EMERGENT HADRON MASS
and PION/KAON/NUCLEON FORM FACTORS

ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
FONDAZIONE BRUNO KESSLER

DANIELE BINOSI

ECT* - FONDAZIONE BRUNO KESSLER

The Complex Structure of Strong Interactions
in Euclidean and Minkowski space

MAY 26 - 30 2025, TRENTO, IT

QCD
(classical)
LAGRANGIAN

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pure-glue QCD displays a **mass gap**

$$m_g \sim 0.5 \text{ GeV}$$

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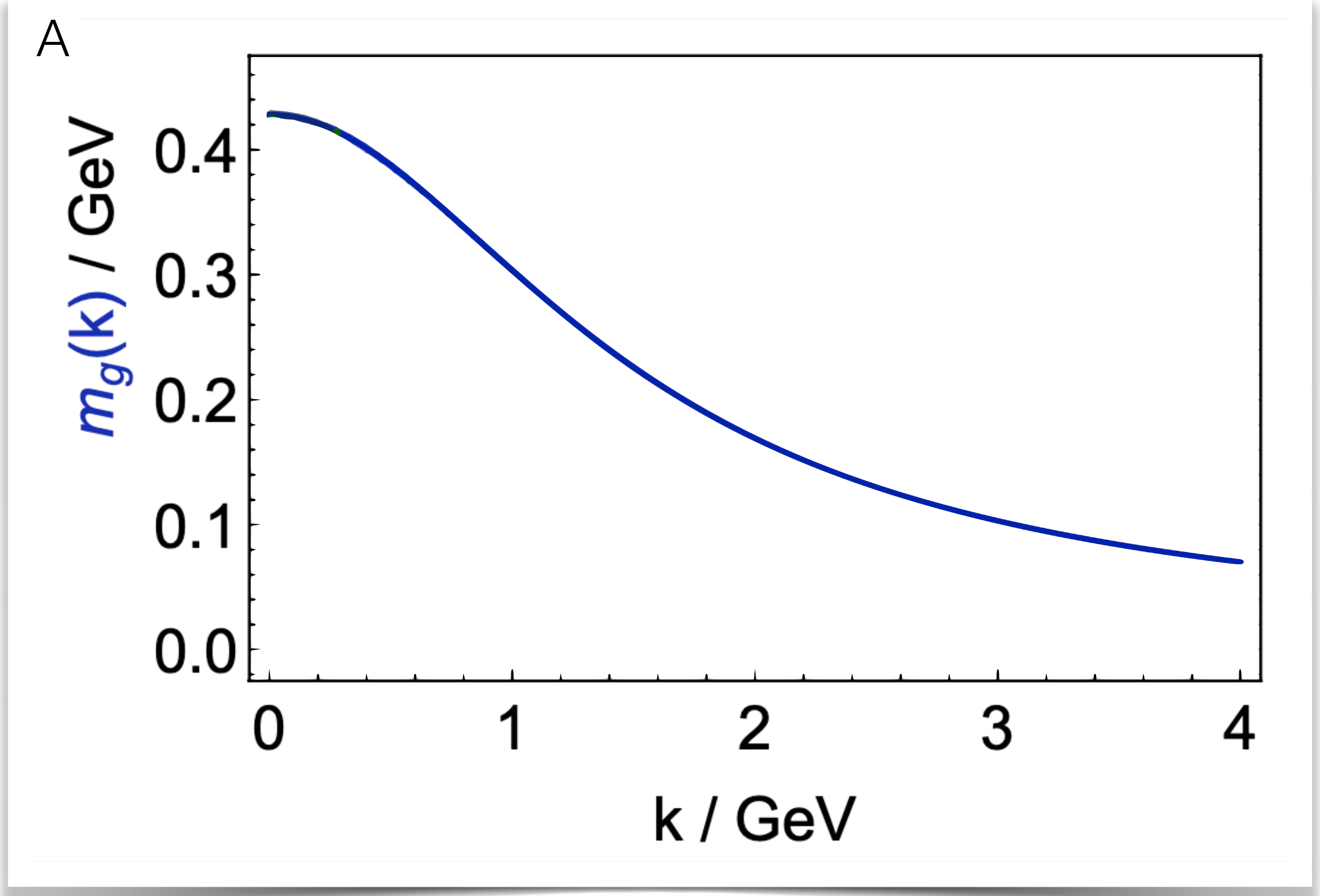

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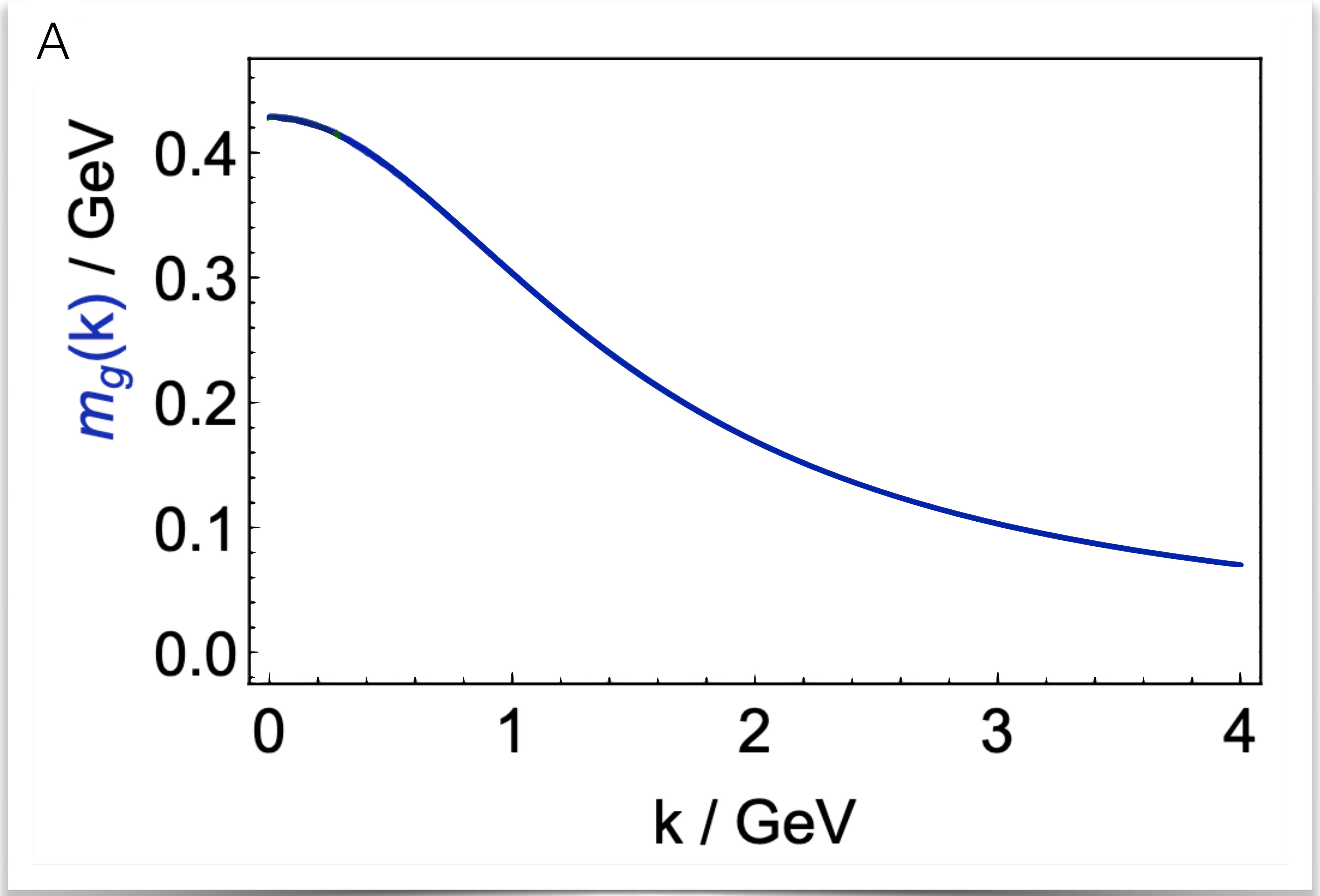

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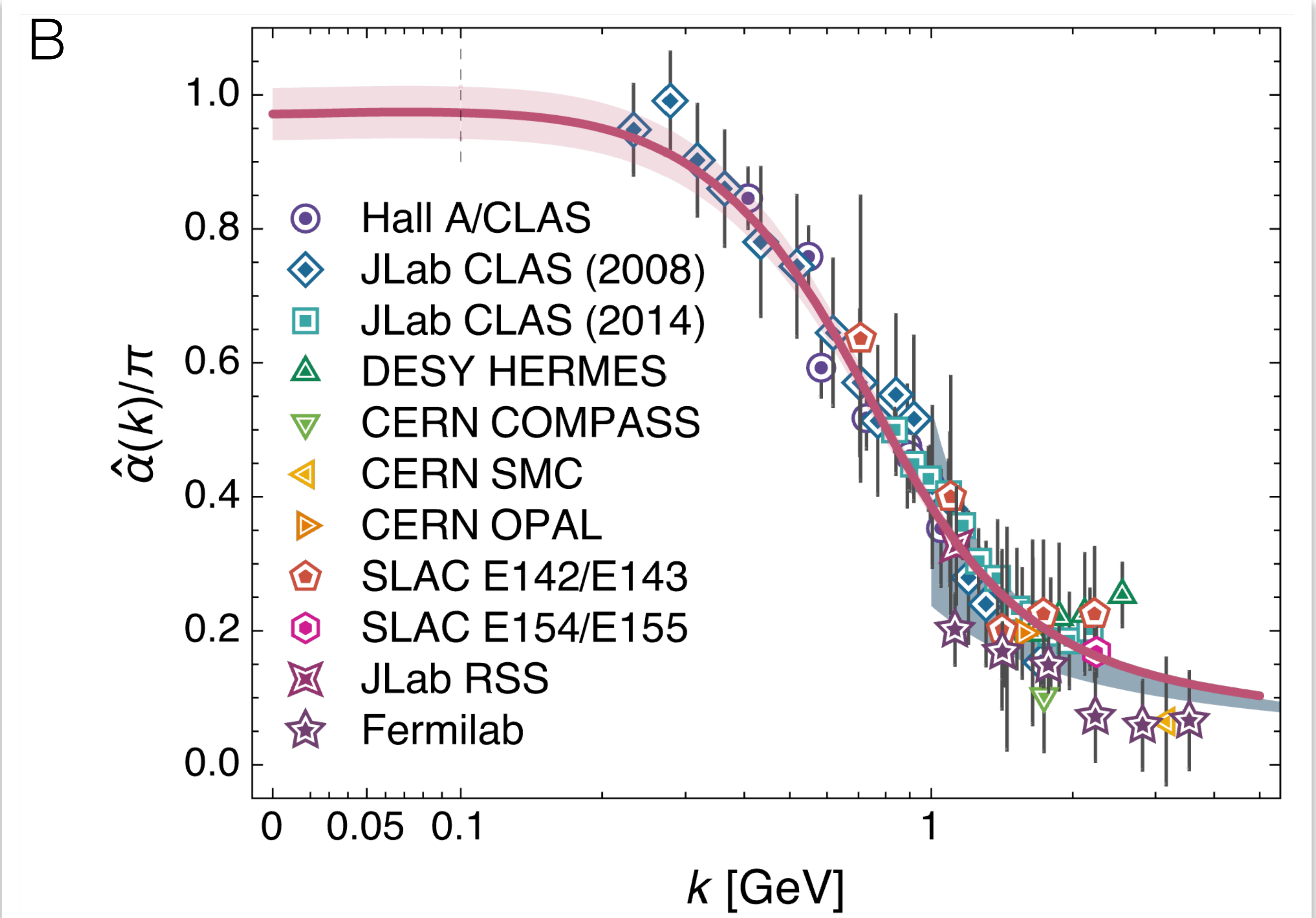

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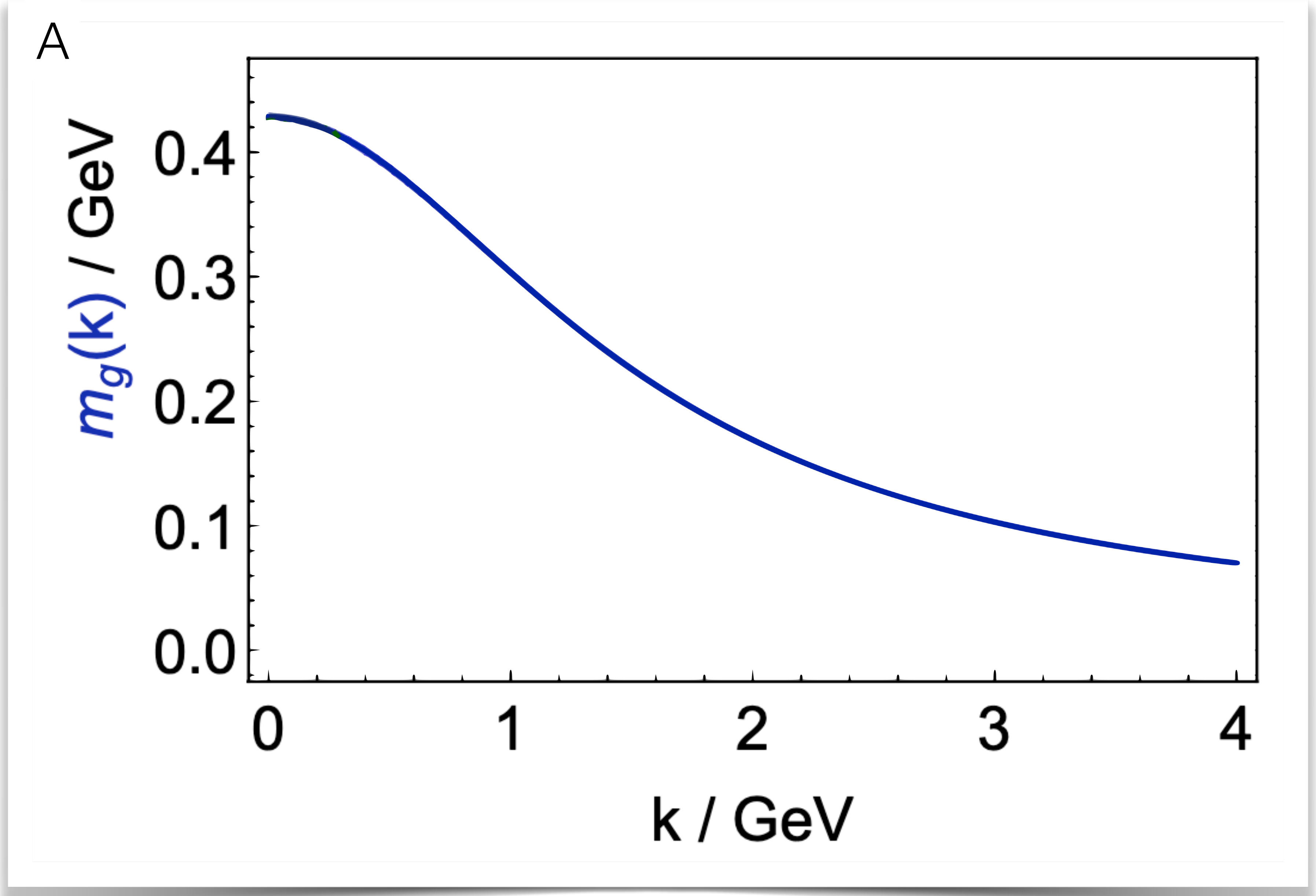

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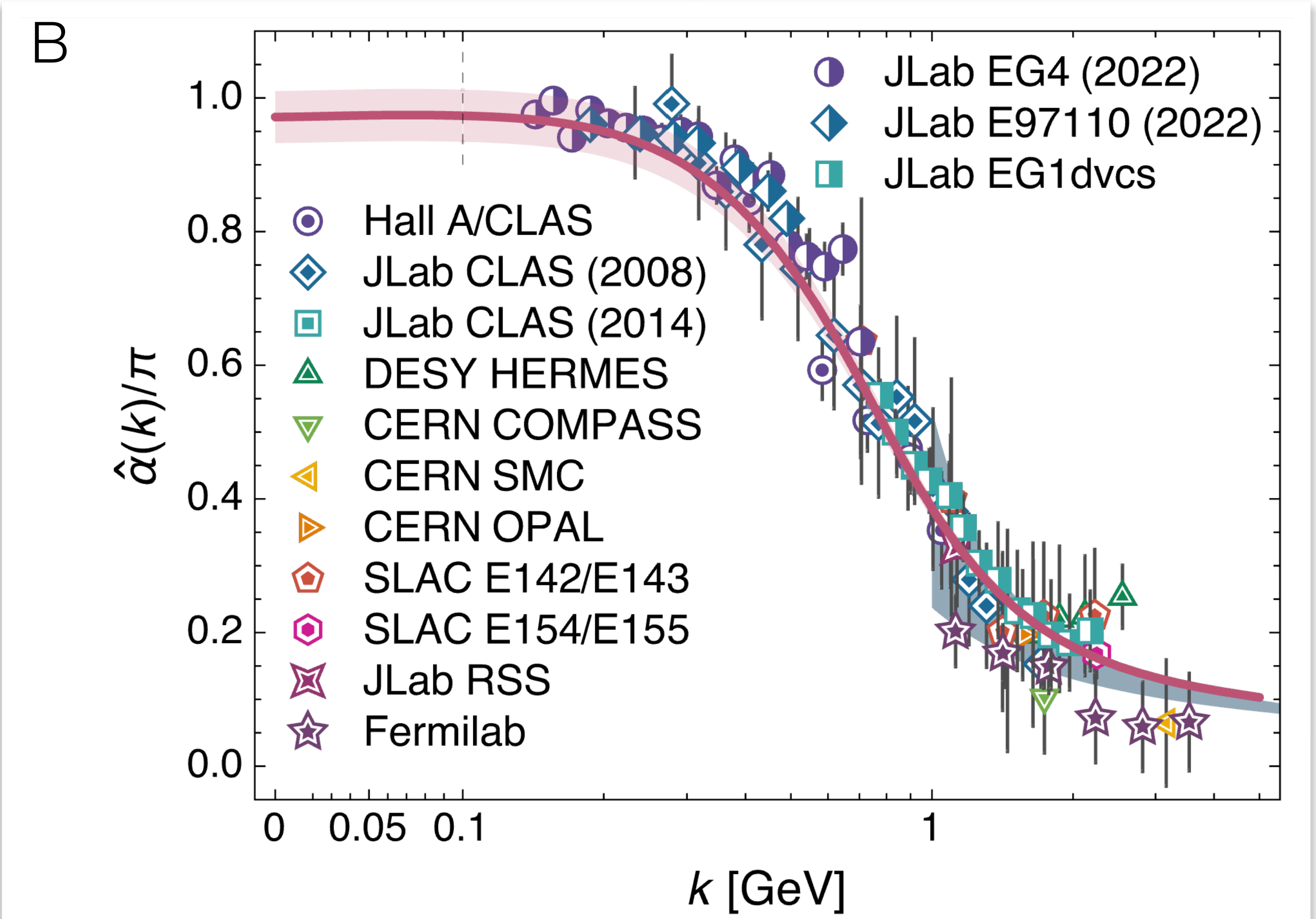

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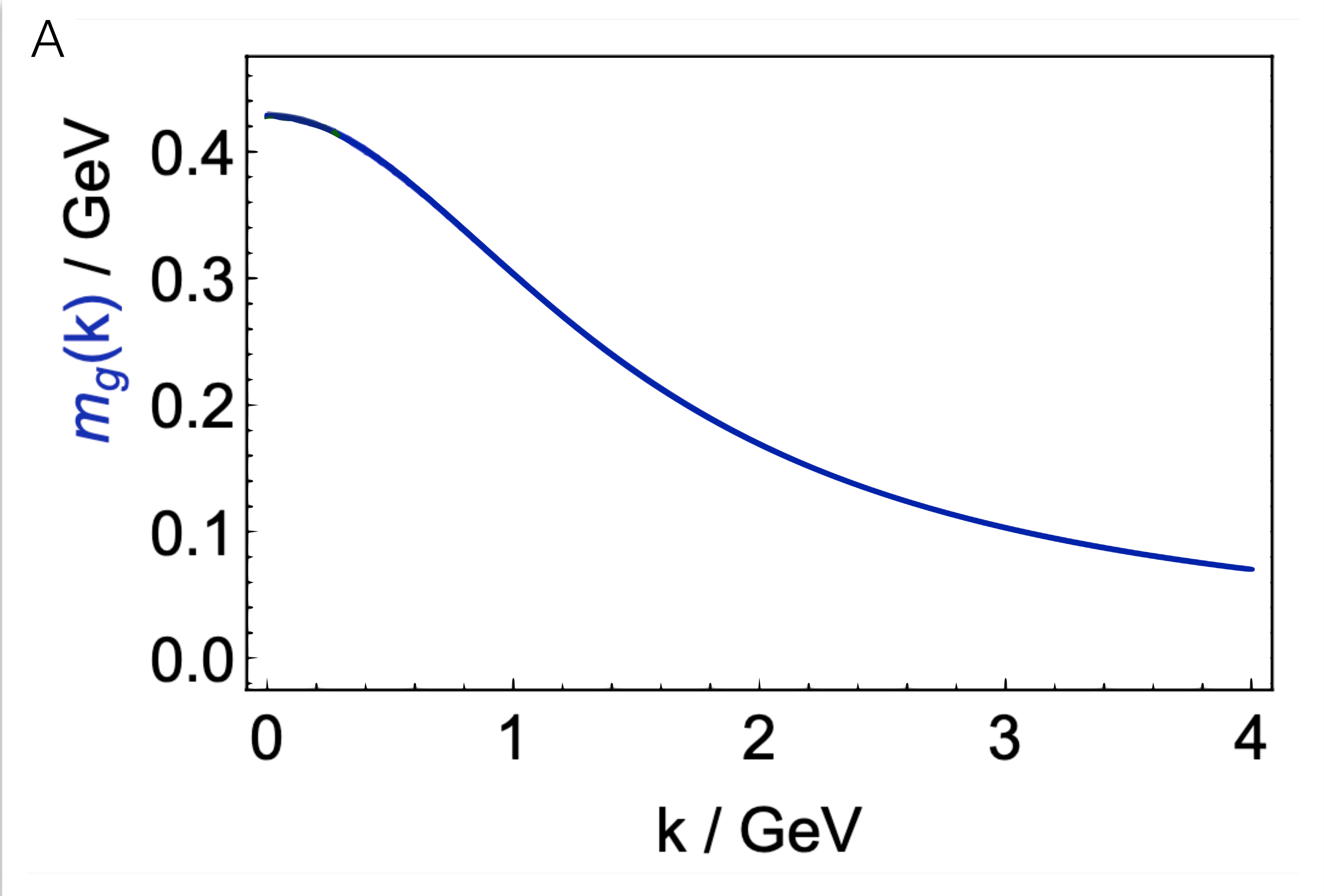

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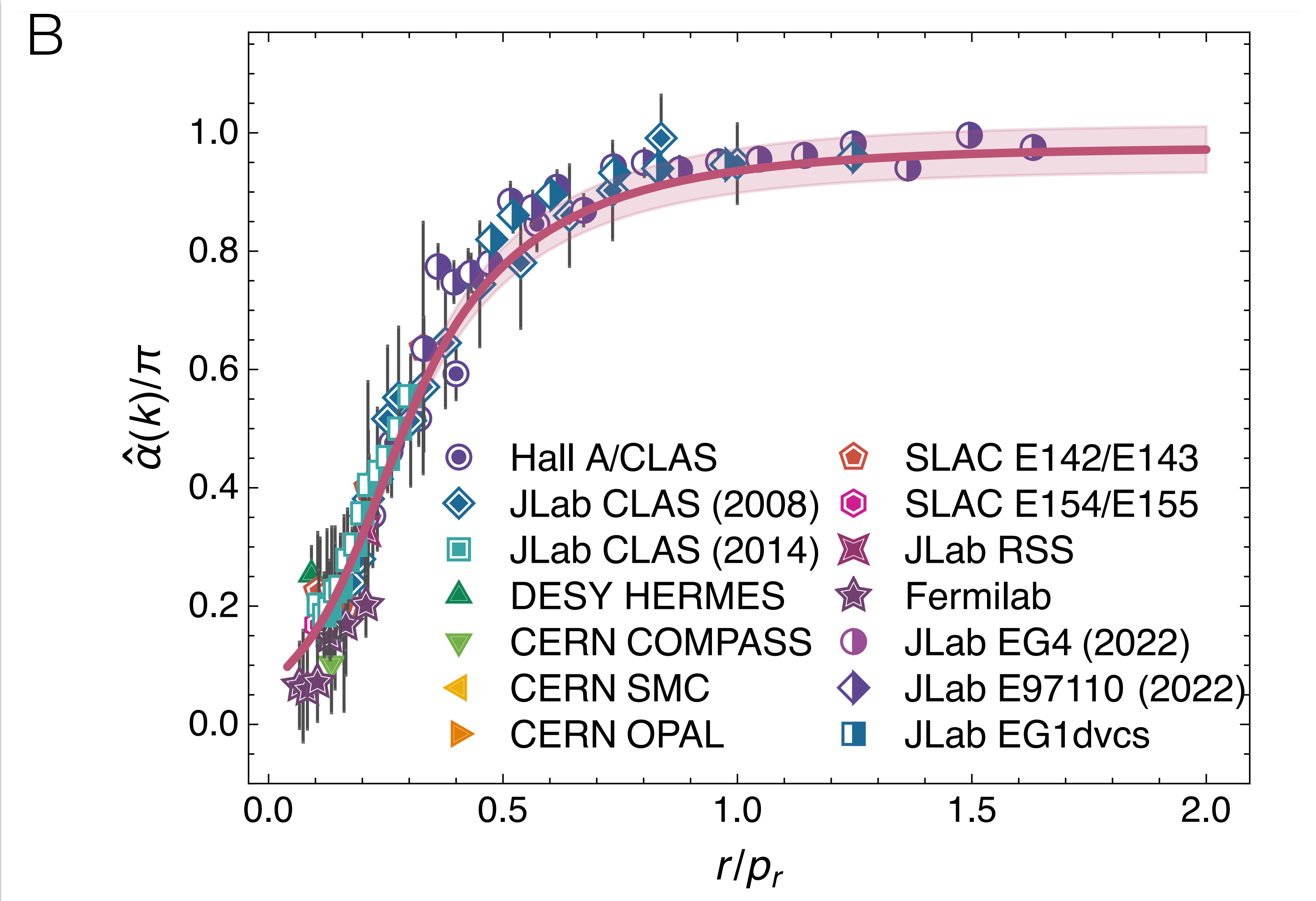

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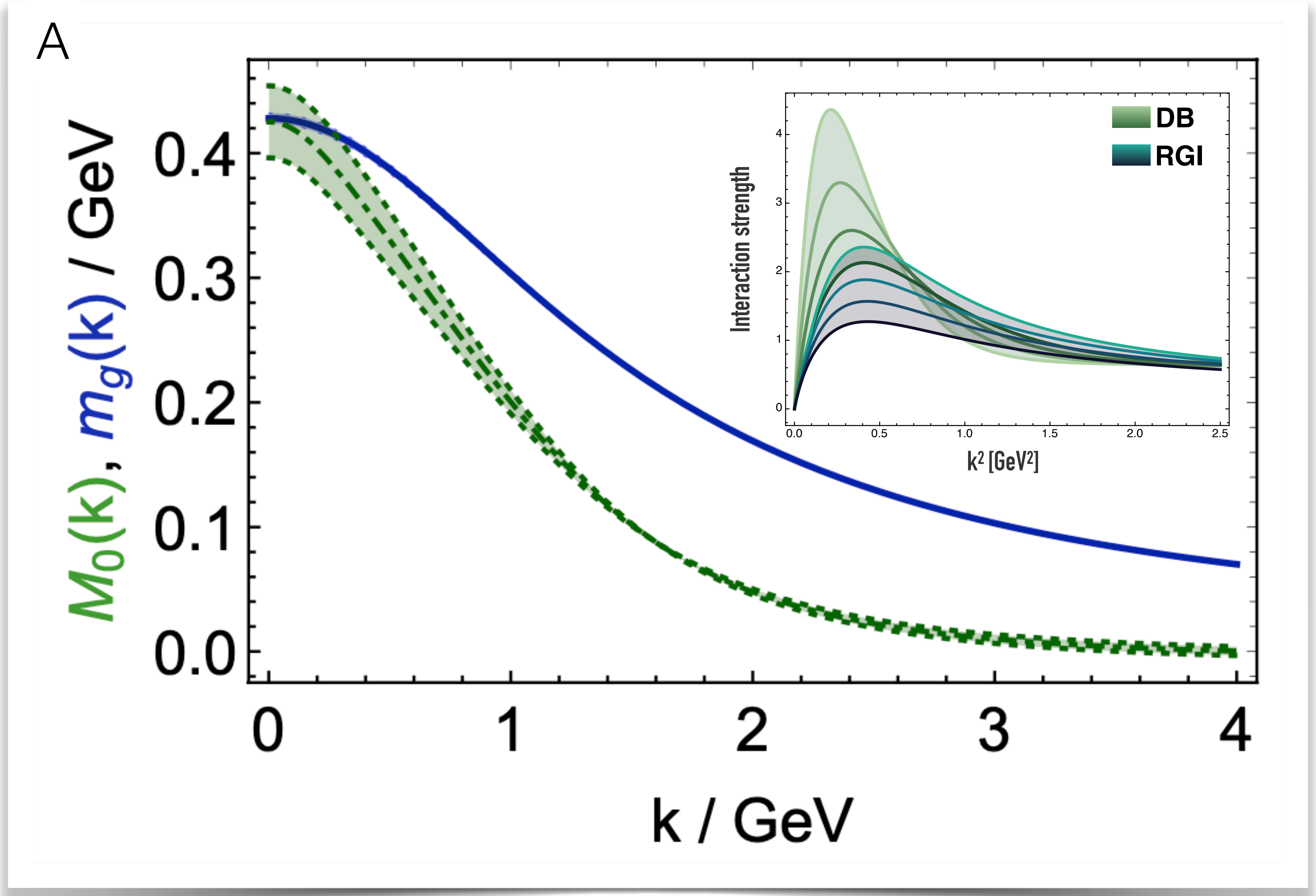

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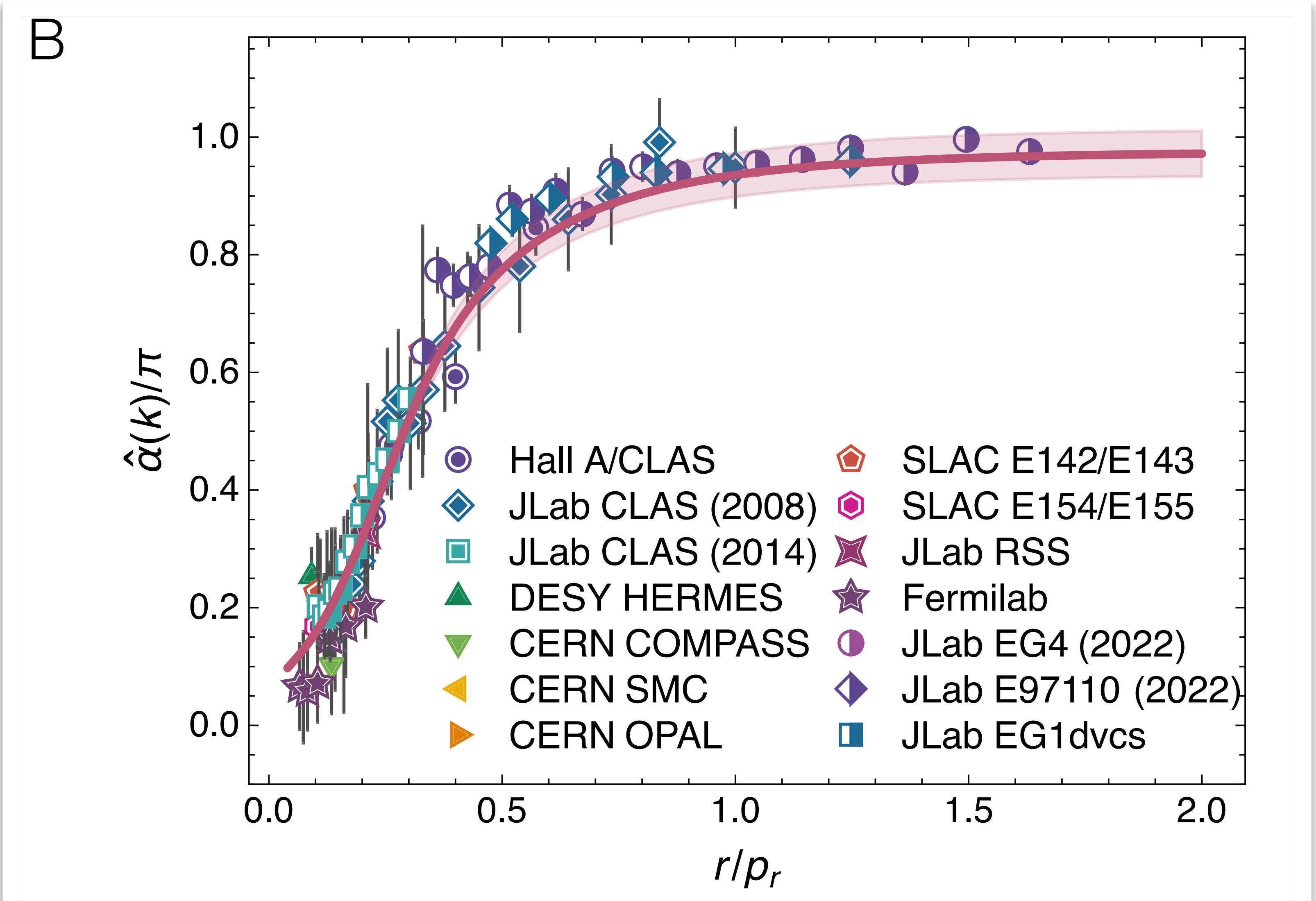

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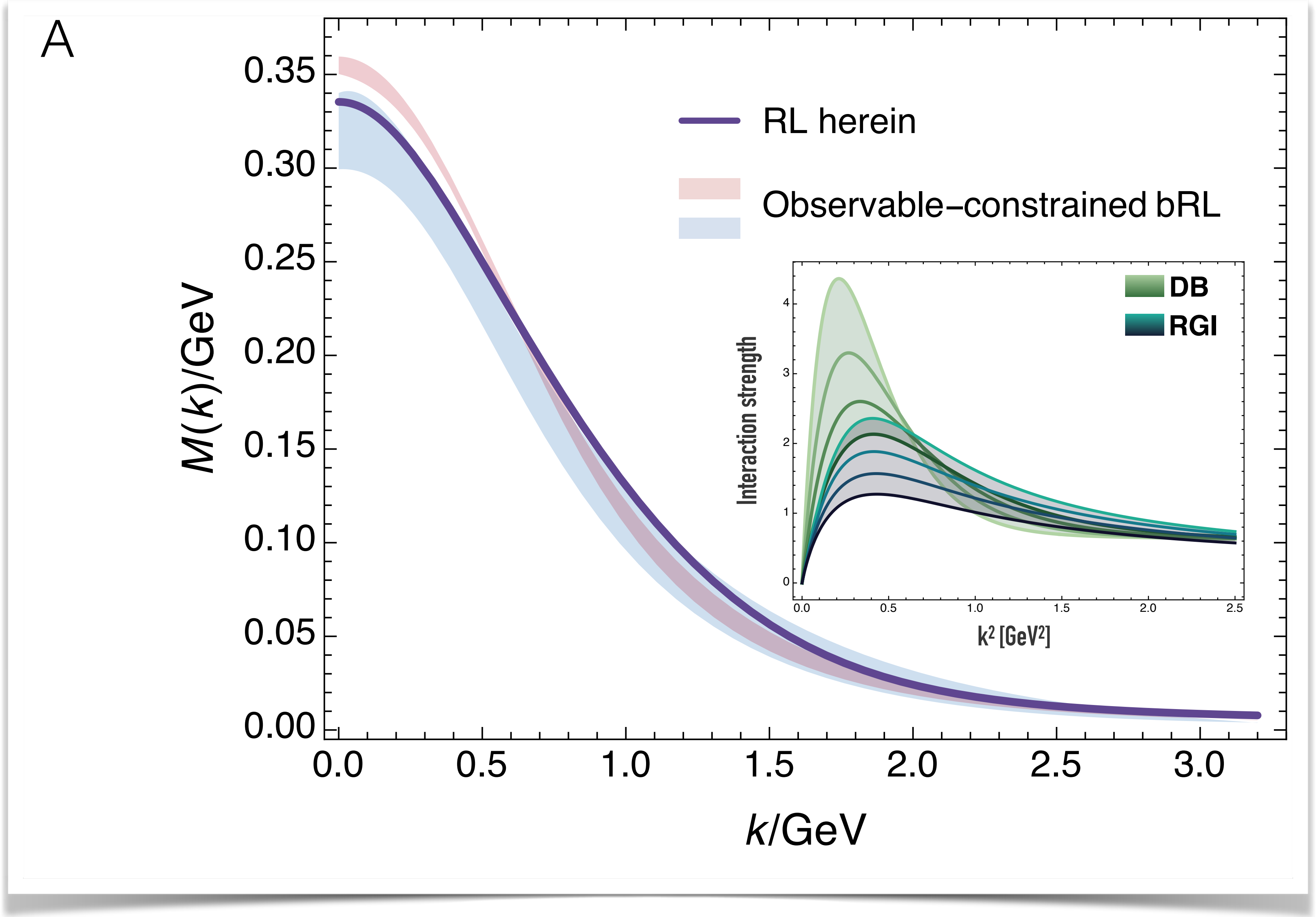

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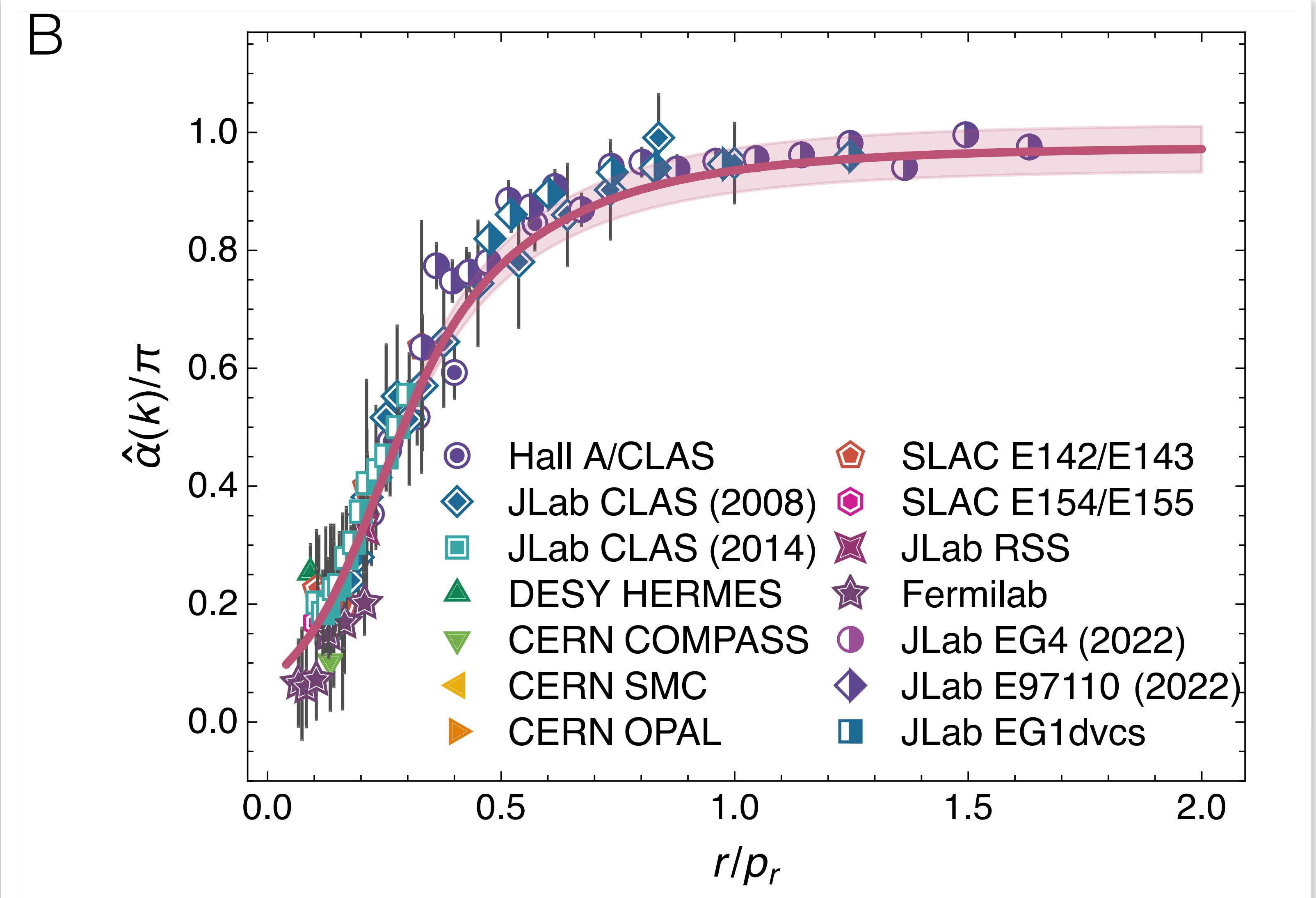

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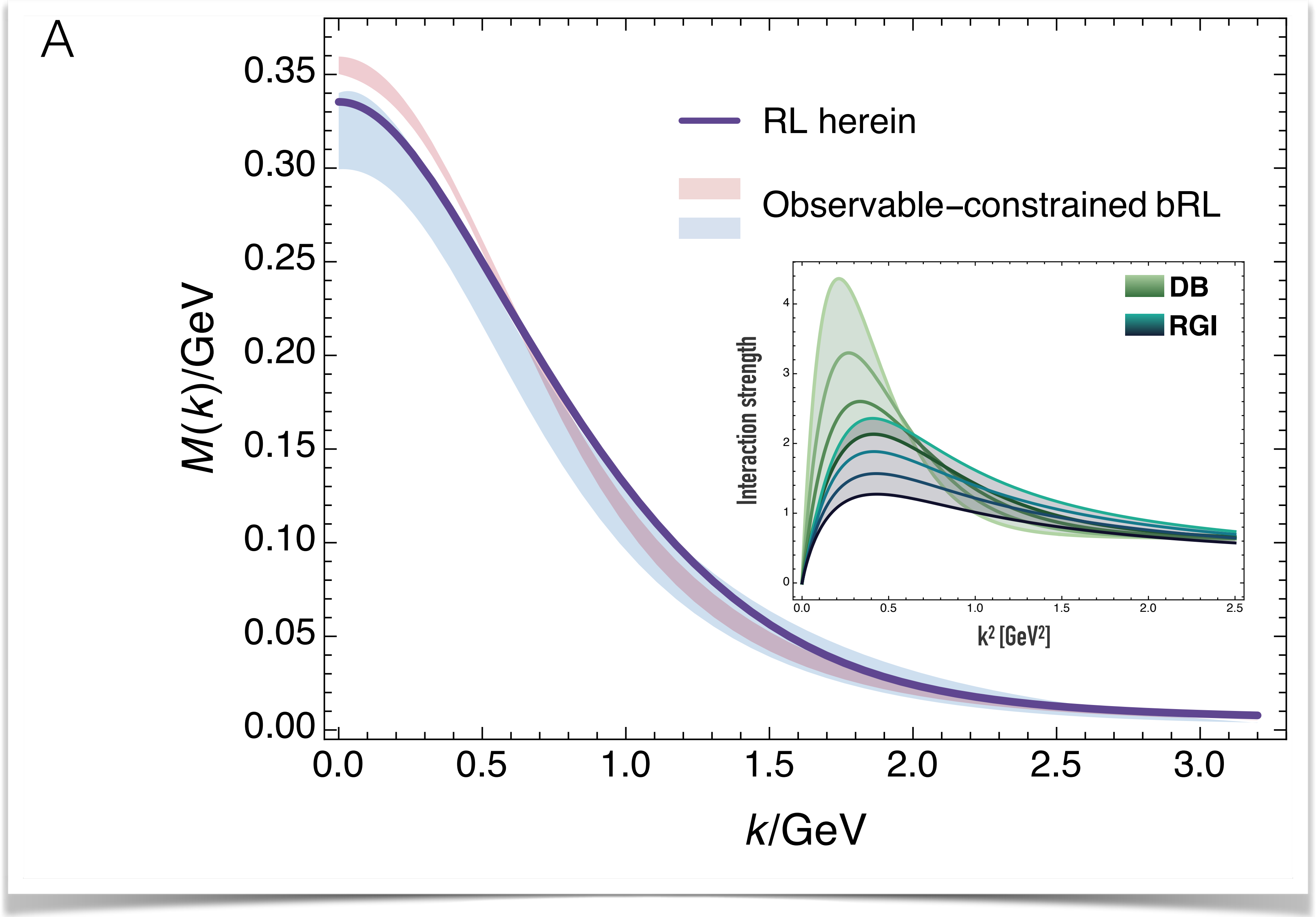

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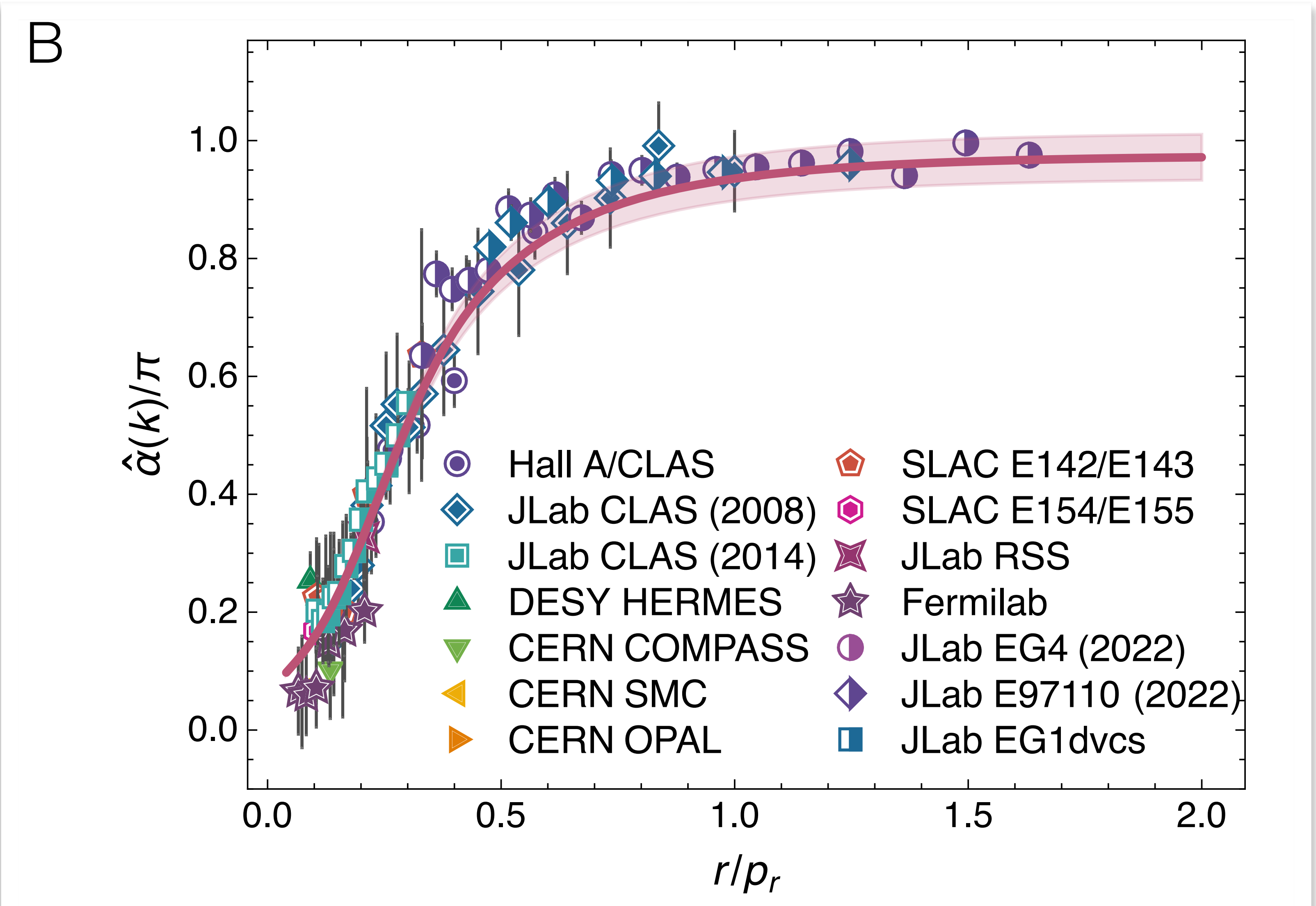

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but no size prescribed…

① RGI MASSES

40 years+ non-perturbative methods uncover the size of the gluon mass

$m_g = 0.43(1)$ GeV

Aguilar et al., EPJC 80 (2020)

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❷ PI EFFECTIVE CHARGE

owing to the emergence of a non-zero gluon mass scale a process independent effective charge emerges

$$\widehat{\alpha}(s) = \frac{4\pi}{(11-2n_f/3)\log[\mathcal{K}^2(s)/\Lambda^2]}, \quad \mathcal{K}^2(s) = \frac{a_0^2 + a_1 s + s^2}{b_0 + s}$$

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Brodsky, Deur, Roberts, SCIAM 05 (2024)

QCD LAGRANGIAN

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,s,d,\ldots} \bar{q}_j \left[\gamma_\mu D_\mu + m_j\right] q_j + \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \partial_\mu \bar{c}^a \partial_\mu c^a + g f^{abc} (\partial_\mu \bar{c}^a) A^b_\mu c^c$$

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A QCD EHM PRIMER

GLUON SELF-INTERACTION

pure-glue QCD displays a **mass gap**

$$m_g \sim 0.5 \text{ GeV}$$

Cornwall, PRD 26 (1982)

GAUGE SYMMETRY IS FINE

2-point STI can be still satisfied with

$$\Delta_{\mu\nu}(q) = \frac{P_{\mu\nu}(q)}{q^2[1+\Pi(q^2)]}, \quad q^\mu P_{\mu\nu}(q) = 0$$

$$\lim_{q^2 \to 0} q^2 \Pi(q^2) = m_g$$

("only" requires the presence of longitudinally coupled massless poles)

Schwinger, PR 125 and 128 (1962)

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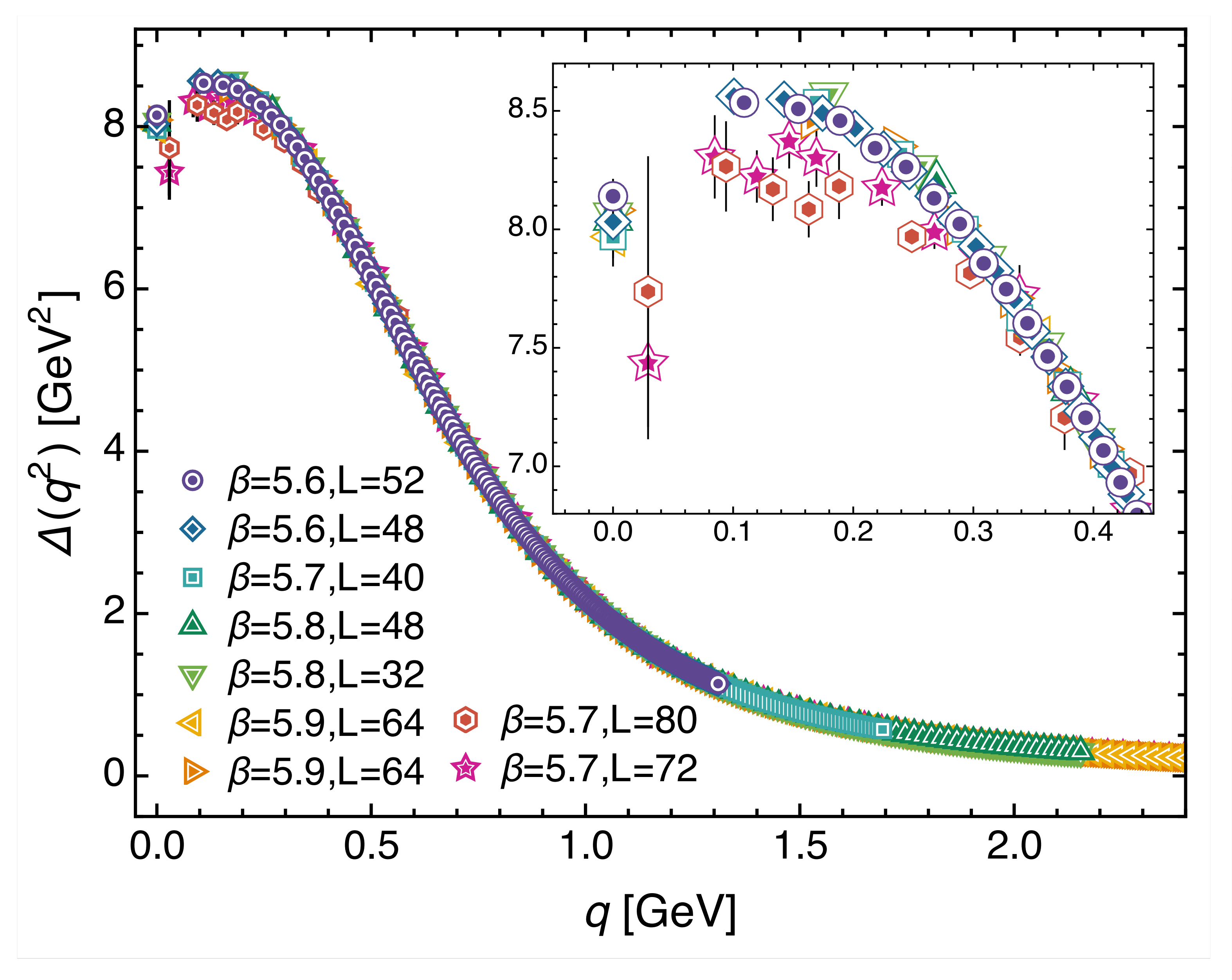

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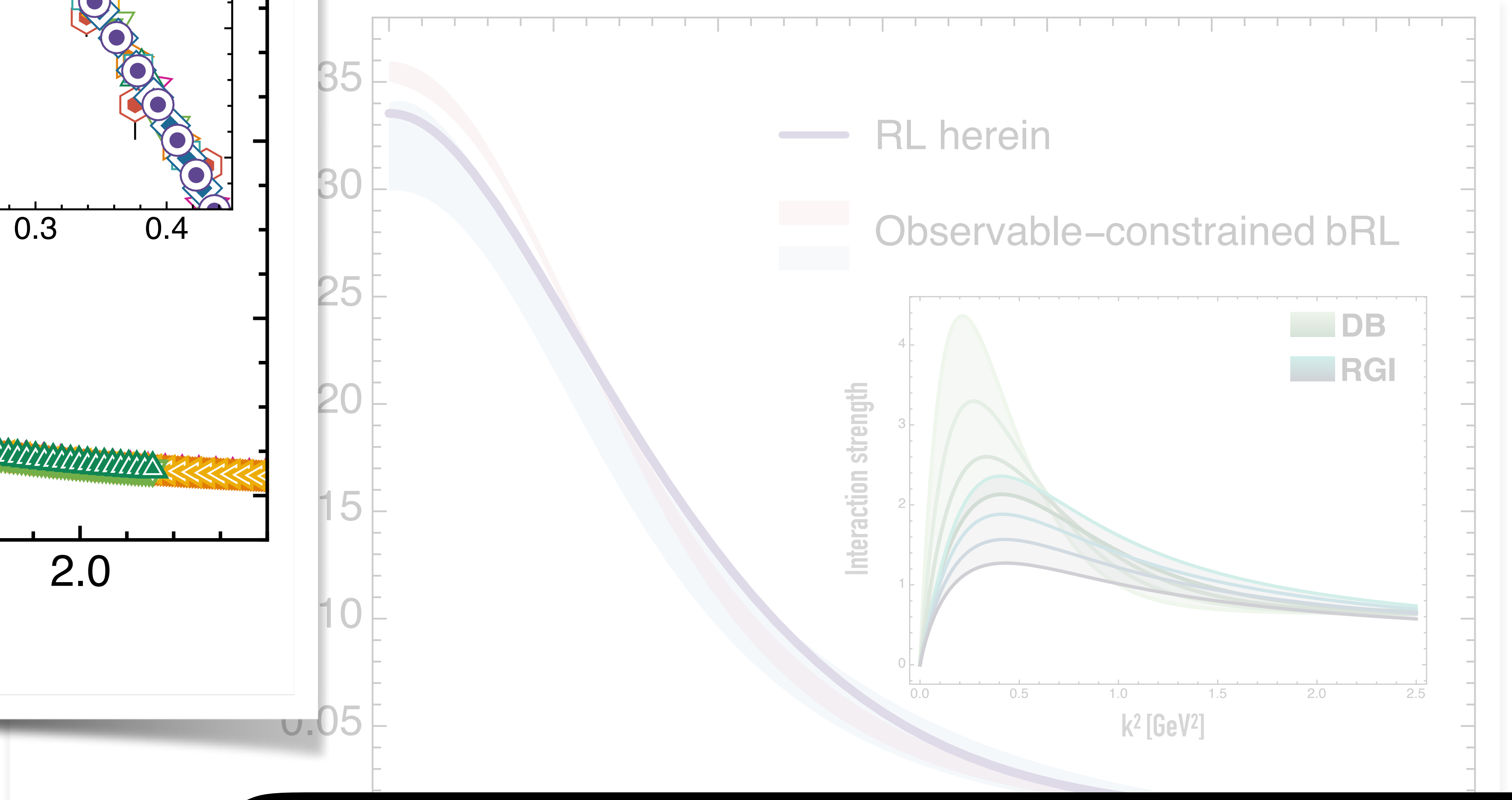

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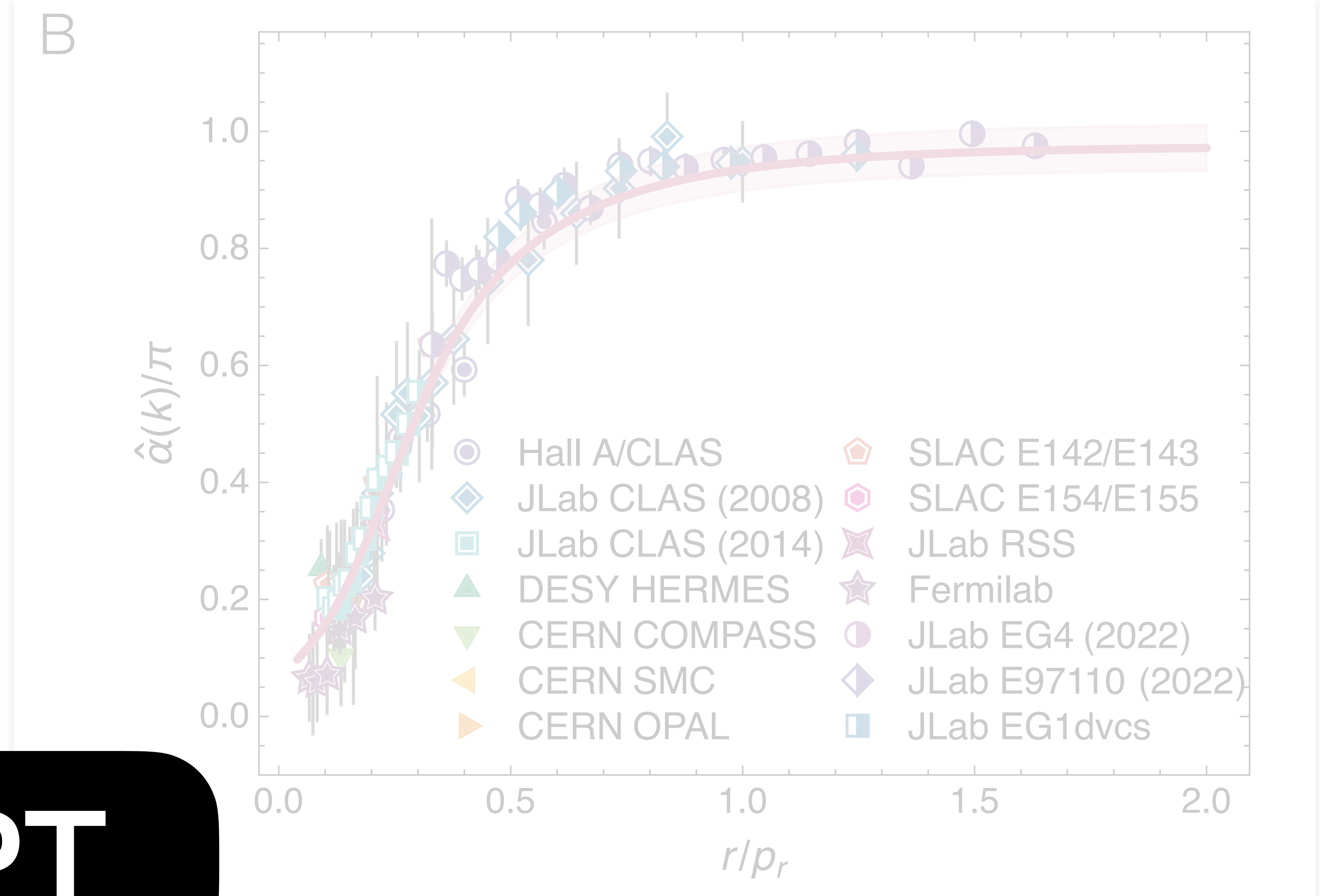

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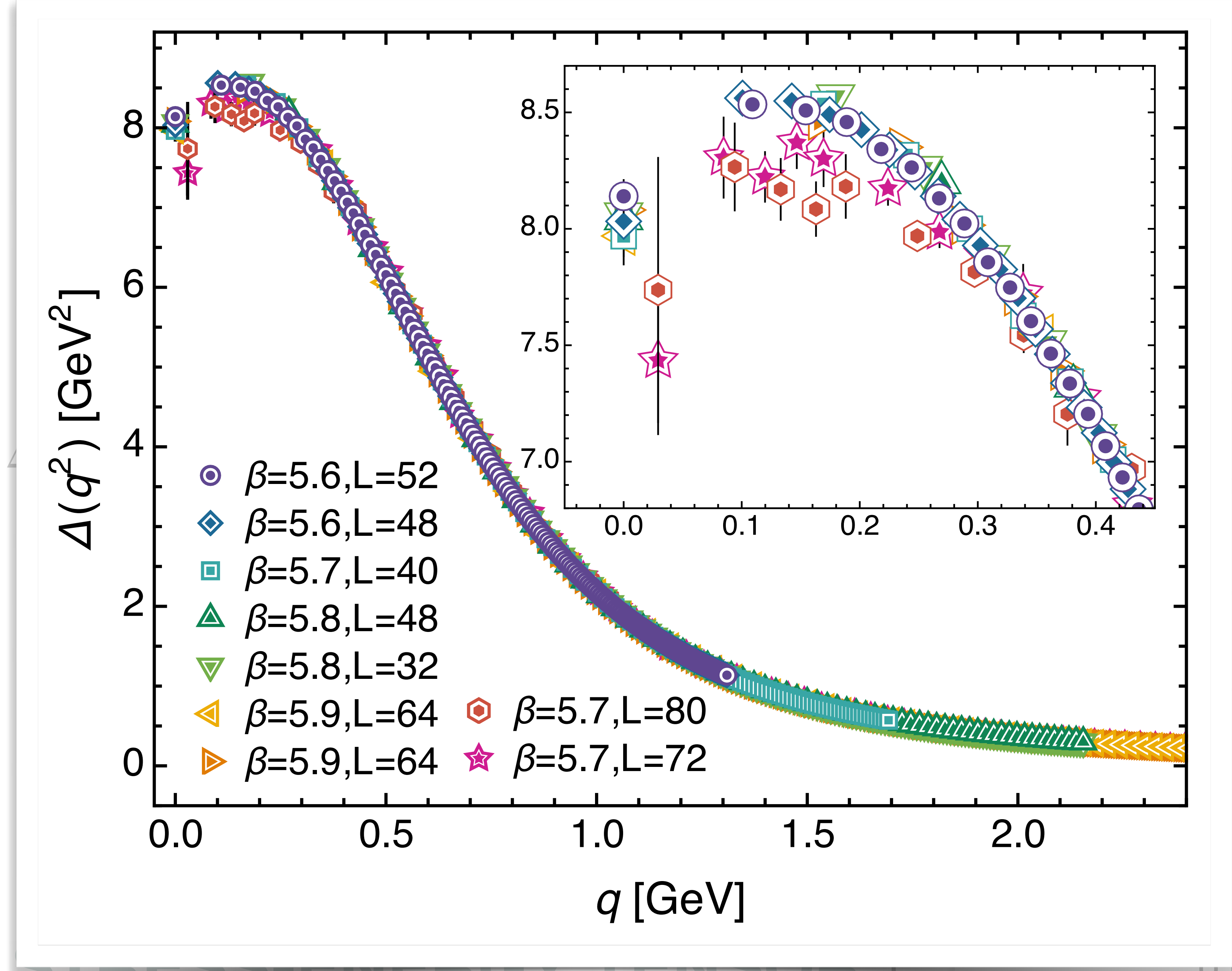

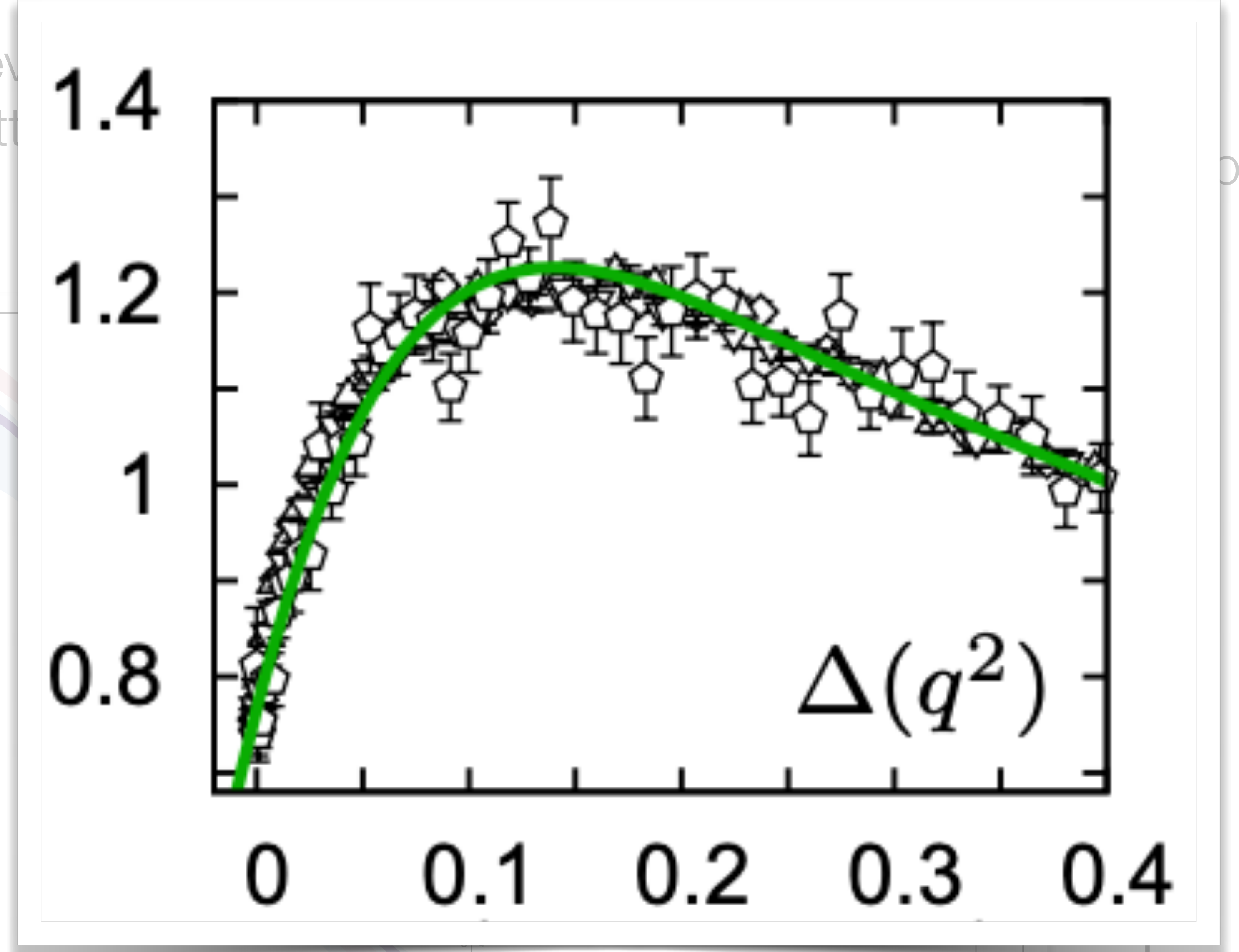

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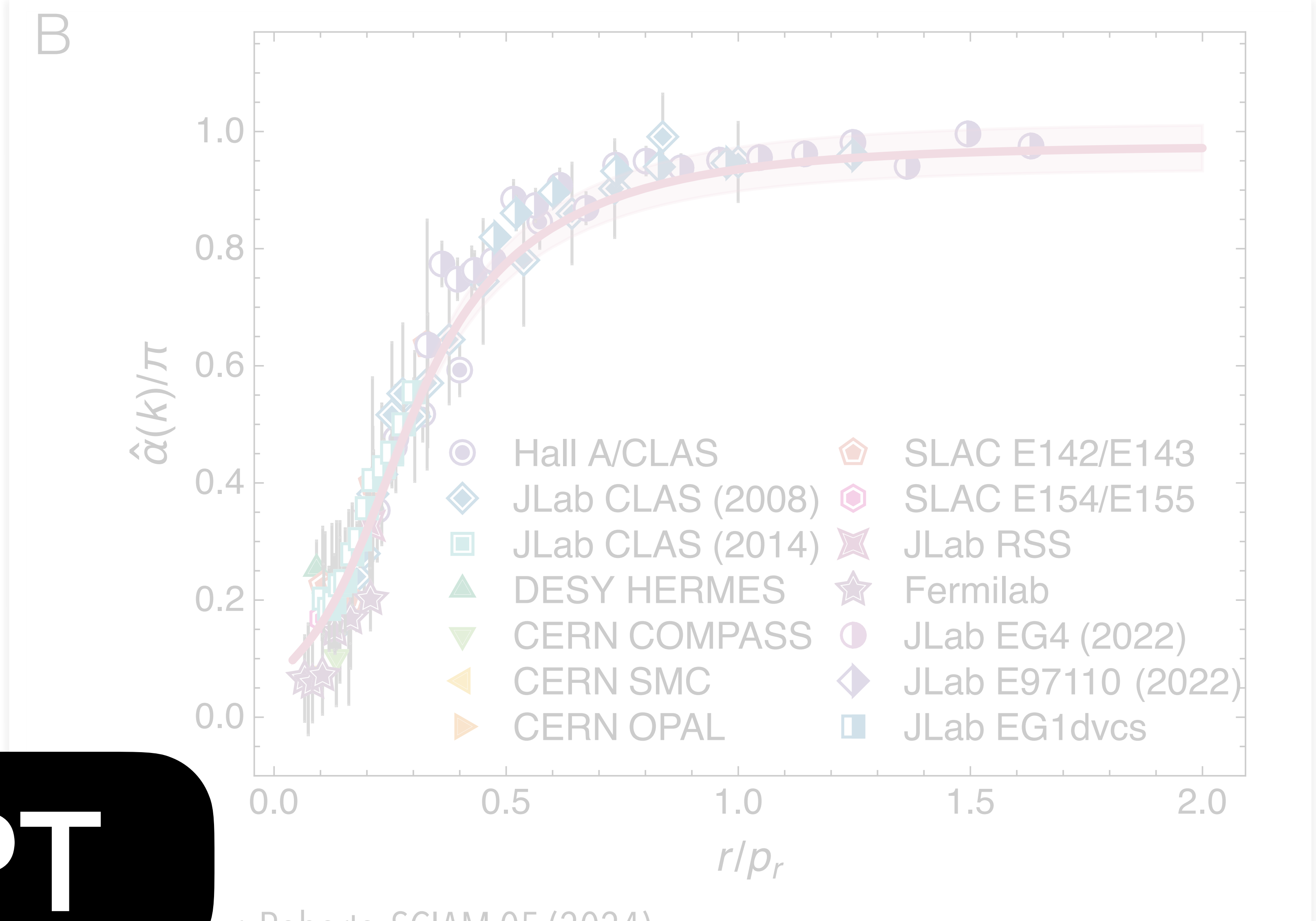

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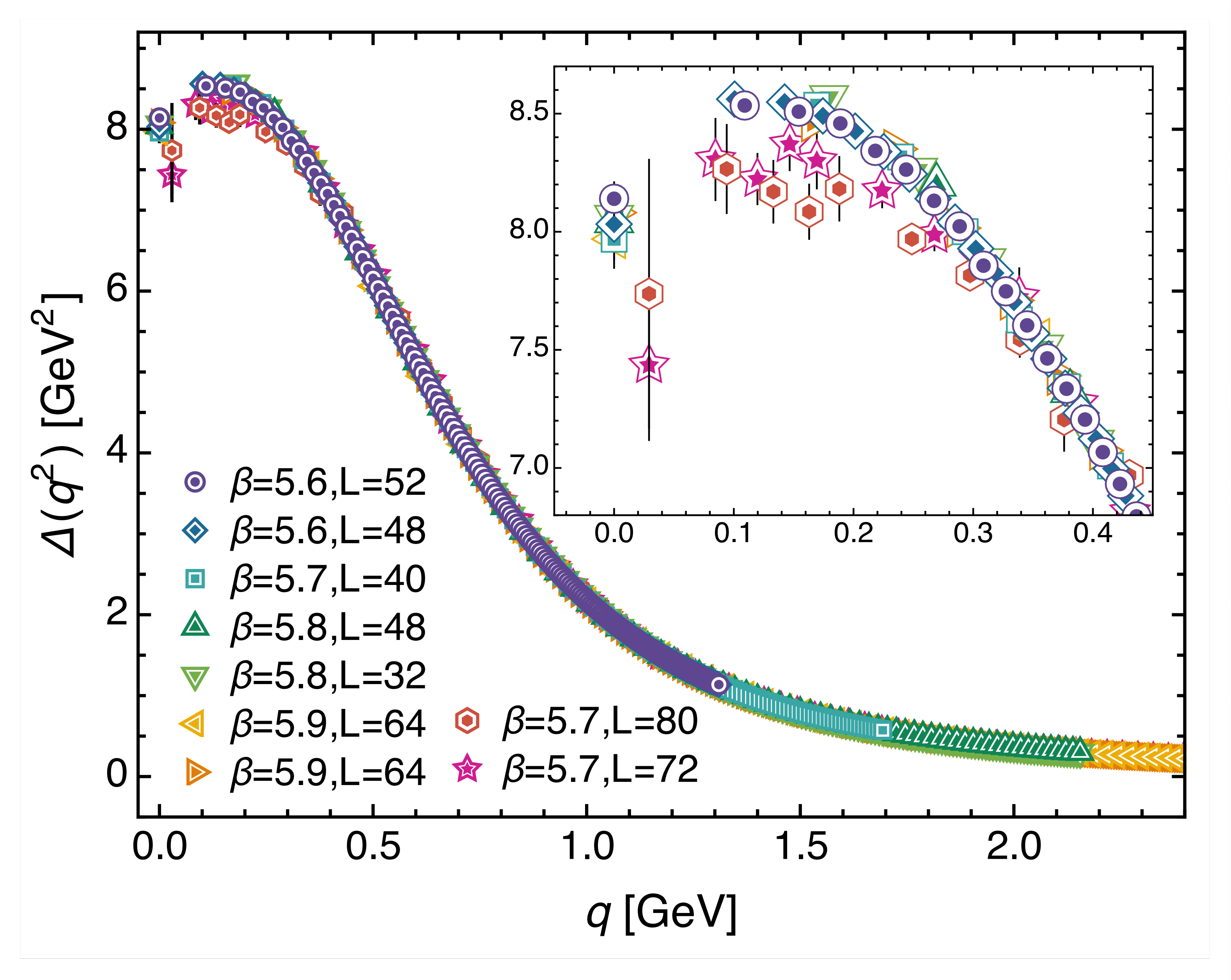

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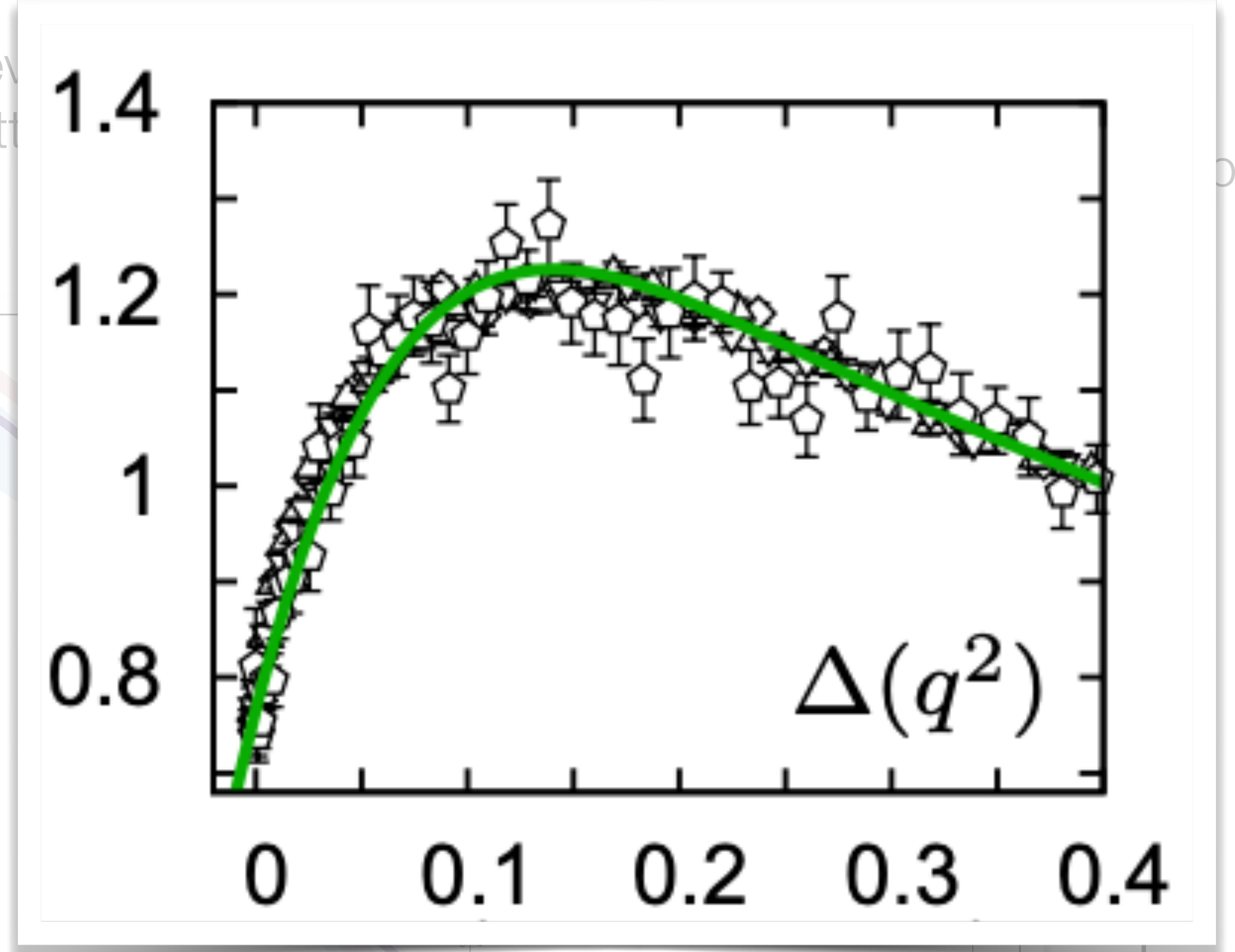

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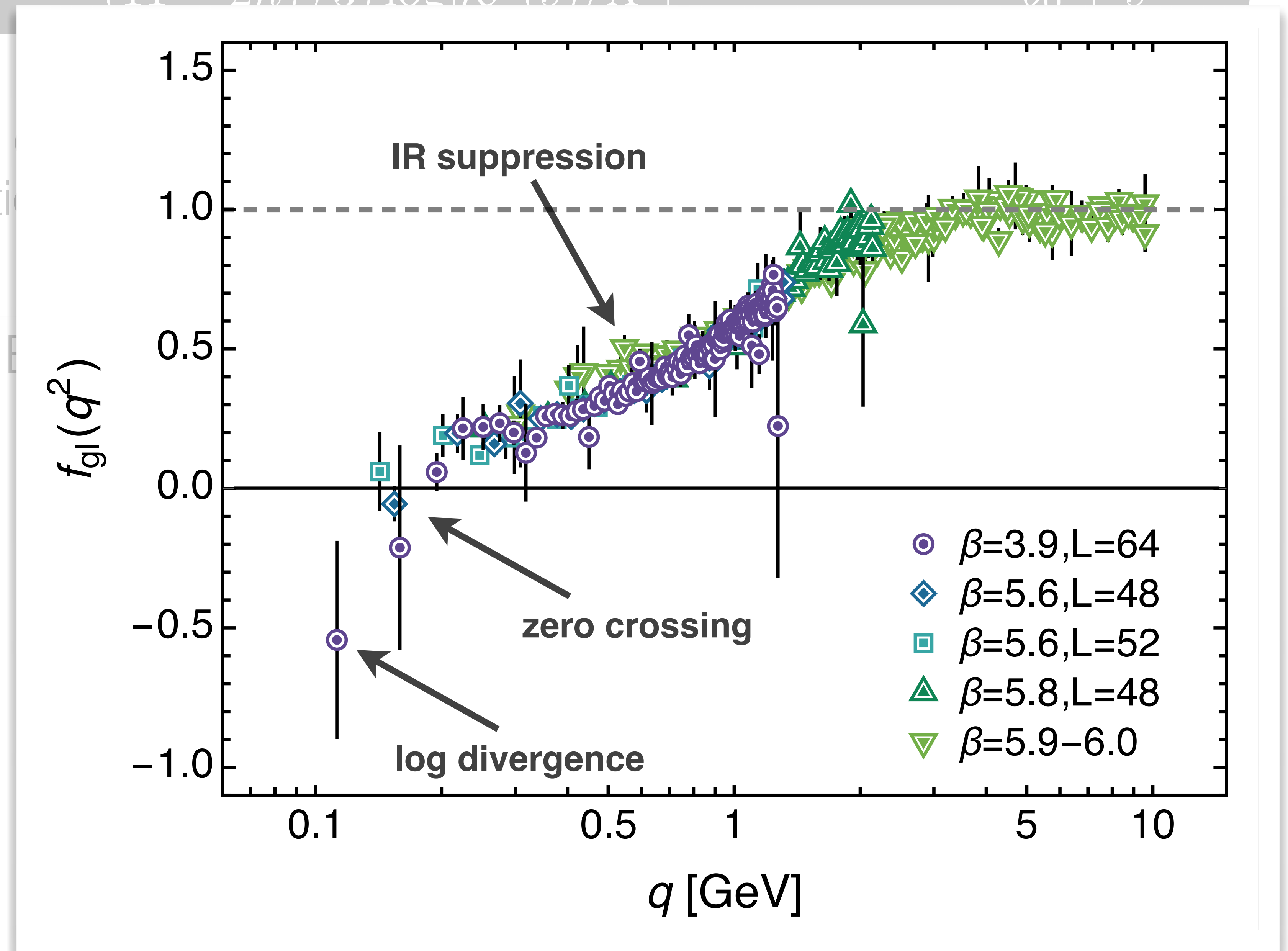


CERN SMC
CERN OPAL
JLab E97110 (2022)
JLab EG1dvcs

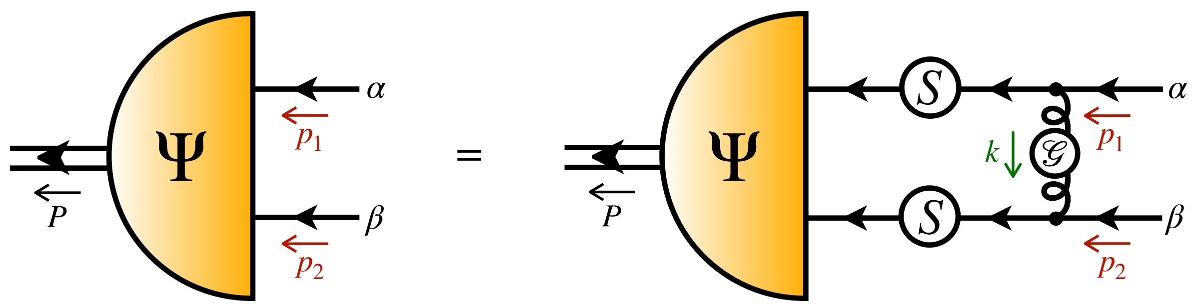
ur, Roberts, SCIAM 05 (2024)

BRST+$\overline{\text{BRST}}$=BFM=PT

π/K FORM FACTORS
ELECTROMAGNETIC

Yao, DB, Roberts, PLB 855 (2024)

BETHE-SALPETER EQUATION



dressed-quark propagators singularities limit mass of bound-state

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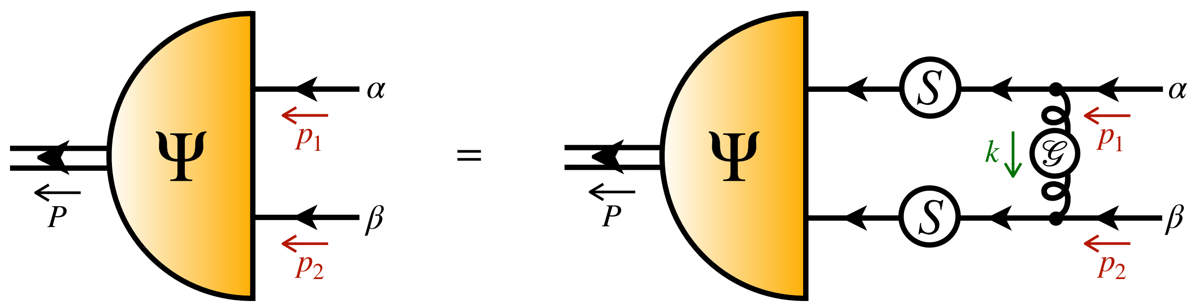

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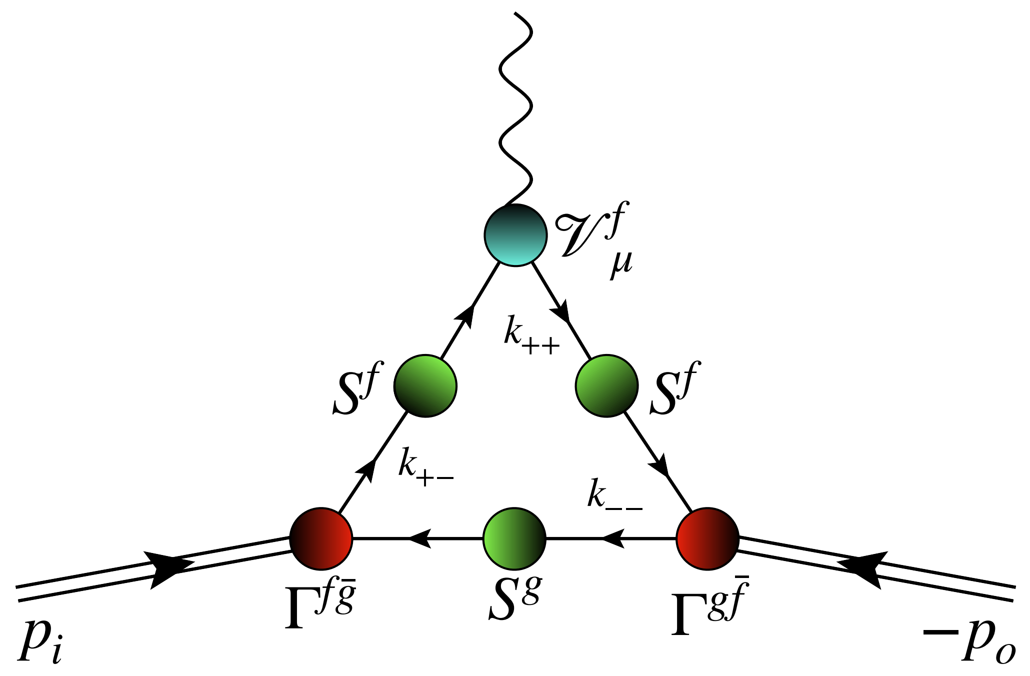
singularities move in the complex k^2 domain sampled by the bound-state equations

Q^2_{max}

TRIANGULAR DIAGRAM



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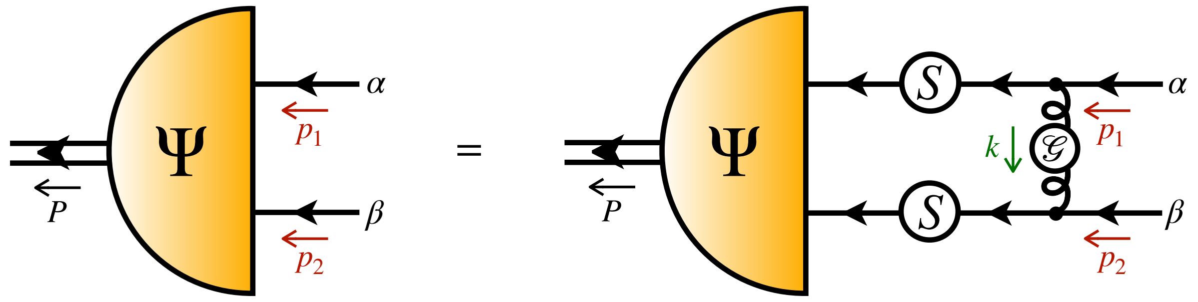

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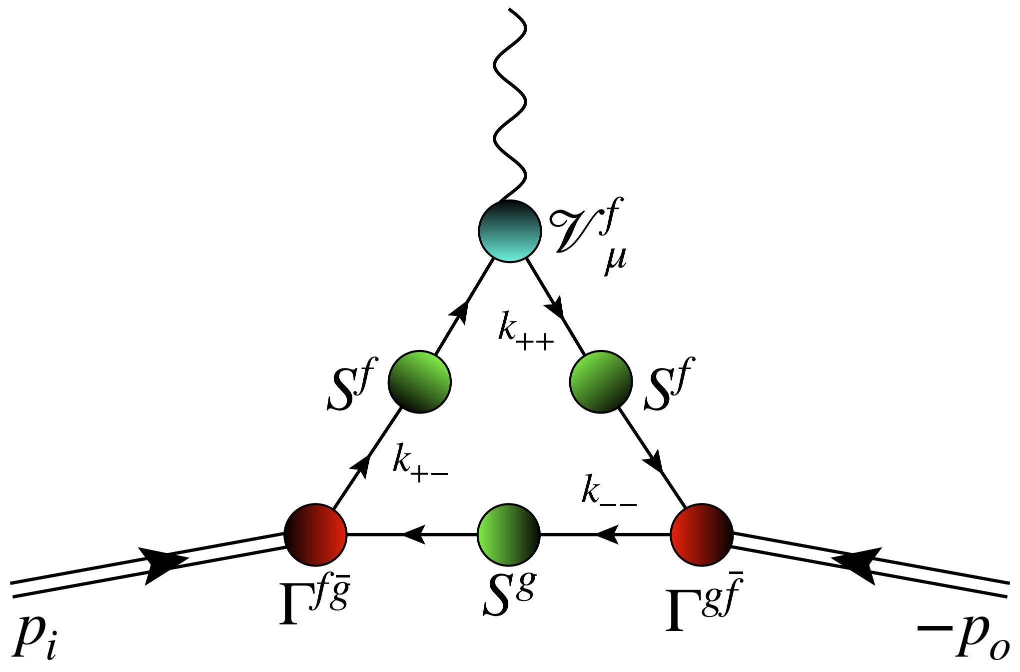

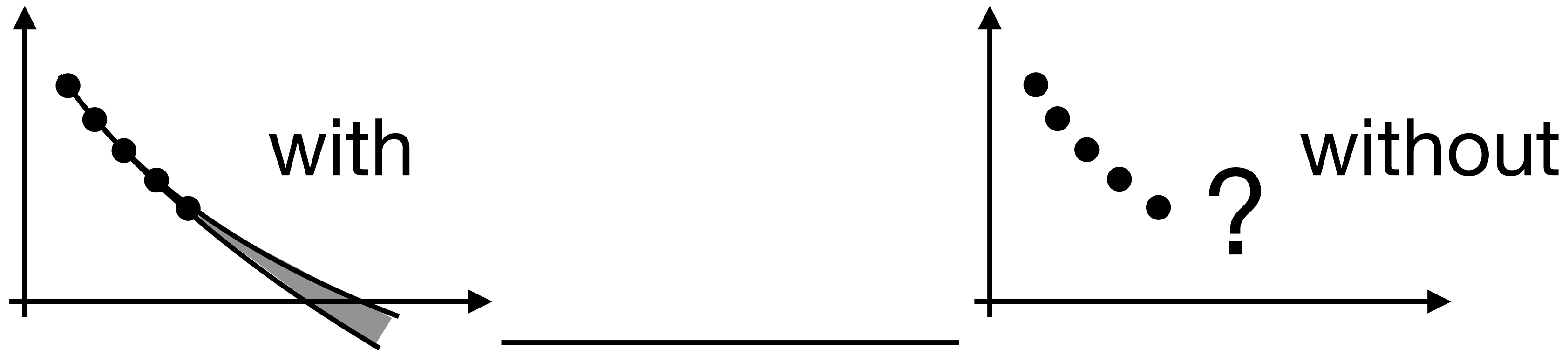

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Schlessinger, PR 167 (1968)

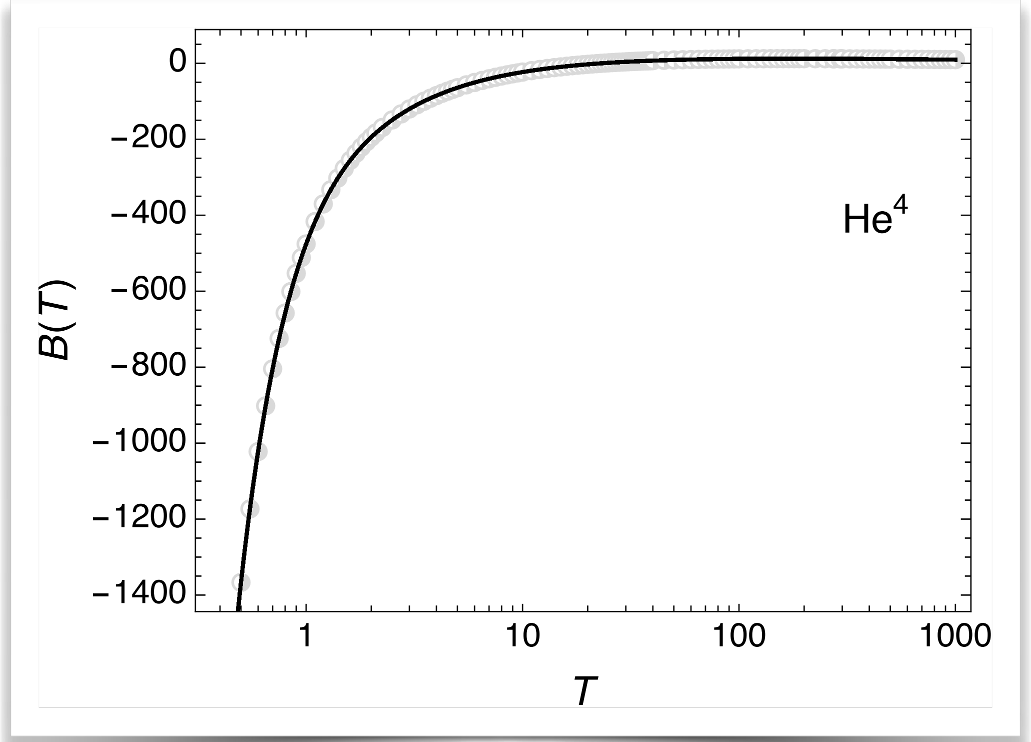
PRECISION

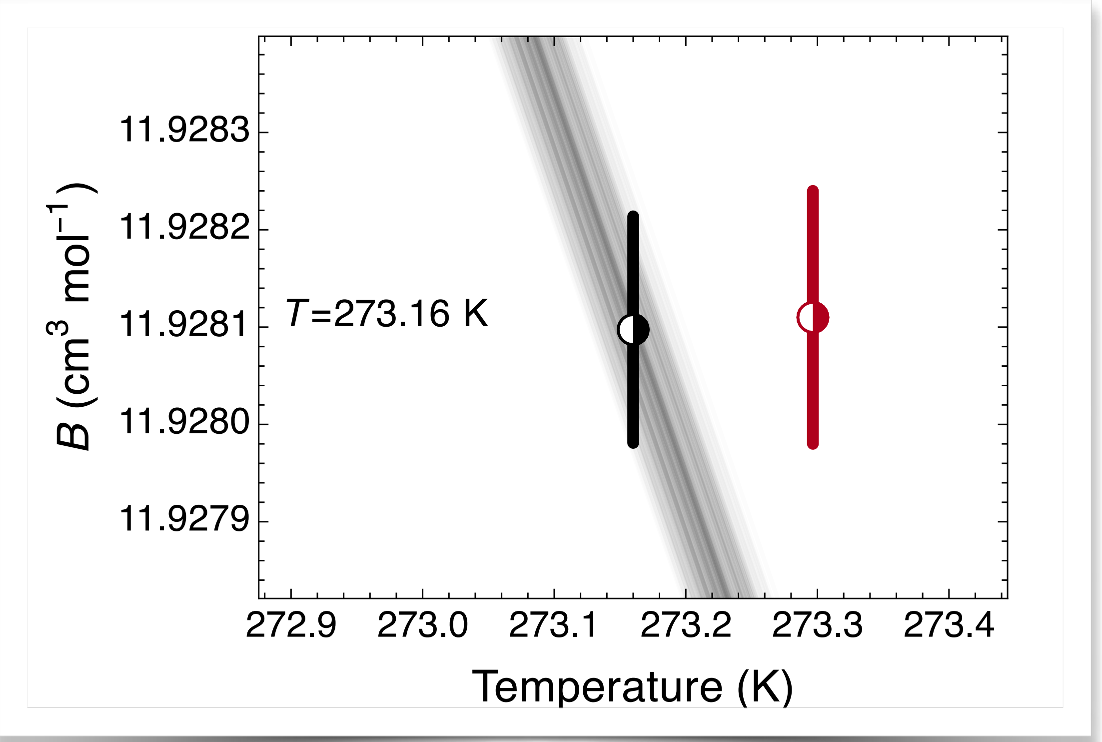
TEMPERATURE METROLOGY

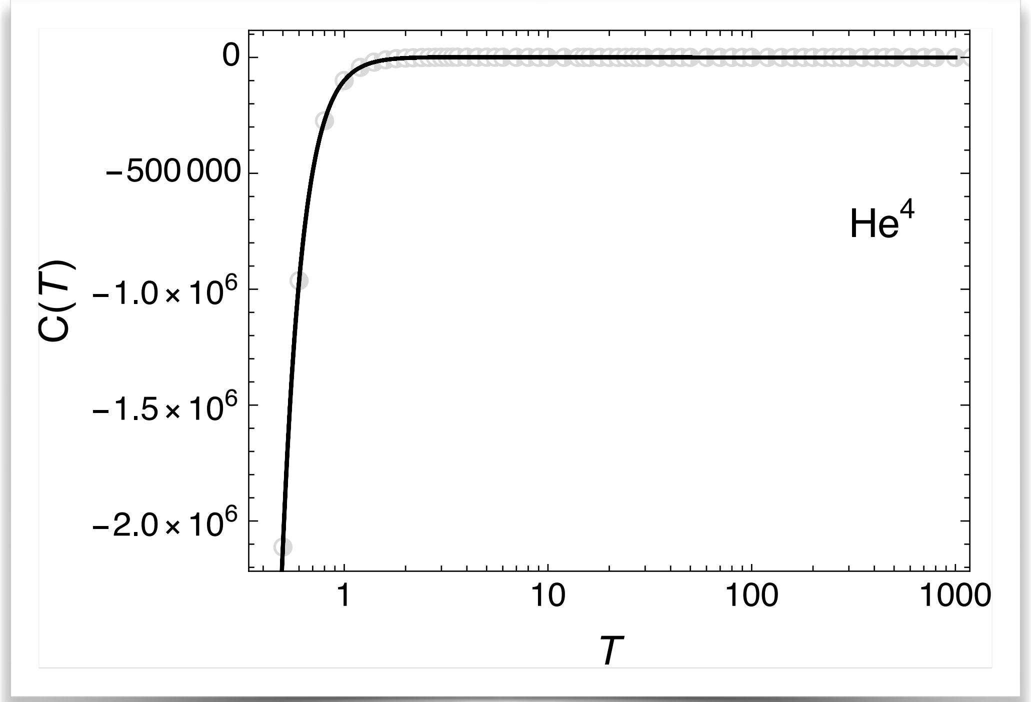
VIRIAL COEFFICIENTS

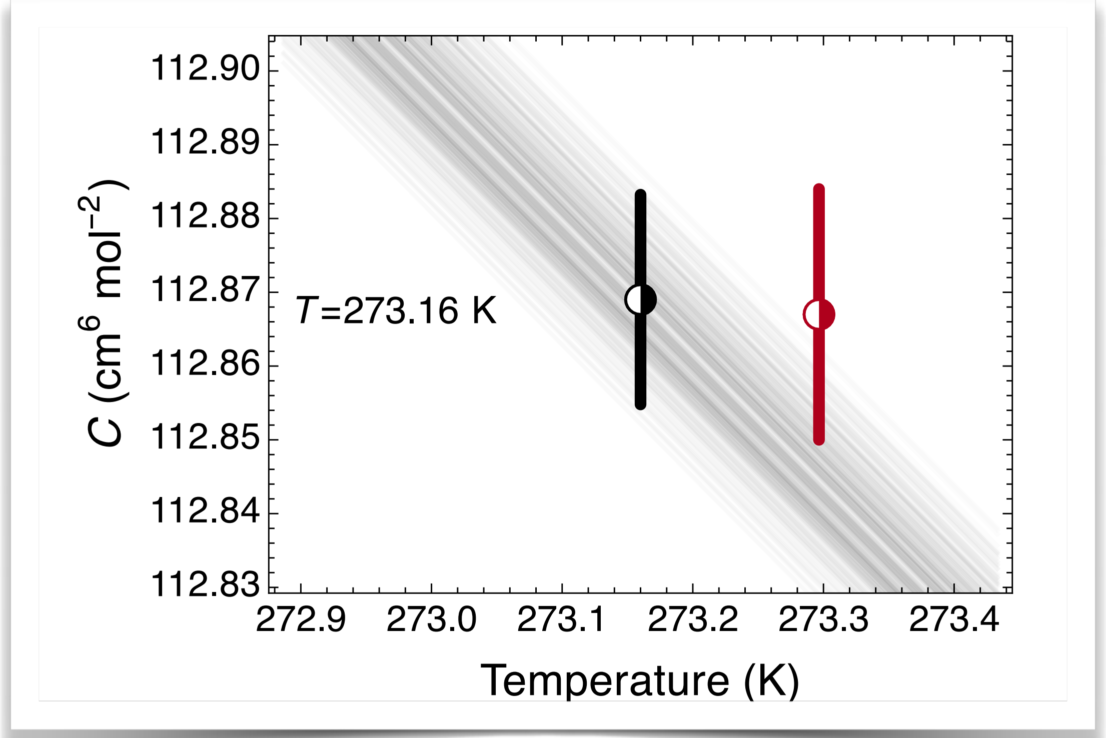
Describe the deviation from ideal-gas behavior

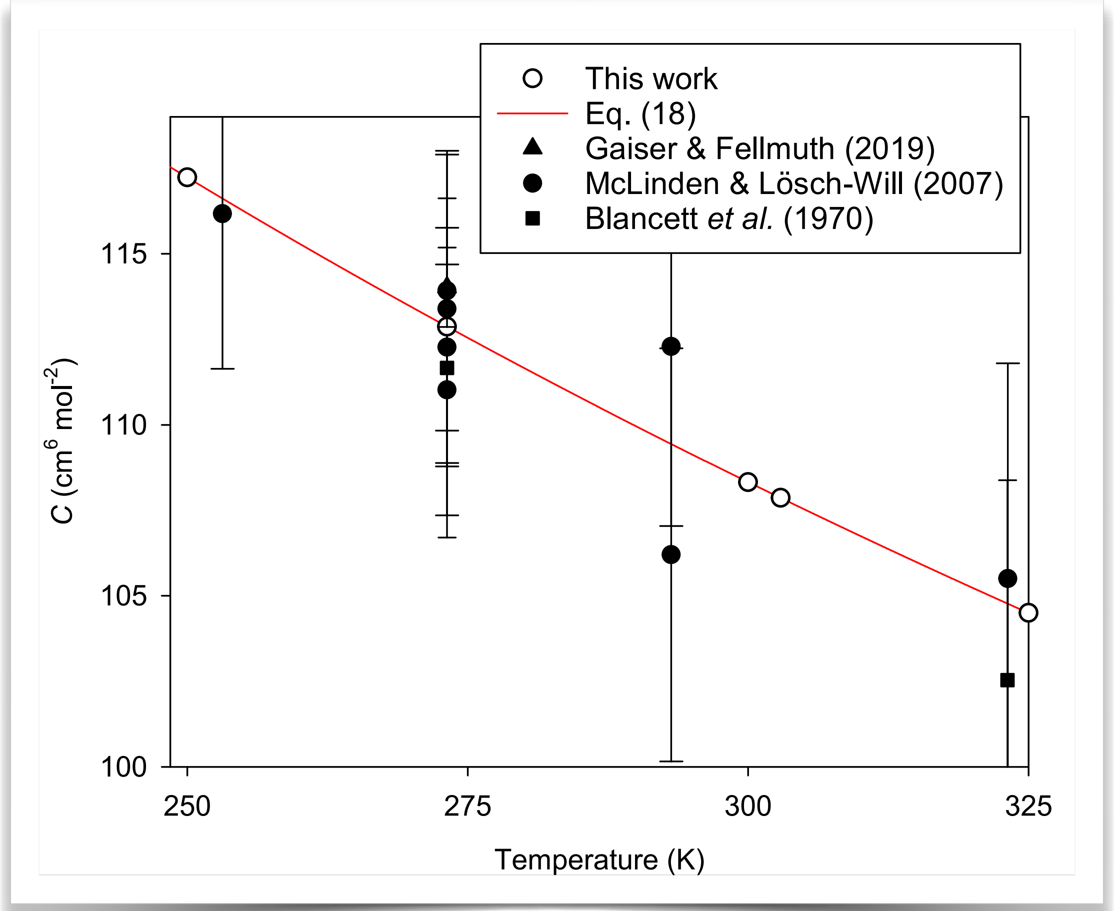
$$\frac{p}{\rho RT} = 1 + B(T)\rho + C(T)\rho^2 + \cdots$$











DB, Garberoglio, Harvey, Jour.Chem.Phys. 160 (24)

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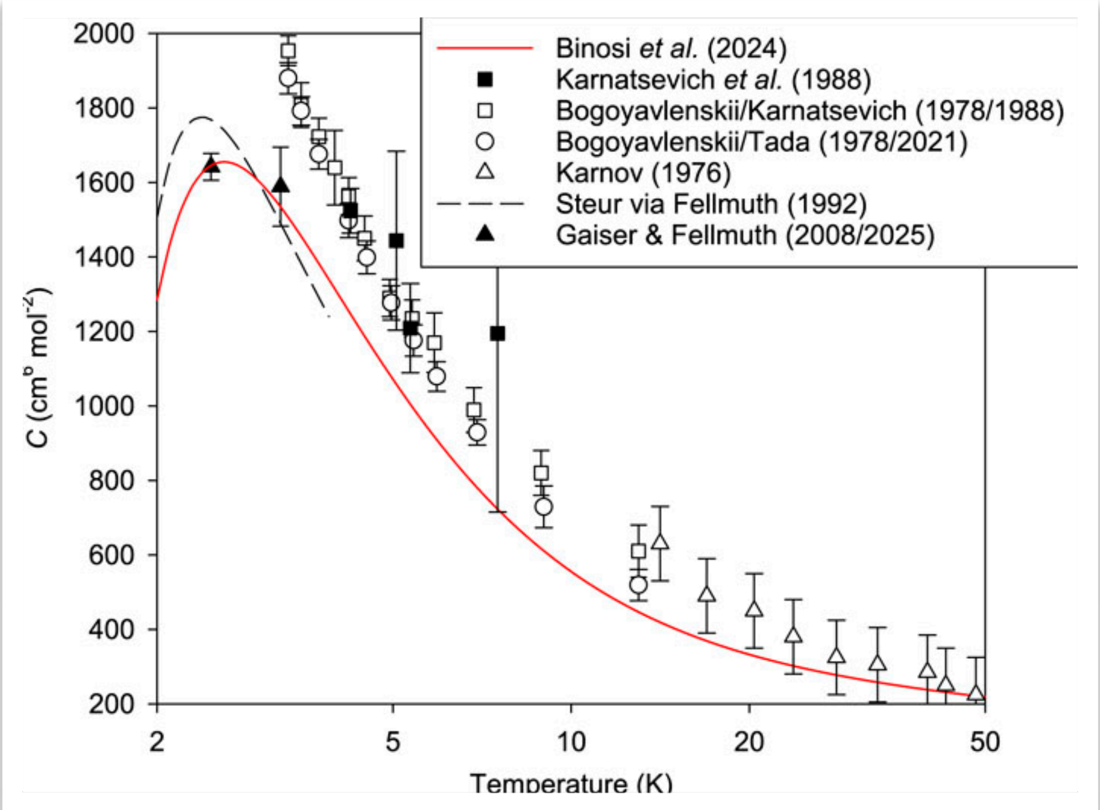

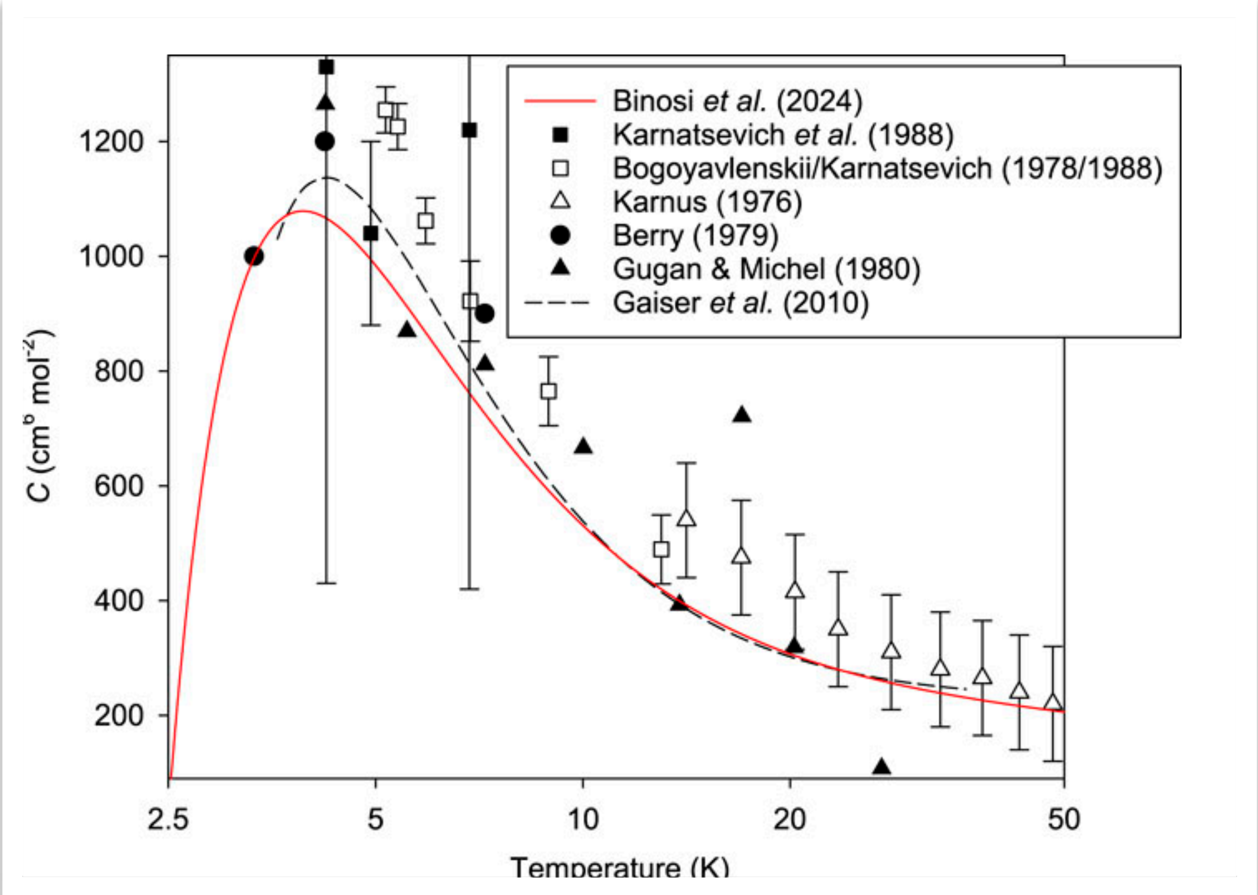

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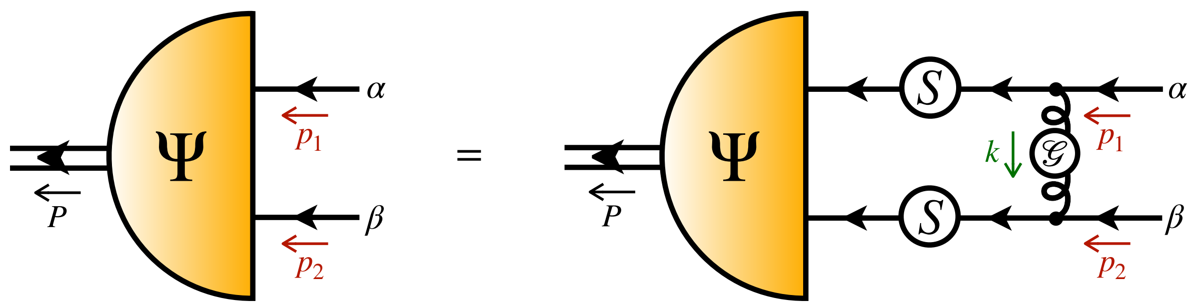

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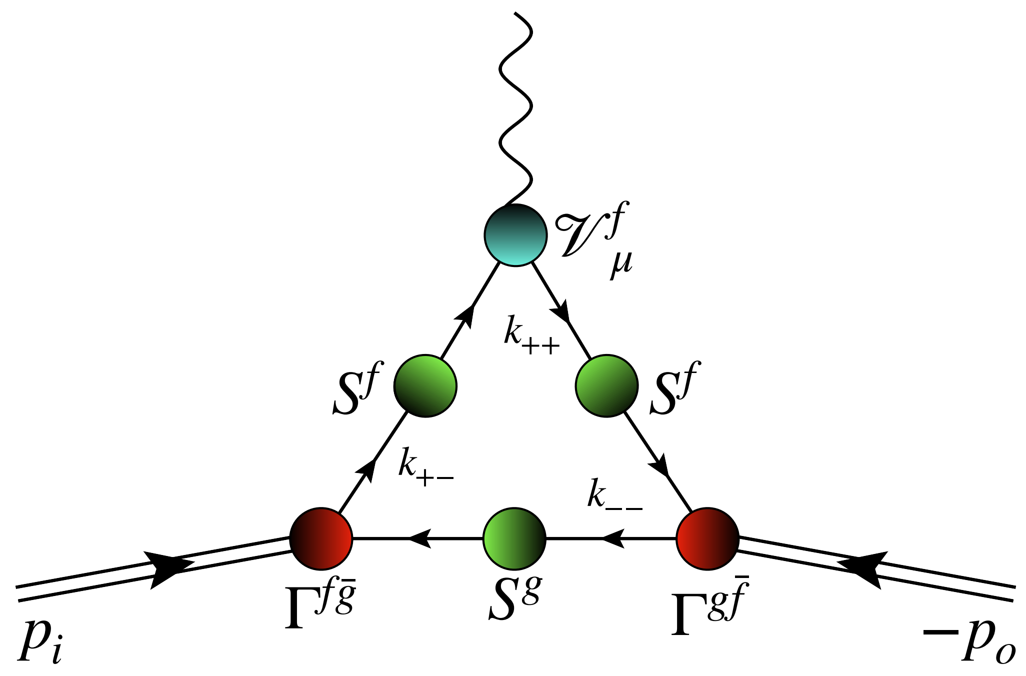

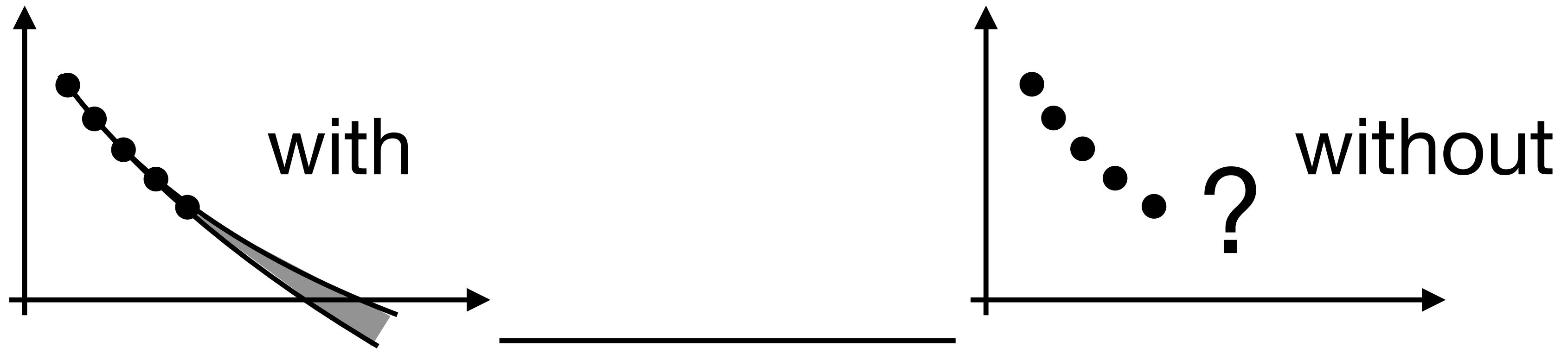

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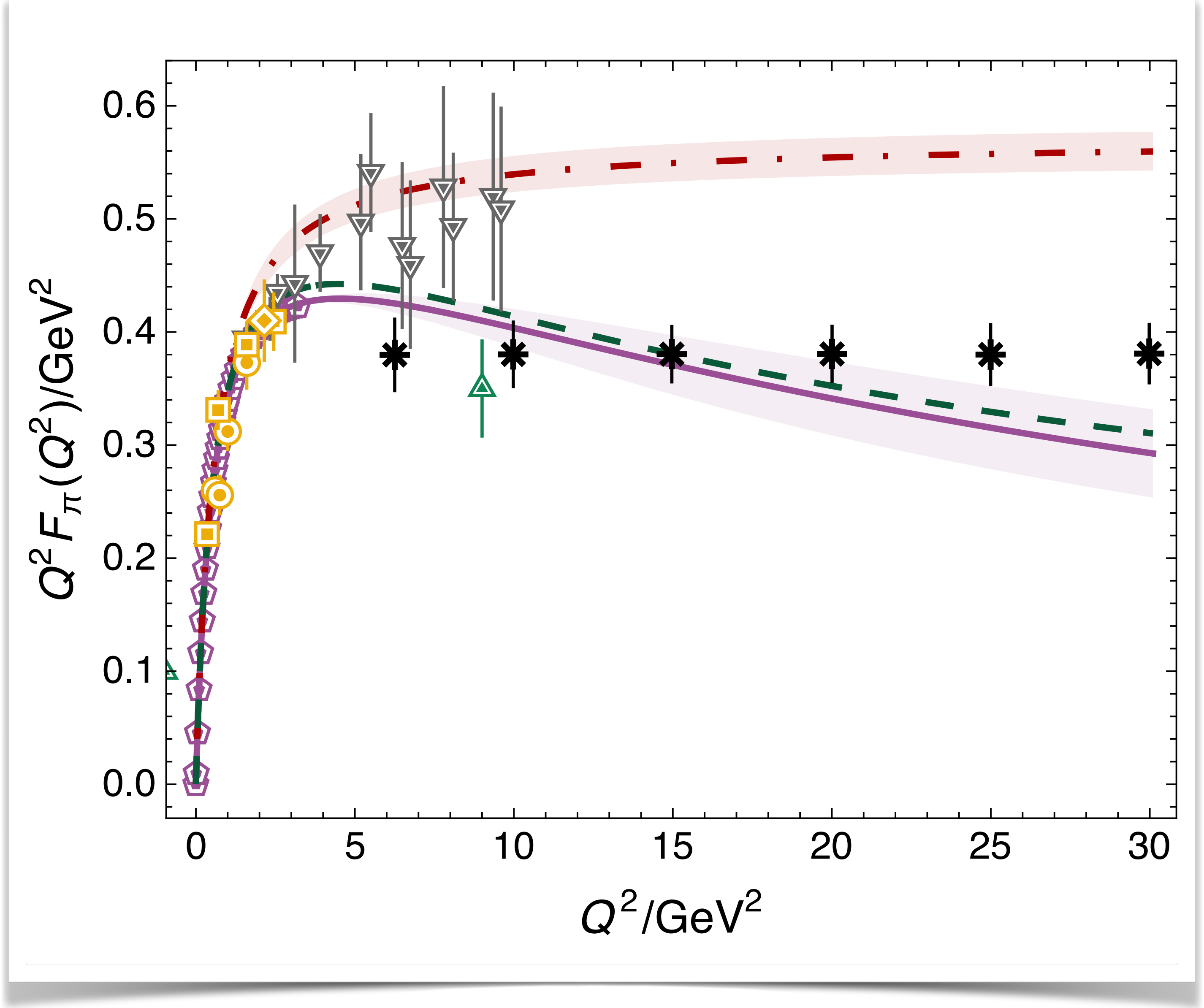

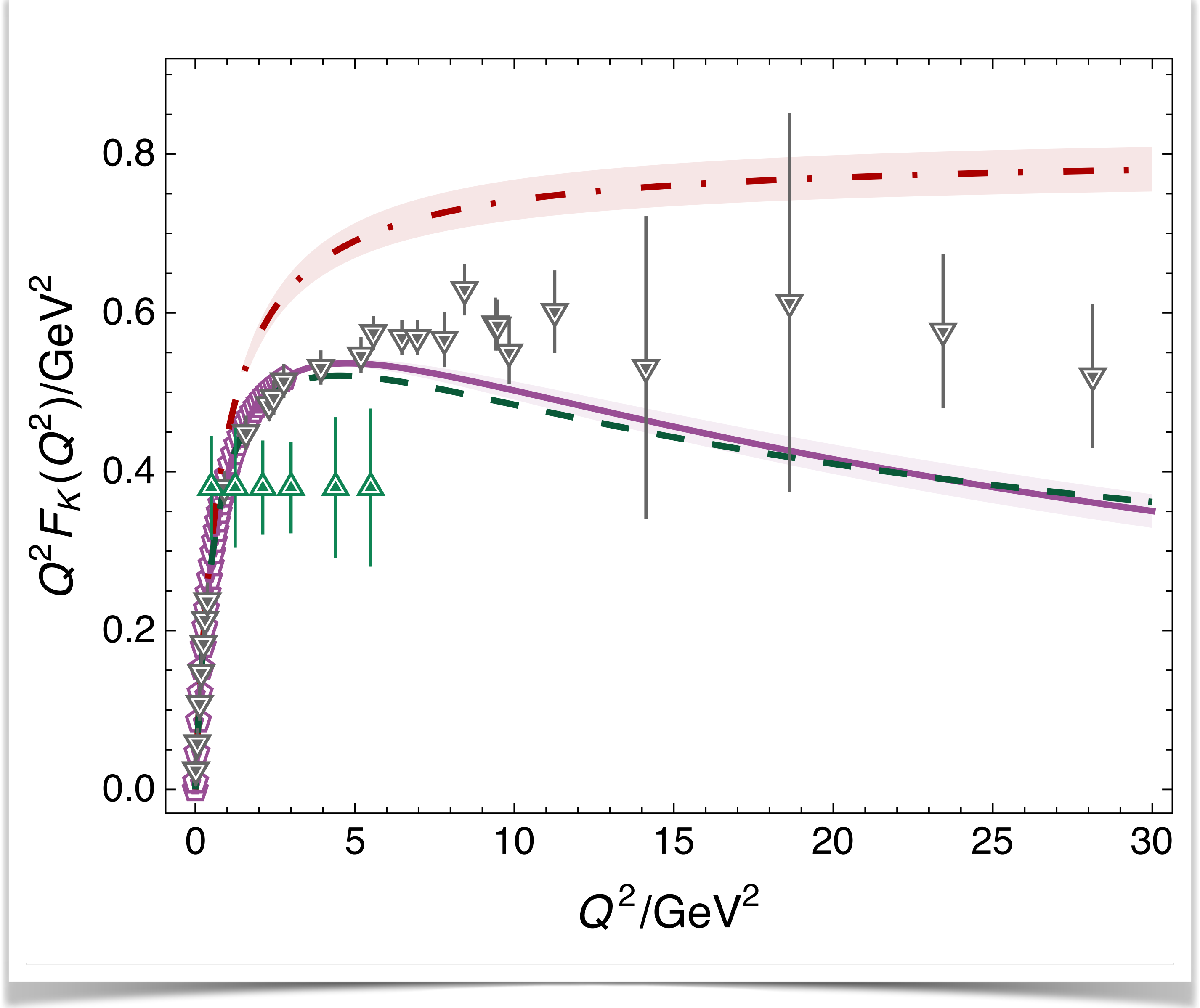

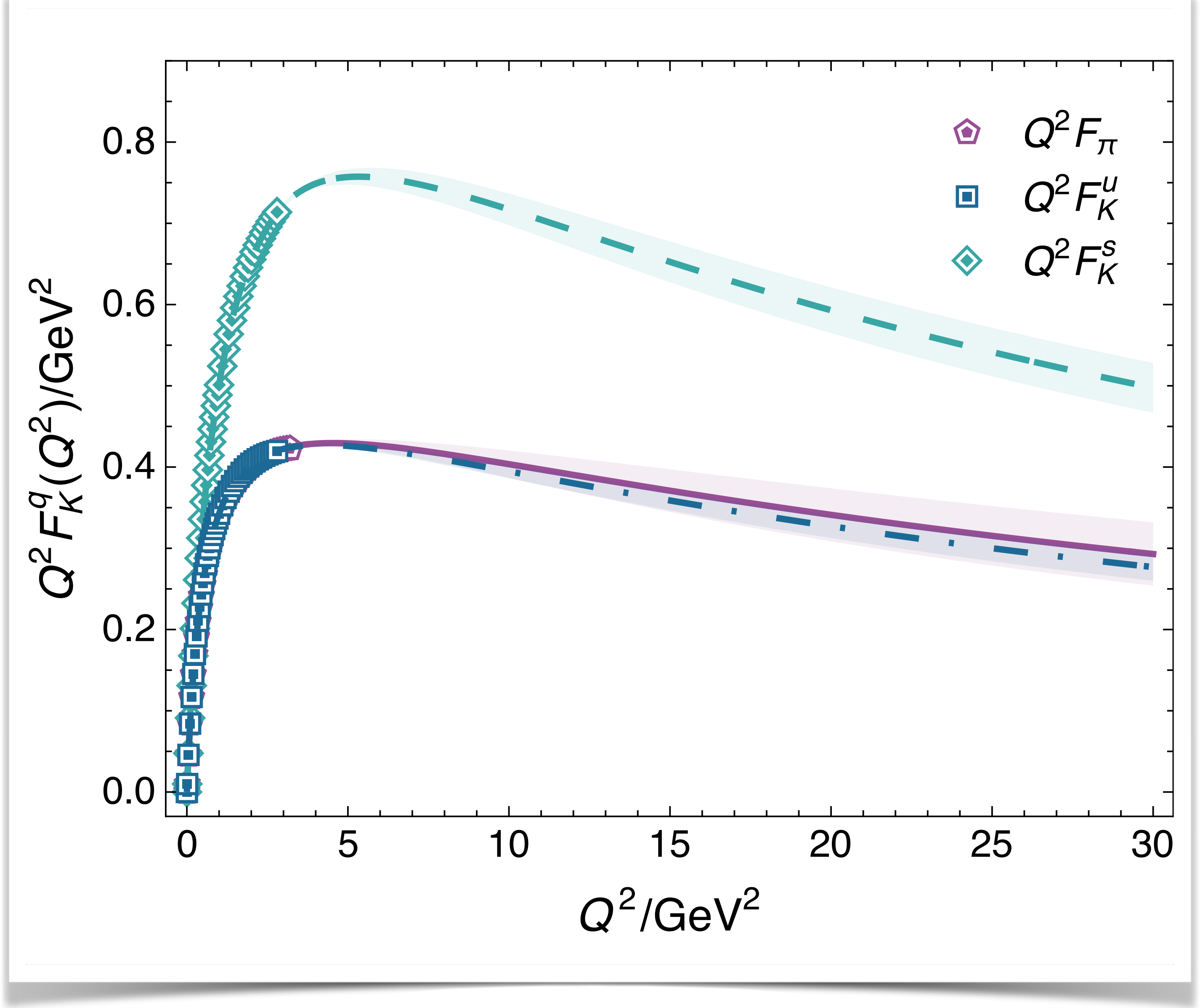

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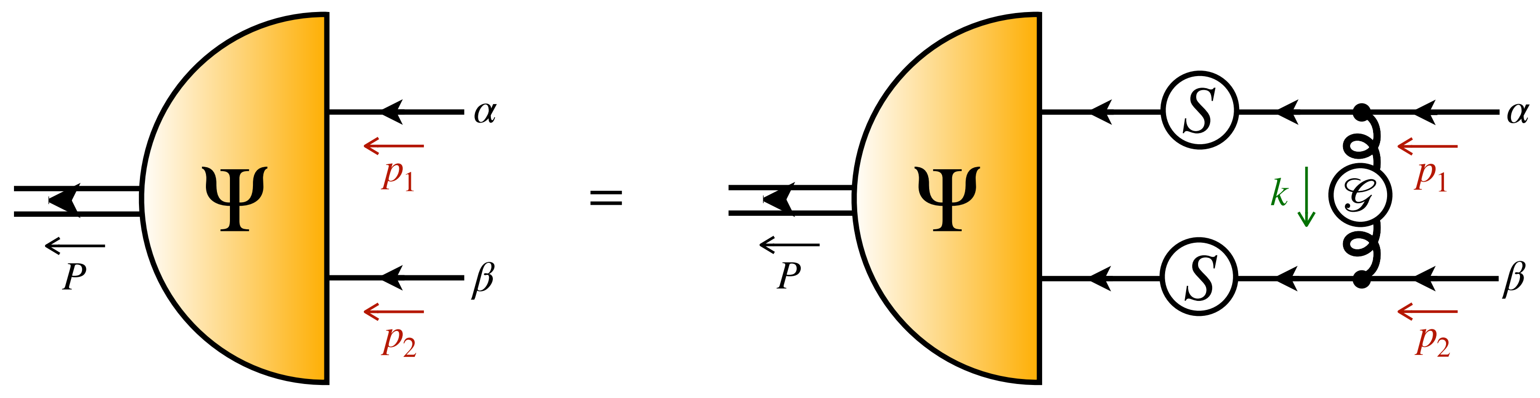

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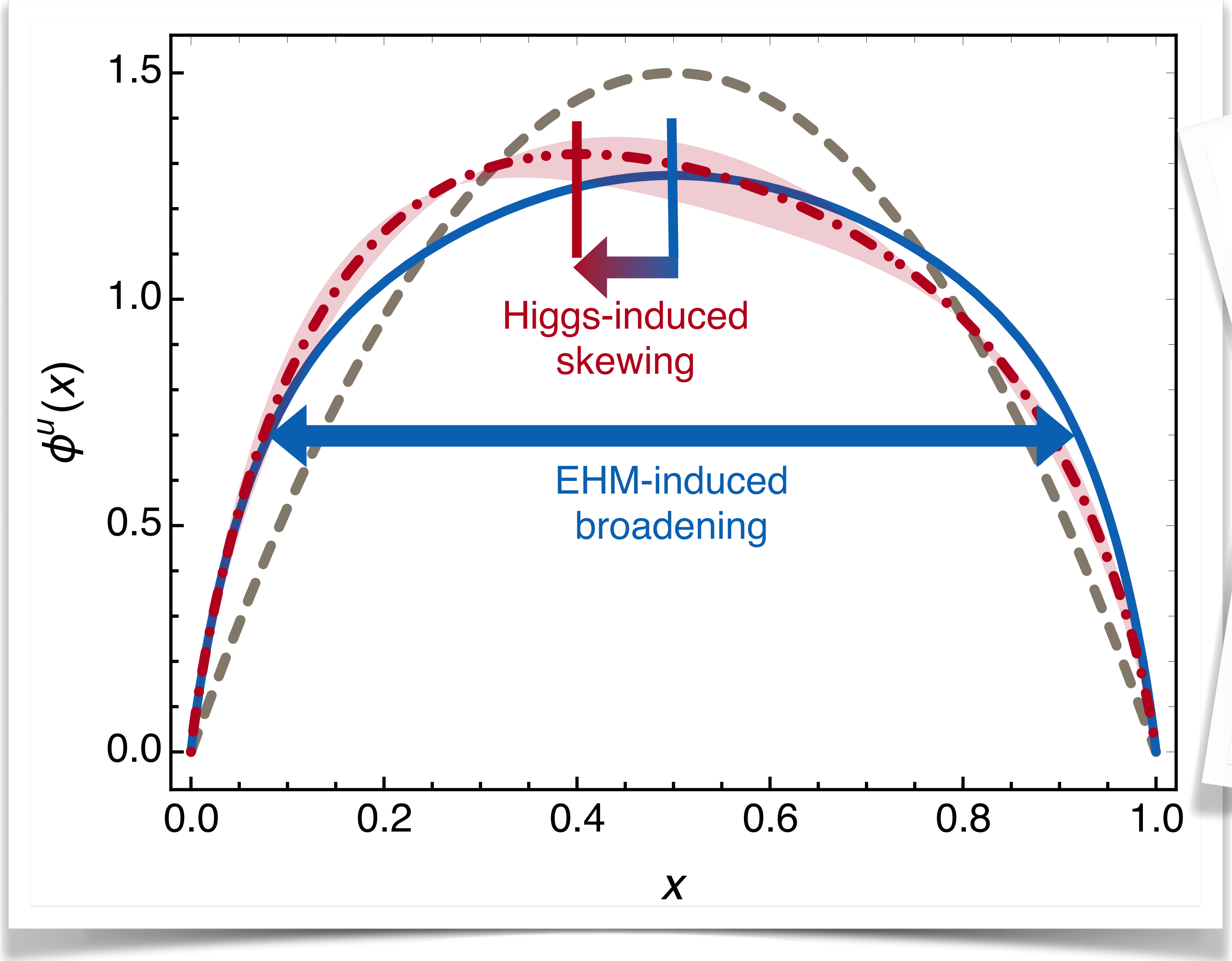
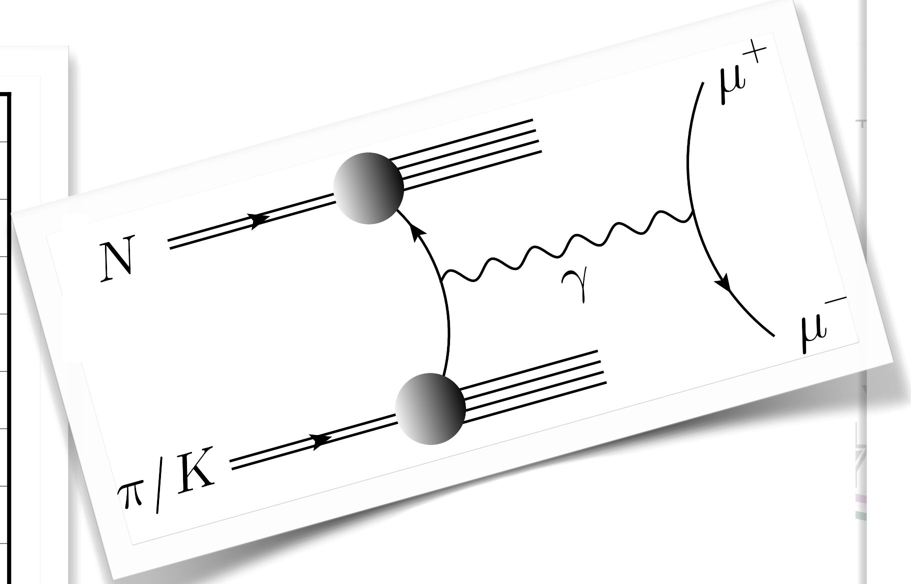
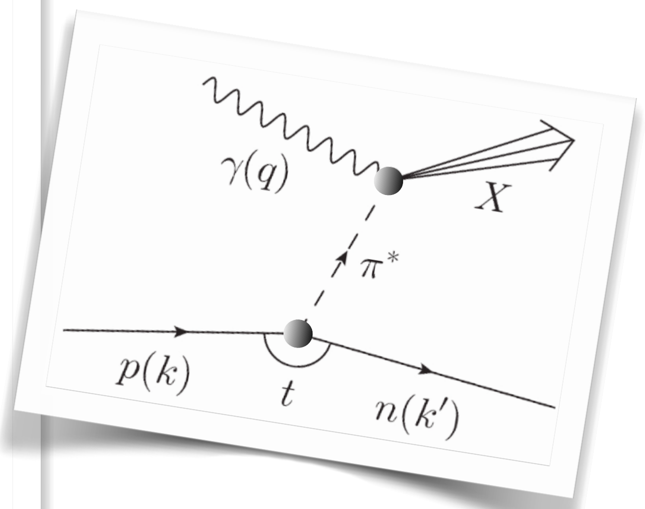
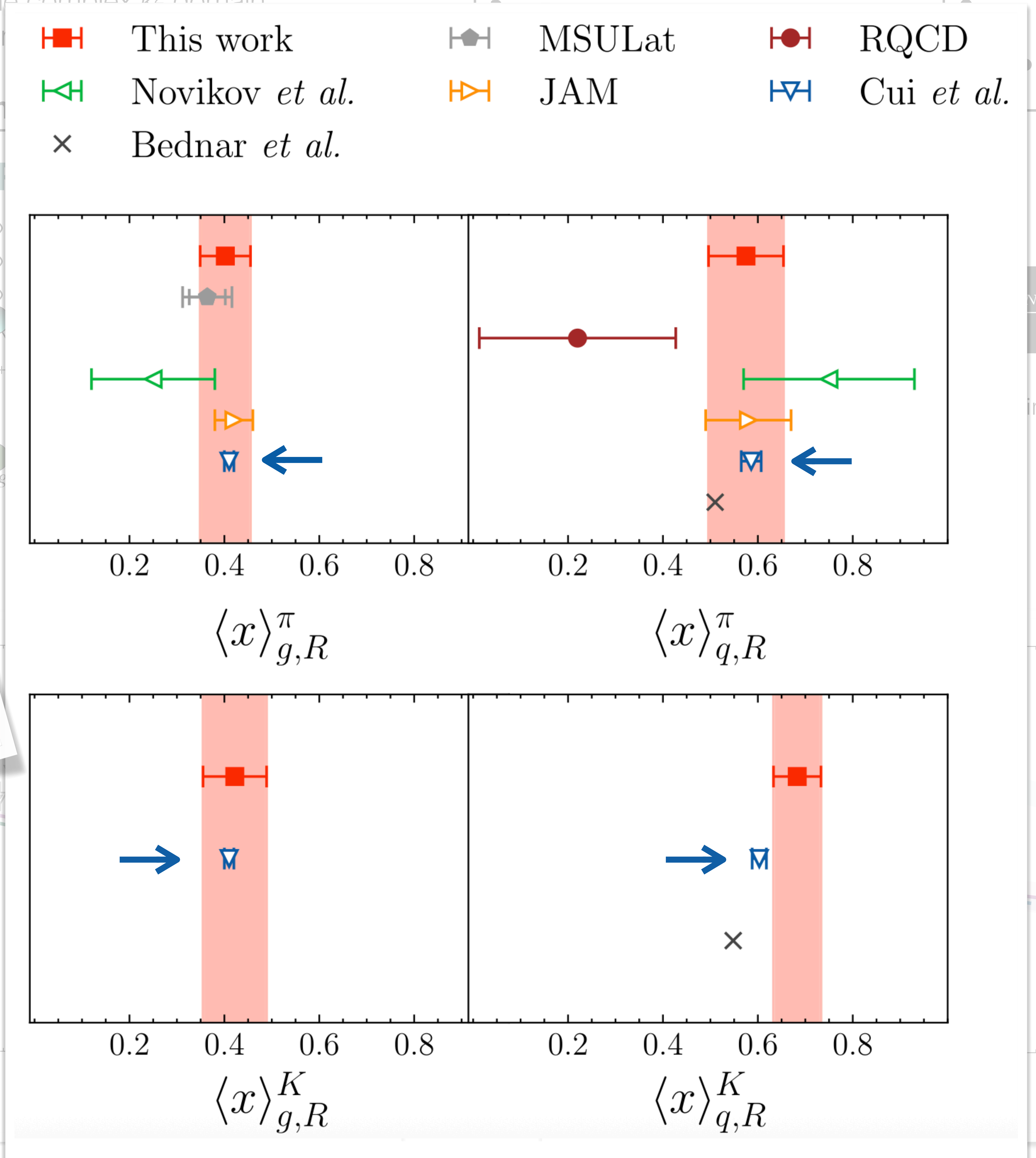






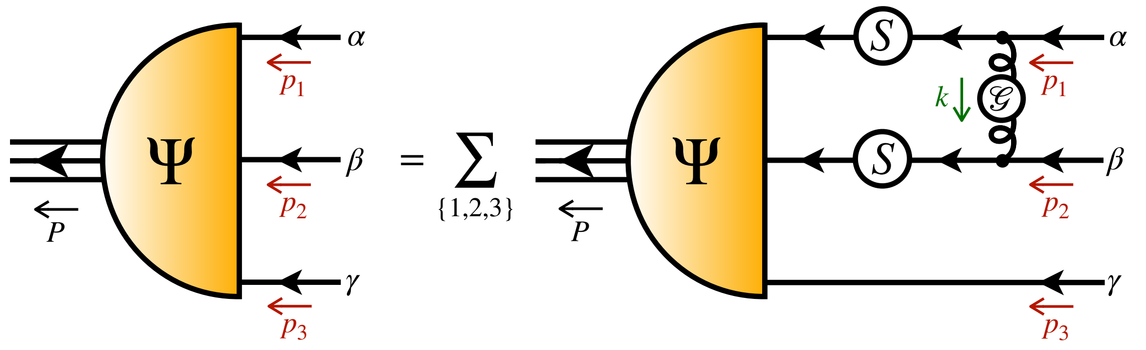


Alexandrou *et al.*, PRL 134 (2025)

p/n ELECTROMAGNETIC FORM FACTORS

Yao, DB, Cui, Roberts, 2403.08088 (FR in press)

FADDEEV EQUATION



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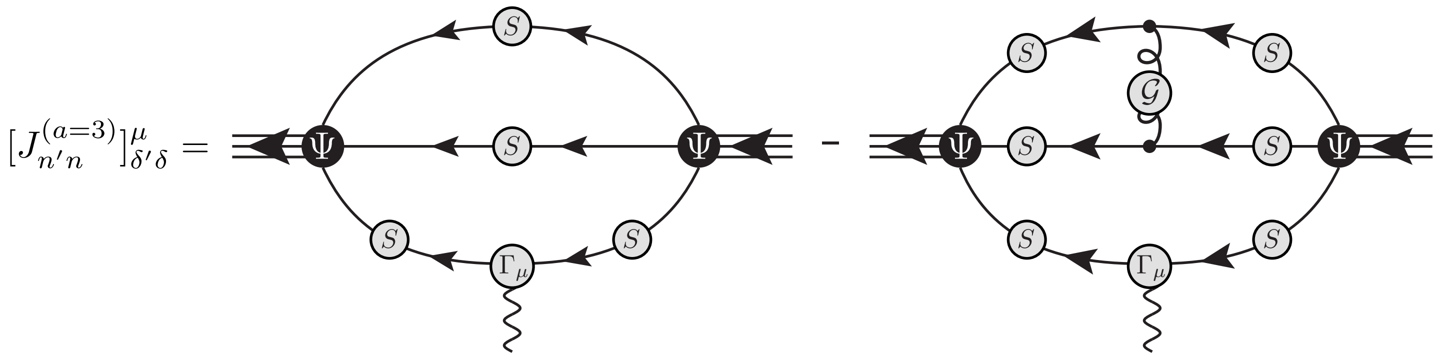

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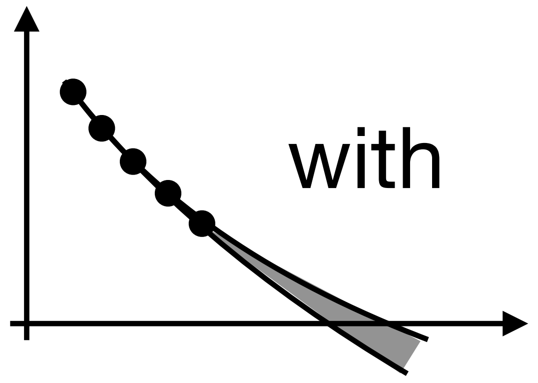

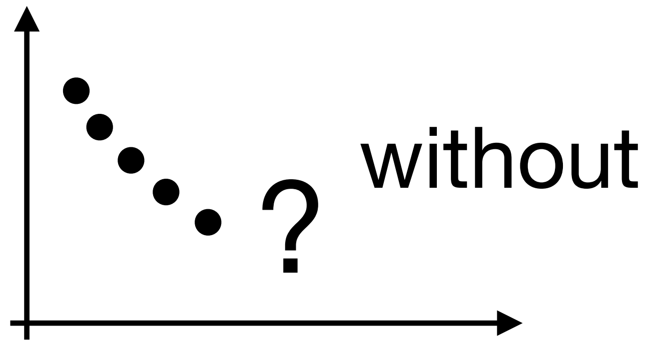

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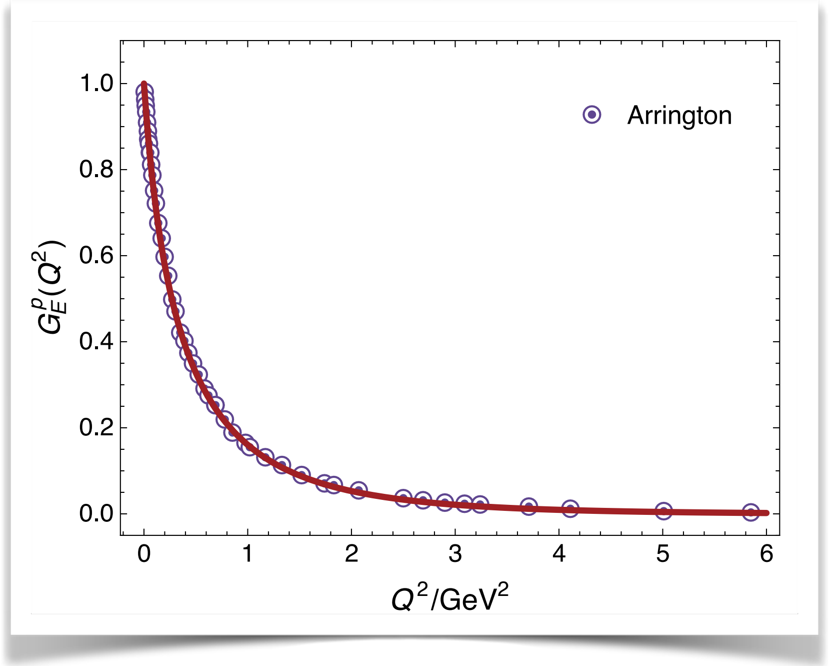
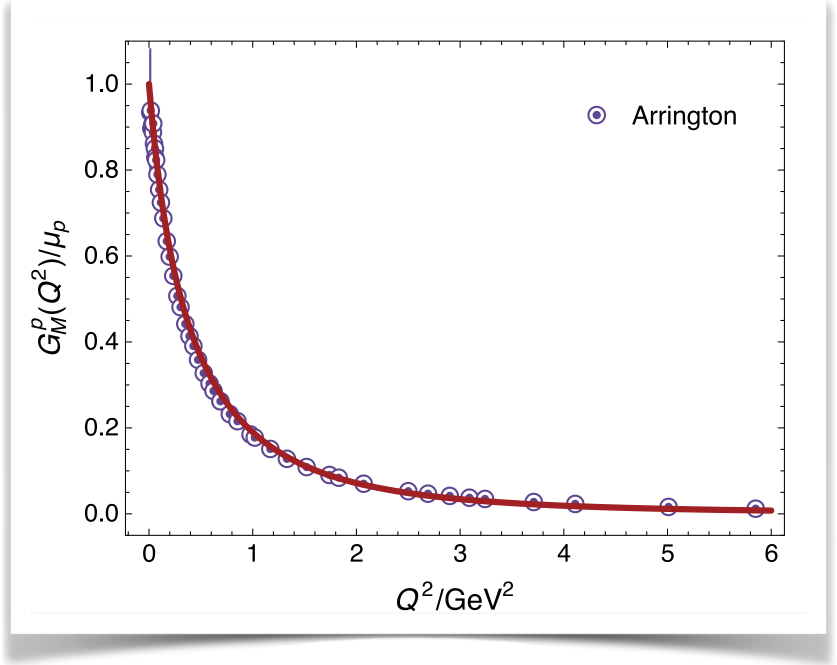
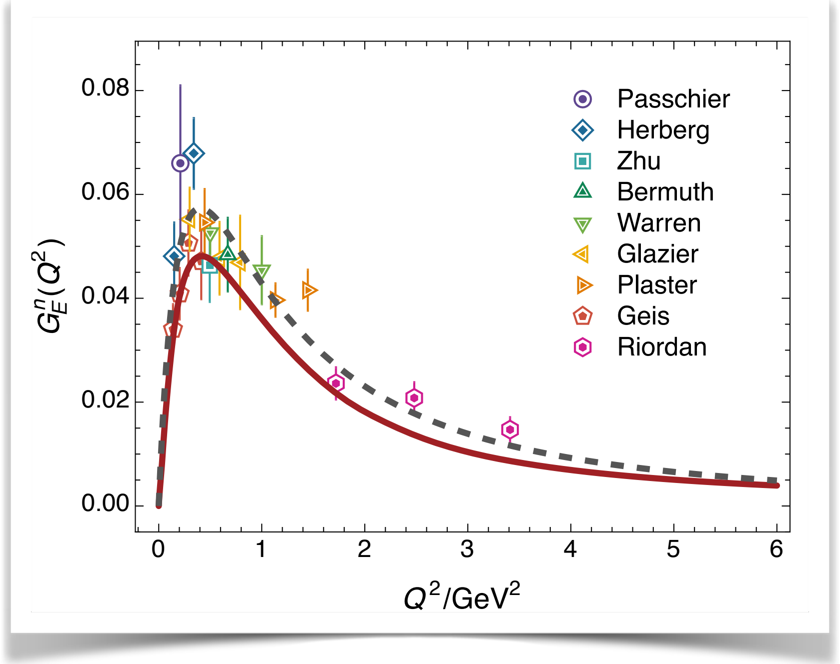
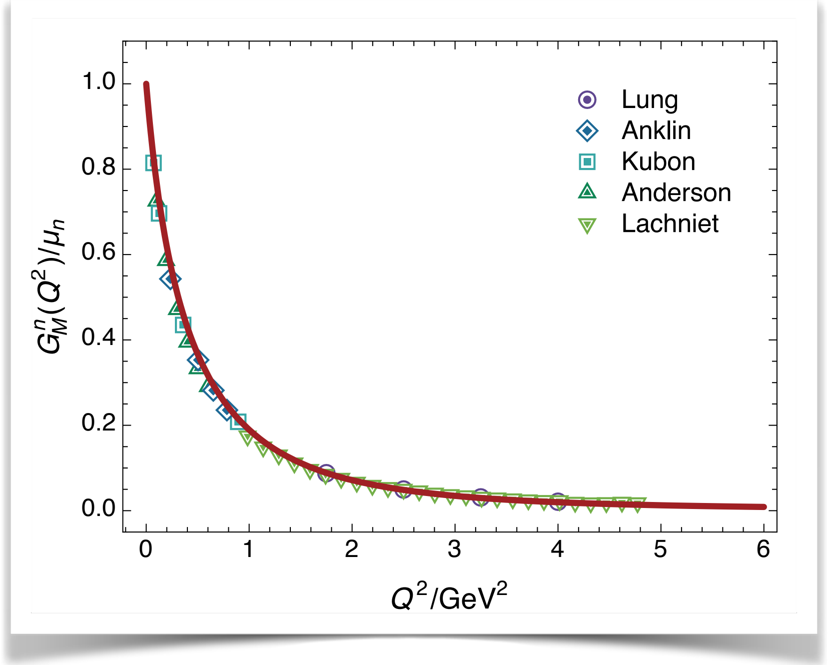
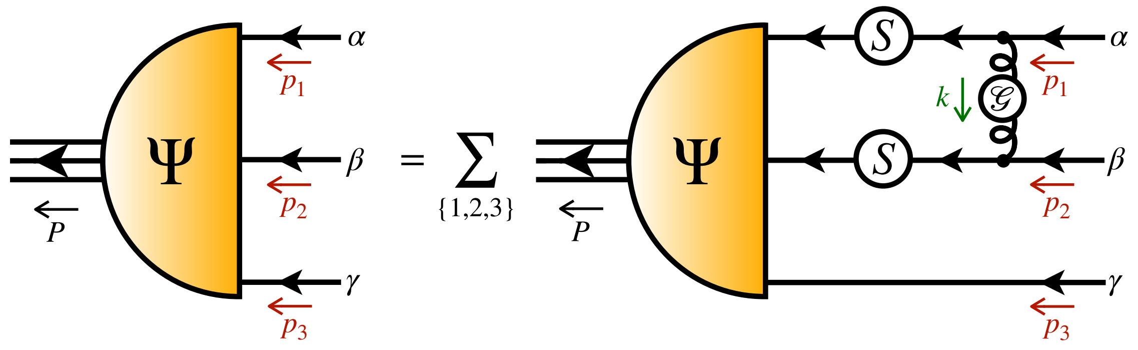






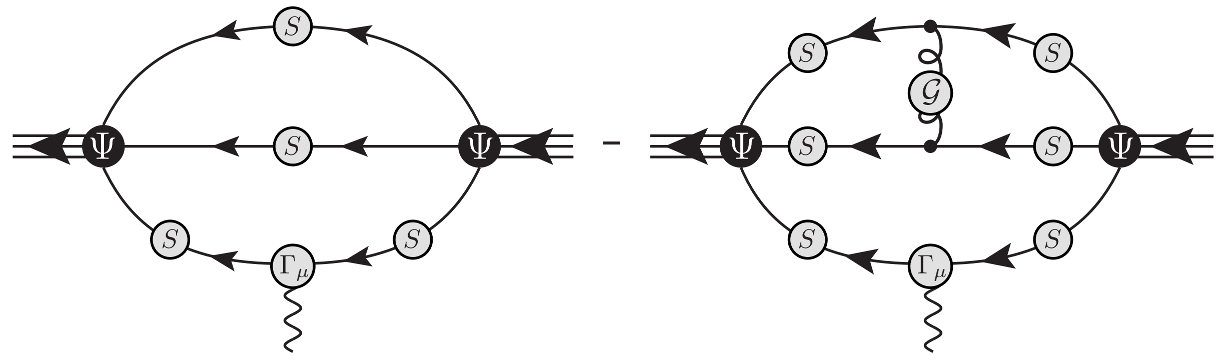


FADDEEV EQUATION



TRIANGULAR DIAGRAM

$[J^{(a=3)}_{n'n}]^{\mu}_{\delta'\delta} =$



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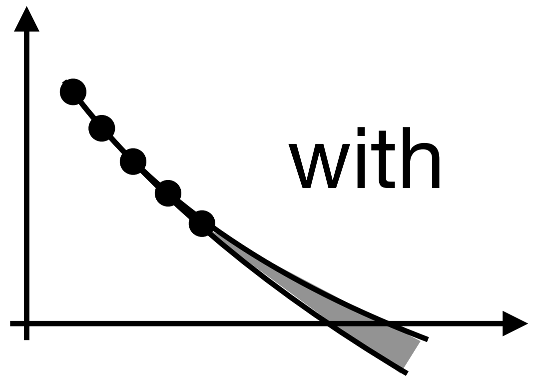
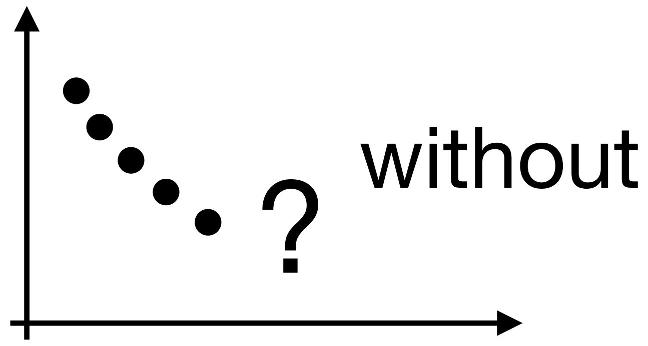
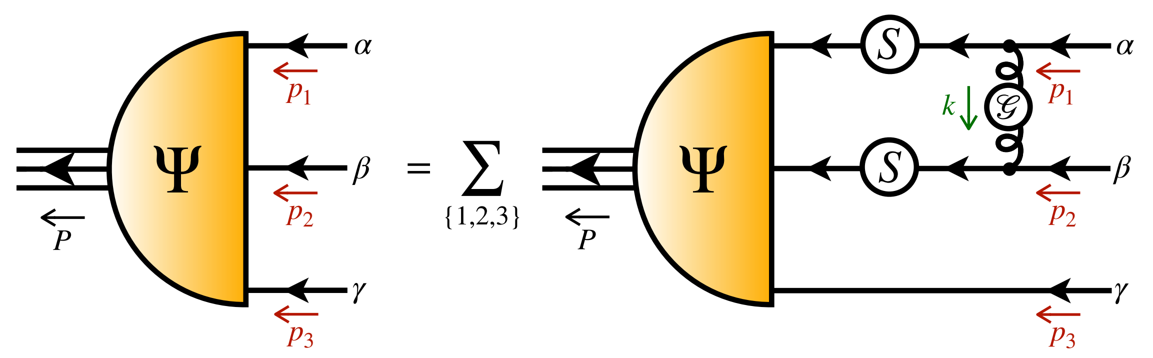




p/n FORM FACTORS
ELECTROMAGNETIC

Yao, DB, Cui, Roberts, 2403.08088 (FR in press)

FADDEEV EQUATION



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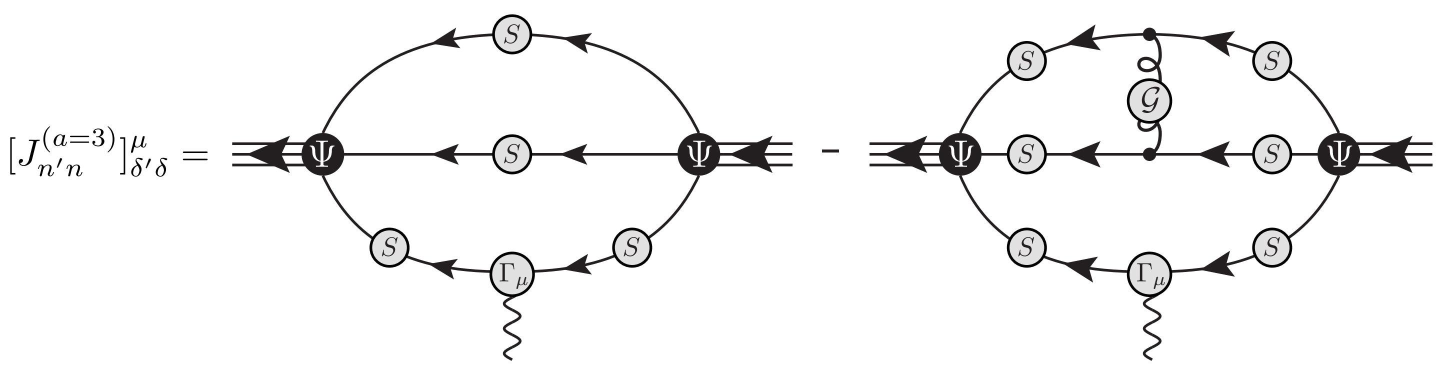

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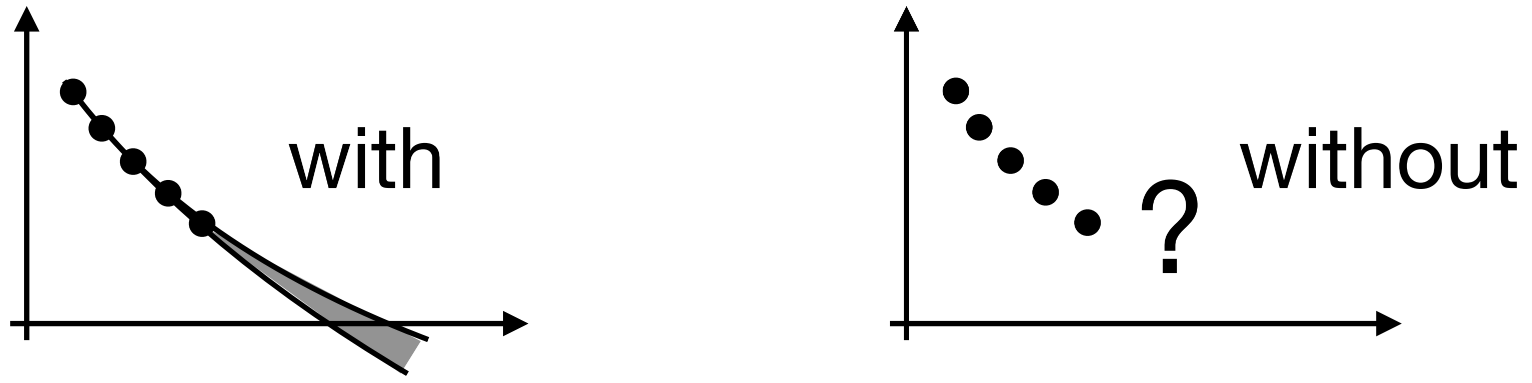

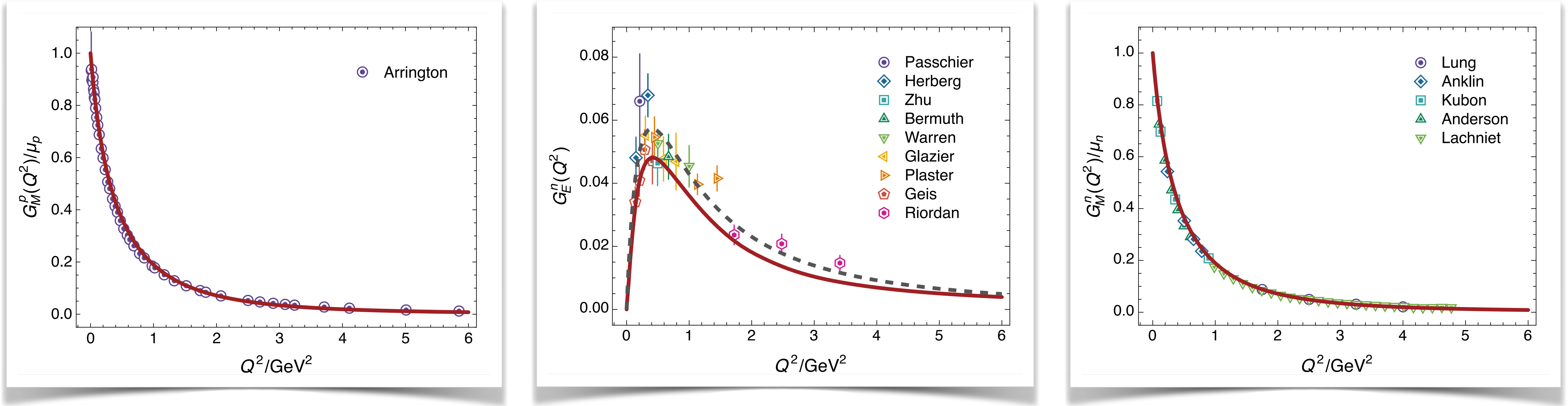

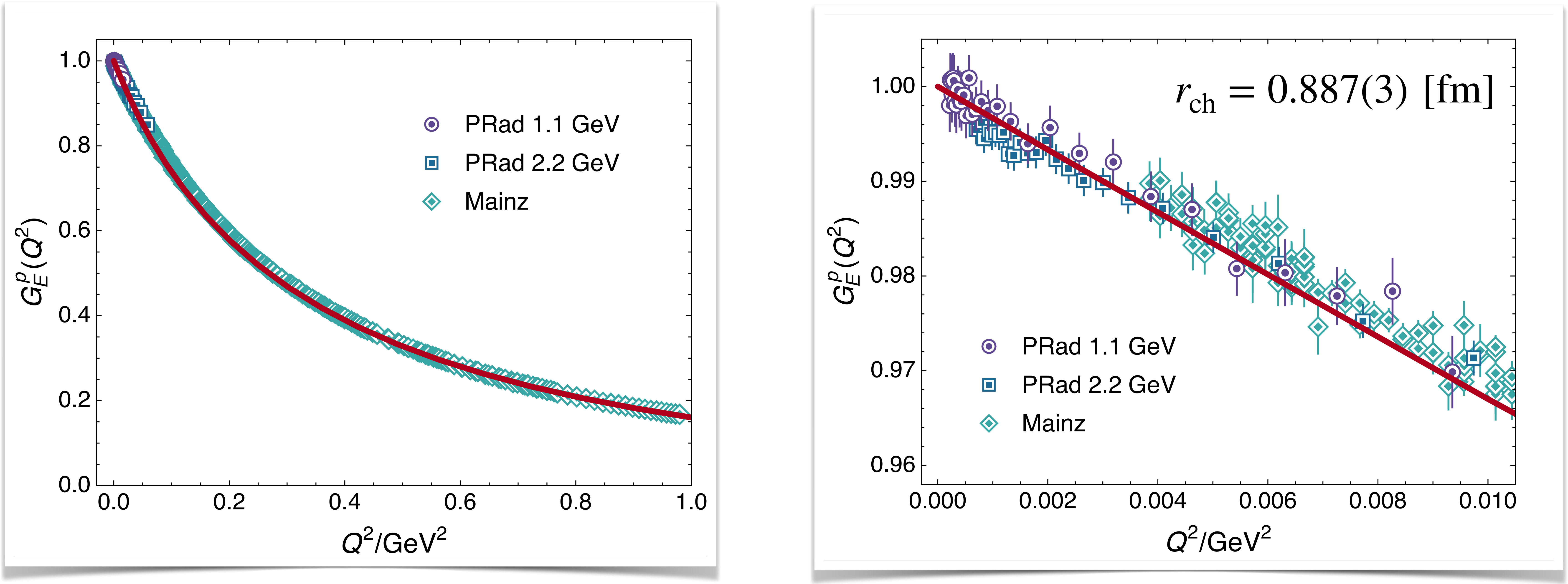

p/n ELECTROMAGNETIC FORM FACTORS

Yao, DB, Cui, Roberts, 2403.08088 (FR in press)

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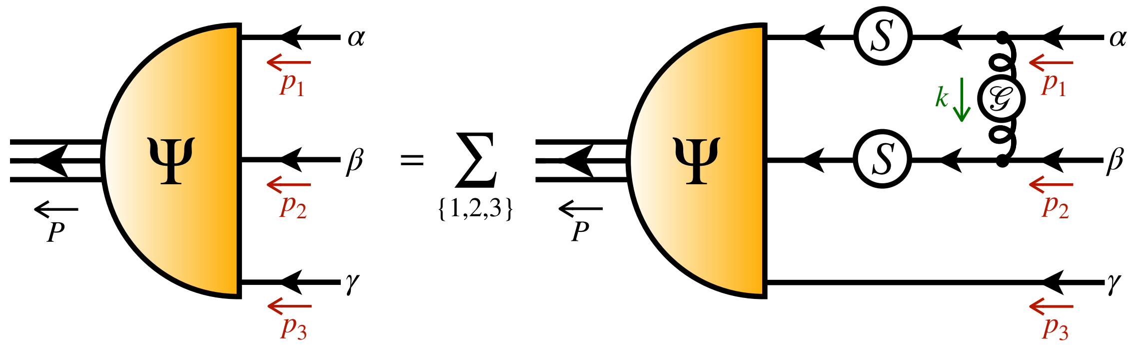

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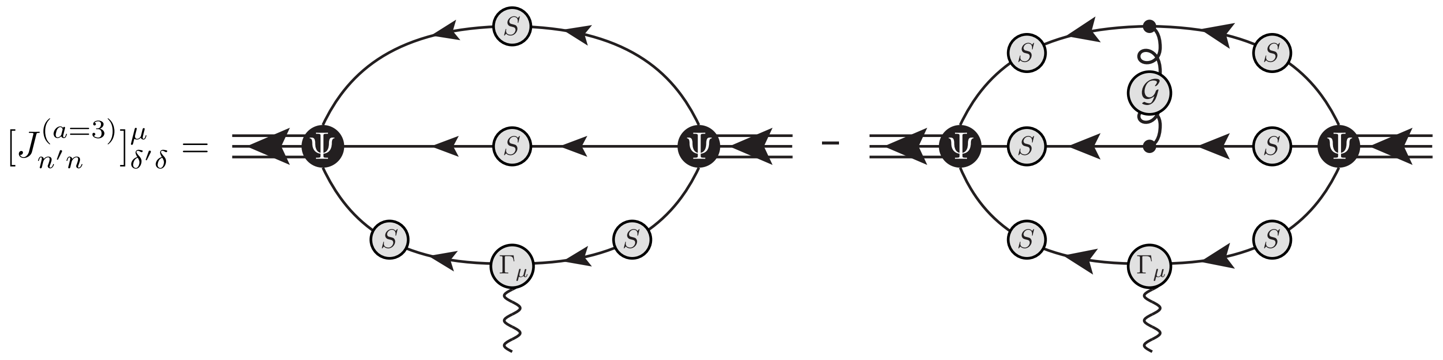

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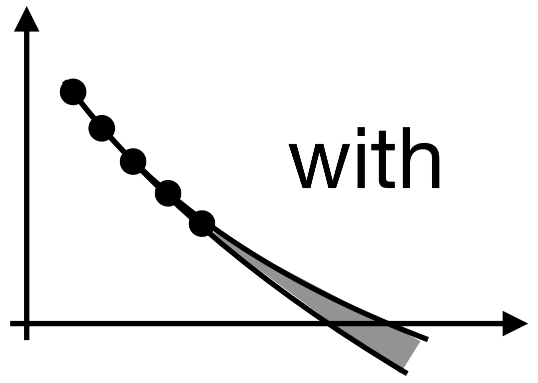

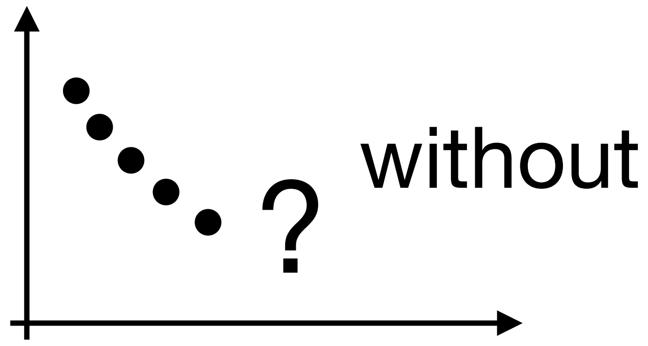

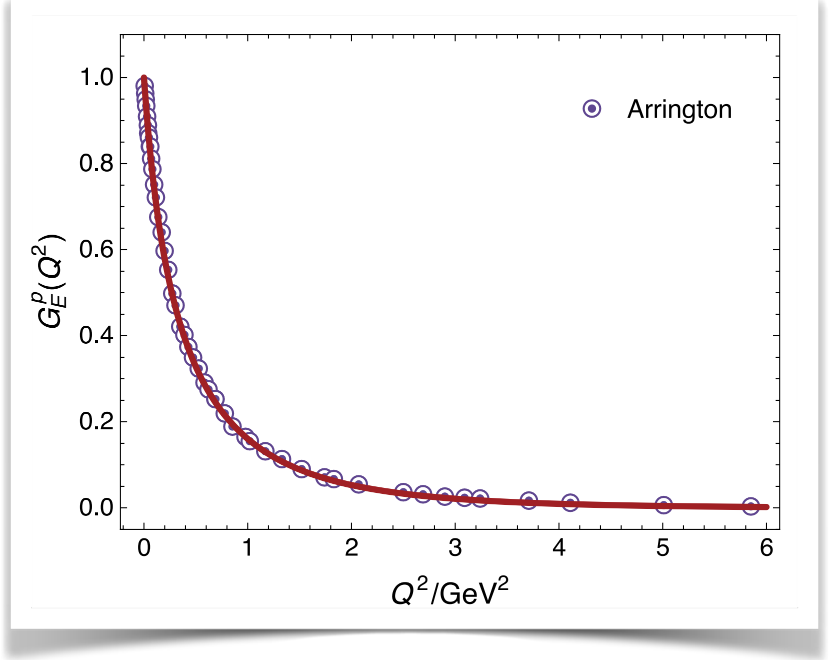

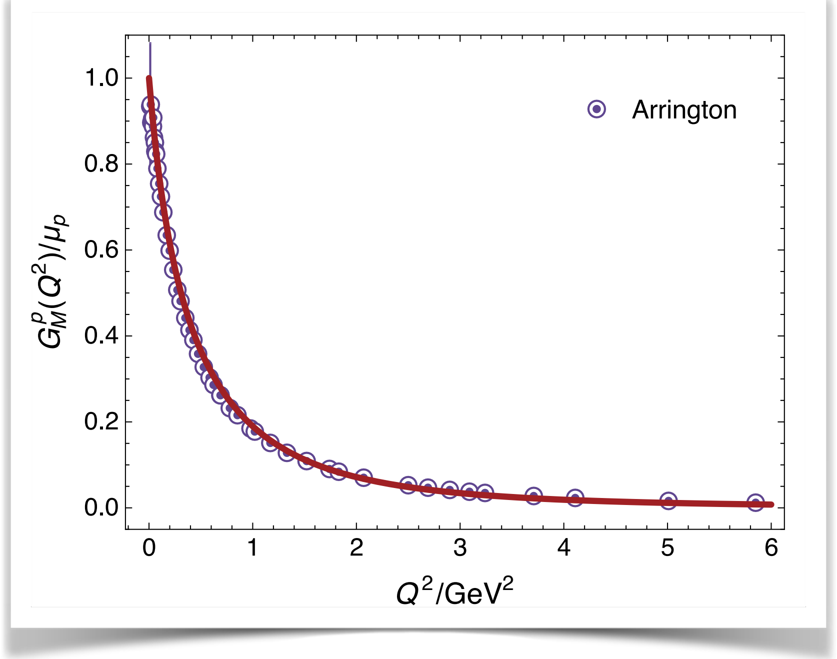

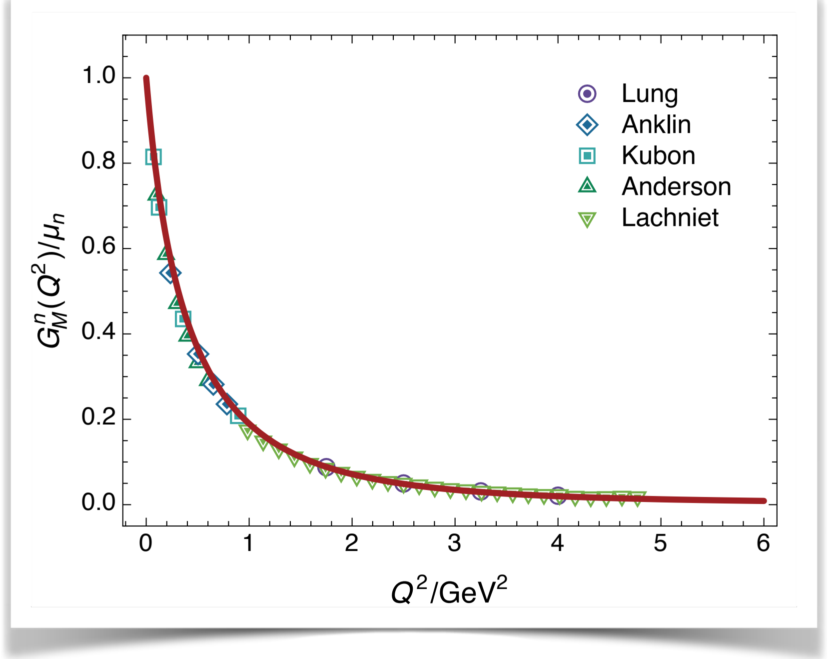

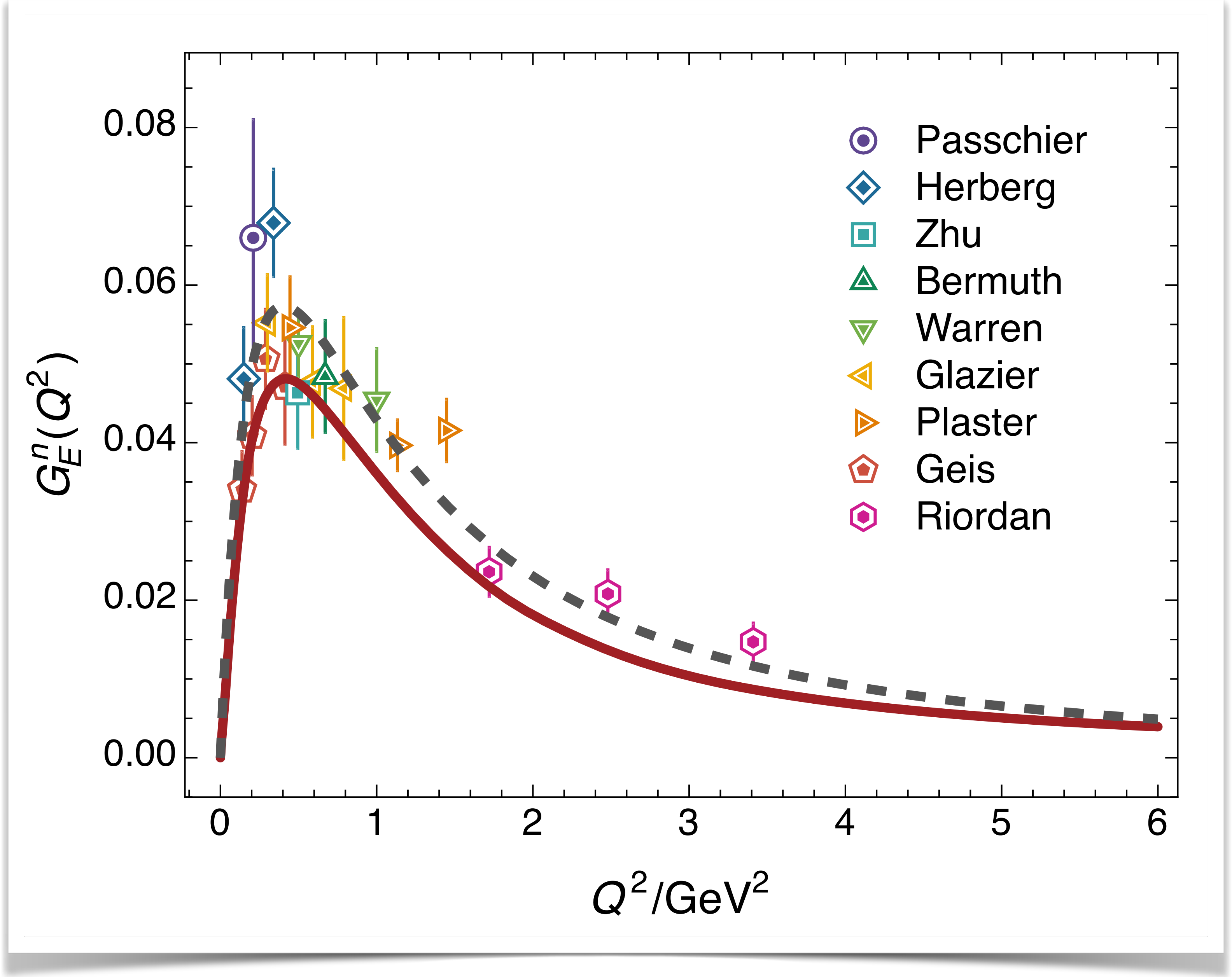

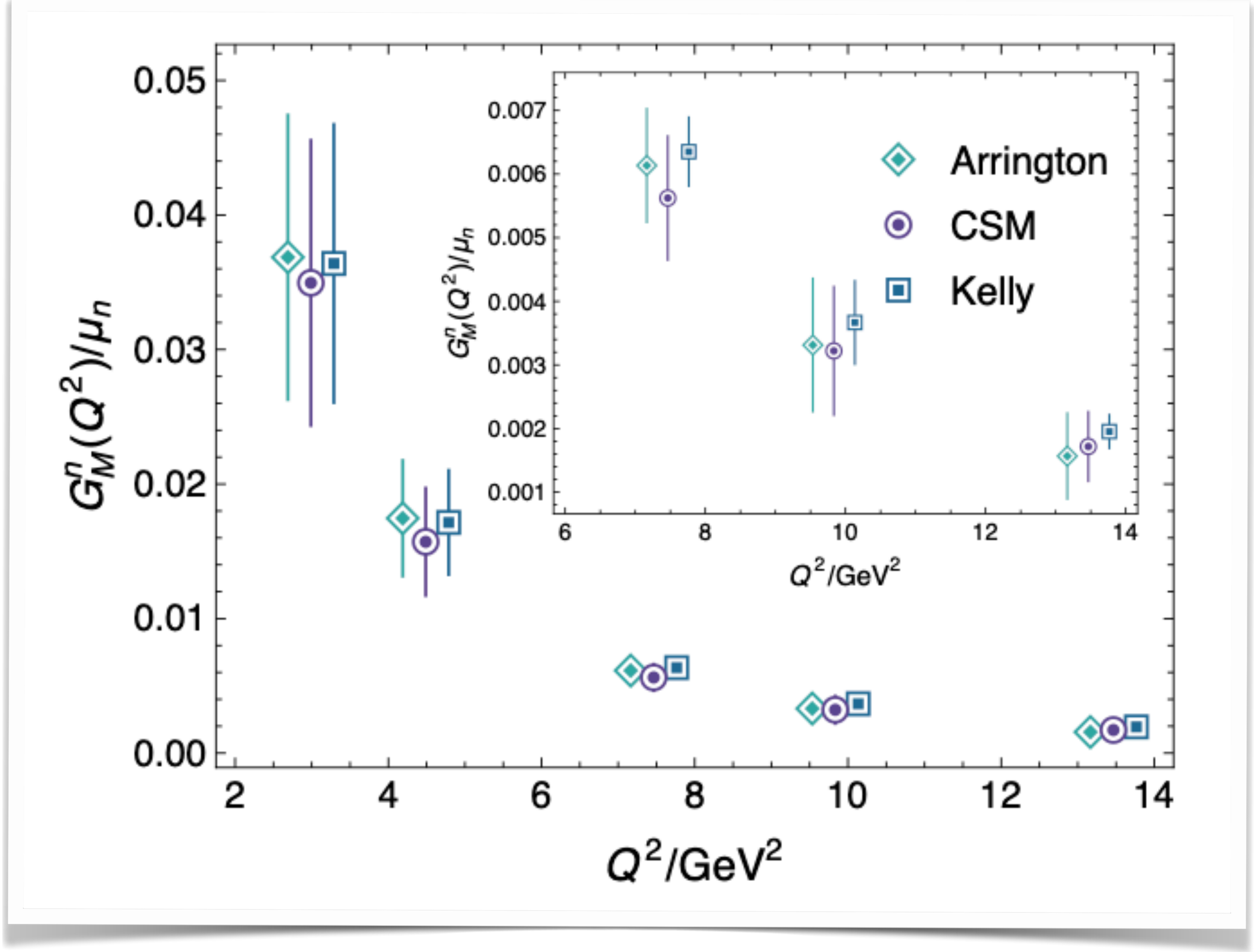

ELECTROMAGNETIC

p/n FORM FACTORS

Yao, DB, Cui, Roberts, 2403.08088 (FR in press)

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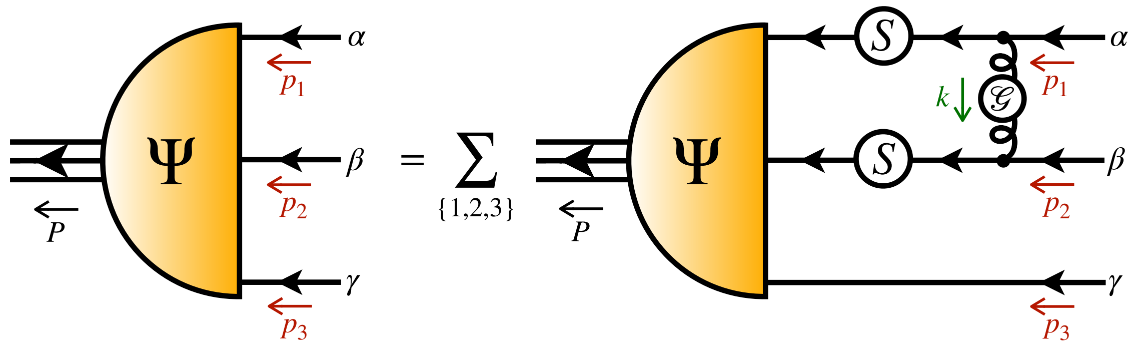

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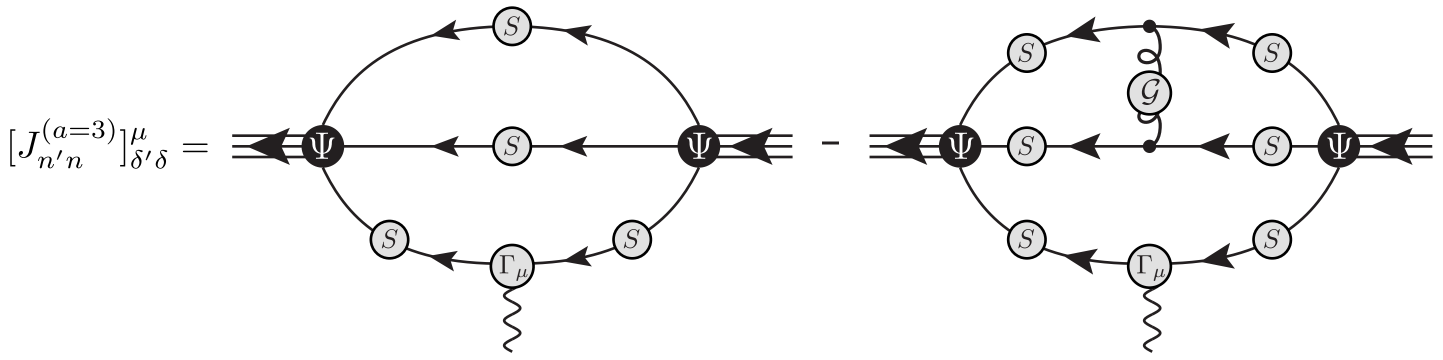

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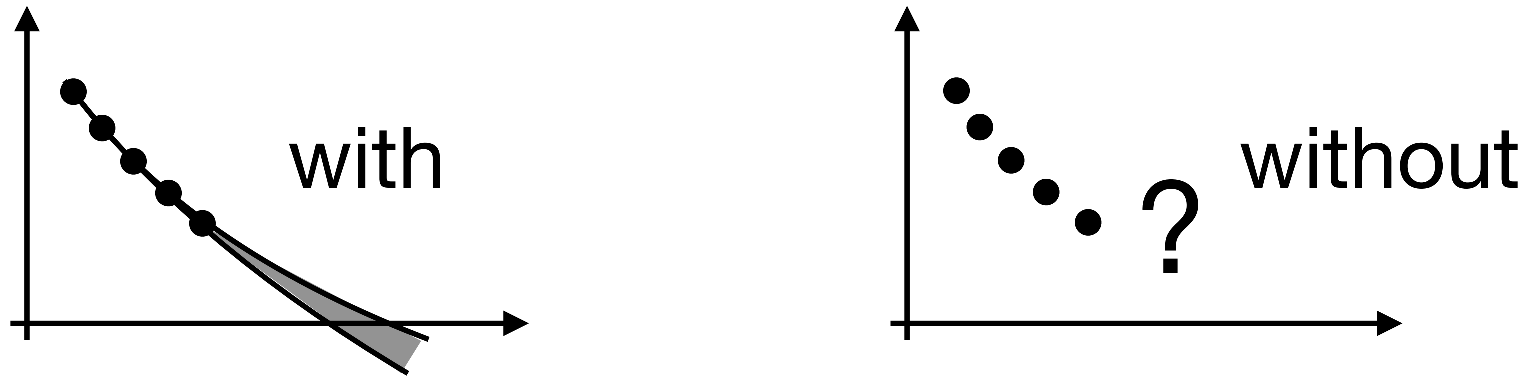

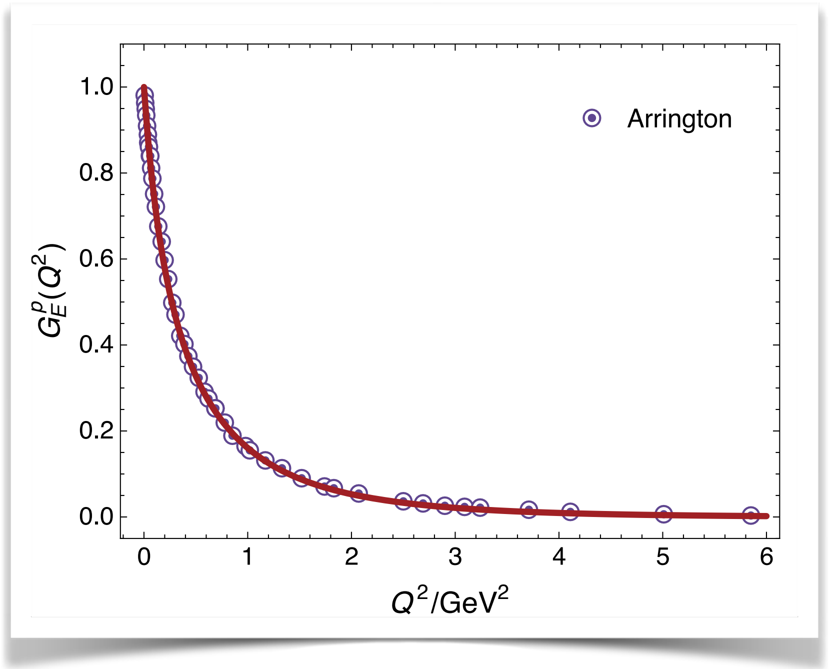

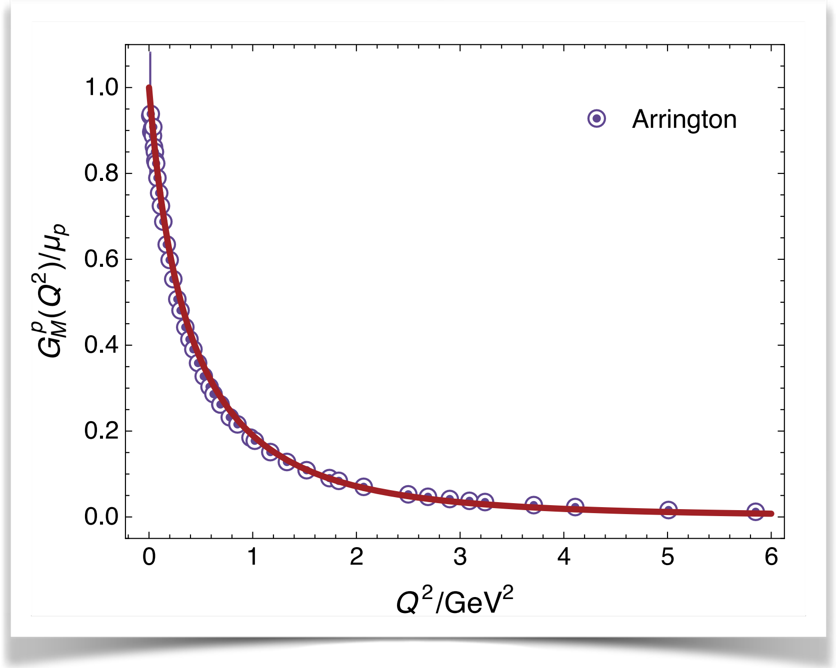

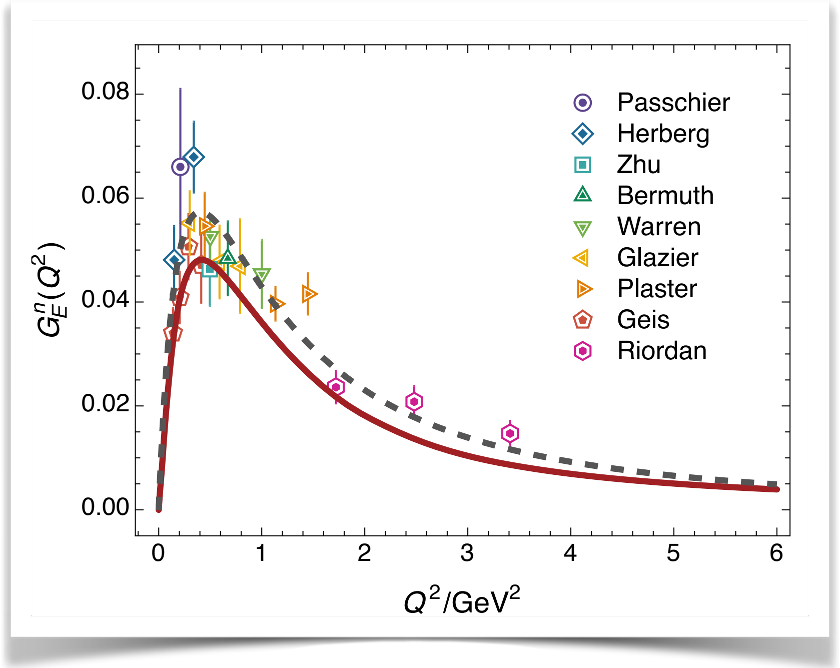

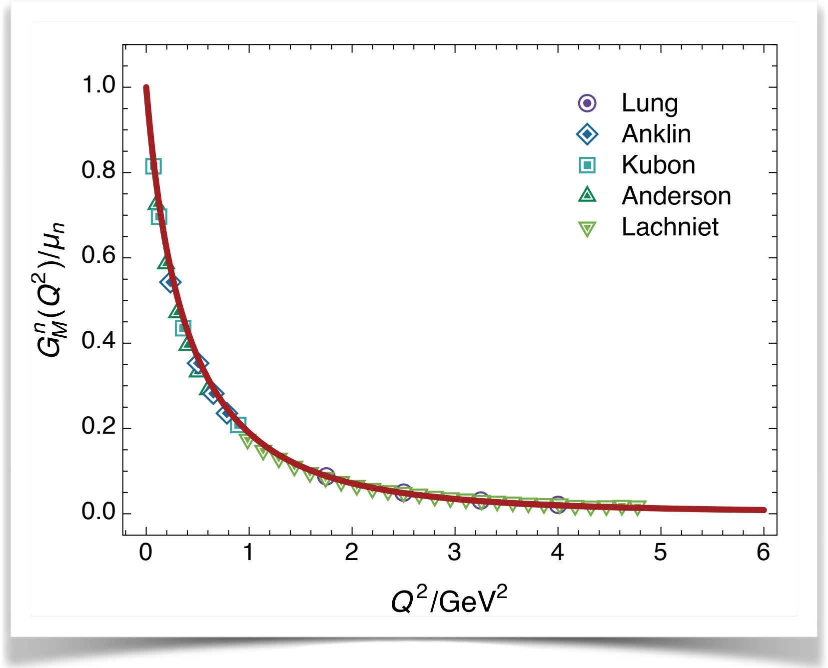

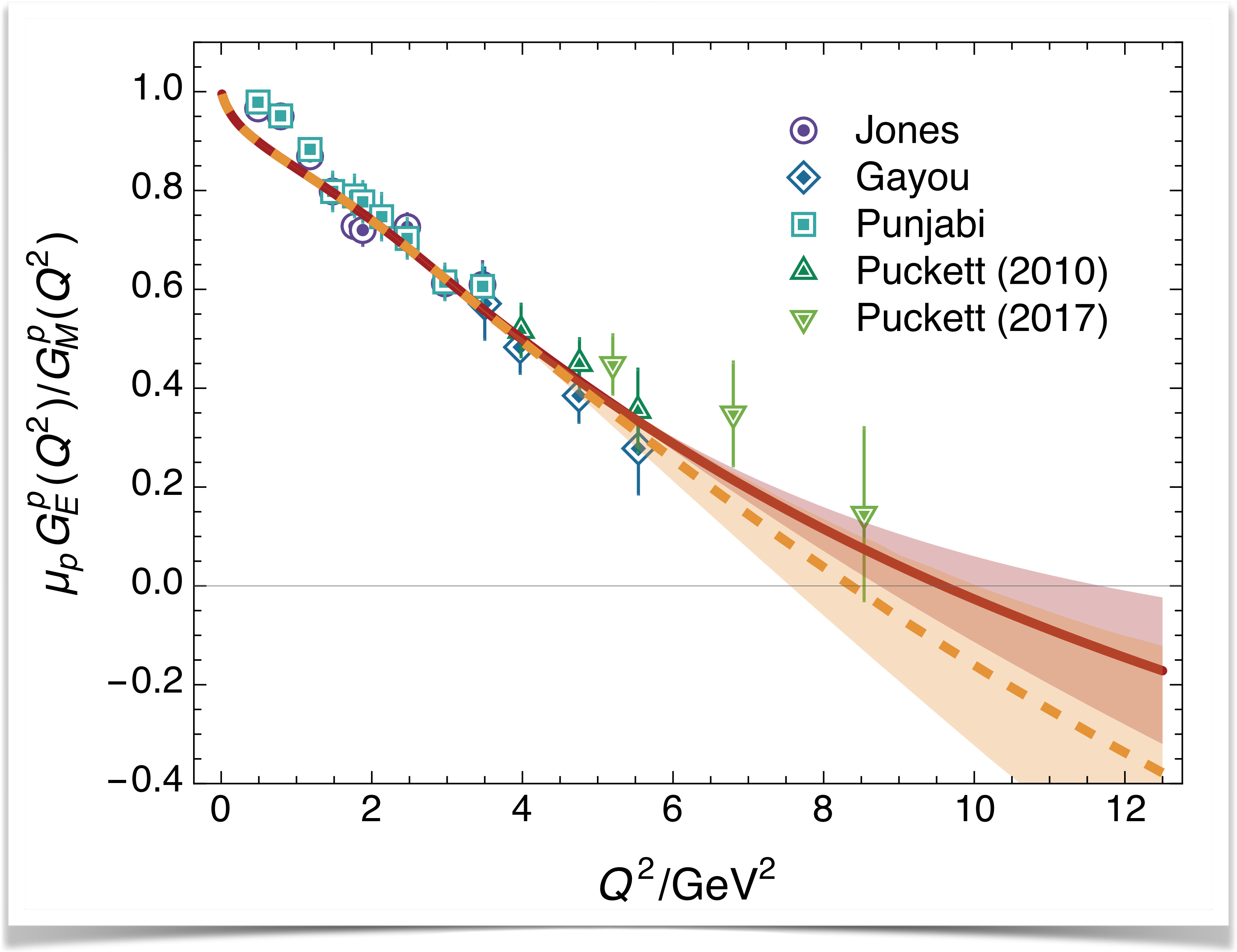

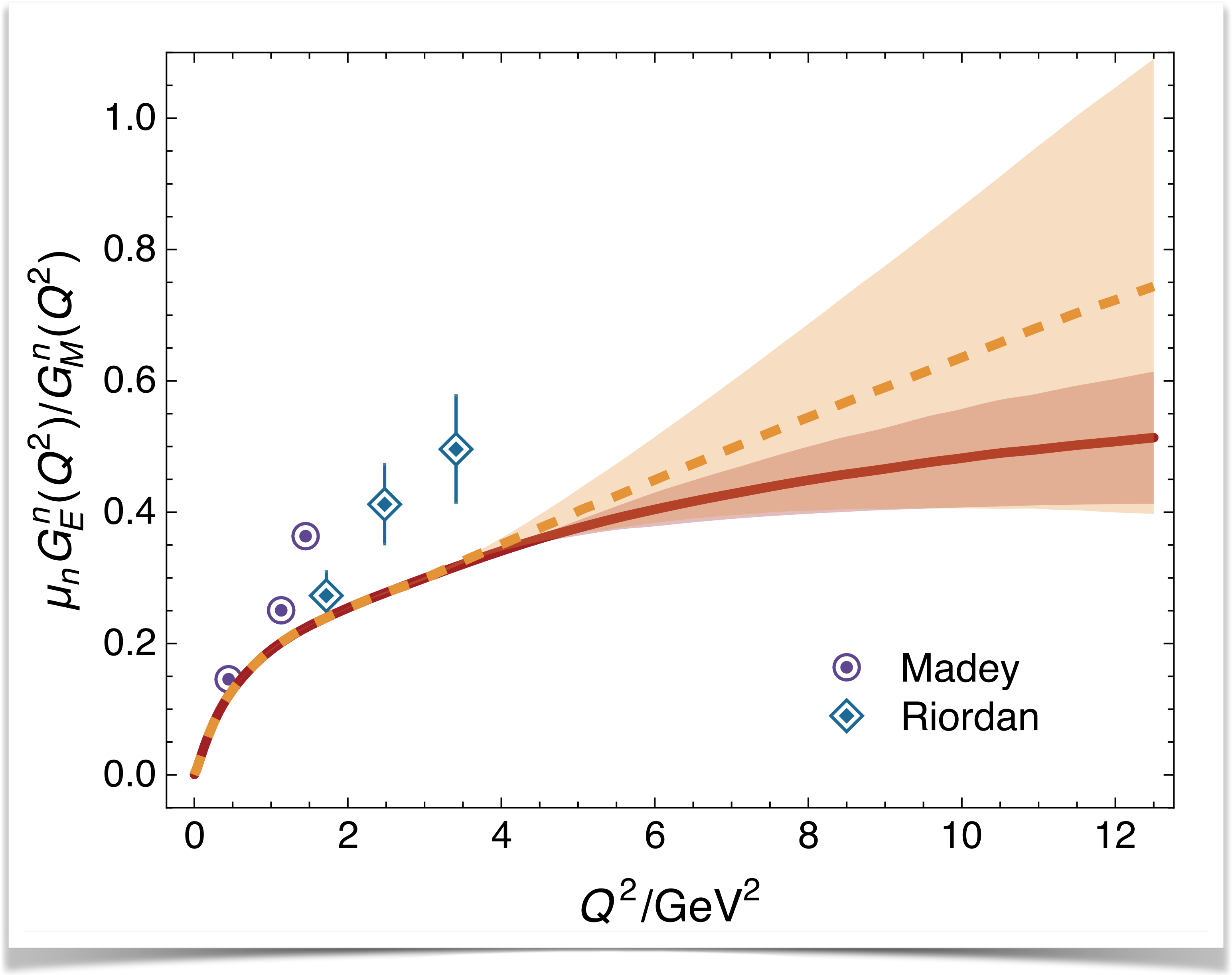

ELECTROMAGNETIC

p/n FORM FACTORS

Yao, DB, Cui, Roberts, 2403.08088 (FR in press)

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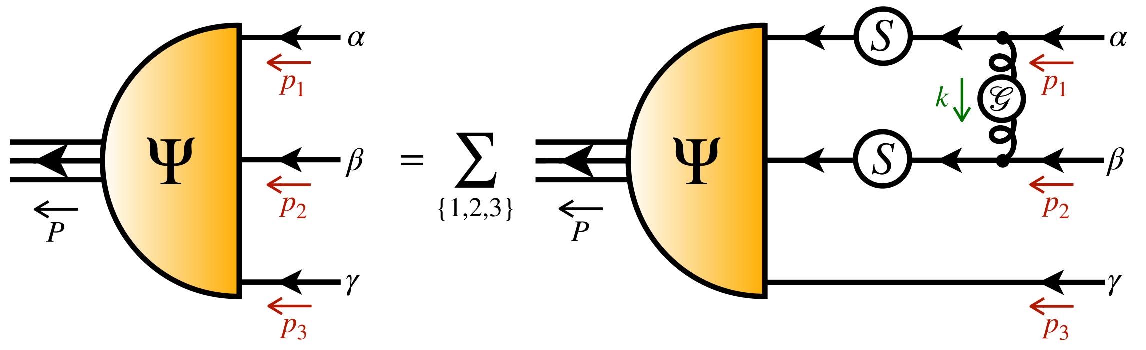

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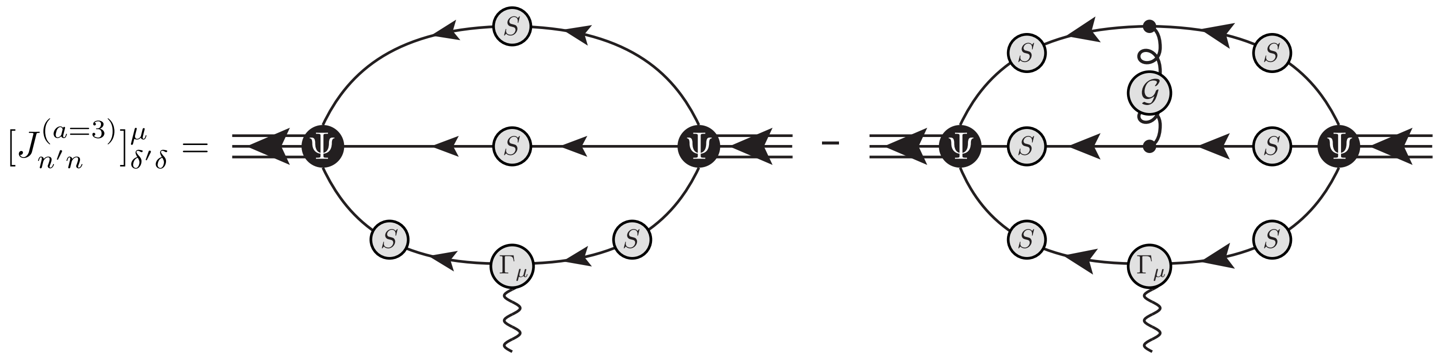

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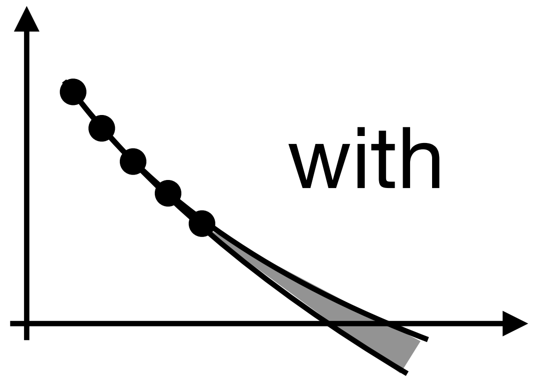
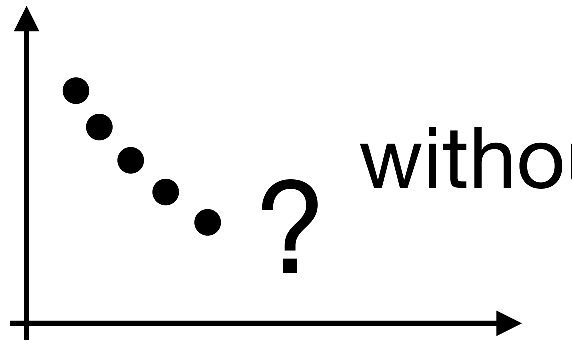
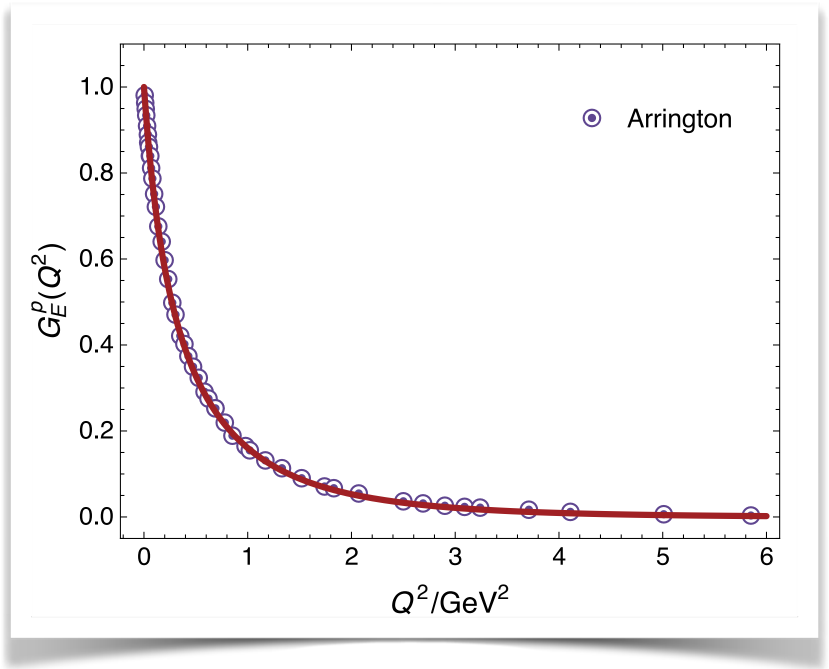
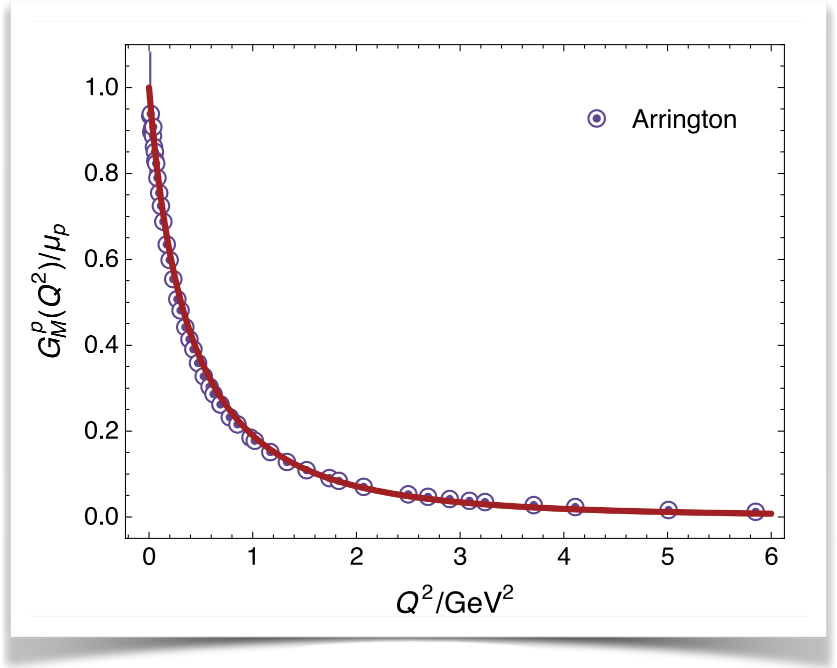
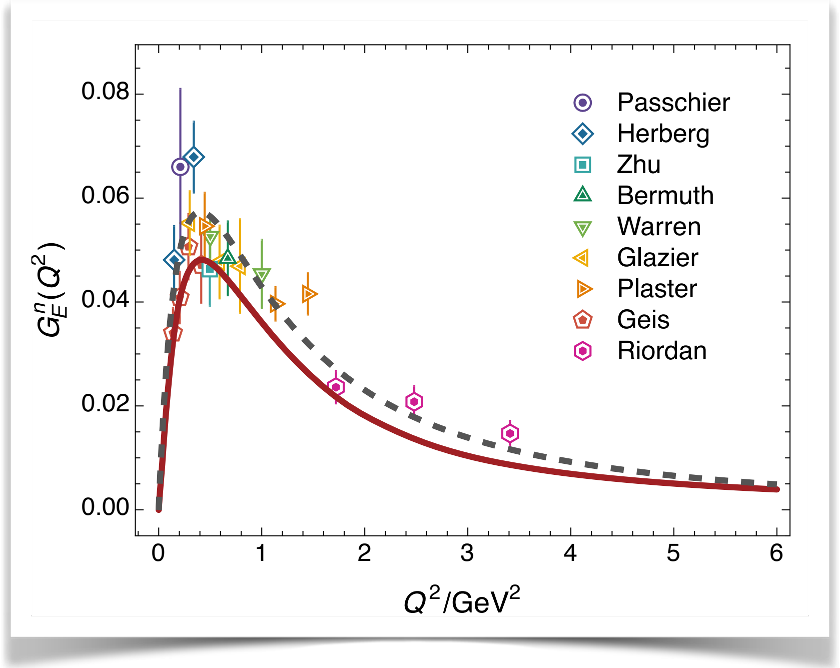
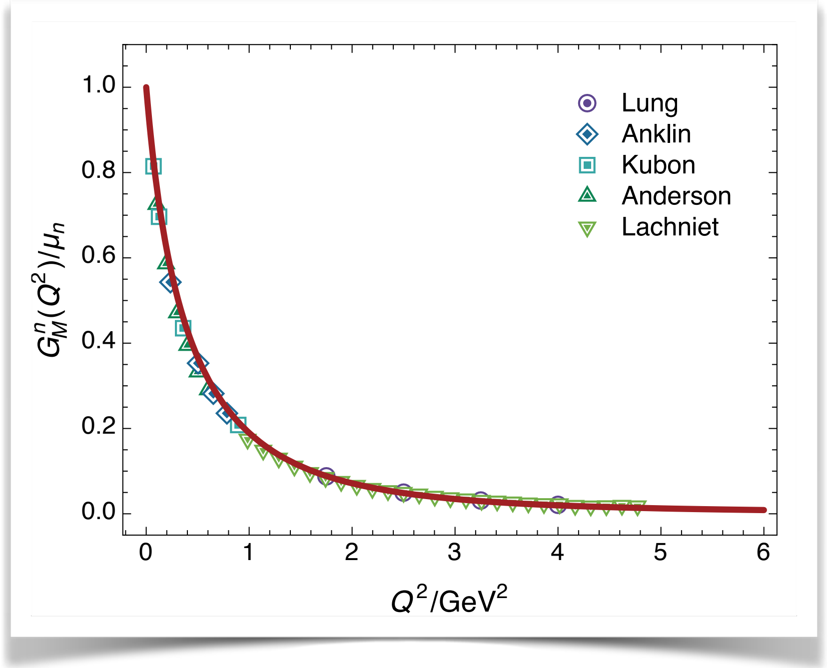


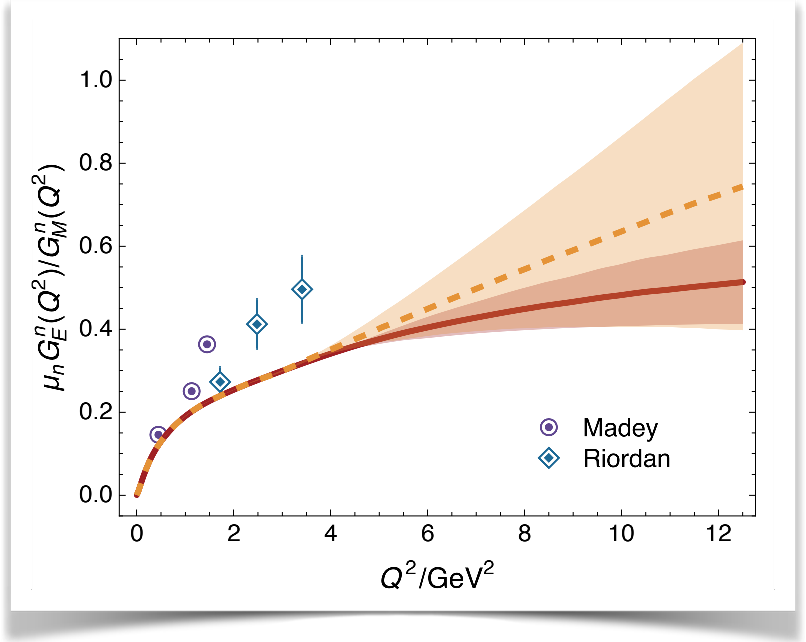


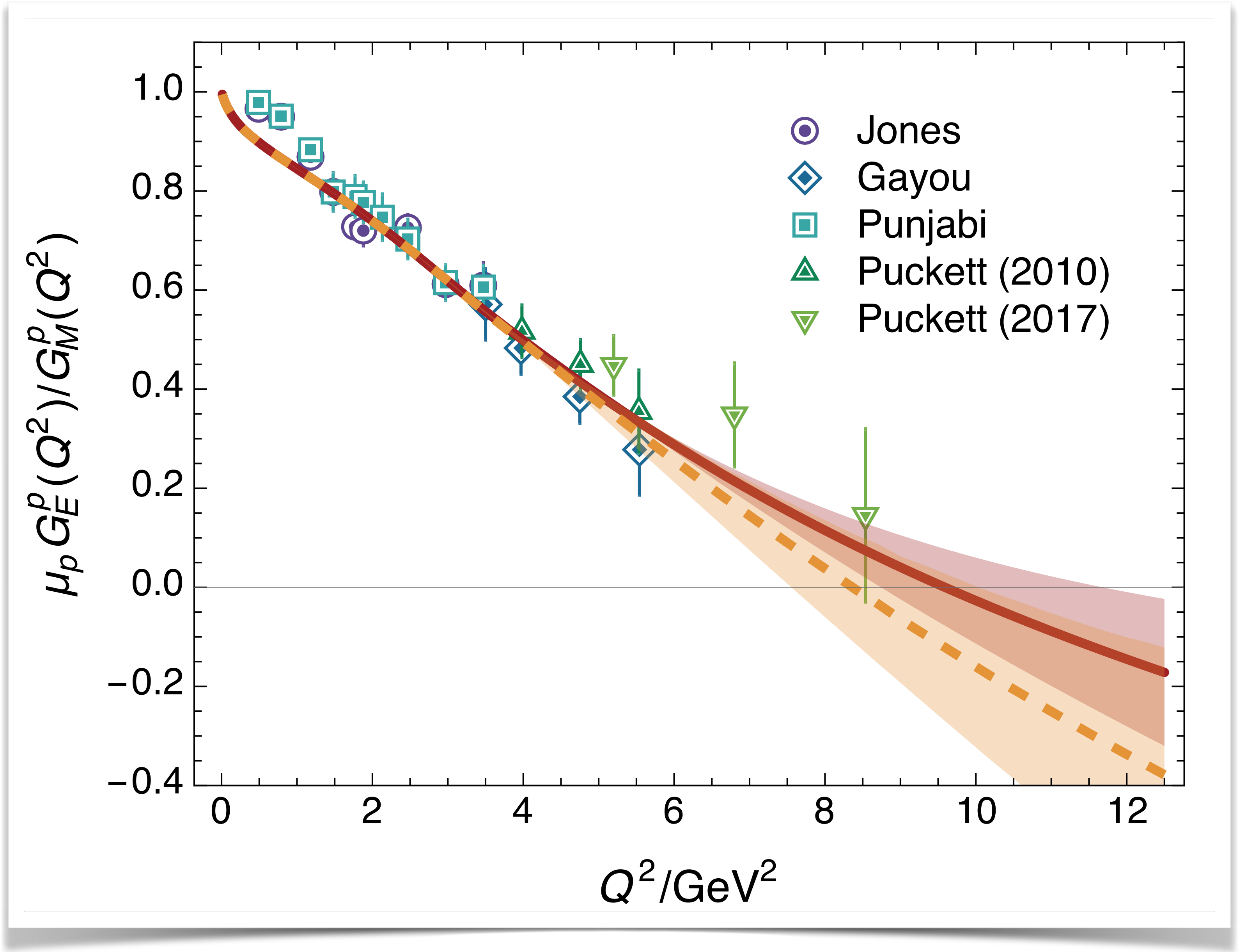


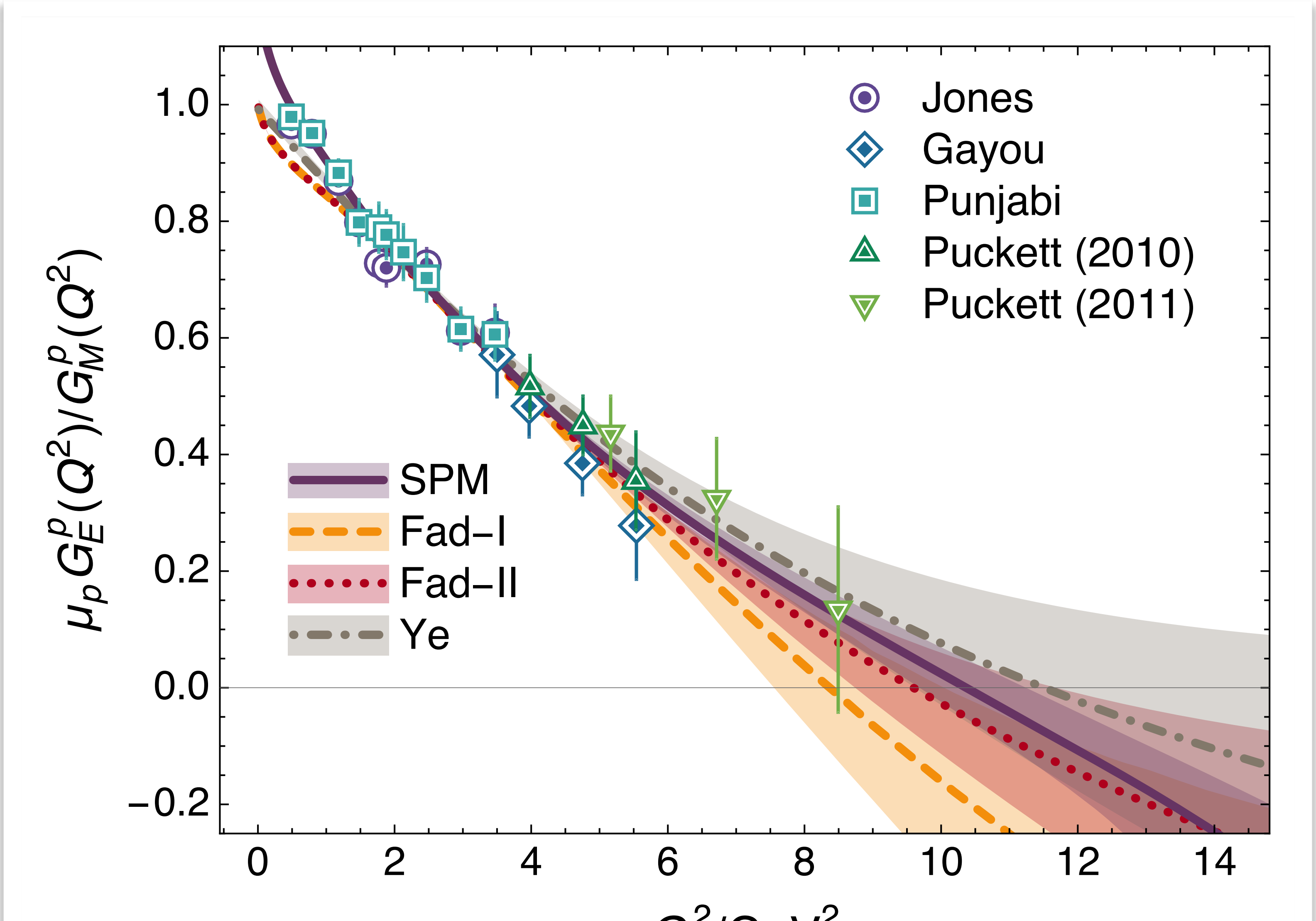












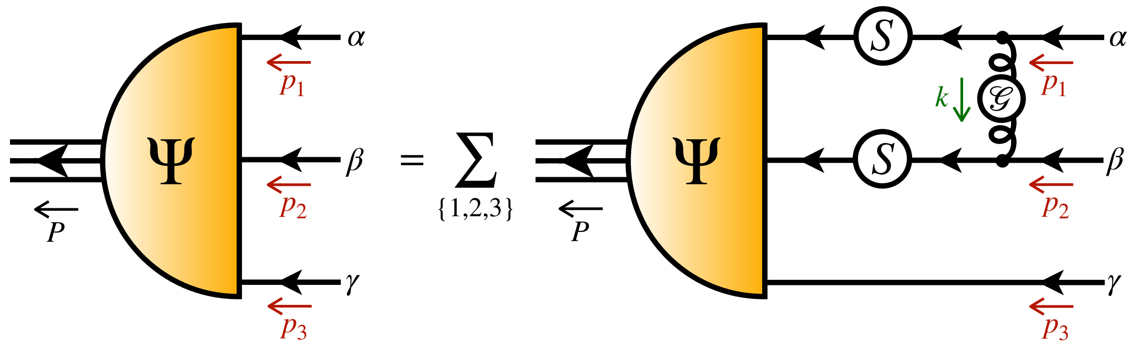
Cheng et al, PLB 862 (2025)

ELECTROMAGNETIC

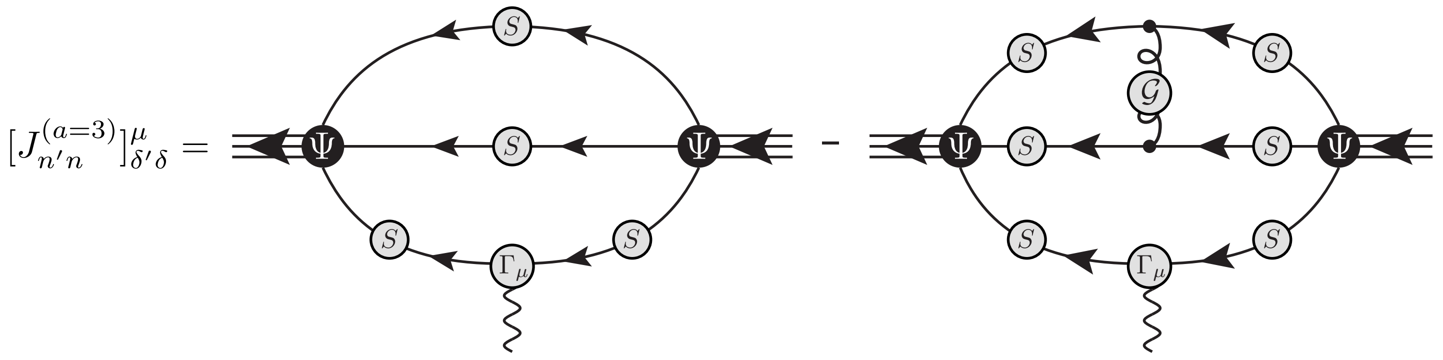
p/n FORM FACTORS

Yao, DB, Cui, Roberts, 2403.08088 (FR in press)

FADDEEV EQUATION



TRIANGULAR DIAGRAM



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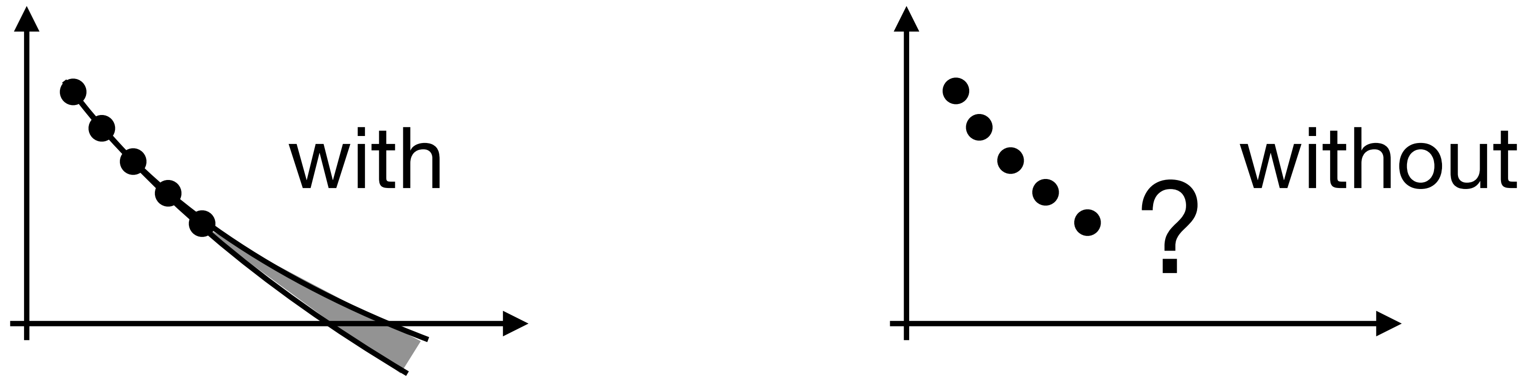

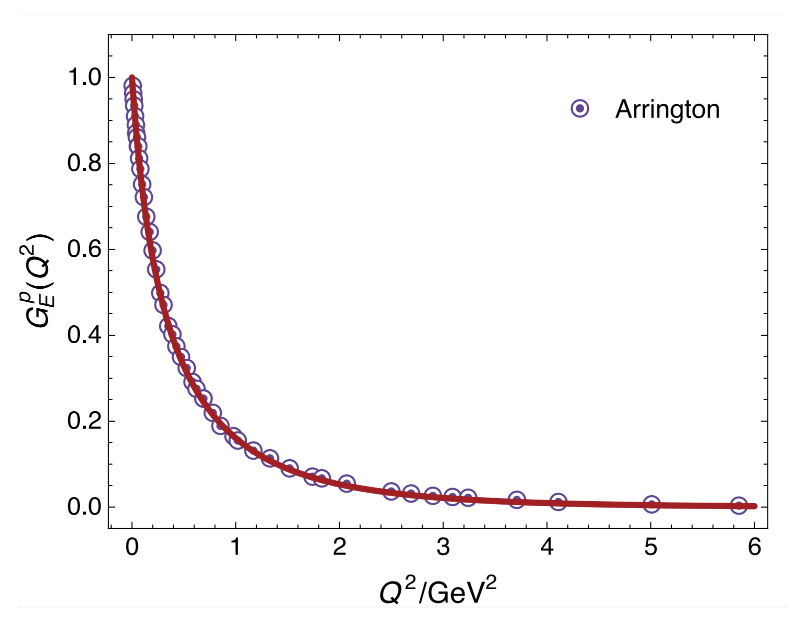

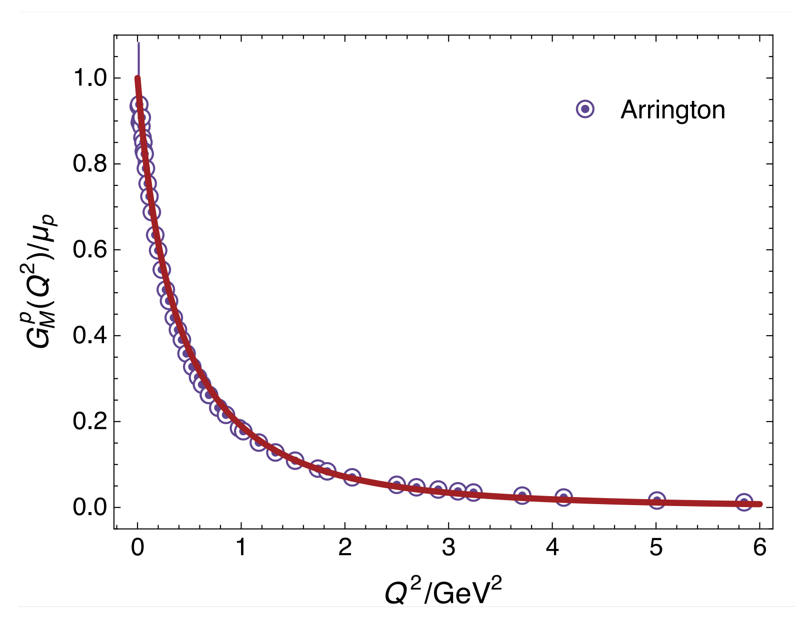

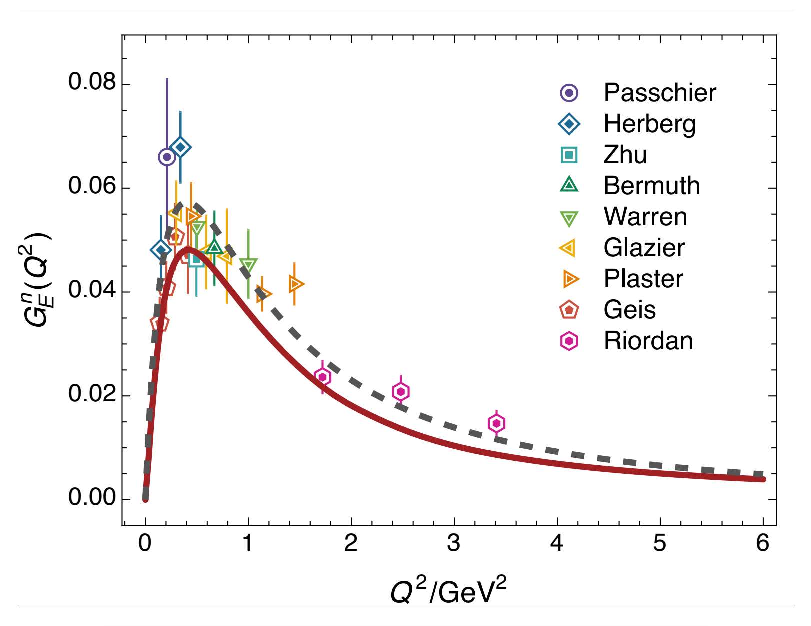

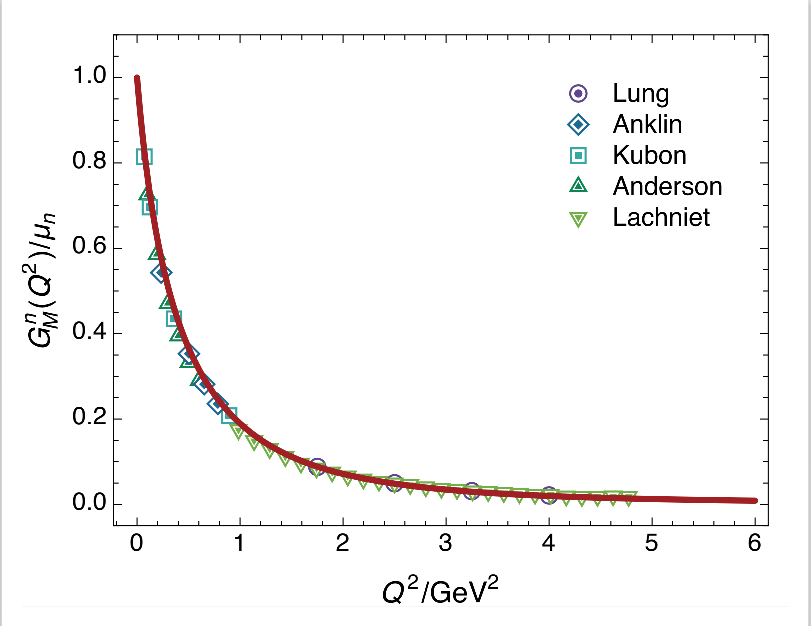

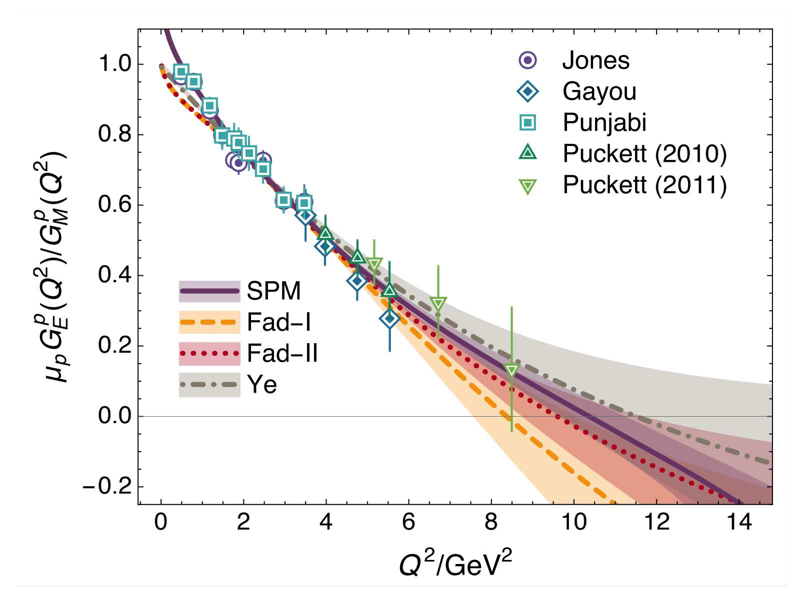

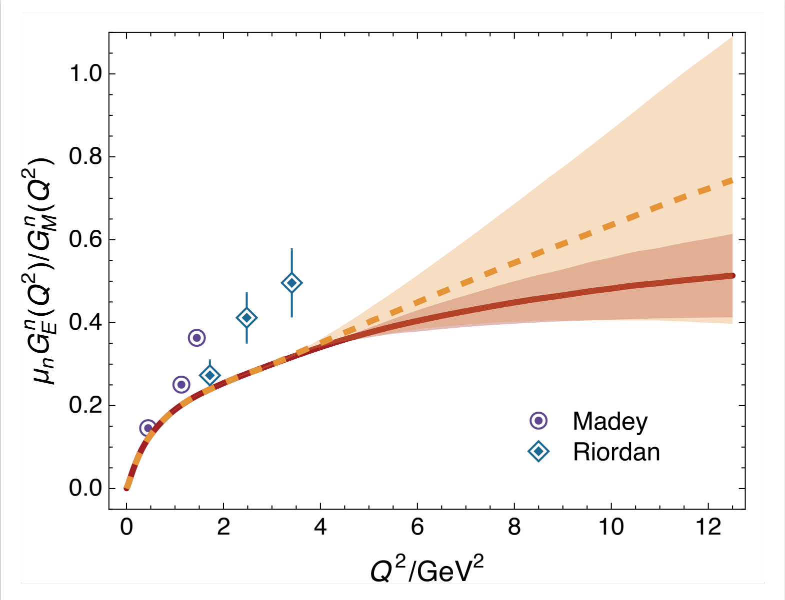

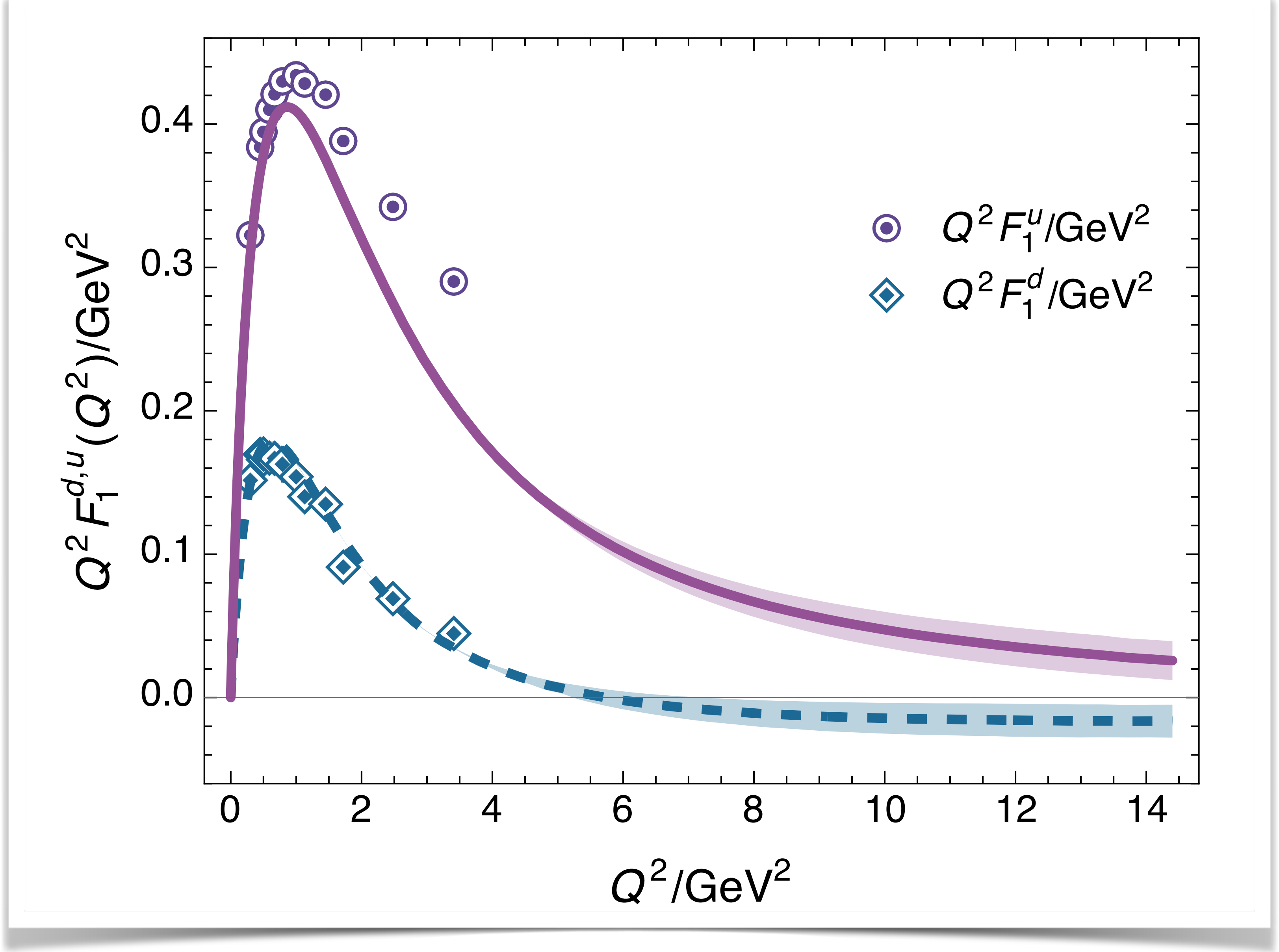

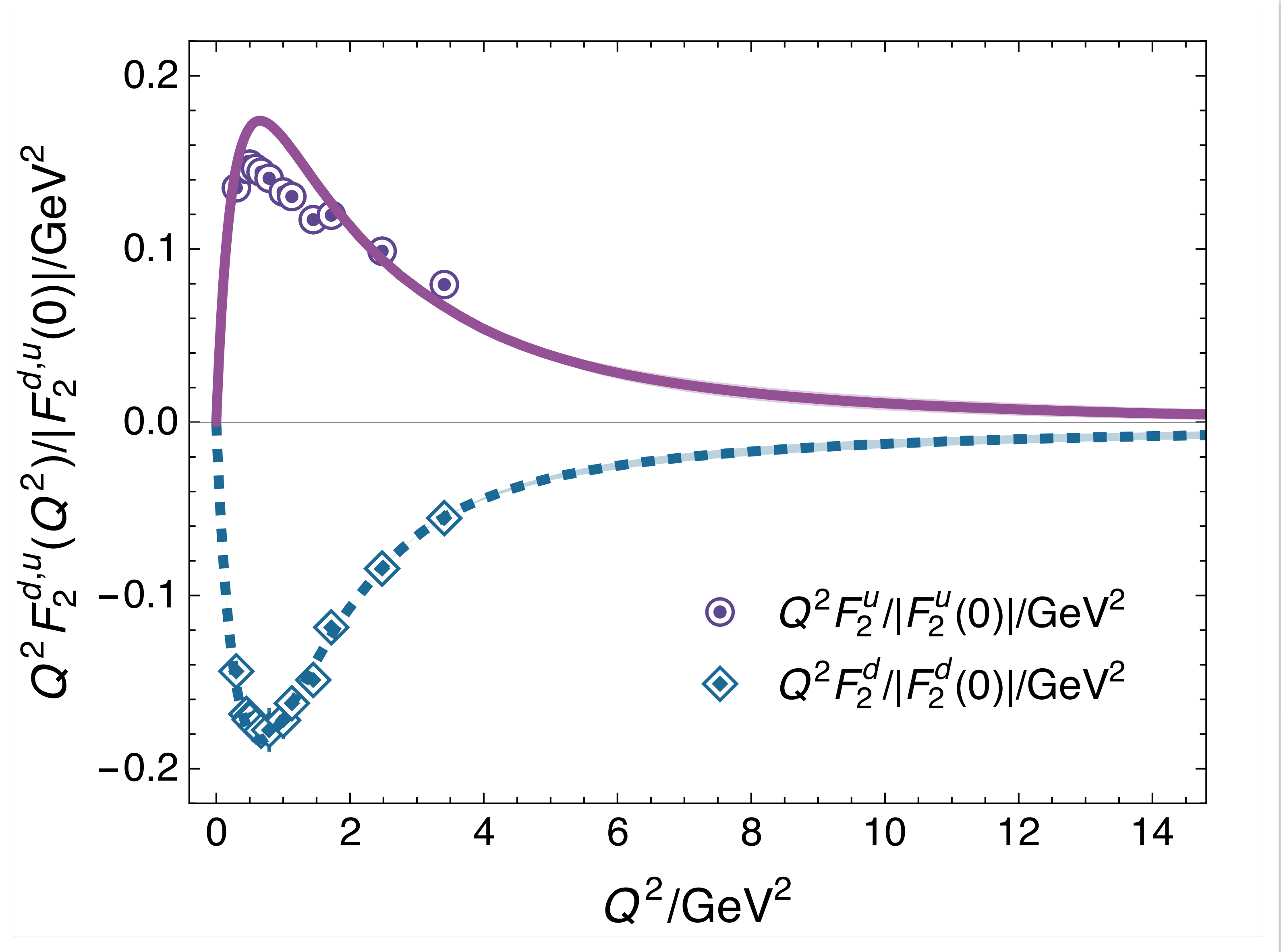

ELECTROMAGNETIC
p/n FORM FACTORS

Yao, DB, Cui, Roberts, 2403.08088 (FR in press)

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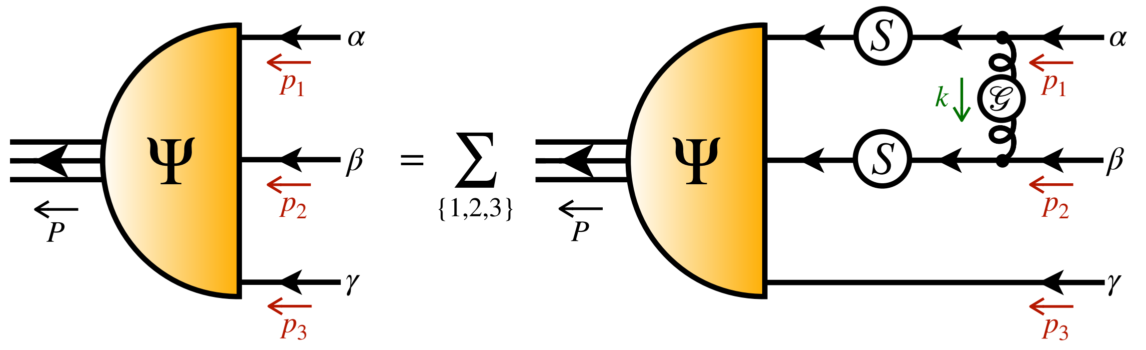

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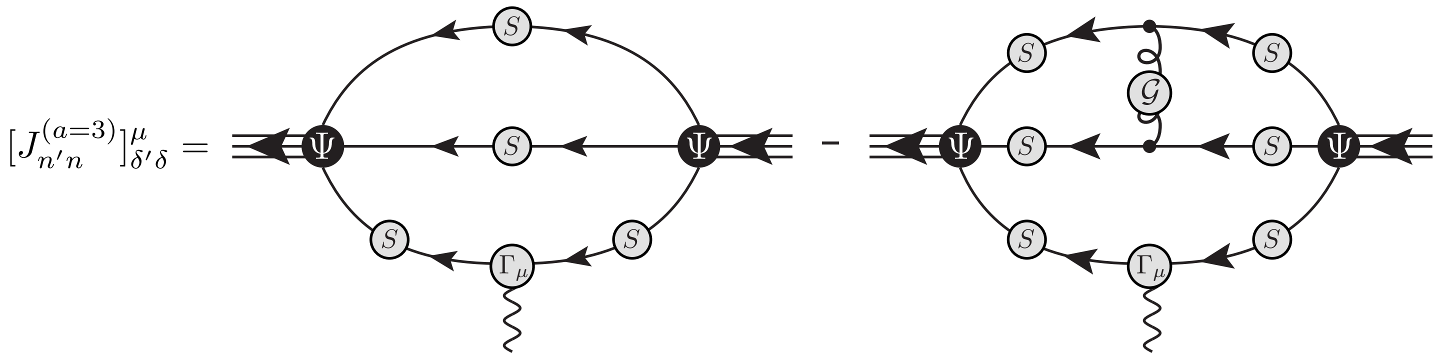

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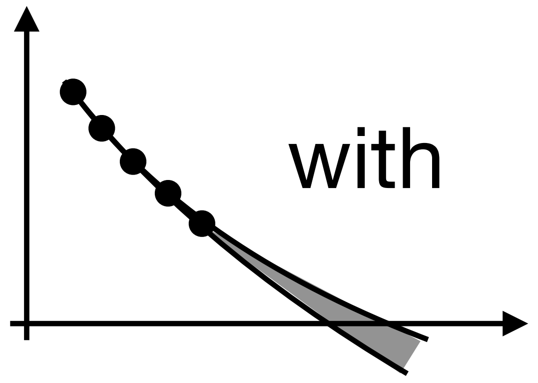
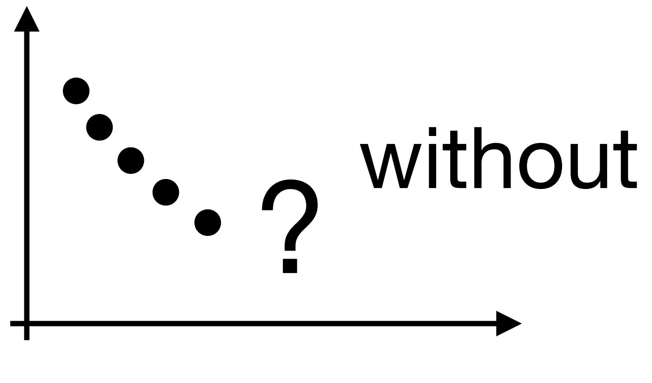
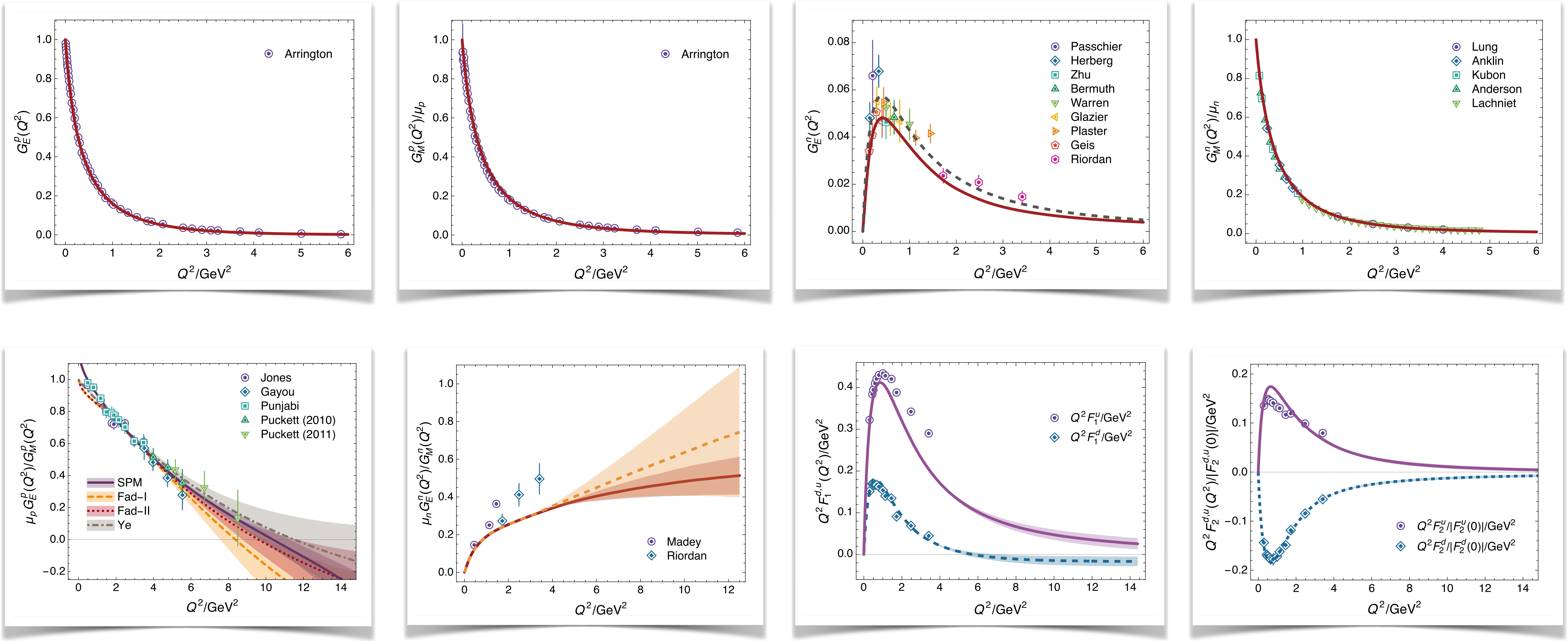
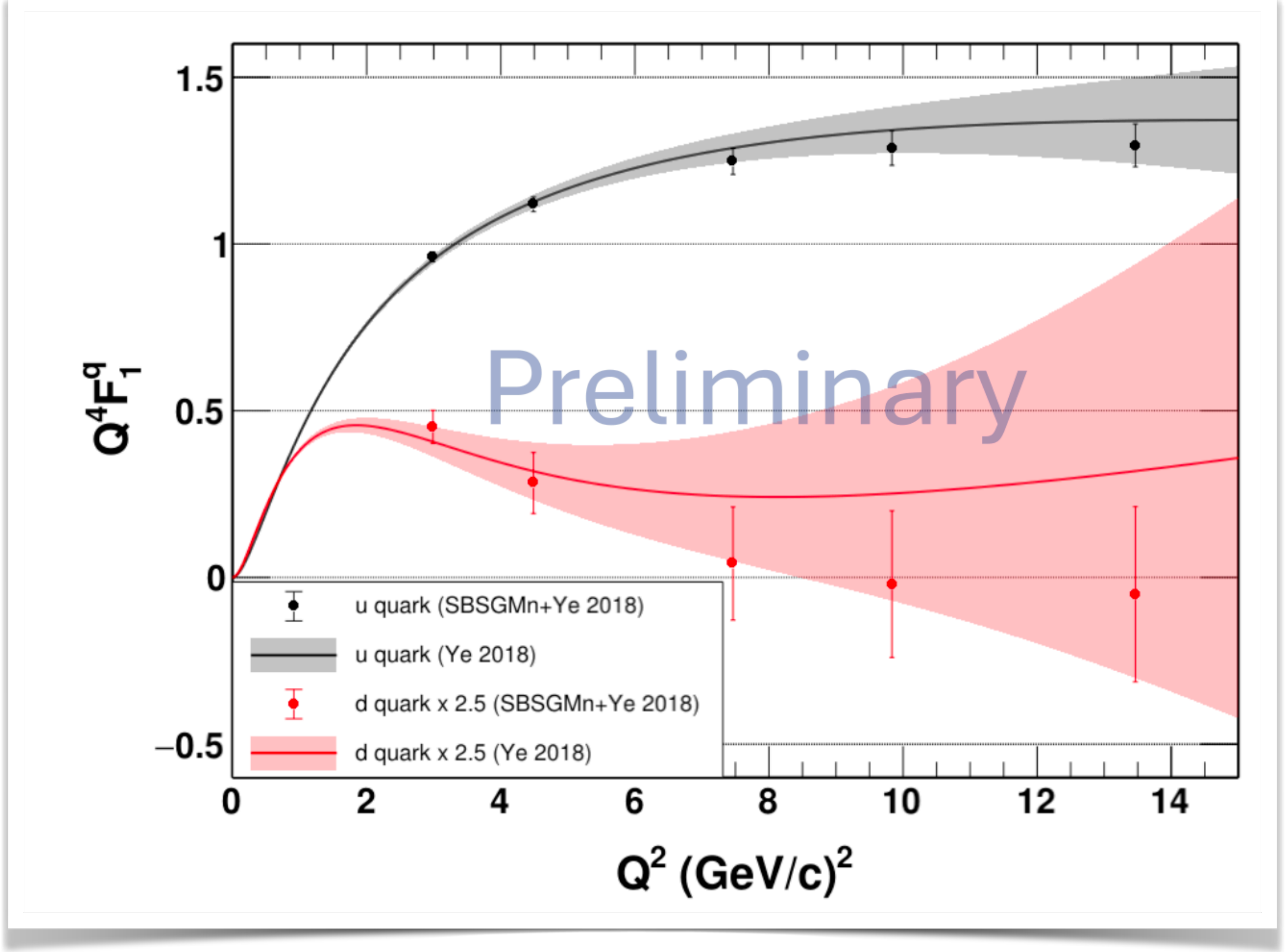








Jefferson Lab
Exploring the Nature of Matter

SUPER BIGBITE SPECTROMETER

Zero-crossing of F_1^d (linear fit to data)

$$Q^2 = 9.8 \pm 1.8\ \mathrm{GeV}^2$$

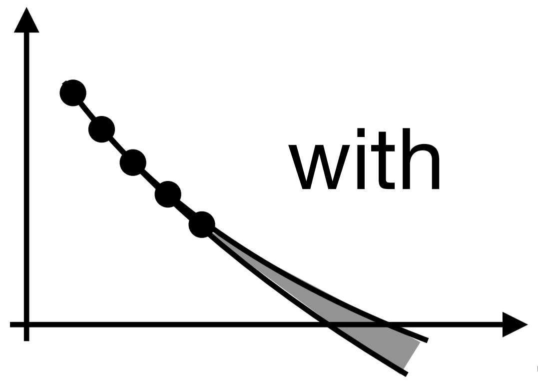
EHM prediction

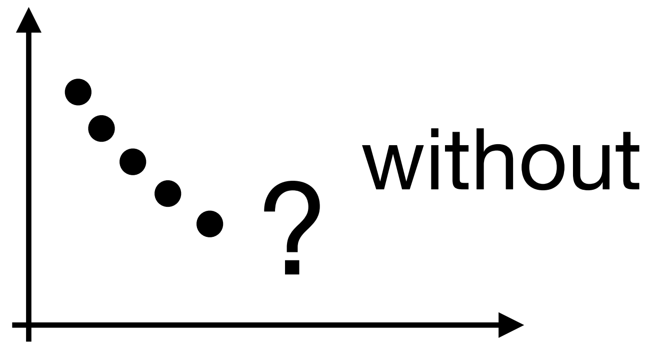
$$Q^2 = 5.7^{+1.5}_{-0.5}\ \mathrm{GeV}^2$$

experiment is only 1.4σ away from our prediction!

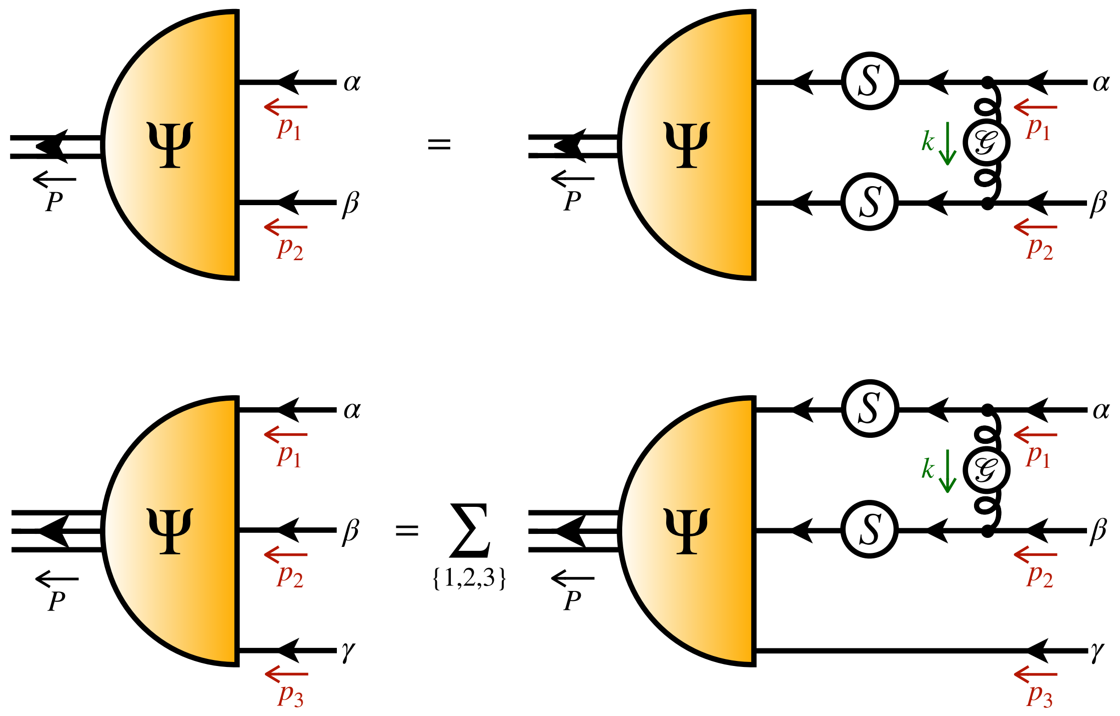
GRAVITATIONAL p/n FORM FACTORS

Yao et al, EPJA 61 (2025)





BETHE-SALPETER + FADDEEV EQUATIONS



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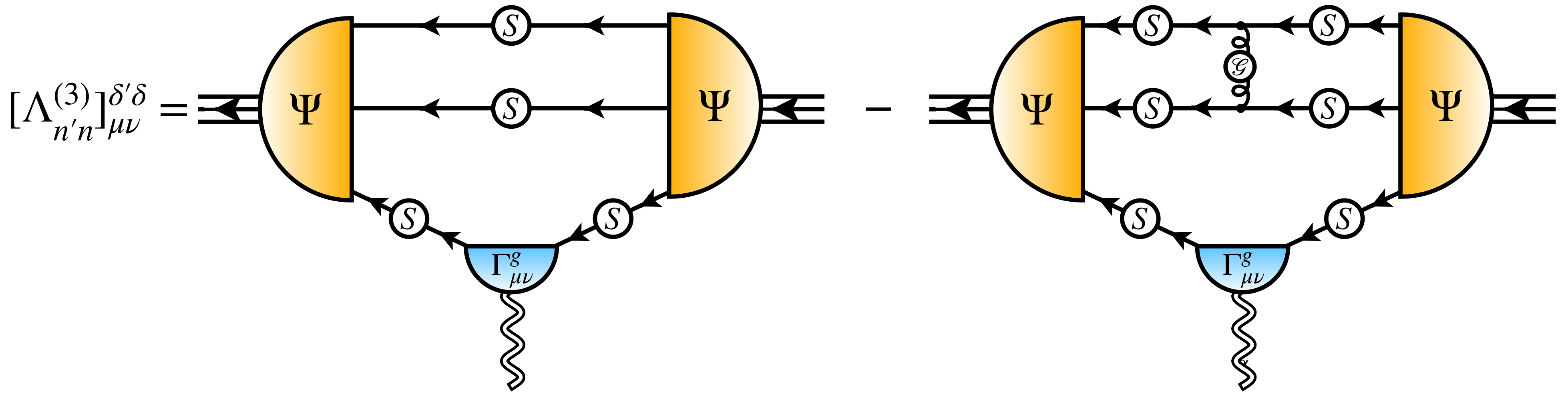

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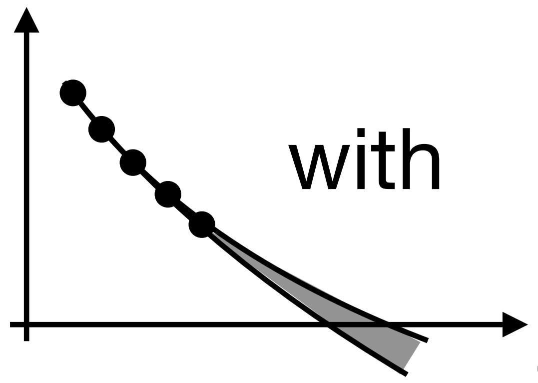

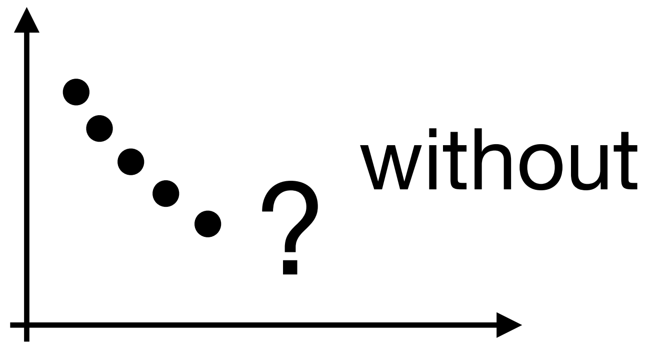

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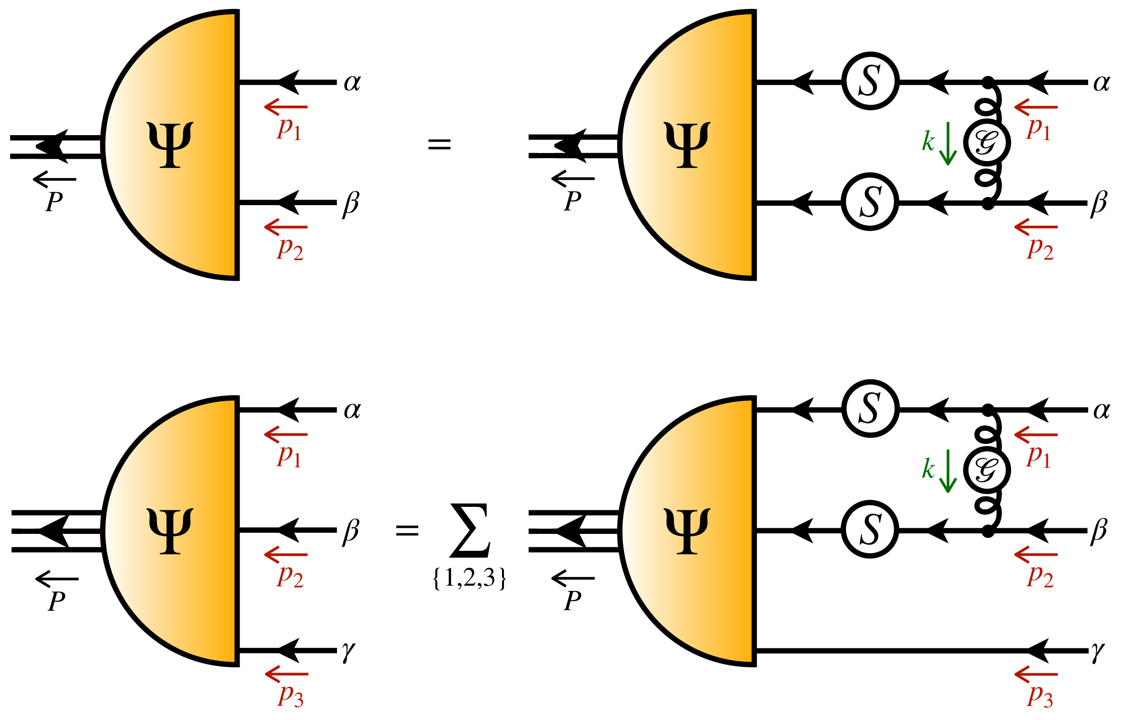

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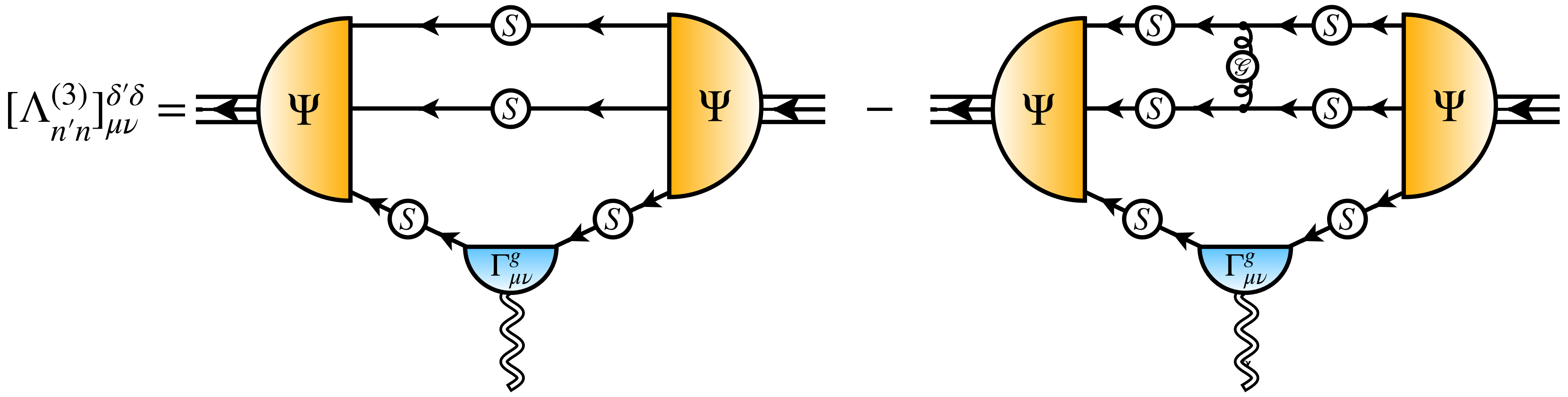

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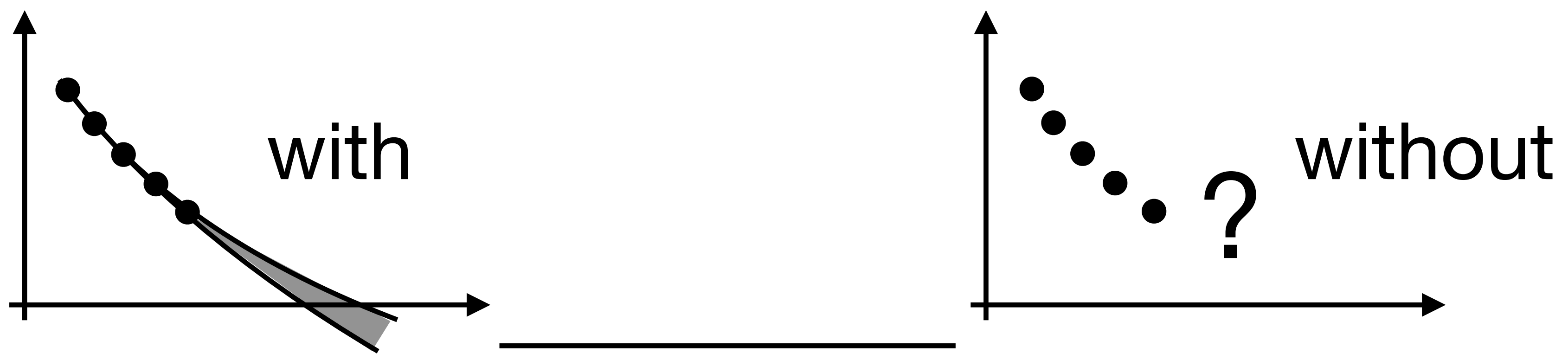

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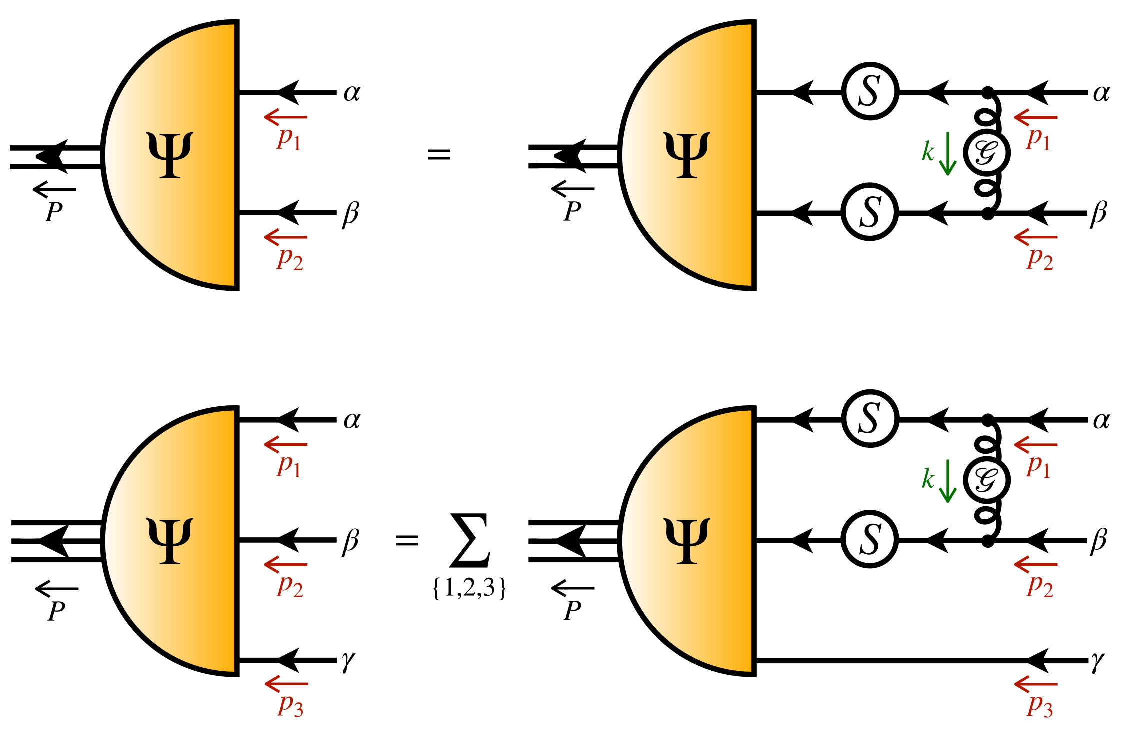

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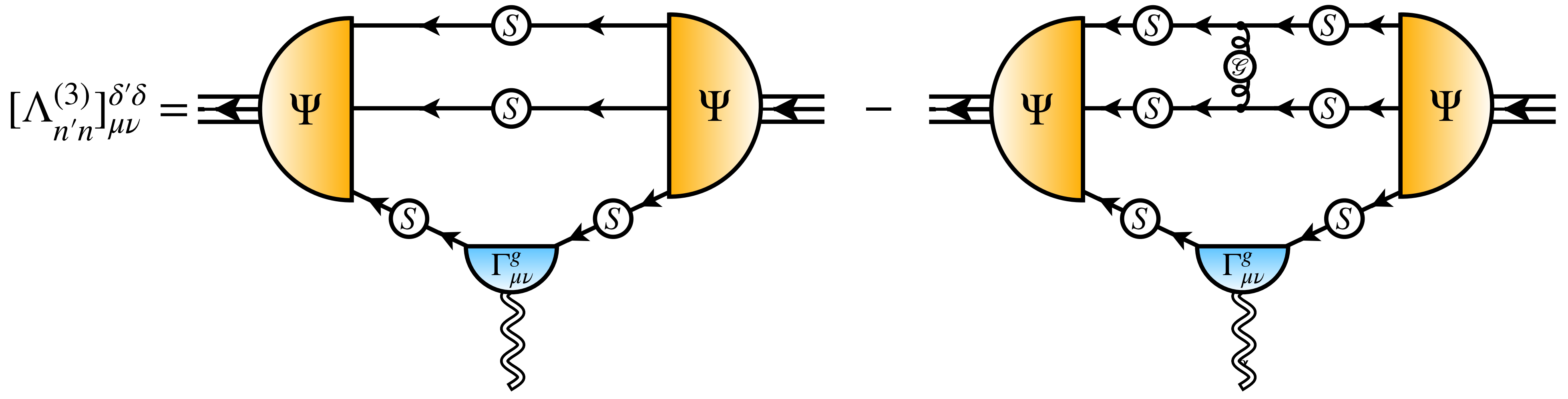

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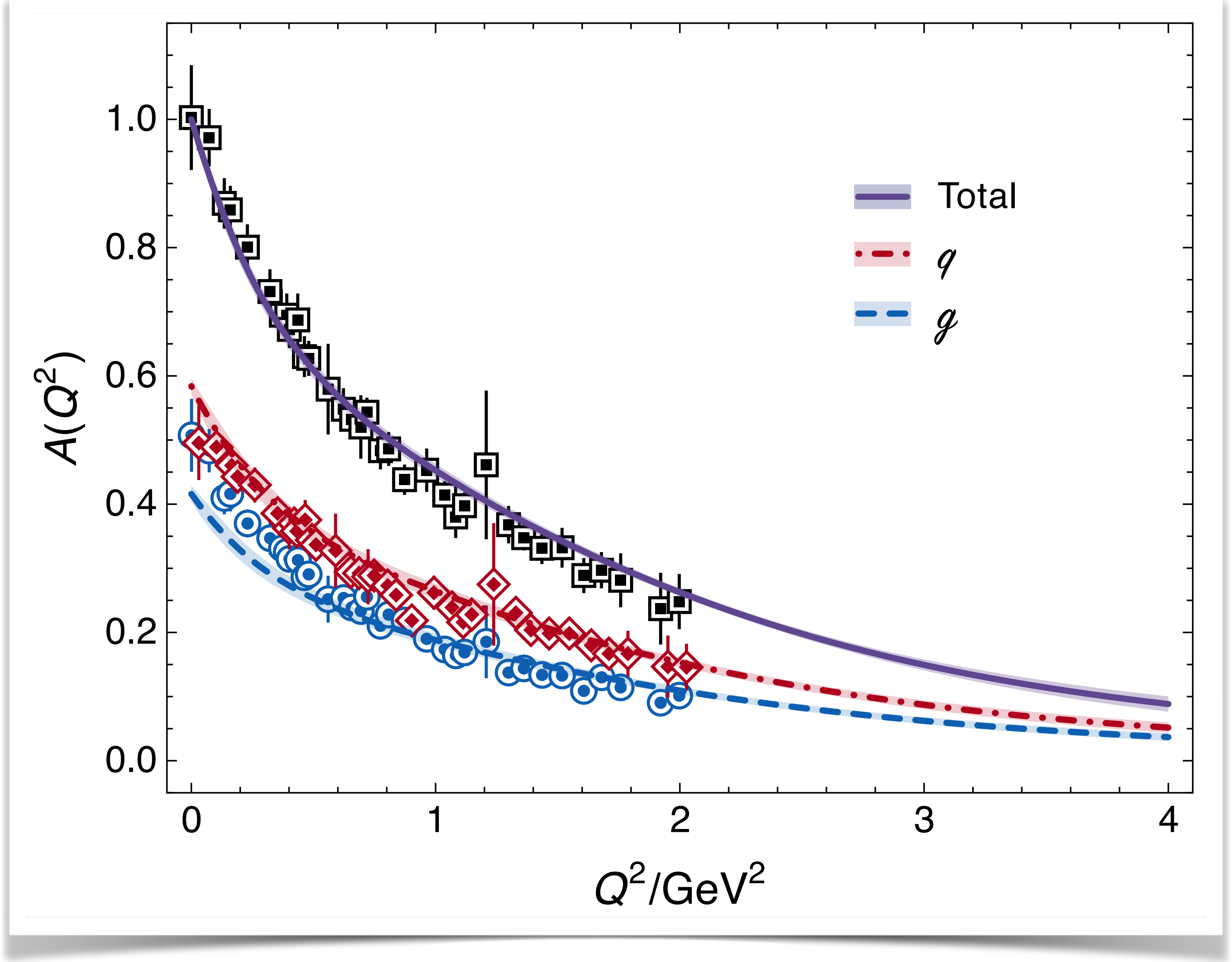

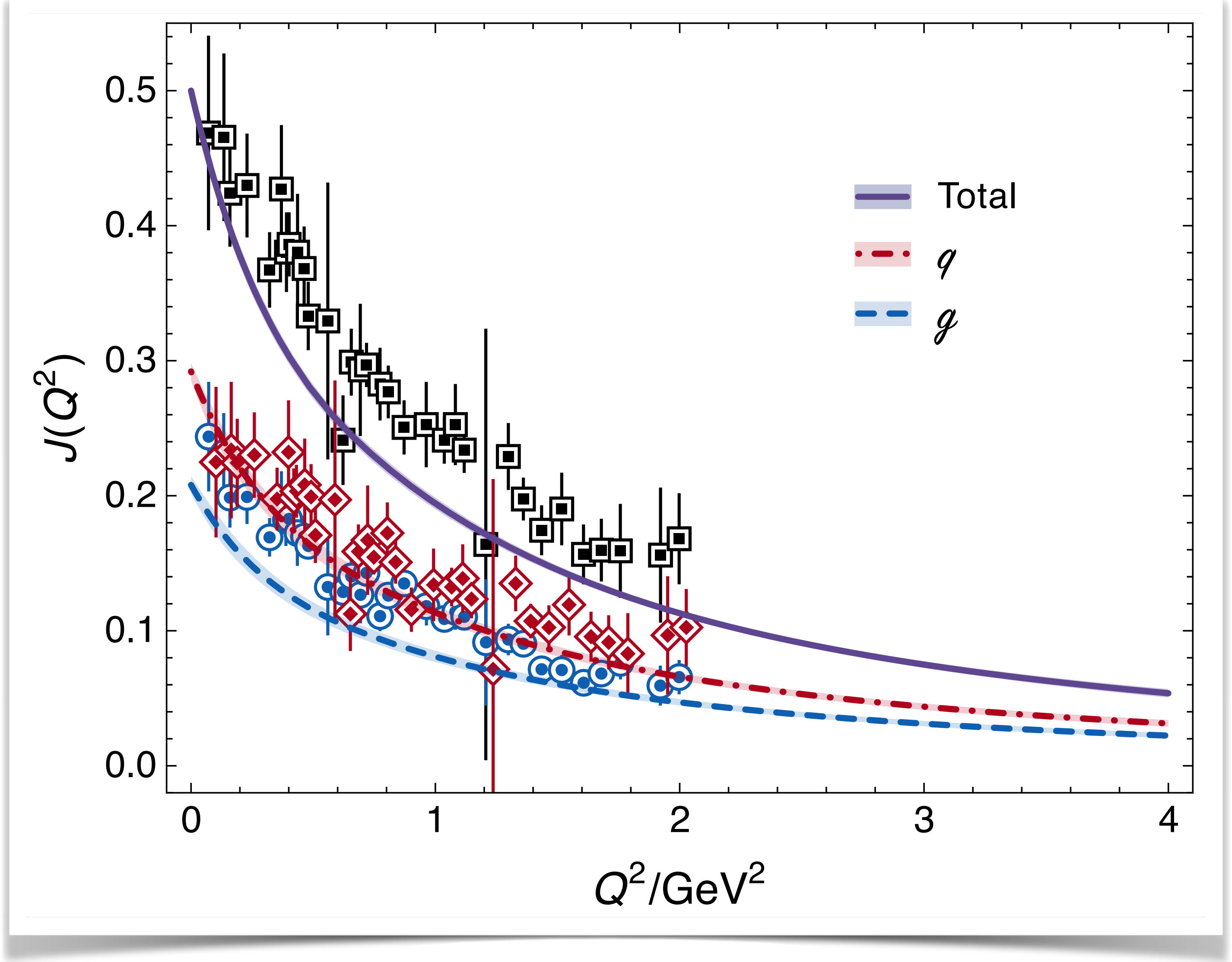

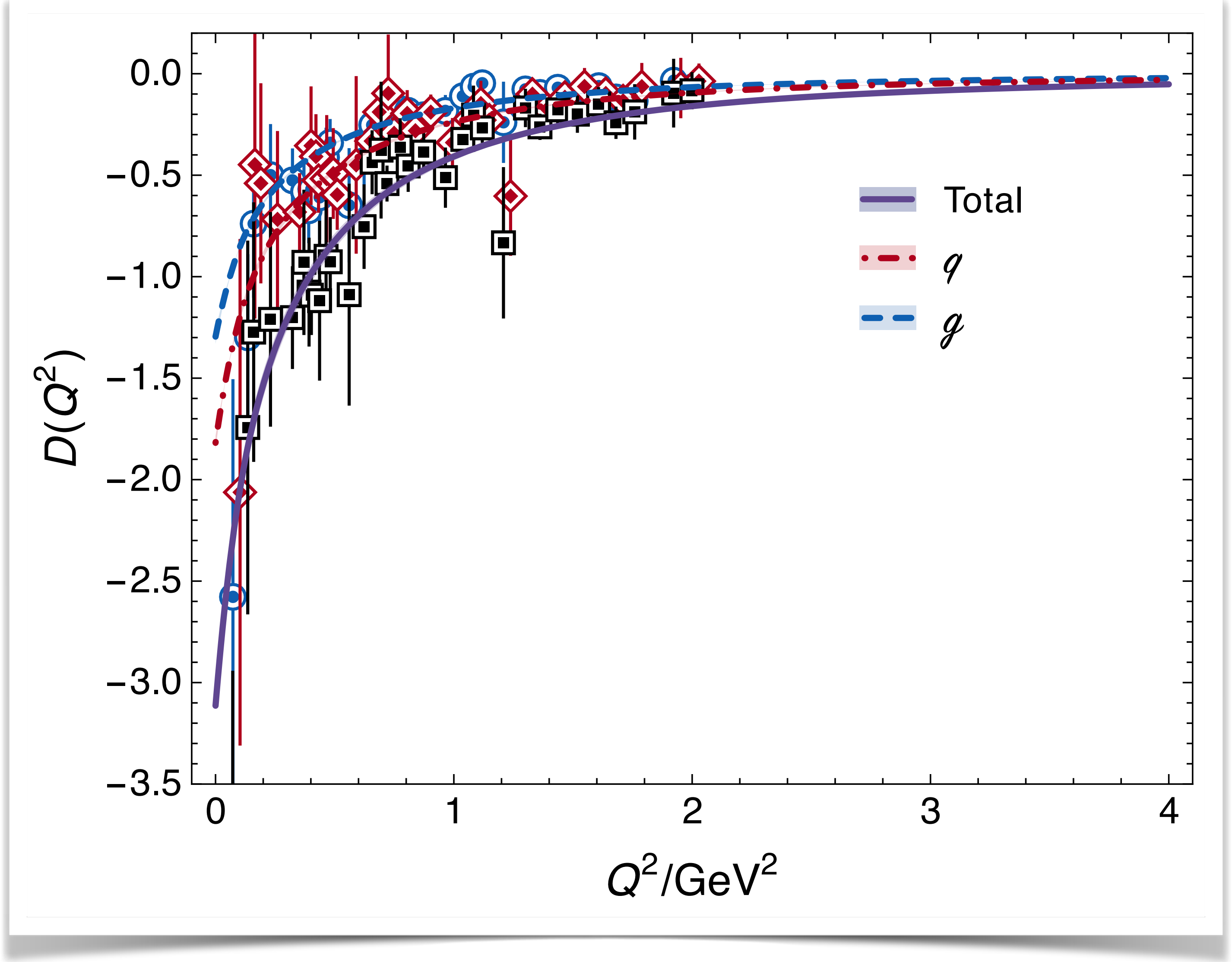

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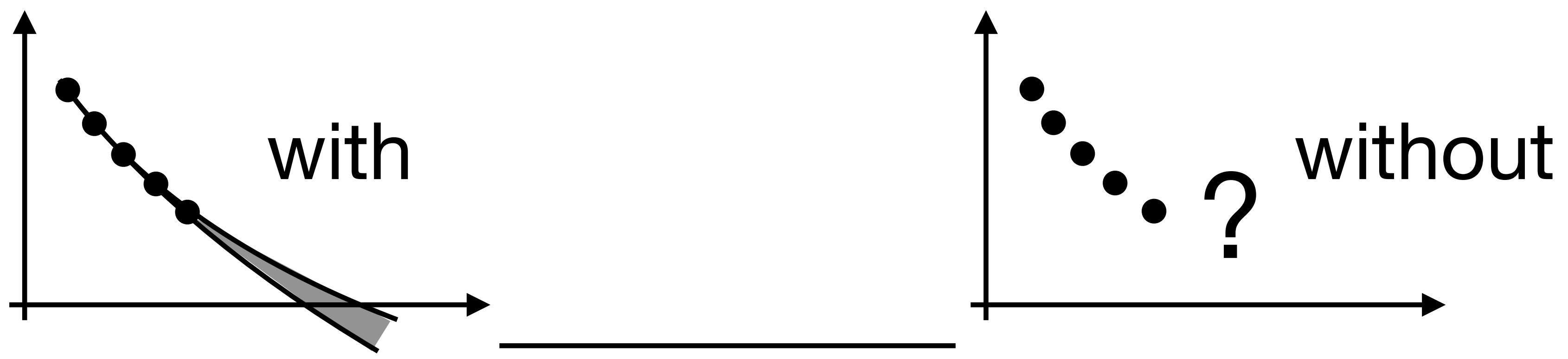

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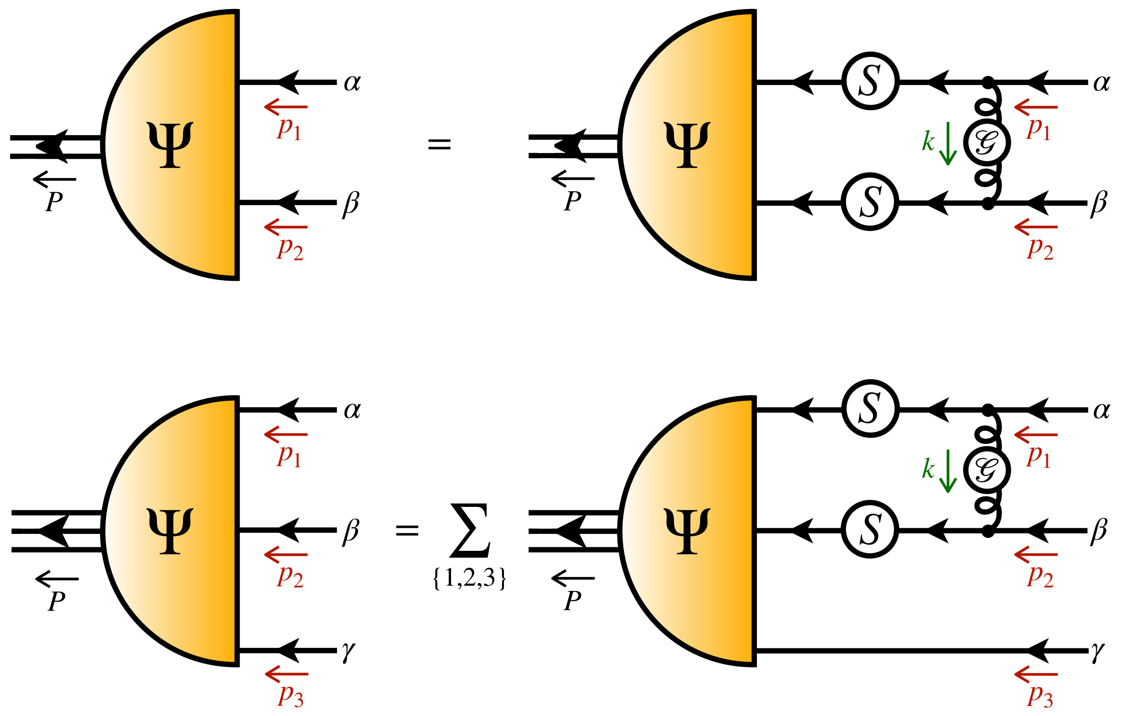

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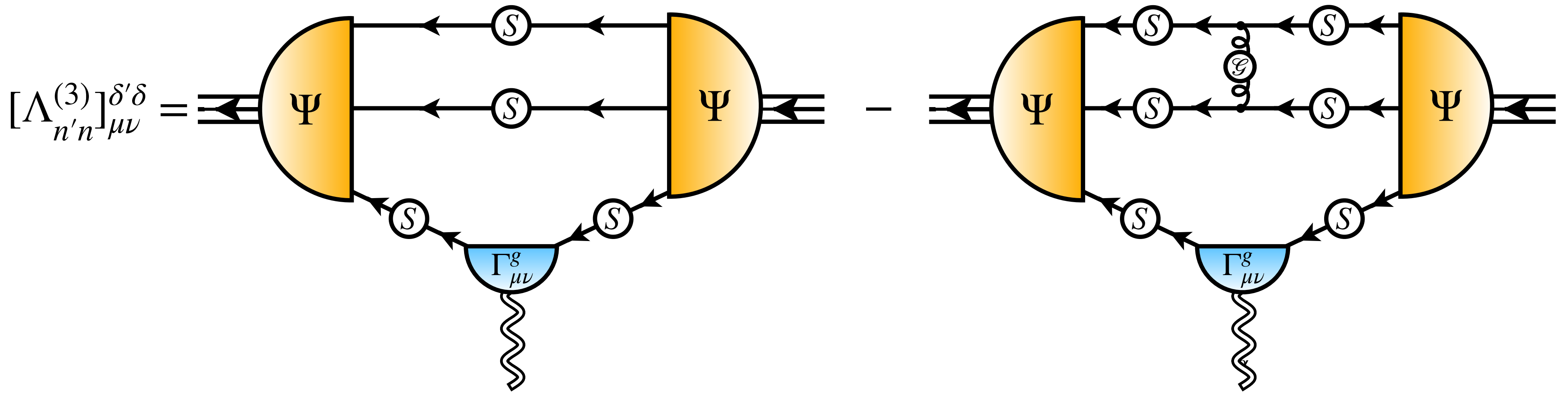

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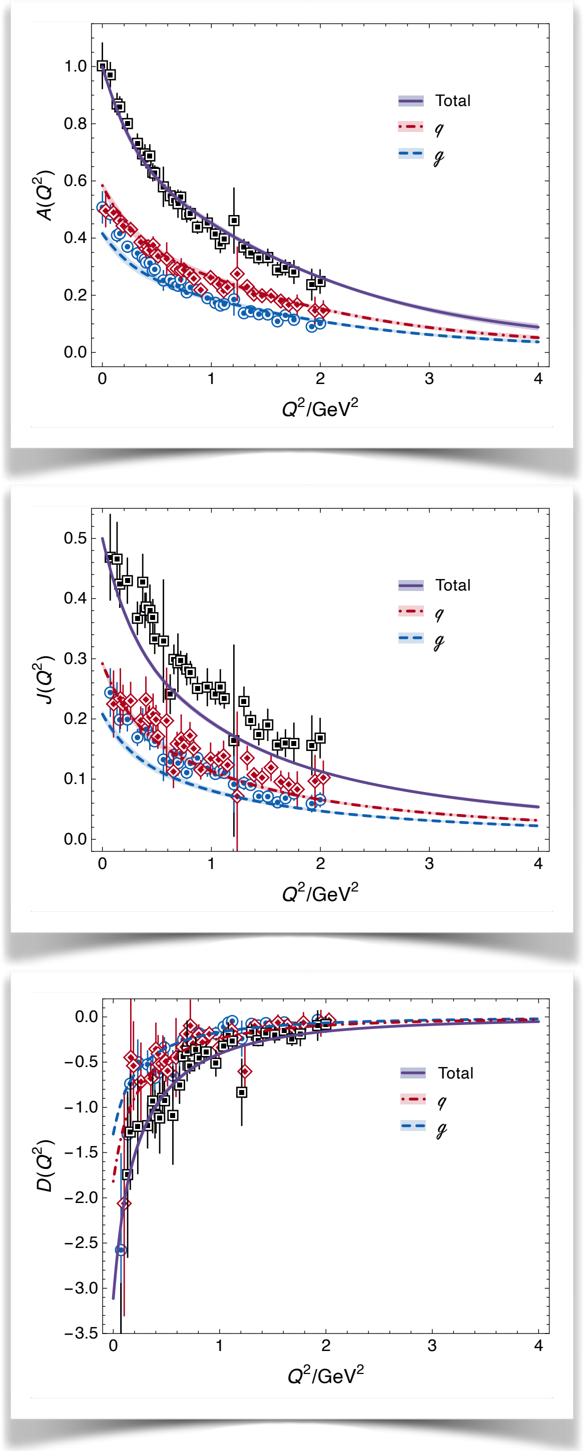

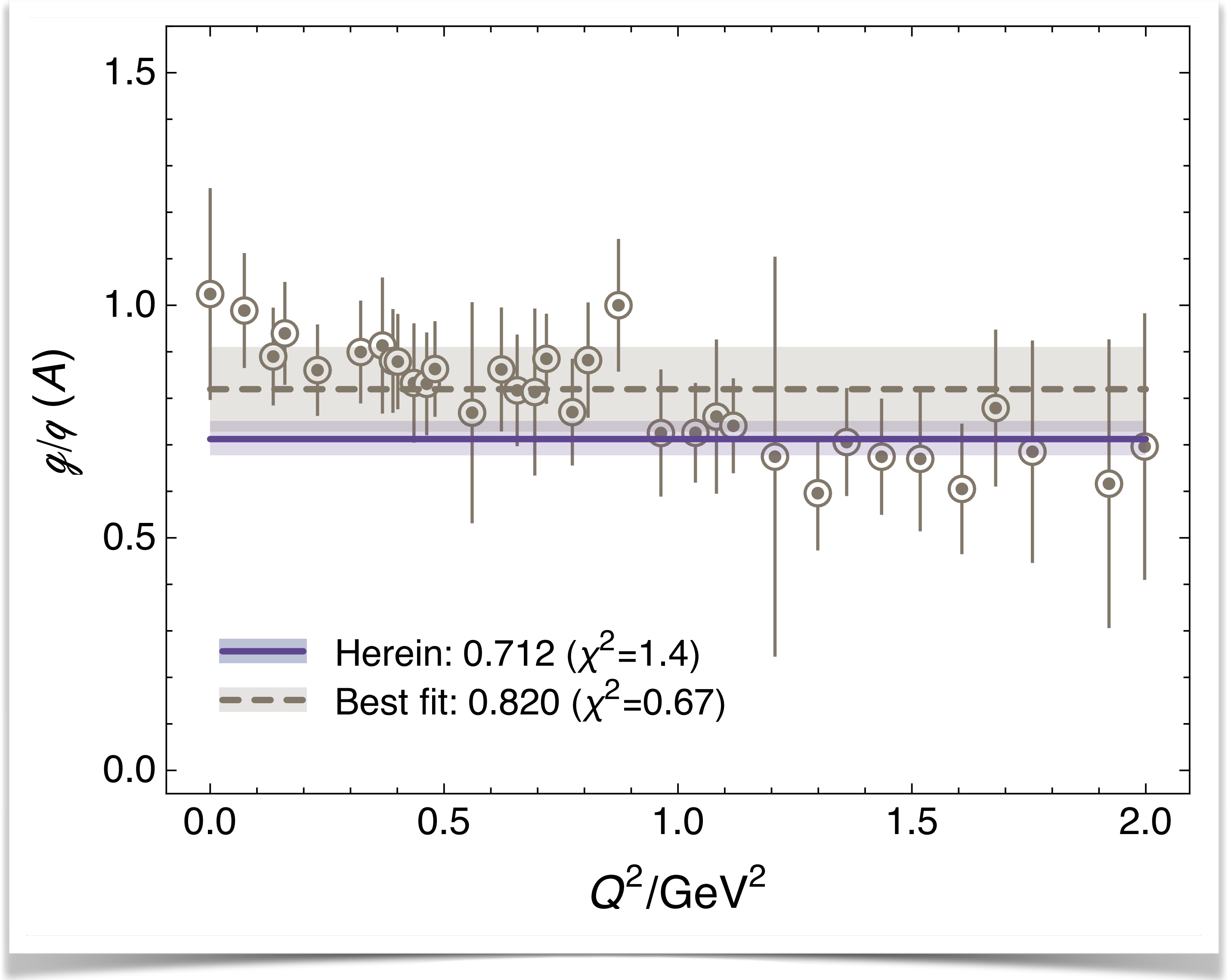

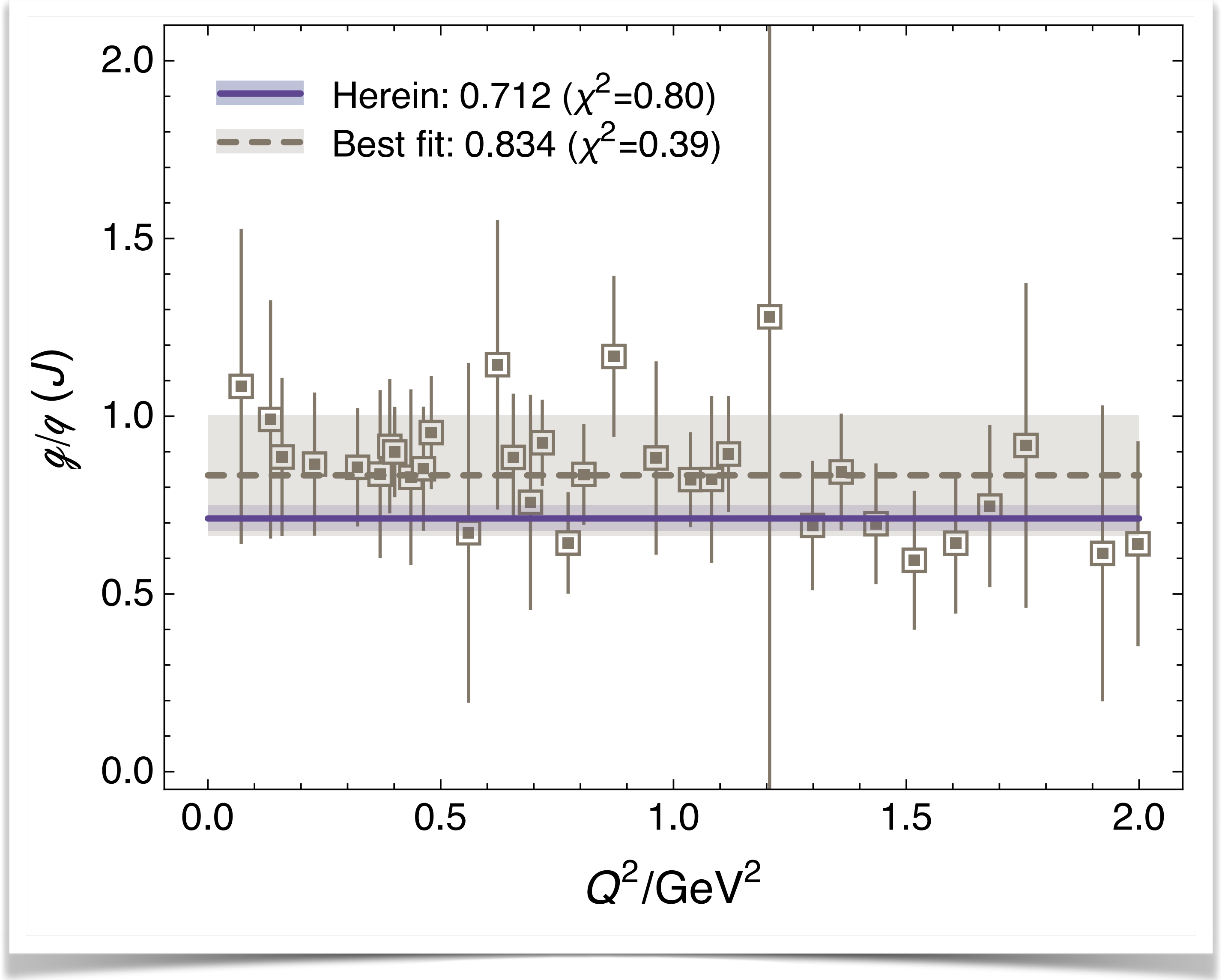

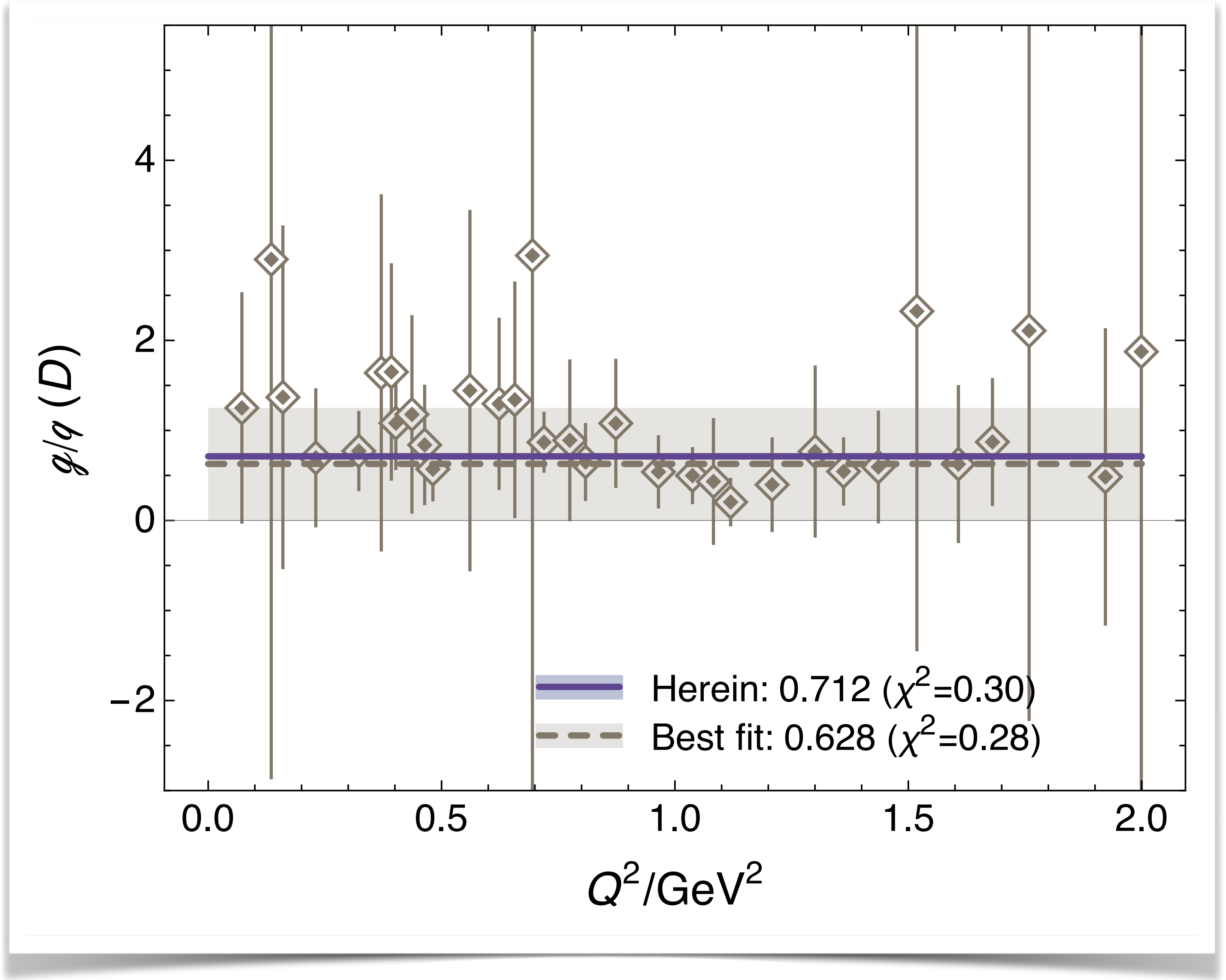

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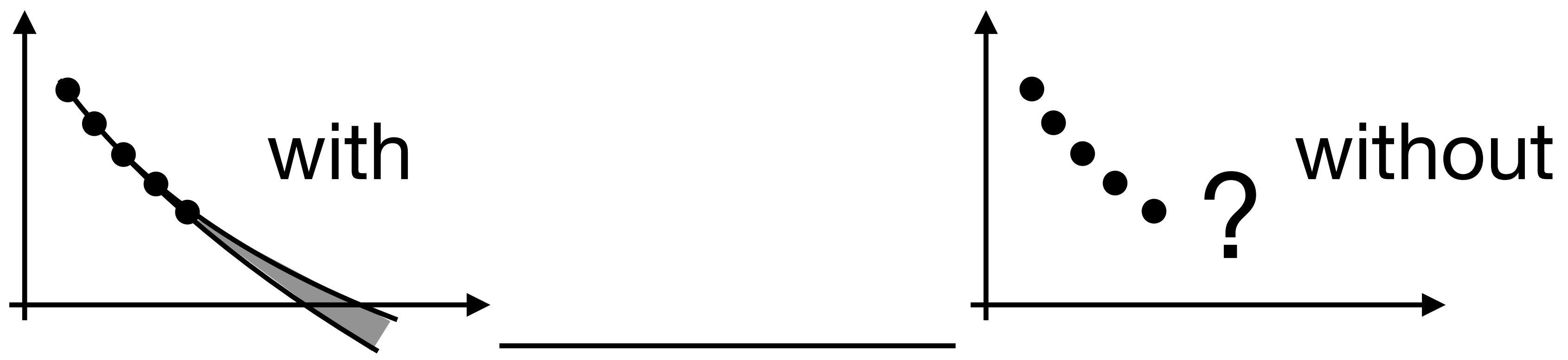

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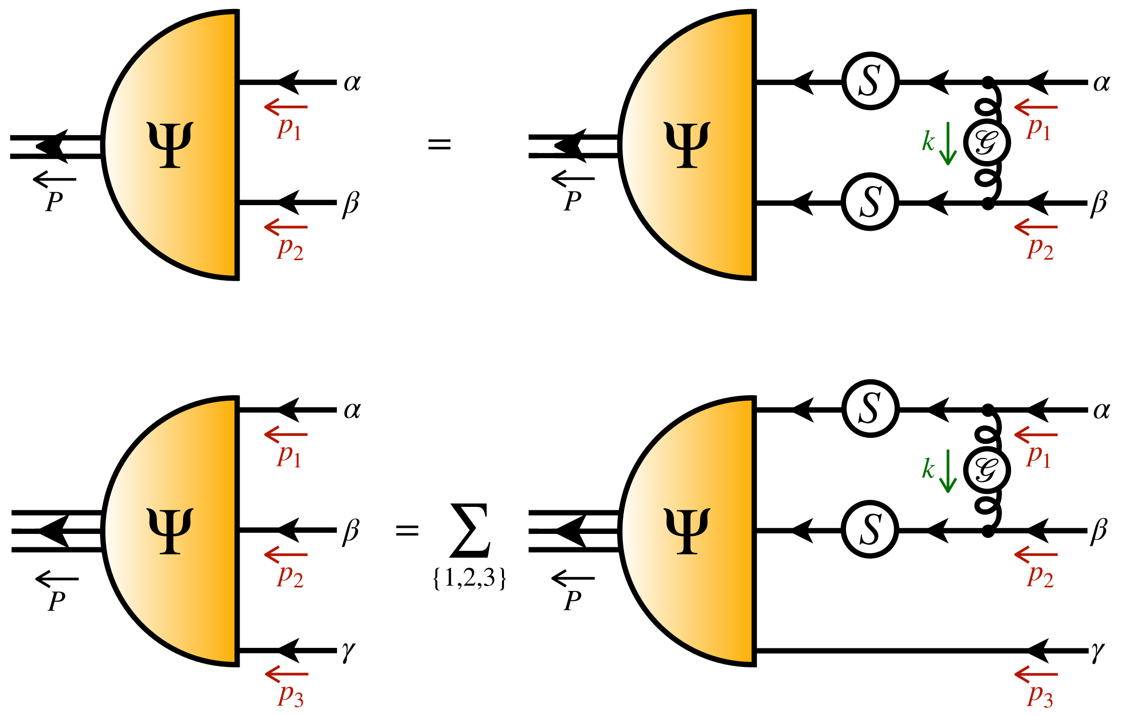

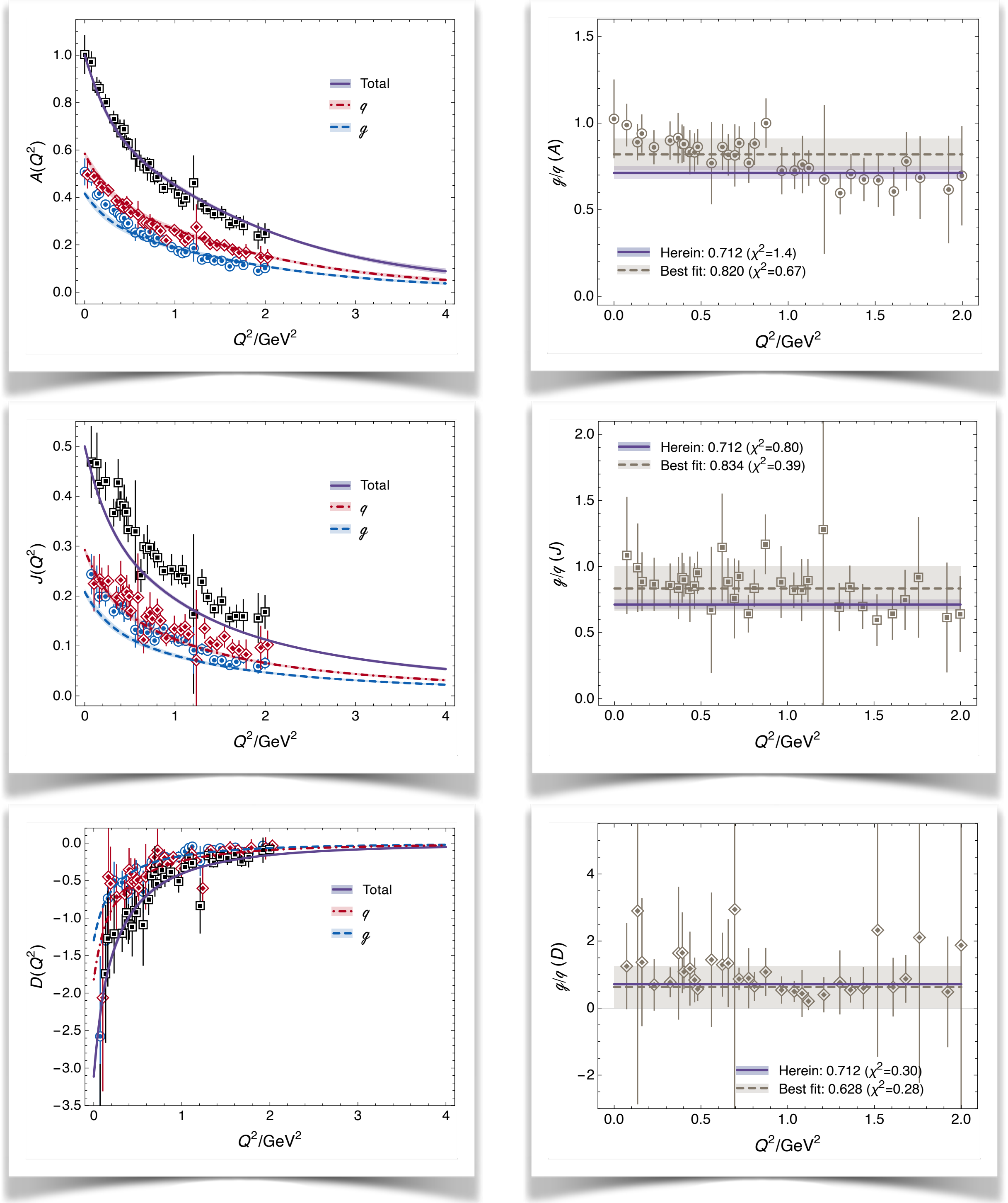

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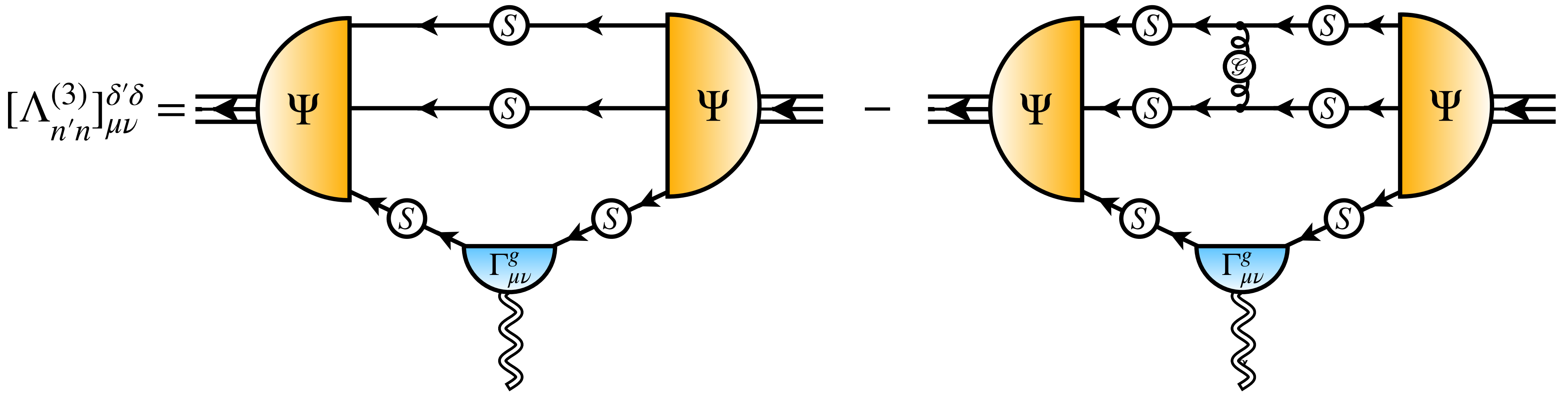

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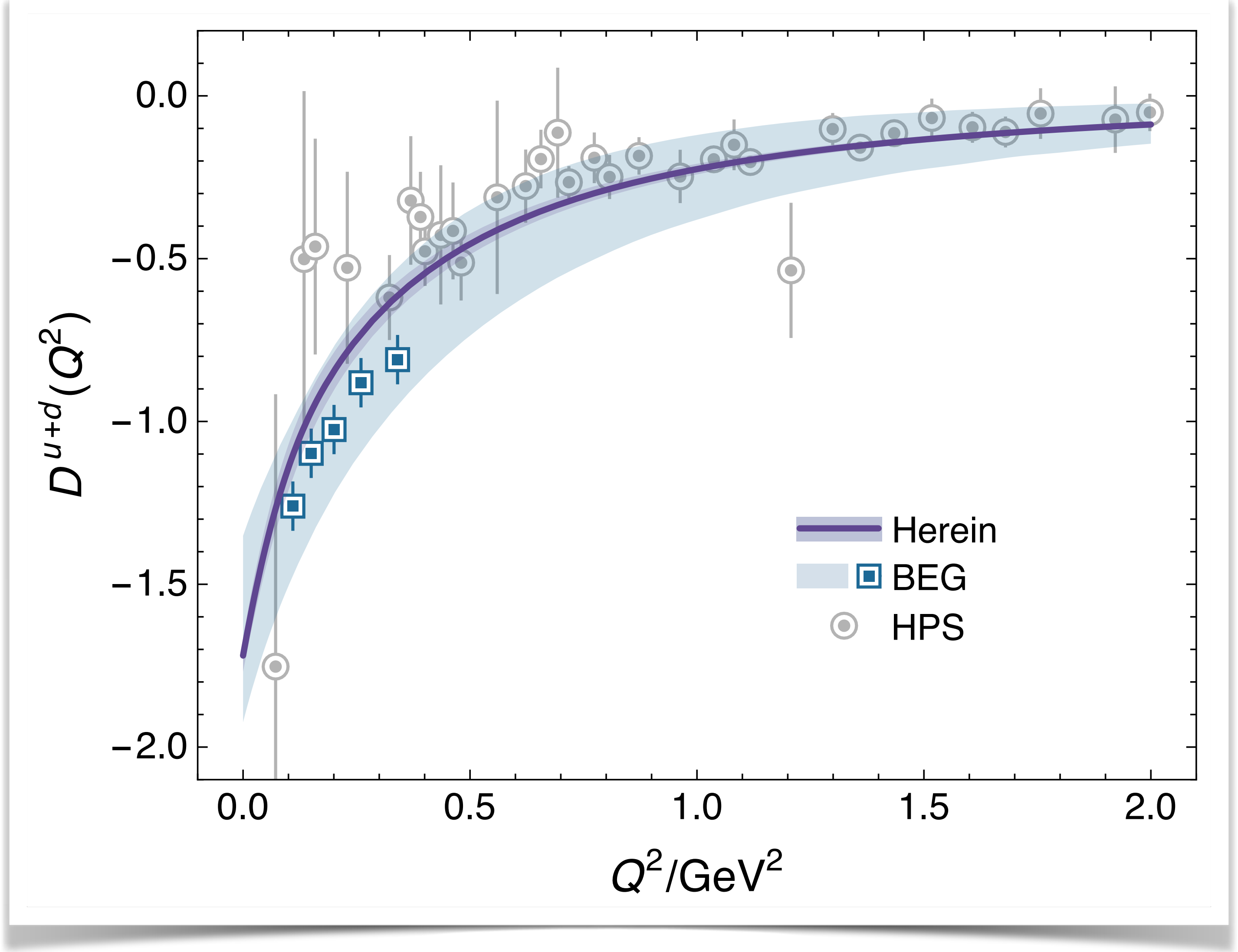

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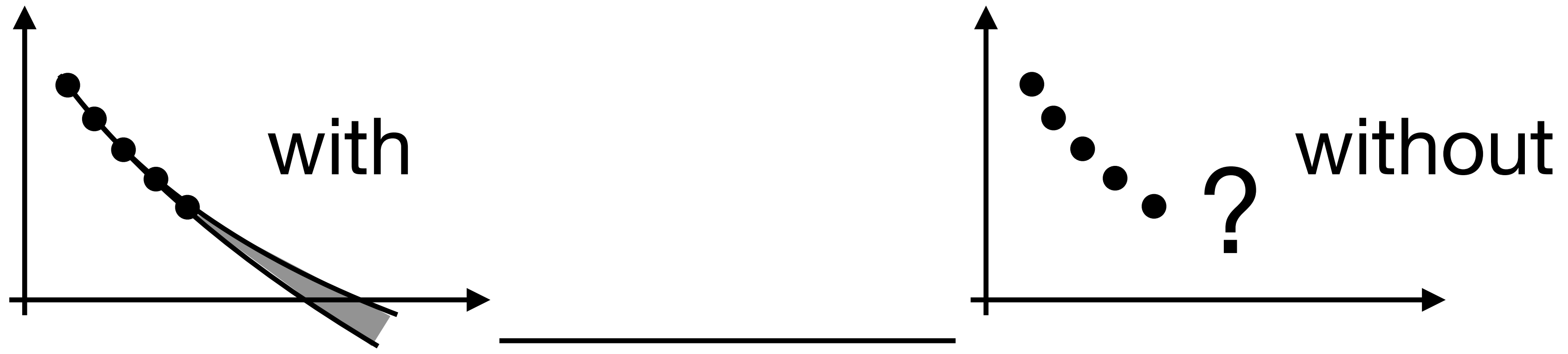

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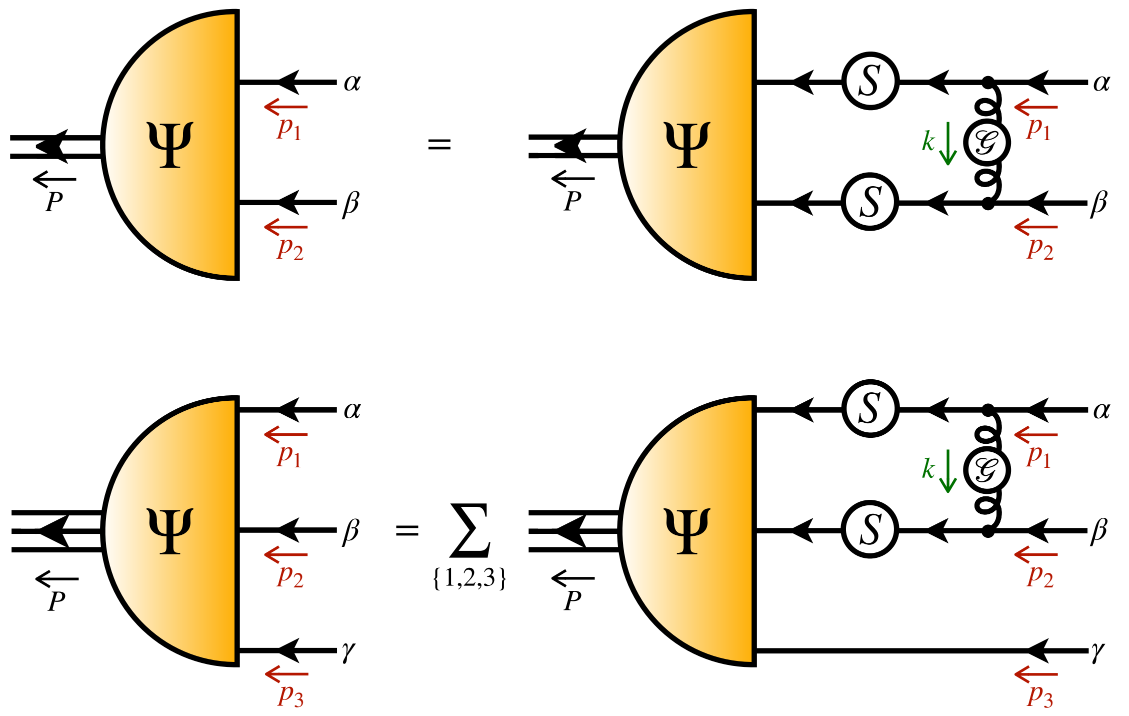

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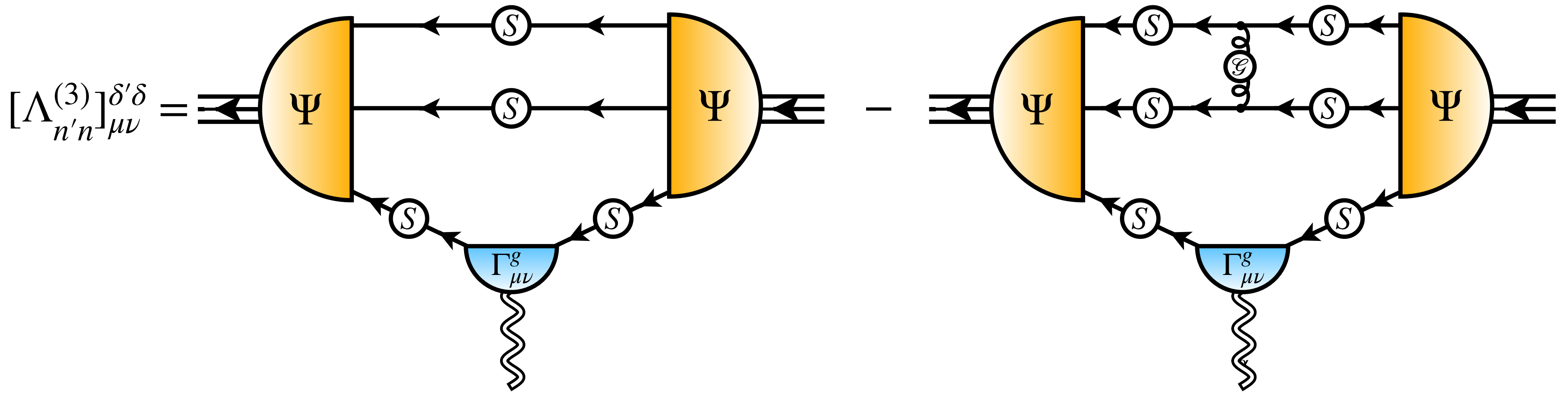

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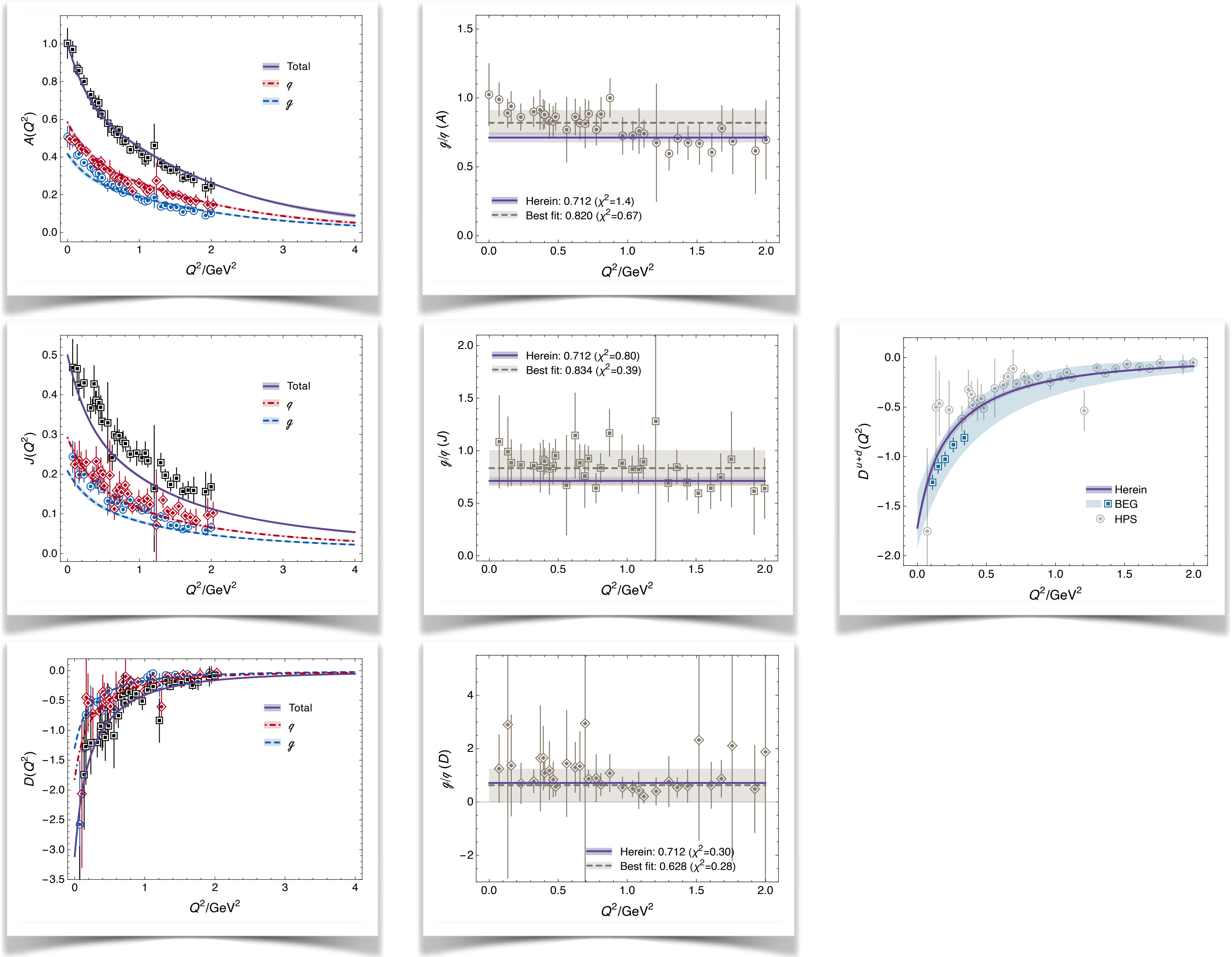

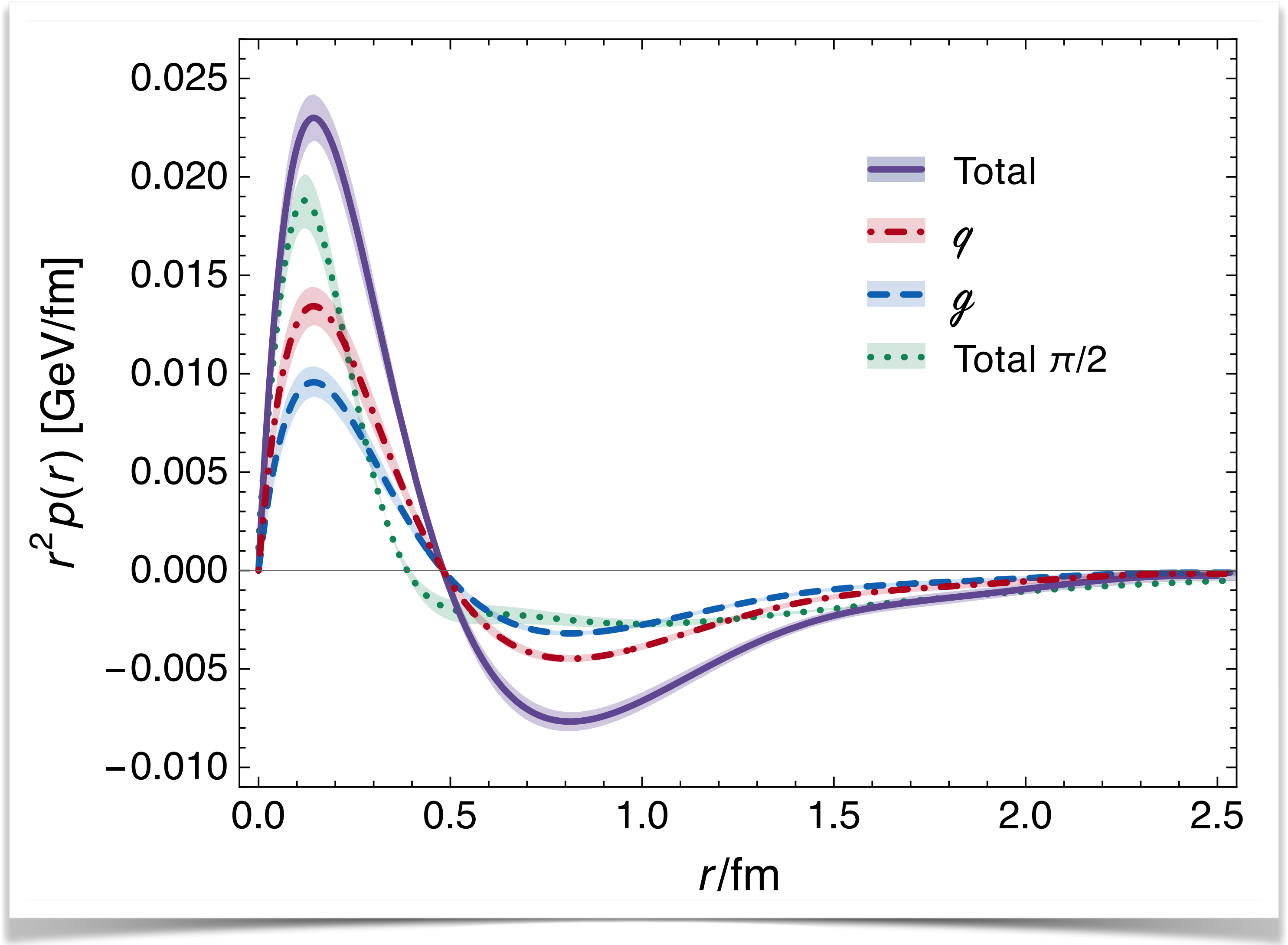

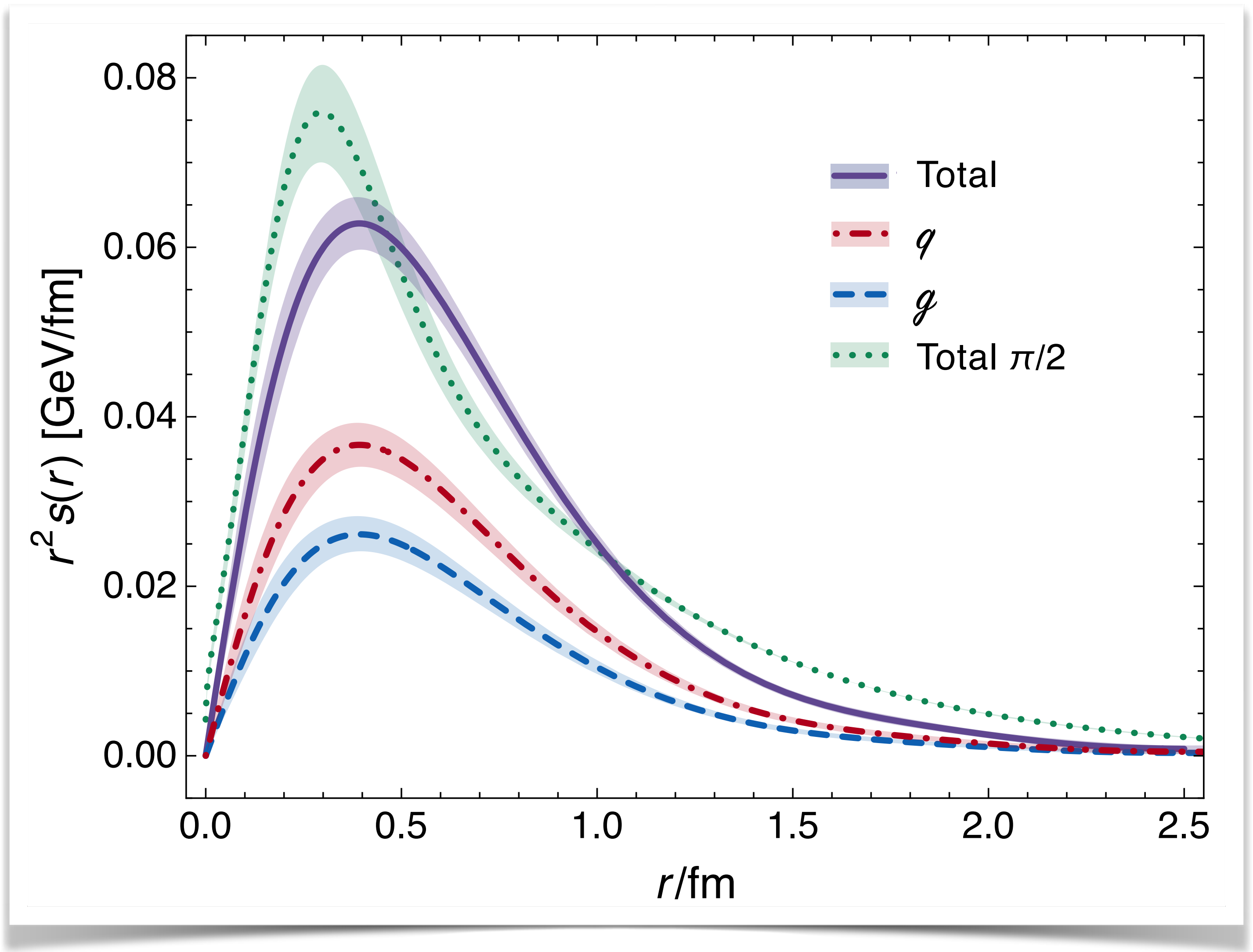

THANKYOU