Exploring three-point functions in the complex plane

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The complex structure of strong interactions in Euclidean and Minkowski space

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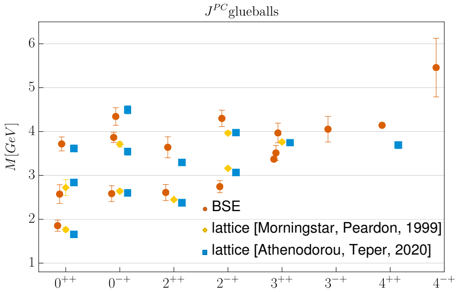
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Deutsche Forschungsgemeinschaft

Propagators, vertices, glueballs

- Glueballs: Existence and properties?
- Bound state equations: Setup and input in form of propagators and vertices?
- Propagators and vertices in the complex plane: Models? Calculations?



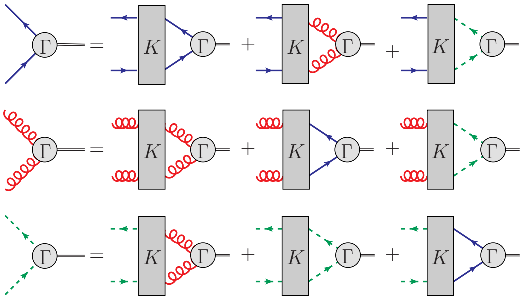
Experiment (BESIII): $f_0(1710)$ [JPAC Coll., Rodas et al., Eur.Phys.J.C 82 (2022)], $f_0(1770)$ [Sarantsev, Denisenko, Thoma, Klempt, Phys. Lett. B 816 (2021)], $X(2370)$ [Ablikim et al. (BESIII), PRL132 (2024)]

Quenched: [Morningstar, Peardon, Phys. Rev. D60 (1999); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80,C81 (2020,2021); Athenodorou, Teper, JHEP11 (2020)]

Bound states

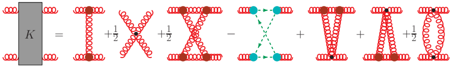
- Calculation of bound states from different methods with individual challenges
- Bound state equations (Bethe-Salpeter, Faddeev, . . .)

Example: Mesons, glueballs



Input:

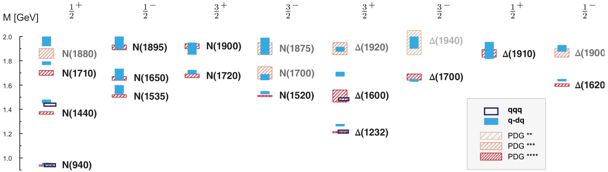
- Nonperturbative propagators: Gluon, ghost, quarks
- Kernels



$\rightarrow$ Nonperturbative vertices

Functional spectrum calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively!



[Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog.Part.Nucl.Phys. 91 (2016); Eichmann, Few Body Syst. 63 (2022)]

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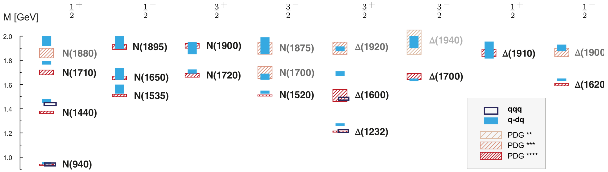

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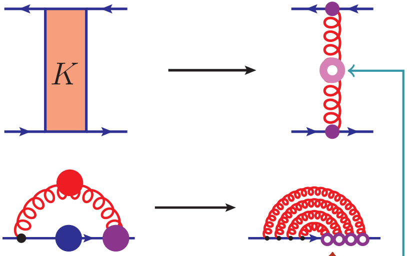
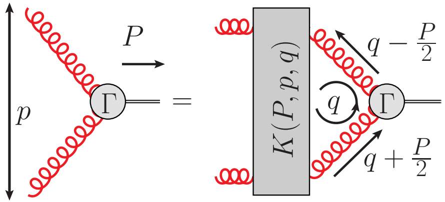


Workhorse for more than 20 years: Rainbow-ladder truncation with an effective interaction, e.g., Maris-Tandy (or similar) which depends only one scale!

restricted structure of equations ($\Gamma_\mu \to \gamma_\mu$)

IR strength + perturbative UV

Correlation functions for complex momenta



(pseudoscalar glueball)

$$\boldsymbol{\lambda(P)}\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

$\rightarrow$ Eigenvalue problem for $\Gamma(P)$:

1. Solve for $\lambda(P)$.
2. Find P with $\lambda(P) = 1$.
   $\Rightarrow M^2 = -P^2$

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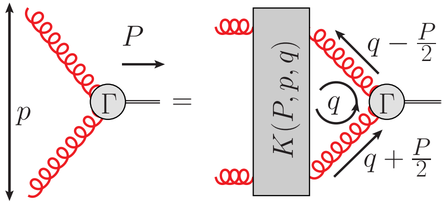

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However:

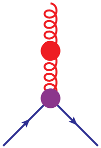
Propagators are probed at $\left(q \pm \frac{P}{2}\right)^2 = \frac{P^2}{4} + q^2 \pm \sqrt{P^2\, q^2} \cos\theta = -\frac{M^2}{4} + q^2 \pm \boldsymbol{i}\, M \sqrt{q^2} \cos\theta$

$\rightarrow$ Complex for $P^2 < 0$!

Time-like quantities ($P^2 < 0$) $\rightarrow$ Correlation functions for complex arguments.

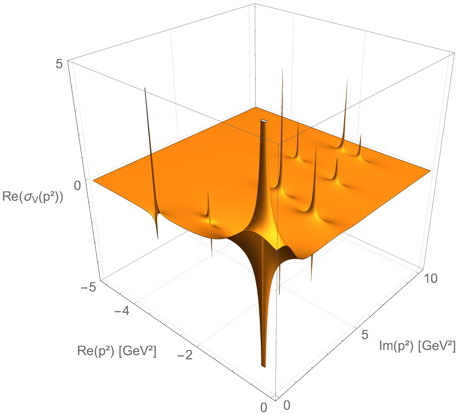
Quark propagator and models

Quark propagator DSE:



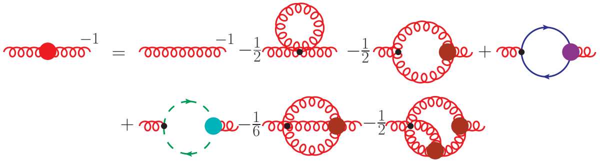
Quark-gluon vertex and the gluon propagator given in closed form. $\rightarrow$ Calculation for complex momenta.

Example: [Windisch, Phys. Rev. C 95 (2017)]



Glueballs with model input

Glueballs? Rainbow-ladder?



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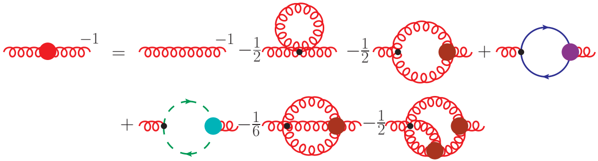

$\boxed{\text{There is no rainbow for gluons!}}$

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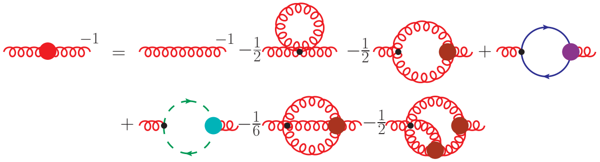

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Model based BSE calculations ($J = 0$):

- [Meyers, Swanson, Phys.Rev.D87 (2013)]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015)]
- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

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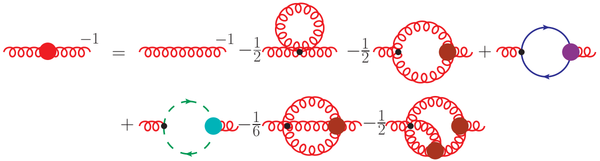

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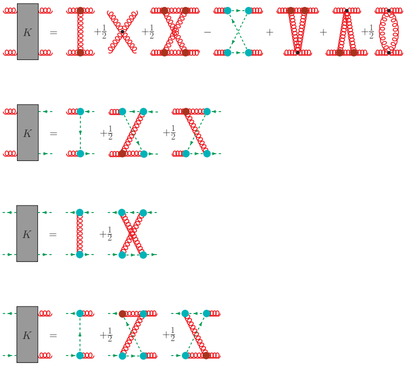
Alternative: <u>Calculated input</u> [MQH, Phys.Rev.D 101 (2020)]

- $J = 0$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]
- $J = 0, 2, 3, 4$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

Extreme sensitivity on input!

Kernels for (pure) glueballs

Systematic derivation from 3PI eff. action: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]
Need propagators and vertices!



[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

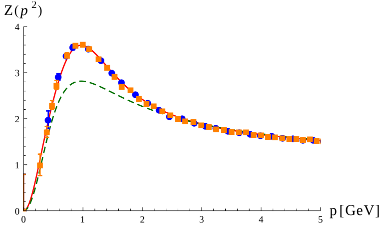
Input for bound state equations from 2013?

Bound state equations require propagators and vertices as input.

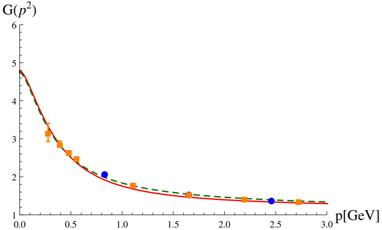
Truncation: Three-gluon vertex model tailored to reproduce lattice results.

Propagators from 2013 [Sternbeck, hep-lat/0609016; MQH, von Smekal, JHEP 04 (2013)]:

Gluon dressing function:



Ghost dressing function:



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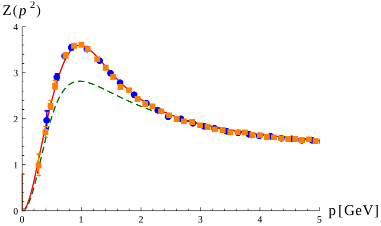
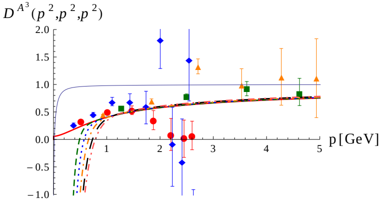


Three-gluon vertex [Cucchieri, Maas, Mendes, Phys. Rev. D 77 (2008); MQH, Phys. Rev. D 101 (2020)]:



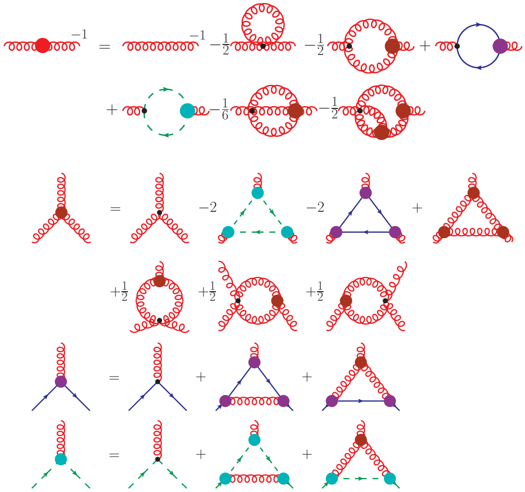
$\rightarrow$ Insufficient for bound states.

Correlation functions of quarks and gluons

Equations of motion of 3PI effective action

→ [Review: MQH, Phys.Rept. 879 (2020)]

Truncation: 3-loop



- Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, ...
- Self-contained: Only parameters are the strong coupling and the quark masses!
- Long way, e.g., ghost-gluon vertex, three-gluon vertex, four-gluon vertex, ...
- → MQH, Phys.Rev.D 101 (2020)

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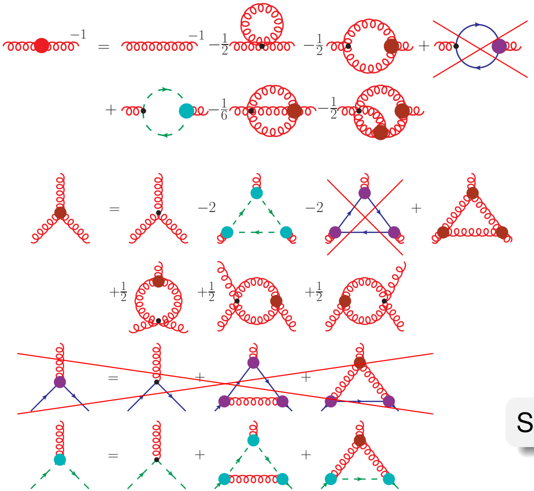

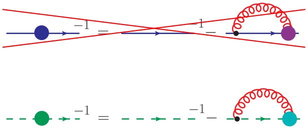

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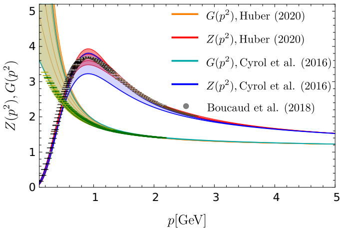
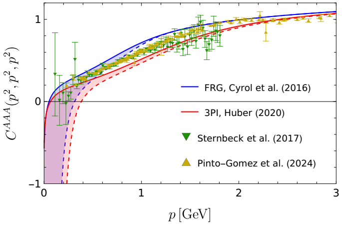
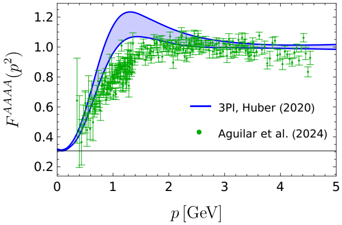
Start with pure gauge theory.

Landau gauge correlation functions

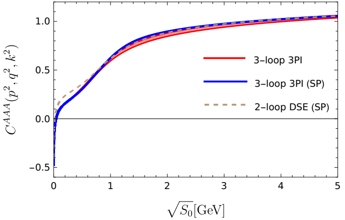
[MQH, Fischer, Sanchis-Alepuz, 2503.03821]







[FRG: Cyrol et al., Phys. Rev. (2016); DSE: MQH, Phys. Rev. D 101 (2020); lattice: Boucaud: Phys. Rev. D 98 (2018)]



[DSE: MQH, Phys. Rev. D 101 (2020); lattice: Aguilar et al., Phys. Lett. B 858 (2024)]

Landau gauge correlation functions: Apparent convergence?

$\longrightarrow$ [MQH, Fischer, Sanchis-Alepuz, 2503.03821]

- Good agreement between lattice and functional methods
- Good agreement between different functional methods
- Further tests:
  - Three-gluon vertex: tree-level dressing dominant [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014)]
  - Influence of other four-point functions tiny [MQH, Eur. Phys.J.C77 (2017)]

$\rightarrow$ Glueballs?

Glueballs as bound states of gluons

Use results for glueball calculations?

All results for spacelike momenta. $\rightarrow$ Not directly.

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All results for spacelike momenta. $\rightarrow$ Not directly.

- Reconstruction from Euclidean results to get correlation functions for complex arguments.

OR

- Extrapolation of the eigenvalue curve. $\rightarrow$ More stable and tests possible.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1+\frac{a_2(x-x_2)}{1+\frac{a_3(x-x_3)}{\cdots}}}}$$

Coefficients a_i can be determined such that $f(x)$ exact at x_i.

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Test extrapolation for solvable system:
Heavy meson [MQH, Sanchis-Alepuz, Fischer, Eur.Phys.J.C 80 (2020)]

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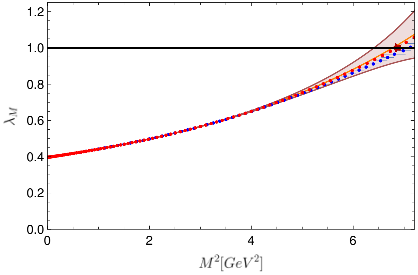
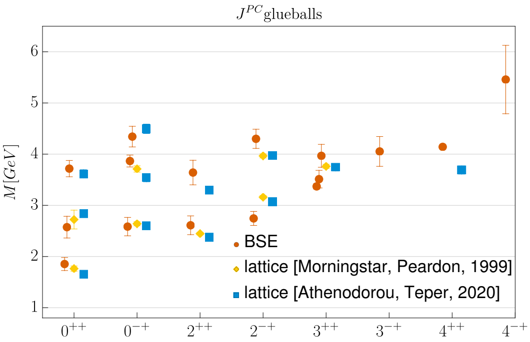
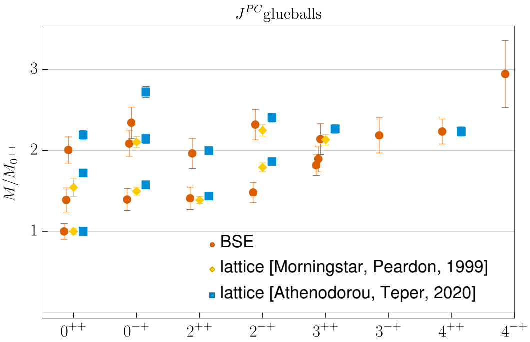


Glueball results





[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

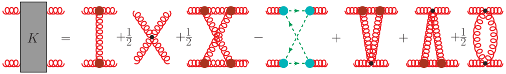
- Agreement with lattice results
- New states: 0^{**++}, 0^{**-+}, 3^{-+}, 4^{-+}

All results for $r_0 = 1/418(5)$ MeV.

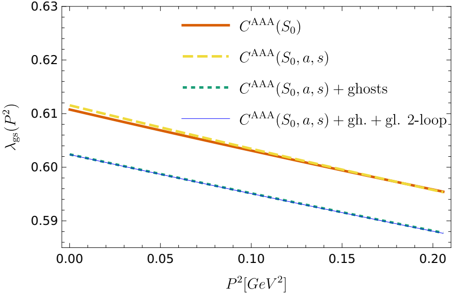
Apparent convergence in glueball calculations

[MQH, Fischer, Sanchis-Alepuz, 2503.03821]

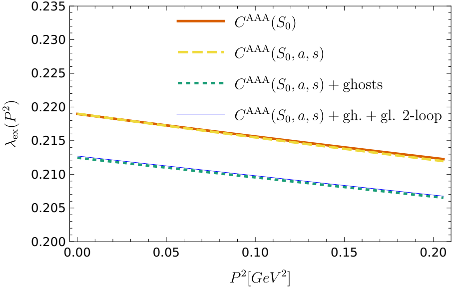
Compare gluons only, one-loop and two-loop kernels.



Ground state:



First excited state:



- Small influence of three-gluon kinematics (S_0 only vs. S_0, a, s) $\to \Delta m \approx 50\,\text{MeV}$
- Ghost diagrams essential: $\Delta m \approx 200 - 350\,\text{MeV}$
- Two-loop diagrams negligible: $\Delta m < 2\%$

Correlation functions in the complex plane

Standard integration techniques fail. ϟ

$$\int d^4q \to \int_{\Lambda_{IR}^2}^{\Lambda_{UV}^2} dq^2 \int d\theta_1$$

Consider example integral:

$$I_2(p^2) = \int dq^2 J(q^2, p^2), \quad J(q^2, p^2) = \int d\theta \sin^2\theta_1 \frac{1}{q^2 + p^2 + \sqrt{p^2}\sqrt{q^2}\cos\theta_1 + m^2} \frac{1}{q^2 + m^2}$$

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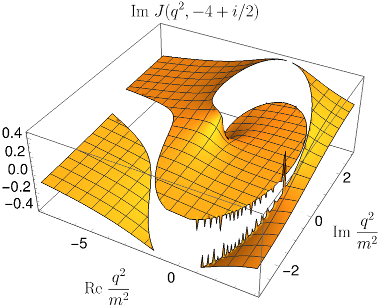
After θ_1 integration:



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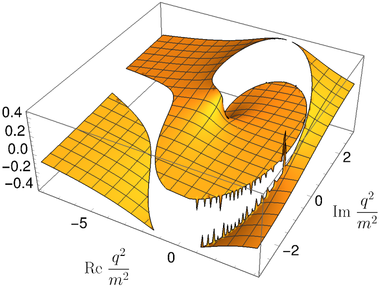
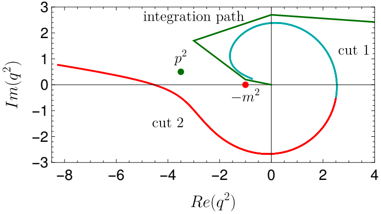
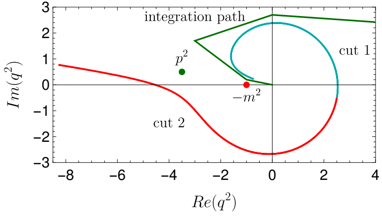


Integration path $\Lambda_{IR}^2 \to \Lambda_{UV}^2$ on real line forbidden. $\to$ Take a detour.



Contour deformation method (CDM)

Originally used for QED: [Maris, Phys.Rev.D52, (1995)]

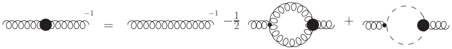


Recent resurgence: massive propagators, three-point functions, e.g.: [Alkofer et al., Phys.Rev.D 70 (2004); Eichmann, Krassnigg, Schwinzerl, Alkofer, Ann.Phys. 323 (2008); Strauss, Fischer, Kellermann, Phys.Rev.Lett. 109 (2012); Windisch, MQH, Alkofer, Phys.Rev.D 87 (2013), Acta Phys.Polon.Supp. 6 (2013); Strodthoff, Phys.Rev.D 95 (2017); Weil, Eichmann, Fischer, Williams, Phys.Rev.D 96 (2017); Pawlowski, Strodthoff, Wink, Phys.Rev.D 98 (2018); Williams, Phys.Lett.B 798 (2019); Miramontes, Sanchis-Alepuz, Eur.Phys.J.A 55 (2019); Eichmann, Duarte, Pena, Stadler, Phys.Rev.D 100 (2019); Fischer, MQH, Phys.Rev.D 102 (2020); Miramontes, Sanchis-Alepuz, Phys.Rev.D 103 (2021); Eichmann, Ferreira, Stadler, Phys.Rev.D 105 (2022); Miramontes, Alkofer, Fischer, Sanchis-Alepuz, Phys.Lett.B 833 (2022); MQH, Kern, Alkofer, Phys.Rev.D 107 (2023); . . .]

Landau conditions: When do singularities arise in external momenta [Landau, Sov. Phys. JETP 10 (1959)]? Directly reflected in possible contours [Windisch, MQH, Alkofer, Acta Phys.Polon.Supp. 6 (2013); MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)].

Landau gauge propagators in the complex plane

Simpler truncation:



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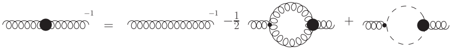
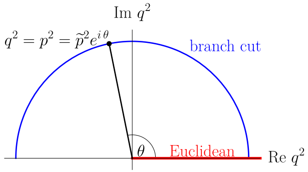


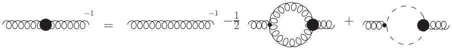
$m = 0$: Branch cuts are circles with one opening.



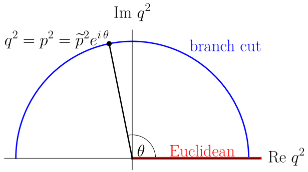
$\rightarrow$ Opening at $q^2 = p^2$.

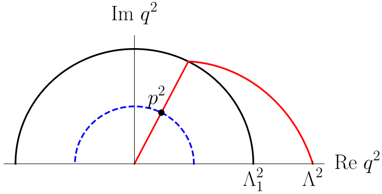
Landau gauge propagators in the complex plane

Simpler truncation:



$m = 0$: Branch cuts are circles with one opening.



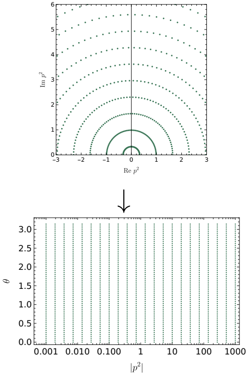


$\rightarrow$ Opening at $q^2 = p^2$.

Rays: $(p+q)^2 \propto e^{i\,\theta}$ $\rightarrow$ Argument dressings all *on* the rays.

Arc: Contribution tiny. Manageable with extra techniques.

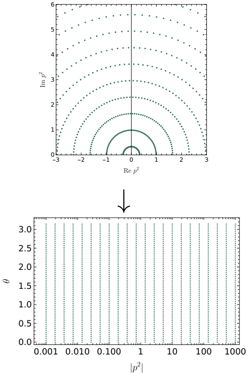
Landau gauge propagators in the complex plane

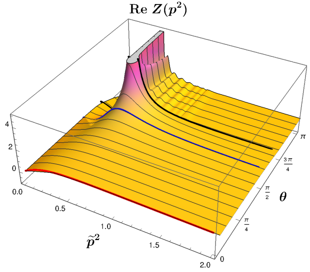


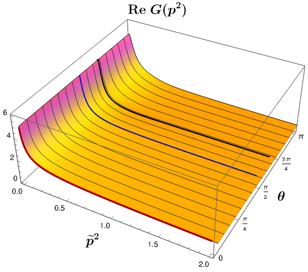
Landau gauge propagators in the complex plane

Ray technique for self-consistent solution of a DSE:

[Fischer, MQH, Phys.Rev.D 102 (2020)]





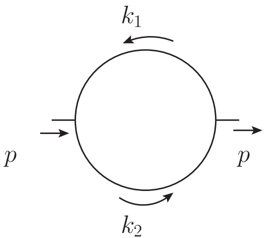


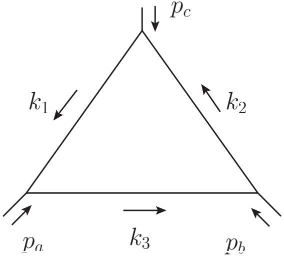
Polar coordinates: $p^2 = \tilde{p}^2 e^{i\,\theta}$

- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction).
- Origin? Effect of dynamic three-point functions?

Kinematics

	2-point I_2	3-point I_3
indep. momenta	1	2
Lorentz inv.	1	3
nontrivial integr.	2	3





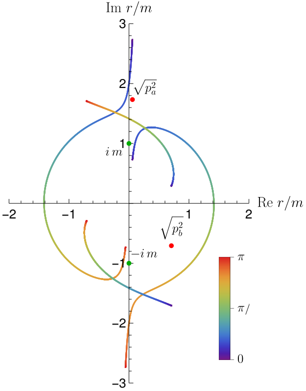
$$I_2(p^2) = \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 + m^2} \frac{1}{q^2 + p^2 - 2\sqrt{p^2}\sqrt{q^2}\cos\theta_1 + m^2}$$

$$I_3(p_a, p_b) = \int dr\, r^3 \int (\sin\theta_1)^2 d\theta_1 \int \sin\theta_2 d\theta_2 \frac{1}{r^2 + m^2} \frac{1}{(q - p_a)^2 + m^2} \frac{1}{(q + p_b)^2 + m^2}$$

$$r = \sqrt{q^2}$$

Singularities in the integrand

Integration over θ_1 and θ_2 creates branch cuts. $\to$ One generic form ($z_1 = \cos\theta_1$):



$r = \sqrt{q^2}$

$$\gamma_\pm(z_1; p^2, m^2) = \sqrt{p^2}\, z_1 \pm i\sqrt{m^2 + p^2(1 - z_1^2)}$$
$$= \sqrt{p^2} \cos\theta_1 \pm i\sqrt{m^2 + p^2 \sin^2\theta_1}$$

3-point

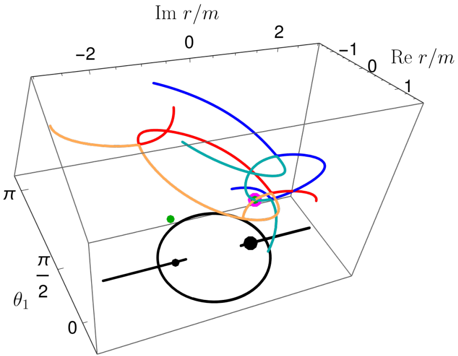
$$k_a \to \gamma_{a\pm}(z_1; p_a^2, m^2) = \gamma_\pm(z_1; p_a^2, m^2),$$
$$k_b \to \gamma_{b\pm}(\tilde{z}; p_b^2, m^2) = \gamma_\pm(-\tilde{z}; p_b^2, m^2)$$

$$\tilde{z} = \cos\tilde{\theta} = \cos\theta\, \cos\theta_1 + \sin\theta\, \sin\theta_1\, \cos\theta_2.$$

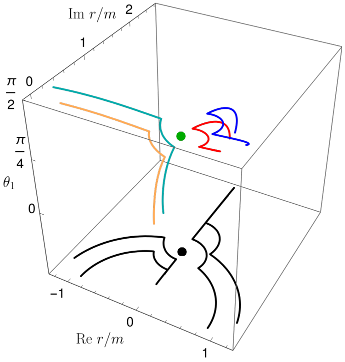
3-point for $p_a^2 = p_b^2 = p^2$

Branch cuts on top of each other:

- $\gamma_{a\pm}$ as for 2-point integral.
- $\gamma_{b\pm}$ is a function of θ_1 AND θ_2.



$p^2 = -3m^2$, $\theta = 2\pi/3$, $\theta_2 = \pi$:
two cuts cross at $i\,m$



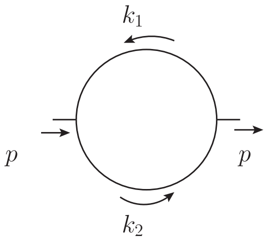
[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]

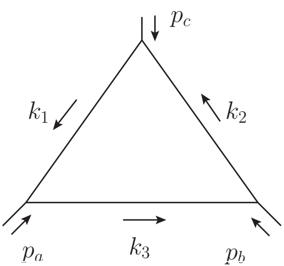
ϕ^3 theory

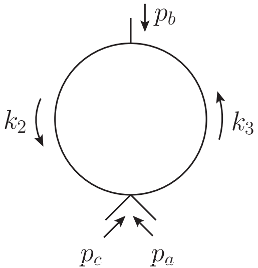
Simple scalar theory with cubic interaction:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3$$

$\rightarrow$ Technical testbed for QCD: 2-point, triangle, swordfish integrals







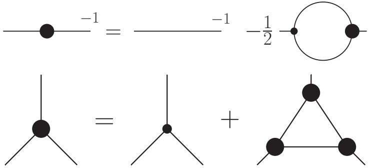
(Ignoring instability of theory.)

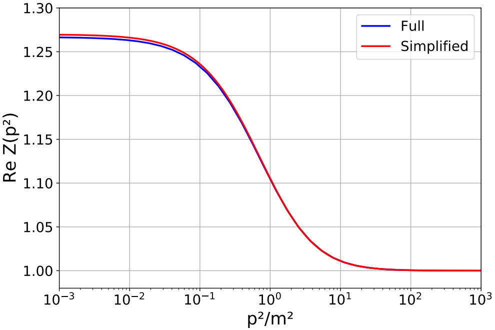
Nonperturbative equations

3PI effective action truncated at 3 loops [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

Propagator:

Equations of motion:





[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]

Simplified kinematics: $p_a^2 = p_b^2 = p^2$

Nonperturbative equations

3PI effective action truncated at 3 loops [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

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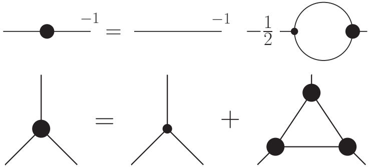
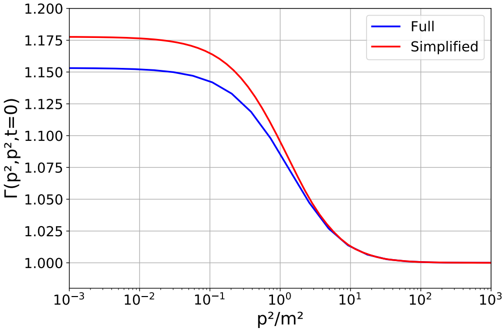


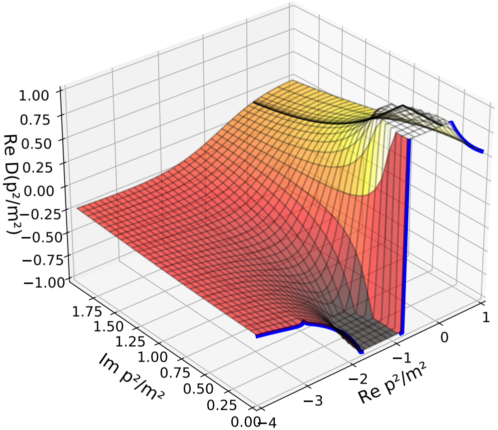
Vertex:

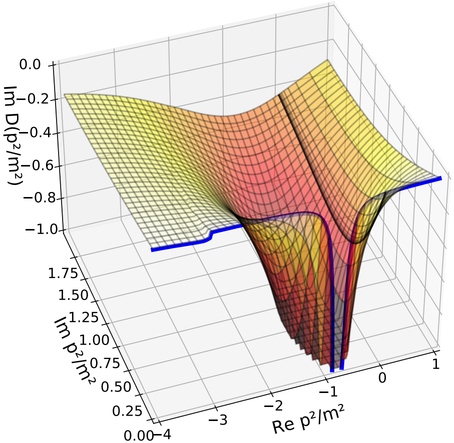


[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]

Simplified kinematics: $p_a^2 = p_b^2 = p^2$

Propagator





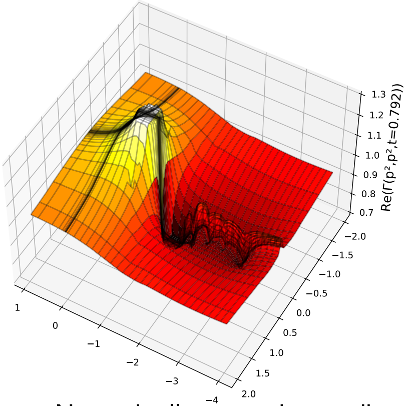
- Nonperturbative pole at $p^2 = -0.75m^2 = -m_r^2$
- A branch cut starting at $-3m^2 = -4m_r^2$ ✓

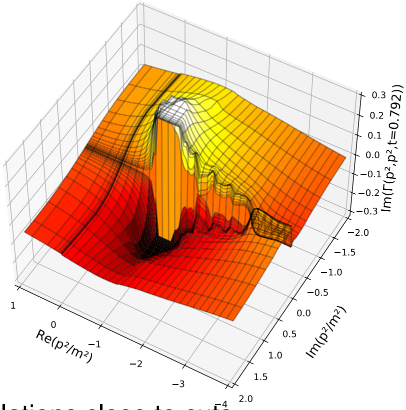
[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]

Vertex

Dressing:

[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]



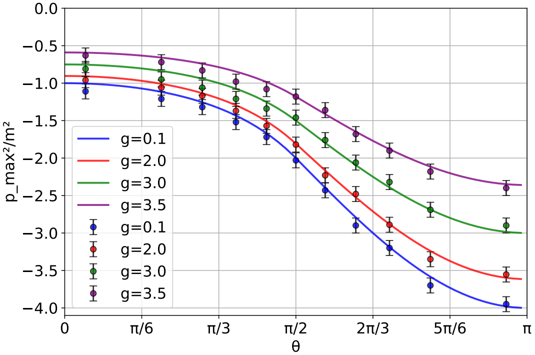


- Numerically more demanding due to calculations close to cuts.

Vertex

Branch point position: Calculation vs. Landau

[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]

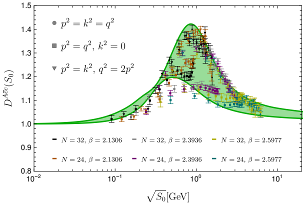


- Numerically more demanding due to calculations close to cuts.
- Branch cuts start close to the predicted value ✓

The simplest three-point function in QCD?

Three-point functions depend on two momenta/three Lorentz invariants.

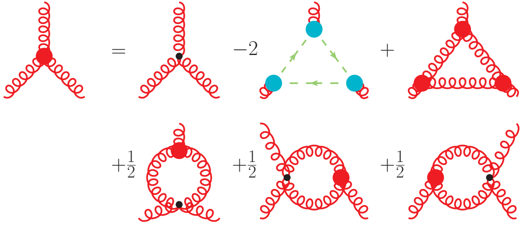
- Quark-gluon vertex: 8 dressings and nontrivial kinematic dependence, e.g., [Williams, Fischer, Heupel, Phys. Rev. D93 (2016); Gao, Papavassiliou, Pawlowski, Phys. Rev. D103 (2021); Cyrol et al. D97 (2018); Kızılersü et al., 2111.13455]
- Ghost-gluon vertex: kinematic dependence [Maas, 1907.10435; MQH, Phys. Rev. D90 (2020)]



- Three-gluon vertex: One dominant dressing, in good approximation only one kinematic variable (planar degeneracy) [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014); MQH, Phys. Rev. D90 (2020); Pinto-Gómez et al., Phys.Lett.B838 (2023); Aguilar et al., Eur.Phys.J.C83 (2023)]

  $\rightarrow$ Explore three-point function without complications of three kinematic variables.

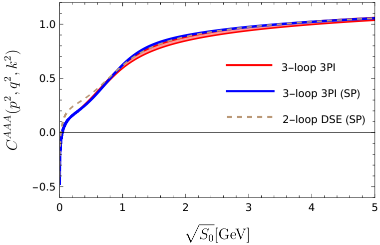
Equation of motion



Kinematics: S_3 singlet only [Eichmann, Williams, Alkofer, Vujinovic, Phys. Rev. D89 (2014)]

$$S_0 = \frac{p_1^2 + p_2^2 + p_3^2}{6}$$

Doublet: $a = s = 0$ (symmetric point)

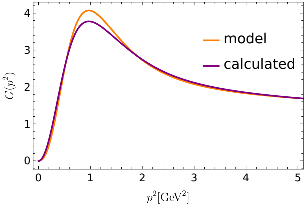


[MQH, Phys. Rev. D90 (2020)]

Input

Gluon propagator

Reconstruction from [Cyrol, Pawlowski, Rothkopf, Wink, Sci. Post 5 (2018)].

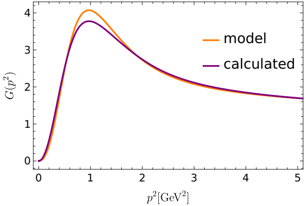


$\rightarrow$ Input has no complex conjugate poles by construction!

Input

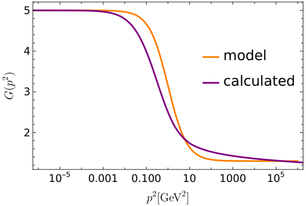
Gluon propagator

Reconstruction from [Cyrol, Pawlowski, Rothkopf, Wink, Sci. Post 5 (2018)].



Ghost propagator

$$G(p^2) = C\frac{p^2+m_2^2}{p^2+m_1^2}$$



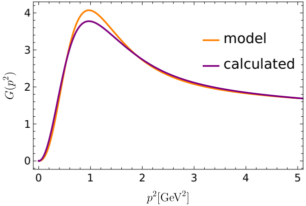
[MQH, Phys. Rev. D90 (2016)]

$\rightarrow$ Input has no complex conjugate poles by construction!

Input

Gluon propagator

Reconstruction from [Cyrol, Pawlowski, Rothkopf, Wink, Sci. Post 5 (2018)].



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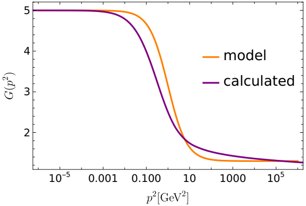
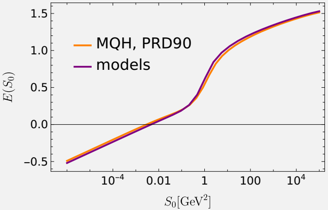


[MQH, Phys. Rev. D90 (2016)]

Ghost-gluon vertex

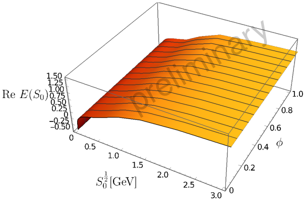
$$\Gamma^{A\bar{c}c}(k; p, q) = 1$$

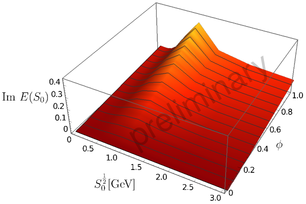
Resulting three-gluon vertex



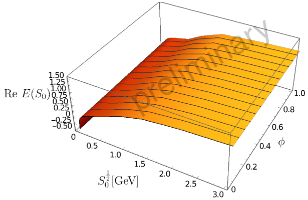
$\rightarrow$ Input has no complex conjugate poles by construction!

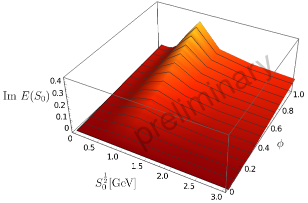
Three-gluon vertex in the complex plane



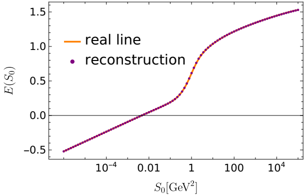


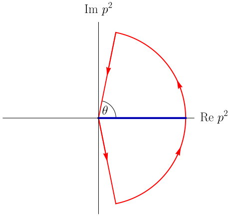
Three-gluon vertex in the complex plane





Reconstruction from rays (analyticity $\to$ Cauchy's integral formula):





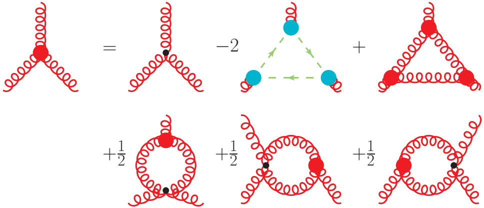
- Real part looks 'flat'.
- Imaginary parts has a ridge. $\to$ Pole?
- Numerical instability increases with ϕ

$$D_c^{\theta}(x) = \frac{\Lambda^2}{\pi}\int_0^{\theta} d\phi \, \mathrm{Re}\left(\frac{e^{i\,\phi} D(\Lambda^2 e^{i\,\phi})}{\Lambda^2 e^{i\,\phi} - x}\right)$$

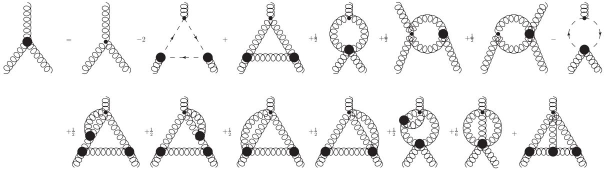
Three-gluon vertex dissection

[MQH, Phys. Rev. D 101 (2020)]

3PI:



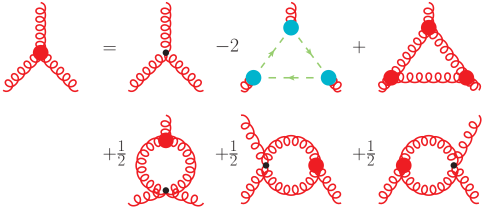
1PI:

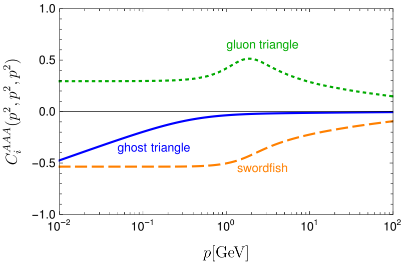


Three-gluon vertex dissection

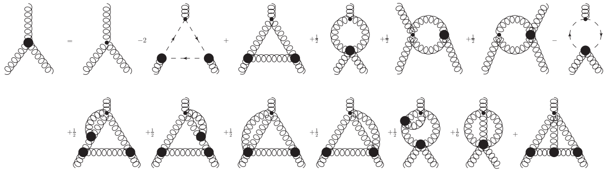
[MQH, Phys. Rev. D 101 (2020)]

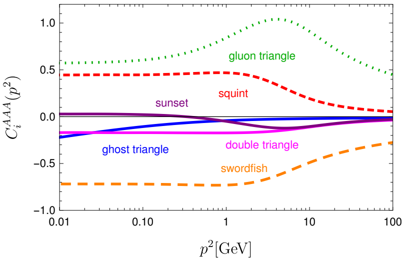
3PI:



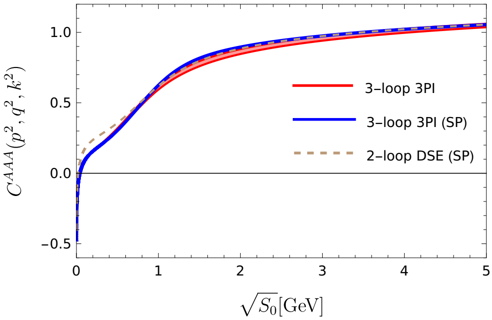


1PI:

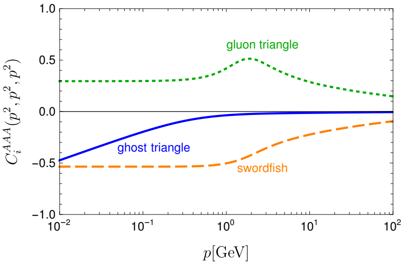


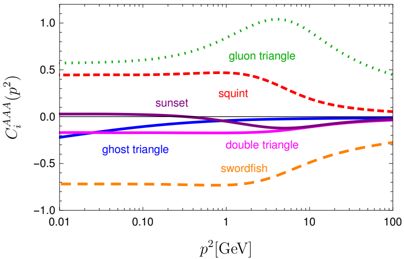


Three-gluon vertex dissection



[MQH, Phys. Rev. D 101 (2020)]

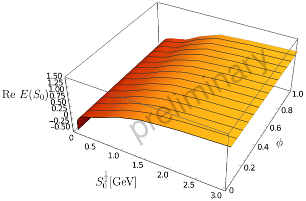


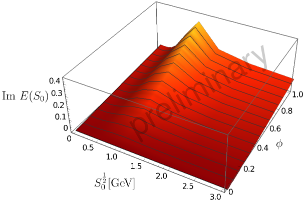


- Different cancellations for 1PI and 3PI with similar results

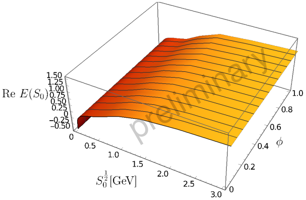
Consistent cancellations in different systems.

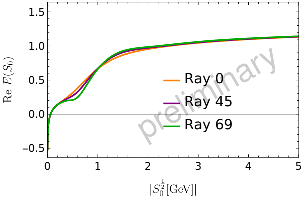
Three-gluon vertex in the complex plane

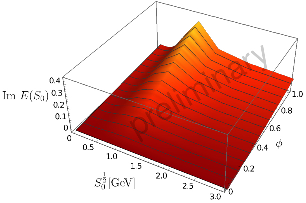


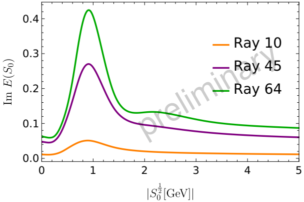


Three-gluon vertex in the complex plane

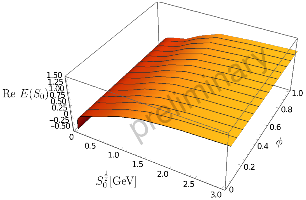


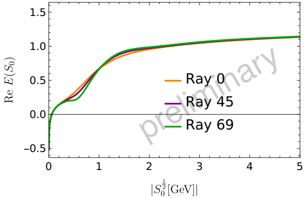


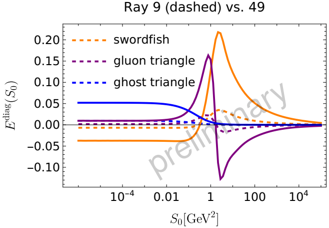


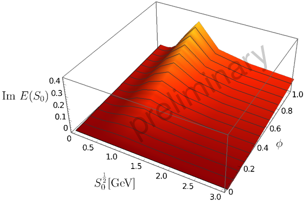


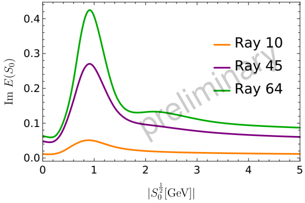
Three-gluon vertex in the complex plane











- Size of individual contributions increase.
- Balance of cancellations important!
- Bug or feature?

Summary and outlook

- Propagators and vertices at complex momenta for bound state studies needed, in particular for resonances, decays.
- Contour deformation method gives access to analytic structure of correlation functions.
- Mechanism of creation of branch cuts $\rightarrow$ "nonperturbative" Landau conditions
- Origin of singularities? Unbalanced cancellations?

- Testbed ϕ^3 theory.
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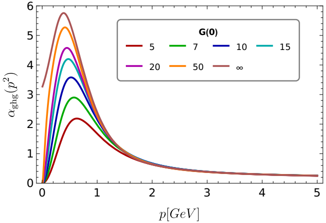
Thank you for your attention.

Spectrum for family of solutions

Couplings:

[MQH, Fischer, Sanchis-Alepuz, 2503.03821]

$$\alpha_{3g}(p^2) = \alpha(\mu^2)\left[D^{3g}(p^2)\right]^2\left[Z(p^2)\right]^3$$



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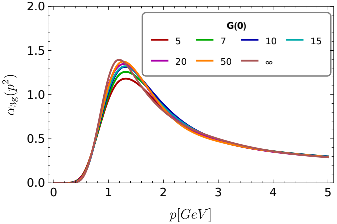

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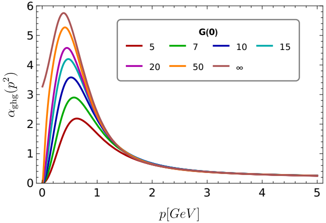
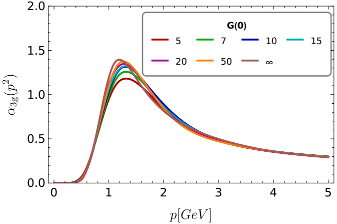
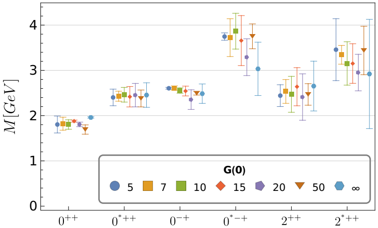


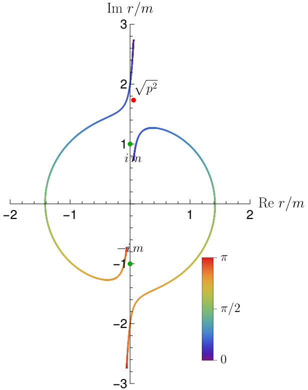




- Invariant spectrum → Nonperturbative gauge hypothesis supported.
- Invariance lost when deforming the input. → Consistency relevant

Creation of branch points in external momentum (2-point)

A branch point arises in the external momenta if the integration contour cannot be deformed.



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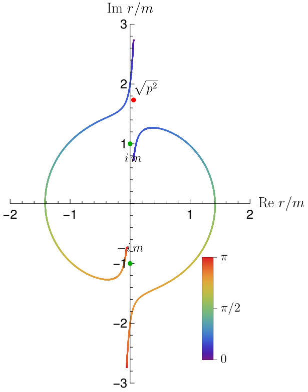
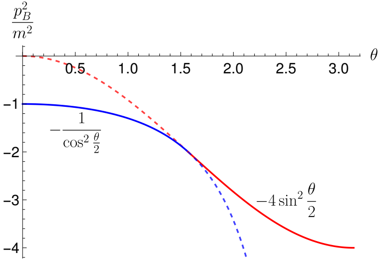


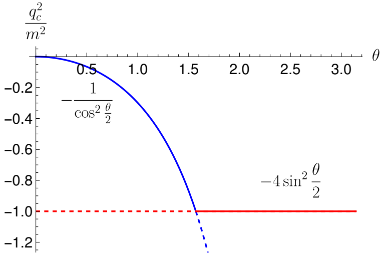
Increasing p^2 until $i\,m$ is at the end point of the branch cut. $\rightarrow$ Contour deformation no longer possible and branch point is created.

- Analytical determination of branch points possible from contour deformations [Windisch, MQH, Alkofer, Acta Phys.Polon.Supp. 6 (2013)]
- $\rightarrow$ Landau conditions [Landau, Sov.Phys.JETP10 (1959)]: $p_B^2 = -(m_1 + m_2)^2$

Critical points

2 solutions: Relevant one is that with the critical point in the *r* plane closer to the origin.





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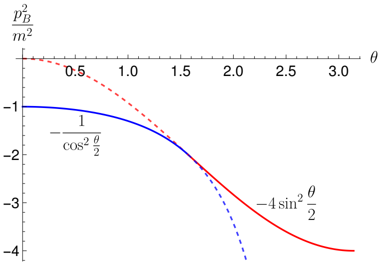

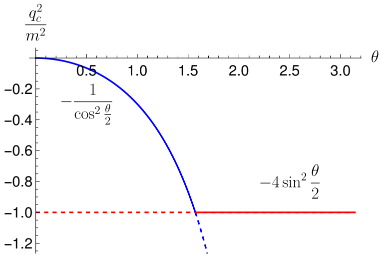

Corresponds to Landau solution:

$$p_B^2 = \begin{cases} -4m^2 \sin\left(\frac{\theta}{2}\right)^2 & \frac{\pi}{2} \leq \theta \leq \pi \\ \frac{-m^2}{\cos\left(\frac{\theta}{2}\right)^2} & 0 \leq \theta \leq \frac{\pi}{2} \end{cases}$$

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3-point: End point in θ_2!

- Two cuts cross for same θ_1 at pole.
  or
- Two cuts meet for θ_1 'inside' of circle.

$$p^2_{B,1} = -4m^2 \sin^2 \frac{\theta}{2}$$

$$p^2_{B,2} = -\frac{m^2}{\cos^2 \frac{\theta}{2}}$$

General kinematics for 3-point integral

Similar analysis [MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]:

- Identify case where all three propagators agree.

$$\rightarrow p_a^2\, p_b^2\, p_c^2 = m^2(p_a^4 + p_b^4 + p_c^4 - 2(p_a^2\, p_b^2 + p_a^2\, p_c^2 + p_b^2\, p_c^2))$$

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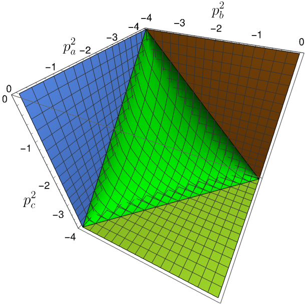


Landau condition

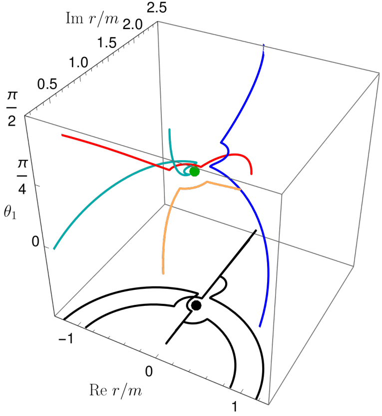
$$p_c^2 = \frac{2m^2(p_a^2 + p_b^2) + p_a^2 p_b^2 + \sqrt{p_a^2(4m^2 + p_a^2)}\sqrt{p_b^2(4m^2 + p_b^2)}}{2m^2}$$

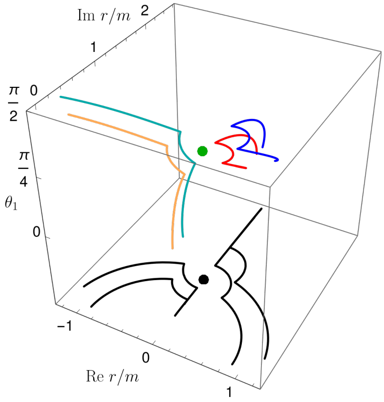
$$\text{for} \quad -4m^2 \le p_a^2, p_b^2 \le 0, \quad \text{and} \quad p_a^2 + p_b^2 \le -4m^2$$

$$p_a^2 = p_b^2 = p_c^2 = -4m^2 \qquad \text{else.}$$

Branch points for general kinematics (3-point)

Exclusion of one solution of the quadratic equation for p_c^2:





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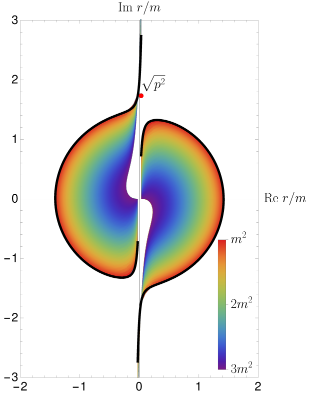

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- Nonperturbative masses ✓
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- Nonperturbative vertices with singularities in dressings: similar branch cuts in integrands as from propagators.

[MQH, Kern, Alkofer, Phys.Rev.D 107 (2023)]