Stabilizing Solitonic Dynamics of Ultracold Gases in the BCS-BEC crossover

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- Motivation
- Typical soliton problem
- Effective theory for time evolution in the BCS-BEC crossover (inspired in LPDA)
- Some preliminary results
- Conclusions

Motivation

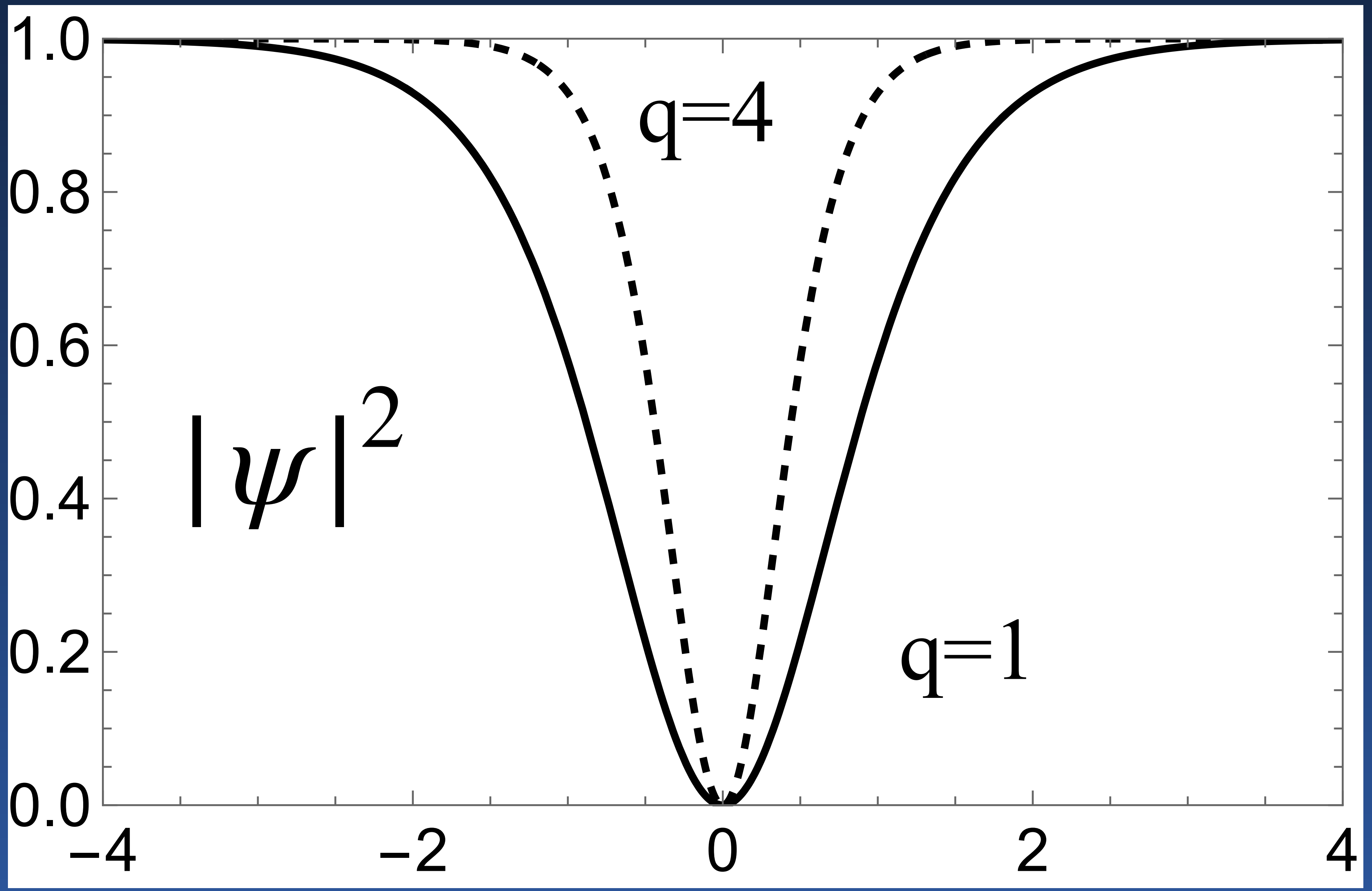
- As it has been shown along the conference it is posible to seed and manipulate defects in ultracold fermionic systems by optical means and one can print density and phase patterns with micro-mirror devices (MMDS) and spatial light modulators (SLM). These are interesting novel conditions to seed dynamical protocols.
- Besides from manipulating vortices in 2D and other topological objects, a simple test setup can be a solitonic solution in an elongated system. There are well know solutions to this problem and has been extensive explored over the past decades. This gives an ideal candidate to explore possible effective theories for effective simulation, as methods like BdG and self consistent density methods can be computationally very expensive to use.
- Recent developments with equilibrium theory (LPDA and related extensions) motivates the exploration of these ideas in a time dependent extension in the non-equilibrium setup.
- We have some preliminary tests we want to share :)

Dark soliton solution of the GPE
(No potential)

$$i\hbar\partial_t\psi = -\frac{1}{2}\partial_x^2\psi + q\,|\psi|^2\psi, \quad q > 0$$

Supports the dark soliton solution:

$$\psi = \exp(-iqt/\hbar)\tanh\left(\sqrt{q}x\right)$$



With trapping potential is not necessary stable…

Theoretical basis (LPDA)

Based on the works of Strinati, Pieri, Simonucci, Pisani and Piselli in the BCS BEC crossover. PRB 89, 054511 (2014) we take as basis.

In the ground state following the BCS-BEC formalism with coarse graining one can write:

$$-\frac{m}{4\pi a_F}\,\Delta(\mathbf{r}) = \mathcal{I}_0(\mathbf{r})\,\Delta(\mathbf{r}) + \mathcal{I}_1(\mathbf{r})\,\frac{\nabla^2}{4m}\Delta(\mathbf{r}) - \mathcal{I}_1(\mathbf{r})\, i\,\frac{\mathbf{A}(\mathbf{r})}{m}\cdot\nabla\Delta(\mathbf{r})$$

$$\xi(\mathbf{k}|\mathbf{r}) = \frac{\mathbf{k}^2}{2m} - \bar{\mu}(\mathbf{r}) \quad E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}) = E(\mathbf{k}|\mathbf{r}) - \frac{\mathbf{k}\cdot\mathbf{A}(\mathbf{r})}{m}$$

$$E(\mathbf{k}|\mathbf{r}) = \sqrt{\xi(\mathbf{k}|\mathbf{r})^2 + |\Delta(\mathbf{r})|^2}$$

$$\mathcal{I}_0(\mathbf{r}) = \int \frac{d\mathbf{k}}{(2\pi)^3}\left\{\frac{1 - 2f_F(E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}))}{2\,E(\mathbf{k}|\mathbf{r})} - \frac{m}{\mathbf{k}^2}\right\}$$

$$\mathcal{I}_1(\mathbf{r}) = \frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^3}\left\{\frac{\xi(\mathbf{k}|\mathbf{r})}{2\,E(\mathbf{k}|\mathbf{r})^3}\left[1 - 2f_F(E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}))\right] + \frac{\xi(\mathbf{k}|\mathbf{r})}{E(\mathbf{k}|\mathbf{r})^2}\,\frac{\partial f_F(E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}))}{\partial E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r})} - \frac{\mathbf{k}\cdot\mathbf{A}(\mathbf{r})}{\mathbf{A}(\mathbf{r})^2}\,\frac{1}{E(\mathbf{k}|\mathbf{r})}\,\frac{\partial f_F(E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}))}{\partial E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r})}\right\}$$

This gives the correct limits in Ginzburg-Landau, BEC and BCS limits as shown in 2014 and subsequent works.

Effective dynamical equation

After some algebra and some redefinitions it is convenient to re-write ($A = 0$):

$$-\frac{1}{2}\nabla^2\Delta + 2V\Delta + \left(\tilde{I}_2|\Delta| - 2\pi\eta\tilde{I}_3\sqrt{|\Delta|} + 2\eta^2\right)\Delta = \mu_B\Delta$$

It follows, we propose the time dependent extension:

$$\boxed{-\frac{1}{2}\nabla^2\Delta + 2V\Delta + \left(\tilde{I}_2|\Delta| - 2\pi\eta\tilde{I}_3\sqrt{|\Delta|} + 2\eta^2\right)\Delta = i\hbar\partial_t\Delta}$$

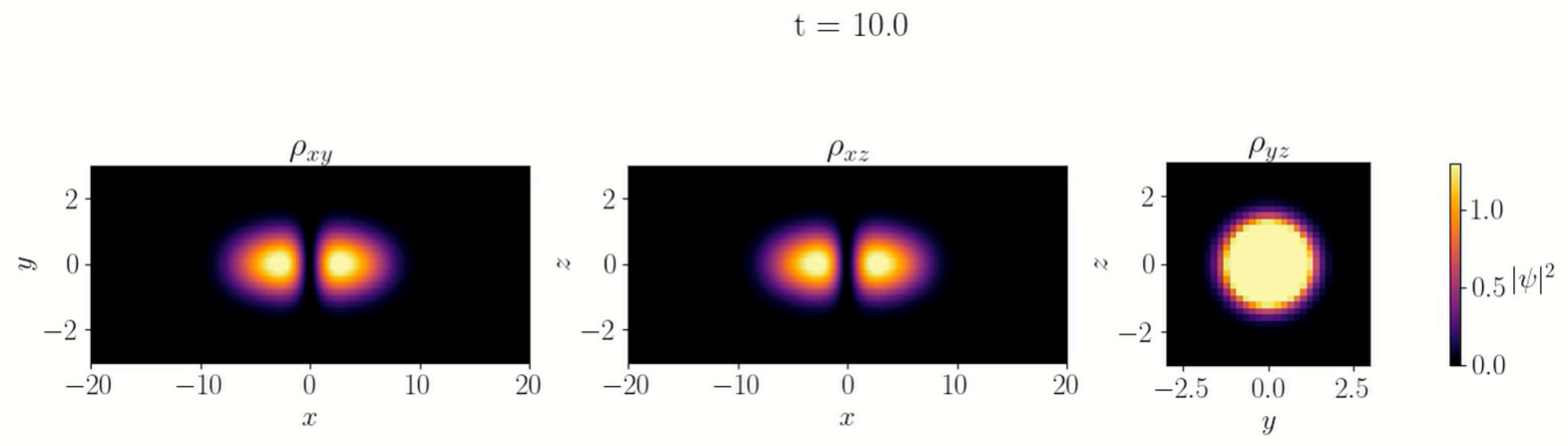
with $\eta = 1/k_F a$ and

$$\tilde{I}_2 = \frac{2\left(\mu - V - \frac{|\nabla\phi|^2}{4}\right)}{\Delta} - \frac{2\mathcal{I}_0}{\mathcal{I}_1}, \quad \tilde{I}_3 = \frac{\pi}{4\mathcal{I}_1}$$

For $T = 0$, $\tilde{I}_{2,3}$ are some non-linear monotonic functions

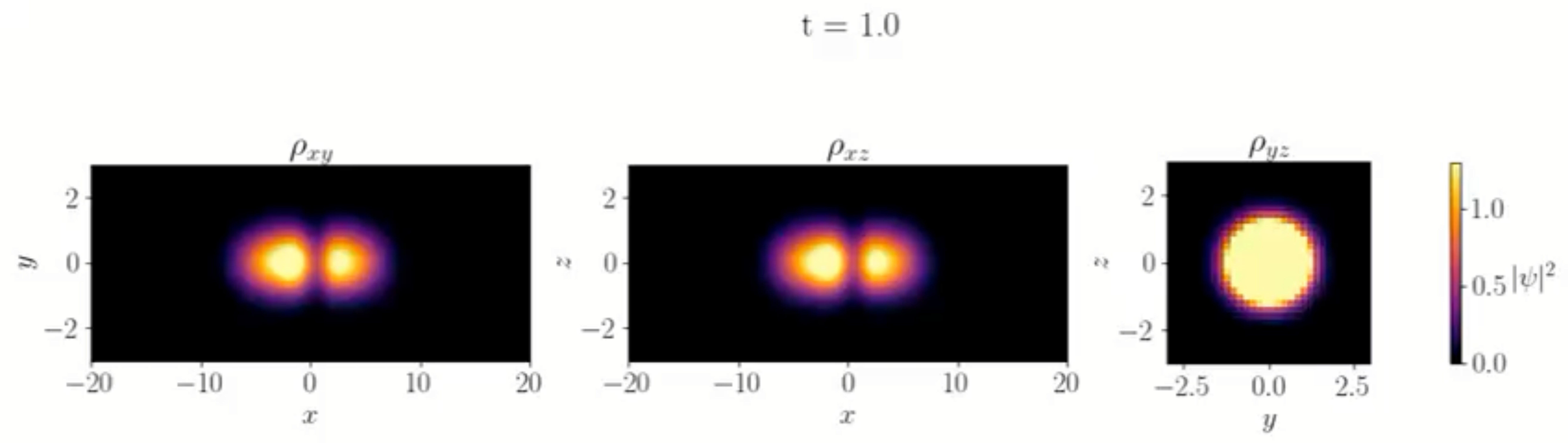
BEC Simulations

Dark Soliton



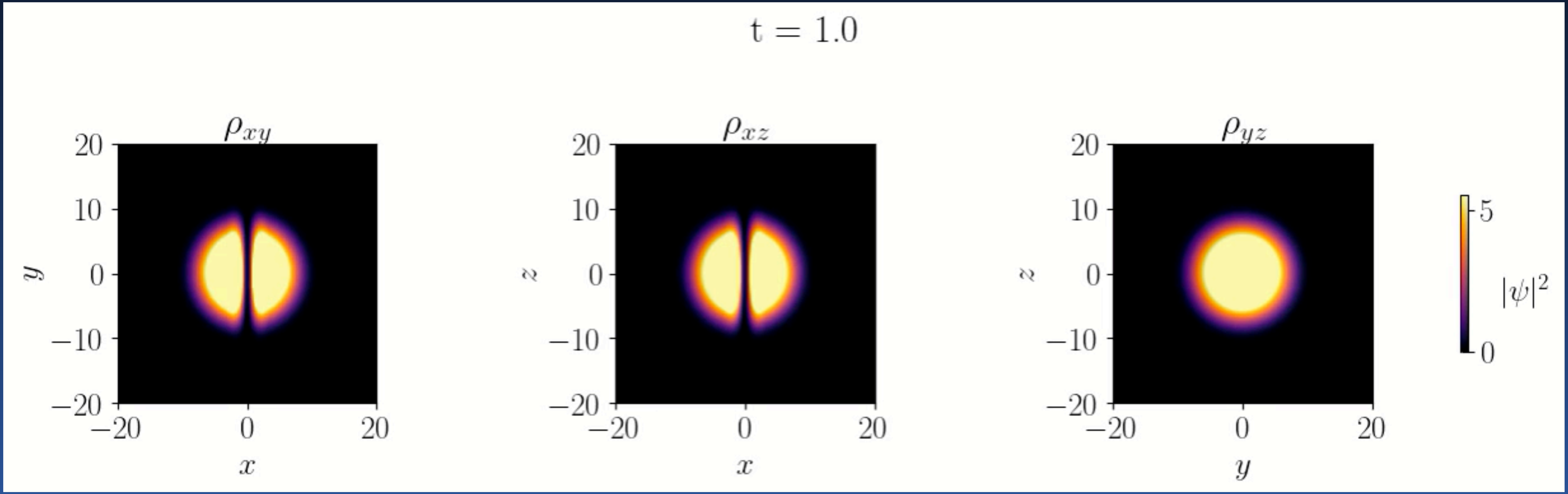
$$\omega_y = \omega_z \quad \omega_x = \frac{\omega_y}{10}$$

Grey Soliton (finite v)



$$\omega_y = \omega_z \quad \omega_x = \frac{\omega_y}{10} \quad v \neq 0$$

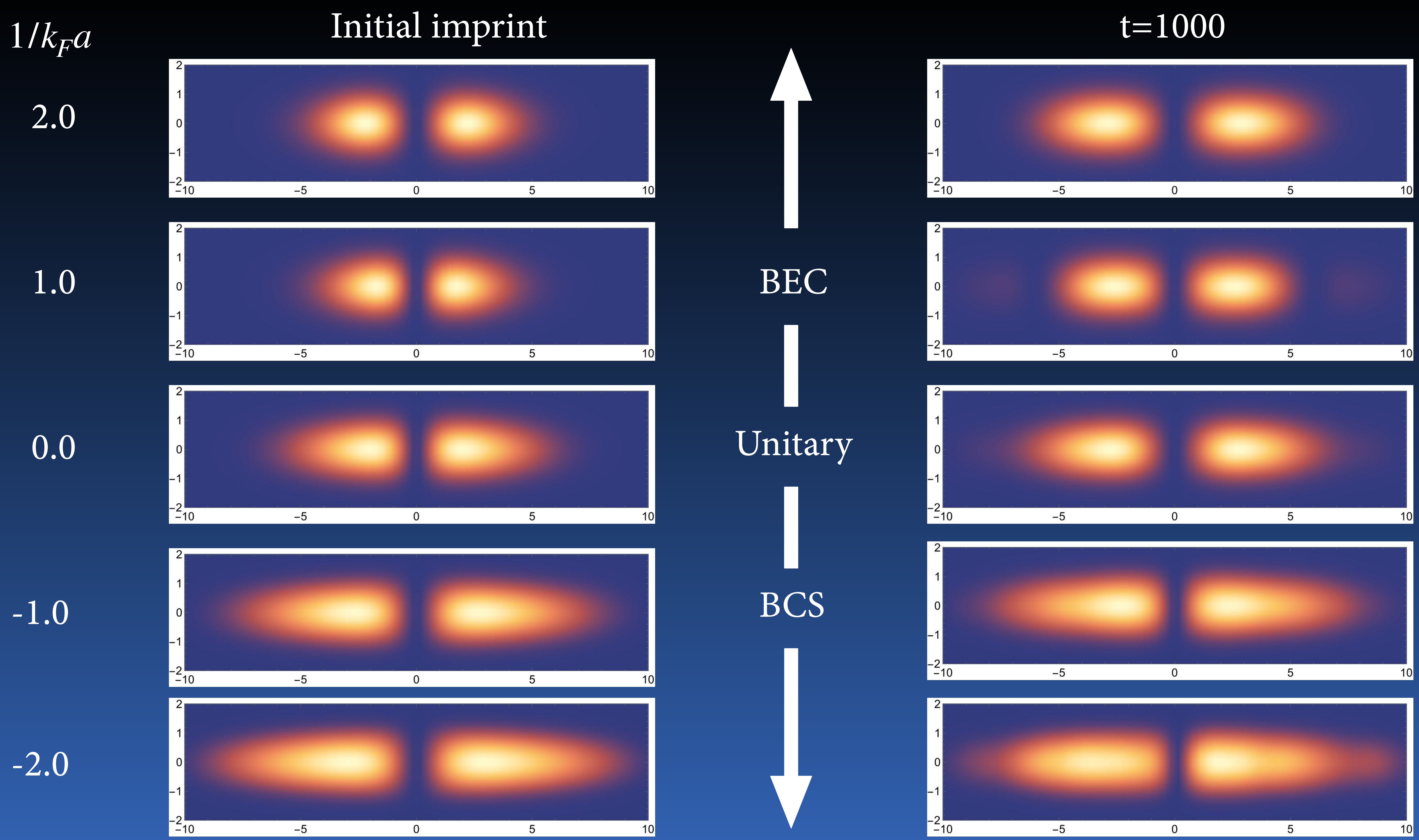
Snake instability



$\omega_y = \omega_z = \omega_x$

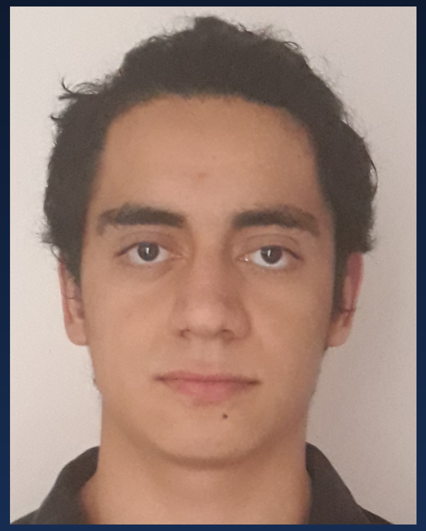
BCS-BEC Simulations

(in 3D)

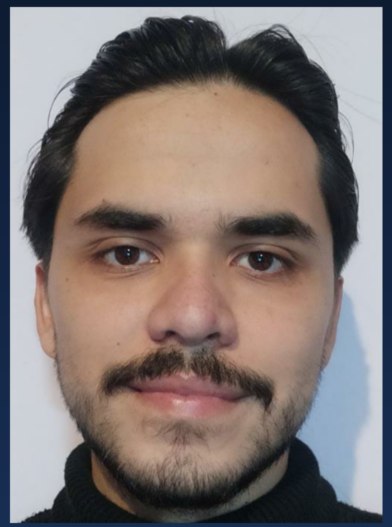

$1/k_F a$
Initial imprint
t=1000
2.0
1.0
0.0
-1.0
-2.0
BEC
Unitary
BCS

Current students

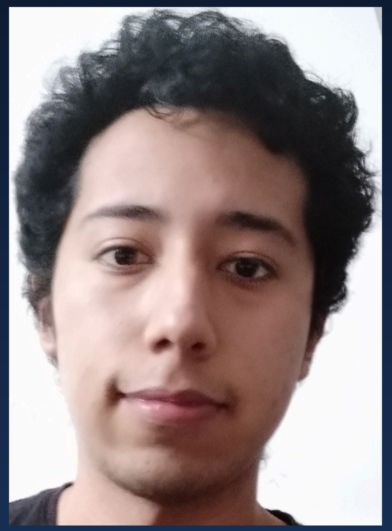
MSc.



Ulises Ramírez



Humberto E. Hernández



Ronaldo Navarro

Computational simulation and algorithm development for quantum systems

Quantum optical lattices
Light-matter interaction

PhD.

Brahyam Ríos

Cesar Terán

Analog Quantum Simulation
Novel states of quantum matter

Conclusions

- We have a potentially useful method to explore dynamics of Fermi systems in the BCS-BEC crossover scenario at $T = 0$.

- The method is amenable to further refine to consider finite T and possibly additional beyond MFT corrections.

- The method allows to have some qualitative simulations to compare with other more sophisticated methods.

- The effective theory has been implemented in GPU's and runs smoothly.

Recent articles:

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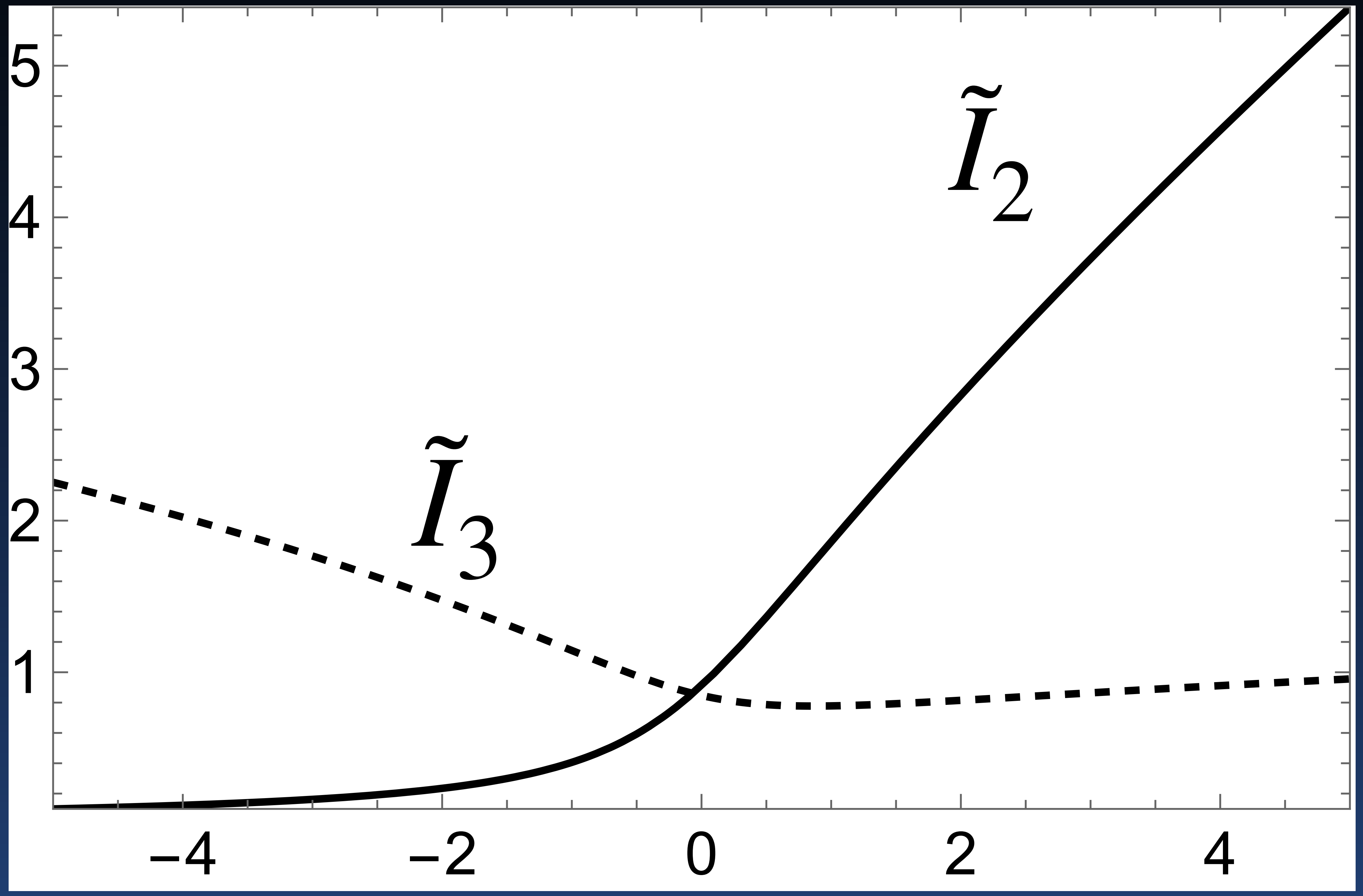
A. Camacho-Guardian, R. Paredes and S. F. Caballero-Benitez, PRA 96, 051602(R)(2017)

¡Gracias!



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