Testing infrared confining models beyond fundamental correlation functions

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FAPERJ
Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

CAPES

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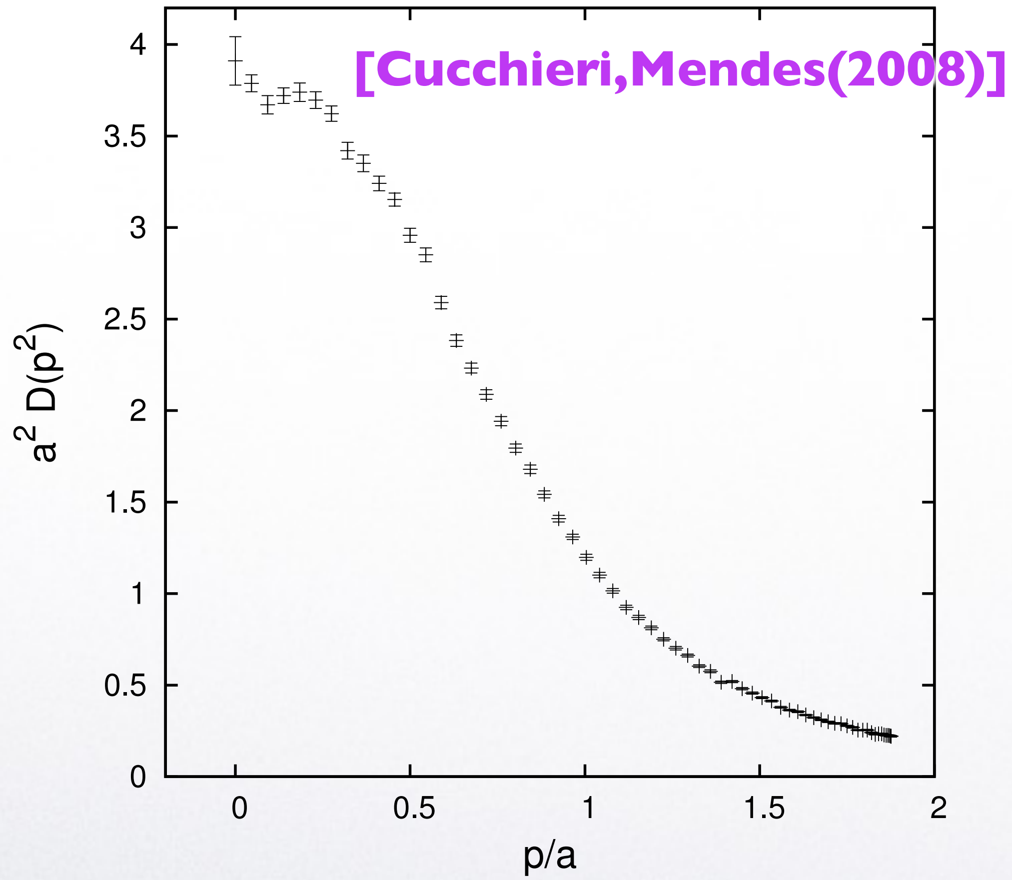
Outline

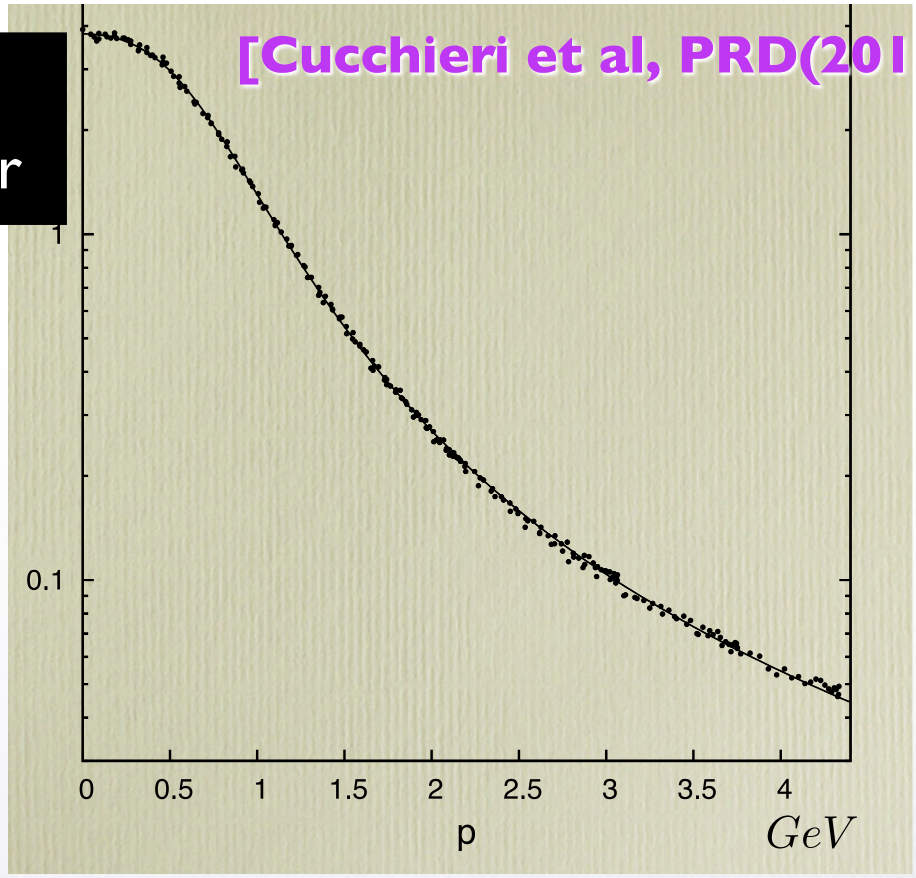
- ***Confining models*** *as an alternative approach to IR QCD*
  - Status of Refined Gribov-Zwanziger framework
- Testing IR confining models in observables and phenomenology:
  - The q-qbar-photon vertex and the anomalous magnetic moment: model contraints from an observable?
  - Color SUC via nonperturbative gluon exchange

Motivation: gluon propagator in the infrared

- **Finite infrared gluon propagator in Landau gauge:**
  - early predictions in Dyson-Schwinger studies [Aguilar, Natale (2004); Frasca (2007)]
  - High-precision lattice YM results for large systems [Cucchieri, Mendes (2008)]


[Cucchieri,Mendes(2008)]

Gluon Propagator


[Cucchieri et al, PRD(2012)]

Also confirmed by other lattice groups: [Bogolubsky et al (2009); Oliveira & Silva (2009)]

- FRG: Cyrol, Fister, Mitter, Pawlowski, Strodthoff (PRD 2016)
- Curci-Ferrari (massive) models: Pelaez, Reinosa, Serreau, Tissier, Wschebor (2015,2016)
- Gluon condensate from lattice QCD: Boucaud, Pene, Rodriguez-Quintero et al (2001)

Quantizing Yang-Mills theories *beyond Pert. Theory?*

[Gribov (1978)]

The Gribov problem:

- In the Landau gauge, for instance, the theory assumes the form

$$\int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}b\, e^{-S_{YM}+S_{gf}}$$

$$S_{gf} = b^a\partial_\mu A^a_\mu - \bar{c}^a\mathcal{M}^{ab}c^b \;, \qquad \mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + gf^{abc}A^c_\mu\right)$$

- Gribov copies → zero eigenvalues of the Faddeev-Popov operator $\mathcal{M}^{ab}$.
- Copies cannot be reached by small fluctuations around $A = 0$ (perturbative vacuum) → pertubation theory works.
- Once large enough gauge field amplitudes have to be considered (non-perturbative domain) the copies will show up enforcing the effective breakdown of the Faddeev-Popov procedure.

Quantizing Yang-Mills theories: the Gribov approach

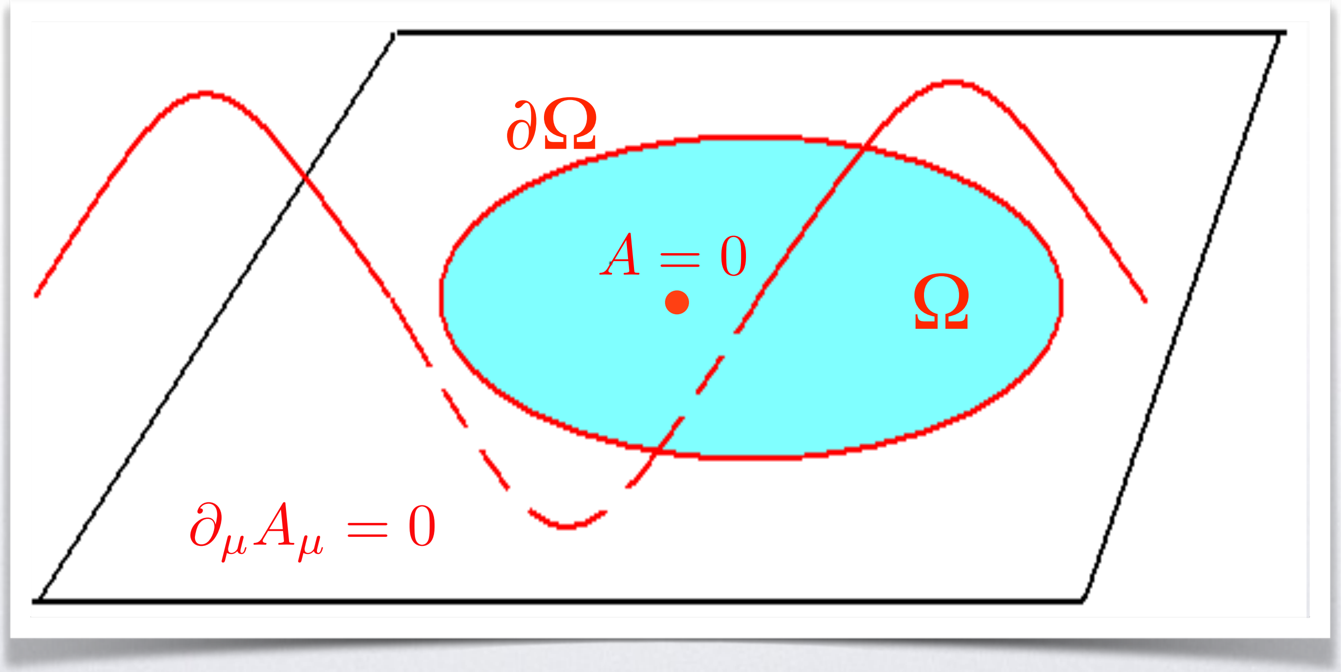
- Gribov proposed a way to eliminate (infinitesimal) Gribov copies from the integration measure over gauge fields: **the restriction to the (first) Gribov region Ω**

$$\int [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\mathrm{YM}}} \longrightarrow \int_\Omega [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\mathrm{YM}}} \qquad S_{\mathrm{YM}} = \frac{1}{4}\int_x F^2$$

with $\Omega = \{A^a_\mu \;\; ; \;\; \partial A^a = 0, \mathcal{M}^{ab} > 0\}$

$$\boxed{\mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + f^{abc}A^c_\mu\right) = -\partial_\mu D^a_\mu}$$

(Faddeev-Popov operator)



The Gribov-Zwanziger action

- The **restriction** can be implemented as a **gap equation** for the vacuum energy obtained as: **[Zwanziger (1989,...)]**

$$Z = e^{-V\mathcal{E}(\gamma)} = \int \mathcal{D}A\, \delta(\partial A)\ \det \mathcal{M}\ e^{-\left(S_{YM} \underbrace{+\gamma^4 H(A) - \gamma^4 V D(N^2-1)}_{=:\ +\gamma^4 \mathcal{H}}\right)}$$

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Gap equation:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = V D(N^2 - 1)$$

$$H(A) = \int_p \int_q A^a_\mu(-p) \left(\mathcal{M}^{ab}\right)^{-1} A^b_\mu(q)$$

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$$H(A) = \int_p \int_q A^a_\mu(-p) \left(\mathcal{M}^{ab}\right)^{-1} A^b_\mu(q)$$

- Using auxiliary fields, this can be cast in a *local* form: $Z = \int [\mathcal{D}\Phi]\, \delta(\partial A)\ \det \mathcal{M}\ e^{-S_{\rm GZ}}$

The Refined Gribov-Zwanziger action

- The GZ theory is unstable against the formation of certain dimension 2 condensates, giving rise to a refinement of the effective IR action:

[Dudal et al (2008)]

$$S_{\mathrm{YM}} \xrightarrow[\text{Gribov restriction(UV →IR)}]{} S_{\mathrm{GZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H}$$

Dynamical generation of dim.2 condensates

$$\boxed{S_{\mathrm{RGZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H} + \frac{m^2}{2} AA - M^2 \left(\overline{\varphi} \varphi - \overline{\omega} \omega \right)}$$

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Gap equation for the Gribov param.:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = V D (N^2 - 1)$$

The parameters *M* and *m* are obtained via minimization of an effective potential for:

$$\langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle \neq 0 \qquad\qquad \langle A^2 \rangle \neq 0$$

- Non-perturbative effects included: $(\gamma, M, m) \propto \mathrm{e}^{-\frac{1}{g^2}}$

A (biased!) checklist for RGZ

- ✓ *(can be cast in a) local and renormalizable action*
- ✓ *reduces to QCD (pure gauge) at high energies?*

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✓ *consistent with gluon 'confinement'? Confining propagator (no physical propagation; violation of reflection positivity)*

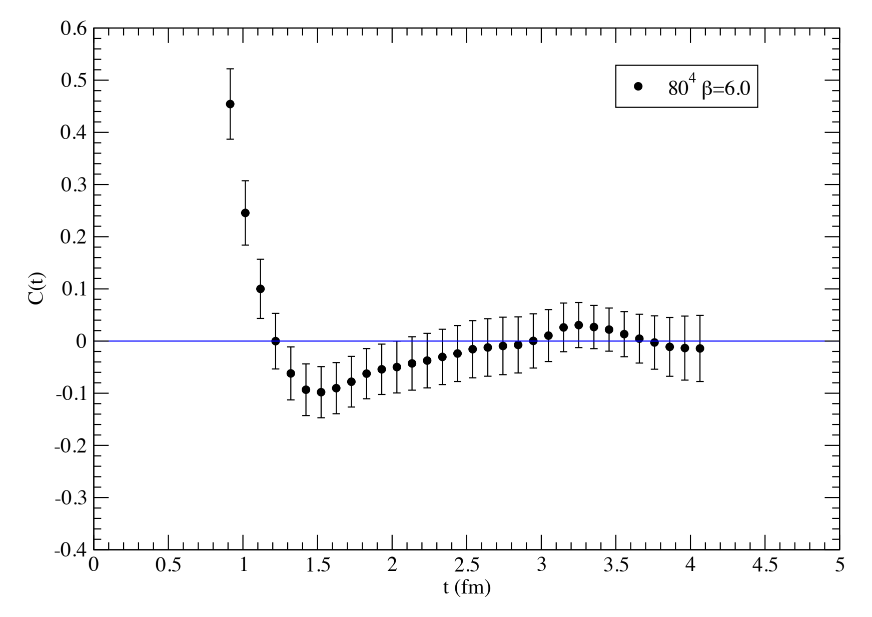
Schwinger function (computed directly from the gluon propagator):

$$C(t) = \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{2\pi} D(p^2) \exp(-ipt).$$

Strictly positive if the gluon spectral function is physical:

$$C(t) = \int_0^{\infty} \mathrm{d}\omega \rho(\omega^2) e^{-\omega t}, \qquad D(p^2) = \int_0^{\infty} \mathrm{d}\mu \frac{\rho(\mu)}{\mu + p^2}.$$

Positivity violation: for the RGZ gluon and in lattice data



SU(3) latt.: [Silva et al (2014)]

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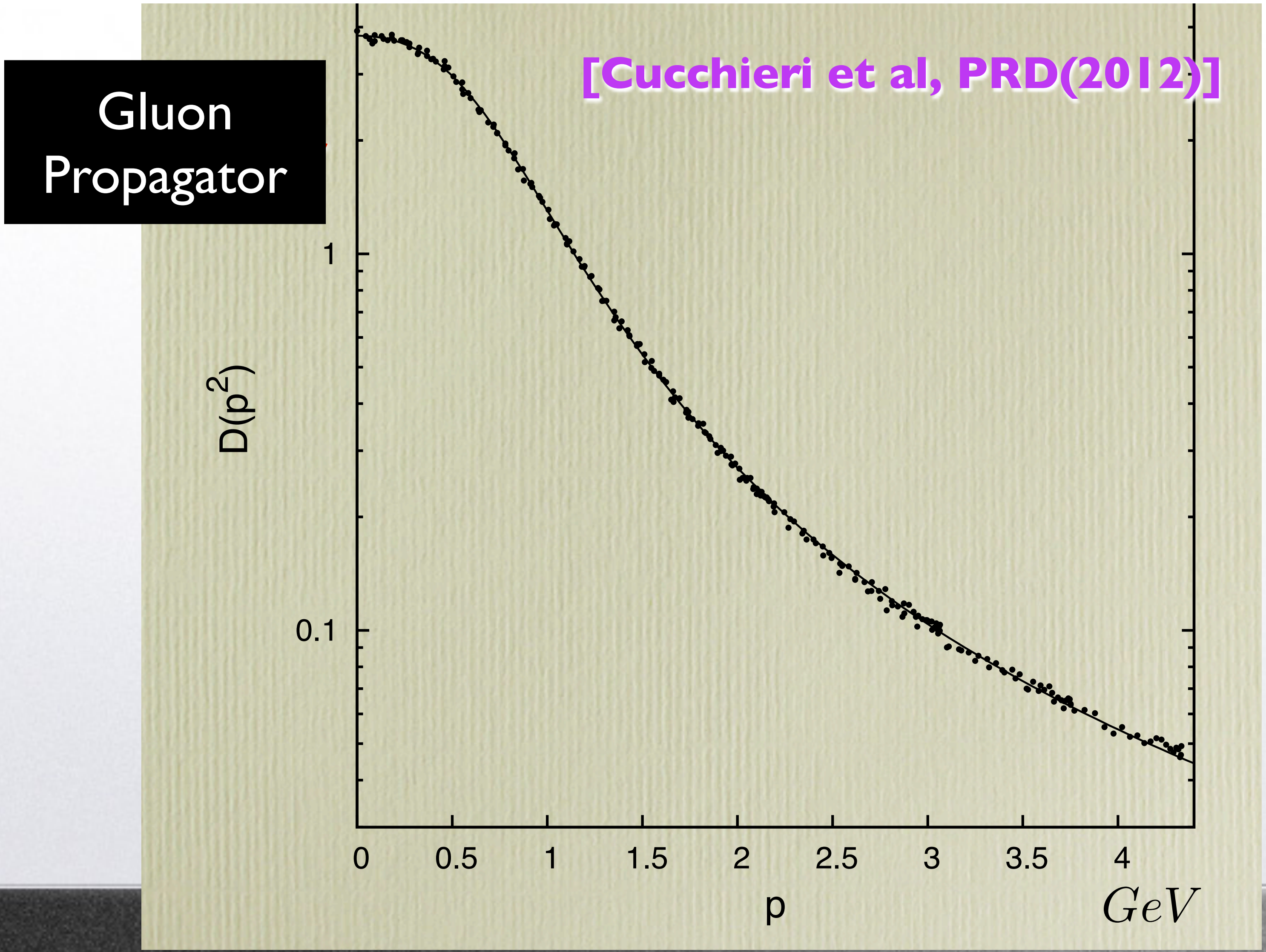

Gluon Propagator

[Cucchieri et al, PRD(2012)]

$$\langle A^a_\mu A^b_\nu \rangle_p = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2)$$

$$D_{\rm fit}(p^2) = C \frac{p^2 + s}{p^4 + u^2 p^2 + t^2}$$

$$C = 0.56(0.01)\,,\; u = 0.53(0.04)\,\mathrm{GeV}\,,$$

$$t = 0.62(0.01)\,\mathrm{GeV}^2\,,\; u = 2.6(0.2)\,\mathrm{GeV}^2$$

poles: $m^2_\pm \;=\; (0.352 \pm 0.522i)\mathrm{GeV}^2$

$$D_{\rm RGZ}(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + 2g^2 N \gamma^4}$$

NB.: Complex conjugated poles!

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✓ *physical spectrum of bound states? Glueballs w/ masses compatible w/ lattice*

[Dudal,Guimaraes,Sorella, PRL(2011), PLB(2014)]

✓ *other applications…*

✓ *Exact modified BRST invariance => gauge-parameter independence*

[Capri et *al* (2016,2017)]

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[Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]

✗ *no quantitative prediction without fitting lattice data for propagators*

✗ *no general definition of physical operators, unitarity*

✗ *quark confinement properties: linear potential, etc…*

✗ *Minkowski space*

✗ *Radiative corrections?*

[Talk by Antonio Pereira]

Radiative Corrections and vertices

- Most applications of RGZ theories are at tree level.
  How do radiative corrections change the results?

- **Recent developments:**

1-loop RGZ propagators **[Talk by Antonio Pereira]**

1-loop ghost-antighost-gluon vertex:

Soft-gluon limit **[Mintz, LFP, Sorella, Pereira PRD (2018)]**

General kinematics + infrared-safe running (ad-hoc model)
[Barrios, Pelaez, Guimaraes, Mintz, LFP PRD (2024)]

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✗ *Minkowski space*

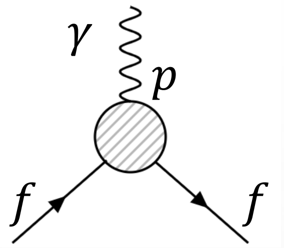
✗ *Radiative corrections,* ***other observables and phenomenological tests?***

[Talk by Antonio Pereira]

Does a theory constructed with positivity-violating fundamental DOFs produce physical phenomenology?

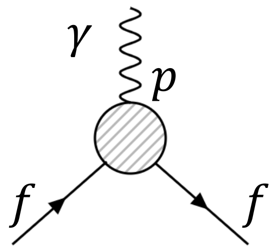
The quark-photon vertex and anomalous mag. mom.

- A different correlation function that is accessible by lattice simulations.
- The soft limit of the F_2 form factor is ***gauge independent*** and, in non-confined fermions, directly related to an ***observable***.



$$= e\, Q_f\, \bar{\boldsymbol{U}}(q_2) \left[\gamma_\mu\, \boldsymbol{F_1}(p^2) + \frac{p_\nu \sigma_{\mu\nu}}{2m}\, \boldsymbol{F_2}(p^2) \right] \boldsymbol{U}(q_1) \Rightarrow g = 2\,[\overset{1}{\cancel{F_1(0)}} + F_2(0)]$$

The quark-photon vertex and anomalous mag. mom.



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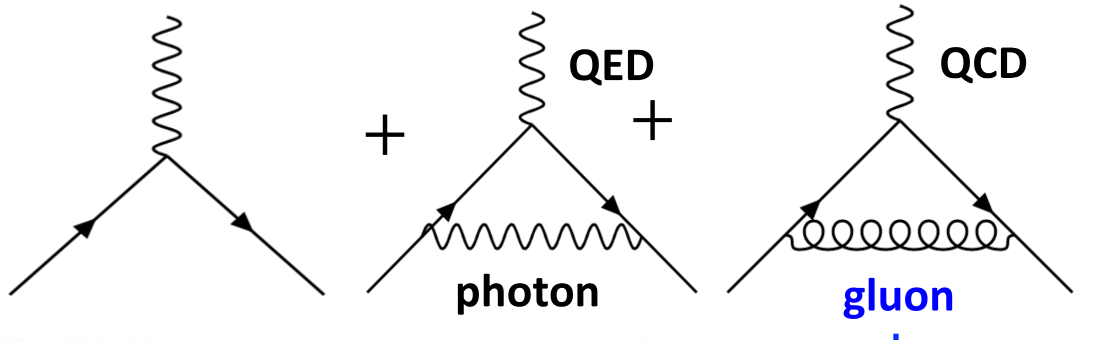
- At one-loop:



…yielding a quark anomalous magnetic moment:

$$\mu_q = Q_q\left(\frac{e}{2m_q}\right)\left(1 + Q_q^2\left(\frac{\alpha}{2\pi}\right) + C_F\left(\frac{\alpha_s}{\pi}\right)\overline{F}_2(0)\right)$$

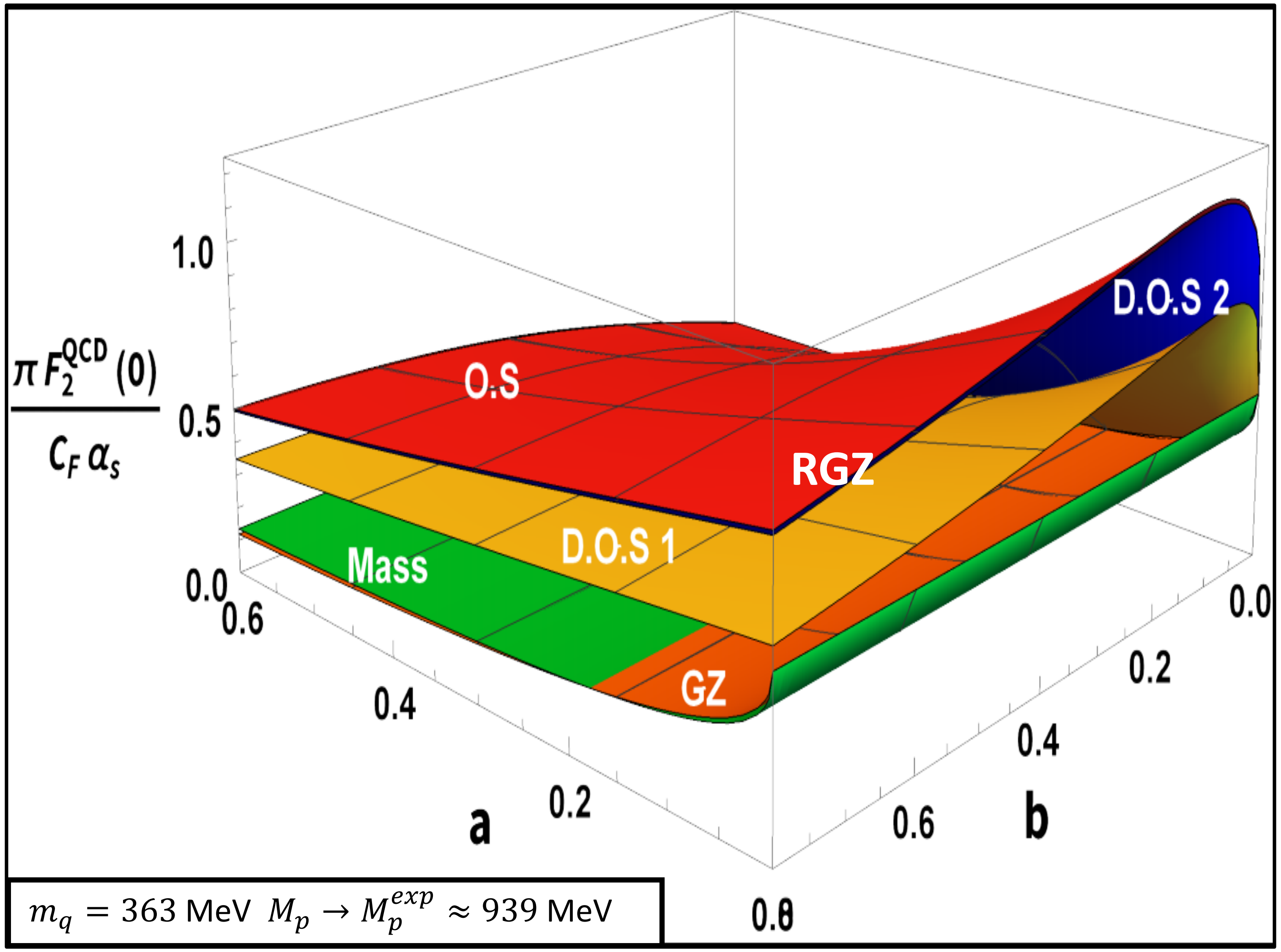
Gluon Confining models	**Massive**	**GZ**	**RGZ**
	$\frac{1}{l^2 + m_g^2}$	$\frac{l^2}{l^4 + \lambda^4}$	$\frac{l^2 + M_1^2}{l^4 + l^2 M_2^2 + M_3^4}$

[Mena & LFP, PRD (2024)]

The quark-photon vertex — soft limit for F_2

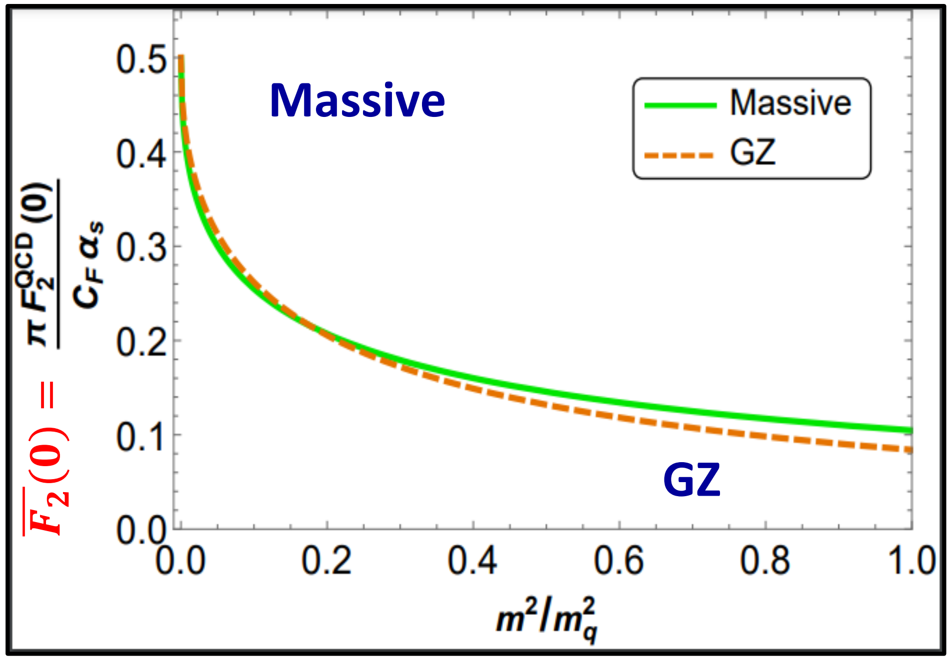
[Mena & LFP, PRD (2024)]

$\overline{F_2}(0)$ RESULTS



m_q = 363 MeV $M_p \rightarrow M_p^{exp} \approx$ 939 MeV

*a and b are ratios of the gluon mass parameters and the quark mass.



RGZ:

DATA	M [GeV]	m [GeV]	λ [GeV]	$\overline{F_2}(0)$
O.S	2.1150	1.9414 i	2.0381	0.2456
D.O.S 2	2.0389	1.8881 i	1.9730	0.2658

Massive parameters fit from lattice data:
Oliveira & Silva, PRD (2012)
Dudal, Oliveira & Silva, Ann. Phys. (2018)

Estimating the proton AMM from confining models...

We adopt the simplest Constituent Quark Model to estimate the effect on the proton AMM:

$$\boxed{\mu_p^{CQM} = \left[\frac{4}{3}\mu_u - \frac{1}{3}\mu_d\right]} \qquad \text{with} \qquad \mu_q = Q_q\left(\frac{e}{2m_q}\right)\left(1 + Q_q^2\left(\frac{\alpha}{2\pi}\right) + C_F\left(\frac{\alpha_s}{\pi}\right)\overline{F}_2(0)\right)$$

- CQM parameters:
  constituent quark mass fixed to proton mass

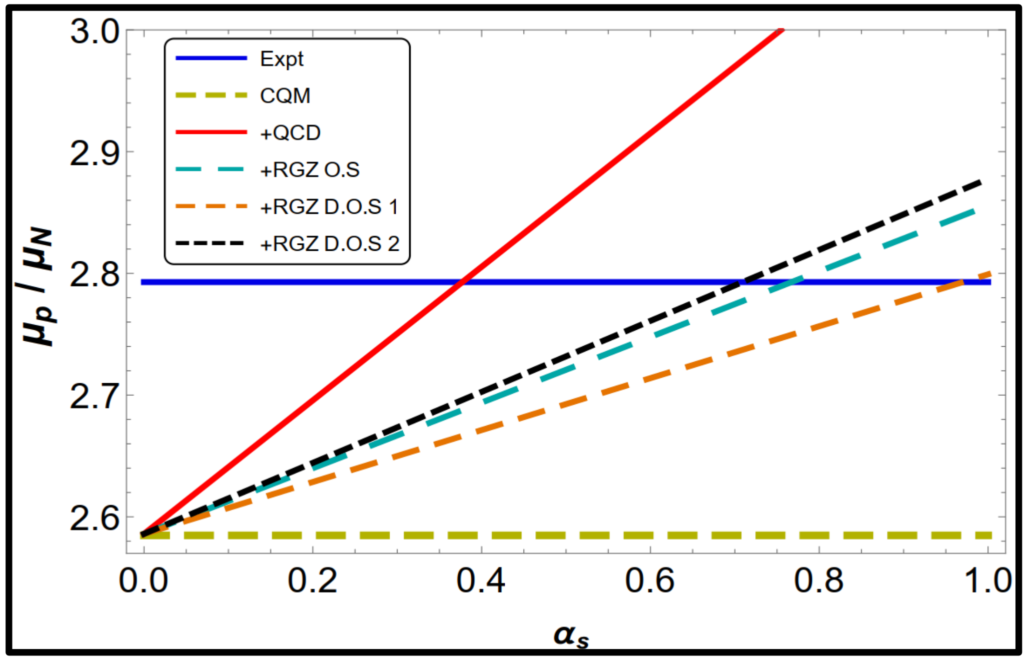
$$m_q = 363 \text{ MeV} \;\; M_p \to M_p^{exp} \approx 939 \text{ MeV}$$

- Confining model parameters:
  dynamically generated gluon mass(es)
  +
  strong coupling in the deep IR

m_g	0 MeV	140 MeV	185.64 MeV	600 MeV
$\alpha_s \parallel \lambda_{CF} = 3\alpha_s/4$	0.38 ‖ 0.091	0.83 ‖ 0.198	1.00 ‖ 0.239	3.24 ‖ 0.773

[Mena & LFP, PRD (2024)]

RGZ model



Estimating the proton AMM from confining models…

1. Confining models — even with complex conjugated poles — yield reasonable results;

2. Dynamically generated gluon masses can be accomodated
   if the strong coupling is large enough in the IR (or changing other CQM parameters…)

3. Still hard to constrain models, but lattice data may help.

- CQM parameters:
  constituent quark mass fixed to proton mass

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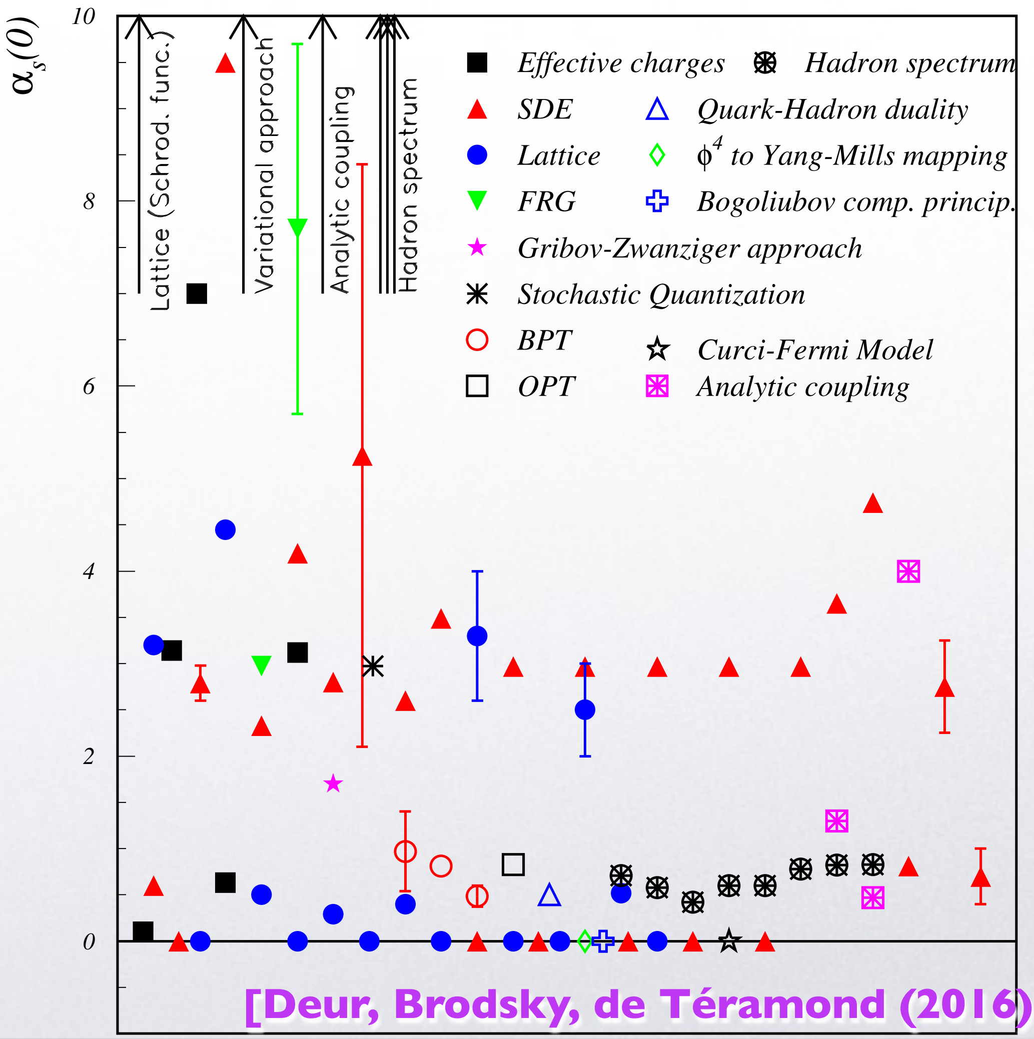


[Deur, Brodsky, de Téramond (2016)]

Testing IR models with color SUC phenomenology

- Color superconductivity mediated by gluons at intermediate to high densities should probably be affected by nonperturbative modifications of the gluon propagator;

- **Aims here:**

  Do complex-conjugated poles generate non-physical features in SUC?

  How is the SUC gap influence by the presence of massive gluon parameters?

  Could one discriminate between the predictions of different IR models?

[Santos & LFP, to appear; Dudal, LFP & Santos, *in progress*]

Toy model for color SUC wiht confining propagators

- For a simple testing ground, we choose a Yukawa theory with the (scalar) bosons displaying propagators with IR modifications (Curci-Ferrari, GZ or RGZ-inspired):

IR-modified boson propagators

$$S = \int_x \mathscr{L} = \int_{x,y} \left[\overline{\psi}(x) G_0^{-1}(x,y) \psi(y) - \frac{1}{2} \varphi(x) D^{-1}(x,y) \varphi(y) \right]$$

$$-g \int_x \overline{\psi}(x) \psi(x) \varphi(x) \,,$$

Dirac (free), finite density => $G_0^{-1}(x,y) = \delta(x-y)(i\gamma^\mu \partial_\mu + \gamma^0 \mu - m)$

Massive	**GZ**	**RGZ**
$\frac{1}{l^2 + m_g^2}$	$\frac{l^2}{l^4 + \lambda^4}$	$\frac{l^2 + M_1^2}{l^4 + l^2 M_2^2 + M_3^4}$

Toy model for color SUC with confining propagators

- Integrating out the boson, one arrives at a 4-fermion theory that is convenient to study SUC:

IR-modified boson propagators

$$S' = \int_{x,y} \left[\overline{\psi}(x) G_0^{-1}(x,y) \psi(y) + \frac{g^2}{2} \overline{\psi}(x)\psi(x) D(x,y) \overline{\psi}(y)\psi(y) \right]$$

- To describe the di-quark condensate, one can transform to Nambu Gorkov space, with a charge-conjugate spinor defined by:

$$\psi_C \equiv C\overline{\psi}^T \;, \qquad\qquad C = i\gamma^2\gamma^0$$

$$\overline{\psi}(x)\psi(x)\overline{\psi}(y)\psi(y) = \frac{1}{2}[\overline{\psi}_C(x)\psi_C(x)\overline{\psi}(y)\psi(y) + \overline{\psi}(x)\psi(x)\overline{\psi}_C(y)\psi_C(y)]$$

$$= -\frac{1}{2}\mathrm{Tr}[\psi_C(x)\overline{\psi}(y)\psi(y)\overline{\psi}_C(x) + \psi(x)\overline{\psi}_C(y)\psi_C(y)\overline{\psi}(x)]$$

Toy model for color SUC with confining propagators

- Introducing the di-quark condensate in mean-field approximation:

$$S'' = \int_{x,y} \left\{ \overline{\psi}(x) G_0^{-1}(x,y) \psi(y) + \frac{1}{2} [\overline{\psi}_C(x) \Phi^+(x,y) \psi(y) + \overline{\psi}(x) \Phi^-(x,y) \psi_C(y)] \right\}$$

$$\boxed{\begin{aligned} \Phi^+(x,y) &\equiv g^2 D(x,y) \langle \psi_C(x) \overline{\psi}(y) \rangle \,, \\ \Phi^-(x,y) &\equiv g^2 D(x,y) \langle \psi(x) \overline{\psi}_C(y) \rangle \,. \end{aligned}}$$

- or, in terms of the inverse propagator matrix in Nambu-Gorkov space:

$$Z = \mathcal{N} Z_{\text{bosons}} Z_0 \int \mathcal{D}\overline{\Psi} \, \mathcal{D}\Psi \, \exp \left[\sum_{k>0} \overline{\Psi}(k) \frac{\mathcal{S}^{-1}(k)}{T} \Psi(k) \right]$$

$$\boxed{\mathcal{S}^{-1}(k) = \begin{pmatrix} [G_0^+(k)]^{-1} & \Phi^-(k) \\ \Phi^+(k) & [G_0^-(k)]^{-1} \end{pmatrix}}$$

$$\boxed{\Psi \equiv \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}}$$

Toy model gap equation

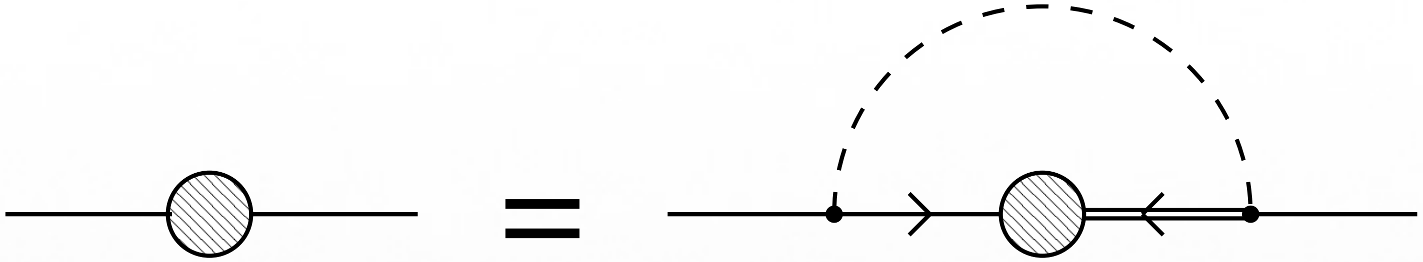
- Implementing a Dyson resummation, the full propagator matrix is written as:

$$\mathbf{S}(k) = \begin{pmatrix} G^+ & F^- \\ F^+ & G^- \end{pmatrix}$$

$$G^\pm \equiv \left([G_0^\pm]^{-1} - \Phi^\mp G_0^\mp \Phi^\pm\right)^{-1}$$

$$F^\pm \equiv -G_0^\mp \Phi^\pm G^\pm$$

$$\boxed{\Phi^+(x,y) = -g^2 D(x,y) F^+(x,y)}$$



$$\boxed{\Phi^+(p) = g^2 \frac{T}{V} \sum_k D(p-k) G_0^- \Phi^+ G^+}$$

IR-modified boson propagators

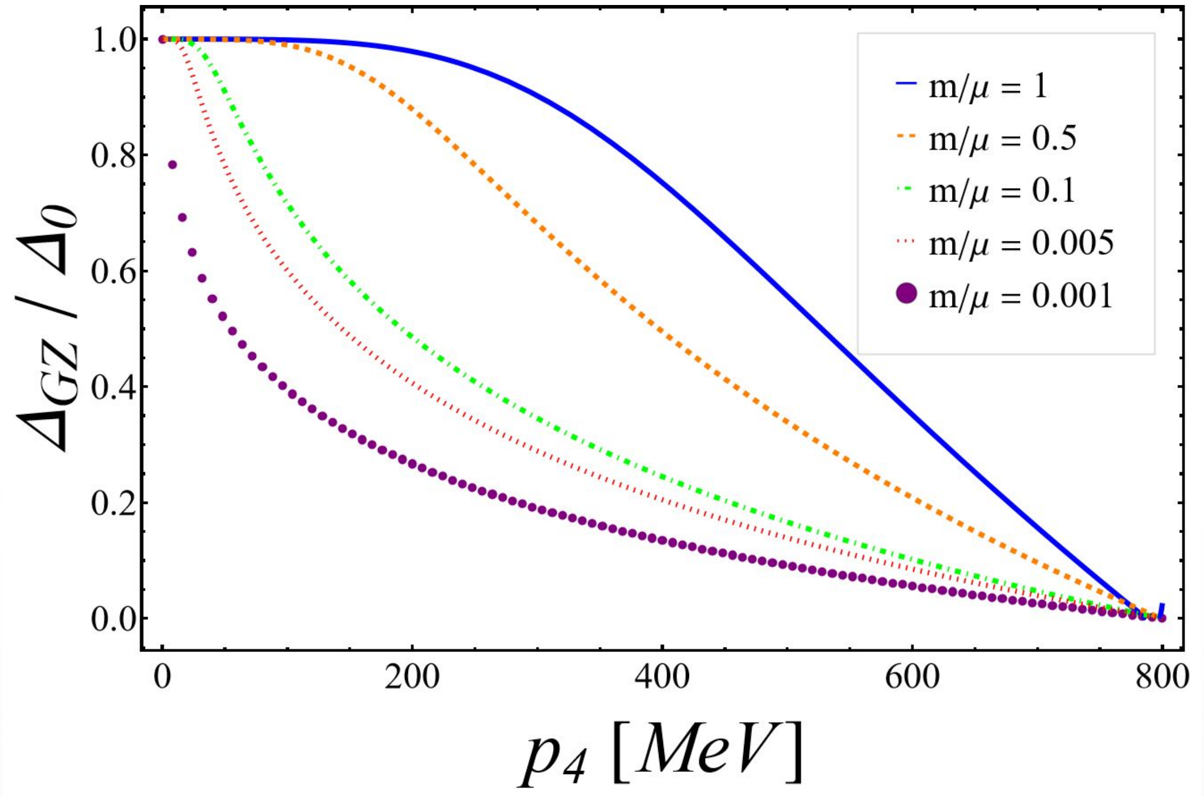
$$\boxed{\Phi^\pm(K) = \pm\Delta(K)\gamma^5\,,}$$

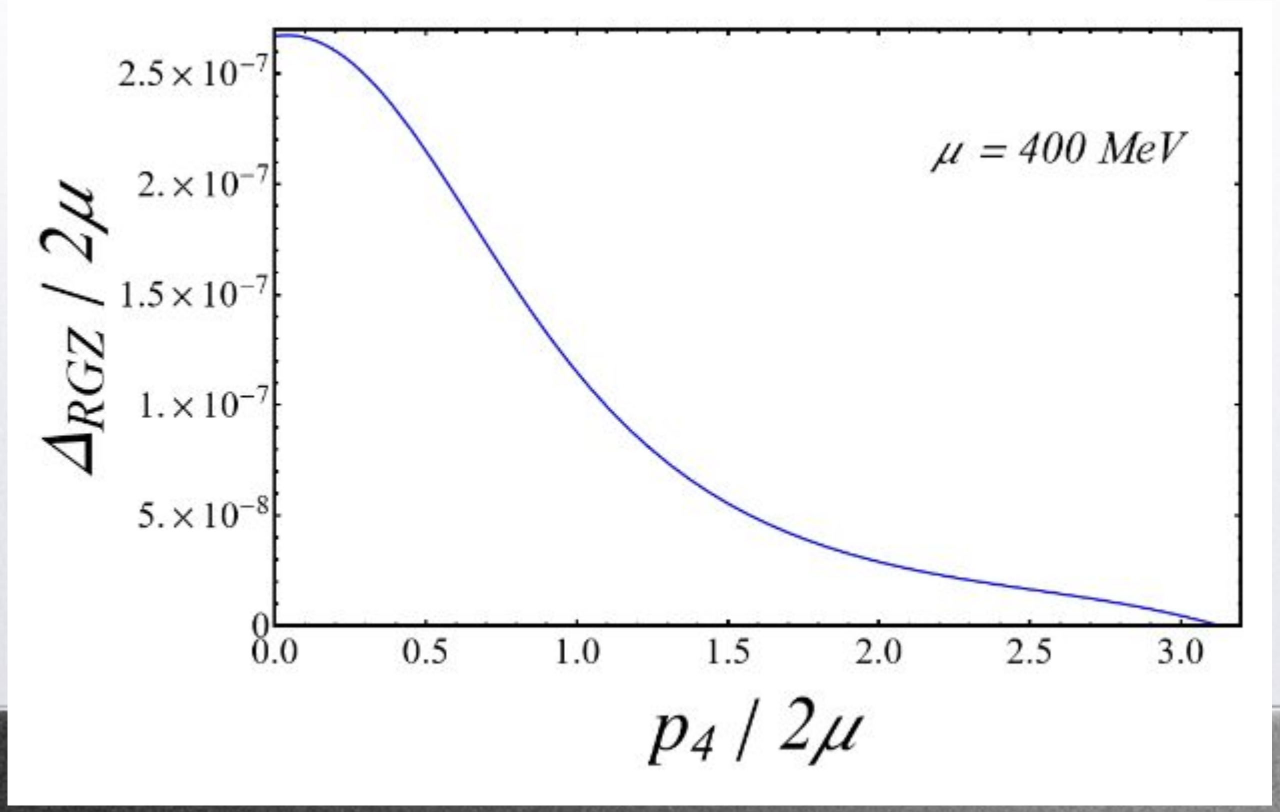
(even parity, spin-singlet pairing)

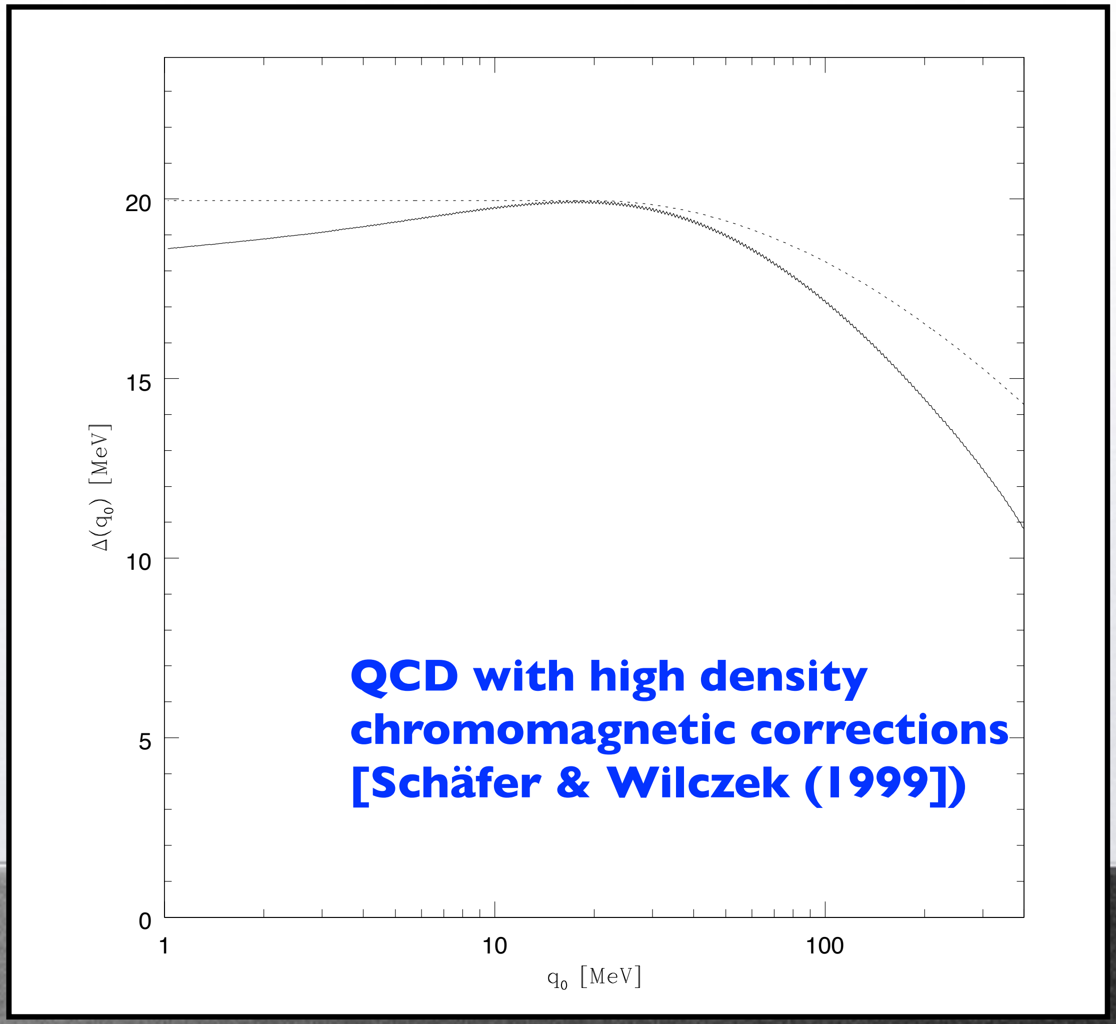
D. Bailin, A. Love, Phys.Rept. **107**, 325 (1984)
R.D. Pisarski, D.H. Rischke, Phys.Rev. **D60**, 094013 (1999)

Results for the SUC gap for confining-type props.

[Santos & LFP, *appear*]

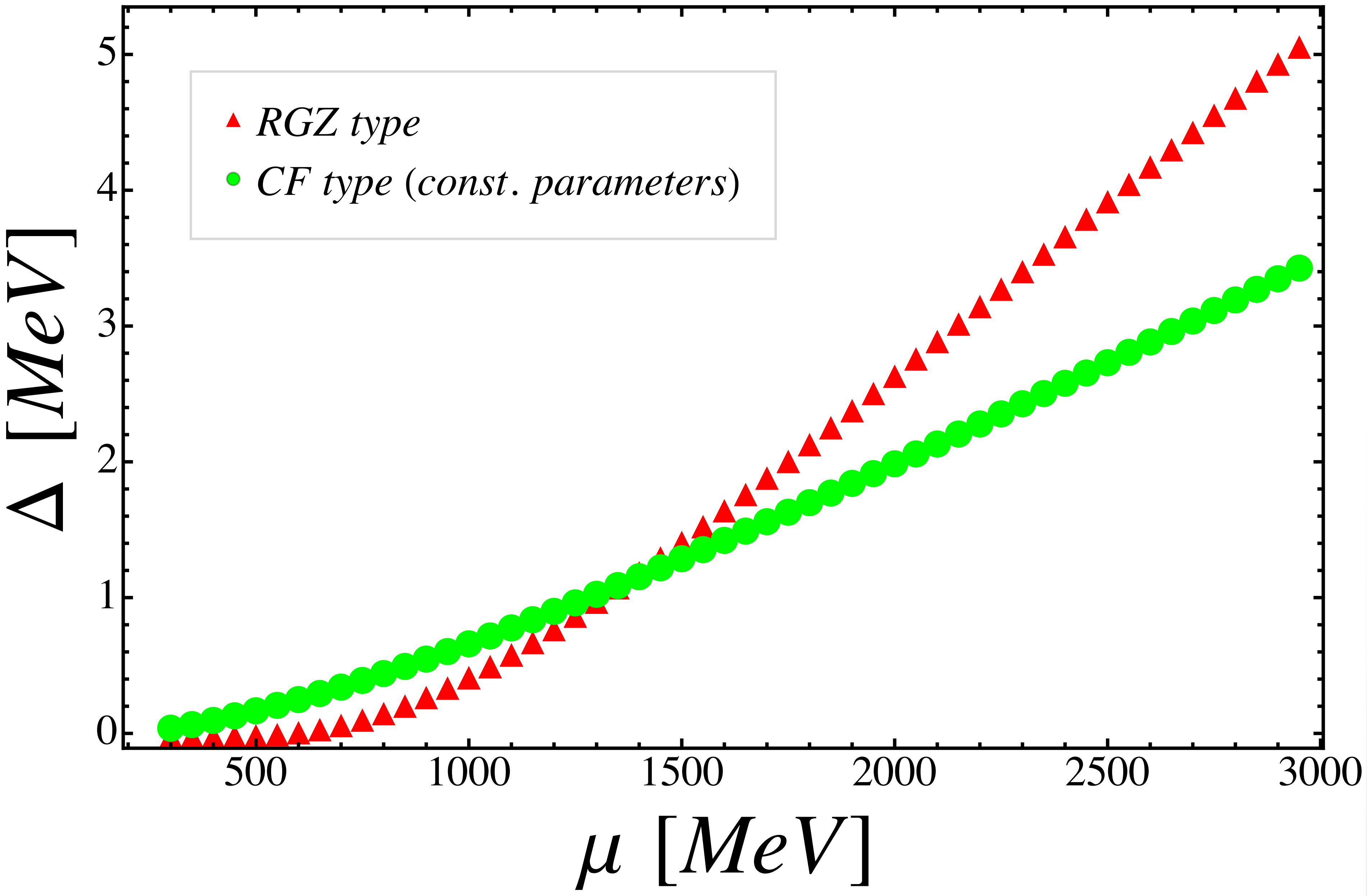






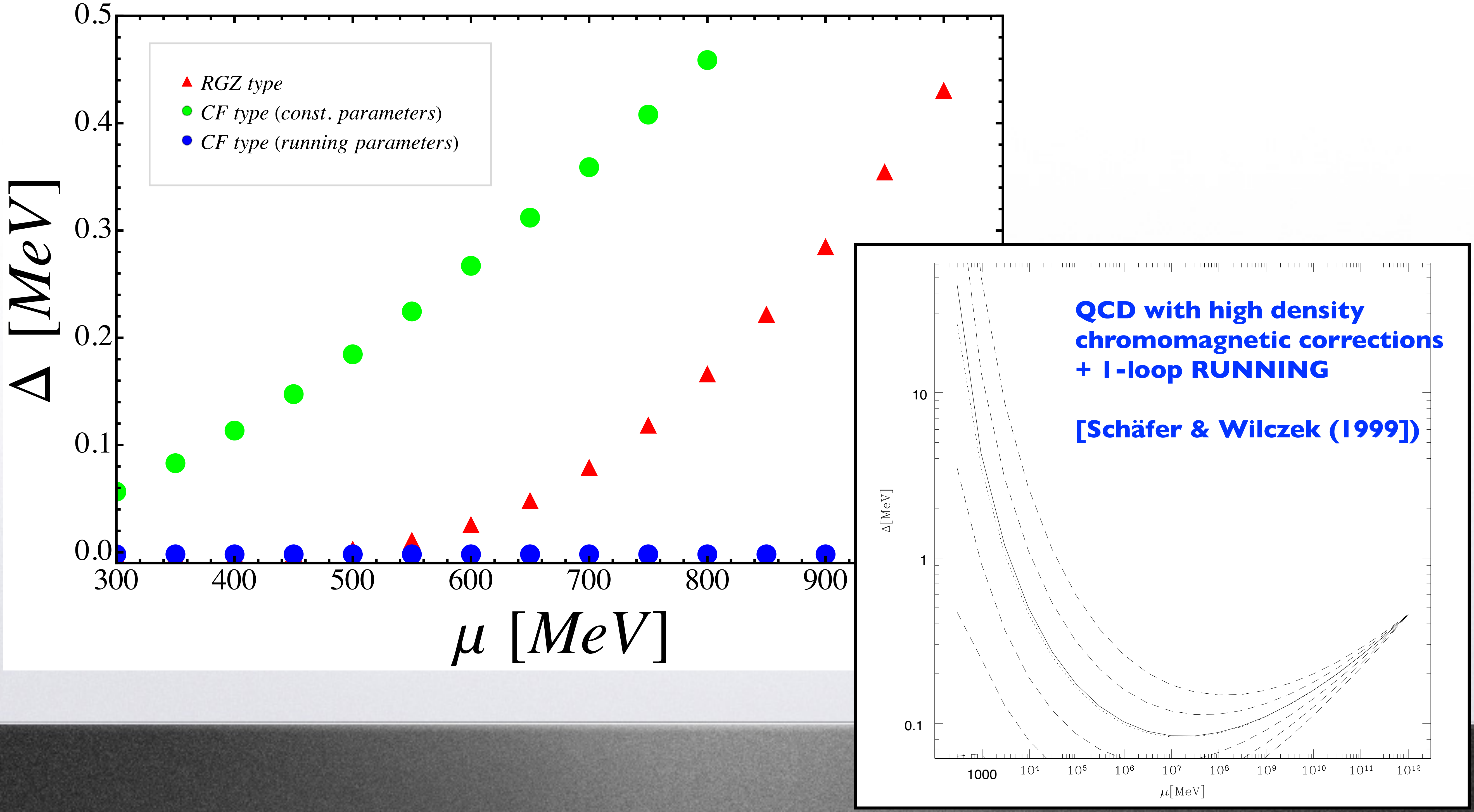
QCD with high density chromomagnetic corrections [Schäfer & Wilczek (1999])

Results for the SUC gap



Toy model for color SUC with confining propagators

- Infrared-safe inspired running suppresses the SUC gap in the Yukawa toy model:



QCD with high density chromomagnetic corrections + 1-loop RUNNING

[Schäfer & Wilczek (1999])

Final comments

- ***Dynamical gluon mass generation should occur in IR YM theories.***
- The ***Gribov problem*** is present and should profoundly affect the IR regime of gauge-fixed non-Abelian gauge theories.
- The ***RGZ framework*** represents a consistent scenario to study the non-perturbative IR physics and has provided ***interesting results for correlation functions in the gluon sector fitting lattice propagators***.
- The q-qbar-photon may be calculated on the lattice and offers a window to observables like the anomalous magnetic moment (possibility of parameter and/or model constraining)
- Color SUC is also sensitive to the nonperturbative gluon mass and IR models yield physical results, with in general a suppression of the value of the gap in the toy model studied.
- Extend calculations to other observables, in order to further test IR model predictions and constrain.

Thank you for your attention!