Mesons, baryons and the confinement/deconfinement transition

Urko Reinosa
Centre de physique théorique
École Polytechnique, CNRS, IP Paris

The complex structure of strong interactions in Euclidean and Minkowski space
ECT*, Trento, 26-30 May, 2025

Work in collaboration with Victor Tomas Mari Surkau.

$\longrightarrow$ arXiv:2504.06459

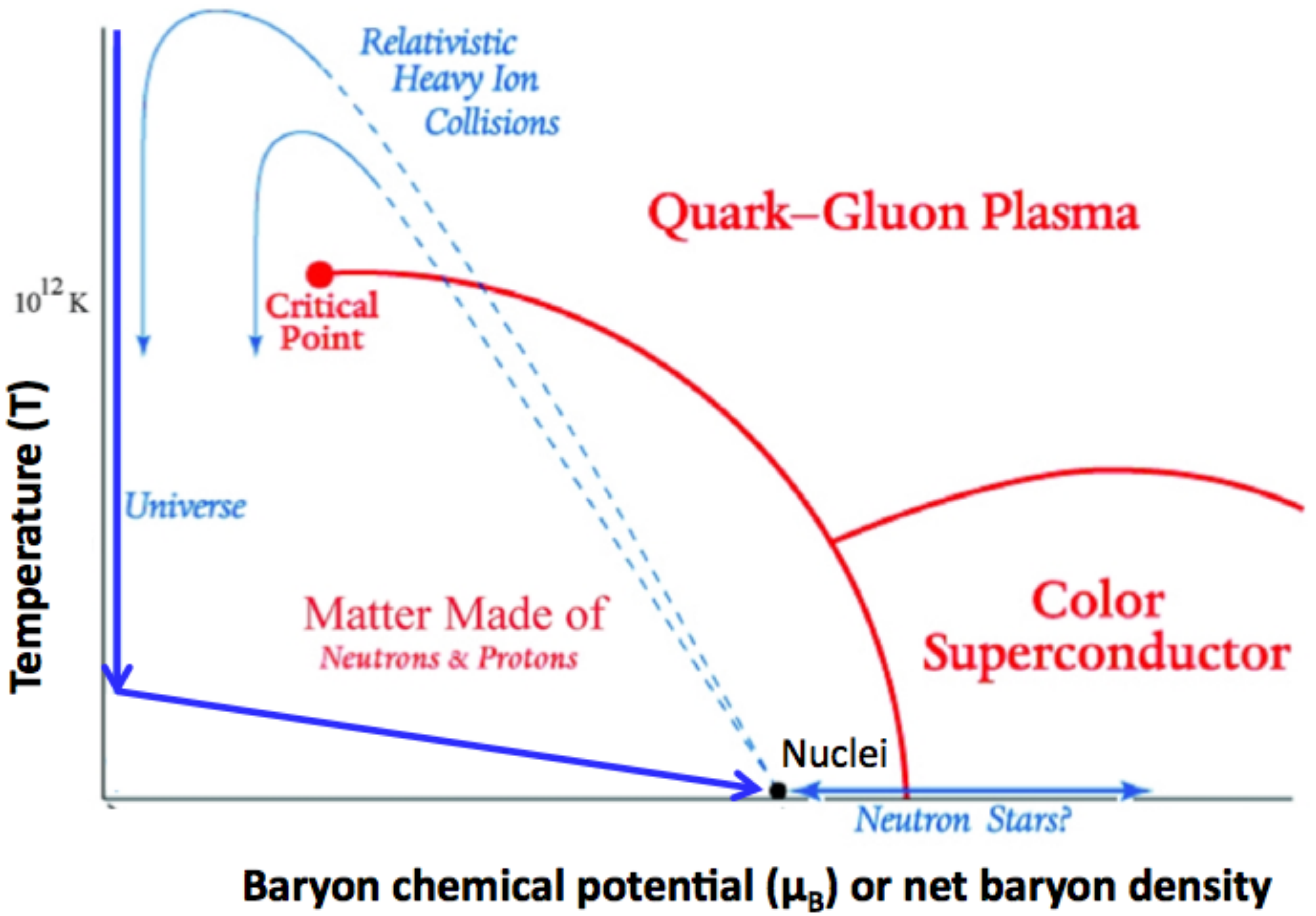
Connection between the Polyakov loop, used to distinguish (qualitatively) between confined and deconfined phases of strongly interacting matter ...

... and the (quantitative) nature of the relevant degrees of freedom in each of these phases.

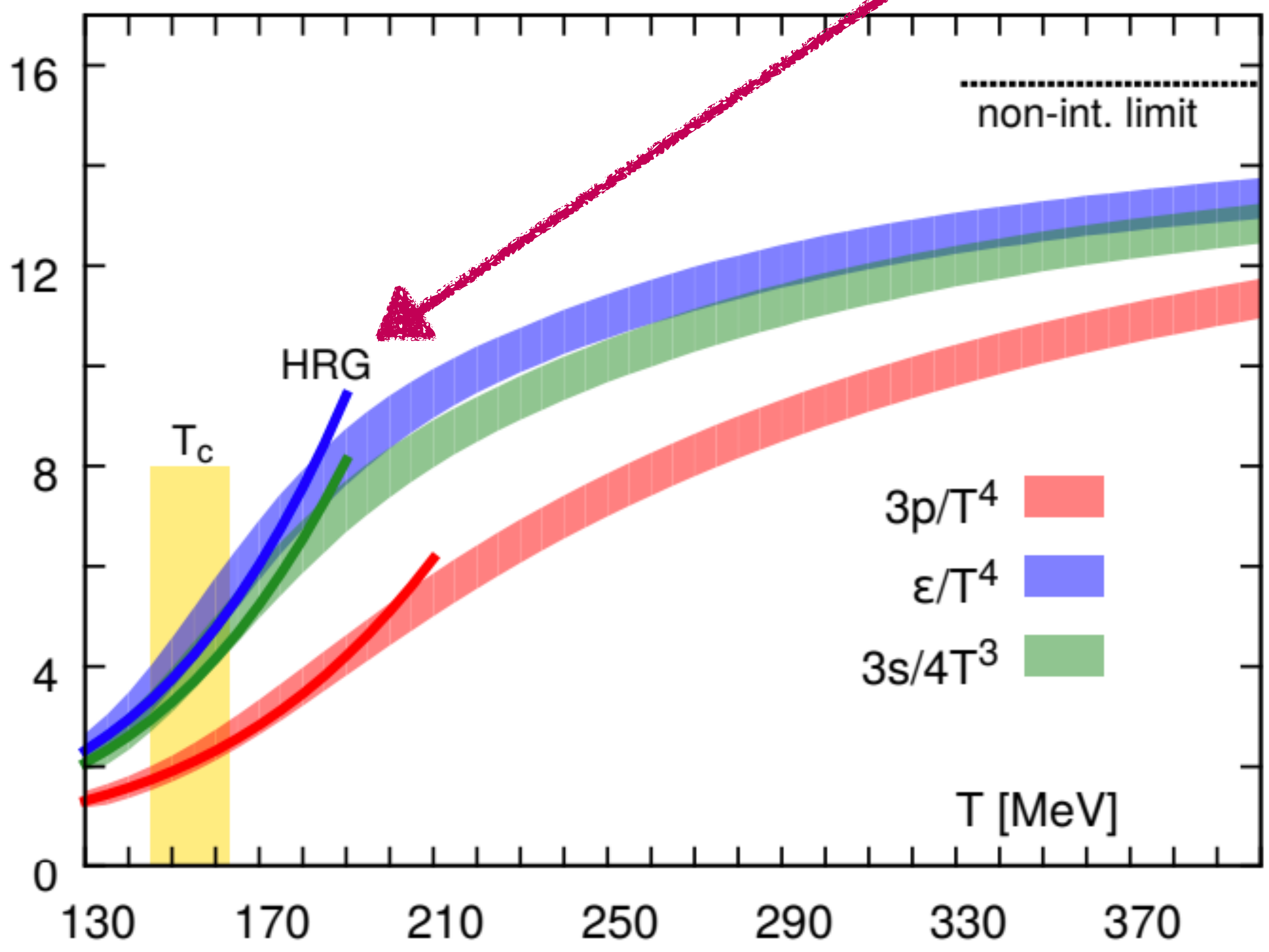
Purely Euclidean (sorry).

But some $\mathbb{C}$-ity will be built in since I will be discussing how the Polyakov depends on the quark chemical potential (complex functional integral).

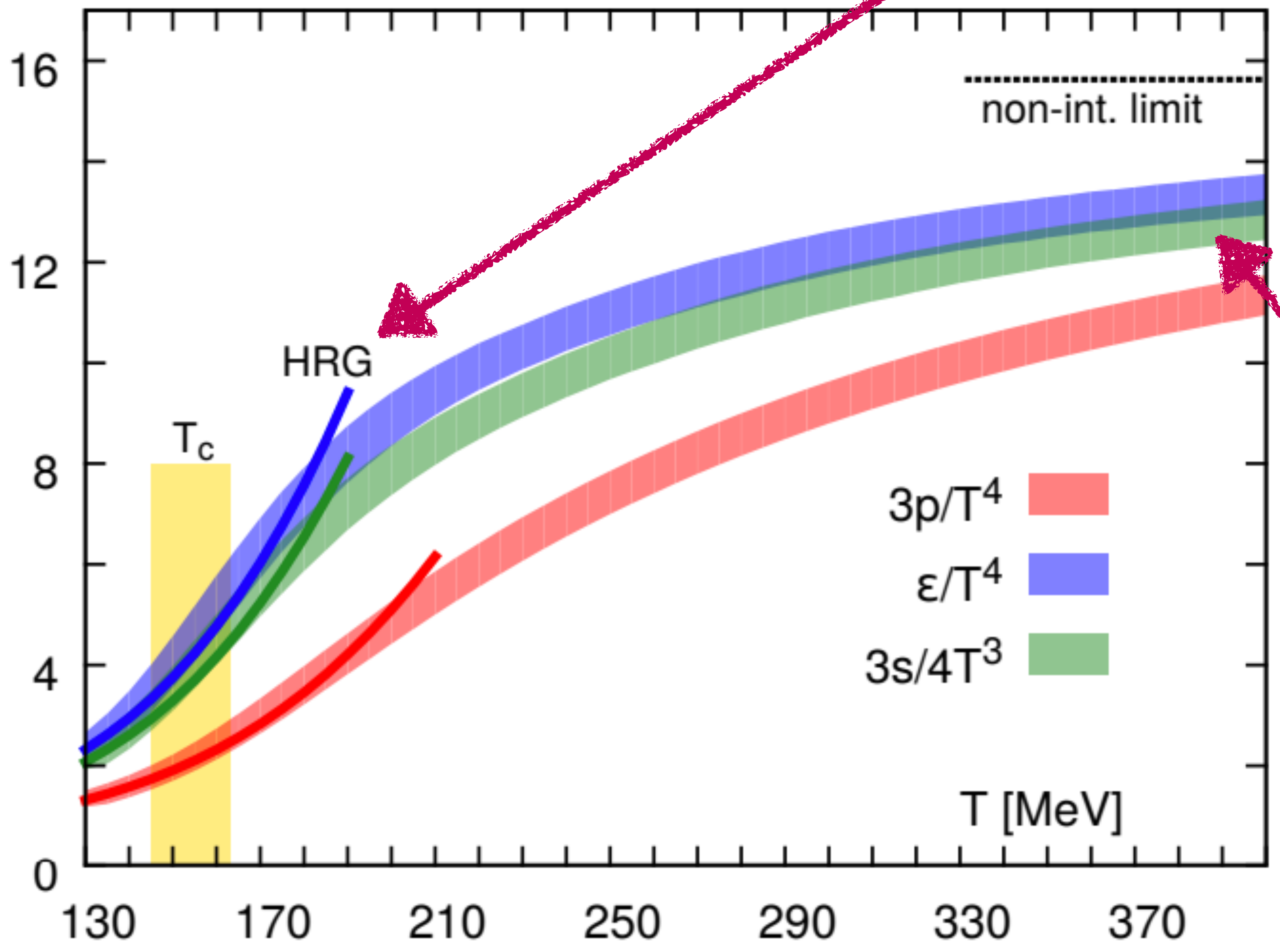
It is well accepted that QCD admits different thermodynamical phases depending on the value of temperature, chemical potential, ...



Evidence from lattice simulations that the low temperature regime is dominated by hadronic degrees of freedom.

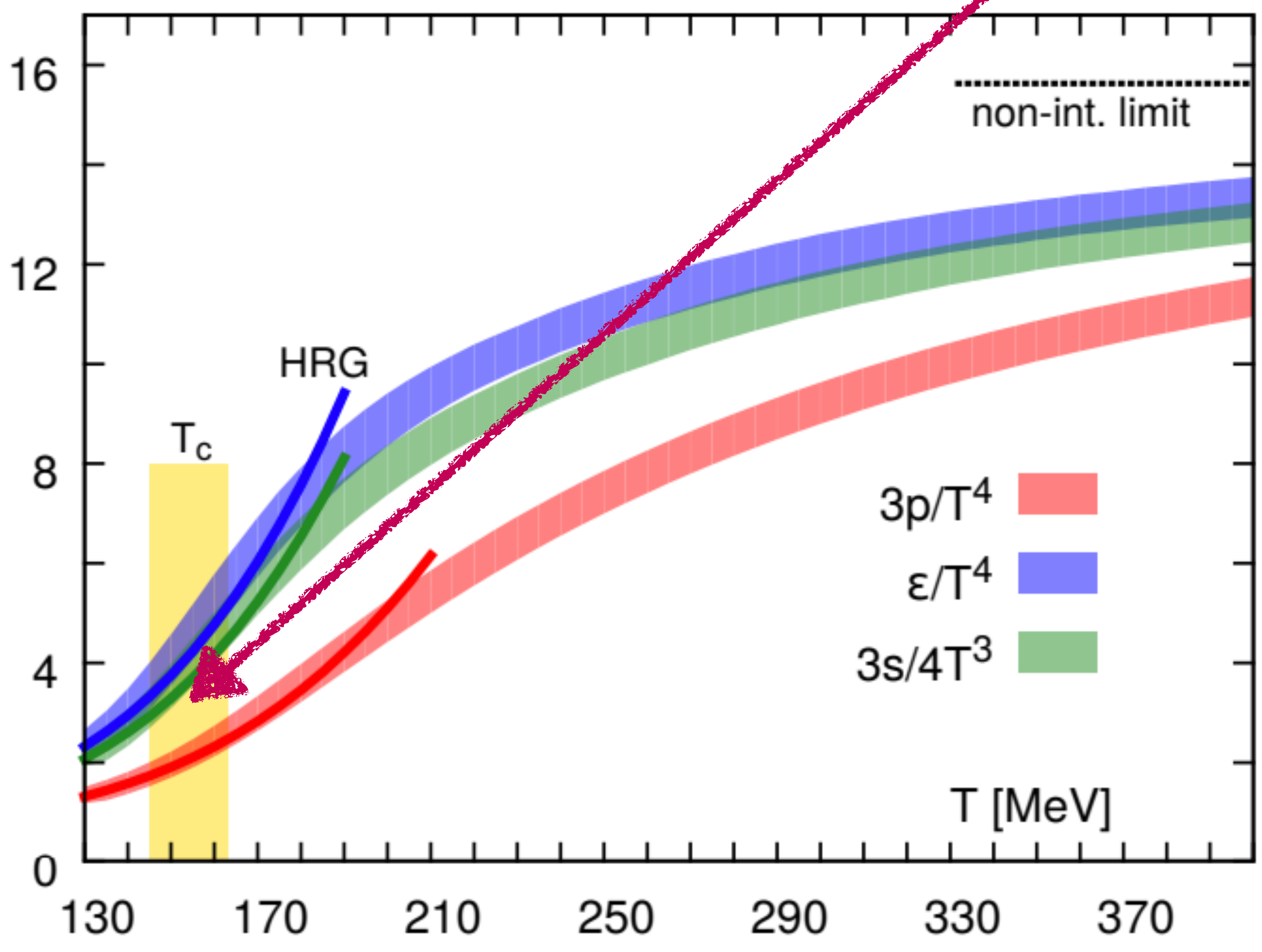


Evidence from lattice simulations that the low temperature regime is dominated by hadronic degrees of freedom.

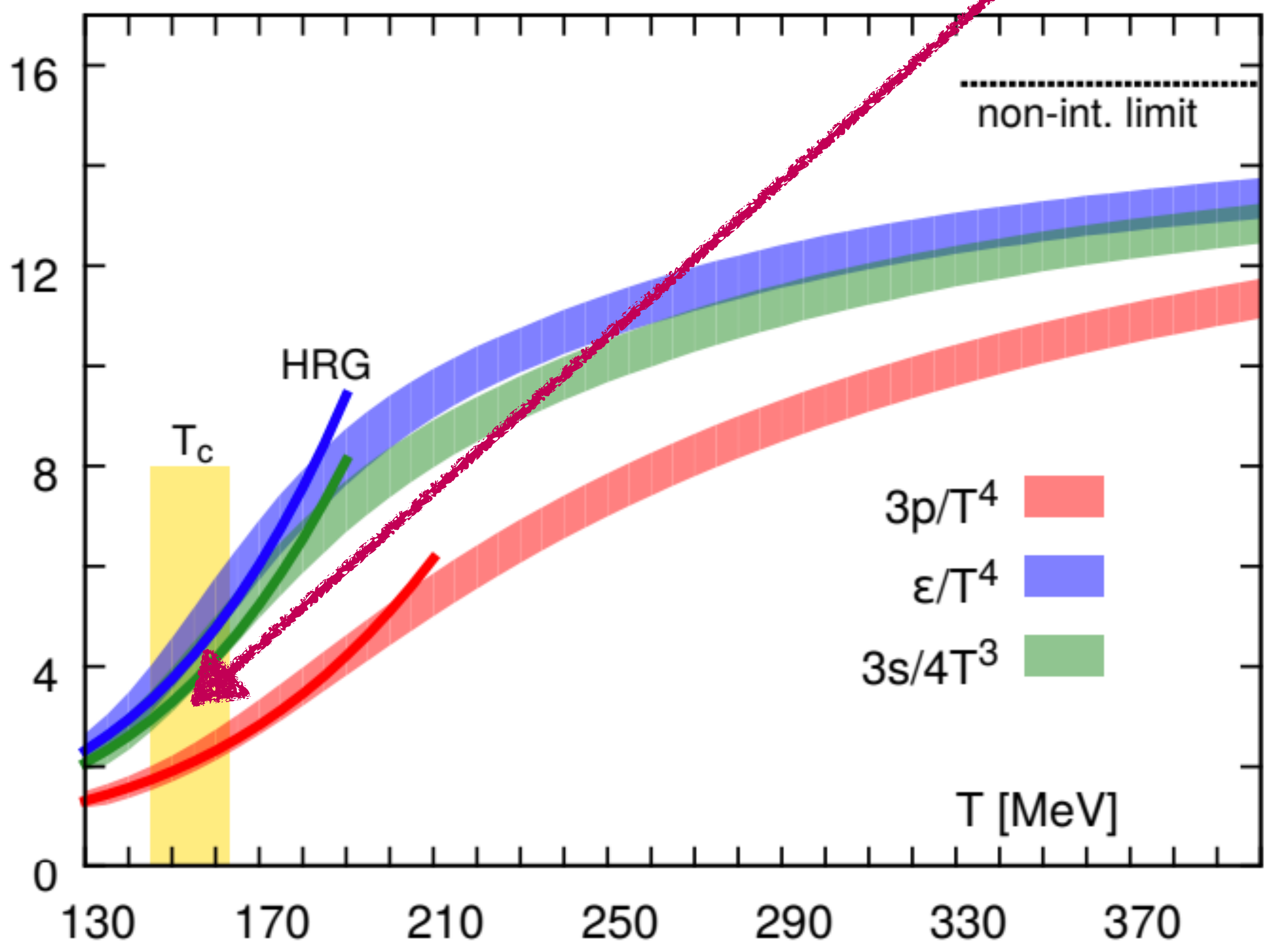


On the contrary, owing to asymptotic freedom, the relevant degrees of freedom in the high temperature regime are quarks and gluons.

The two regimes are connected by a smooth crossover.

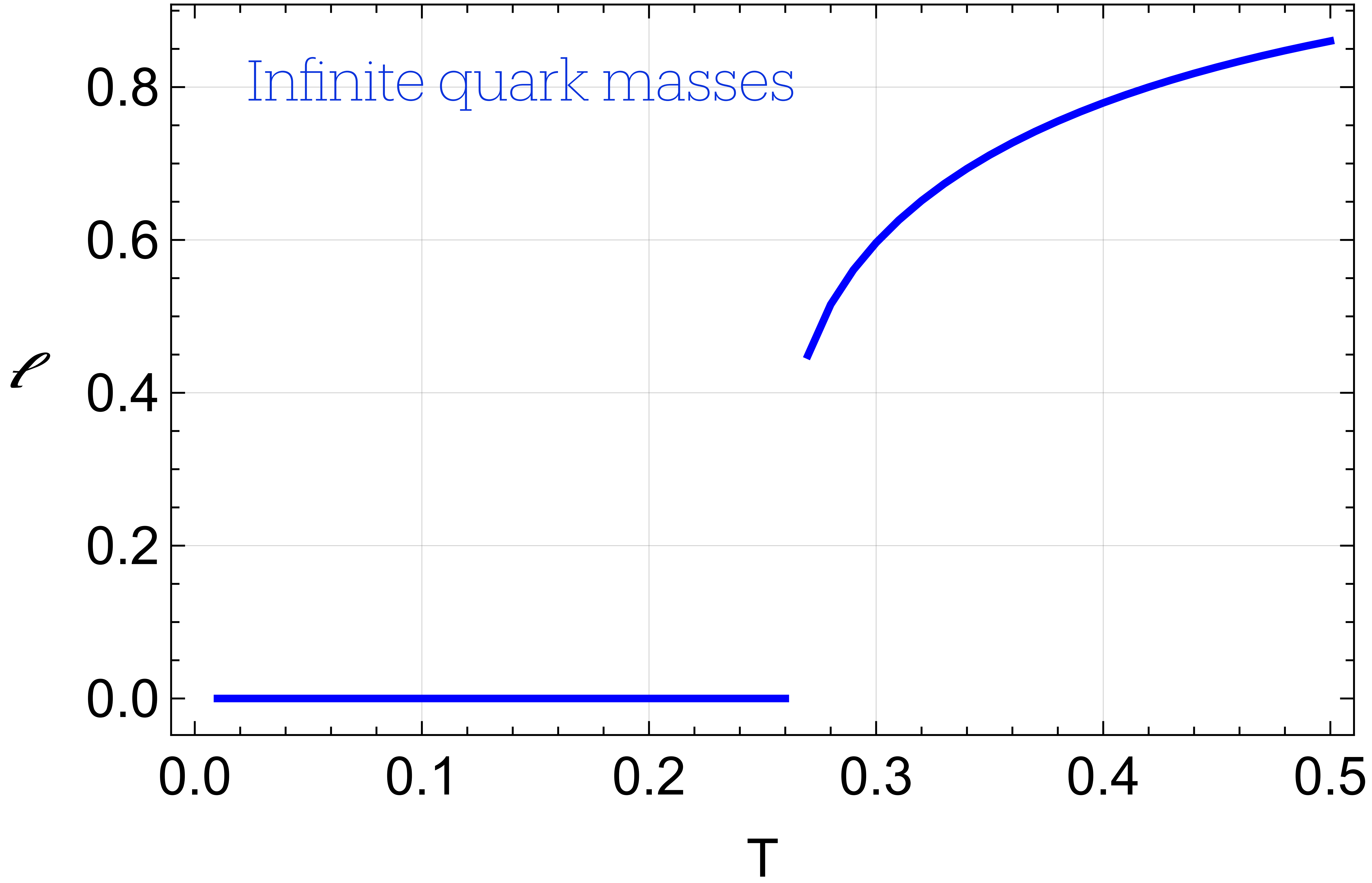


The two regimes are connected by a smooth crossover.

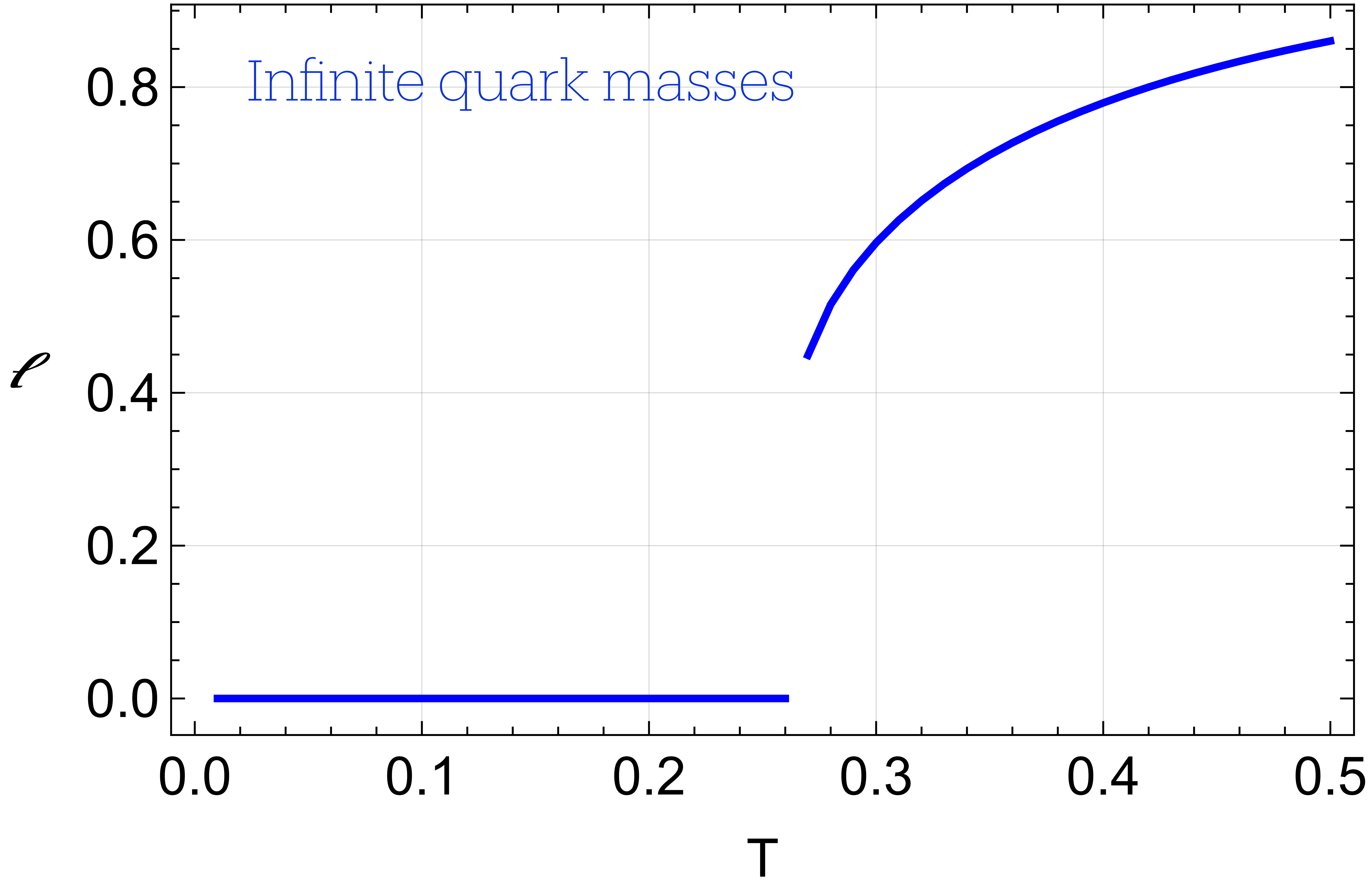


Becomes a sharp transition in the formal limit of infinitely heavy quarks, a.k.a. pure Yang-Mills theory.

In this case, the transition can be understood in terms of the breaking of a symmetry, known as center symmetry, probed by means of an order parameter, the Polyakov loop ℓ.



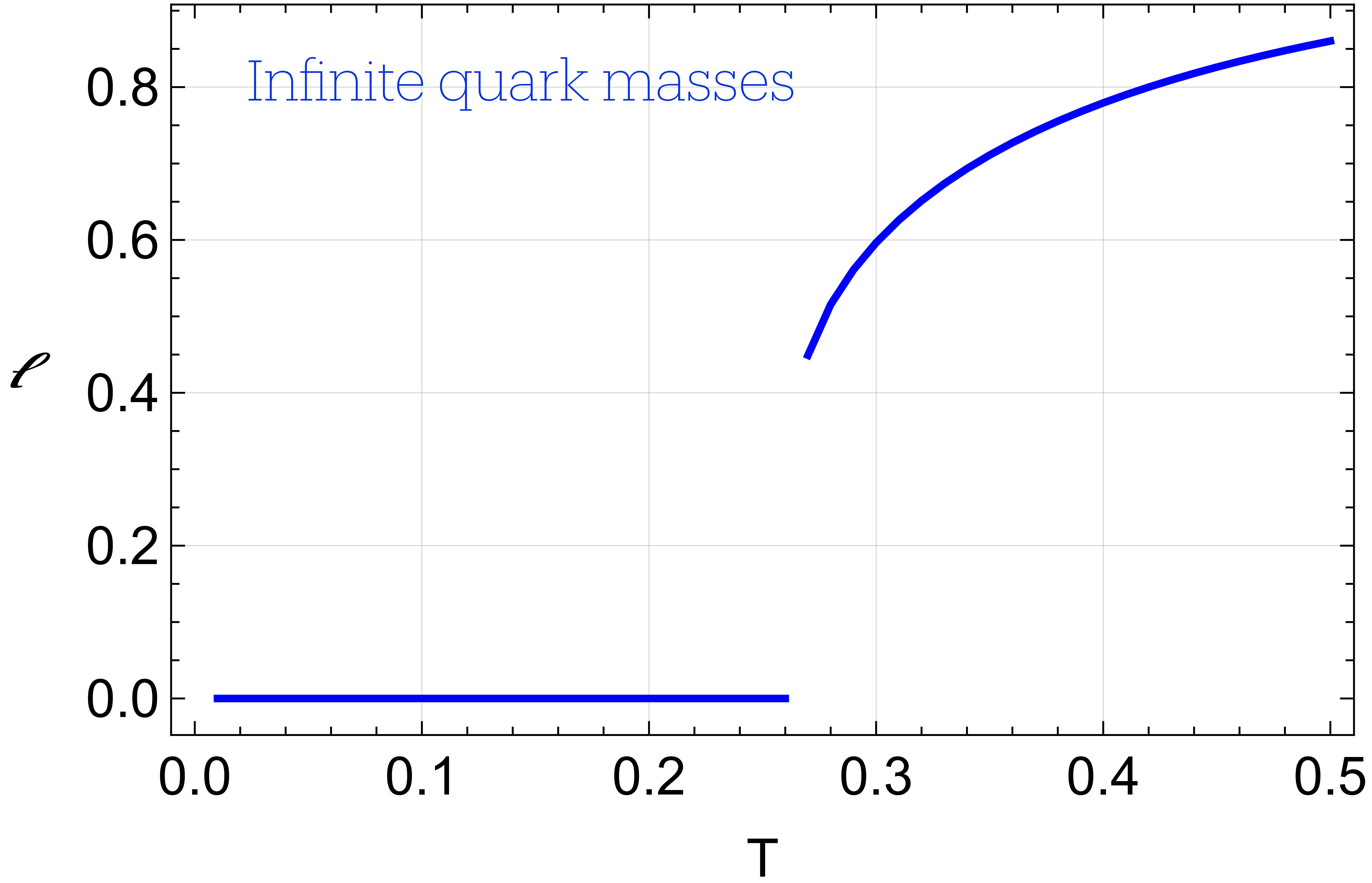
In this case, the transition can be understood in terms of the breaking of a symmetry, known as center symmetry, probed by means of an order parameter, the Polyakov loop ℓ.



Related to the energy cost ΔF_q for bringing a quark probe q into the medium:

$$\ell = \exp\left(-\frac{\Delta F_q}{T}\right)$$

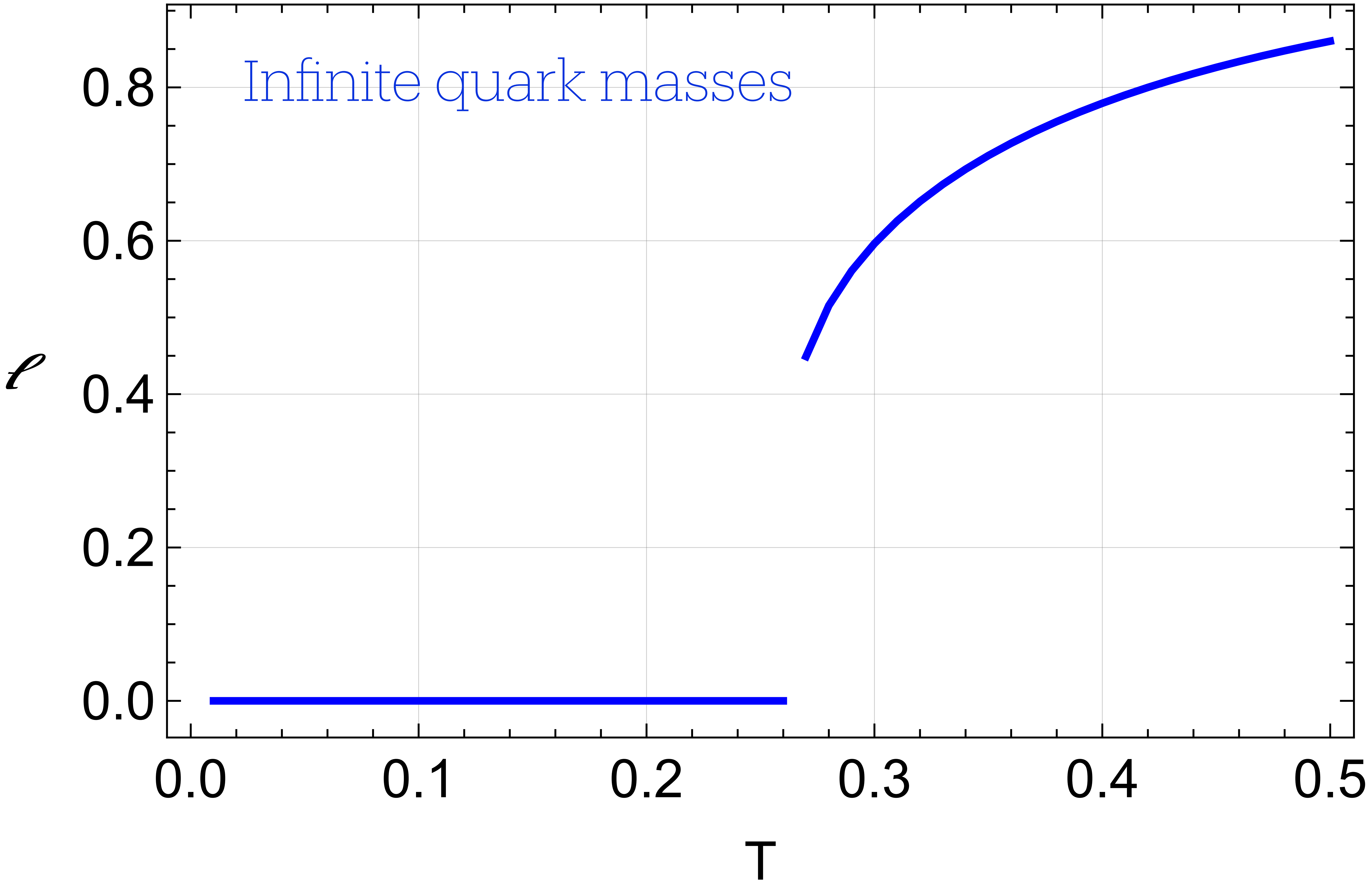
In this case, the transition can be understood in terms of the breaking of a symmetry, known as center symmetry, probed by means of an order parameter, the Polyakov loop ℓ.



Related to the energy cost ΔF_q for bringing a quark probe q into the medium:

$$\ell = \exp\left(-\frac{\Delta F_q}{T}\right)$$

In this case, the transition can be understood in terms of the breaking of a symmetry, known as center symmetry, probed by means of an order parameter, the Polyakov loop ℓ.



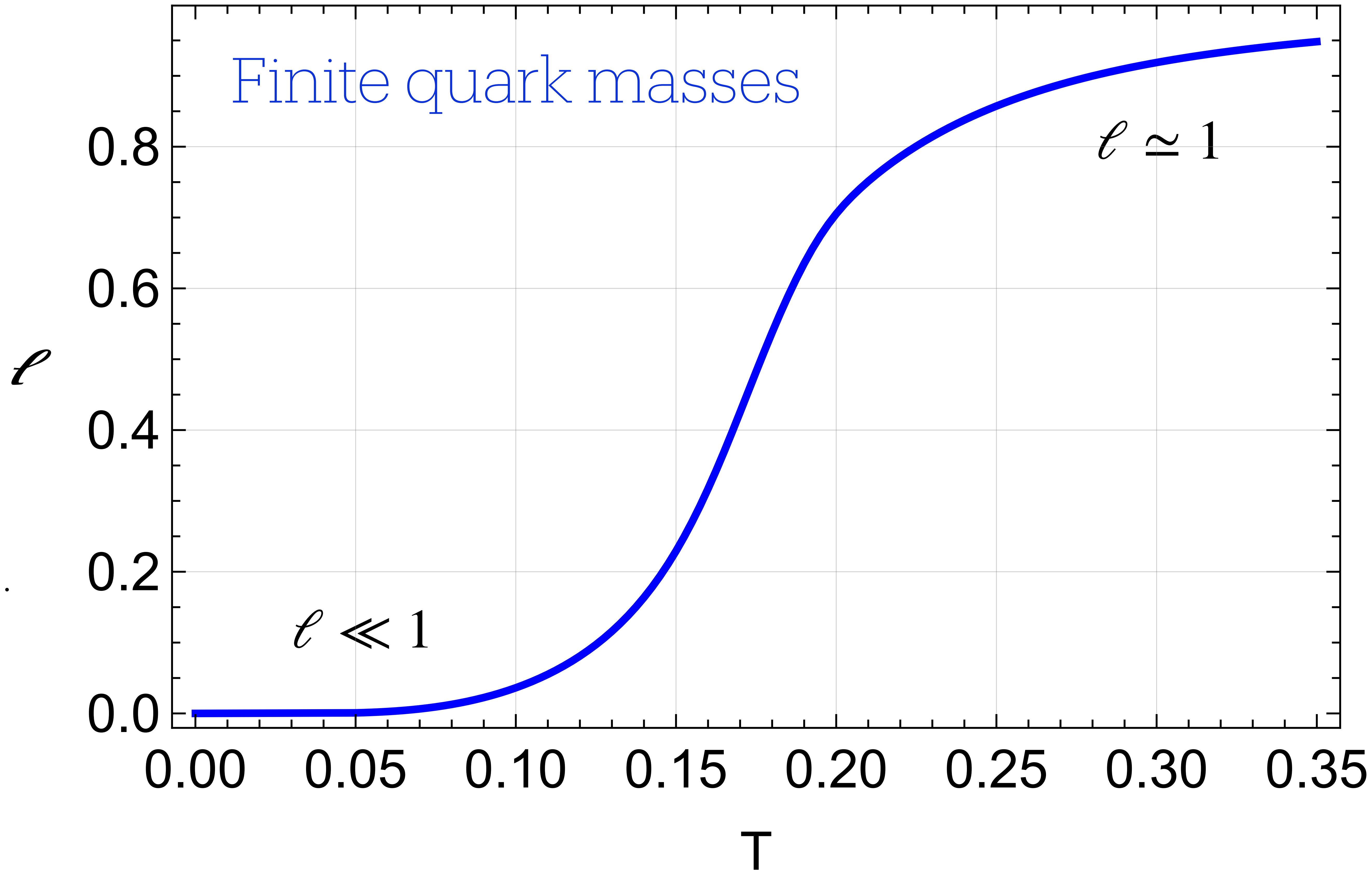
Related to the energy cost ΔF_q for bringing a quark probe q into the medium:

$$\ell = \exp\left(-\frac{\Delta F_q}{T}\right)$$

In QCD, the Polyakov is not a strict order parameter as it is never exactly equal to $\mathbf{0}$.

But it can still be used to distinguish between two phases characterized by either $\ell \ll 1$ or $\ell \simeq 1$

This leads to a little paradox though ...



If the Polyakov loop is never 0, this means that, in principle, a quark can always be added to the medium, even in the low temperature phase.

If the Polyakov loop is never 0, this means that, in principle, a quark can always be added to the medium, even in the low temperature phase.

How is this compatible with a picture where the low temperature phase is made of hadrons?

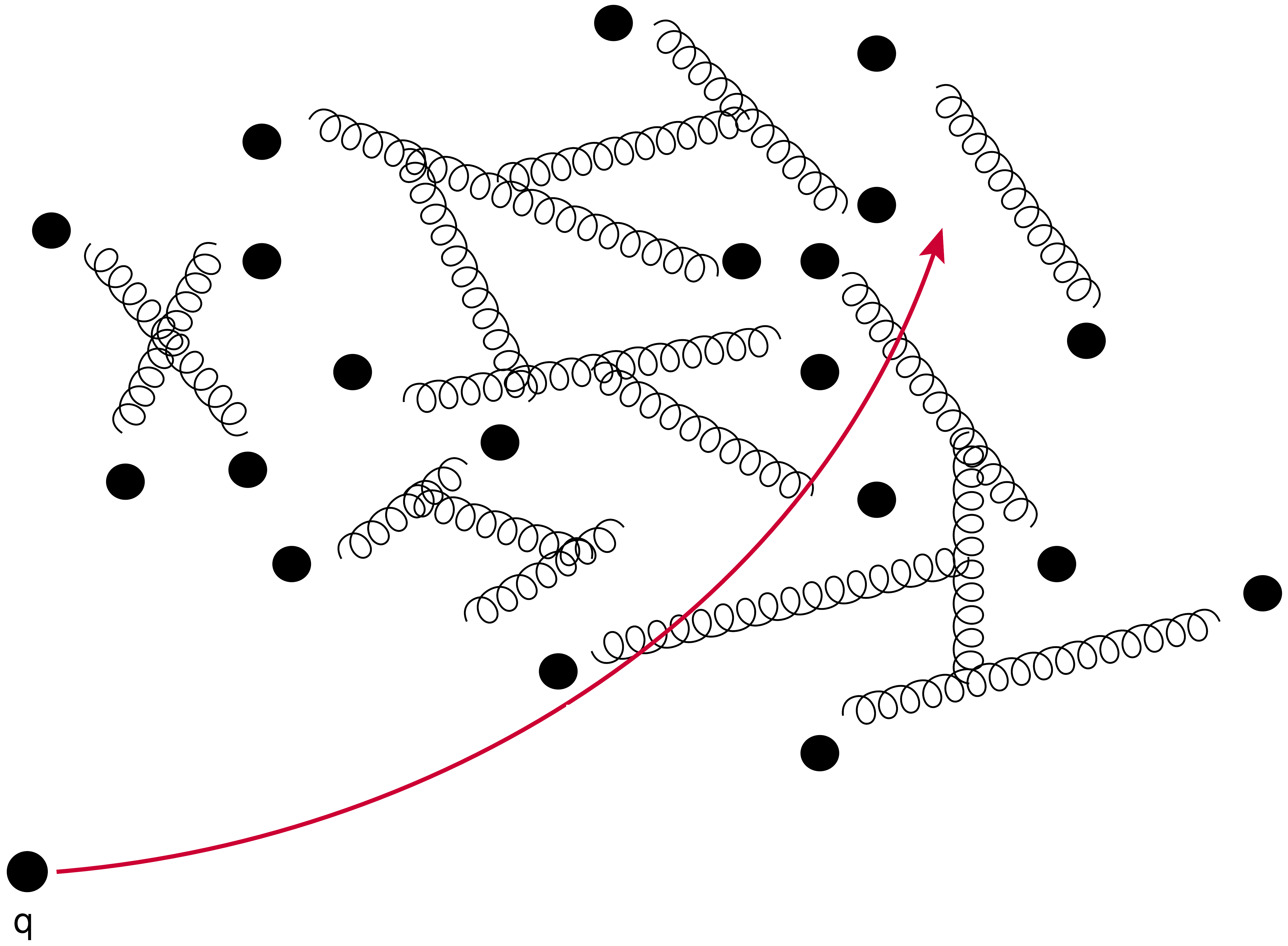
If the Polyakov loop is never 0, this means that, in principle, a quark can always be added to the medium, even in the low temperature phase.

How is this compatible with a picture where the low temperature phase is made of hadrons?

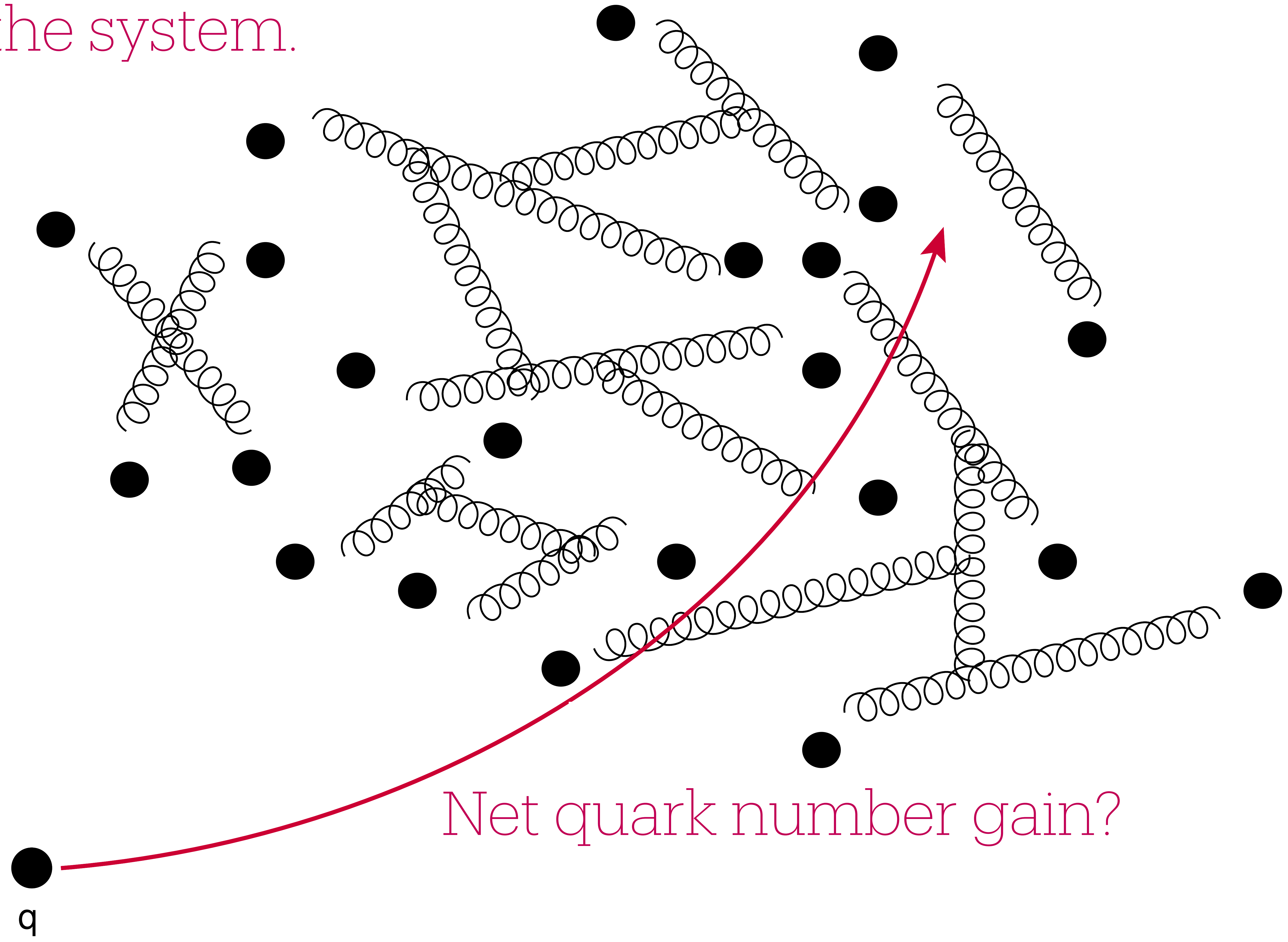
This talks aims at answering this riddle, or, at least contributing to a possible answer.

To that aim, I will consider the following thought experiment.

To that aim, I will consider the following thought experiment. After bringing a quark into the medium,



To that aim, I will consider the following thought experiment. After bringing a quark into the medium, I will measure (or evaluate), the (net) quark number gained by the system.



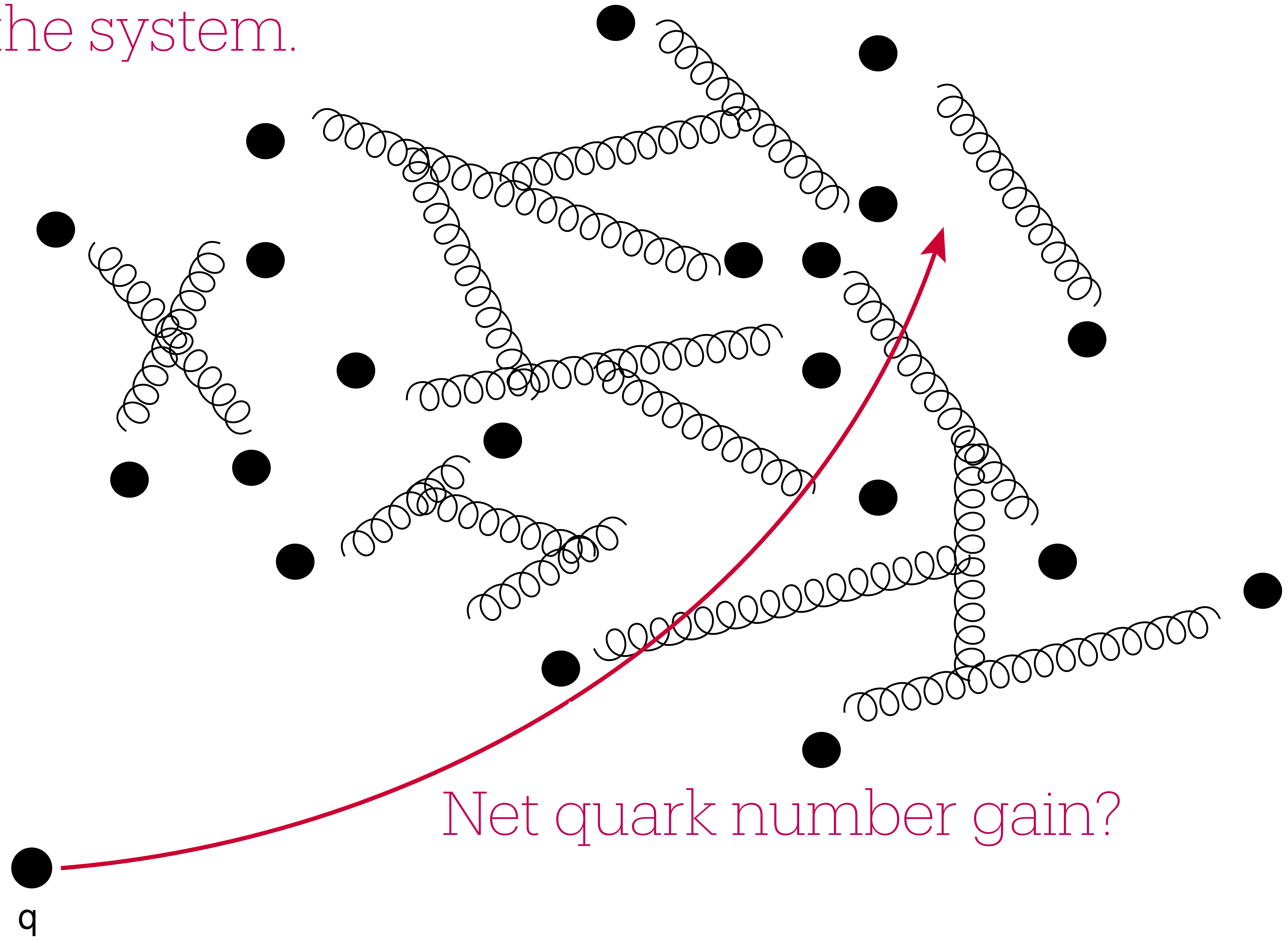
To that aim, I will consider the following thought experiment. After bringing a quark into the medium, I will measure (or evaluate), the (net) quark number gained by the system.

This gain writes:

$$1 + \Delta Q_q$$

Quark number of the quark probe

(Net) quark number response of the medium



In the high temperature, deconfined phase, it is usually found that the response of the medium is $\Delta Q_q = 0$ and thus $1 + \Delta Q_q = 1$.

In the high temperature, deconfined phase, it is usually found that the response of the medium is $\Delta Q_q = 0$ and thus $1 + \Delta Q_q = 1$.

On the contrary, in the low temperature, confined phase,
I will argue that the response of the medium can be :

- either $\Delta Q_q = -1$ corresponding to $1 + \Delta Q_q = 0$, interpreted as the quark probe being screened into a meson-like state;

- or $\Delta Q_q = 2$ corresponding to $1 + \Delta Q_q = 3$, interpreted as the quark probe being screened into a baryon-like state.

I say meson-like or baryon-like because the best I will be able to conclude from this calculation is that the states that are formed have the same net quark number as actual mesons or baryons.

I say meson-like or baryon-like because the best I will be able to conclude from this calculation is that the states that are formed have the same net quark number as actual mesons or baryons.

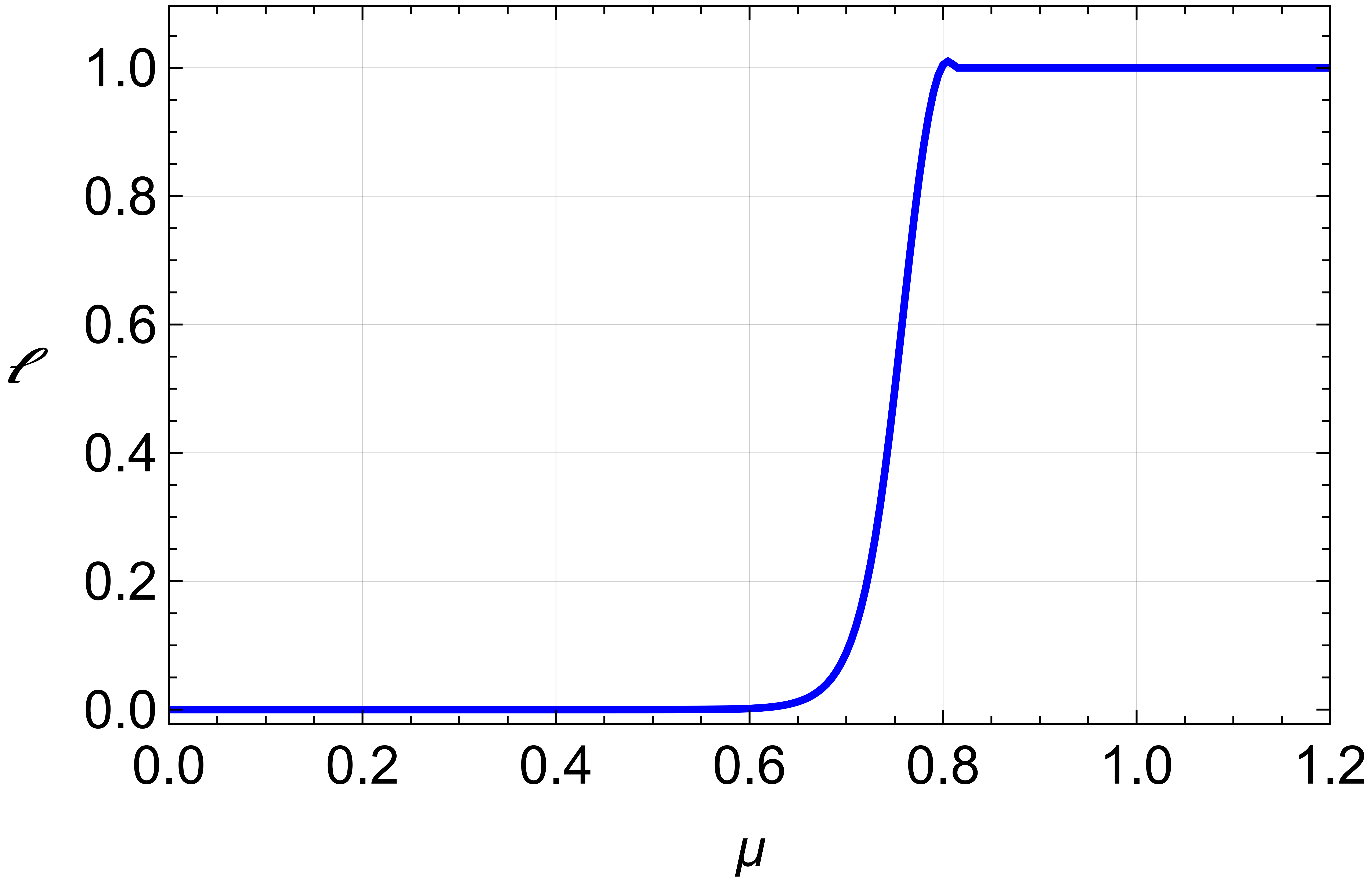
A refinement would involve computing the color charge, or better the Casimir operator (color charge squared) on these states but this is beyond the scope of this talk [work in progress].

I say meson-like or baryon-like because the best I will be able to conclude from this calculation is that the states that are formed have the same net quark number as actual mesons or baryons.

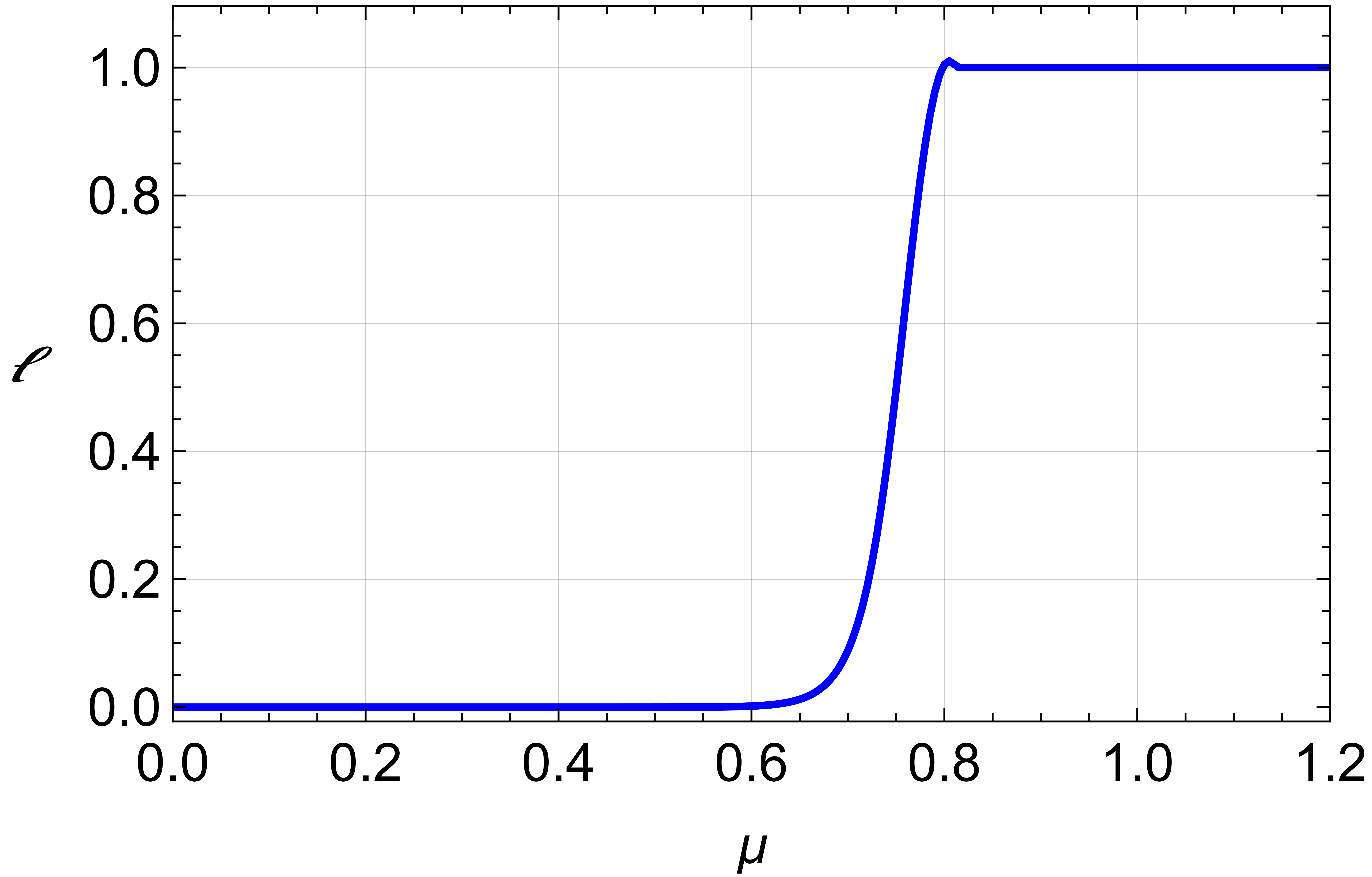
A refinement would involve computing the color charge, or better the Casimir operator (color charge squared) on these states but this is beyond the scope of this talk [work in progress].

The net quark number gain will be extracted from the Polyakov loop. Arguments will rely on very few, model-independent ingredients.

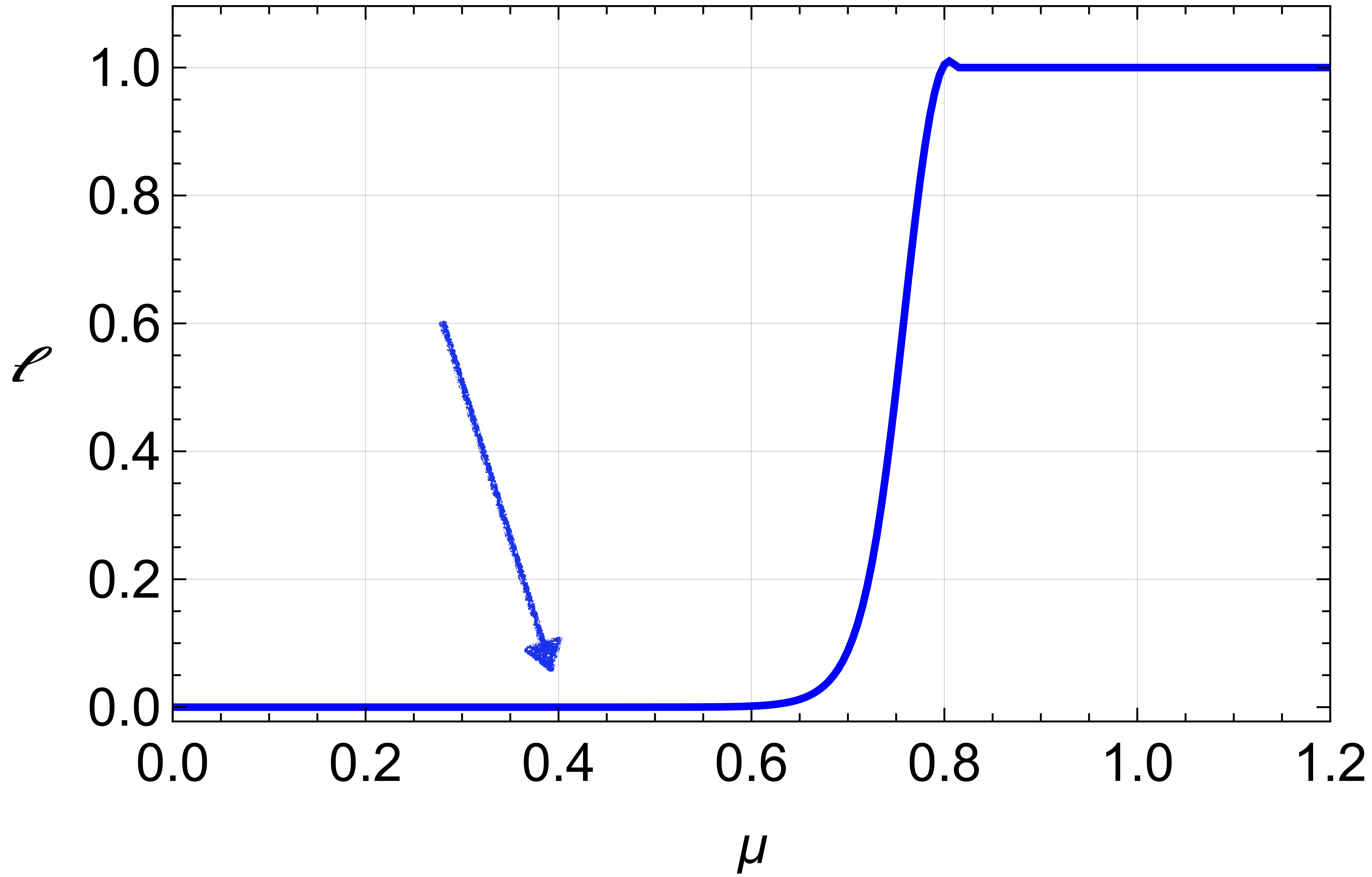
How come that this information can be extracted from the Polyakov loop?



How come that this information can be extracted from the Polyakov loop?

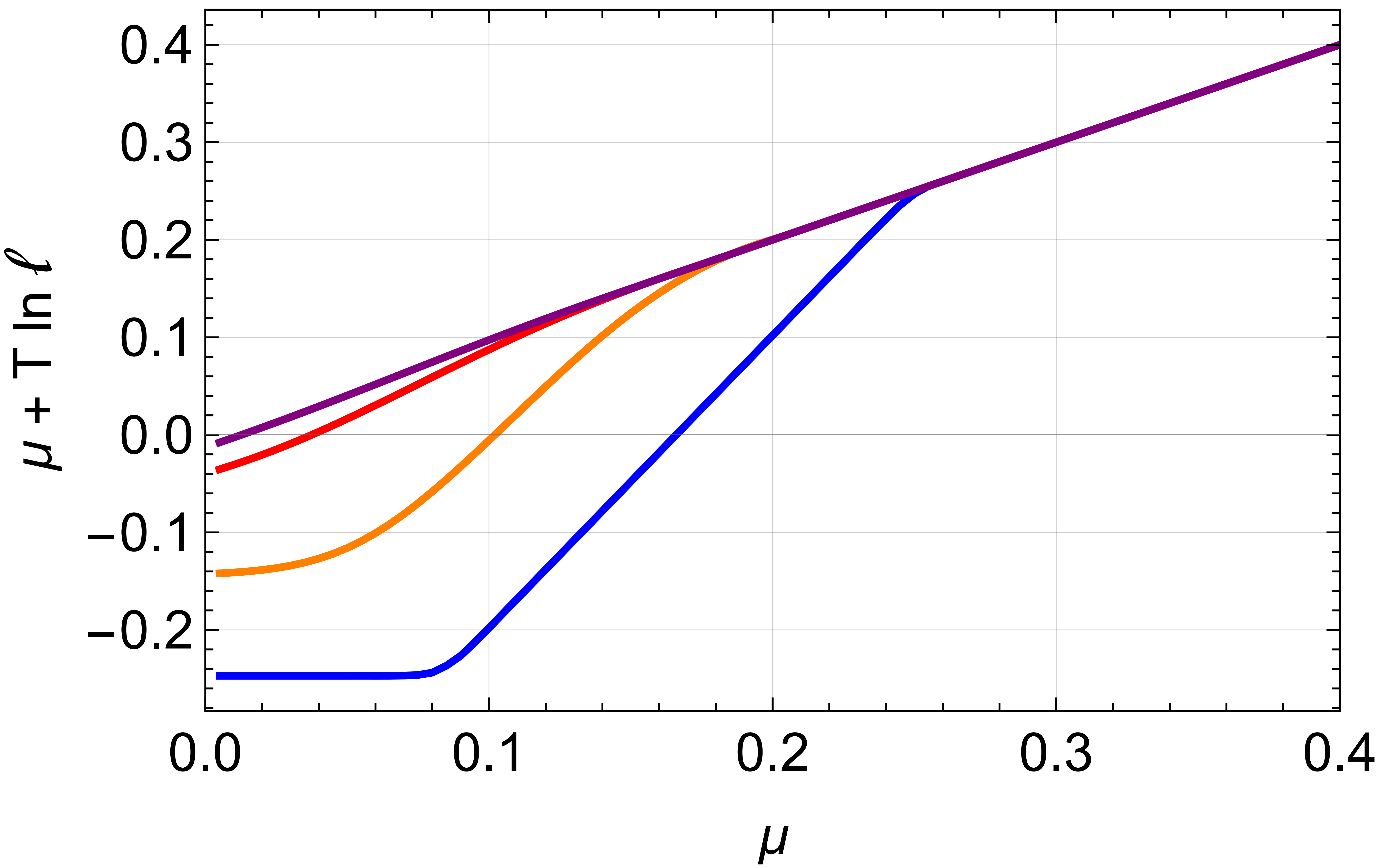


How come that this information can be extracted from the Polyakov loop?

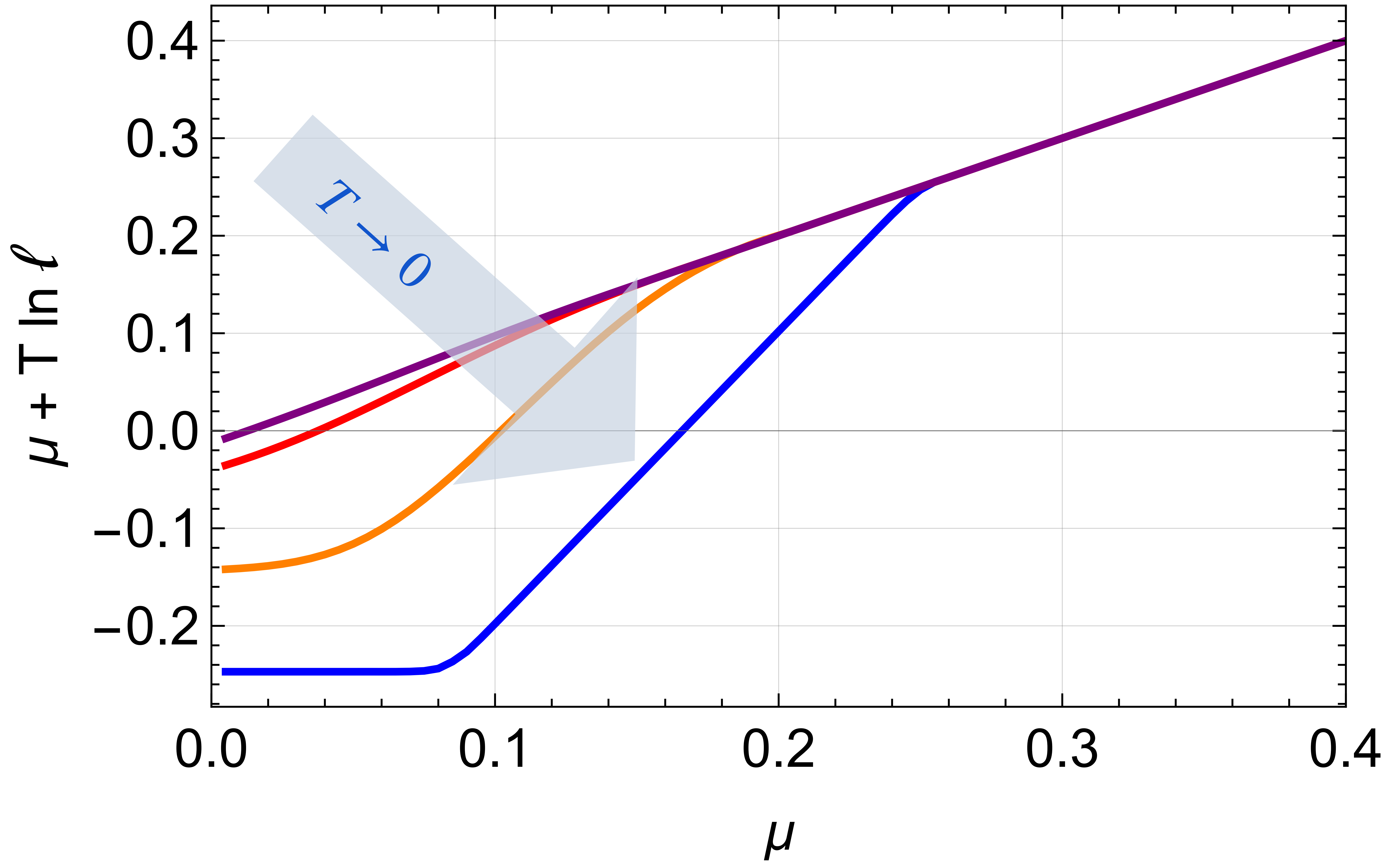


To give more weight to this region, take the logarithm: $\mu + T \ln \ell$

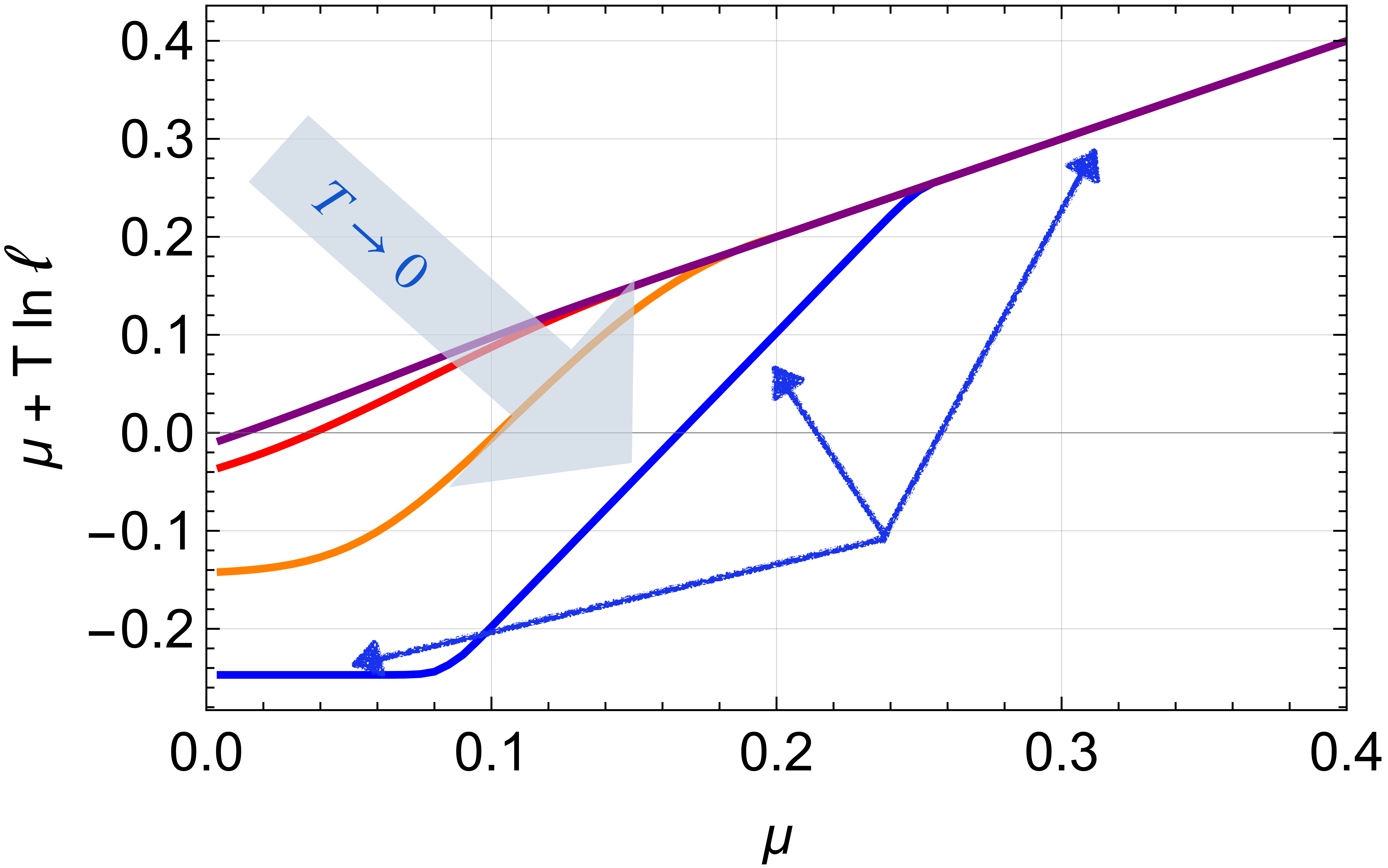
To give more weight to this region, take the logarithm: $\mu + T \ln \ell$



To give more weight to this region, take the logarithm: $\mu + T \ln \ell$

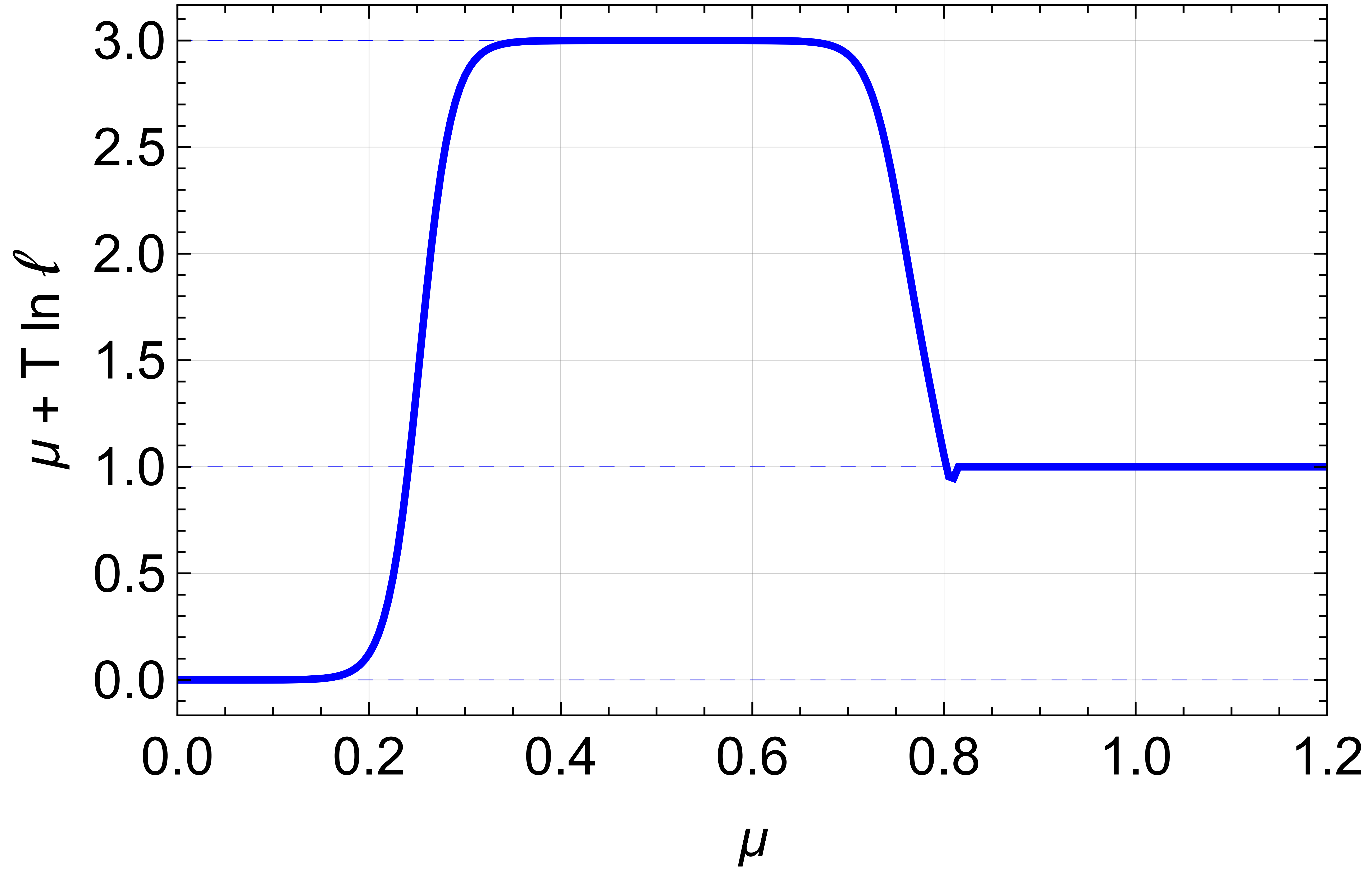


To give more weight to this region, take the logarithm: $\mu + T \ln \ell$

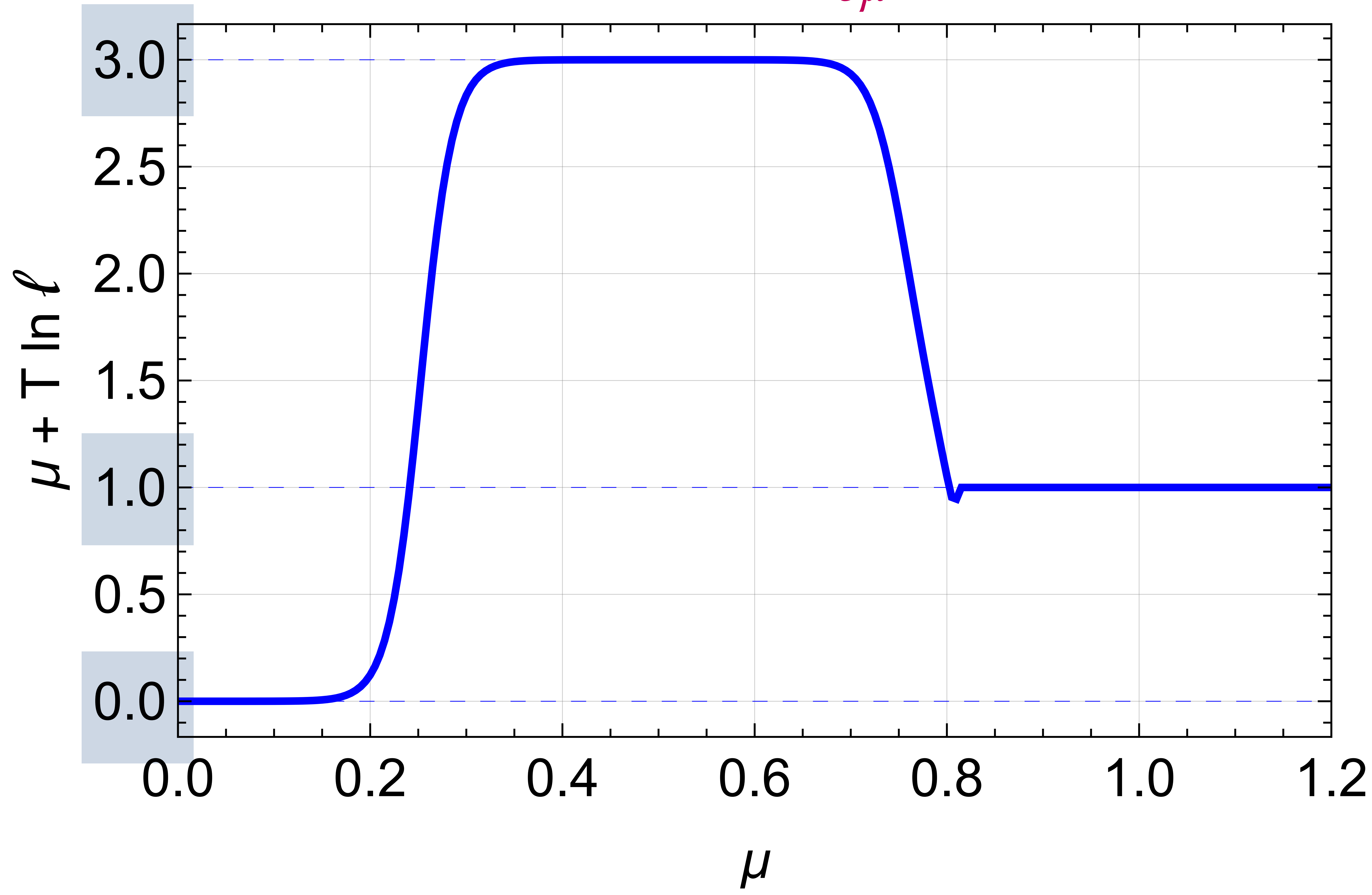


To access the slopes take a $\boldsymbol{\mu}$-derivative: $\dfrac{\partial}{\partial \mu}(\mu + T \ln \ell)$

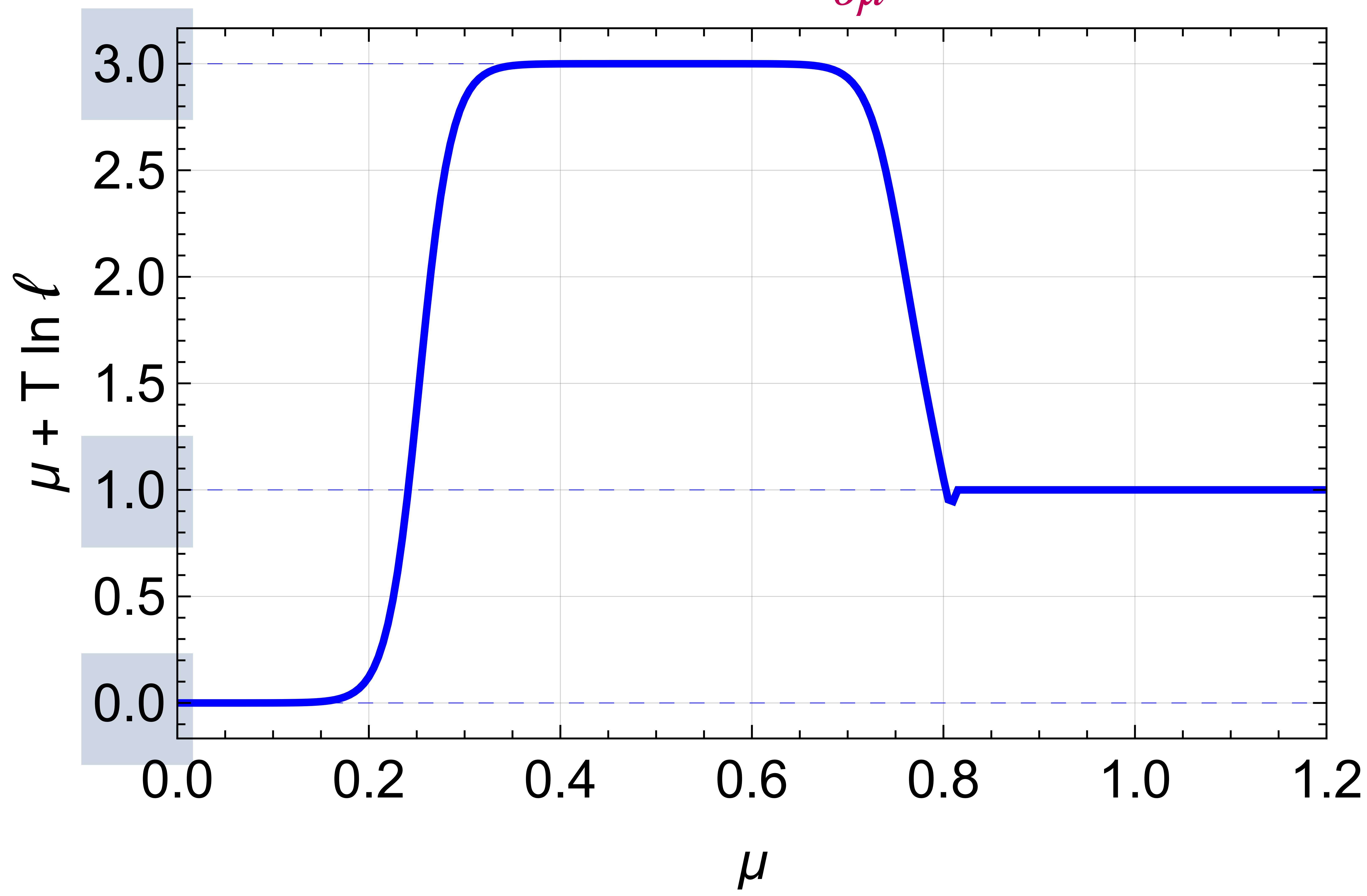
To access the slopes take a $\boldsymbol{\mu}$-derivative: $\dfrac{\partial}{\partial \mu}(\mu + T \ln \ell)$



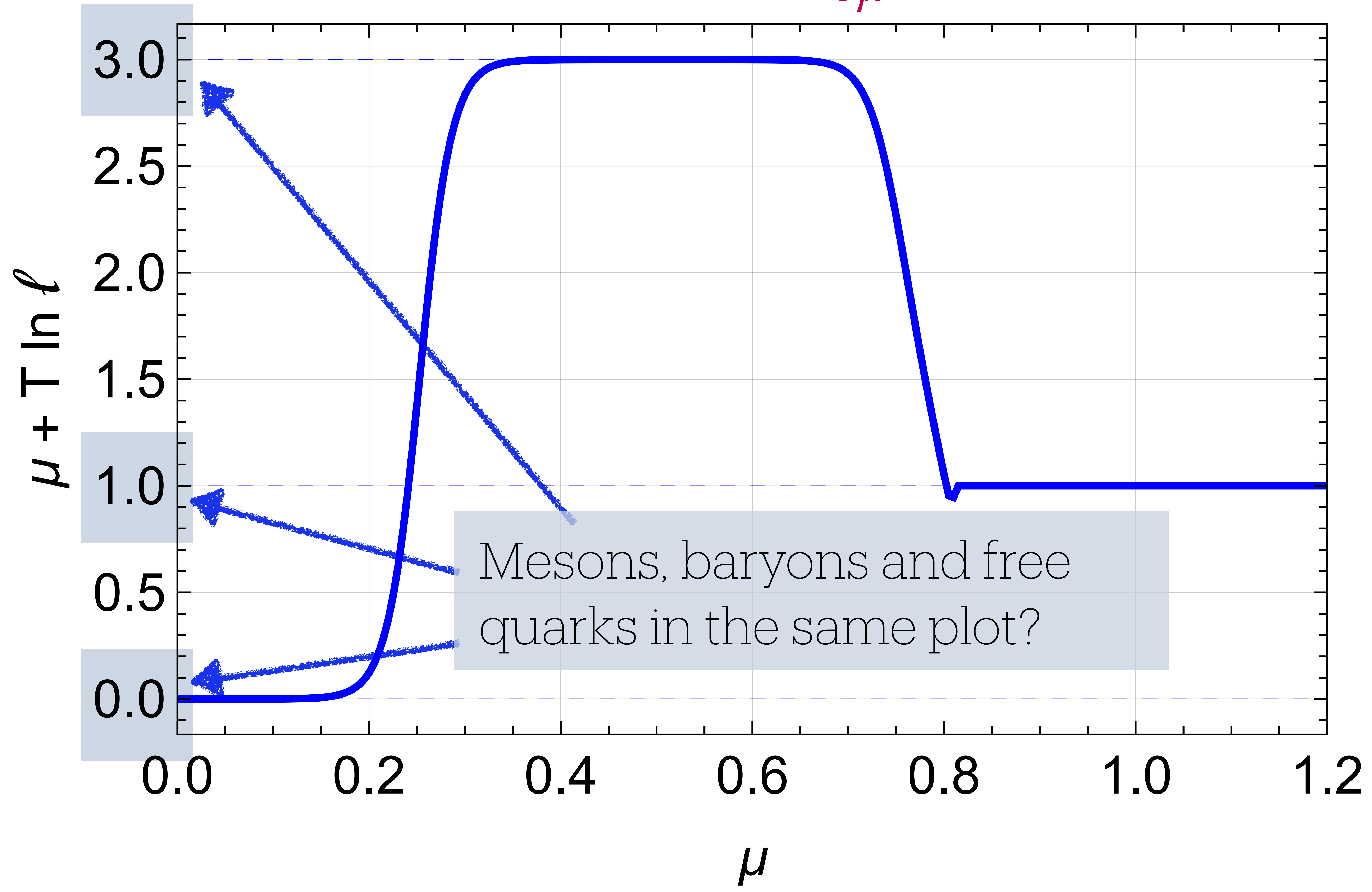
To access the slopes take a $\boldsymbol{\mu}$-derivative: $\dfrac{\partial}{\partial\mu}(\mu + T \ln \ell)$



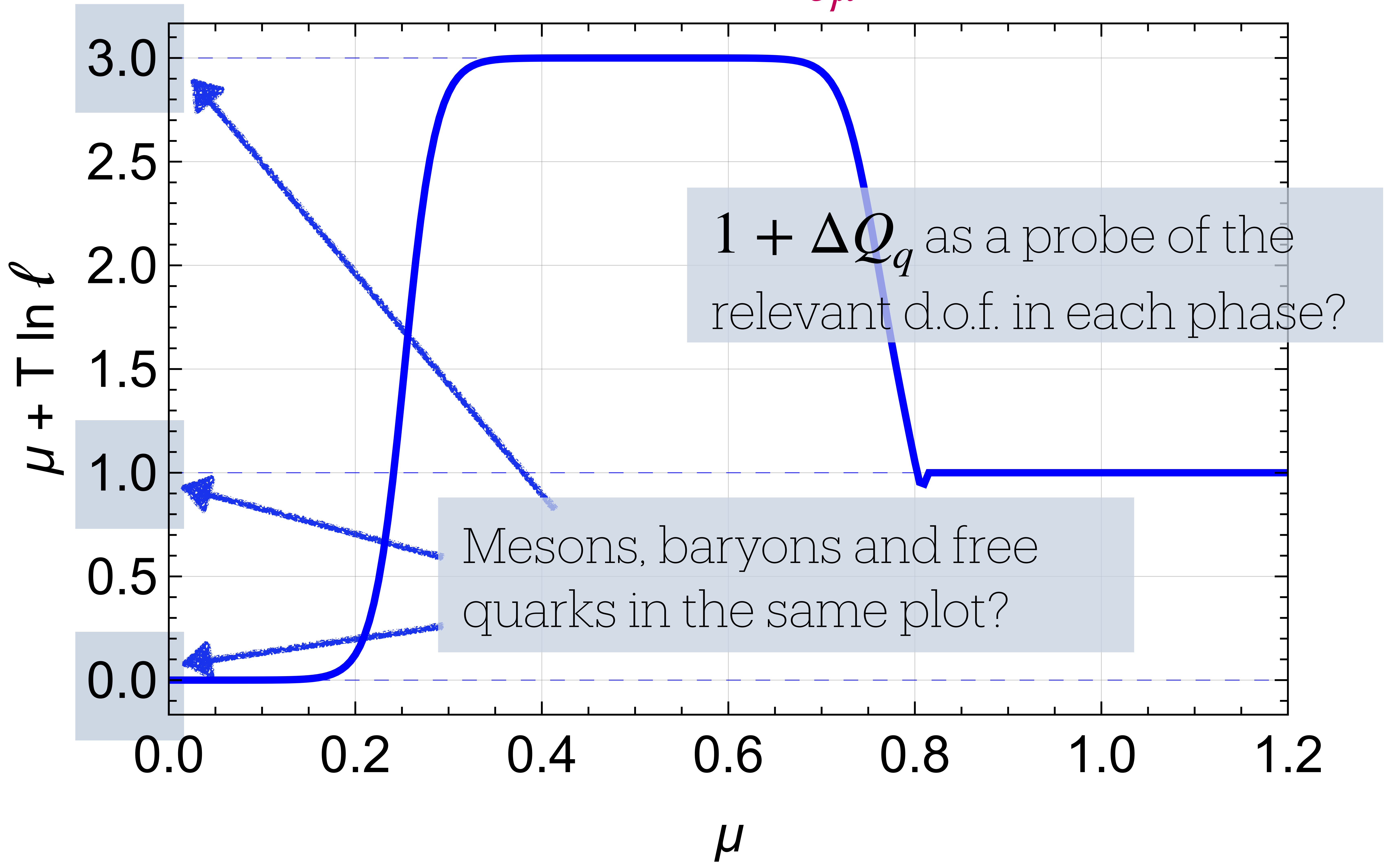
To access the slopes take a μ-derivative: $\dfrac{\partial}{\partial\mu}(\mu + T\ln\ell) = 1 + \Delta Q_q$



To access the slopes take a $\boldsymbol{\mu}$-derivative: $\frac{\partial}{\partial \mu}(\mu + T \ln \ell) = 1 + \Delta Q_q$



To access the slopes take a $\boldsymbol{\mu}$-derivative: $\dfrac{\partial}{\partial\mu}(\mu + T\ln\ell) = 1 + \Delta Q_q$



Outline

I. Motivation ✓

II. Relating the Polyakov loop and the Net Quark Number Response

III. Quark Number Response at Low Temperatures

IV. Generalization to Nc Colors

I. Motivation ✓

II. Relating the Polyakov loop and the Net Quark Number Response

III. Quark Number Response at Low Temperatures

IV. Generalization to Nc Colors

Basic thermodynamics

Consider a bath of quarks and gluons at temperature T and quark chemical potential μ. The partition function writes:

$$Z = \mathrm{Tr}\ \exp\{-\beta(H - \mu Q)\}$$

with Q the net quark number operator.

Basic thermodynamics

Consider a bath of quarks and gluons at temperature T and quark chemical potential μ. The partition function writes:

$$Z = \mathrm{Tr}\ \exp\{-\beta(H - \mu Q)\}$$

with Q the net quark number operator.

The average net quark number is then given by:

$$\langle Q \rangle = T \frac{\partial \ln Z}{\partial \mu} = -\frac{\partial F}{\partial \mu}$$

with F the free energy.

Quark number response of the bath

Recall now that the Polyakov loop gives access to the energy cost for bringing a quark probe into the bath

$$T \ln \ell = -\Delta F_q$$

This energy cost is just the difference in the bath free energy before and after bringing the probe.

Quark number response of the bath

Recall now that the Polyakov loop gives access to the energy cost for bringing a quark probe into the bath

$$T \ln \ell = -\Delta F_q$$

This energy cost is just the difference in the bath free energy before and after bringing the probe.

It follows that the quark number response of the medium is

$$T \frac{\partial \ln \ell}{\partial \mu} = -\frac{\partial \Delta F_q}{\partial \mu} = \Delta Q_q$$

Quark number gain of the system

The net quark number gain when bringing one quark $\boldsymbol{q}$ is then

$$1 + \Delta Q_q = 1 + T\frac{\partial \ln \ell}{\partial \mu} = \frac{\partial}{\partial \mu}(\mu + T\ln \ell)$$

Similarly the net quark number gain when bringing one antiquark $\bar{\boldsymbol{q}}$ can be expressed in terms of the anti Polyakov loop. One finds:

$$-1 + \Delta Q_{\bar{q}} = -1 + T\frac{\partial \ln \bar{\ell}}{\partial \mu} = \frac{\partial}{\partial \mu}(-\mu + T\ln \bar{\ell})$$

Quark number gain of the system

The net quark number gain when bringing one quark $\boldsymbol{q}$ is then

$$1 + \Delta Q_q = 1 + T\frac{\partial \ln \ell}{\partial \mu} = \frac{\partial}{\partial \mu}(\mu + T\ln \ell)$$

Similarly the net quark number gain when bringing one antiquark $\bar{\boldsymbol{q}}$ can be expressed in terms of the anti Polyakov loop. One finds:

$$-1 + \Delta Q_{\bar{q}} = -1 + T\frac{\partial \ln \bar{\ell}}{\partial \mu} = \frac{\partial}{\partial \mu}(-\mu + T\ln \bar{\ell})$$

Both gauge-invariant and RG invariant!

Polyakov loop potential

To access the quark number gains $1 + \Delta Q_q$ and -$1 + \Delta Q_{\bar{q}}$, we need access to ℓ and $\bar{\ell}$, and more precisely to their μ-dependence.

Polyakov loop potential

To access the quark number gains $1 + \Delta Q_q$ and $-1 + \Delta Q_{\bar{q}}$, we need access to ℓ and $\bar{\ell}$, and more precisely to their μ-dependence.

As with any order parameter, the values of the Polyakov loops are obtained from the extremization of a potential $V(\ell, \bar{\ell})$.

Polyakov loop potential

To access the quark number gains $1 + \Delta Q_q$ and -$1 + \Delta Q_{\bar{q}}$, we need access to ℓ and $\bar{\ell}$, and more precisely to their μ-dependence.

As with any order parameter, the values of the Polyakov loops are obtained from the extremization of a potential $V(\ell, \bar{\ell})$.

To access the behavior of $1 + \Delta Q_q$ and -$1 + \Delta Q_{\bar{q}}$ in the low temperature phase, we will only need a few, well established properties of $V(\ell, \bar{\ell})$.

I. Motivation ✓

II. Relating the Polyakov loop and the Net Quark Number Response ✓

III. Quark Number Response at Low Temperatures

IV. Generalization to Nc Colors

Matter contribution

At low T, quarks acquire a constituent mass. For T much below this mass, the matter contribution to $V(\ell,\bar\ell)$ is well approximated by its one-loop expression:

$$
\begin{aligned}
V_{\text{matter}}(\ell,\bar\ell) = &-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln\left[1+3\ell e^{-\beta(\varepsilon_q-\mu)}+3\bar\ell e^{-2\beta(\varepsilon_q-\mu)}+e^{-3\beta(\varepsilon_q-\mu)}\right]\\
&-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln\left[1+3\bar\ell e^{-\beta(\varepsilon_q+\mu)}+3\ell e^{-2\beta(\varepsilon_q+\mu)}+e^{-3\beta(\varepsilon_q+\mu)}\right]
\end{aligned}
$$

for N_f degenerate quarks of constituent mass M,

with $\varepsilon_q \equiv \sqrt{q^2+M^2}$ and $\beta \equiv 1/T$.

Glue contribution

The glue contribution $V_{\rm glue}(\ell,\bar\ell)$ is more complicated but I will not be needing it explicitly. I will just need to know that:

- it is center-symmetric : $V_{\rm glue}(\ell,\bar\ell)=V_{\rm glue}(e^{i\frac{2\pi}{3}}\ell,e^{-i\frac{2\pi}{3}}\bar\ell)$;

- its physical extremum at low T is located at $\ell=\bar\ell=0$.

Simplified potential

At the end of the day, the relevant potential at low temperatures is:

$$V(\ell,\bar{\ell}) = V_{\text{glue}}(\ell,\bar{\ell})$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln\left[1 + 3\ell e^{-\beta(\varepsilon_q-\mu)} + 3\bar{\ell} e^{-2\beta(\varepsilon_q-\mu)} + e^{-3\beta(\varepsilon_q-\mu)}\right]$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln\left[1 + 3\bar{\ell} e^{-\beta(\varepsilon_q+\mu)} + 3\ell e^{-2\beta(\varepsilon_q+\mu)} + e^{-3\beta(\varepsilon_q+\mu)}\right]$$

with $V_{\text{glue}}(\ell,\bar{\ell})$ center-symmetric and confining.

Simplified potential

Suppose first that $|\mu| < M$.

$$V(\ell,\bar{\ell}) = V_{\text{glue}}(\ell,\bar{\ell})$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln\left[1 + 3\ell e^{-\beta(\varepsilon_q-\mu)} + 3\bar{\ell} e^{-2\beta(\varepsilon_q-\mu)} + e^{-3\beta(\varepsilon_q-\mu)}\right]$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln\left[1 + 3\bar{\ell} e^{-\beta(\varepsilon_q+\mu)} + 3\ell e^{-2\beta(\varepsilon_q+\mu)} + e^{-3\beta(\varepsilon_q+\mu)}\right]$$

with $V_{\text{glue}}(\ell,\bar{\ell})$ center-symmetric and confining.

Simplified potential

Suppose first that $|\mu| < M$. Then all exponential are suppressed.

$$V(\ell,\bar{\ell}) = V_{\text{glue}}(\ell,\bar{\ell})$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln\left[1 + 3\ell \boxed{e^{-\beta(\varepsilon_q-\mu)}} + 3\bar{\ell}\boxed{e^{-2\beta(\varepsilon_q-\mu)}} + \boxed{e^{-3\beta(\varepsilon_q-\mu)}}\right]$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln\left[1 + 3\bar{\ell} \boxed{e^{-\beta(\varepsilon_q+\mu)}} + 3\ell\boxed{e^{-2\beta(\varepsilon_q+\mu)}} + \boxed{e^{-3\beta(\varepsilon_q+\mu)}}\right]$$

with $V_{\text{glue}}(\ell,\bar{\ell})$ center-symmetric and confining.

Simplified potential

Suppose first that $|\mu| < M$. Then all exponential are suppressed.

$$V(\ell,\bar{\ell}) = V_{\rm glue}(\ell,\bar{\ell})$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2\left[3\ell e^{-\beta(\varepsilon_q-\mu)} + 3\bar{\ell} e^{-2\beta(\varepsilon_q-\mu)} + e^{-3\beta(\varepsilon_q-\mu)}\right]$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2\left[3\bar{\ell} e^{-\beta(\varepsilon_q+\mu)} + 3\ell e^{-2\beta(\varepsilon_q+\mu)} + e^{-3\beta(\varepsilon_q+\mu)}\right]$$

with $V_{\rm glue}(\ell,\bar{\ell})$ center-symmetric and confining.

Equations of motion for ℓ and $\bar{\ell}$

The equations fixing ℓ and $\bar{\ell}$ are then

$$0 = \frac{\partial V_{\text{glue}}}{\partial \ell} - \frac{3N_f T}{\pi^2}\int_0^\infty dq\, q^2 \left[e^{-\beta(\varepsilon_q-\mu)} + e^{-2\beta(\varepsilon_q+\mu)}\right]$$

$$0 = \frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} - \frac{3N_f T}{\pi^2}\int_0^\infty dq\, q^2 \left[e^{-2\beta(\varepsilon_q-\mu)} + e^{-\beta(\varepsilon_q+\mu)}\right]$$

Equations of motion for ℓ and $\bar{\ell}$

The equations fixing ℓ and $\bar{\ell}$ are then

$$0 = \frac{\partial V_{\text{glue}}}{\partial \ell} - \frac{3N_f T}{\pi^2} \int_0^\infty dq\, q^2 \left[e^{-\beta\varepsilon_q} e^{\beta\mu} + e^{-2\beta\varepsilon_q} e^{-2\beta\mu} \right]$$

$$0 = \frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} - \frac{3N_f T}{\pi^2} \int_0^\infty dq\, q^2 \left[e^{-2\beta\varepsilon_q} e^{2\beta\mu} + e^{-\beta\varepsilon_q} e^{-\beta\mu} \right]$$

Note that the μ-dependence can be pulled out of the integrals.

Equations of motion for ℓ and $\bar{\ell}$

This becomes

$$0 = \frac{\partial V_{\text{glue}}}{\partial \ell} - C\left[e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}\right]$$

$$0 = \frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} - C\left[e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}\right]$$

with $C = 3N_f T M^3$ and

$$f_y \equiv \frac{1}{\pi^2}\int_0^\infty dx\, x^2\, e^{-y\sqrt{x^2+1}} \sim \frac{y^{-3/2}}{\sqrt{2}\pi^{3/2}} e^{-y} \text{ for } y \to \infty$$

Equations of motion for ℓ and $\bar{\ell}$

This becomes

$$0 = \frac{\partial V_{\text{glue}}}{\partial \ell} - C\left[e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}\right]$$

$$0 = \frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} - C\left[e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}\right]$$

Exponentially suppressed at low T

with $C = 3N_f T M^3$ and

$$f_y \equiv \frac{1}{\pi^2}\int_0^\infty dx\, x^2\, e^{-y\sqrt{x^2+1}} \sim \frac{y^{-3/2}}{\sqrt{2}\pi^{3/2}} e^{-y} \quad \text{for } y \to \infty$$

Equations of motion for ℓ and $\bar{\ell}$

This becomes

$$0 = \frac{\partial V_{\text{glue}}}{\partial \ell} - C\left[e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}\right]$$

$$0 = \frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} - C\left[e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}\right]$$

Exponentially suppressed at low T

ℓ and $\bar{\ell}$ approach the pure glue solution $\ell = \bar{\ell} = 0$

with $C = 3N_f T M^3$ and

$$f_y \equiv \frac{1}{\pi^2}\int_0^\infty dx\, x^2\, e^{-y\sqrt{x^2+1}} \sim \frac{y^{-3/2}}{\sqrt{2}\pi^{3/2}} e^{-y} \quad \text{for } y \to \infty$$

Equations of motion for ℓ and $\bar{\ell}$

This becomes

$$0 = \frac{\partial V_{\text{glue}}}{\partial \ell} - C\left[e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}\right]$$

$$0 = \frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} - C\left[e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}\right]$$

Exponentially suppressed at low T

with $C = 3N_f T M^3$ and

$$f_y \equiv \frac{1}{\pi^2}\int_0^\infty dx\, x^2\, e^{-y\sqrt{x^2+1}} \sim \frac{y^{-3/2}}{\sqrt{2}\pi^{3/2}} e^{-y} \quad \text{for } y \to \infty$$

ℓ and $\bar{\ell}$ approach the pure glue solution $\ell = \bar{\ell} = 0$

The equations can be linearized around that point!

Center symmetry at play

The linearization proceeds by writing

$$\frac{\partial V_{\text{glue}}}{\partial \ell} = \left.\frac{\partial V_{\text{glue}}}{\partial \ell}\right|_{\ell=\bar{\ell}=0} + \ell \left.\frac{\partial^2 V_{\text{glue}}}{\partial \ell \partial \ell}\right|_{\ell=\bar{\ell}=0} + \bar{\ell} \left.\frac{\partial^2 V_{\text{glue}}}{\partial \ell \partial \bar{\ell}}\right|_{\ell=\bar{\ell}=0}$$

$$\frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} = \left.\frac{\partial V_{\text{glue}}}{\partial \bar{\ell}}\right|_{\ell=\bar{\ell}=0} + \ell \left.\frac{\partial^2 V_{\text{glue}}}{\partial \ell \partial \bar{\ell}}\right|_{\ell=\bar{\ell}=0} + \bar{\ell} \left.\frac{\partial^2 V_{\text{glue}}}{\partial \bar{\ell} \partial \bar{\ell}}\right|_{\ell=\bar{\ell}=0}$$

Center symmetry at play

The linearization proceeds by writing

$$\frac{\partial V_{\text{glue}}}{\partial \ell} = \left.\frac{\partial V_{\text{glue}}}{\partial \ell}\right|_{\ell=\bar{\ell}=0} + \ell \left.\frac{\partial^2 V_{\text{glue}}}{\partial \ell \partial \ell}\right|_{\ell=\bar{\ell}=0} + \bar{\ell} \left.\frac{\partial^2 V_{\text{glue}}}{\partial \ell \partial \bar{\ell}}\right|_{\ell=\bar{\ell}=0}$$

$$\frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} = \left.\frac{\partial V_{\text{glue}}}{\partial \bar{\ell}}\right|_{\ell=\bar{\ell}=0} + \ell \left.\frac{\partial^2 V_{\text{glue}}}{\partial \ell \partial \bar{\ell}}\right|_{\ell=\bar{\ell}=0} + \bar{\ell} \left.\frac{\partial^2 V_{\text{glue}}}{\partial \bar{\ell} \partial \bar{\ell}}\right|_{\ell=\bar{\ell}=0}$$

But center symmetry $V_{\text{glue}}(\ell, \bar{\ell}) = V_{\text{glue}}(e^{i\frac{2\pi}{3}}\ell, e^{-i\frac{2\pi}{3}}\bar{\ell})$ imposes that the only non-vanishing derivatives are those with equal number of ℓ and $\bar{\ell}$.

Center symmetry at play

The linearization proceeds by writing

$$\frac{\partial V_{\text{glue}}}{\partial \ell} = \bar{\ell}\, \frac{\partial^2 V_{\text{glue}}}{\partial \ell\, \partial \bar{\ell}}\bigg|_{\ell=\bar{\ell}=0}$$

$$\frac{\partial V_{\text{glue}}}{\partial \bar{\ell}} = \ell\, \frac{\partial^2 V_{\text{glue}}}{\partial \ell\, \partial \bar{\ell}}\bigg|_{\ell=\bar{\ell}=0}$$

But center symmetry $V_{\text{glue}}(\ell, \bar{\ell}) = V_{\text{glue}}(e^{i\frac{2\pi}{3}}\ell, e^{-i\frac{2\pi}{3}}\bar{\ell})$ imposes that the only non-vanishing derivatives are those with equal number of ℓ and $\bar{\ell}$.

Back to the equations of motion

Back to the equations of motion, they become

$$0 = \bar{\ell}\, \frac{\partial V^2_{\text{glue}}}{\partial \bar{\ell}\, \partial \ell}\bigg|_{\ell=\bar{\ell}=0} - C\left[e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}\right]$$

$$0 = \ell\, \frac{\partial^2 V_{\text{glue}}}{\partial \ell\, \partial \bar{\ell}}\bigg|_{\ell=\bar{\ell}=0} - C\left[e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}\right]$$

Back to the equations of motion

And are explicitly solved as

$$\bar{\ell} = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M} \right]$$

$$\ell = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M} \right]$$

Back to the equations of motion

And are explicitly solved as

$$\bar{\ell} = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M} \right]$$
$$\ell = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M} \right]$$

The μ-derivatives are trivially evaluated

$$T \frac{\partial \bar{\ell}}{\partial \mu} = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M} \right]$$
$$T \frac{\partial \ell}{\partial \mu} = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M} \right]$$

Quark number responses

And are explicitly solved as

$$\bar{\ell} = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M} \right]$$

$$\ell = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M} \right]$$

The μ-derivatives are trivially evaluated

$$T \frac{\partial \bar{\ell}}{\partial \mu} = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M} \right]$$

$$T \frac{\partial \ell}{\partial \mu} = \frac{C}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}} \left[-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M} \right]$$

We deduce the quark number responses

$$\boxed{\begin{aligned} \Delta Q_{\bar{q}} &= \frac{T}{\bar{\ell}} \frac{\partial \bar{\ell}}{\partial \mu} = \frac{e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M}}{e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}} \\ \Delta Q_q &= \frac{T}{\ell} \frac{\partial \ell}{\partial \mu} = \frac{-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M}}{e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}} \end{aligned}}$$

$\boxed{V_{\text{glue}} \text{ has dropped!}}$

Quark number gains

From the quark number reponses ...

$$\Delta Q_{\bar{q}} = \frac{e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M}}{e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}}$$

$$\Delta Q_q = \frac{-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M}}{e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}}$$

Quark number gains

From the quark number reponses ...

$$\Delta Q_{\bar{q}} = \frac{e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M}}{e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}}$$

$$\Delta Q_{q} = \frac{-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M}}{e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}}$$

... we get the quark number gains

$$-1 + \Delta Q_{\bar{q}} = \frac{-3e^{-2\beta\mu} f_{2\beta M}}{e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}}$$

$$1 + \Delta Q_{q} = \frac{+3e^{2\beta\mu} f_{2\beta M}}{e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}}$$

Quark number gains

From the quark number reponses ...

$$\Delta Q_{\bar{q}} = \frac{e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M}}{e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}}$$

$$\Delta Q_{q} = \frac{-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M}}{e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}}$$

... we get the quark number gains

$$-1 + \Delta Q_{\bar{q}} = \frac{-3}{1 + e^{3\beta\mu} f_{\beta M} / f_{2\beta M}}$$

$$1 + \Delta Q_{q} = \frac{+3}{1 + e^{-3\beta\mu} f_{\beta M} / f_{2\beta M}}$$

Quark number gains

From the quark number reponses ...

$$\Delta Q_{\bar{q}} = \frac{e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M}}{e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}}$$

$$\Delta Q_q = \frac{-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M}}{e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}}$$

... we get the quark number gains

$$-1 + \Delta Q_{\bar{q}} = \frac{-3}{1 + e^{3\beta\mu} f_{\beta M} / f_{2\beta M}}$$

$$1 + \Delta Q_q = \frac{+3}{1 + e^{-3\beta\mu} f_{\beta M} / f_{2\beta M}}$$

Recall that $f(y) \propto e^{-y}$

Quark number gains

From the quark number reponses ...

$$\Delta Q_{\bar{q}} = \frac{e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M}}{e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}}$$

$$\Delta Q_{q} = \frac{-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M}}{e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}}$$

... we get the quark number gains

$$-1 + \Delta Q_{\bar{q}} = \frac{-3}{1 + ce^{\beta(3\mu+M)}}$$

$$1 + \Delta Q_{q} = \frac{+3}{1 + ce^{-\beta(3\mu-M)}}$$

Recall that $f(y) \propto e^{-y}$

Quark number gains

From the quark number reponses ...

$$\Delta Q_{\bar{q}} = \frac{e^{\beta\mu} f_{\beta M} - 2e^{-2\beta\mu} f_{2\beta M}}{e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}}$$

$$\Delta Q_{q} = \frac{-e^{-\beta\mu} f_{\beta M} + 2e^{2\beta\mu} f_{2\beta M}}{e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M}}$$

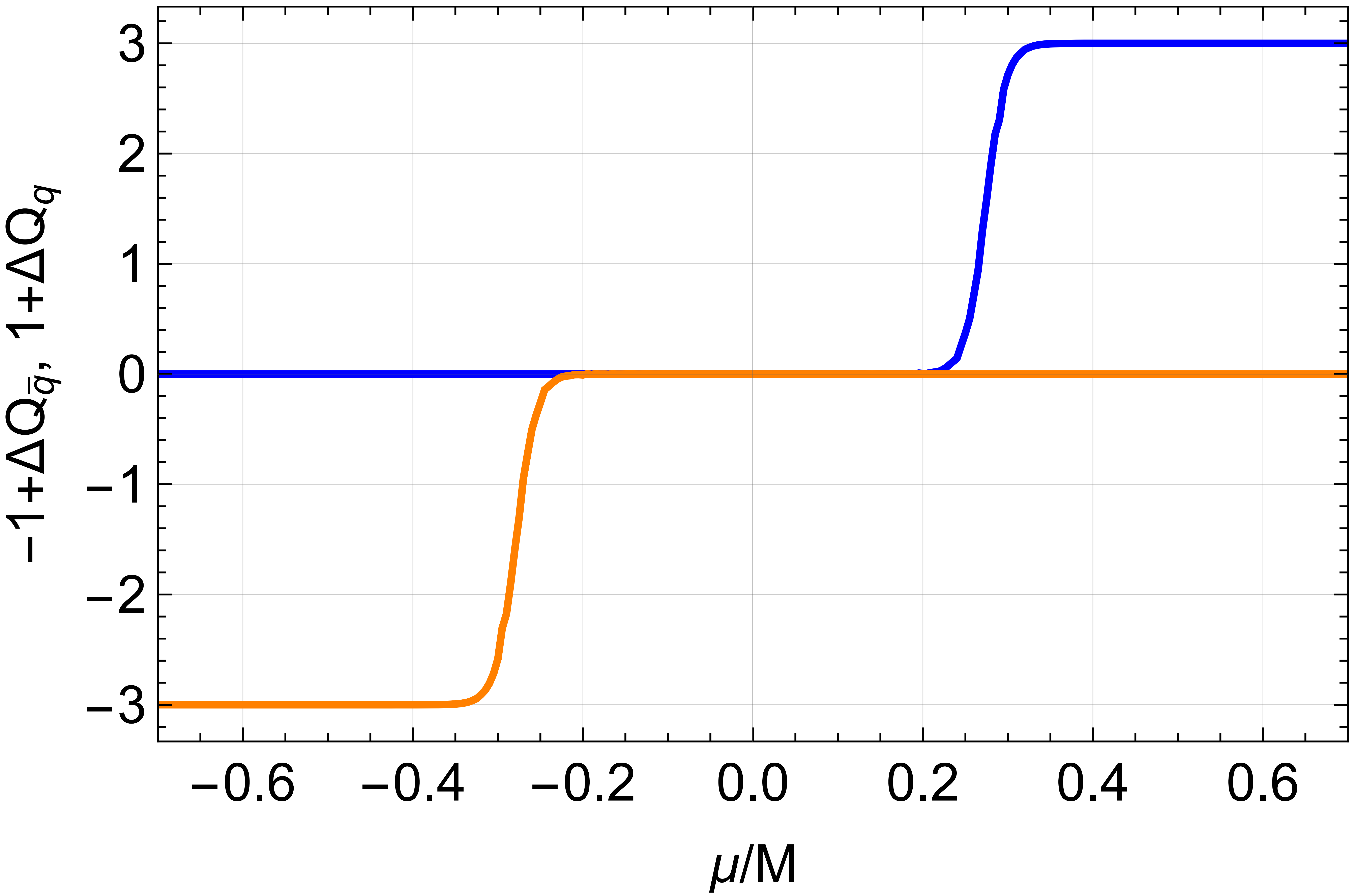
Recall that $f(y) \propto e^{-y}$

... we get the quark number gains

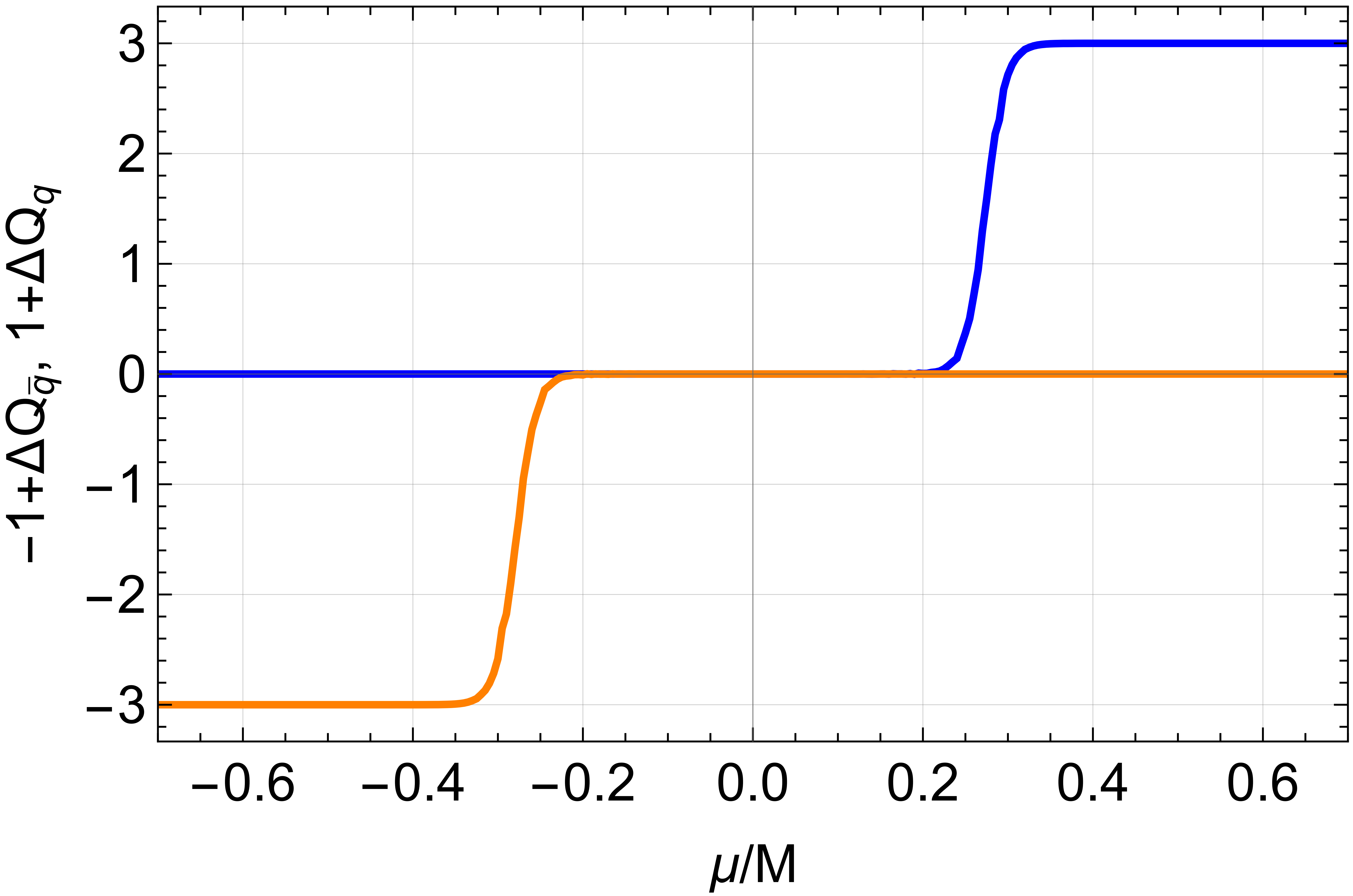
$$-1 + \Delta Q_{\bar{q}} = \frac{-3}{1 + ce^{\beta(3\mu+M)}} \begin{cases} 0 & \text{for } \mu > -M/3 \\ -3 & \text{for } \mu < -M/3 \end{cases}$$

$$1 + \Delta Q_{q} = \frac{+3}{1 + ce^{-\beta(3\mu-M)}} \begin{cases} 3 & \text{for } \mu > M/3 \\ 0 & \text{for } \mu < M/3 \end{cases}$$

Quark number gains

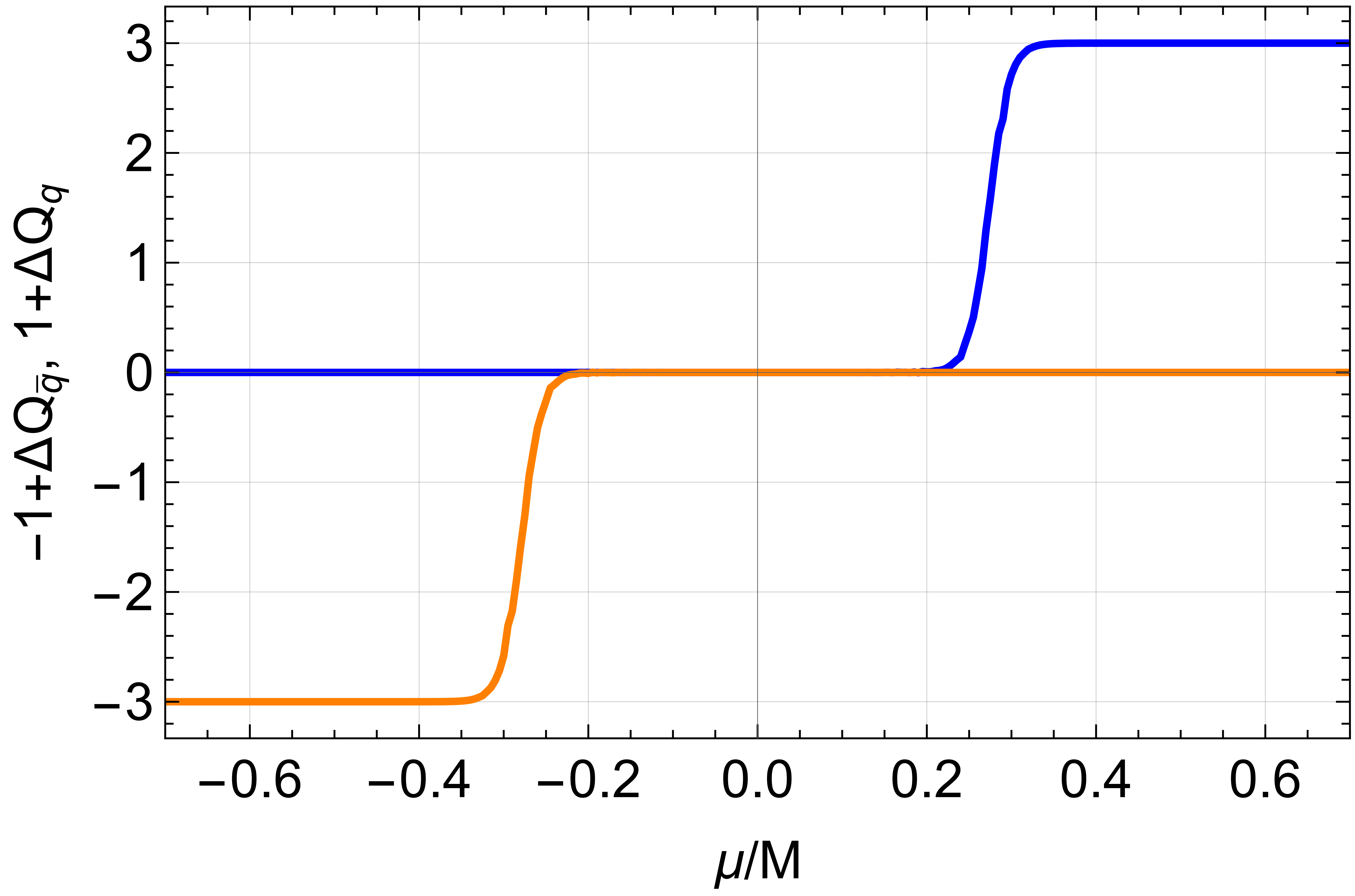


Quark number gains



- quark probe

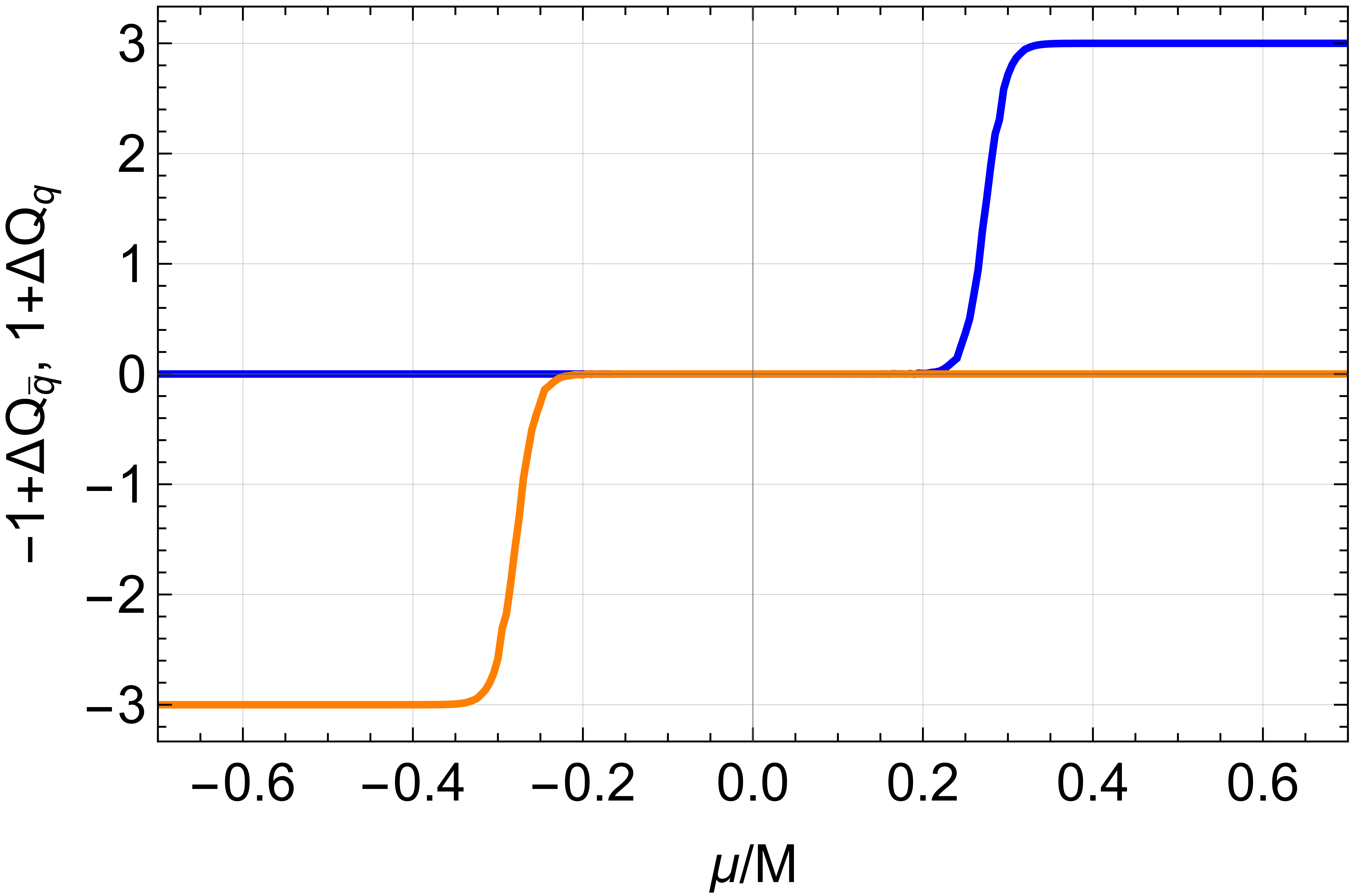
Quark number gains



quark probe

antiquark probe

Quark number gains

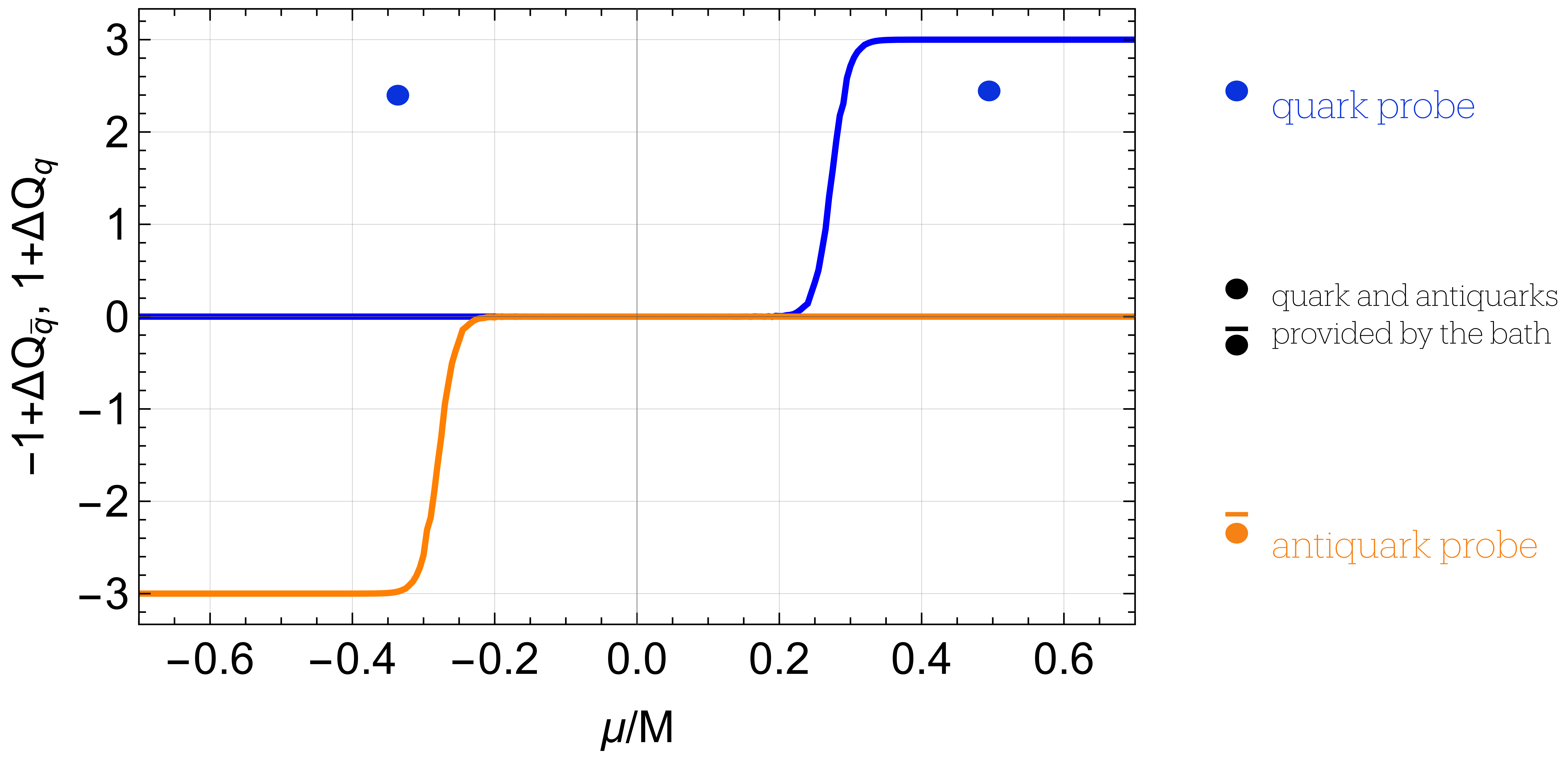


quark probe

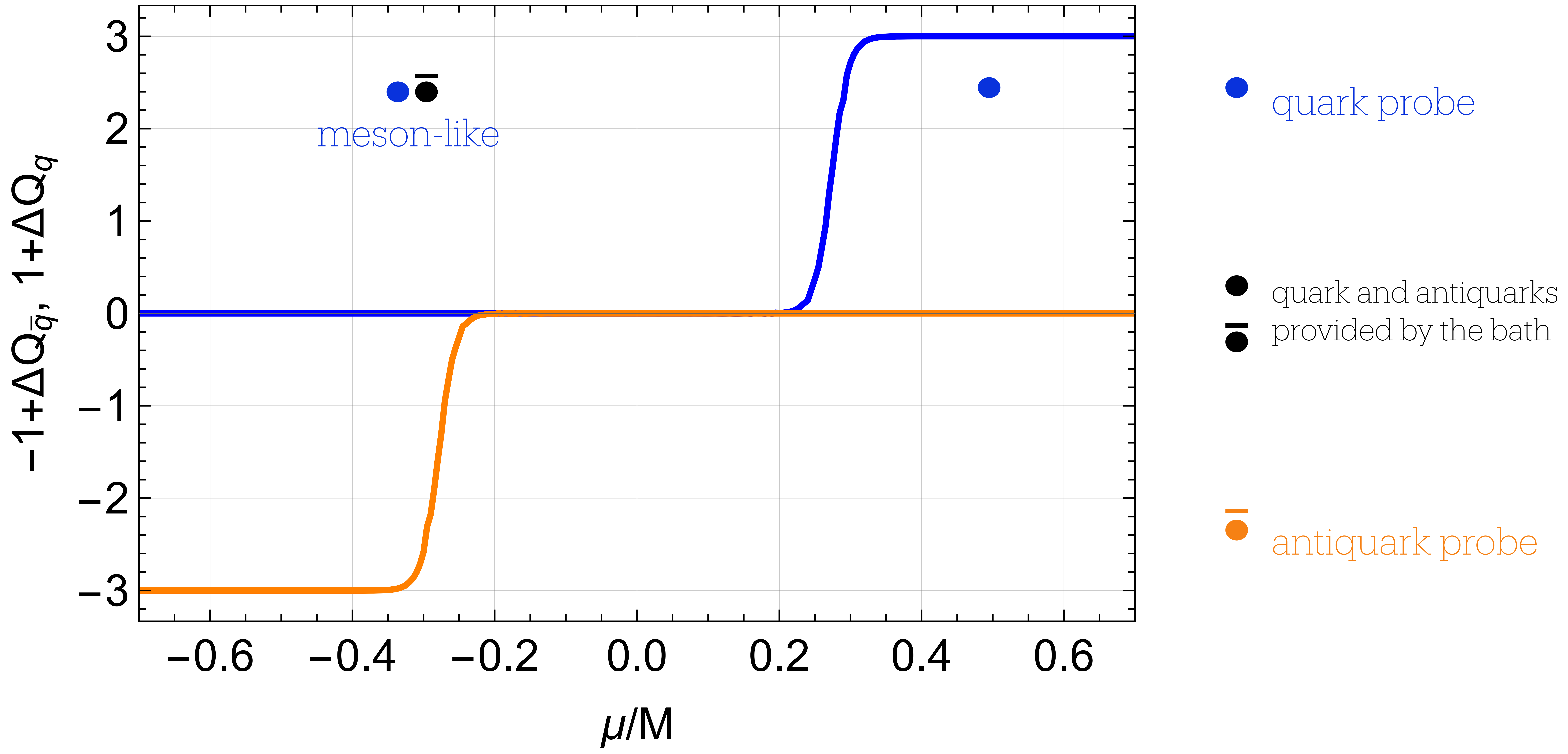
quark and antiquarks
provided by the bath

antiquark probe

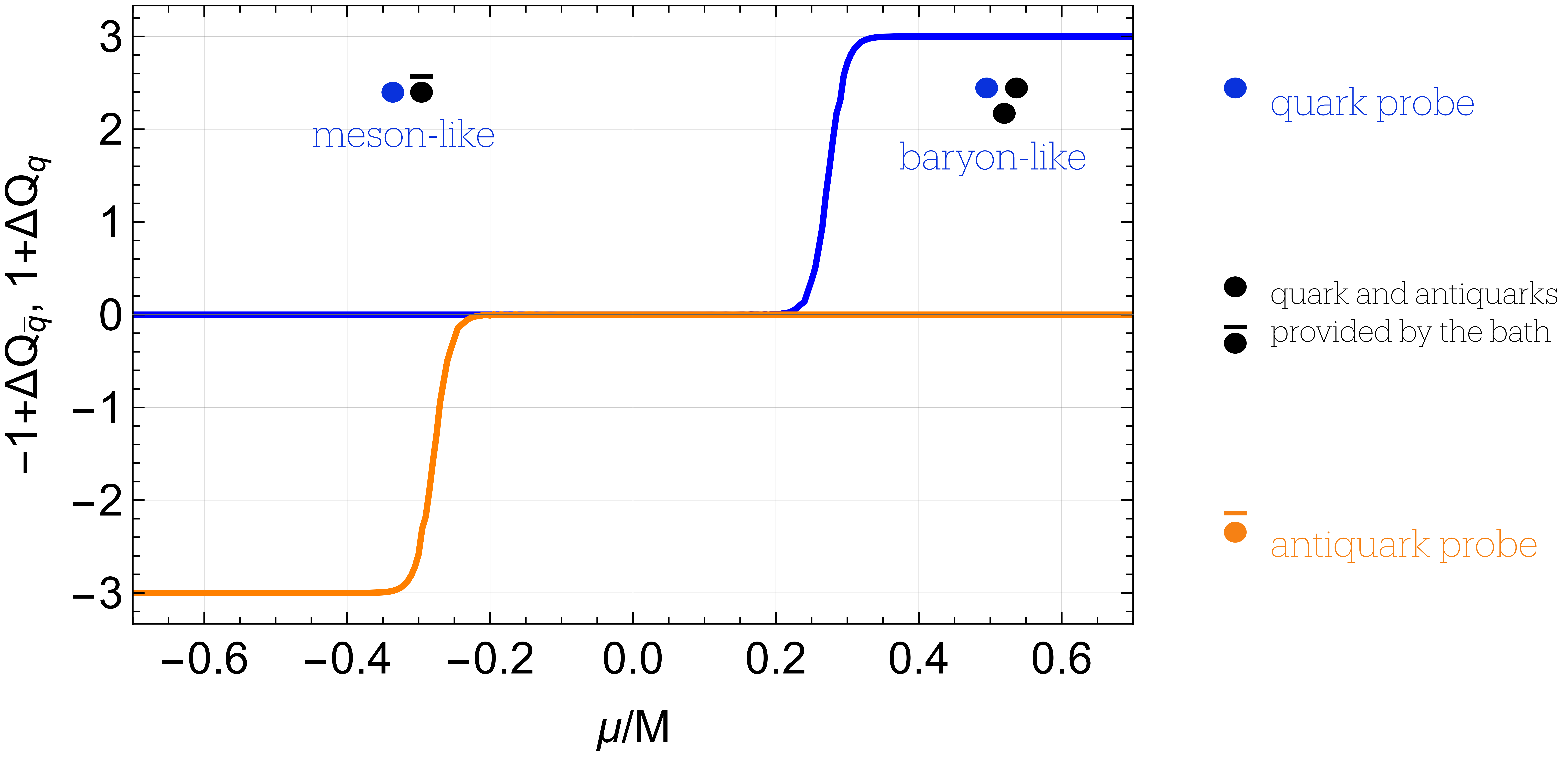
Quark number gains



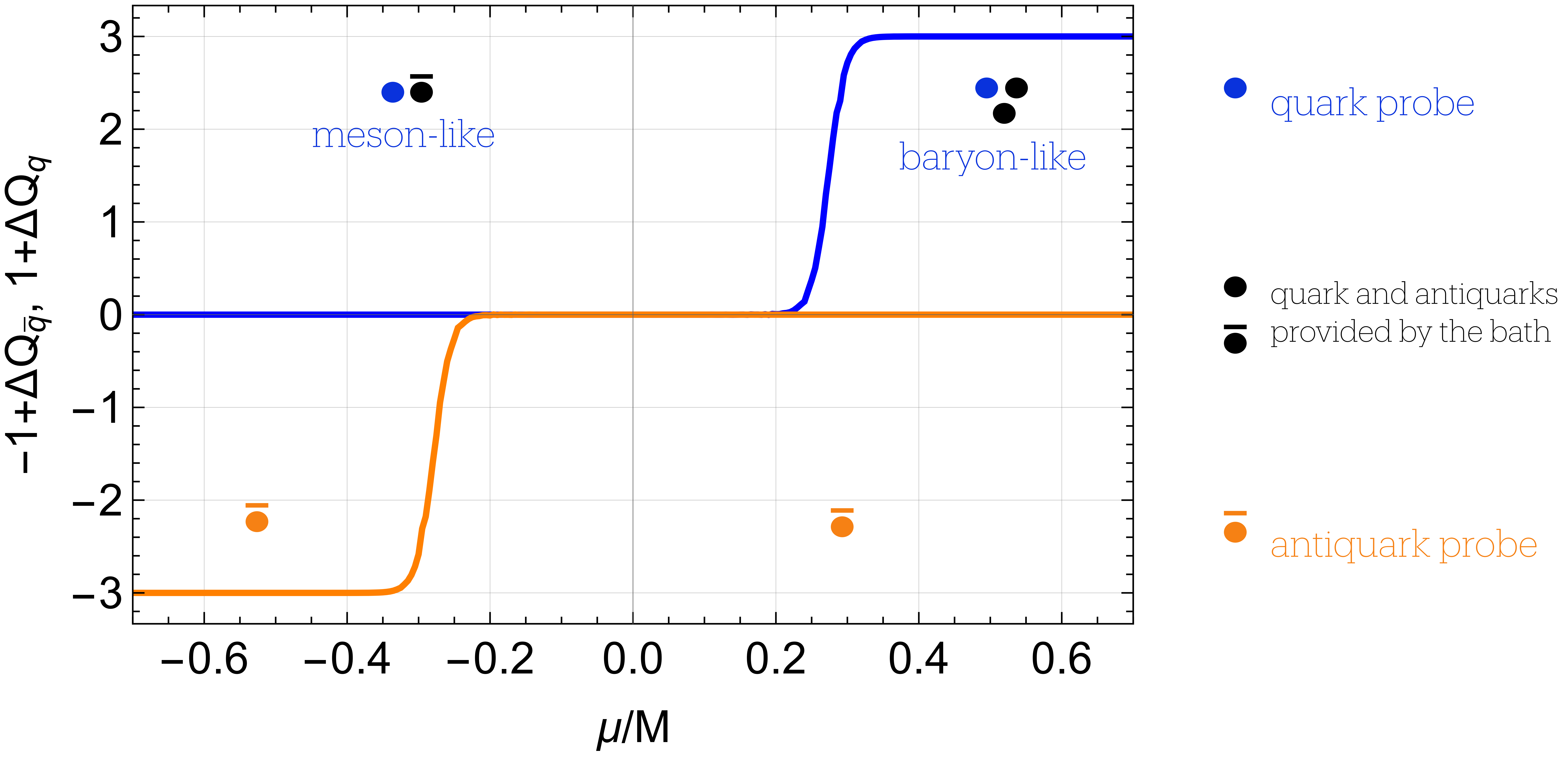
Quark number gains



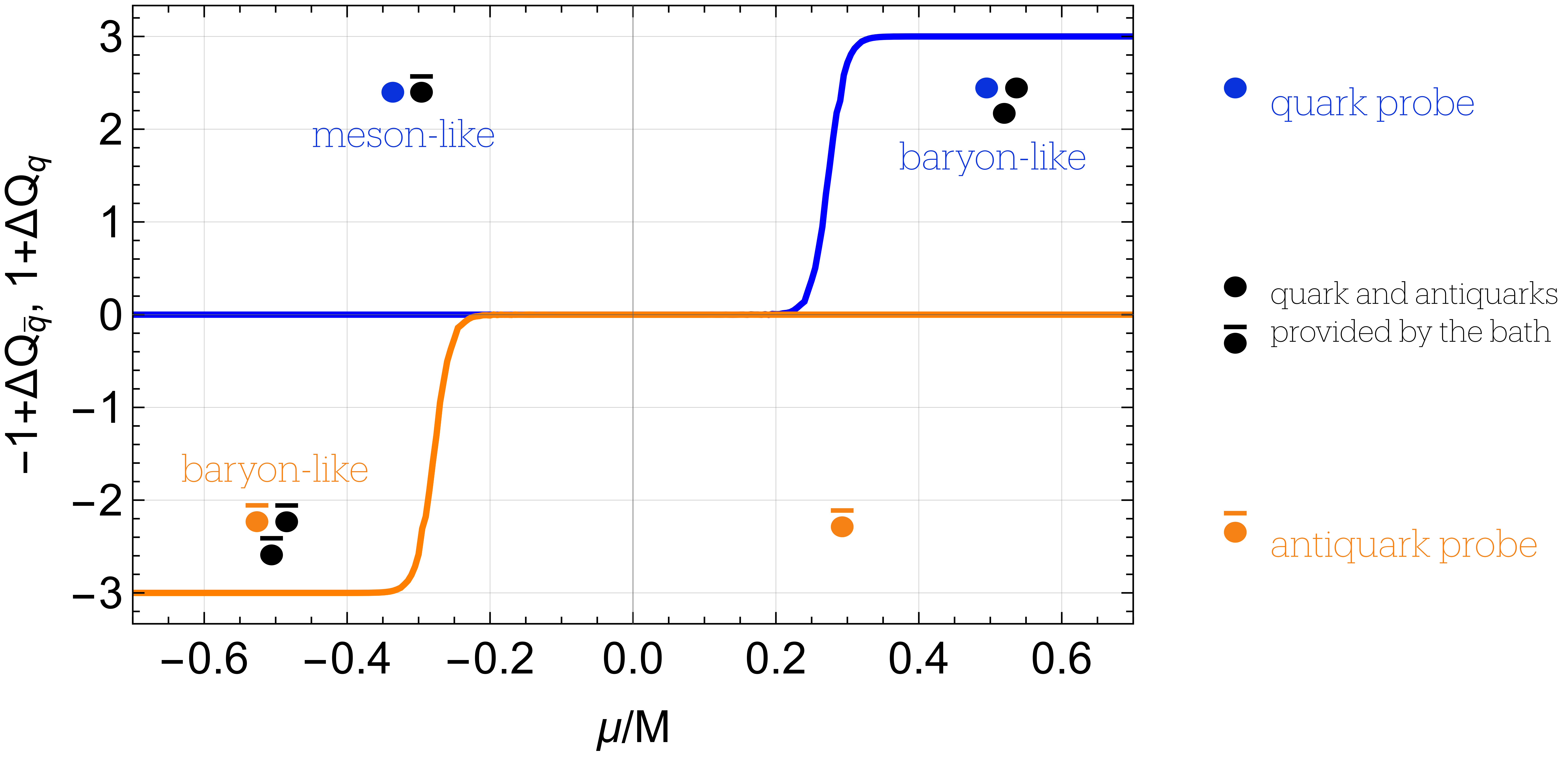
Quark number gains



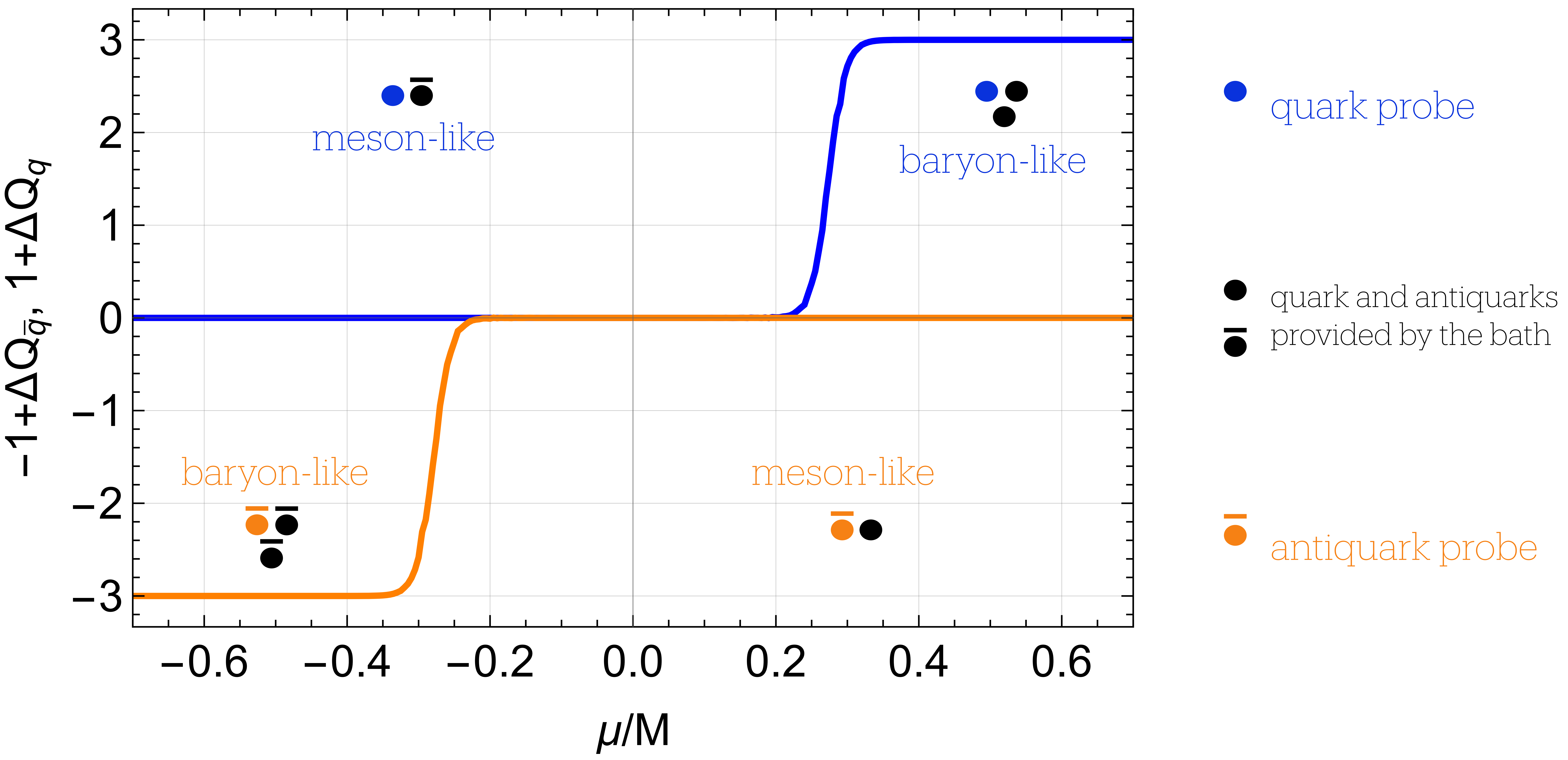
Quark number gains



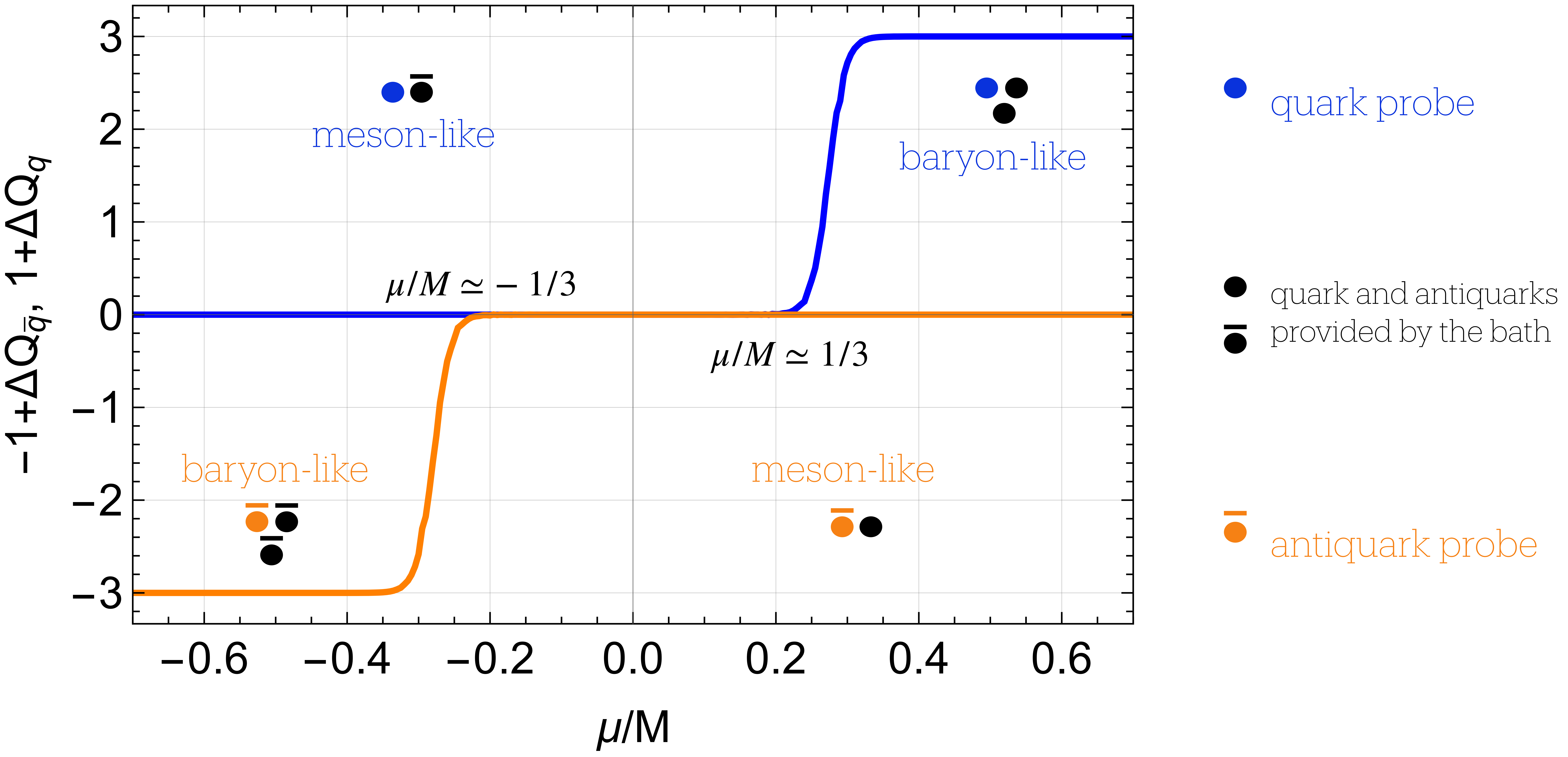
Quark number gains



Quark number gains



Quark number gains



Mesons vs Baryons

The value of $\mu = M/3$ above which a quark probe combines preferentially into a baryon-like state can be understood from basic thermodynamics.

Mesons vs Baryons

The value of $\mu = M/3$ above which a quark probe combines preferentially into a baryon-like state can be understood from basic thermodynamics.

At low T, $Z = \mathrm{Tr}\,\exp\{-\beta(H - \mu Q)\}$ is dominated by those states that minimize $H - \mu Q$.

Mesons vs Baryons

The value of $\mu = M/3$ above which a quark probe combines preferentially into a baryon-like state can be understood from basic thermodynamics.

At low T, $Z = \mathrm{Tr}\,\exp\{-\beta(H - \mu Q)\}$ is dominated by those states that minimize $H - \mu Q$.

When the quark probe combines into a baryon-like state, one has

$$H - \mu Q \simeq 2M - \mu(+2) = 2(M - \mu)$$

Mesons vs Baryons

The value of $\mu = M/3$ above which a quark probe combines preferentially into a baryon-like state can be understood from basic thermodynamics.

At low T, $Z = \mathrm{Tr}\, \exp\{-\beta(H - \mu Q)\}$ is dominated by those states that minimize $H - \mu Q$.

When the quark probe combines into a baryon-like state, one has

$$H - \mu Q \simeq 2M - \mu(+2) = 2(M - \mu)$$

When the quark probe combines into a meson-like state, one has

$$H - \mu Q \simeq M - \mu(-1) = M + \mu$$

Mesons vs Baryons

The value of $\mu = M/3$ above which a quark probe combines preferentially into a baryon-like state can be understood from basic thermodynamics.

At low T, $Z = \mathrm{Tr}\,\exp\{-\beta(H - \mu Q)\}$ is dominated by those states that minimize $H - \mu Q$.

When the quark probe combines into a baryon-like state, one has

$$H - \mu Q \simeq 2M - \mu(+2) = 2(M - \mu)$$

When the quark probe combines into a meson-like state, one has

$$H - \mu Q \simeq M - \mu(-1) = M + \mu$$

The two are equal when $M + \mu = 2(M - \mu)$ that is precisely at $\mu = M/3$.

Large chemical potential

So far, the discussion concerned $|\mu| < M$. What about $|\mu| > M$?

Large chemical potential

So far, the discussion concerned $|\mu| < M$. What about $|\mu| > M$?
We expect the system to deconfine.

Large chemical potential

So far, the discussion concerned $|\mu| < M$. What about $|\mu| > M$?
We expect the system to deconfine.

The discussion is more delicate since we also expect the formation of diquarks.
But our approximated $V_{\mathrm{matter}}(\ell, \bar{\ell})$ does not include that possibility.

Large chemical potential

So far, the discussion concerned $|\mu| < M$. What about $|\mu| > M$?
We expect the system to deconfine.

The discussion is more delicate since we also expect the formation of diquarks.
But our approximated $V_{\rm matter}(\ell, \bar{\ell})$ does not include that possibility.

If we ignore this for the moment, do we get a picture in terms of $1 + \Delta Q_q$ compatible with a deconfined phase at large mu?

Large chemical potential

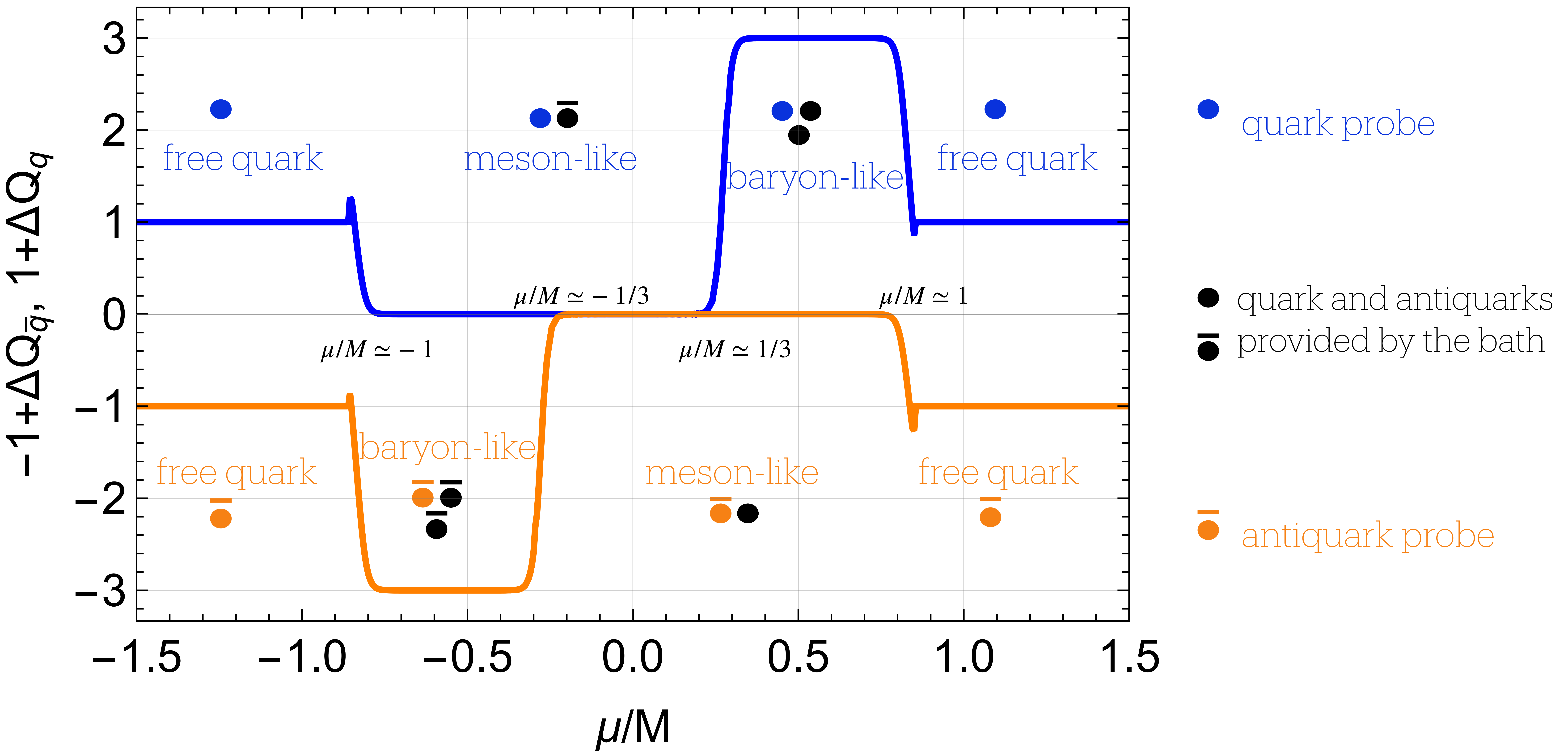
So far, the discussion concerned $|\mu| < M$. What about $|\mu| > M$?
We expect the system to deconfine.

The discussion is more delicate since we also expect the formation of diquarks.
But our approximated $V_{\text{matter}}(\ell, \bar{\ell})$ does not include that possibility.

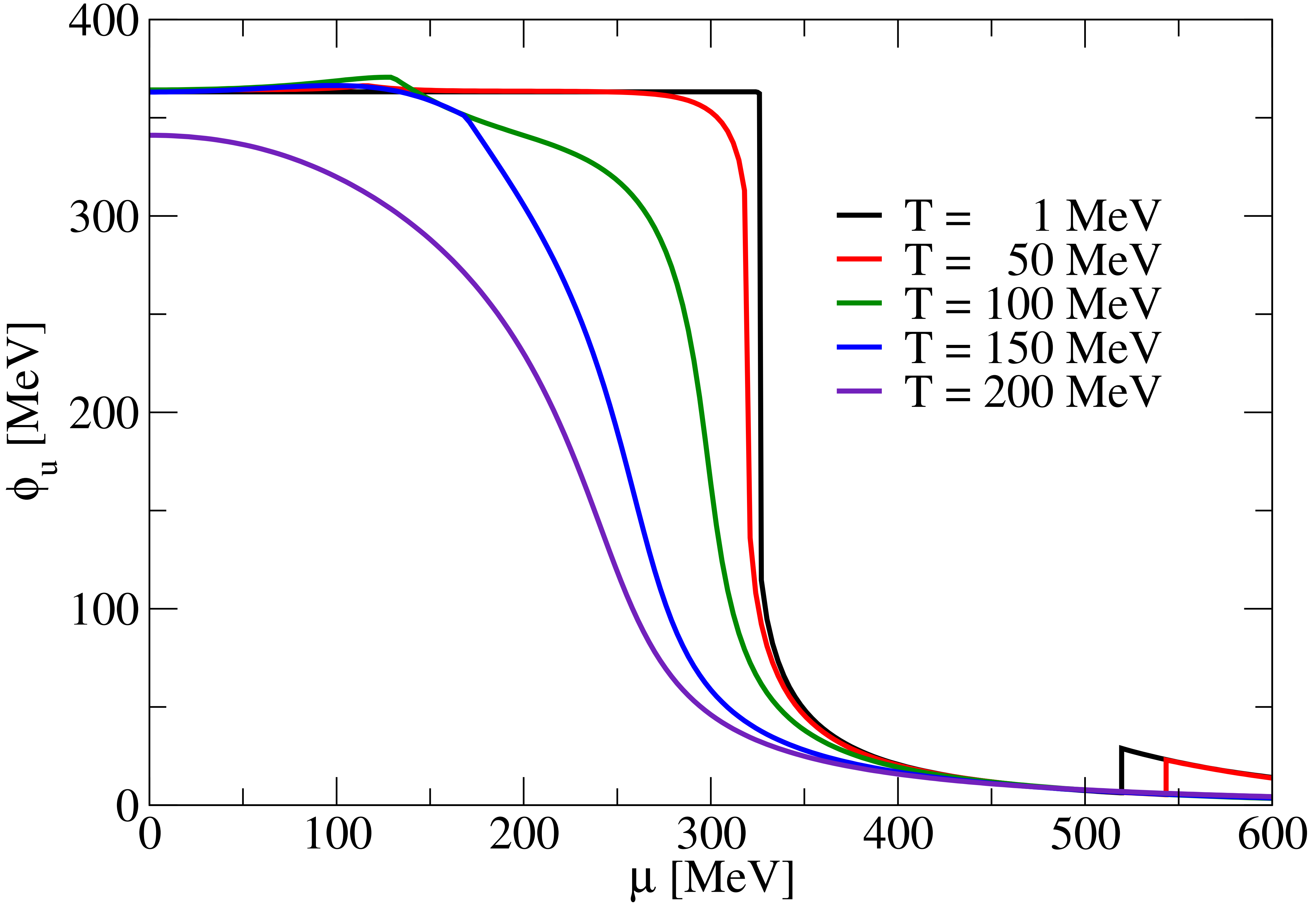
If we ignore this for the moment, do we get a picture in terms of $1 + \Delta Q_q$ compatible with a deconfined phase at large mu? The answer is yes!
One finds:

$$\ell \propto T^2 |\mu| (\mu^2 - M^2)^{1/2} \quad \text{and thus} \quad \Delta Q_q = T \frac{\partial \ln \ell}{\partial \mu} \propto T \to 0$$

Large chemical potential



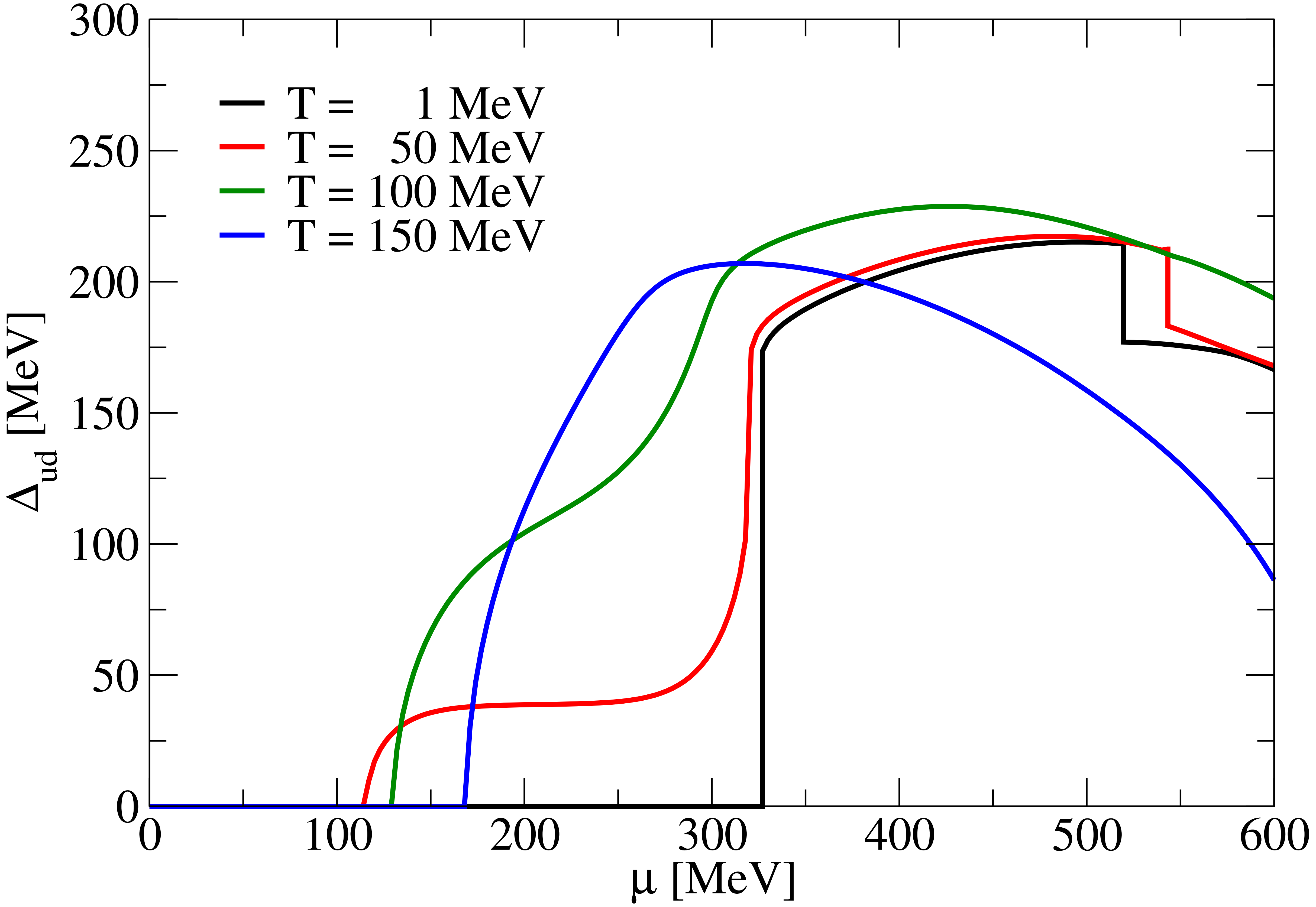
Quark number gain and Cooper pairing?



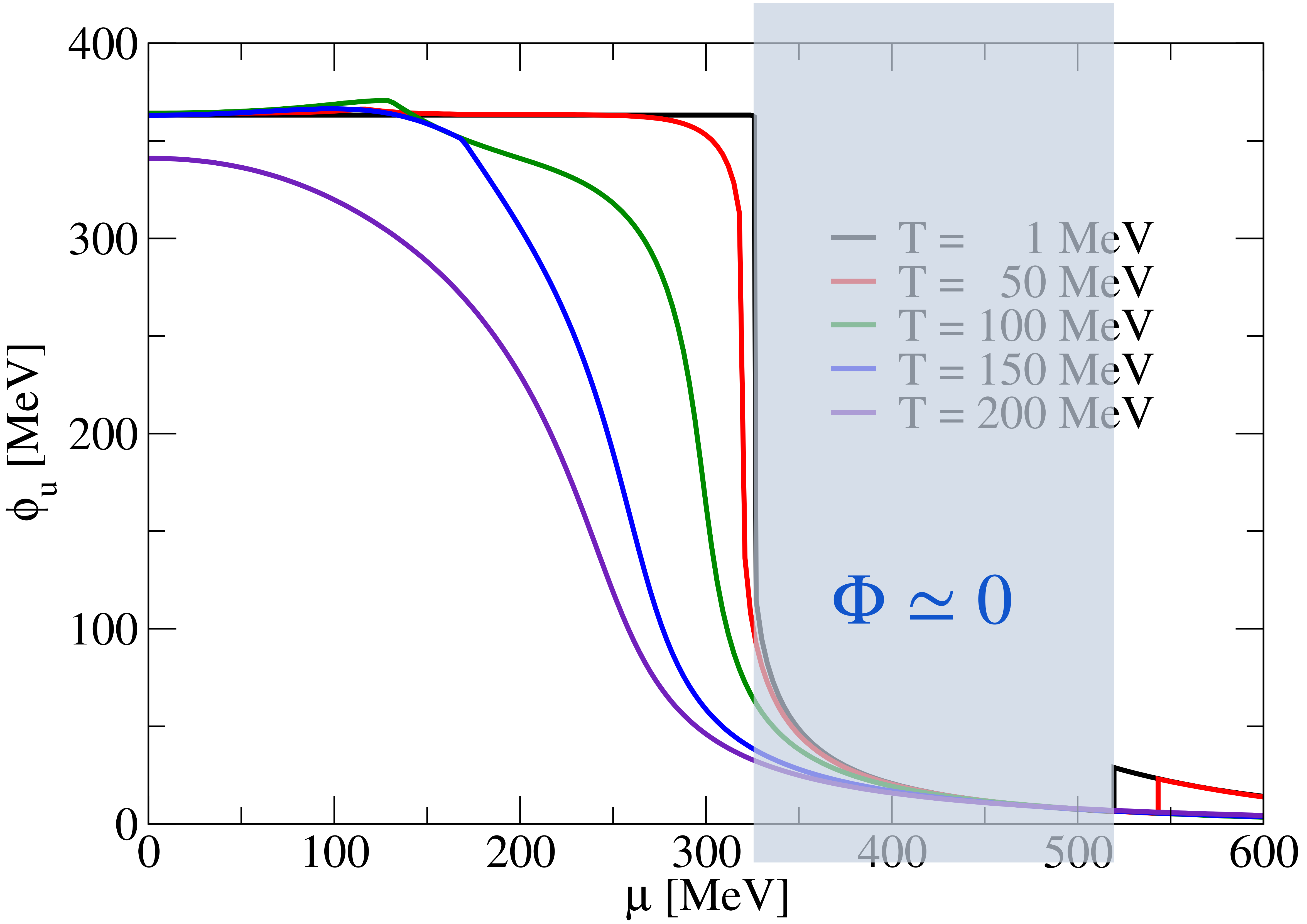
Chiral condensate

3-flavor PNJL model, A. Ayriyan et al, arXiv:1608.7875

Super-conducting gap



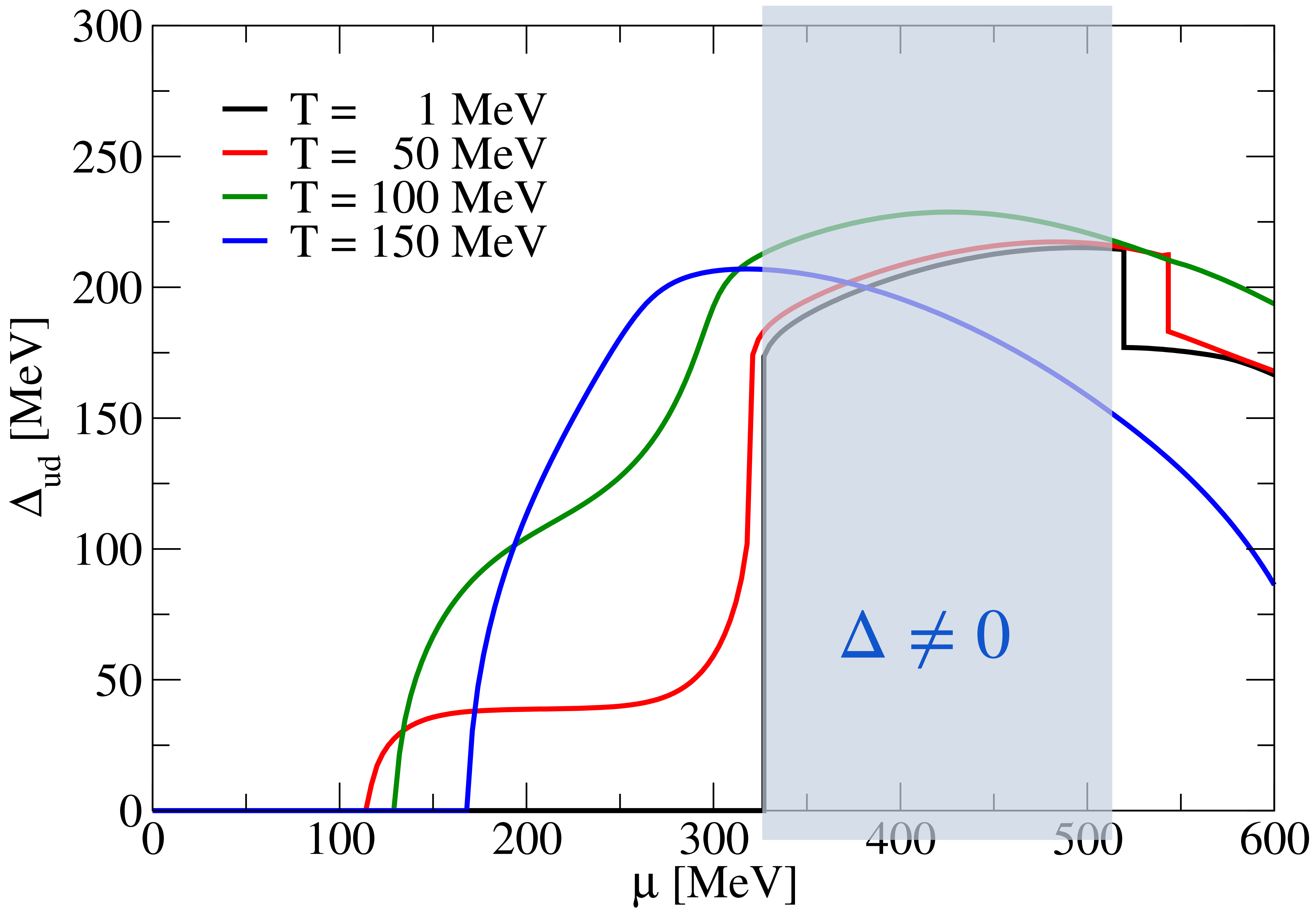
Quark number gain and Cooper pairing?



Chiral condensate

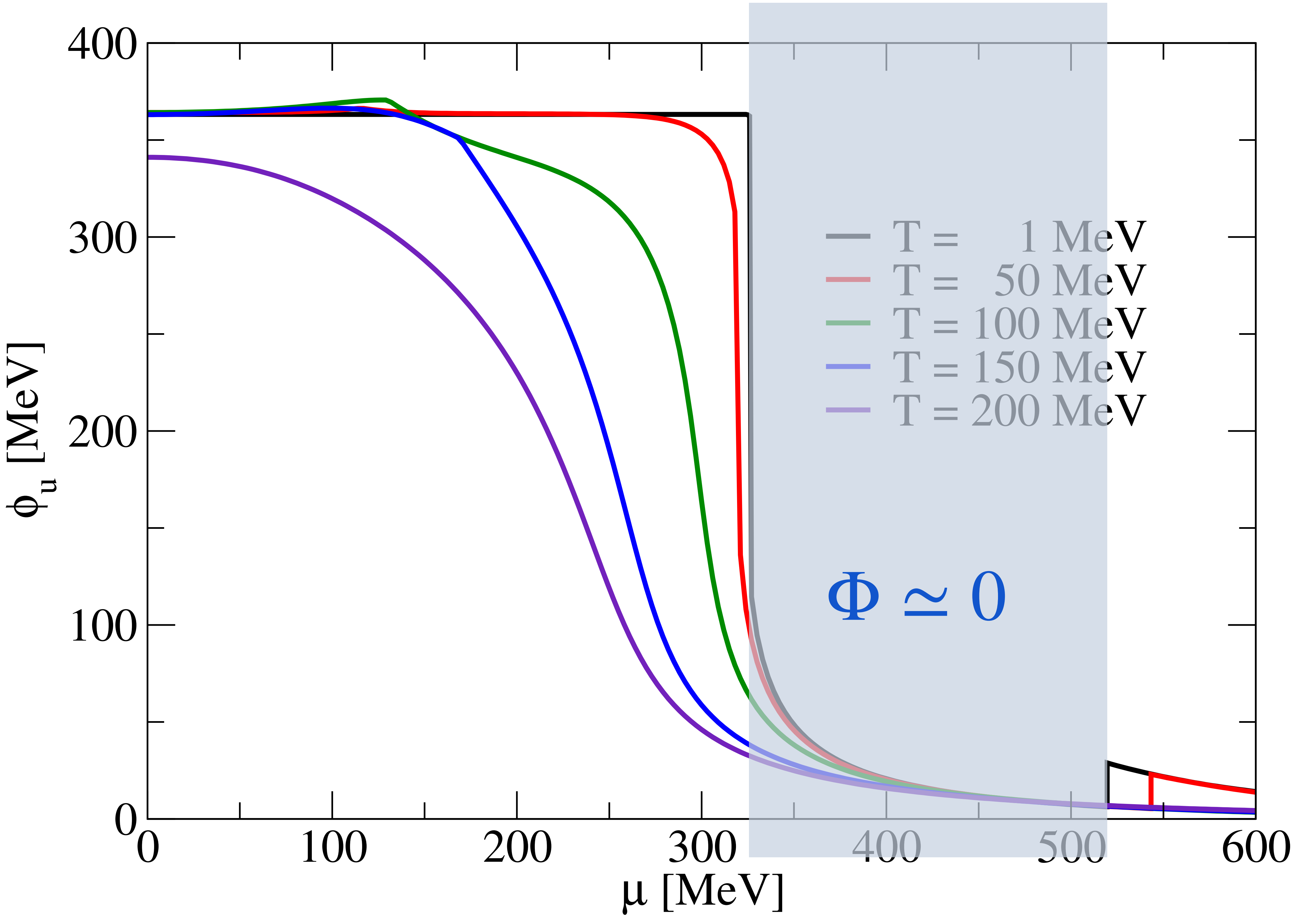
3-flavor PNJL model, A. Ayriyan et al, arXiv:1608.7875

Super-conducting gap



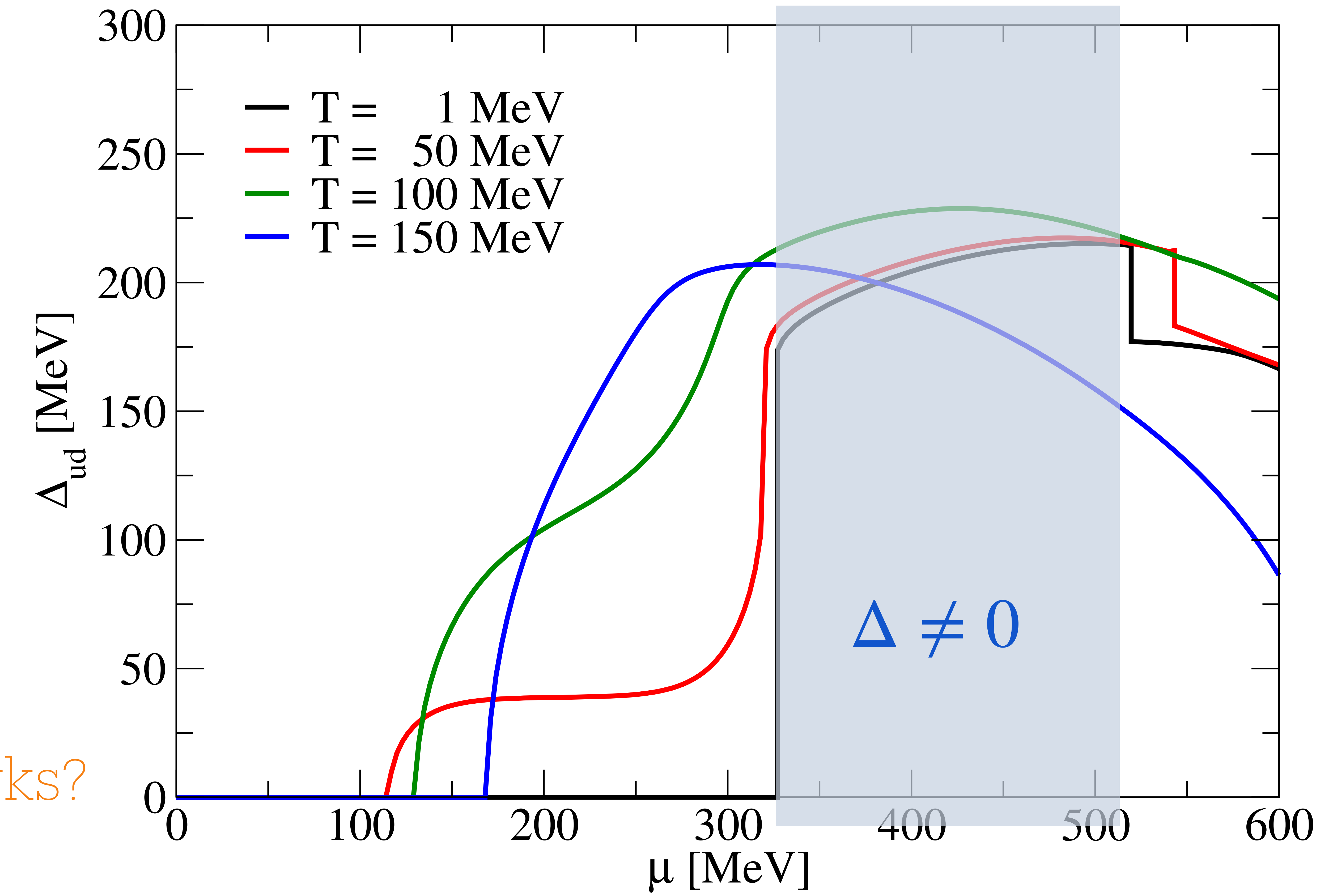
Quark number gain and Cooper pairing?

3-flavor PNJL model, A. Ayriyan et al, arXiv:1608.7875



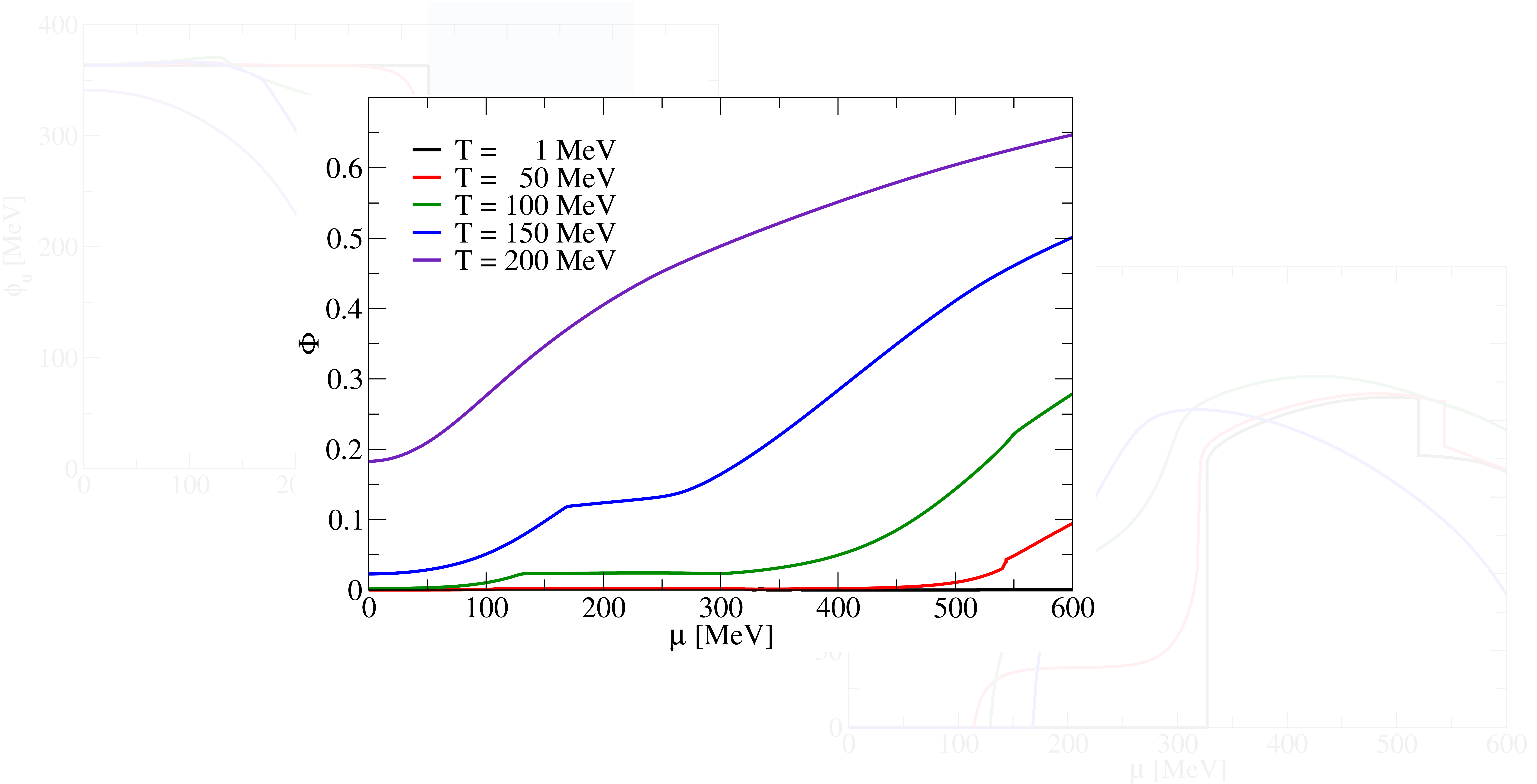
Chiral condensate

Super-conducting gap

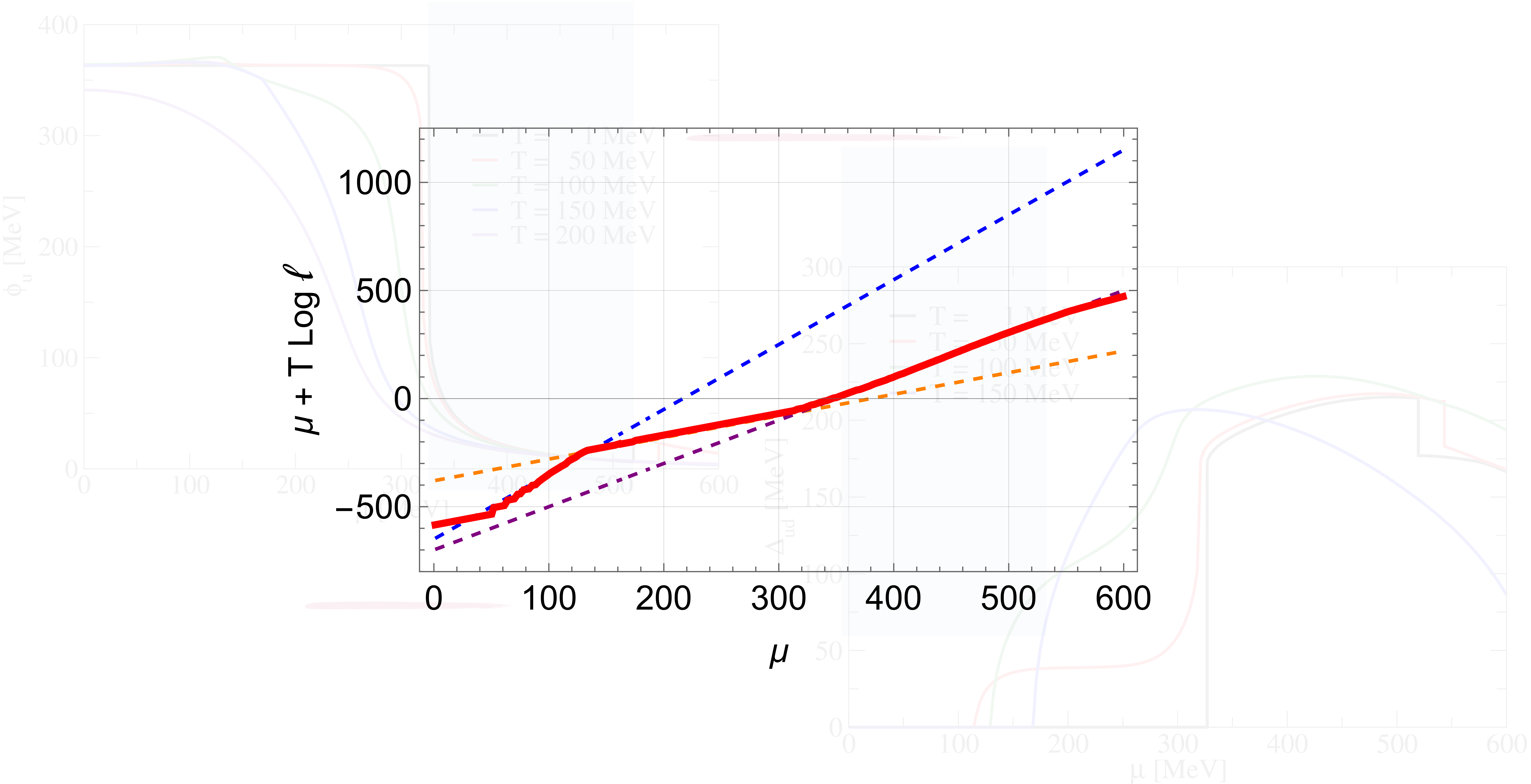


Is $1 + \Delta Q_q$ sensitive to the formation of diquarks?

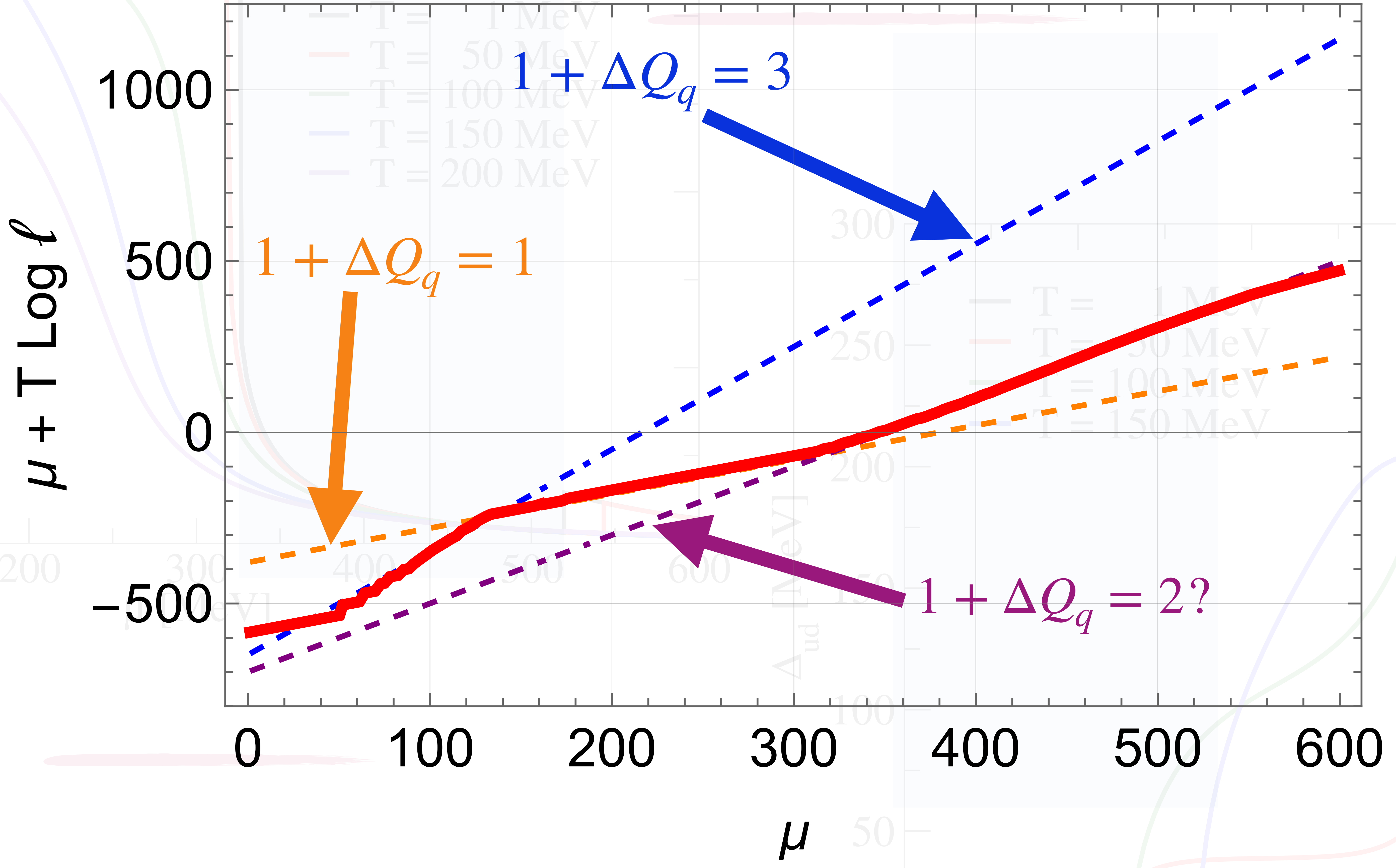
Quark number gain and Cooper pairing?



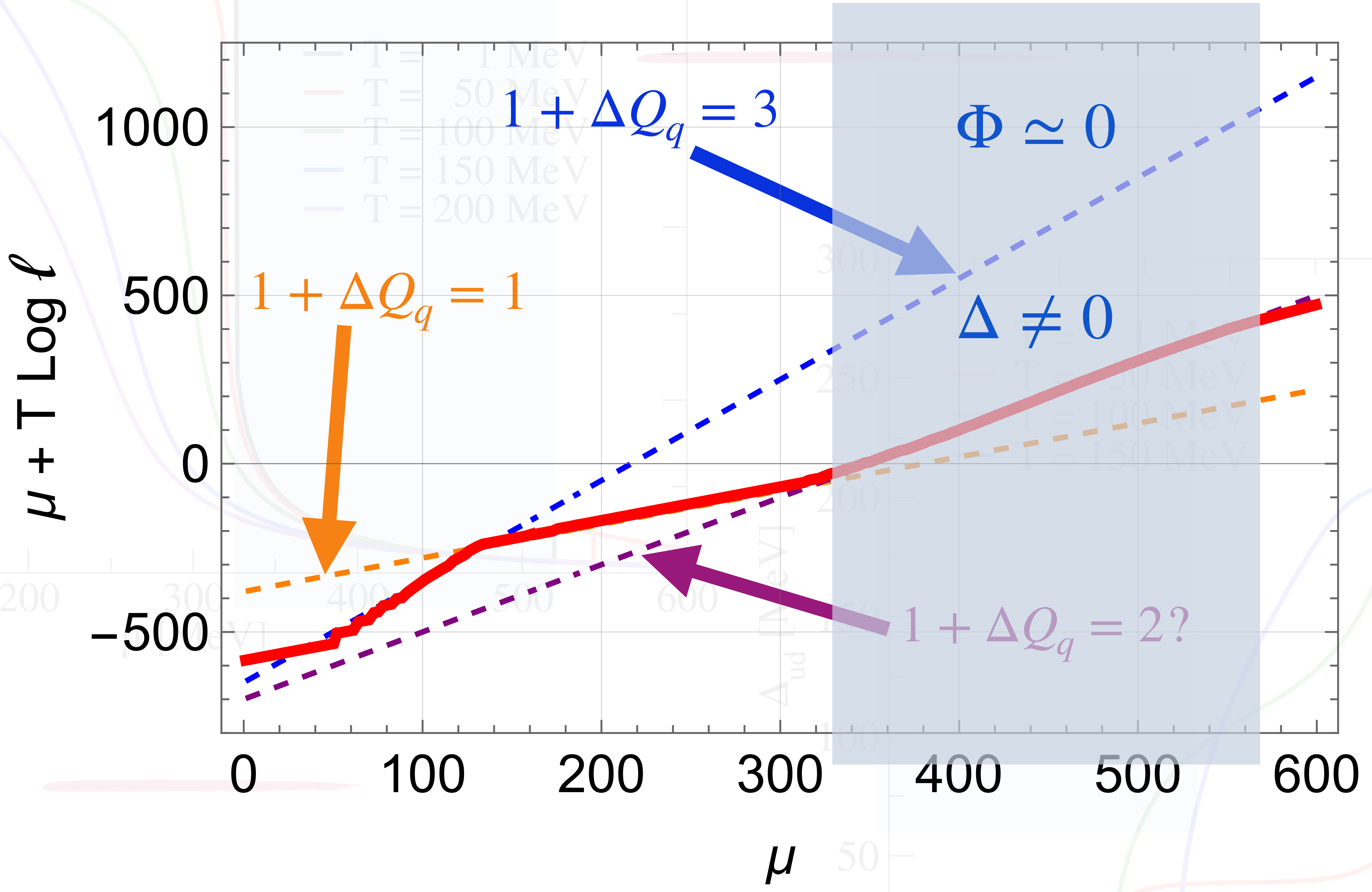
Quark number gain and Cooper pairing?



Quark number gain and Cooper pairing?



Quark number gain and Cooper pairing?



I. Motivation ✓

II. Relating the Polyakov loop and the Net Quark Number Response ✓

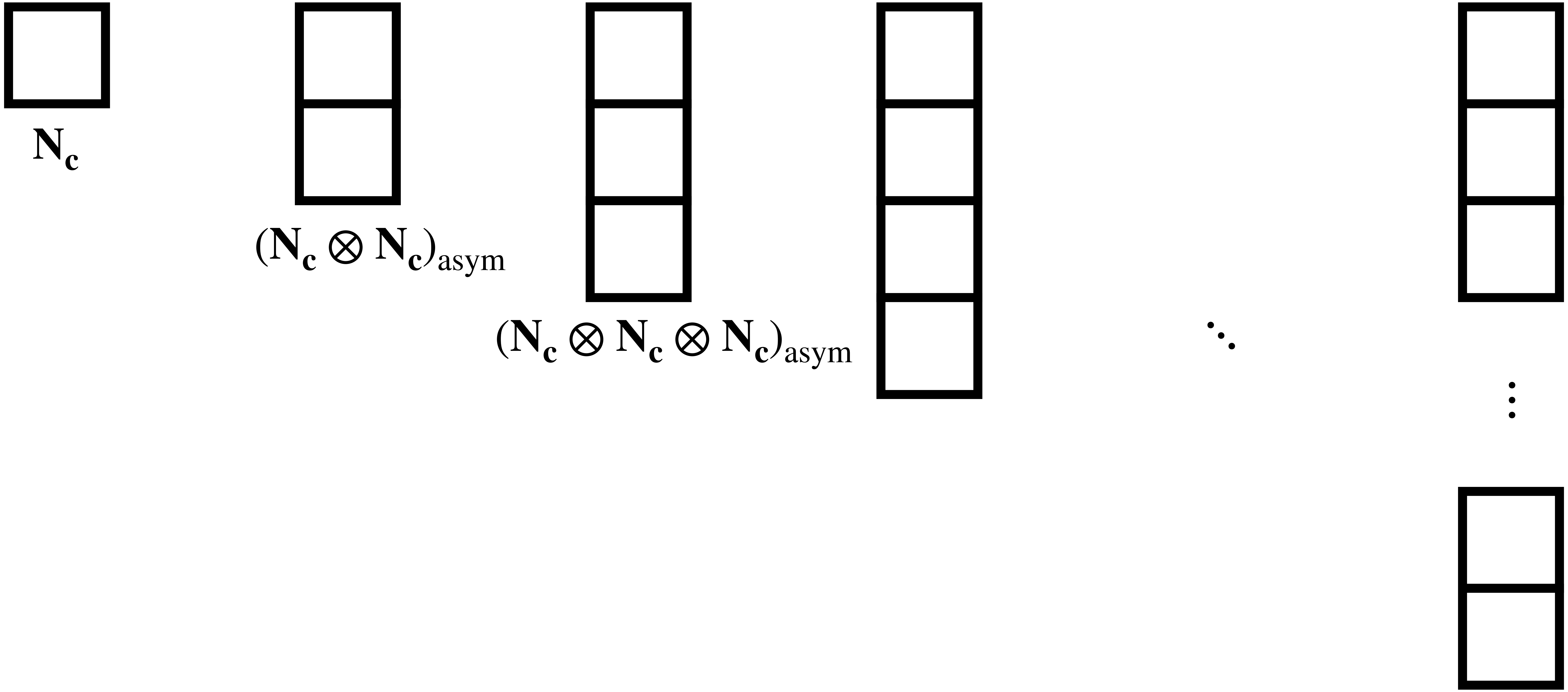
III. Quark Number Response at High Temperatures ✓

IV. Quark Number Response at Low Temperatures ✓

V. Generalization to Nc Colors

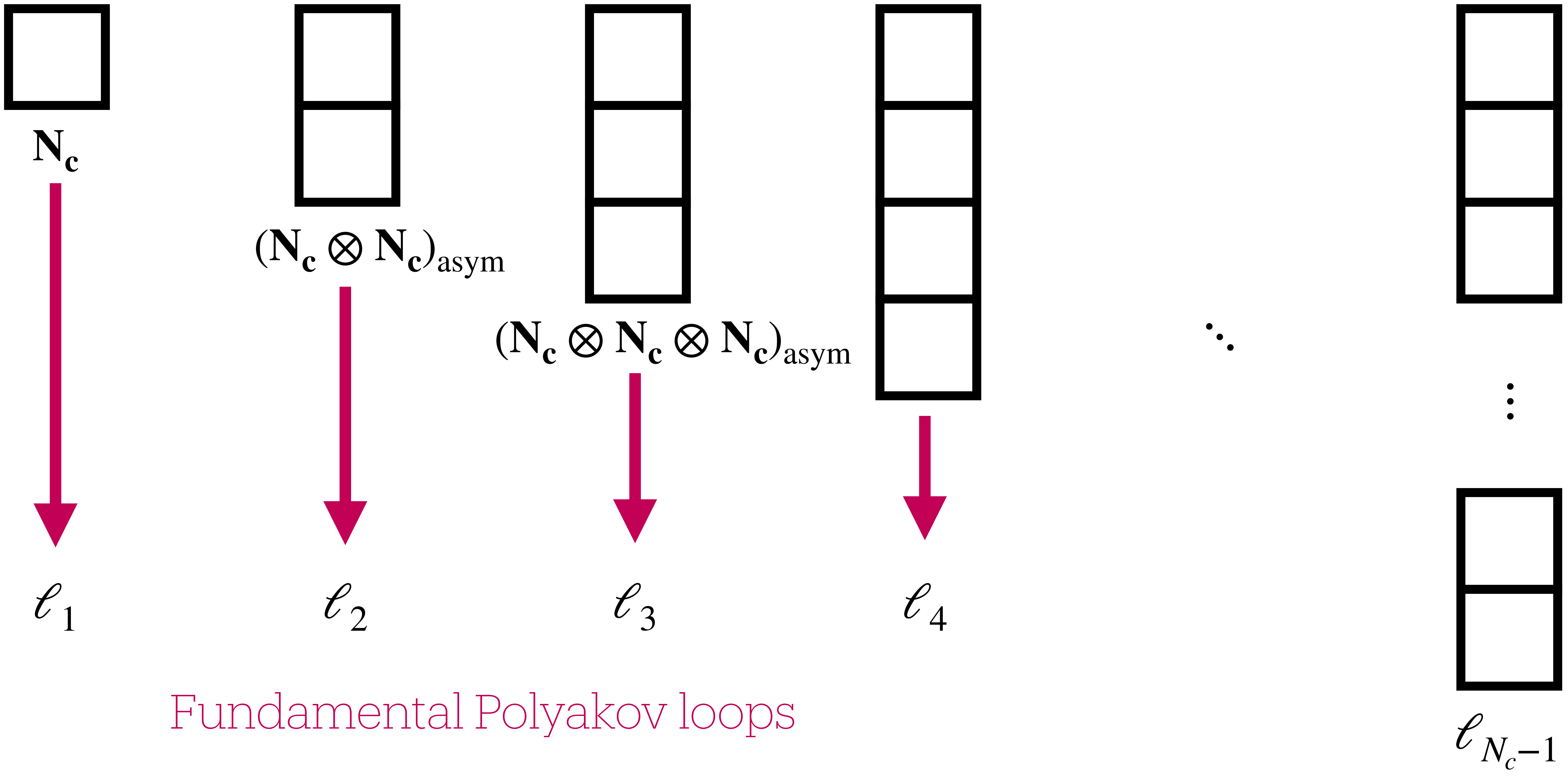
Fundamental representations

Horizontal Young tableaux: antisymmetrized product of ν quarks



Fundamental representations

Horizontal Young tableaux: antisymmetrized product of ν quarks



Polyakov loop potential

Function of the fundamental Polyakov loops: $V(\{\ell_\nu\})$

The glue contribution is center-symmetric: $V_{\text{glue}}(\{\ell_\nu\}) = V_{\text{glue}}\left(\left\{e^{i\frac{2\pi}{N_c}\nu}\ell_\nu\right\}\right)$

The matter contribution is well approximated by:

$$V_{\text{matter}}(\{\ell_\nu\}) = -\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln \sum_{\nu=0}^{N_c} \frac{N_c!}{\nu!(N_c-\nu)!}\ell_\nu\, e^{-\nu\beta(\varepsilon_q-\mu)}$$

$$-\frac{N_f T}{\pi^2}\int_0^\infty dq\, q^2 \ln \sum_{\nu=0}^{N_c} \frac{N_c!}{\nu!(N_c-\nu)!}\ell_{N_c-\nu}\, e^{-\nu\beta(\varepsilon_q+\mu)}$$

Quark number gains

We find that, bringing ν quarks in the fundamental representation ν

$$\nu + \Delta Q_\nu = \frac{N_c}{1 + e^{-N_c\beta\mu} f_{\nu\beta M}/f_{(N_c-\nu)\beta M}} \simeq \frac{N_c}{1 + c e^{-\beta(N_c\mu-(N_c-2\nu)M}}$$

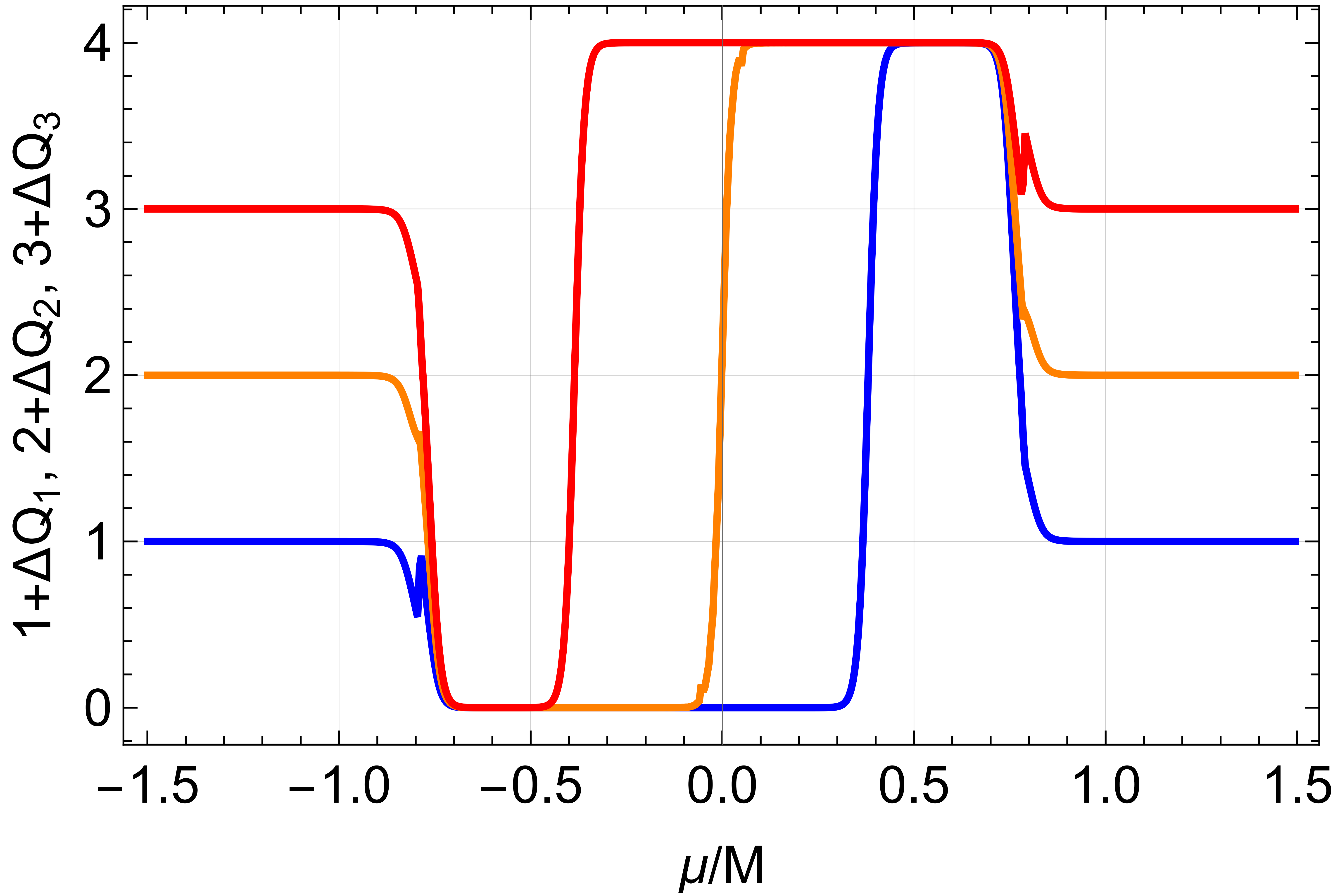
In the limit $T \to 0$, one finds a step function equal to:

- 0 for $\frac{\mu}{M} < 1 - \frac{2\nu}{N_c}$, the ν quarks are absorbed into ν meson-like states;

- N_c for $\frac{\mu}{M} > 1 - \frac{2\nu}{N_c}$, the ν quarks are absorbed into 1 baryon-like state.

Example of SU(4)

ν can take the values 1, 2 or 3 and thus $1 - 2\nu/N_c$ equals $1/2$, 0 or $-1/2$.



Conclusions

- We have identified a (theoretical) observable that is sensitive to the net quark number content of the relevant degrees of freedom in the confined and deconfined phases of strongly interacting matter.

- In particular, it points to the fact that the relevant degrees of freedom in the confined phase have the same net quark number than actual mesons or baryons.

- It would be interesting to test this observable in other regions of the phase diagram, such as for instance the super-conducting phases.

- It could be possible to test our claims in the formal heavy-quark regime of QCD where numerical simulations at finite μ are possible.

Conclusions

- One out of many observables that one could consider to probe the nature of the relevant degrees of freedom:

$$\int \mathscr{D}[A,\psi,\bar{\psi}]\, \mathscr{P}\, \exp\left\{ i\int_0^\beta d\tau A_0^a(\tau,\vec{x})t^a \right\} \times Q_{\text{net quark number}}$$

$$\int \mathscr{D}[A,\psi,\bar{\psi}]\, \mathscr{P}\, \exp\left\{ i\int_0^\beta d\tau A_0^a(\tau,\vec{x})t^a \right\} \times Q_{\text{any other conserved charge}}$$

or $$\int \mathscr{D}[A,\psi,\bar{\psi}]\, \mathscr{P}\, \exp\left\{ i\int_0^\beta d\tau A_0^a(\tau,\vec{x})t^a \right\} \times Q^2_{\text{any other conserved charge}}$$

THANK YOU!