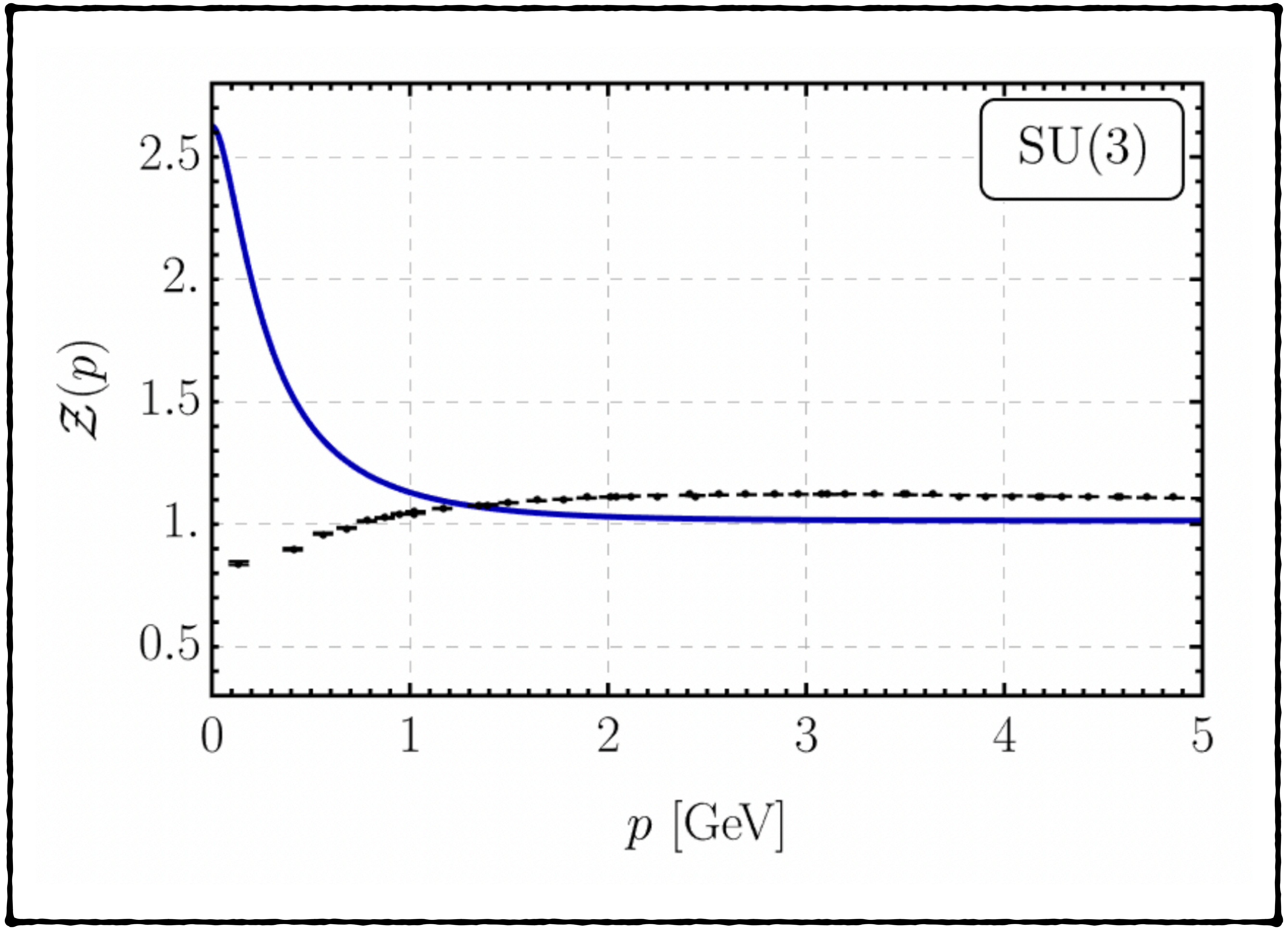

[de Brito, de Fabritiis, AP 2025]

[Lattice data: Oliveira, Silva, Skullerud and Sternbeck]

The quark dressing function is not even in qualitative agreement with the lattice data in the IR

This is very similar to what happens in the Curci-Ferrari model. In fact, we checked that in the limit $\gamma \to 0$, our results coincide with Curci-Ferrari results at one-loop order

[Pelaez, Tisser and Wschebor 2014]

Two-loop computations are needed to improve the results (as it was shown in the Curci-Ferrari case) - Challenging task in the RGZ

[Barrios, Gracey, Pelaez and Reinosa 2021]

4. Going beyond pure YM-RGZ

Chiral-symmetry breaking effects require some sort “beyond”-perturbative treatment.

There are several ways to tackle this problem, but there are no results in the context of the RGZ coupled to matter in a minimal way

The results here presented can be viewed as an encouragement that the elimination of Gribov copies in the gauge sector is sufficient to transfer the “necessary” IR effects to the matter sector via gluon and localizing auxiliary fields loops.

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The results here presented can be viewed as an encouragement that the elimination of Gribov copies in the gauge sector is sufficient to transfer the "necessary" IR effects to the matter sector via gluon and localizing auxiliary fields loops.

Since the Curci-Ferrari model can be viewed as a minimalistic approach to mimic the removal of Gribov copies, the results here presented should be, at least, in qualitative agreement with such a model. They are indeed.

It is important to emphasize that the Curci-Ferrari model is a particular case of the present setup in which the Gribov parameter γ is set to zero

RGZ expressions for the propagators at one-loop are much more complicated than the CF ones. This makes the fitting procedure much more involved.

The non-perturbative matter coupling can be cast in a BRST-invariant framework by dressing the matter fields

[Capri, Fiorentini, AP, Sorella 2017]

5. Conclusions

The Gribov problem is around for almost 50 years

So far, we have managed to understand how to partially deal with it for computationally advantageous gauges

It is unclear if its resolution will give any new physical information in YM theories

We have evidence for its relevance in the IR for correlation functions involving gauge fields and FP ghosts

There exists a local and renormalizable framework, the RGZ, that effectively removes infinitesimal Gribov copies and takes into account the formation of condensates

Its original formulation in the Landau gauge has been extended to an entire new class of gauges thanks to a BRST-invariant formulation of the RGZ

5. Conclusions

The RGZ provides a non-perturbatively-informed tree-level structure

What happens if loop corrections are introduced? This requires a systematic investigation and we have presented some recent contributions in this direction

See also [Mintz, Palhares, AP, Sorella 2018]

[Barrios, Guimaraes, Mintz, Palhares, Pelaez 2024]

Should the matter sector also be non-perturbatively-informed or the gluon (and auxiliary fields) loops will take care of the IR behavior of matter fields?

We have found evidence supporting the minimal-coupling prescription

However, the non-perturbative RGZ-matter coupling should be investigated further as well. Even if it is not a necessary ingredient, it can be an efficient effective description of matter

A lot of work to be done. Cross-fertilization with functional methods would be fantastic!

Obrigado