Observation of distinct dynamical scalings in turbulent Bose-Einstein condensates

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Nonequilibrium phenomena in superfluid systems: atomic nuclei, liquid helium, ultracold gases, and neutron stars , May 12–16 2025, ECT*

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CePOF

- Group Leader: Prof. Vanderlei Bagnato
- Optics and Photonics Research Center
- University of São Paulo (USP) at São Carlos, Brazil

Examples of turbulent flows

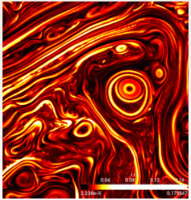
Airplanes (link)

Ocean waves (link)

Wind turbines (link)

Smoke (link)

Von Karman vortices (link)



Solar wind (link)

Interstellar medium (link)

Interest in turbulence

Mathematics

- Solutions of the Navier-Stokes equations
  - Regularity
  - Uniqueness
  - Limiting cases
- One of the Millennium Prize Problems

Physics

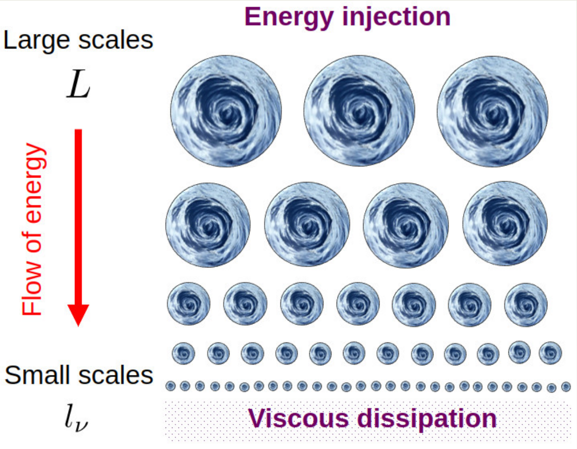
- Microscopical understanding
- Multiscale modeling
- Predicting statistics for non-equilibrium turbulent flows
- Transition to turbulence (and the other way around)
- ...

Engineering

- Turbulent models for use in industry: mixing
- Weather forecast
- Drag reduction
- ...

Richardson's cascade

- Richardson was the first to propose that the phenomenology of 3D turbulence is related to a forward cascade
- An external source injects the kinetic energy at large length scales
- The eddies interact, break up into smaller structures, and the process repeats itself down to small length scales, where energy is dissipated by viscous forces



Courtesy of J. Laurie

L.F. Richardson, Weather Prediction by Numerical Process (Cambridge University Press, Cambridge, 1922), Vol. 5.

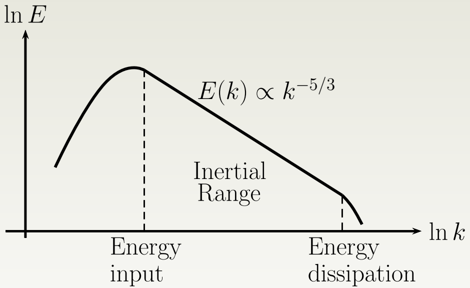
Kolmogorov's theory of turbulence

- Kolmogorov developed a theory to predict the statistical properties of turbulent flows
- Assumptions
  - Inertial range
  - Large Reynolds number
  - Homogeneous and isotropic fluid
- Energy in momentum space:

$$E_{\text{total}} = \int d^3k \, E_{3\text{D}}(\mathbf{k}) = \int dk \, E(k)$$

- Kolmogorov's energy spectrum:

$$\boxed{E(k) = C\varepsilon^{2/3} k^{-5/3}}$$



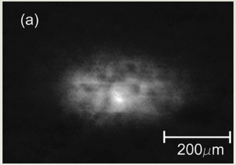
A. N. Kolmogorov, Dokl. Akad. Nauk SSSR 30, 299 (1941)

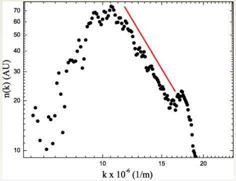
Turbulence in quantum fluids - why?

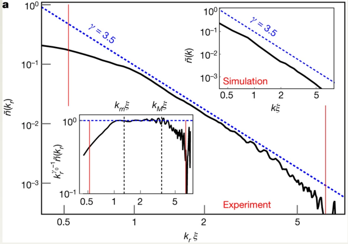
- Larger inertial range: larger system or fluids with smaller viscosities
- Superfluid helium and atomic Bose-Einstein condensates
- Superfluidity: flow without viscous dissipation
  - Other mechanisms for dissipation
- Different types of turbulence
  - With/without classical analogues
- Vortices are quantized

Quantum turbulence in trapped BECs - experiments

- Observation of QT in a trapped BEC
- Power-law in the momentum distribution
- São Carlos group: harmonic trap
- Cambridge group: box trap

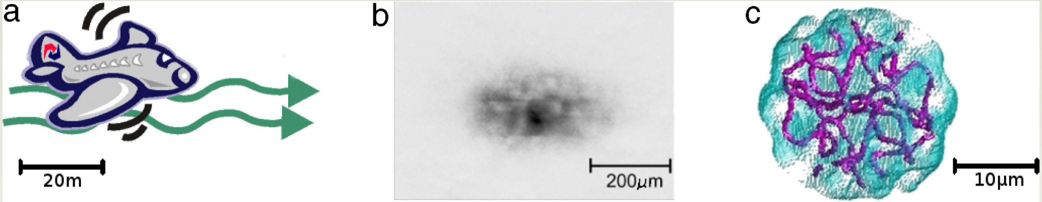






E. A. L. Henn et al., "Emergence of turbulence in an oscillating Bose-Einstein condensate", Phys. Rev. Lett. **103** (2009).
K. J. Thompson et al., "Evidence of power law behavior in the momentum distribution of a turbulent trapped Bose–Einstein condensate", Laser Phys. Lett. **11** (2014).
N. Navon et al., "Emergence of a turbulent cascade in a quantum gas", Nature **539** (2016).

Quantum turbulence in trapped BECs



- Trapped BECs - Intrinsic difficulties
  - finite size
  - probing the turbulent cloud
  - type of turbulence
  - inhomogeneity
- Power-law behavior: $n(k) \propto k^{-\delta}$

Objective

- Find alternative ways to identify and characterize quantum turbulence in trapped BECs

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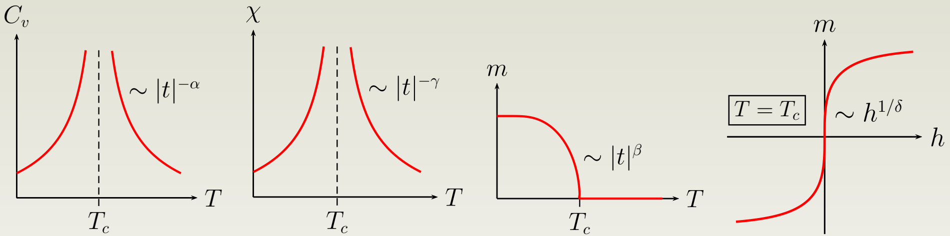
M. C. Tsatsos et al., “Quantum turbulence in trapped atomic Bose–Einstein condensates”, Phys. Rep. **622** (2016).

Renormalization group theory in equilibrium critical phenomena



- In the vicinity of a phase transition: correlations are self-similar (independent of the resolution)
- Scaling the spatial resolution by a parameter s
  - Correlation function that depends on a distance x between two points
  - $C(x;s) = s^{\zeta} f(x/s)$
  - The correlation is characterized by a universal exponent ζ and function f
  - Fixed-point: varying s does not change $C(x;s) \to f(x) \propto x^{\zeta}$

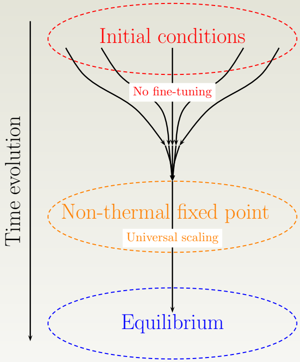
C.-M. Schmied et al., "Non-thermal fixed points: Universal dynamics far from equilibrium", Int. J. Mod. Phys. A **34** (2019).

Non-thermal fixed points

- Analogy with RG theory in equilibrium critical phenomena
- Far-from-equilibrium closed quantum system
- Some observables are insensitive to initial conditions and system parameters → universal scaling → presence of a NTFP
- NTFP: the time t is the scale parameter
- Correlations: $C(x,t) = t^{-a} f(t^{-b} x)$
- Spatio-temporal scaling emerges:

$$n(k,t) = \left(\frac{t}{t_0}\right)^{\alpha} n\left[\left(\frac{t}{t_0}\right)^{\beta} k, t_0\right]$$

- Universality class: α and β



C.-M. Schmied et al., "Non-thermal fixed points: Universal dynamics far from equilibrium", Int. J. Mod. Phys. A **34** (2019).

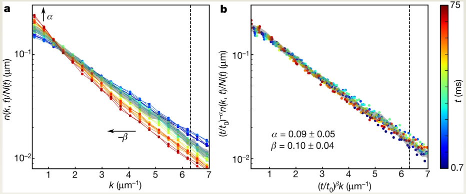
Observing universal scaling

- There are several theoretical investigations: here I will focus on experiments with cold gases

Universal dynamics in an isolated one-dimensional Bose gas far from equilibrium

Sebastian Erne, Robert Bücker, Thomas Gasenzer, Jürgen Berges & Jörg Schmiedmayer

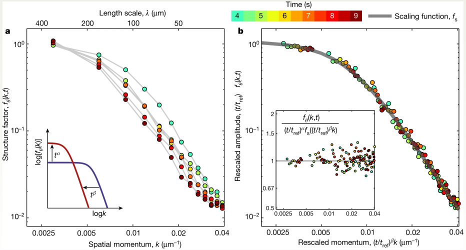
Nature **563**, 225–229 (2018) | Cite this article



Observation of universal dynamics in a spinor Bose gas far from equilibrium

Maximilian Prüfer, Philipp Kunkel, Helmut Strobel, Stefan Lannig, Daniel Linnemann, Christian-Marcel Schmied, Jürgen Berges, Thomas Gasenzer & Markus K. Oberthaler

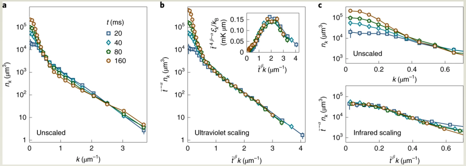
Nature **563**, 217–220 (2018) | Cite this article



Bidirectional dynamic scaling in an isolated Bose gas far from equilibrium

Jake A. P. Glidden, Christoph Eigen, Lena H. Dogra, Timon A. Hilker, Robert P. Smith & Zoran Hadzibabic

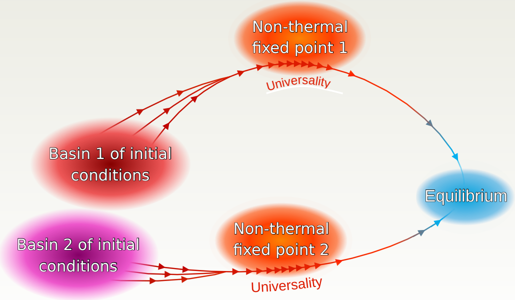
Nature Physics 17, 457–461 (2021) | Cite this article

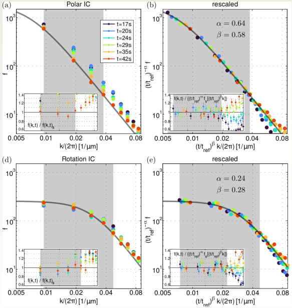


[Submitted on 28 Jun 2023]

Observation of two non-thermal fixed points for the same microscopic symmetry

Stefan Lannig, Maximilian Prüfer, Yannick Deller, Ido Siovitz, Jan Dreher, Thomas Gasenzer, Helmut Strobel, Markus K. Oberthaler





Turbulent superfluid gas

- Is the decaying quantum turbulence regime part of the far-from-equilibrium systems that display dynamical scaling due to NTFPs?

Turbulent superfluid gas

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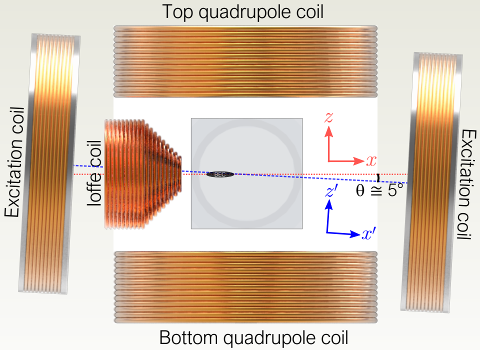
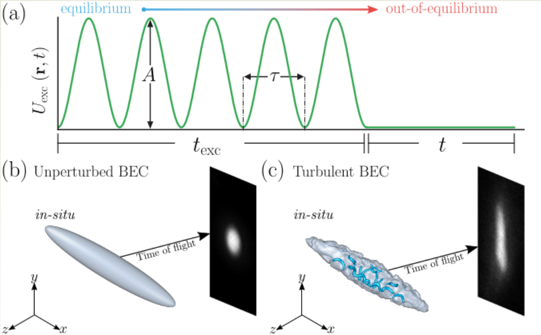
Universal dynamics of a turbulent superfluid Bose gas

A. D. García-Orozco, L. Madeira, M. A. Moreno-Armijos, A. R. Fritsch, P. E. S. Tavares, P. C. M. Castilho, A. Cidrim, G. Roati, and V. S. Bagnato
Phys. Rev. A **106**, 023314 – Published 22 August 2022

- ^{87}Rb BEC with $\approx 3.5 \times 10^5$ atoms
- Exciting a trapped BEC: time-varying magnetic field gradient

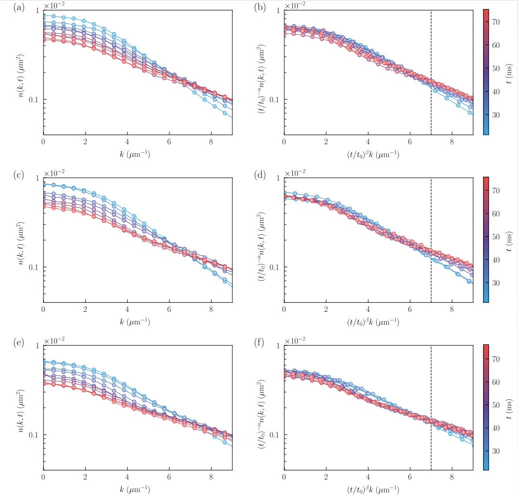




A. D. García-Orozco et al., "Universal dynamics of a turbulent superfluid Bose gas", Phys. Rev. A **106** (2022).

$$n(k,t) = \left(\frac{t}{t_{\text{ref}}}\right)^{\alpha} n\left[\left(\frac{t}{t_{\text{ref}}}\right)^{\beta} k, t_{\text{ref}}\right]$$

- $A = 1.8, 2.0, 2.2\ \mu_0$
- Same scaling exponents:
  - $\boxed{\alpha = -0.5(1) \text{ and } \beta = -0.25(7)}$
- Different initial states → same universality class



A. D. García-Orozco et al., "Universal dynamics of a turbulent superfluid Bose gas", Phys. Rev. A **106** (2022).
M. A. Moreno-Armijos et al., "Observation of relaxation stages in a nonequilibrium closed quantum system: decaying turbulence in a trapped superfluid", Phys. Rev. Lett. **134** (2025).

Increasing/decreasing the population of the condensate

- So far, the excitation protocol promoted a continuous depletion of the condensate
- Is it possible to tune our excitation protocol to observe particle transport in the opposite direction?

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- Is it possible to tune our excitation protocol to observe particle transport in the opposite direction?

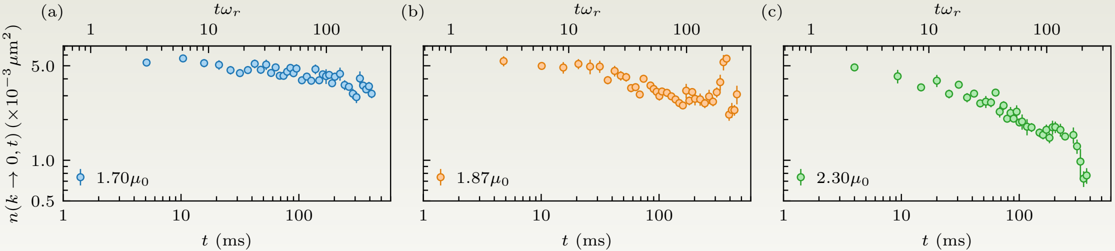
Observation of Relaxation Stages in a Nonequilibrium Closed Quantum System: Decaying Turbulence in a Trapped Superfluid

M. A. Moreno-Armijos[1,*], A. R. Fritsch[1], A. D. García-Orozco[1], S. Sab[1], G. Telles[1], Y. Zhu[2], L. Madeira[1], S. Nazarenko[2], V. I. Yukalov[1,3] V. S. Bagnato[1,4,5]

Phys. Rev. Lett. **134**, 023401 – **Published 14 January, 2025**

The $k \to 0$ limit

- $n(k \to 0, t)$ is the relevant quantity
- We varied the excitation amplitude



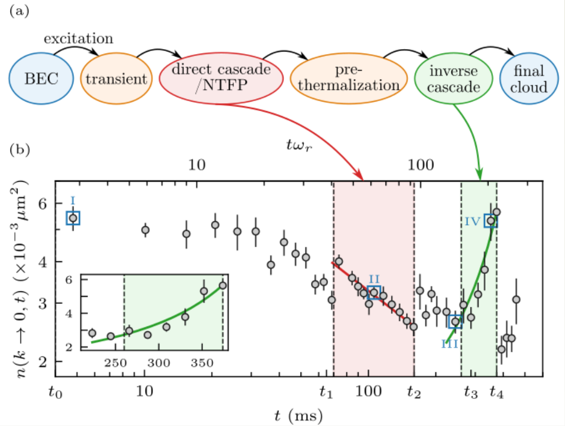
M. A. Moreno-Armijos et al., "Observation of relaxation stages in a nonequilibrium closed quantum system: decaying turbulence in a trapped superfluid", Phys. Rev. Lett. **134** (2025).

Relaxation stages

- **Direct cascade (NTFP)**

$$n(k,t) = \left(\frac{t}{t_{\text{ref}}}\right)^{\alpha} n\left[\left(\frac{t}{t_{\text{ref}}}\right)^{\beta} k, t_{\text{ref}}\right]$$

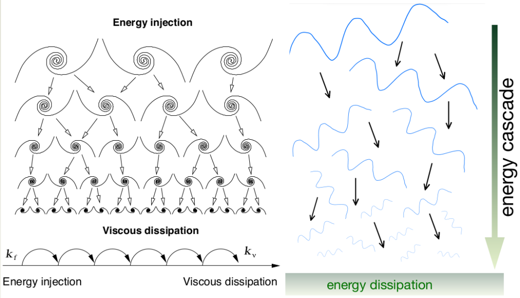
- $n(k \to 0, t) \propto t^{\alpha} \to \boxed{\alpha = -0.52(6)}$
- **Inverse cascade**



L. Madeira et al., "Universal scaling in far-from-equilibrium quantum systems: an equivalent differential approach", Proceedings of the National Academy of Sciences **121** (2024).
M. A. Moreno-Armijos et al., "Observation of relaxation stages in a nonequilibrium closed quantum system: decaying turbulence in a trapped superfluid", Phys. Rev. Lett. **134** (2025).

Wave turbulence

- Wave Turbulence (WT) is a non-equilibrium statistical system of many randomly interacting waves
- The kinetic equations of WT describe evolution of the wave energy in momentum space
- Different processes (4-waves, 3-waves, ...)
- Direct energy cascade Kolmogorov-Zakharov spectrum
  - $n(k) \propto k^{-3} \ln(k/k_f)$



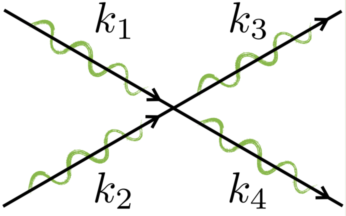
Courtesy of S. Nazarenko

S. Nazarenko, *Wave turbulence*, Vol. 825 (Springer Berlin Heidelberg, 2011).
Y. Zhu et al., “Direct and inverse cascades in turbulent Bose-Einstein condensates”, Phys. Rev. Lett. **130** (2023).
Y. Zhu et al., “Self-similar evolution of wave turbulence in Gross-Pitaevskii system”, Phys. Rev. E **108** (2023).

Wave turbulence

- Wave-kinetic equation with four-wave interactions

$$\frac{\partial n_k^{\mathrm{rad}}}{\partial t} = 2\pi \int \frac{\min(k, k_1, k_2, k_3)}{k k_1 k_2 k_3} n_k^{\mathrm{rad}} n_{k_1}^{\mathrm{rad}} n_{k_2}^{\mathrm{rad}} n_{k_3}^{\mathrm{rad}}$$
$$\times\, \delta(k^2 + k_1^2 - k_2^2 - k_3^2) \left(\frac{k^2}{n_k^{\mathrm{rad}}} + \frac{k_1^2}{n_{k_1}^{\mathrm{rad}}} - \frac{k_2^2}{n_{k_2}^{\mathrm{rad}}} - \frac{k_3^2}{n_{k_3}^{\mathrm{rad}}} \right) dk_1 dk_2 dk_3$$



Courtesy of Y. Zhu

- **Self-similar solutions**
- **First-kind (direct cascade):**
  - $n^{\mathrm{rad}}(k,t) = t^{-a} f(t^{-b} k)$
  - Same form as NTFPs!
- **Second-kind (inverse cascade):**
  - $n^{\mathrm{rad}}(k,t) = \tau^{-r} g(\tau^{-m} k)$
  - $\tau = t^* - t$

Y. Zhu et al., "Self-similar evolution of wave turbulence in Gross-Pitaevskii system", Phys. Rev. E **108** (2023).

Relaxation stages

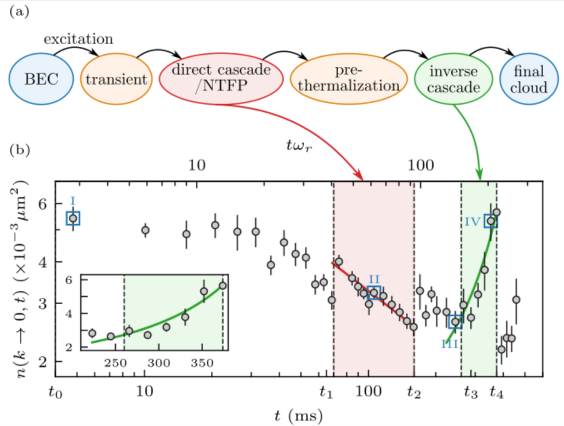
- **Direct cascade (NTFP)**

$$n(k,t) = \left(\frac{t}{t_{\text{ref}}}\right)^{\alpha} n\left[\left(\frac{t}{t_{\text{ref}}}\right)^{\beta} k, t_{\text{ref}}\right]$$

- $n(k \to 0, t) \propto t^{\alpha}$
- **Inverse cascade**

$$n(k,t) = \tau^{\lambda} n(\tau^{\mu} k, t_{\text{ref}}),\ \tau = \frac{(t_b - t)}{(t_b - t_{\text{ref}})}$$

- $n(k \to 0, t) \propto (t_b - t)^{\lambda}$



M. A. Moreno-Armijos et al., "Observation of relaxation stages in a nonequilibrium closed quantum system: decaying turbulence in a trapped superfluid", Phys. Rev. Lett. **134** (2025).

Dynamical scaling

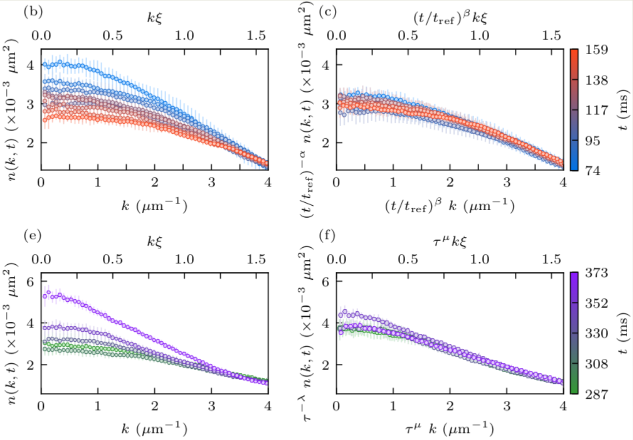
- **Direct cascade (NTFP)**

$$n(k,t) = (t/t_{\mathrm{ref}})^{\alpha} n\left[(t/t_{\mathrm{ref}})^{\beta} k, t_{\mathrm{ref}}\right]$$

- $\boxed{\alpha = -0.5(1) \text{ and } \beta = -0.25(7)}$
- **Inverse cascade**

$$n(k,t) = \tau^{\lambda} n(\tau^{\mu} k, t_{\mathrm{ref}}), \ \tau = \frac{(t_b - t)}{(t_b - t_{\mathrm{ref}})}$$

- $\boxed{\lambda = -1.5(5) \text{ and } \mu = -0.9(3)}$
- Wave turbulence theory (infinite and homogeneous system)
- $\lambda \approx -1.46$ and μ from -1.14 to -1.04



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Conclusions

- Overview of quantum turbulence and its challenges
- Alternative ways of identifying and characterizing quantum turbulence
- We observed universal behavior in an atomic superfluid driven far from equilibrium towards a turbulent state while in the vicinity of a NTFP
- Our results suggest merging the quantum turbulence regime in trapped atomic gases into a class of systems that present dynamical universality by scaling
- We observed two types of dynamical scaling in a single experimental realization due to the same microscopical process

Collaborators

- São Carlos Institute of Physics - University of São Paulo
  - Sarah Sab
  - Michelle Moreno-Armijos
  - Arnol García-Orozco
  - Amilson Fritsch
  - Gustavo Telles
  - Patricia Castilho
  - Vanderlei Bagnato
- Pedro Tavares (IFSP)
- André Cidrim (UFSCar)
- Giacomo Roati (INO-CNR)
- Ying Zhu (INPHYNI)
- Sergey Nazarenko (INPHYNI)
- Vyacheslav Yukalov (JINR)