ECT* workshop: Complex structure of strong interactions in Euclidean and Minkowski space

Goldstone bosons at finite temperature

(Based on: PL, O. Philipsen, 2501.17120, 2506.XXXX)

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Outline

1. Goldstone's theorem in vacuum
2. Generalisation to finite temperature
3. A thermal particle? $\rightarrow$ "Thermoparticle"
4. Thermal Goldstone bosons
5. Goldstone signatures from the lattice

1. Goldstone's theorem in vacuum

- In words: Goldstone's theorem states that the spontaneous breaking of a continuous symmetry implies the existence of massless (Goldstone) bosons

- QFT language: if j^μ is the conserved current associated with the symmetry, and A is some local field whose transformation under the symmetry has a non-trivial vev: $<\delta A> = \lim_{R\to\infty} <[Q_R, A]> \neq 0$, then:

  → *The Fourier transform of* $<[j^0(x), A(y)]>$ *contains a* $\delta(p^2)$ *singularity*

- In fact... current conservation and field locality means that $<\delta A> \neq 0$ implies the Fourier transform of $<[j^0(x), A(y)]>$ contains a $\delta(\omega)$ component as $\boldsymbol{p} \to 0$. This is ***independent*** of the properties of the background state*

- The Goldstone "quasi-particle" $\delta(\omega)$ becomes a stable massless particle state $\delta(p^2)$ for relativistic QFTs

* See: [F. Strocchi, Symmetry Breaking, Lect. Notes Phys. 732 (2008)]

2. Generalisation to finite temperature

- For $T = 1/\beta > 0$, one defines: $<\Phi(x_1)\ldots\Phi(x_n)>_\beta = Z^{-1}\,\mathrm{Tr}\, e^{-\beta H}\,\Phi(x_1)\ldots\Phi(x_n)$
- There are some immediate implications:
  - **Lorentz invariance** ✘ $\rightarrow$ but can retain rotational invariance
  - **Spectral condition ($H > 0$)** ✘ $\rightarrow$ replaced by KMS condition
  - **Field locality (**causality**)** ✓ $\rightarrow$ this is important!
- Since current conservation and field locality are unaffected by T, the Fourier transform of $<[j^0(x), A(y)]>_\beta$ ***still*** contains a $\delta(\omega)$ as $\boldsymbol{p} \rightarrow 0$
- Can we learn anything else about the properties of thermal Goldstone modes, e.g. what happens for $\boldsymbol{p} > 0$?

Yes! The key is to determine how T modifies spectral functions $\rho_{AB}(\omega,\boldsymbol{p})$, the Fourier transform of the thermal expectation values $<[\Phi_A(x), \Phi_B(y)]>_\beta$ [Bros, Buchholz, *PRD* 58 (1998)]

2. Generalisation to finite temperature

- For (complex) scalar fields, the constraints imposed for $T > 0$ imply that the spectral function has the representation*

$$\rho(\omega,\vec{p}) = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\, \epsilon(\omega)\, \delta\big(\omega^2 - (\vec{p}-\vec{u})^2 - s\big)\, \widetilde{D}_\beta(\vec{u},s)$$

"Thermal spectral density"

This is the $T > 0$ generalisation of the textbook *Källén-Lehmann* representation!

$$\rho(\omega,\vec{p}) = 2\pi\epsilon(\omega)\int_0^\infty ds\, \delta\big(p^2 - s\big)\, \varrho(s)$$

- $T > 0$ effects amount to understanding: $\rho(s) \rightarrow \widetilde{D}_\beta(\boldsymbol{u},s)$, which tells us about the possible excitations that can exist in a thermal medium
- The non-trivial $\boldsymbol{u}$ dependence of $\widetilde{D}_\beta(\boldsymbol{u},s)$ controls the extent to which the spectral function can be off the mass-shell $p^2 = s$
- The s dependence determines whether the spectral function has energy ω thresholds, much like in the $T = 0$ case

* See: [J. Bros and D Buchholz, *Z. Phys. C* 55 (1992); *Ann. Inst. H.Poincare Phys.Theor.* 64 (1996)]

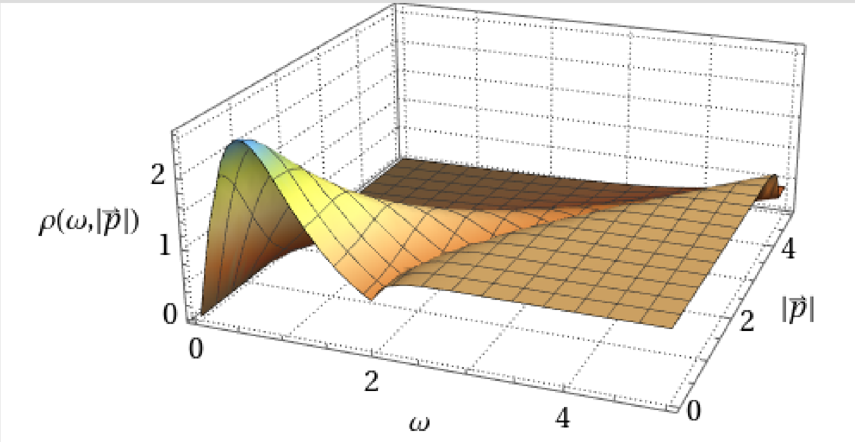
3. A thermal particle? → "Thermoparticle"

Proposition: the medium contains "Thermoparticles": particle-like excitations which differ from collective quasi-particle modes, and show up as ***discrete*** contributions to $\widetilde{D}_\beta(\boldsymbol{u},s)$ [Bros, Buchholz, *NPB* 627 (2002)]

$$\widetilde{D}_\beta(\vec{u},s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s-m^2) + \widetilde{D}_{c,\beta}(\vec{u},s)$$

→ Thermoparticle components $\widetilde{D}_\beta(\boldsymbol{u})\delta(s\text{-}m^2)$ reduce to those of a vacuum particle with mass m in the limit $T \to 0$

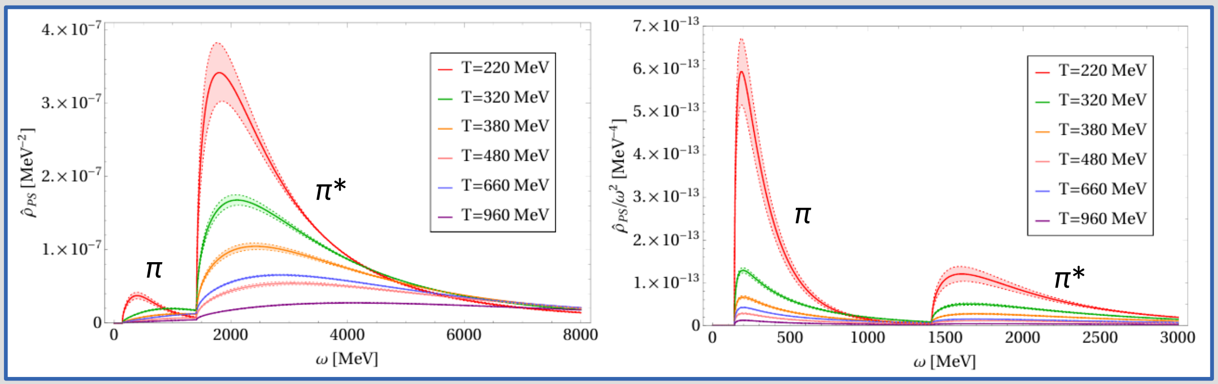
→ Non-trivial "Damping factor" $\widetilde{D}_\beta(\boldsymbol{u})$ results in thermally-broadened peaks in the spectral function: this parametrises the effects of collisional broadening



→ Component $\widetilde{D}_{c,\beta}(\boldsymbol{u},s)$ contains all other types of excitations, including those that are *continuous* in s

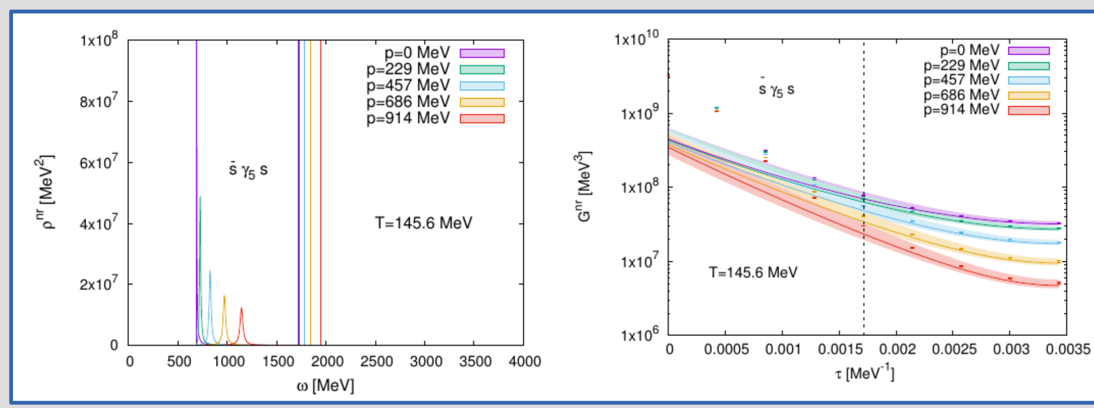
3. A thermal particle? → "Thermoparticle"

- There is mounting evidence for low-energy thermoparticle excitations, e.g. spatial correlator $C_{\mathrm{PS}}(z)$ of the pseudo-scalar meson operator $\mathcal{O}^a_{\mathrm{PS}} = \overline{\psi}\gamma_5 \frac{\tau^a}{2}\psi$ in lattice QCD
- Studies extracting pseudo-scalar spectral function in various channels:



Light-light pseudo-scalar meson (pion) channel [P.L., O. Philipsen, *JHEP* 10, 161 (2022)]

Light-strange (kaon) and strange-strange (eta) pseudo-scalar meson channels [D. Bala, O. Kaczmarek, P. L., O. Philipsen, and T. Ueding, *JHEP* 05, 332 (2024)]



Data in *all* channels consistent with a thermoparticle-type ground state: suggests light pseudo-scalar mesons (pions, kaons,..) still have a bound-state-like structure, even at high T

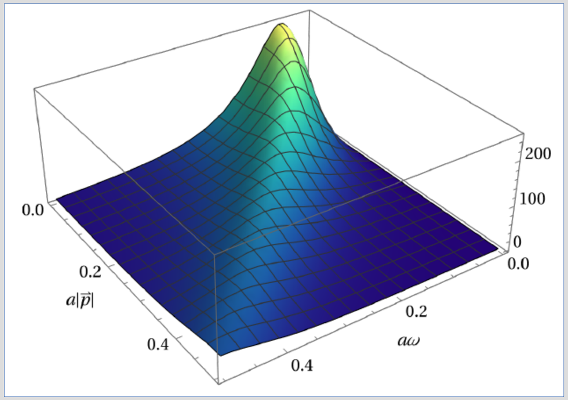
4. Thermal Goldstone bosons

- Using the extra information given by the $T > 0$ spectral representation, in [Bros, Buchholz, 1998] the authors were able to prove that the SSB condition $<\delta A> \neq 0$ implies that $<[j^0(x), A(0)]>_\beta$ contains a *massless* thermoparticle component, which in position space has the form:

$$D_\beta^G(\vec{x}, s) = D_\beta^G(\vec{x})\delta(s)$$

- This a **thermal Goldstone boson**: in the $T \to 0$ limit $D^G(\mathbf{x}) \to$ const, hence the current-field spectral function contains a vacuum Goldstone component $\delta(p^2)$, as expected!

- When the damping factor $D^G(\mathbf{x})$ is non-trivial, this causes the stable massless Goldstone peak at $p^2 = 0$ to become broadened



→ Describes the dissipative effects of the Goldstone moving through the thermal medium

4. Thermal Goldstone bosons

- This analysis reveals some very important characteristics:
  - → The thermal Goldstone mode is never on-shell for $T > 0$
  - → This component can persist at ***any*** temperature, even if the symmetry is restored when $T > T_c$
  - → The order parameter v depends ***entirely*** on the functional form of the damping factor $D^G(\boldsymbol{x})$

"Strong dissipation", $D^G(\boldsymbol{x}) \to 0$ for $|\boldsymbol{x}| \to \infty$, implies $v = 0$, **Restored phase**

$T = T_c$

"Weak dissipation", $D^G(\boldsymbol{x}) > 0$ for $|\boldsymbol{x}| \to \infty$, implies $v > 0$, **Broken phase**

$T = 0$ $D^G(\boldsymbol{x}) = \text{const}$, i.e. no dissipative effects, hence $p^2 = 0$ mode

- This captures the physics! Sufficiently strong dissipative effects destroy the long-range order and lead to symmetry restoration

4. Thermal Goldstone bosons

- The phase of the theory is determined *entirely* by the dissipative effects experienced by the thermal Goldstone mode
- Only $D^{G}(\boldsymbol{x}) \to 0$ for $|\boldsymbol{x}| \to \infty$ is required to ensure the symmetry is restored, but this can happen at *any* rate
- If the damping factor has the functional form $D^{G}(\boldsymbol{x}) \sim |\boldsymbol{x}|^{-\varepsilon}$ with some $\boldsymbol{\varepsilon} > 0$ at large $|\boldsymbol{x}|$, the two-point function decays as a pure power-law

> **Interesting possibility**: a symmetry could be restored at high temperatures without there being a finite correlation length!

- Analysis of *massive* thermoparticle states [Bros, Buchholz, (2002)] demonstrates that damping factors are fixed by the (asymptotic) dynamics of the theory
- If this is also the case for thermal Goldstone modes, this suggests that universality arguments alone may not be sufficient to fully characterise finite-temperature phase transitions

4. Thermal Goldstone bosons

- If thermal Goldstone modes are present one can look for their signatures in (Euclidean) correlation functions
- For simplicity, consider the QFT of a single complex scalar field at finite temperature, with two-point function $C(\tau,\vec{x}) \;=\; \langle \phi(\tau,\vec{x})\phi^{\dagger}(0)\rangle_{\beta}$
- If a thermal Goldstone mode is present, it follows from the thermoparticle structure, and the spectral function representation, that:

$$C^{G}(0,\vec{x}) = \frac{\coth\left(\frac{\pi|\vec{x}|}{\beta}\right)}{4\pi\beta|\vec{x}|} D^{G}_{\beta}(\vec{x})$$

- The mode dissipation is determined by the damping factor $D^{G}(\boldsymbol{x})$

  → For $T \to 0$ the vacuum behaviour is recovered: $C^{G}(0,\vec{x}) \xrightarrow{T\to 0} \frac{\alpha_0}{4\pi^2|\vec{x}|^2}$

- For the spatial correlator

$$C(z) = \int dx\, dy\, d\tau\, C(\tau,\vec{x}) = \frac{1}{2}\int_0^{\infty} ds \int_{|z|}^{\infty} dR\; e^{-R\sqrt{s}} D_{\beta}(R,s)$$

$$\Rightarrow \quad C^{G}(z) = \frac{1}{2}\int_{|z|}^{\infty} dR\; D^{G}_{\beta}(R)$$

See: [PL,O. Philipsen, 2022]

5. Goldstone signatures from the lattice

- Now we know what the signatures of thermal Goldstone modes are, one can look for them in lattice data

- Consider a simple model with SSB: U(1) complex scalar field theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^\dagger \partial^\mu \phi - \frac{1}{2}m^2 \phi^\dagger \phi - \frac{\lambda}{4!}(\phi^\dagger \phi)^2$$

- In the broken phase at $T=0$ the model contains a massless Goldstone boson and a resonance-like mode

  → Model expected* to undergo a second-order phase transition: for $T > T_c$ the U(1) symmetry is restored, and $|v|^2 = <\Phi><\Phi^\dagger> = 0$

- Investigate theory on a $L_\tau \times L^3$ lattice ($L_\tau = aN_\tau$, $L = aN_s$) with action

$$S = a^4 \sum_{x \in \Lambda_a} \left[\sum_\mu \left(\frac{1}{2} \Delta^f_\mu \phi^*(x) \Delta^f_\mu \phi(x) \right) + \frac{m_0^2}{2} \phi^*(x)\phi(x) + \frac{g_0}{4!} \left(\phi^*(x)\phi(x)\right)^2 \right]$$

  → Avoid potential triviality by keeping lattice spacing fixed throughout, hence $T = (aN_\tau)^{-1}$ is varied in discrete steps

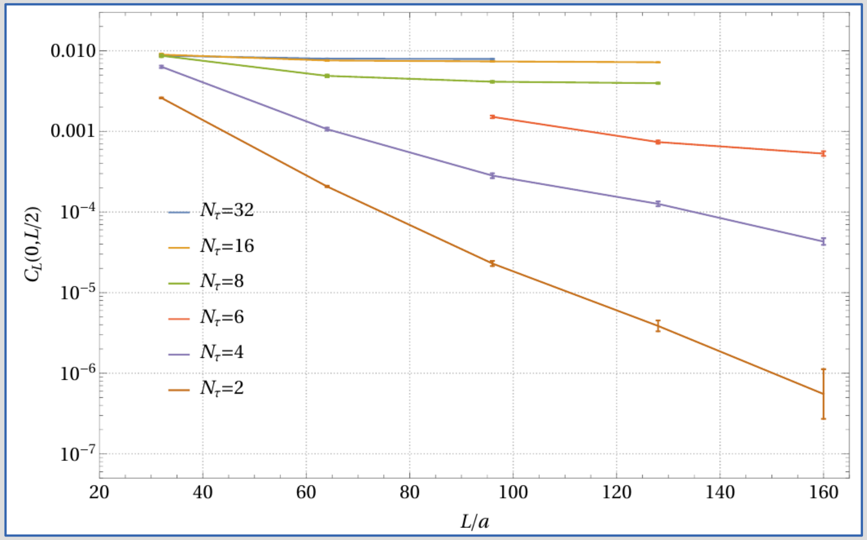
* See eg. [J. I. Kapusta and C. Gale, *Finite-temperature Field Theory*]

5. Goldstone signatures from the lattice

- SSB does not occur in a finite spatial volume $V = L^3$, i.e. there is no notion of a "vev" on the lattice

  $\rightarrow$ One needs to perform an $L \to \infty$ extrapolation of lattice results!

- For the finite-volume correlator $C_L(\tau, \boldsymbol{x})$: $\lim_{L\to\infty} C_L(\tau, \vec{x}) \xrightarrow{|\vec{x}|\to\infty} |v|^2$
- Based on this property there are different approaches* for extracting $|v|^2$
- Given periodic spatial boundary conditions one can use: $|v|^2 = \lim_{L\to\infty} C_L(0, |\vec{x}| = L/2)$



- $N_\tau = 8, 16, 32$ non-zero for large $L \rightarrow$ **U(1)** ***broken***
- $N_\tau = 2, 4, 6$ vanishing for small $L \rightarrow$ **U(1)** ***restored***
- Infinite-volume extrapolation requires a parametrisation for $C_L(\tau{=}0,\ |\boldsymbol{x}| = z)$
- Use finite-volume version of $C^G(0,\vec{x}) = \dfrac{\coth\left(\frac{\pi|\vec{x}|}{\beta}\right)}{4\pi\beta|\vec{x}|} D^G_\beta(\vec{x})$

* See eg. [H. Neuberger, *PRL* 60,(1988).]

5. Goldstone signatures from the lattice

Broken phase ($N_\tau = 8, 16, 32$):

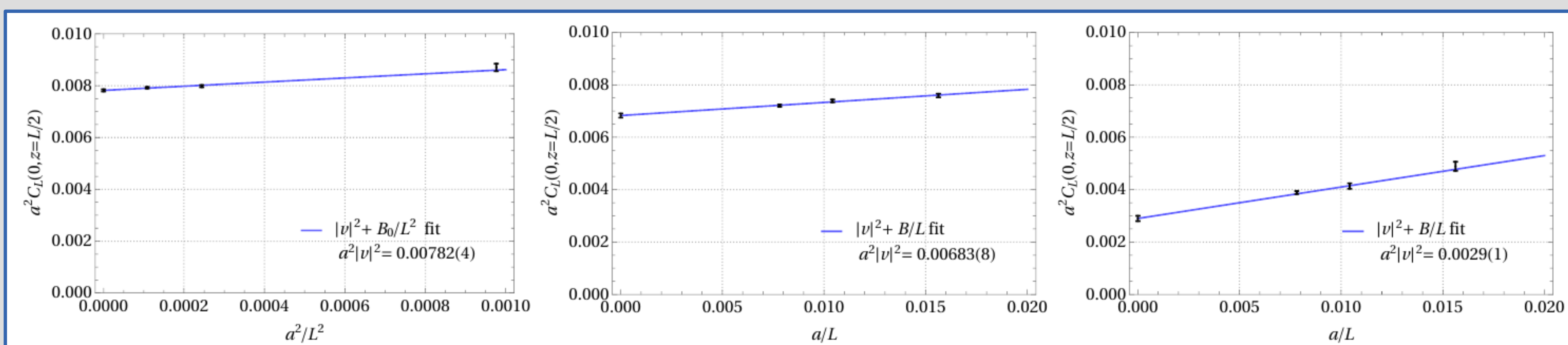
- In this case we fit the ansatz (assuming $D^G(\boldsymbol{x}) \sim \text{const}$):

$$C_L(0,z) = c_L + b_L \left[\frac{\coth\left(\frac{\pi z}{\beta}\right)}{z} + \{z \to (L-z)\} \right]$$

Non-zero if in broken phase

Finite L symmetrisation

- Functional form provides very good description of the data for each volume considered ($L/a = 32, 64, 96, 128$). Use this to do $L \to \infty$ extrapolation:



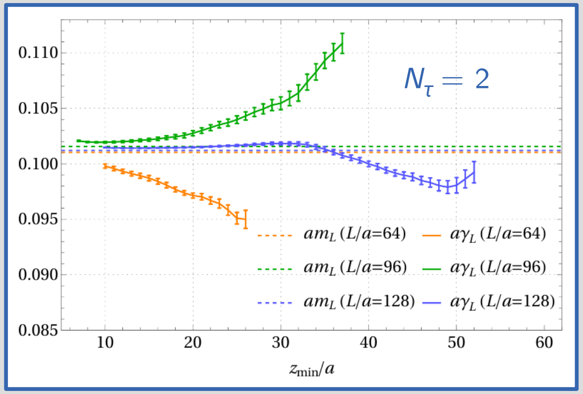
5. Goldstone signatures from the lattice

Restored phase ($N_\tau = 2, 4, 6$):

- As outlined previously, the Goldstone mode can still persist in this regime, and would have the properties of a massless thermoparticle
- What is the structure of the damping factor $D^G(\boldsymbol{x})$? Use *spatial* correlator:

$$C_L(z) = d_L\left[e^{-m_L z} + \{z \to (L-z)\}\right] \quad \longrightarrow \quad D^G_\beta(\vec{x}) = \alpha\, e^{-\gamma|\vec{x}|} \quad \longrightarrow \quad C^G(0,\vec{x}) = \frac{\coth\left(\frac{\pi|\vec{x}|}{\beta}\right)}{4\pi\beta|\vec{x}|}\alpha\, e^{-\gamma|\vec{x}|}$$

- Spatial correlator fits provide excellent description of data over full range $[0, L/2]$ for each of the (large) volumes considered ($L/a = 64, 96, 128, 160$)
- Test damping by fitting: $C_L(0,z) = b_L\left[\frac{\coth\left(\frac{\pi z}{\beta}\right)}{z}e^{-\gamma_L z} + \{z \to (L-z)\}\right]$
- Consistency of the thermoparticle hypothesis requires that the fit parameters γ_L and m_L approach one another in the infinite-volume $L \to \infty$ limit



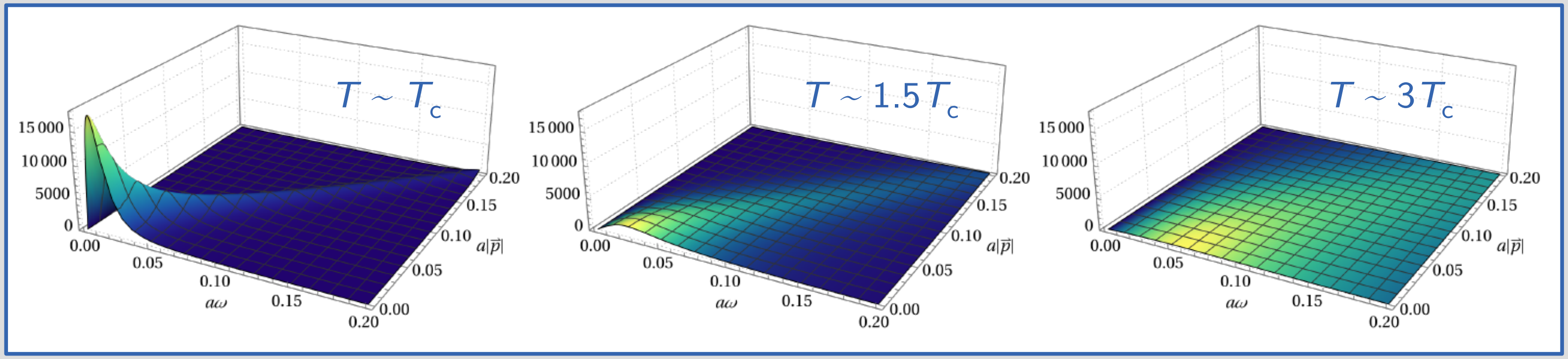
Restored phase ($N_\tau = 2, 4, 6$):

- Data at each value of N_τ consistent with the correlator being dominated by an exponentially-damped massless thermoparticle → *a thermal Goldstone!*

- One can use the extracted damping factor $D^G(\boldsymbol{x}) = \alpha\, e^{-\gamma|\boldsymbol{x}|}$ to compute the spectral function $\rho_G(\omega,\boldsymbol{p})$ of the Goldstone mode

$$\rho_G(\omega,\vec{p}) = \frac{4\alpha\,\omega\gamma}{(\omega^2 - |\vec{p}|^2 - \gamma^2)^2 + 4\omega^2\gamma^2}$$

- Spectral properties are very different to the vacuum case $\rho_G(\omega,\boldsymbol{p}) \sim \delta(p^2)$

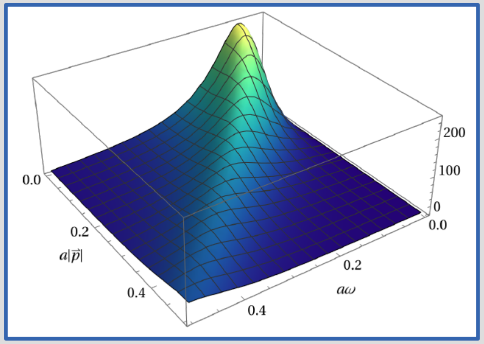


- Broadened peak structure around $p^2 = 0$ increases with temperature

  → Represents the increasingly strong dissipative effects of the medium!

Summary & outlook

- Goldstone's theorem in vacuum has well-known consequences, but at finite temperature there still remain open questions
- One can use the non-perturbative constraints imposed by *causality* to gain new insights $\rightarrow$ $\rho(\omega,\boldsymbol{p})$ have spectral representations. This narrows down the potential excitations that exist $\rightarrow$ "Thermoparticles"
- SSB for $T>0$ implies existence of Goldstone modes which have the structure of *massless* thermoparticles $\rightarrow$ these modes can persist at *any* temperature, even if the symmetry is restored
- Phase structure is determined by the damping of the Goldstone mode: (i) Weak dissipation $\rightarrow$ SSB, (ii) Strong dissipation $\rightarrow$ symmetry restoration
- We find evidence for the existence of such modes in the U(1) complex scalar field theory on the lattice below *and* above T_c $\rightarrow$ General feature of $T>0$ QFTs



Backup: Rigorous definition of SSB

- In order to define SSB rigorously one needs to define a regularised charged operator Q_R, and this only converges for $R \to \infty$ within commutator $[Q_R, A]$

$$Q_{\delta R} = \int d^4x \, \alpha_\delta(t) g_R(\vec{x}) j_0(t, \vec{x})$$

(i) $g_R(\boldsymbol{x})=1$ for $|\boldsymbol{x}| \leq R$, $g_R(\boldsymbol{x})=0$ for $|\boldsymbol{x}| \geq R(1+\varepsilon)$

(ii) $\alpha_\delta(t)$ has compact support, and $\alpha_\delta(t) \to \delta(t)$

- The condition: $\lim_{R\to\infty} <[Q_R, A]>=q$ is then ***always*** well-defined, and $q=0$ is a necessary and sufficient condition for the existence of a charge operator Q, defined by: $<u, Qv> := \lim_{R\to\infty} <u, Q_R v>$

$\rightarrow$ If this is non-vanishing for *any* A then no such charge exists, i.e. SSB!

- SSB condition: $$\lim_{\delta\to 0}\lim_{R\to\infty}\int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^3}\frac{d^3\vec{p}}{(2\pi)^3}\, \widetilde{D}^{(+)}_\beta(\vec{u},s)\,\tilde{g}(\vec{p})\,\tilde{\alpha}\left(\delta R\sqrt{((\vec{p}/R)-\vec{u})^2+s}\right) = iq\tilde{\alpha}(0)$$

- In the vacuum case this implies: $\widetilde{D}^{(+)}(\boldsymbol{u},s) \to iq\,(2\pi)^3\,\delta(\boldsymbol{u})\delta(s)$

- Value of q is ***determined*** by Goldstone damping factor $D^{(+)}(\boldsymbol{x})$ for $|\boldsymbol{x}| \to \infty$

Backup: NRT theorem

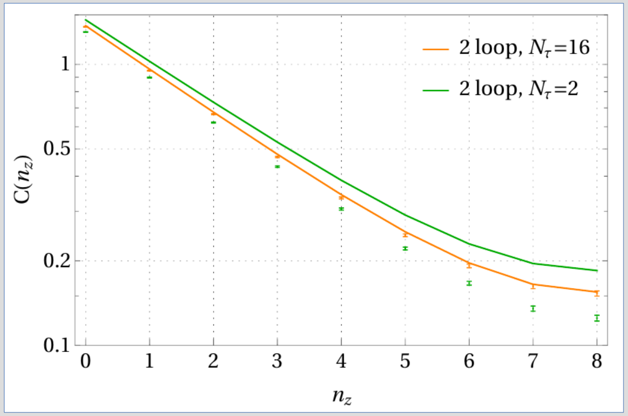
- The NRT theorem implies that states with purely real dispersion relations $\omega = E(\boldsymbol{p})$ ***cannot exist*** in interacting theories when $T > 0$

  QFT reason: *thermal states satisfy the KMS condition, and this gives rise to very different spectral constraints than in the vacuum case*

  Physics reason: *Dissipative effects of the thermal medium are everywhere-present $\rightarrow$ always need to take these into account (i.e. always a width!)*

- This has significant implications for perturbation theory: *neither free field, nor quasi-particle propagators with real poles can form the basis of finite-temperature perturbative expansions* [Landsman, *Ann. Phys.* 186, 141 (1988)]

- There is both analytic [Weldon, *PRD* 65 (2002)] and now numerical [PL, O. Philipsen, JHEP 08, (2024)] evidence for this perturbative breakdown



Backup: Perturbation theory

- How might one resolve inconsistencies of $T > 0$ perturbation theory?
- In the vacuum theory we know that the basis of perturbation theory is the Gell-Mann/Low formula [Landsman, *Ann. Phys.* 186, 141 (1988)]:

$$iG(x_1 \cdots x_n) = \sum_{i_1 \cdots i_n} Z_{i_1}^{1/2} \cdots Z_{i_n}^{1/2} \frac{\langle \Omega | \, T[\Phi_{i_1}(x_1) \cdots \Phi_{i_n}(x_n) \, U(\infty, -\infty)] \, | \Omega \rangle}{\langle \Omega | \, U(\infty, -\infty) \, | \Omega \rangle}.$$

$$U(t_1, t_2) = e^{iH_0[\Phi] t_1} e^{-iH[\Phi](t_1 - t_2)} e^{-iH_0[\Phi] t_2} = T \exp\left(-i \int_{t_2}^{t_1} dt \, H_I(t)\right),$$

- Correlation functions of *interacting* fields can be computed from correlators of *free* fields $\rightarrow$ This is *derived* from the fact that at large times the interactions between fields diminish: the asymptotic fields/states are free!
- The standard perturbative series is defined by expanding the exponential in the evolution operator as a series in the coupling parameter $\rightarrow$ each term in the expansion is determined by the propagators of the asymptotic fields

<u>Key point</u>: free field propagators form the basis of perturbation theory at $T = 0$ because the large-time states experience no interactions

Backup: Thermoparticles in QCD data

<u>Goal</u>: Extract information about the finite T spectral function $\rho_\Gamma(\omega,\mathbf{p})$ from data of *Euclidean* correlator $C_\Gamma(\tau,\vec{x}) = \langle O_\Gamma(\tau,\vec{x})\, O_\Gamma(0,\vec{0})\rangle_T$ O_Γ = scalar operator

- <u>Standard approach</u>: extract $\rho_\Gamma(\omega,\mathbf{p})$ from temporal correlator $\widetilde{C}_\Gamma(\tau,\mathbf{p})$

$$\widetilde{C}_\Gamma(\tau,\vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh\left[\left(\frac{\beta}{2}-|\tau|\right)\omega\right]}{\sinh\left(\frac{\beta}{2}\omega\right)} \rho_\Gamma(\omega,\vec{p})$$

→ *Problem is ill-conditioned, need more information!*

- Instead, one can use the ***spatial*** correlator, where one integrates $C_\Gamma(\tau,\mathbf{x})$ over $\{\tau,x,y\}$ and fixes a spatial direction z

$$C_\Gamma(z) = \int_{-\infty}^\infty \frac{dp_z}{2\pi} e^{ip_z z} \int_0^\infty \frac{d\omega}{\pi\omega}\, \rho_\Gamma(\omega, p_x = p_y = 0, p_z)$$

- It turns out that *if* thermoparticles exist, then they will give a distinct contribution to $C(z)$

$$C(z) \approx \frac{1}{2}\int_{|z|}^\infty dR\, e^{-mR} D_{m,\beta}(R)$$

[P.L., *PRD* 106 (2022); P.L., O. Philipsen, *JHEP* 10, 161 (2022)]

→ This component can be extracted *directly* from data

Backup: Thermoparticle characteristics

Given a specific QFT, what form should the damping factors take?

Idea: thermal scattering states are defined by imposing an asymptotic field condition [Bros, Buchholz, (2002)]:

Asymptotic fields Φ_0 are assumed to satisfy dynamical equations, but only at large x_0

In Φ^4 theory

$$(\partial^2 + m^2)\phi_0(x) + \frac{\lambda}{3!}\phi_0^3(x) \xrightarrow{|x_0|\to\infty} 0$$

- Since thermoparticles dominate the large-time behaviour of correlators, they are natural candidates for describing such states. It turns out that their damping factors $\widetilde{D}_{m,\beta}(\boldsymbol{u})$ are **uniquely fixed** by the asymptotic condition
- In Φ^4 theory one finds (where κ is a thermal width):

For $g < 0$:

$$\widetilde{D}^{(-)}_{m,\beta}(\vec{u}) = \frac{2\pi^2}{\kappa^2}\delta(|\vec{u}| - \kappa),$$

$$\widetilde{G}^{(-)}_{\beta}(k_0,\vec{p}) = \frac{1}{4|\vec{p}|\kappa} \ln\left[\frac{-k_0^2 + m^2 + (|\vec{p}| + \kappa)^2}{-k_0^2 + m^2 + (|\vec{p}| - \kappa)^2}\right]$$

For $g > 0$:

$$\widetilde{D}^{(+)}_{m,\beta}(\vec{u}) = \frac{4\pi}{\kappa_0\left(|\vec{u}|^2 + \kappa^2\right)},$$

$$\widetilde{G}^{(+)}_{\beta}(k_0,\vec{p}) = \frac{i}{2|\vec{p}|\kappa_0} \ln\left[\frac{\sqrt{-k_0^2 + m^2} - i|\vec{p}| + \kappa}{\sqrt{-k_0^2 + m^2} + i|\vec{p}| + \kappa}\right]$$

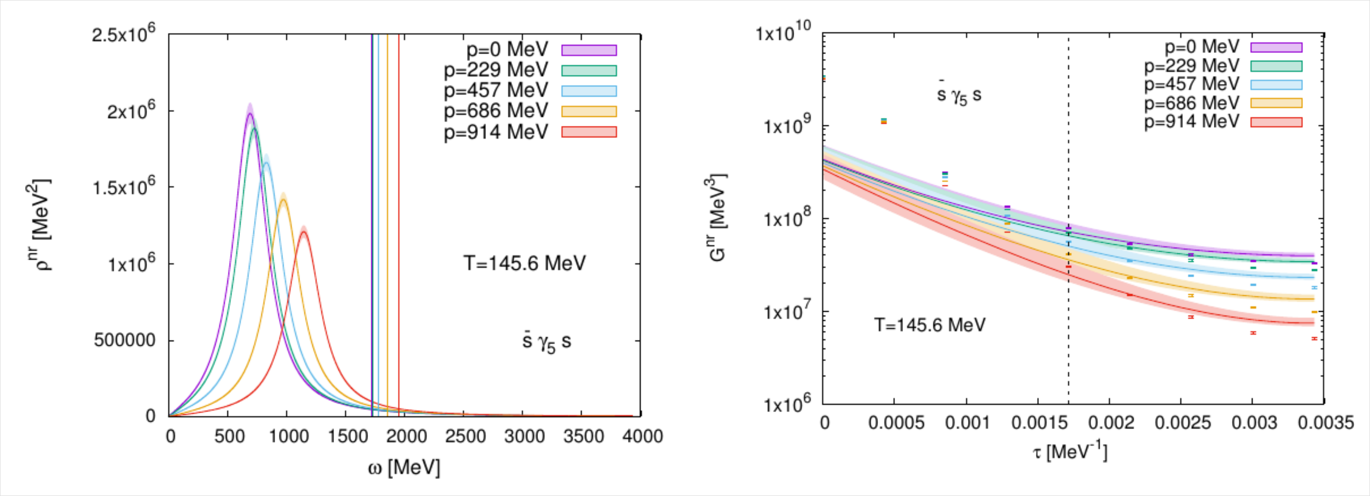
Backup: Robustness of spectral approach

- The robustness of the thermoparticle hypothesis can also be tested by comparing with different causal models, e.g. a Breit Wigner

$$\rho_{\mathrm{BW}}(\omega,\vec{p}) = \frac{4\omega\Gamma}{(\omega^2 - |\vec{p}|^2 - m^2 - \Gamma^2)^2 + 4\omega^2\Gamma^2},$$

$$C_{\mathrm{BW}}(z) = \frac{e^{-\sqrt{m^2+\Gamma^2}|z|}}{2\sqrt{m^2+\Gamma^2}}.$$

- Same procedure as with the thermoparticle case: (i) extract the width parameter Γ and coefficient from the spatial lattice data (ii) use this to predict the corresponding temporal correlator



→ Data is *not* consistent with a Breit-Wigner-type ground state!