Excited charm/charmonium mesons and exotics from lattice QCD

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ECT* workshop on "*The complex structure of strong interactions in Euclidean and Minkowski space*"
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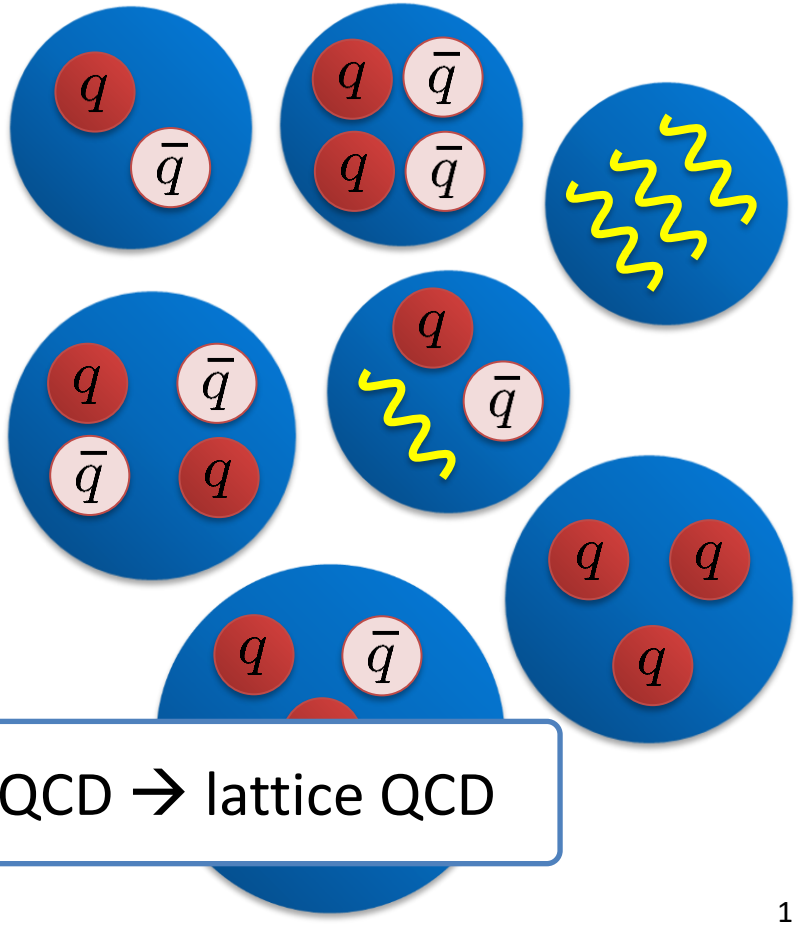
UNIVERSITY OF CAMBRIDGE

hadspec

Hadron spectroscopy

Intriguing observations, e.g. $X(3872)$, $Y(4230)$, $Z_c^+(4430)$, $Z_c^+(3900)$, $X(6900)$, $T_{cc}(3875)$, $D_{s0}(2317)$, $T_{cs}(2900)$, Z_b^+, light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

Belle II, BESIII, ATLAS, panda, LHCb, CMS, COMPASS, KLOE(-2), CLAS12, GlueX



Exotic (
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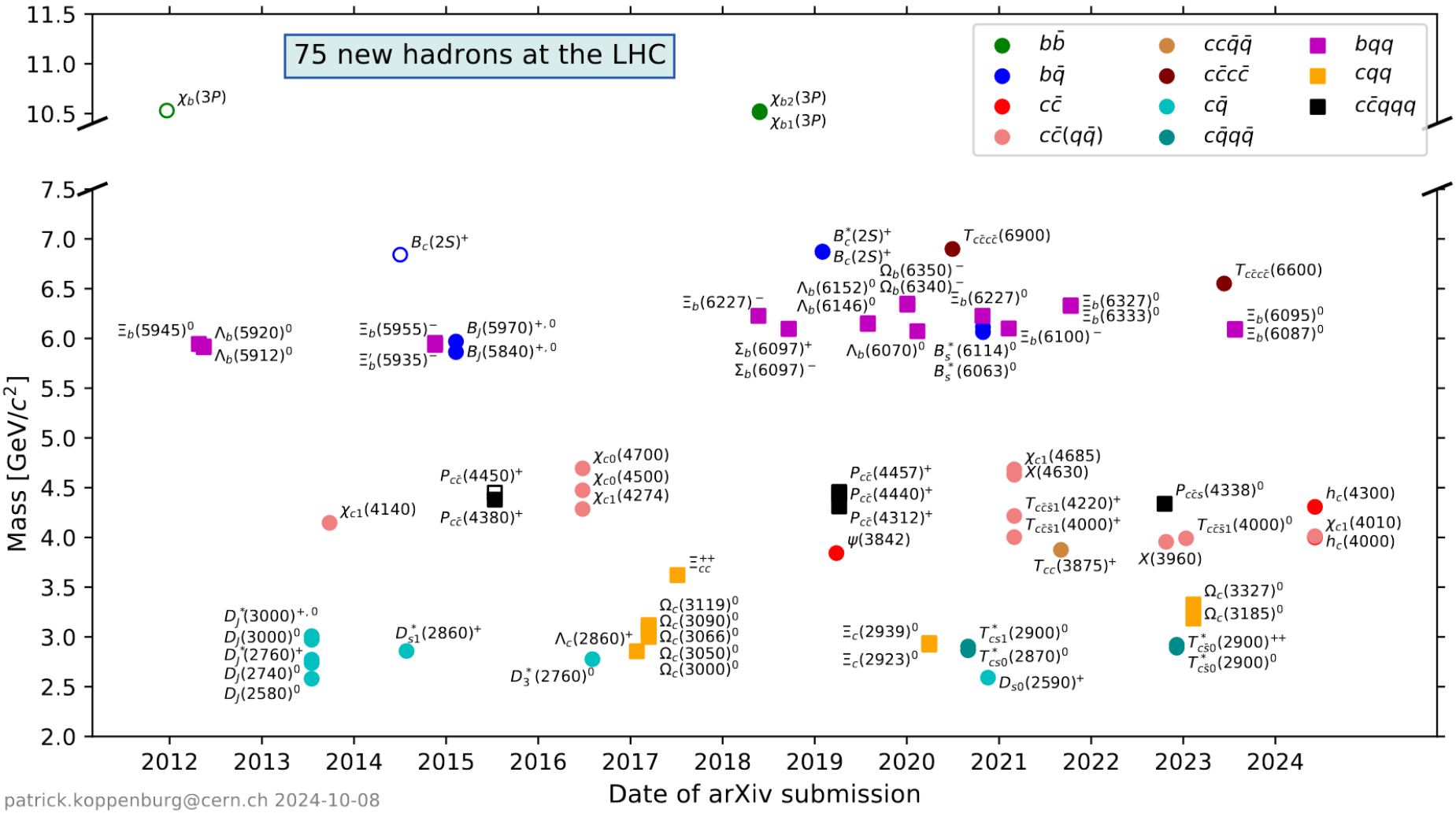
First-principles calculations in QCD → lattice QCD

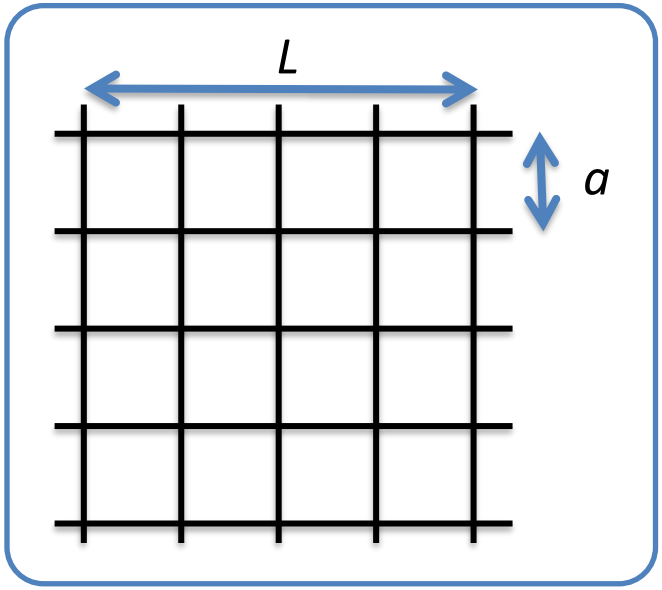
Hadron spectroscopy



patrick.koppenburg@cern.ch 2024-10-08

Lattice QCD

Systematically-improvable
first-principles calculations in QCD

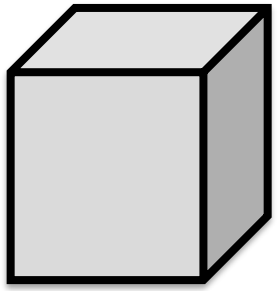


- **Discretise** spacetime in a **finite volume**
- Compute correlation fns. numerically
  (Euclidean time, $t \rightarrow i\,t$)

Note:

- Finite a and L
- Possibly heavy u, d quarks
  ($\rightarrow$ unphysical m_π)

Lattice QCD spectroscopy



Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

$$= \sum_n \frac{e^{-E_n t}}{2\ E_n} \langle 0|\mathcal{O}_i(0)|n\rangle\langle n|\mathcal{O}_j^\dagger(0)|0\rangle$$

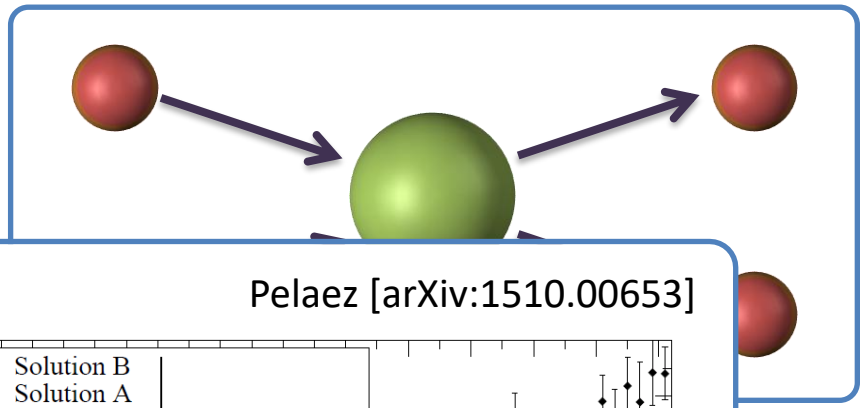
Lower-lying hadrons in each flavour sector are well determined (including isospin breaking, QED).

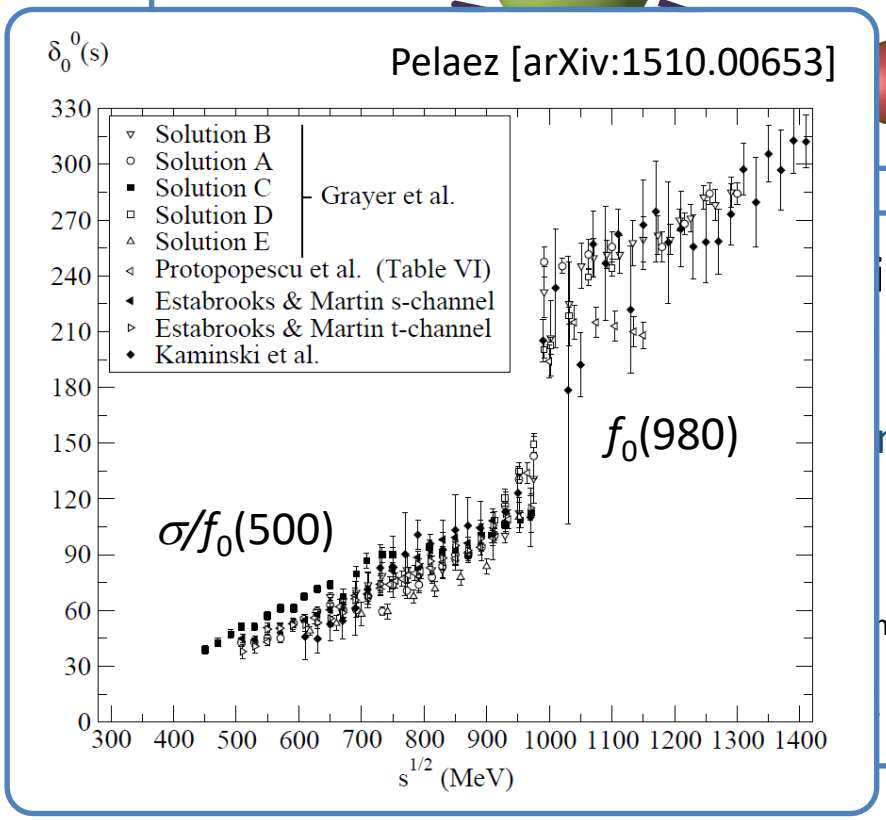
Excited states: in each symmetry channel compute matrix of correlators for **large bases of interpolating operators** with appropriate variety of structures.

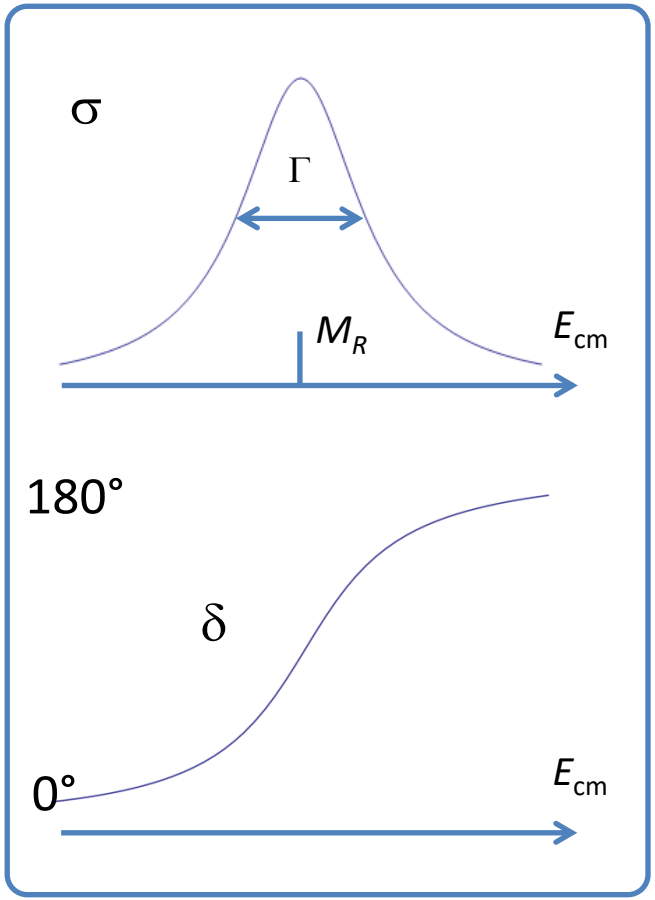
Variational method (generalised eigenvalue problem) → $\{E_n\}$

Scattering and resonances in infinite-volume continuum

Most hadrons are resonances and decay strongly to lighter hadrons



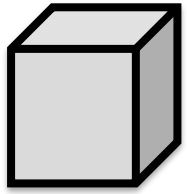




Scattering and resonances in lattice QCD

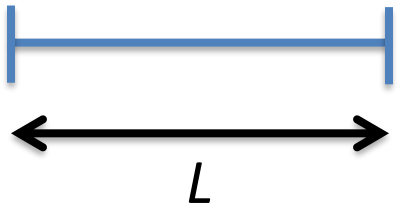
Can't directly compute scattering amplitudes in lattice QCD

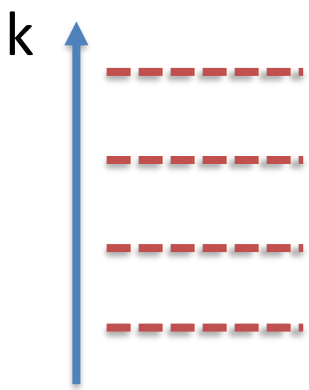
Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\text{cm}}\}$ to **infinite-volume scattering *t*-matrix**.

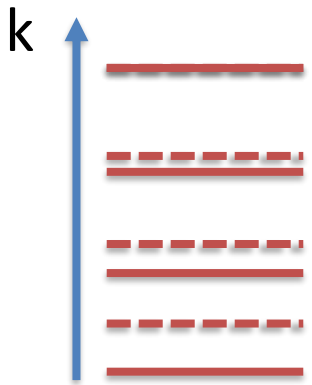


$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

c.f. 1-dim: $k = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$ (periodic b.c.s)







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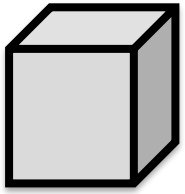


$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\rho(E_{\text{cm}})\,t(E_{\text{cm}})\left(1 + i\,\mathcal{M}^{\vec{P}}(E_{\text{cm}}, L)\right)\right] = 0$$

Infinite-volume scattering *t*-matrix
$S = 1 + 2i\sqrt{\rho}\,t\sqrt{\rho}$

Effect of finite volume (including reduced sym.)

Elastic …e mapping …

Couple… E_{cm}): solutio…

But we … (E_{cm}) – under-constrained problem

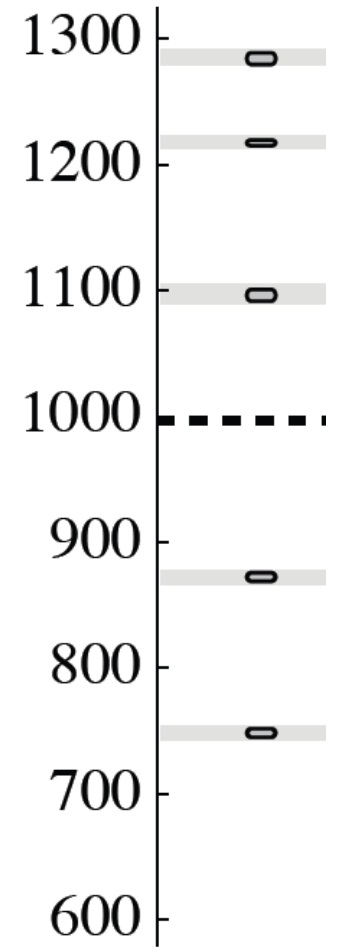
(each E_{cm} constrains *t*-matrix at that E_{cm})

Analytically continue $t(E_{\text{cm}})$ in complex E_{cm} plane, look for poles.

Demonstrated in calcs. of ρ, light scalars, b_1, charm mesons, …

The ρ resonance in ππ scattering

($J^{PC} = 1^{--}$, I = 1)



P = [0,0,0]

$J^P = 1^-$ [ℓ = 1]

Reduced sym. → other partial waves can mix in [T_1^-]

$K\bar{K}$ thresh.

$m_\pi \approx 236$ MeV

Experimentally

$$BR(\rho \to \pi\pi) \sim 100\%$$

Use many different operators

$$\bar{\psi}\Gamma D \ldots \psi$$

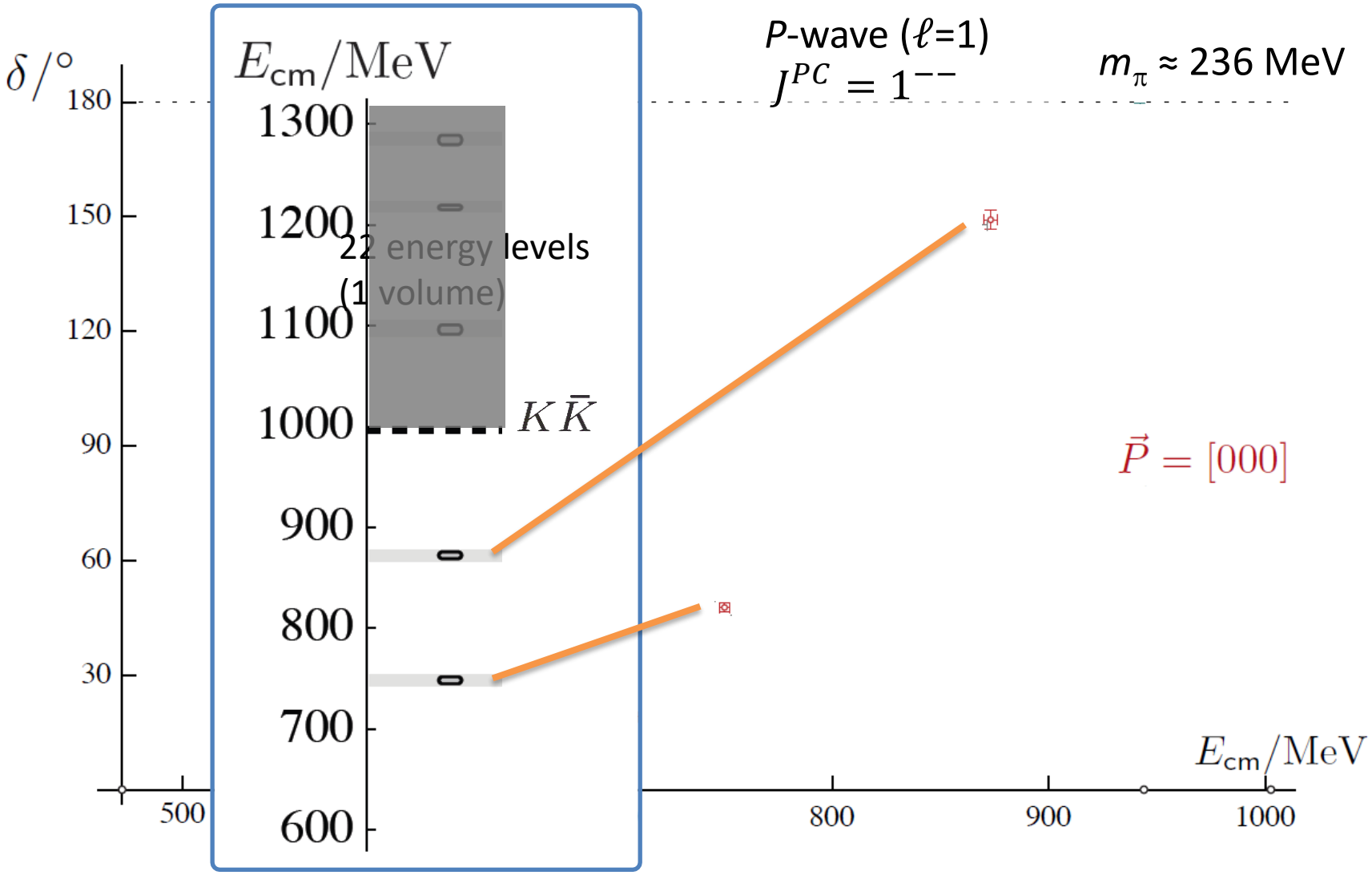
$$\sum_{\vec{p}_1,\vec{p}_2} C(\vec{P},\vec{p}_1,\vec{p}_2)\pi(\vec{p}_1)\pi(\vec{p}_2)$$

$$\sum_{\vec{p}_1,\vec{p}_2} C(\vec{P},\vec{p}_1,\vec{p}_2)K(\vec{p}_1)\bar{K}(\vec{p}_2)$$

built from *optimised* π & *K* ops

Wilson *et al* (HadSpec) [PR D92, 094502 (2015)] and Dudek, Edwards, CT (HadSpec) [PR D87, 034505 (2013)]

The ρ resonance: **elastic** ππ scattering

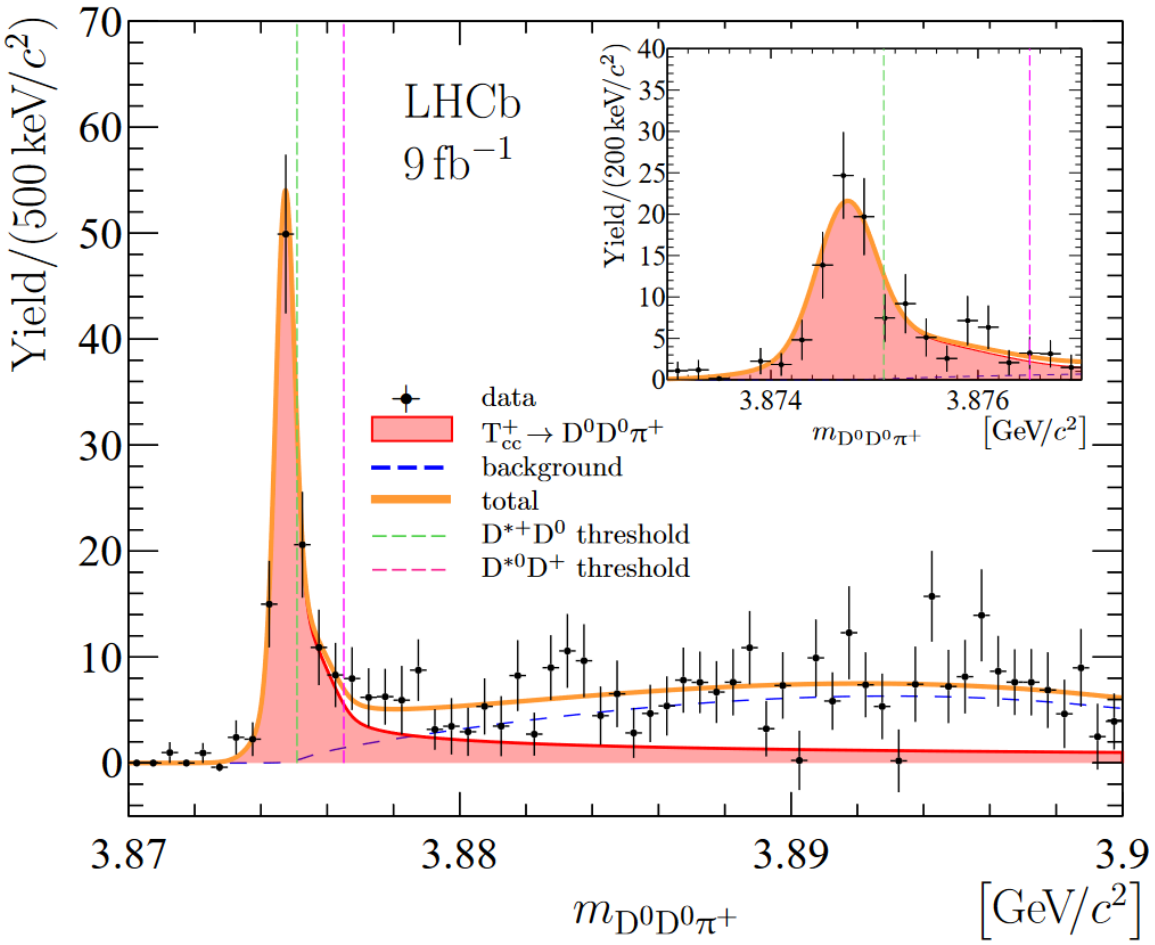


(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

T_{cc}

T_{cc}^+ seen in $D^0D^0\pi^+$ at LHCb [2109.01038, 2109.01056]

Close to DD^* threshold, $J^P=1^+$, I=0, **exotic flavour** ($cc\bar{u}\bar{d}$)



What about higher energies (coupled DD^*, D^*D^*)?

Other lattice calcs:

- Padmanath & Prelovsek [2202.10110, PRL];
- Chen *et al* [2206.06185, PLB];
- Lyu *et al* (HAL QCD) [2302.04505, PRL];
- Collins *et al* [2402.14715, PRD];
- Meng *et al* [2411.06266, PRD];

See also:

- Du *et al* [2303.09441, PRL];
- Meng *et al* [2312.01930, PRD].
- Gil-Domínguez & Molina [2409.15141, PRD].
- Dawid *et al* [2409.17059, JHEP].

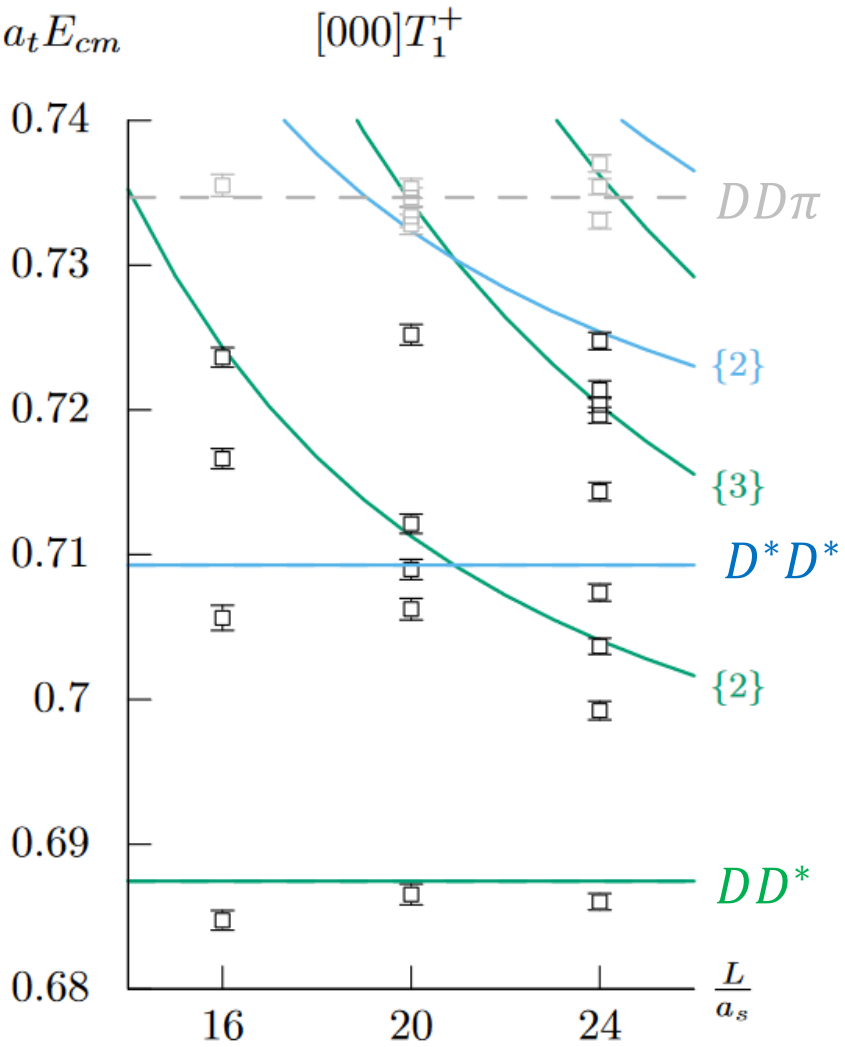
Coupled DD^*, D^*D^* scattering

P = [0,0,0] $J^P = 1^+$, ...

First lattice QCD calculation of coupled DD^*, D^*D^* scattering

$m_\pi \approx 391$ MeV, $m_D \approx 1890$ MeV, $m_{D^*} \approx 2010$ MeV (D^* is stable), 3 lattice volumes ($L \approx 2 - 3$ fm)

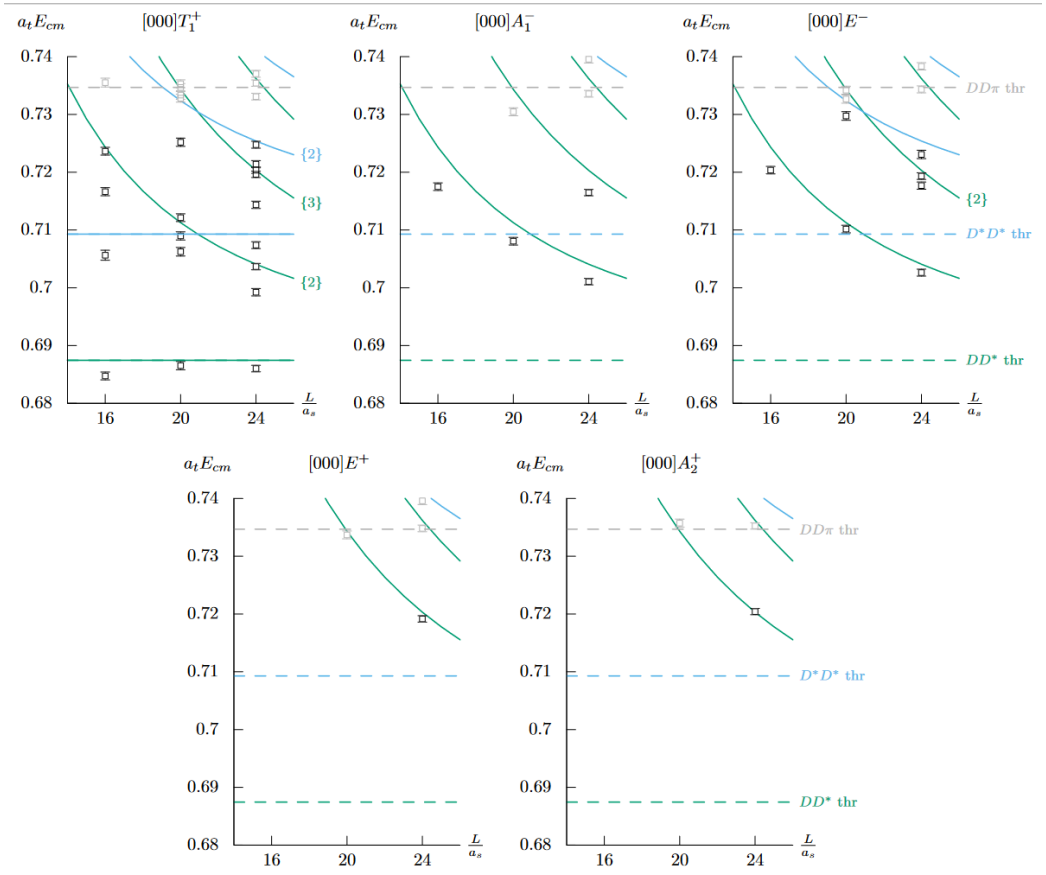
Matrices of correlation functions with many meson-meson-like DD^* and D^*D^* ops (I = 0)

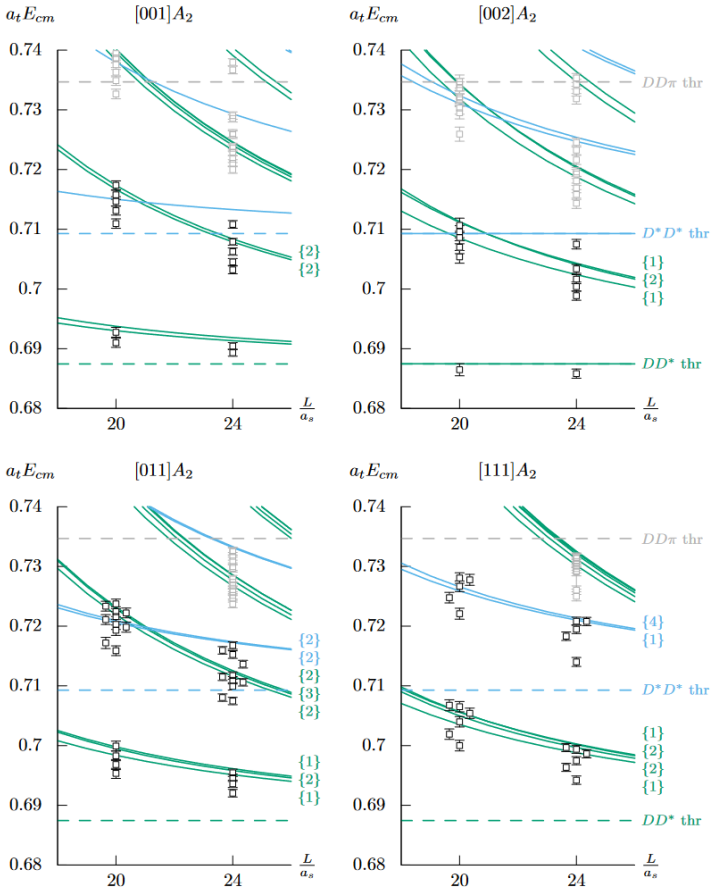


[**Whyte**, Wilson, Thomas (HadSpec), 2405.15741 (PRD)]

Coupled DD^*, D^*D^* scattering

[2405.15741]





Use 109 energy levels

Coupled DD^*, D^*D^* scattering

[2405.15741]

Partial wave amplitudes for $J^P = 1^+$:

DD^* $^3\mathrm{S}_1$ $(l = 0, S = 1)$, $^3\mathrm{D}_1$ $(l = 2, S = 1)$

D^*D^* $^3\mathrm{S}_1$ $(l = 0, S = 1)$, $^3\mathrm{D}_1$ $(l = 2, S = 1)$

and 'background' partial waves

K-matrix param. – respects unitarity (conserve prob.) and flexible

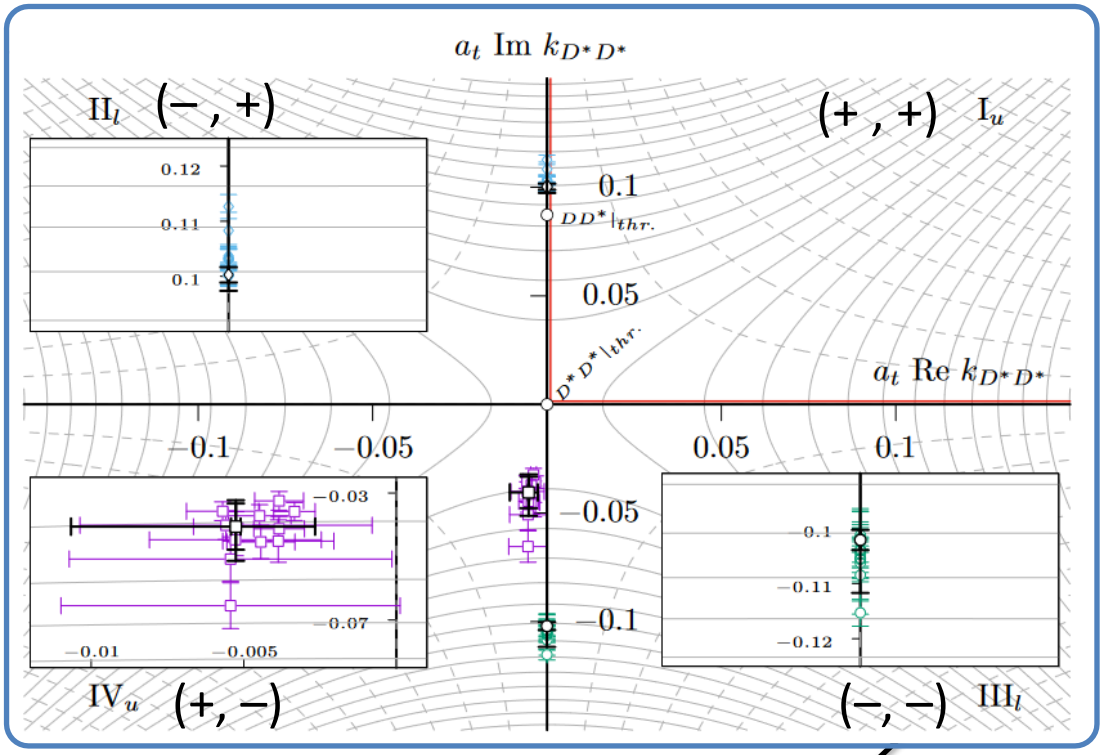
$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s) \qquad \mathrm{Im}[I_{ij}(s)] = -\delta_{ij}\rho_i(s)$$

$$\rho_i(E_{\mathrm{cm}}) = \frac{2k_i}{E_{\mathrm{cm}}}$$

In this work $K(s)_{\ell SJa,\ell' S' Jb} = \sum_n \gamma^{(n)}_{\ell SJa,\ell' S' Jb} s^n$,

T_{cc} and T'_{cc} in coupled DD^*, D^*D^* scattering

[2405.15741]



Virtual bound state (T_{cc}) below DD^* threshold

$\sqrt{s} \approx 3834(31)$ MeV

Resonance (T'_{cc}) pole on (+ , –) sheet below D^*D^* threshold – prediction of new state

Dependence on m_π?

Effect of left hand cut from π exchange (≈18 MeV below DD^* thresh)?

Charmonium scalar (0^{++}) and tensor (2^{++}) resonances

Experimental situation:

- Ground state $\chi_{c0}(1P)$ (0^{++}) and $\chi_{c2}(1P)$ (2^{++}) below $D\overline{D}$ threshold. Above that it is less clear…

- $\chi_{c0}(3860) \to D\overline{D}$ (Belle). Not seen in $B^+ \to D^+D^-K^+$ (LHCb). Theoretical reanalyses: may be from pole below $D\overline{D}$ thresh.
- $\chi_{c0}(3930) \to D\overline{D}$ (LHCb)
- $\chi_{c0}(3960) \to D_s\overline{D}_s$ (LHCb)
- $X(3915) \to J/\psi\omega$ (Belle)
- $\chi_{c2}(3930) \to D\overline{D}$ (Belle, BABAR, LHCb)

Charmonium 0^{++} and 2^{++} resonances

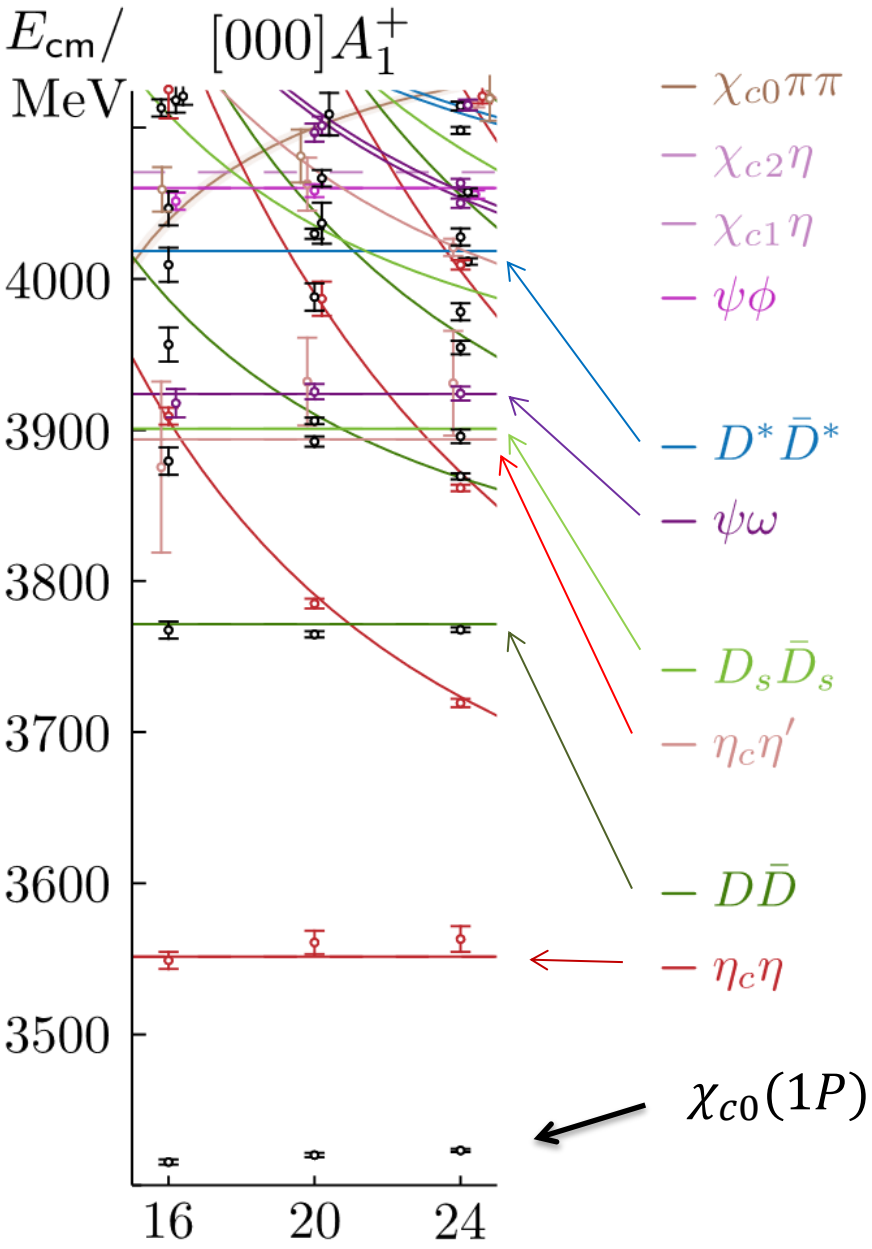
$m_\pi \approx 391$ MeV,
3 lattice volumes ($L \approx 2 - 3$ fm)
No c - $\bar{c}$ annihilation.

Use many
fermion-bilinear ($\bar{c}\,\Gamma\,\mathrm{D}\,\ldots\,c$)
and meson-meson-like ops
($\eta_c\eta$, $D\bar{D}$, $\eta_c\eta'$, $D_s\bar{D}_s$, $D\bar{D}^*$, $D_s\bar{D}_s^*$, $\psi\omega$, $D^*\bar{D}^*$, $\psi\phi$, $\eta_c\sigma$, $\chi_{c0,2}\sigma$, …)

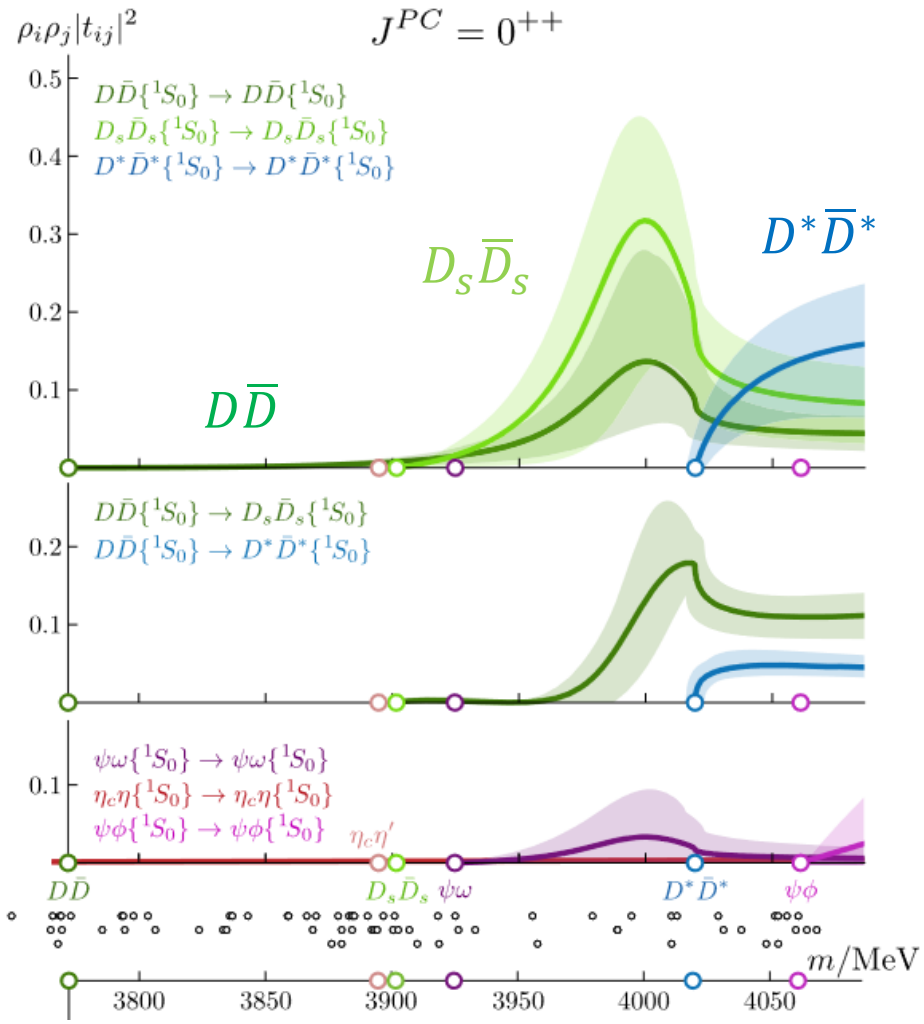
First ‘complete’ lattice study of this energy region.

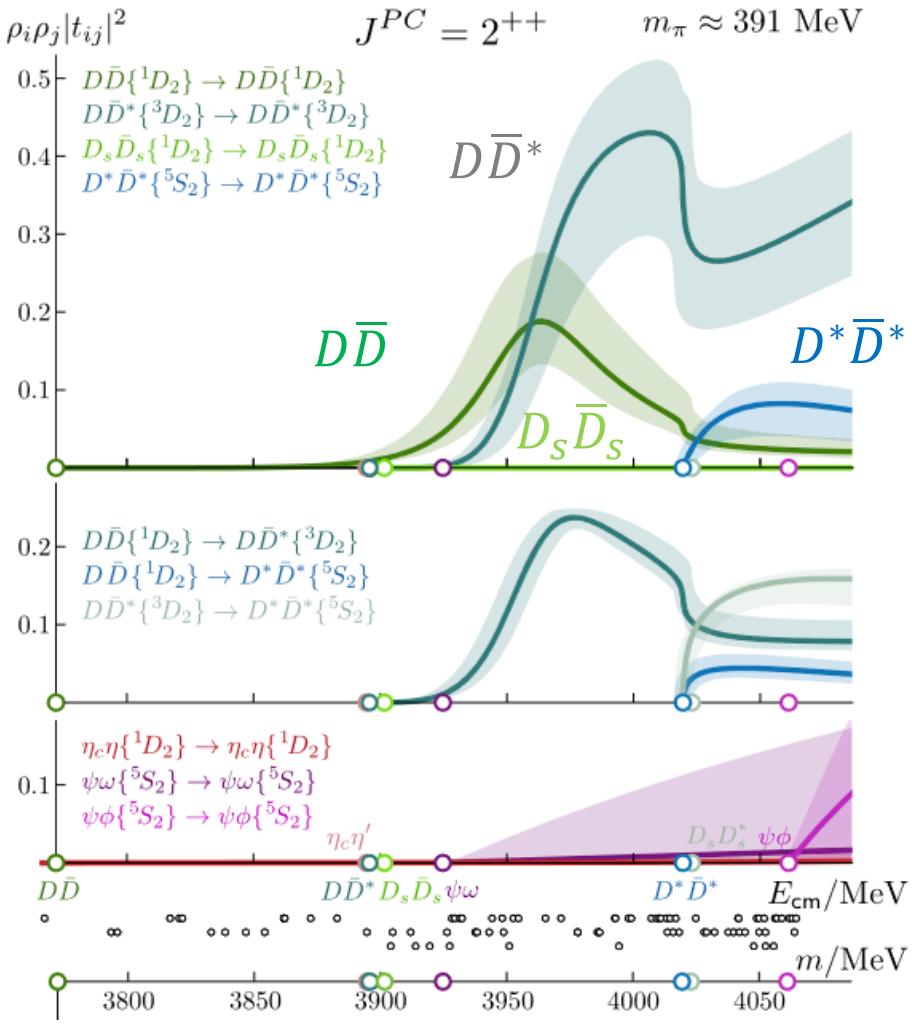
[**Wilson**, Thomas, Dudek, Edwards (HadSpec), 2309.14070 (PRL), 2309.14071 (PRD)]

P = [0,0,0] $J^P = 0^+, (4^+, \ldots)$



0^{++} and 2^{++} scattering amplitudes



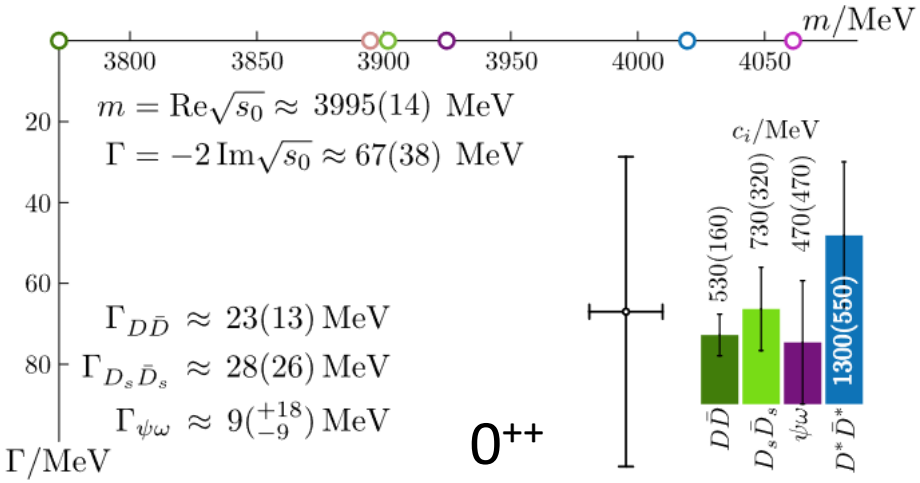


[2309.14070, 2309.14071]

Use more than 200 energy levels

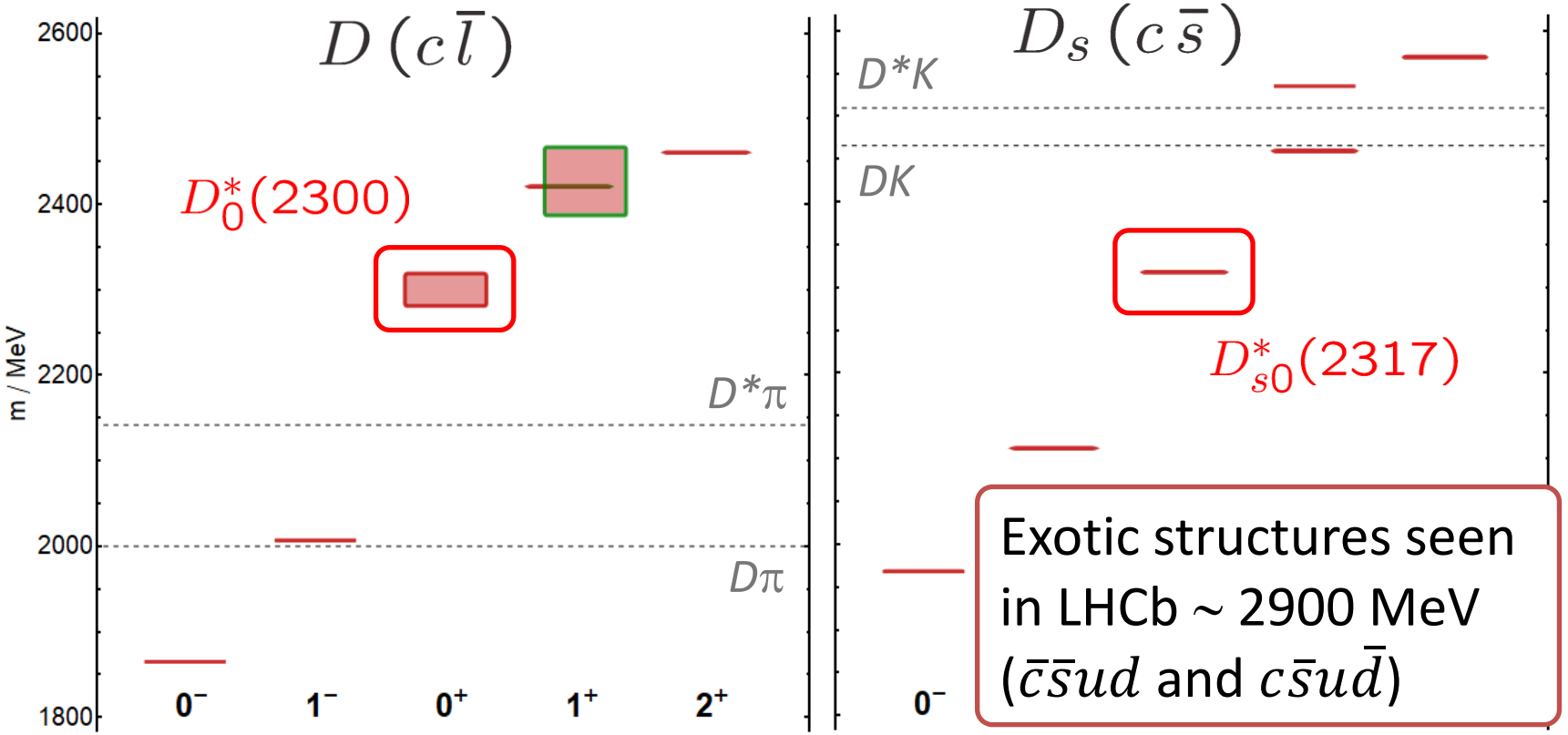
Charmonium 0^{++} and 2^{++} resonances

[2309.14070, 2309.14071]



- Only one 0^{++} and one 2^{++} resonance up to ≈ 4100 MeV, c.f. quark model (2P states)
- No large scattering amps in channels with $\bar{c}c$ + light meson (OZI)
- Above ground state χ_{c0} no other 0^{++} bound states or near-$D\overline{D}$ / $D_s\overline{D}_s$ threshold resonances.
  c.f. claims for extra $\chi_{c0}(3860)$ by Belle [1704.01872], lattice calc ($m_\pi \approx 280$ MeV) [Prelovsek *et al,* 2011.02542], some models, …)
- (Also bound state in 2^{-+} and narrow resonance in 3^{++}.)

Charm (D) and charm-strange (D_s) mesons



DK/π with $SU(3)_F$ sym

[**Yeo**, Thomas, Wilson (HadSpec), 2403.10498 (JHEP)]

$SU(3)_F$ flavour symmetry (m_u=m_d=m_s)

$D_{\bar{3}}$ (D, D_s) ≈ 1960 MeV; η_8 (π, K, …) ≈ 690 MeV

η_1 ≈ 940 MeV

Elastic $D_{\bar{3}}\,\eta_8$ scattering: $\bar{\mathbf{3}} \otimes \mathbf{8} = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$

(S-wave, $J^{PC} = 0^{++}$)

Exotic flavour (can't be just $\bar{q}c$)

- Study dependence of DK/π scattering on light-quark mass
- Disentangle different $SU(3)_F$ multiplets

[See also PR D87, 014508 (2013) (1208.4535); PL B767, 465 (2017) (1610.06727); PR D98, 094018 (2018) (1712.07957); *PR D*98 014510 (2018) (1801.10122); *EPJ C79, 13 (2019)* (1811.05585); arXiv:2106.15391; PR D107, L031505 (2023)]

DK/π with $SU(3)_F$ sym

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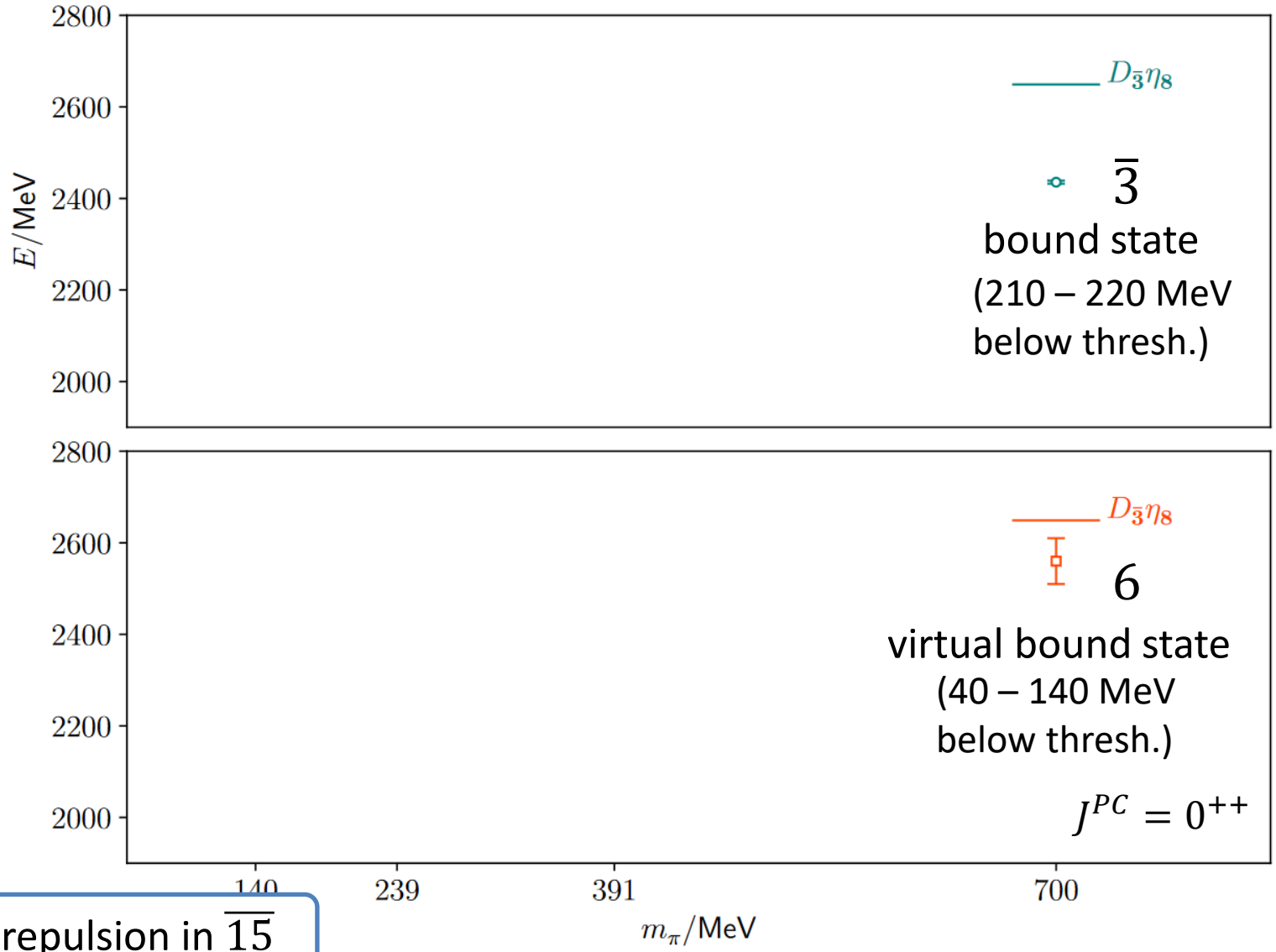
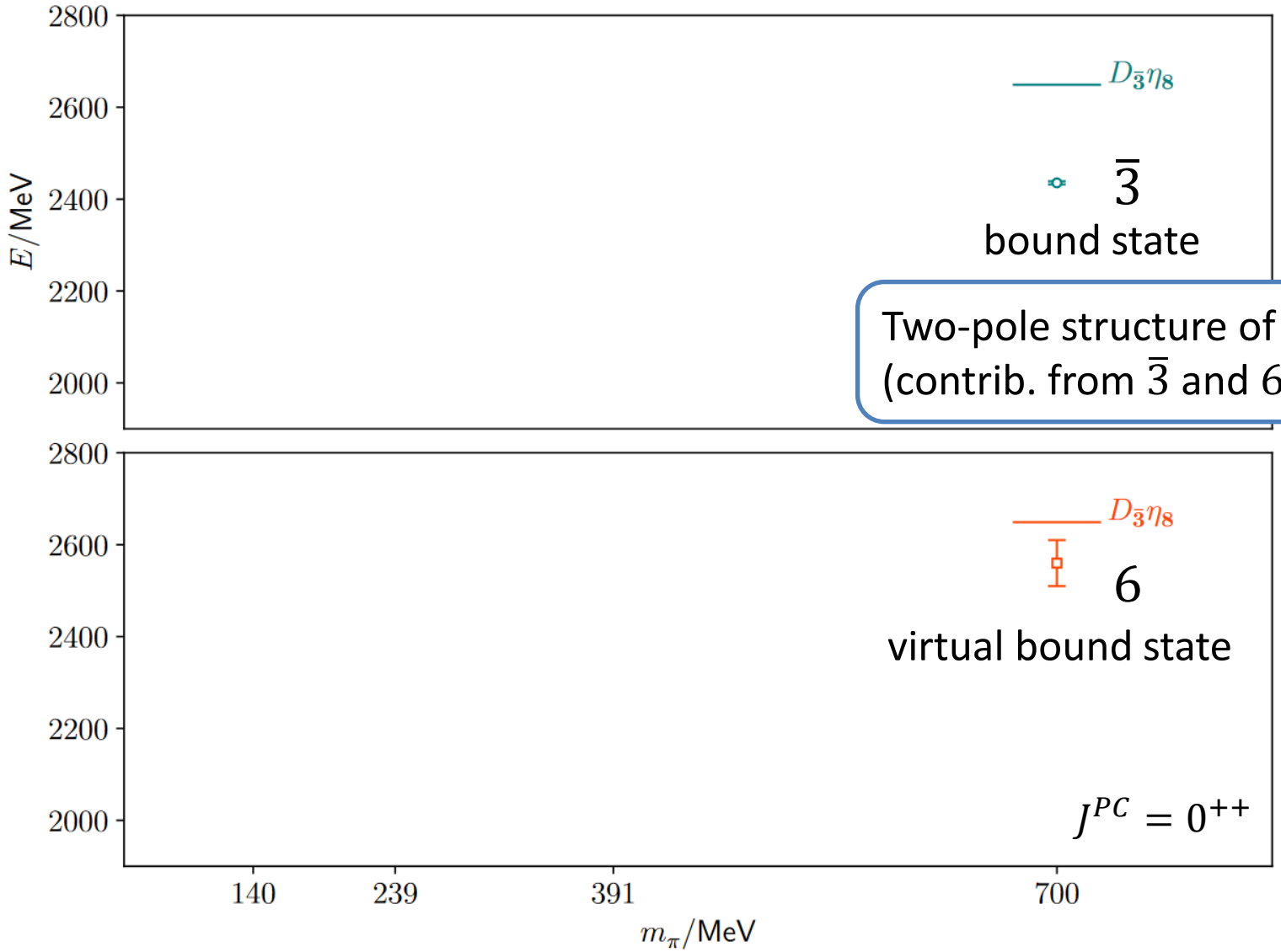


Weak repulsion in $\overline{15}$

DK/π – dependence on m_π

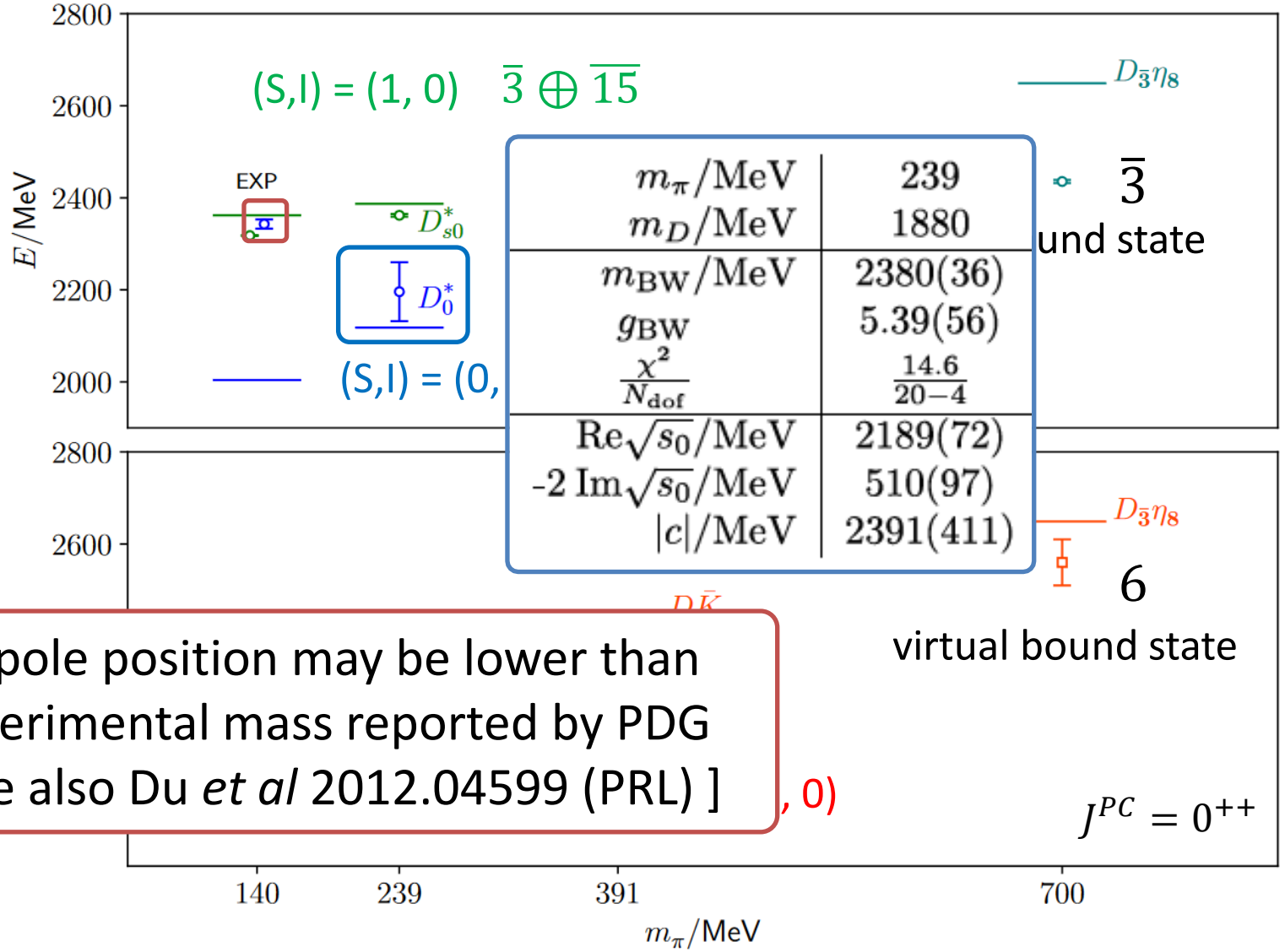
[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]



Two-pole structure of D_0^*?
(contrib. from $\bar{3}$ and 6)

DK/π – dependence on m_π

[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]

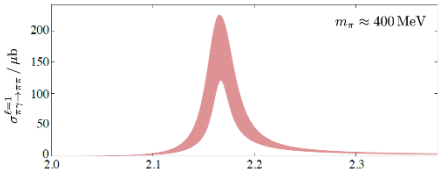


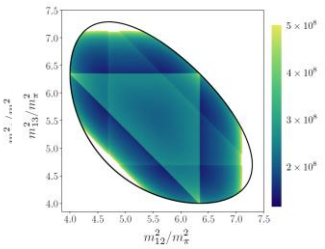
D_0^* pole position may be lower than experimental mass reported by PDG [See also Du *et al* 2012.04599 (PRL)]

See Lang and Wilson, 2205.05026 (PRL) and 2502.04232 for some recent work on axial-vector (D_1) charm mesons ($m_\pi \approx 391$ MeV).

Summary

- Significant progress in LQCD calculations of excited/exotic hadrons.
- Some recent examples:
  - T_{cc} and T'_{cc} in coupled DD^*, D^*D^* scattering.
  - Scalar (0^{++}) and tensor (2^{++}) charmonium resonances.
  - [DK/π at SU(3)$_F$ sym. point and dependence on m_π]
- Many other calcs, e.g. D_1, π_1 (exotic $J^{PC} = 1^{-+}$), b_1, light scalars.
- Outlook:
  - Other channels and vary light-quark mass.
  - Effect of left hand cut?
  - Three (or more!?) hadron scattering.
  - Probe structure, e.g. transitions and form factors.





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