SURPRISING VORTEX DYNAMICS IN SUPERFLUID ^{3}He: STRONG PINNING, VORTEX SHEETS AND WAVE TURBULENCE

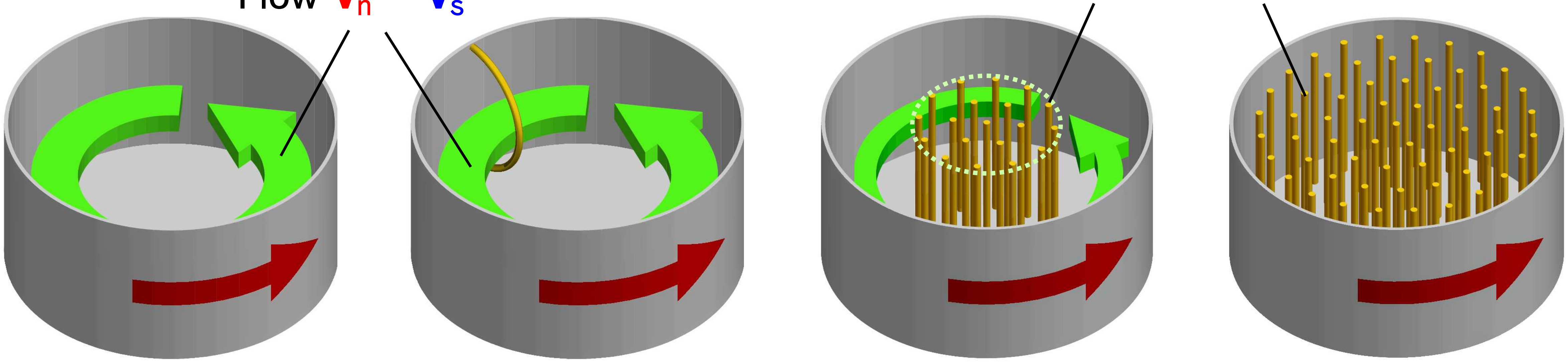
Vladimir Eltsov

Department of Applied Physics, Aalto University, Finland

Aalto University

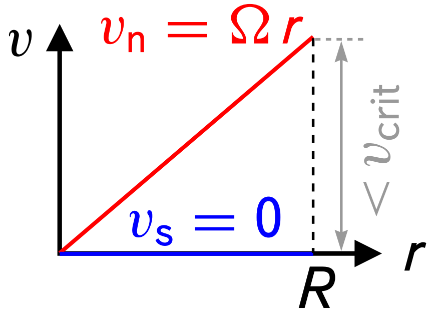
TYPICAL VORTEX DYNAMICS IN ROTATING SUPERFLUID

$\mathrm{Re}_\alpha < 1$:



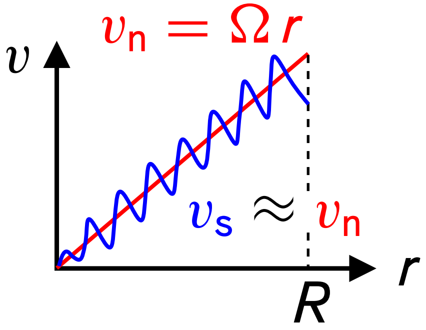
Flow $\mathbf{v}_\mathrm{n} - \mathbf{v}_\mathrm{s}$

Vortex array with density $n_\mathrm{v} = 2\Omega/\kappa$



Vortex velocity

$$\mathbf{v}_\mathrm{L} = \mathbf{v}_\mathrm{s} + \alpha'(\mathbf{v}_\mathrm{n} - \mathbf{v}_\mathrm{s})_\perp + \alpha\, \hat{\mathbf{s}} \times (\mathbf{v}_\mathrm{n} - \mathbf{v}_\mathrm{s})$$



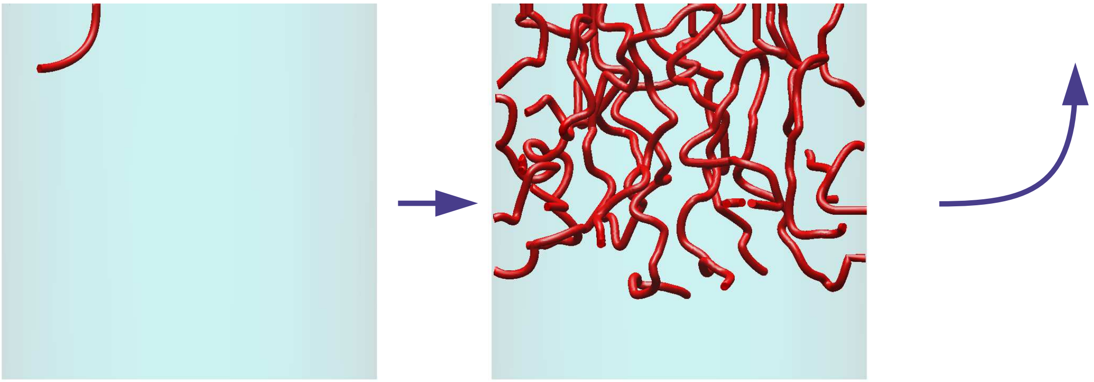
Equivalent Reynolds number

$$\mathrm{Re}_\alpha = \frac{\text{inertial}}{\text{viscous}} = \frac{1-\alpha'}{\alpha}$$

Finne *et al*, Nature **424**, 1022 (2003)

VE *et al*, PNAS **111**, 4711 (2014)

$\mathrm{Re}_\alpha \gg 1$:

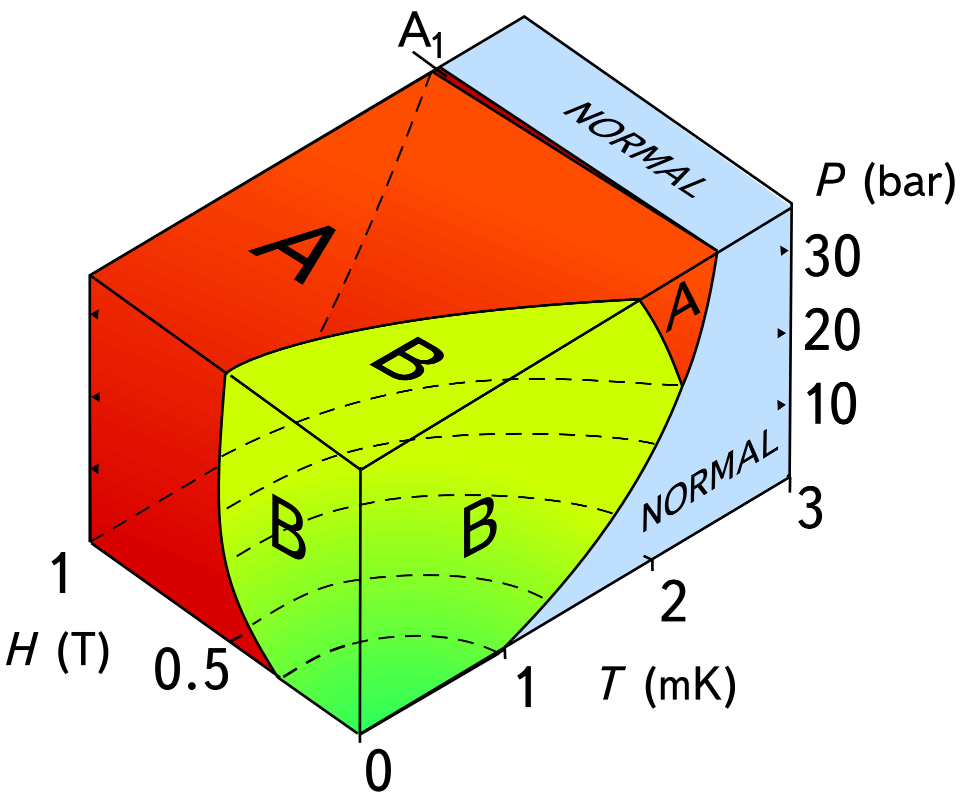


Relaxation towards equilibrium

Multiplication of a vortex via turbulent burst

TOPOLOGICAL SUPERFLUID ^{3}He

Fermi system with pairing in $L = 1$, $S = 1$ state: Several superfluid phases with multitude of topological defects including single-, double- and half-quantum vortices.



- **High viscosity of the normal component**

  $\eta \propto T^{-2}$, at 1 mK oil-like: Normal component is never turbulent

  — Clamped to walls in a typical experiment

- **Non-singular vortices with the core size $\gtrsim$ 50 nm**

  — Engineering pinning and surface friction including elimination

  — Link to the reference frame via volume interaction

- **Mutual friction from the vortex-core-bound fermions**

  — Wide range of friction from $\alpha \ll 1$ to $\alpha \gg 1$

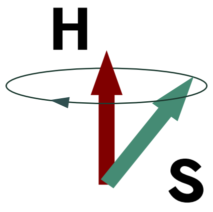
  — Link to the physics of topological matter

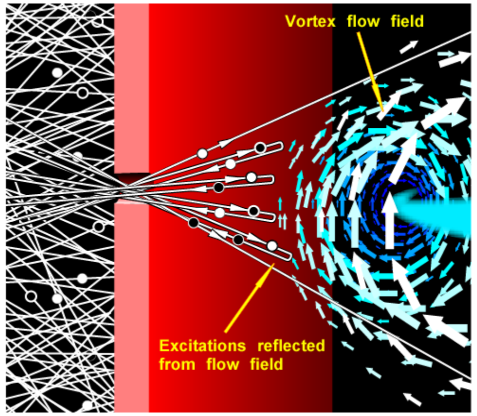
- **Versatile flow measurements**

  — Frequency shift in NMR

$$\frac{\partial \mathbf{S}}{\partial t} = \gamma \mathbf{S} \times \mathbf{H} + \mathbf{R}_D$$

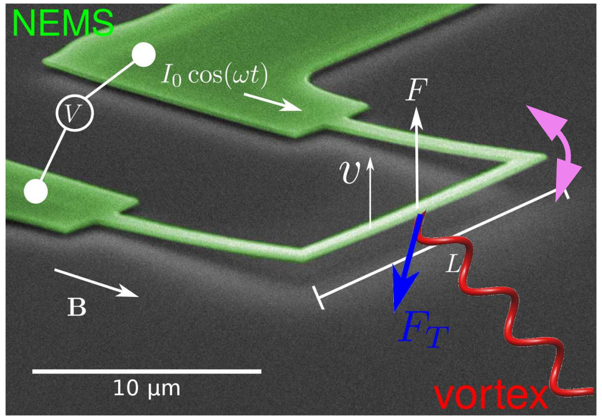
spin-orbit torque



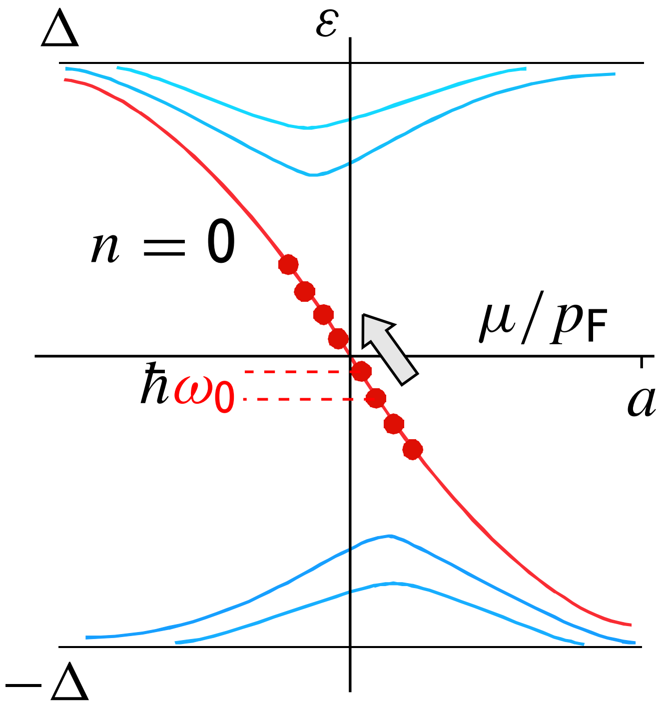


— Andreev reflection of quasiparticles

— Nanomechanics



MUTUAL FRICTION FROM CORE-BOUND FERMIONS



Core-bound states

Vortex motion leads to pumping of core-bound quasiparticles along anomalous branch with minigap ω_0.

Relaxation (τ) towards equilibrium distribution via interaction with bulk quasiparticles $\Rightarrow$ force $\mathbf{F}_N = D(\mathbf{v}_n - \mathbf{v}_L)_\perp + D'\hat{\mathbf{z}} \times (\mathbf{v}_n - \mathbf{v}_L)$

$$D = \rho\kappa \frac{\omega_0 \tau}{1+\omega_0^2\tau^2} \tanh \frac{\Delta(T)}{2T}$$

Kopnin force

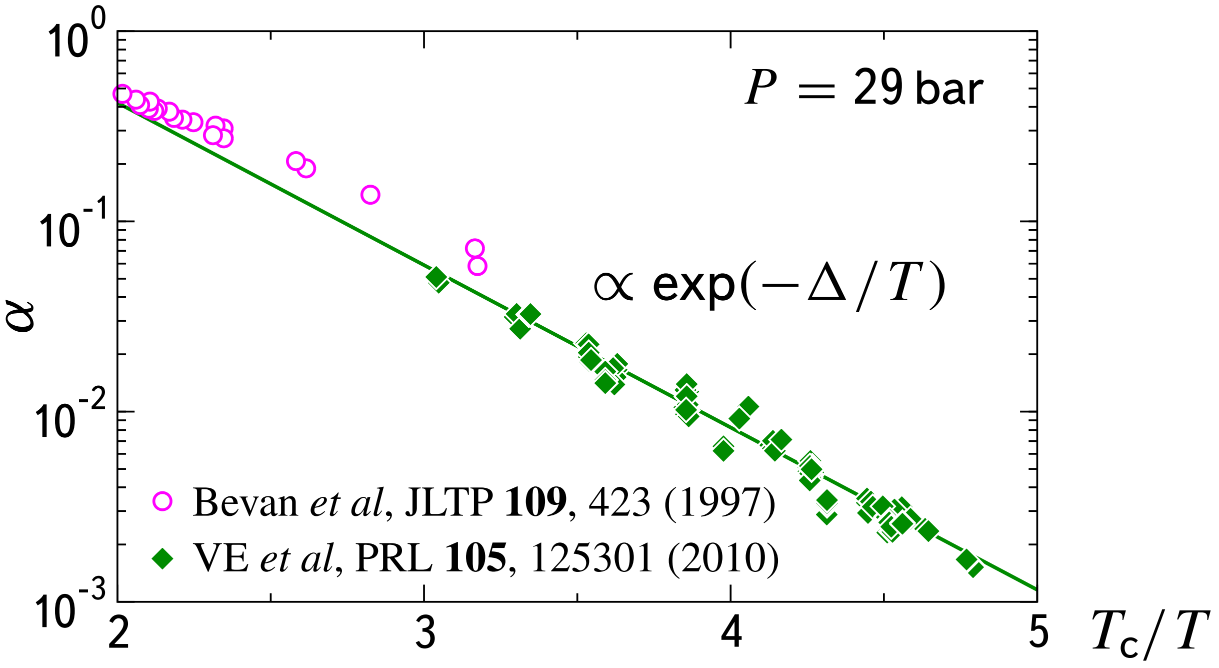
$$D' = \rho\kappa \left[1 - \frac{\omega_0^2\tau^2}{1+\omega_0^2\tau^2} \tanh \frac{\Delta(T)}{2T} \right] - \rho_n \kappa$$

Iordanskii force

$$\mathrm{Re}_\alpha = \frac{1-\alpha'}{\alpha} = \frac{1 - D'/\kappa\rho_s}{D/\kappa\rho_s} = \omega_0 \tau$$

$T \rightarrow T_c$: $\omega_0 \sim \Delta^2 / E_F \rightarrow 0 \quad \Rightarrow \mathrm{Re}_\alpha \rightarrow 0$

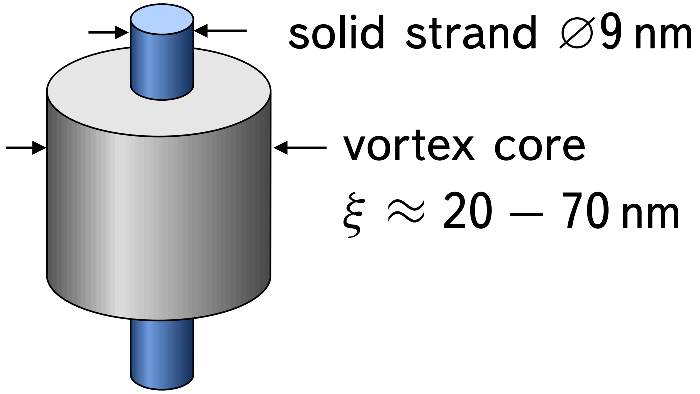
$T \rightarrow 0$: $\tau \sim \tau_n \exp \frac{\Delta}{T} \rightarrow \infty \Rightarrow \mathrm{Re}_\alpha \rightarrow \infty$



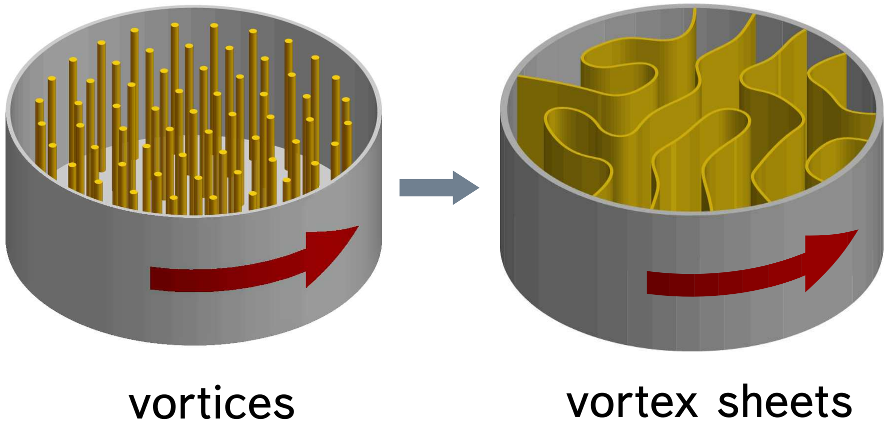
Applies: Straight vortex moving with constant velocity in infinite superfluid.

TALK FOCUS: LESS USUAL VORTEX DYNAMICS IN SUPERFLUIDS

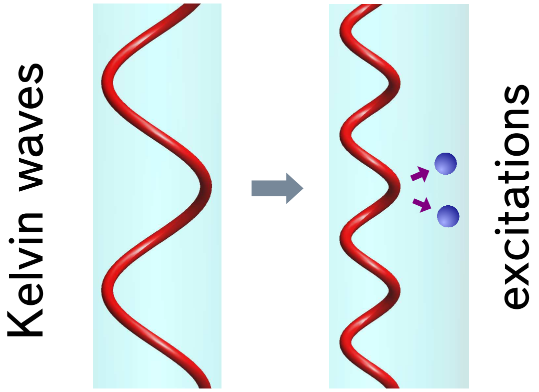
- In the polar phase, created by nanostructured confinement of superfluid ^{3}He, dynamics of single- and half-quantum vortices is drastically altered by strong pinning on the confining strands.



- In the A phase, double-quantum vortices are replaced by vortex sheets under strong dynamic drive.

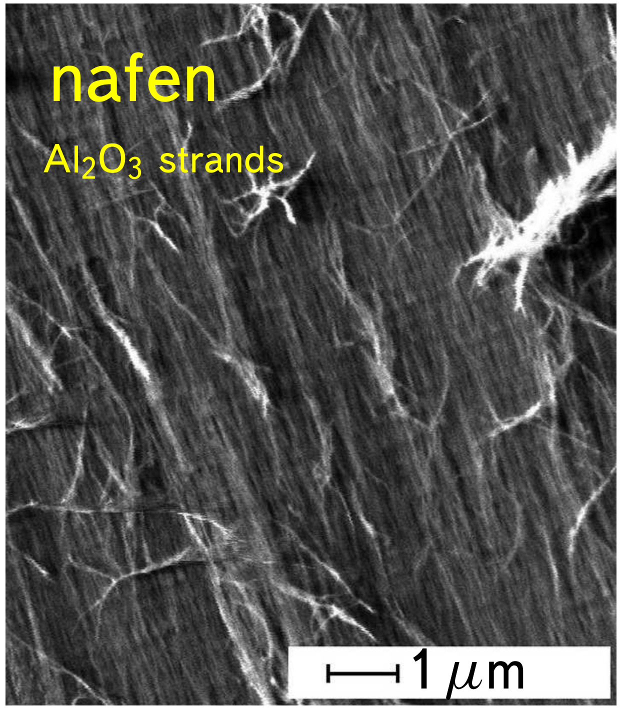


- In the B phase at the lowest temperatures, Kelvin waves can be controllaby excited on vortcies and evidence for the Kelvin-wave cascade and overheating of vortex-core-bound states is obtained.



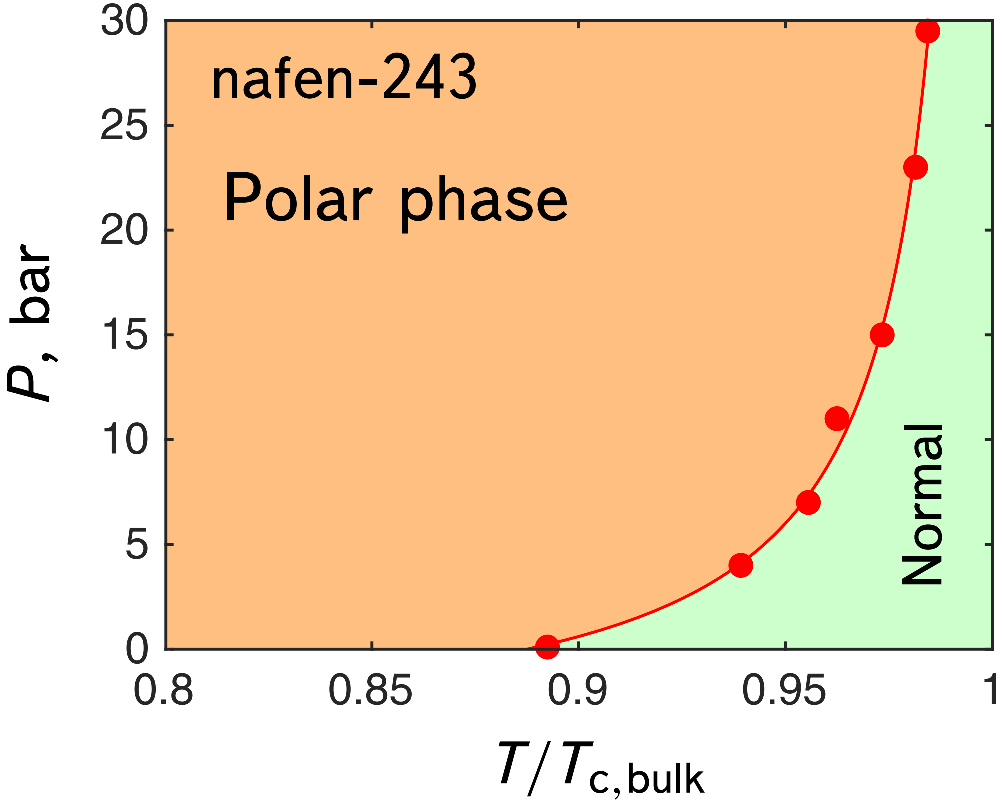
CONFINED ^{3}He: SUPERFLUID WITH STRONG VORTEX PINNING

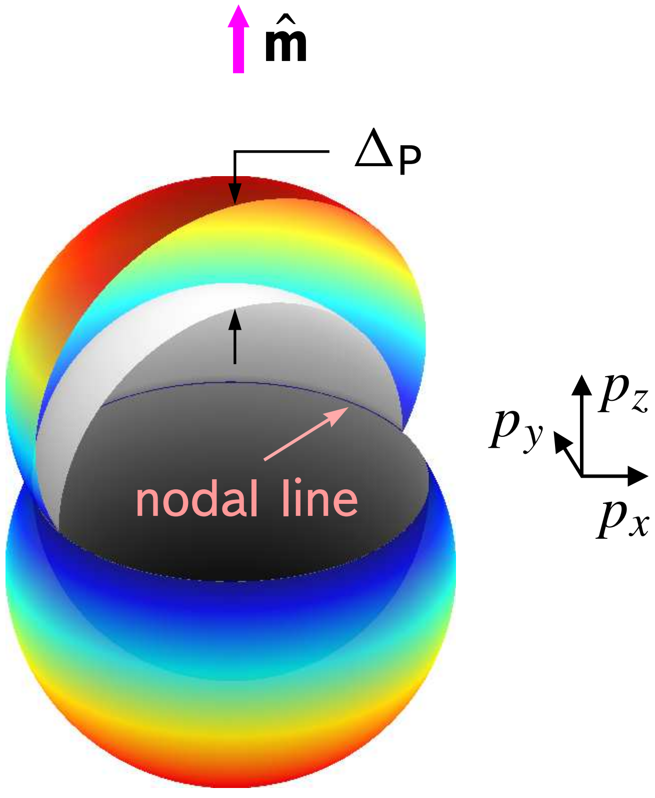
POLAR PHASE OF SUPERFLUID ^{3}He

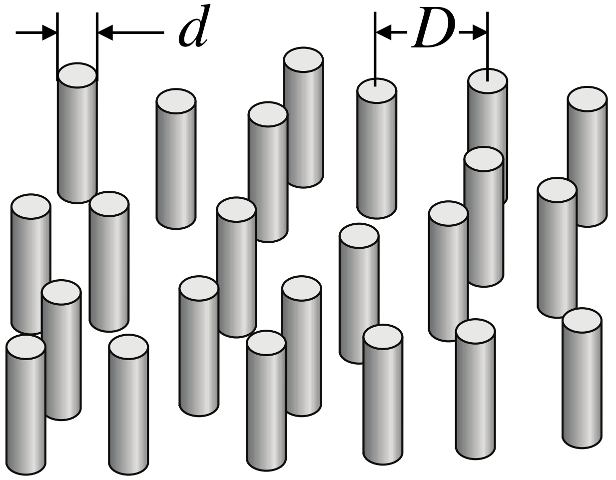


Stabilized with confinement between parallel nanostrands.

$$A_{\mu j} = \Delta_{\mathrm{P}}\, e^{i\phi}\, \hat{\mathbf{d}}_\mu\, \hat{\mathbf{m}}_j$$







	open	d, nm	$\langle D \rangle$, nm
nafen-90	98%	8	47
nafen-243	94%	9	32

With random impurities of this density superfluidity in ^{3}He will be completely suppressed!

Aoyama & Ikeda, PRB **73**, 060504 (2006) ; Dmitriev *et al*, PRL **115**, 165304 (2015)

EVIDENCE FOR ROBUST NODE LINE IN THE POLAR PHASE

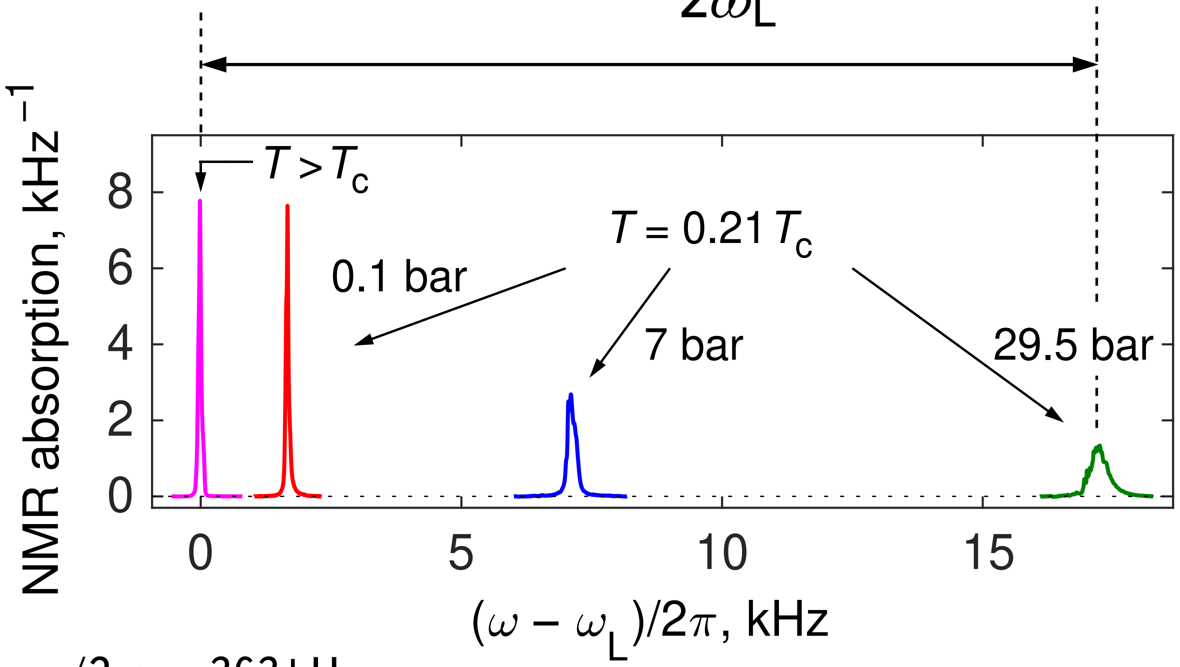
With node line in the energy spectrum: $\Delta(T)/\Delta(0) = 1 - a\,(T/T_\mathrm{c})^3$, $T \ll T_\mathrm{c}$, $a \sim 1$.

Xu, Yip & Sauls, PRB **51**, 16233 (1995)

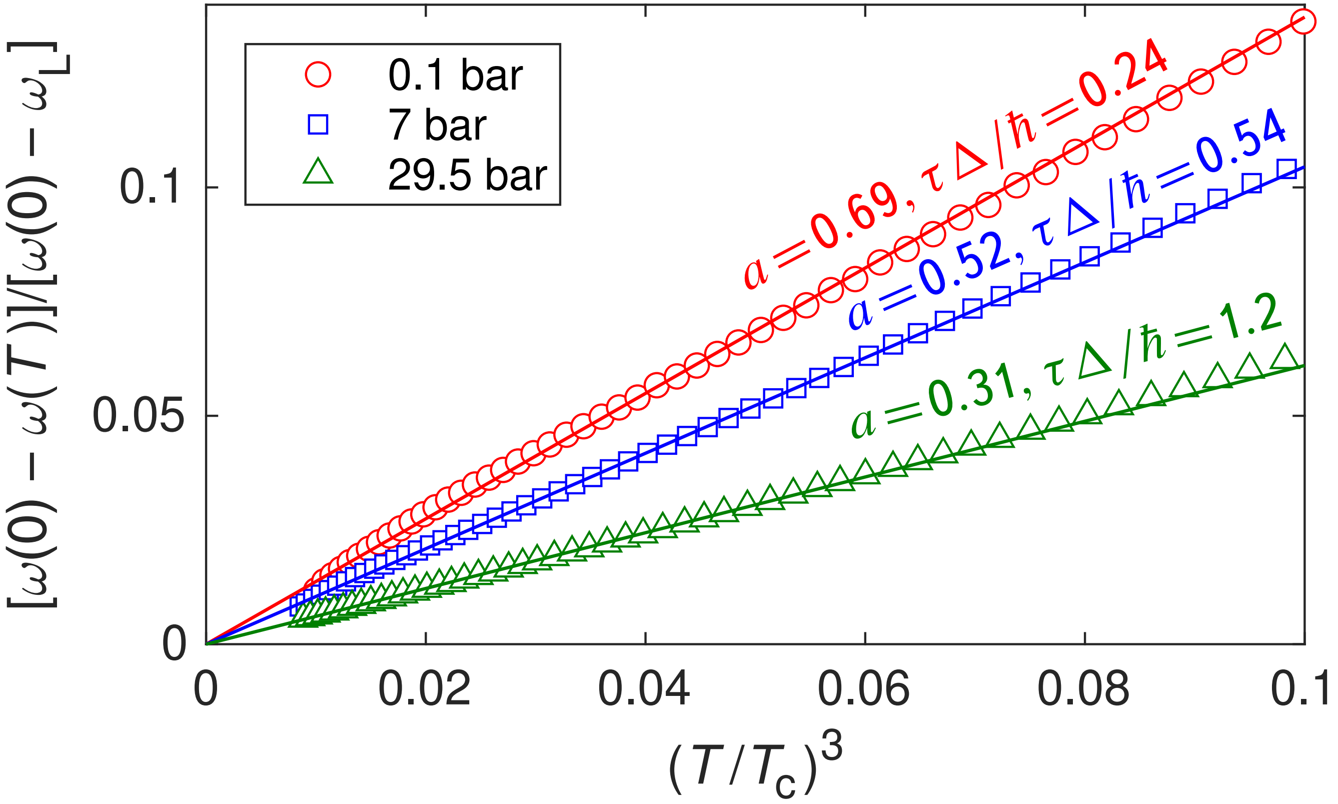
NMR frequency ω for $\mathbf{H} \parallel \hat{\mathbf{m}}$ is

$$\omega(T) - \omega_\mathrm{L} \approx \frac{\Omega_\mathrm{P}^2(T)}{2\omega_\mathrm{L}} \propto \Delta(T)^2$$

Thus $$\frac{\omega(0) - \omega(T)}{\omega(0) - \omega_\mathrm{L}} = 2a \left(\frac{T}{T_\mathrm{c}}\right)^3$$



$\omega_\mathrm{L}/2\pi = 363\,\mathrm{kHz}$



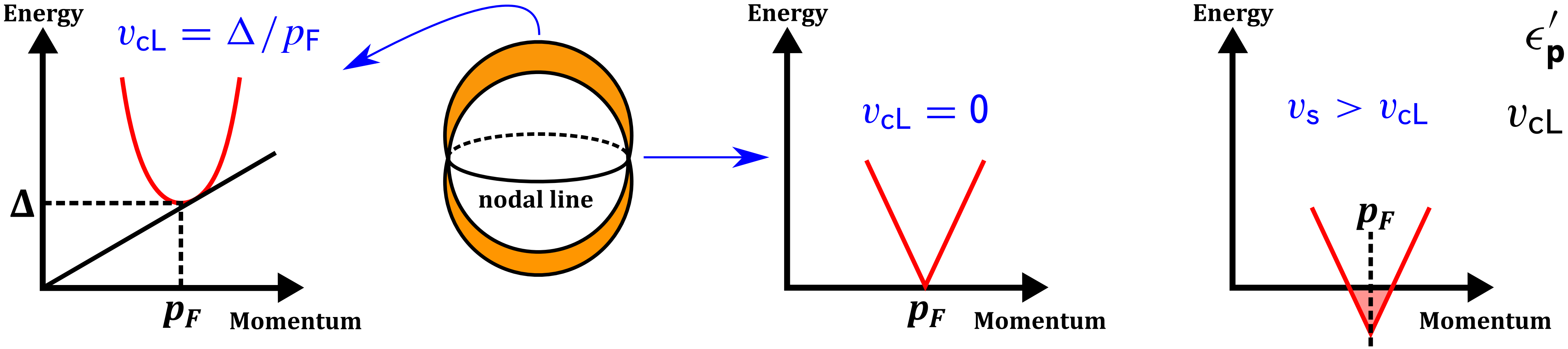
Clean-limit BCS: $a = \frac{9\pi}{2}\zeta(3)\left(\frac{T_\mathrm{c}}{\Delta(0)}\right)^3 = 0.57.$ In reality scattering is strong: $\tau\Delta/\hbar < 1$.

Nodal line is robust to impurity scattering due to extension of the Anderson theorem.

Kamppinen *et al*, Nature Commun. **14**, 4276 (2023)

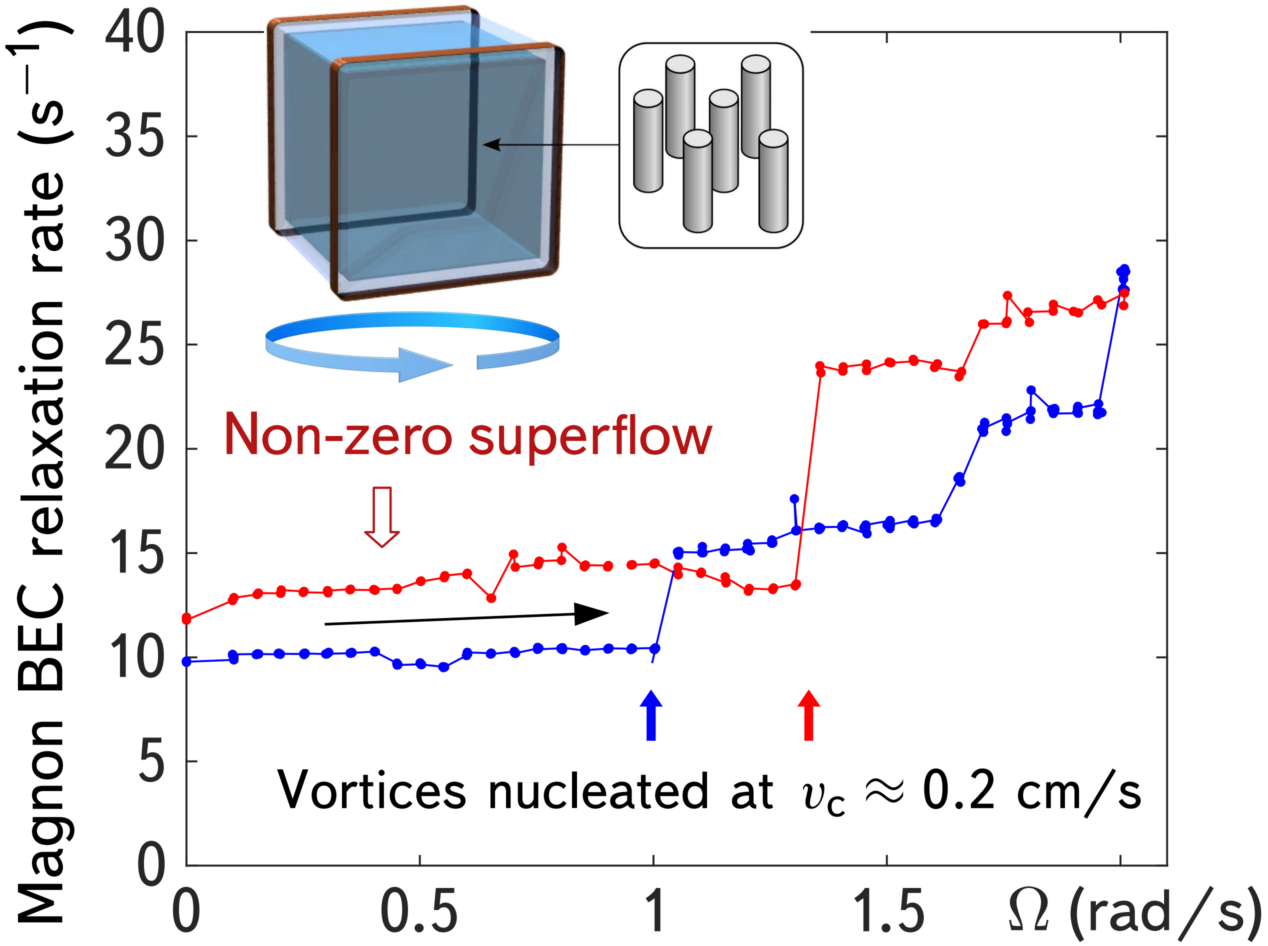
FROM THE NODAL LINE TO A BOGOLIUBOV FERMI SURFACE

In a nodal-line superfluid Landau critical velocity $v_{\mathrm{cL}} = 0$ but superflow is stable.



$$\epsilon'_{\mathbf{p}} = \epsilon_{\mathbf{p}} + \mathbf{p}\mathbf{v}_{\mathrm{s}}$$

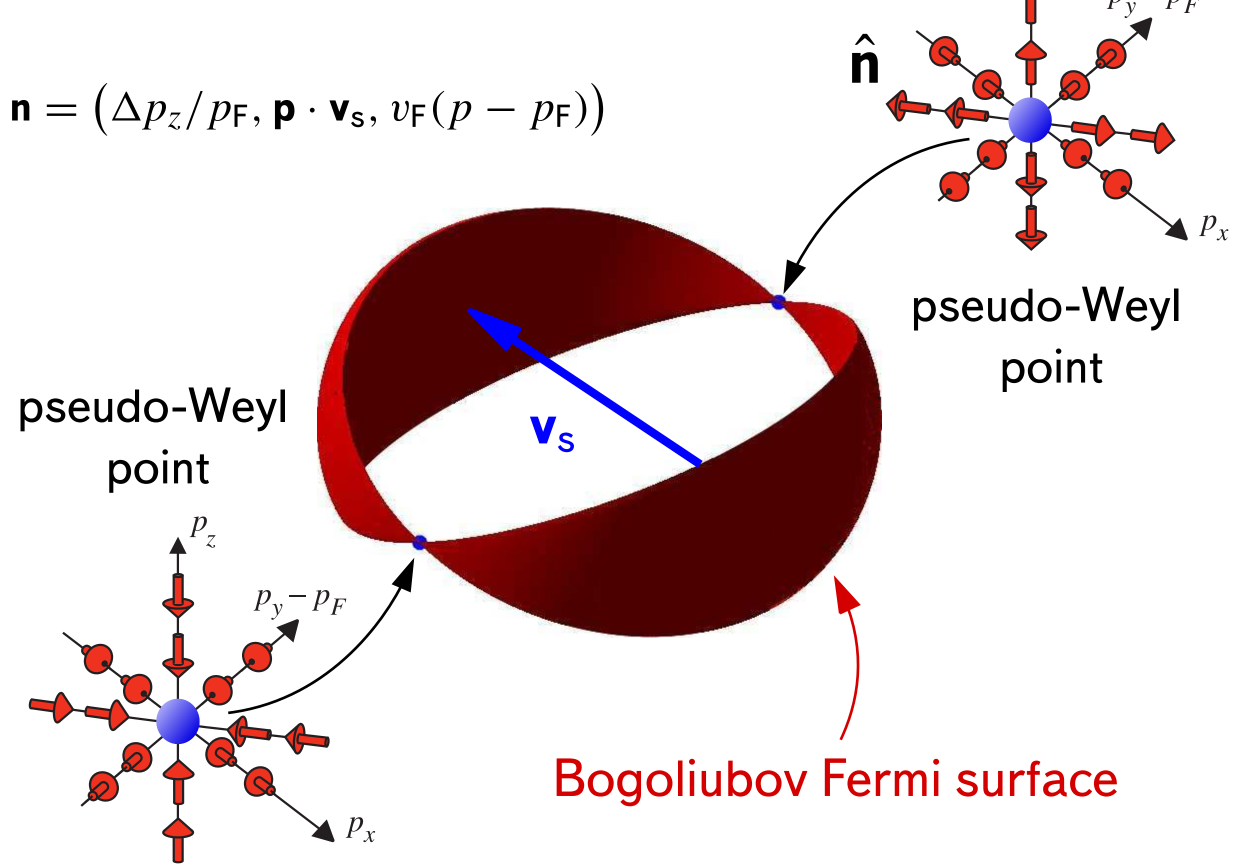
$$v_{\mathrm{cL}} = \min(\epsilon_{\mathbf{p}}/p)$$



Non-zero superflow

Vortices nucleated at $v_{\mathrm{c}} \approx 0.2$ cm/s

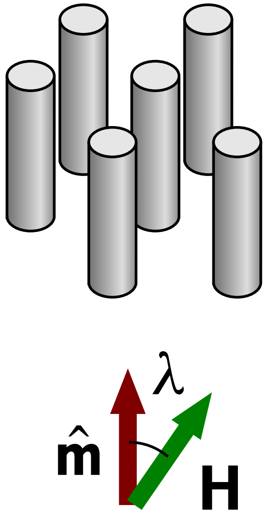
$$\mathbf{n} = \left(\Delta p_z/p_{\mathrm{F}}, \mathbf{p}\cdot\mathbf{v}_{\mathrm{s}}, v_{\mathrm{F}}(p - p_{\mathrm{F}})\right)$$

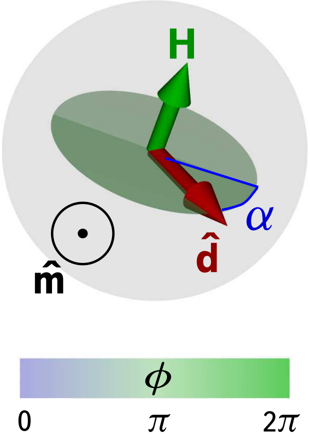


Autti *et al*, Phys. Rev. Res. **2**, 033013 (2020)

VORTICES IN THE POLAR PHASE

Order parameter $A = \Delta\, \hat{\mathbf{d}}\, \hat{\mathbf{m}}\, e^{i\phi} = \Delta\, (\cos\alpha\, \hat{\mathbf{e}}_1 + \sin\alpha\, \hat{\mathbf{e}}_2)\, \hat{\mathbf{m}}\, e^{i\phi}$

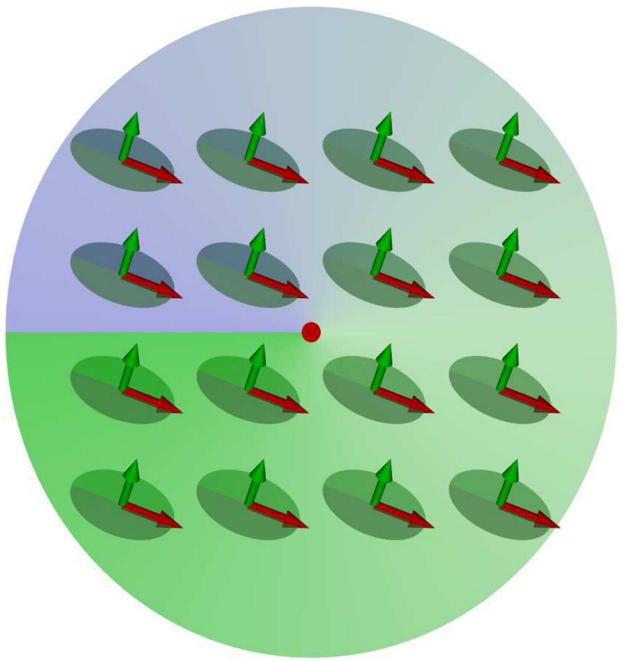




Zeeman energy $\propto (\hat{\mathbf{d}} \cdot \mathbf{H})^2$

$\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2 \perp \mathbf{H}$

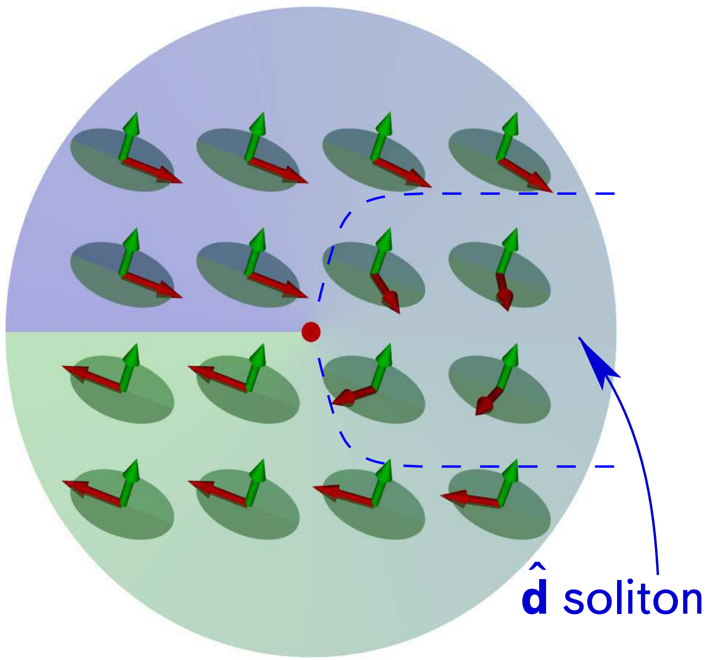
Single-quantum vortex



$$\phi \to \phi + 2\pi$$

$$\alpha = \text{const}$$

Half-quantum vortex

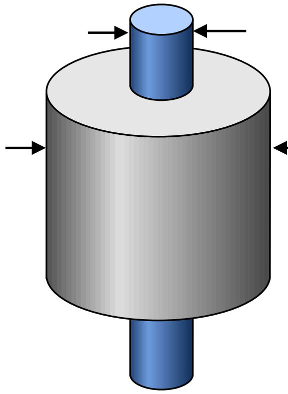


$$\phi \to \phi + \pi$$

$$\alpha \to \alpha + \pi$$

Spin-orbit interaction $\propto (\hat{\mathbf{d}} \cdot \hat{\mathbf{m}})^2 \Rightarrow$ Sine-Gordon equation $\quad \nabla^2\alpha = \dfrac{\sin^2\lambda}{2\xi_D^2} \sin 2\alpha$

VORTEX "DYNAMICS" WITH STRONG PINNING

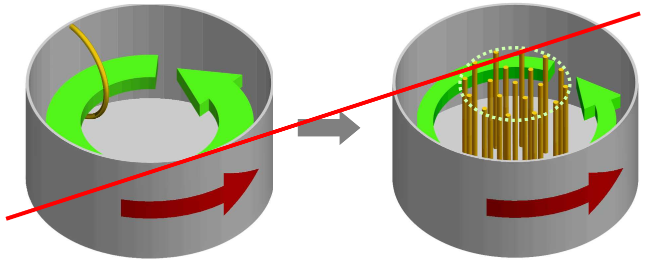


strand $\varnothing 9$ nm

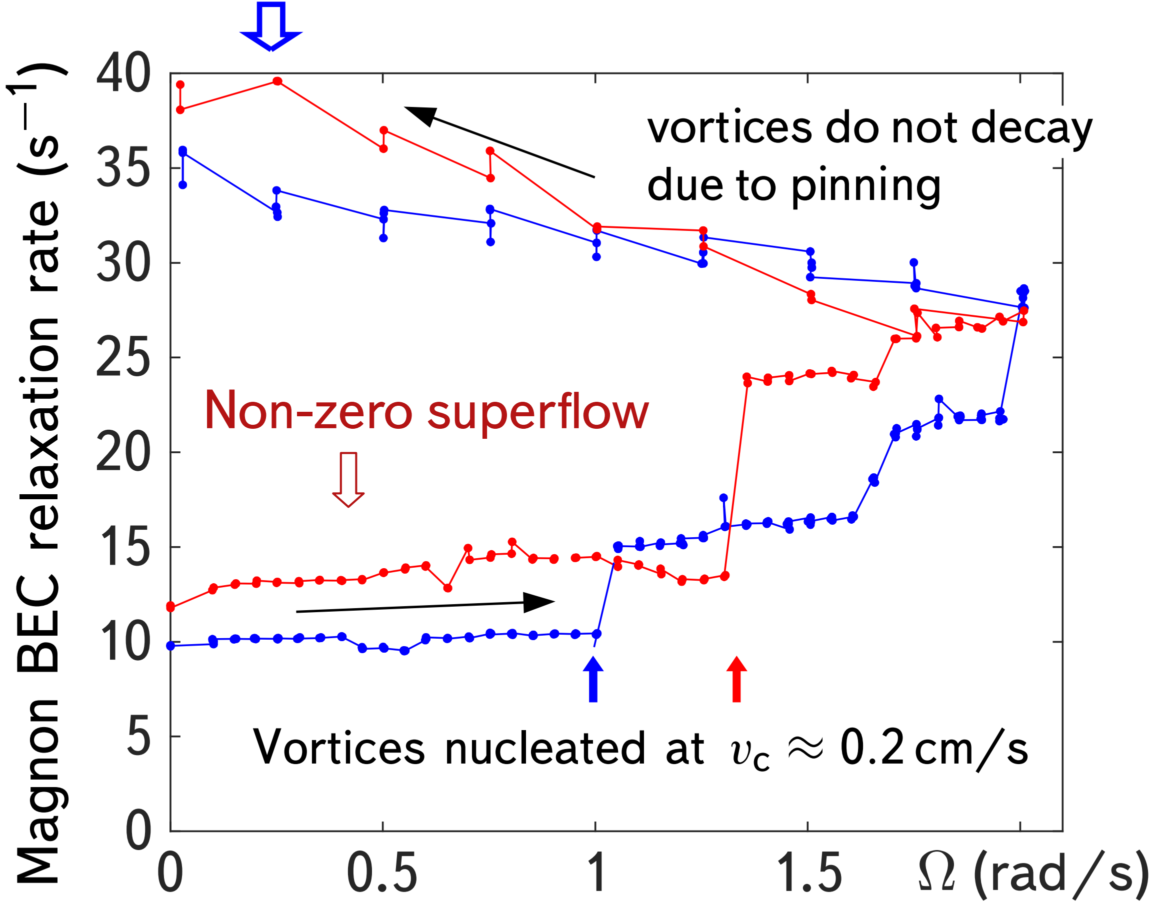
vortex core

$\xi \approx 20 - 70$ nm

HQVs and SQVs are strongly pinned by confining strands.



Experiment

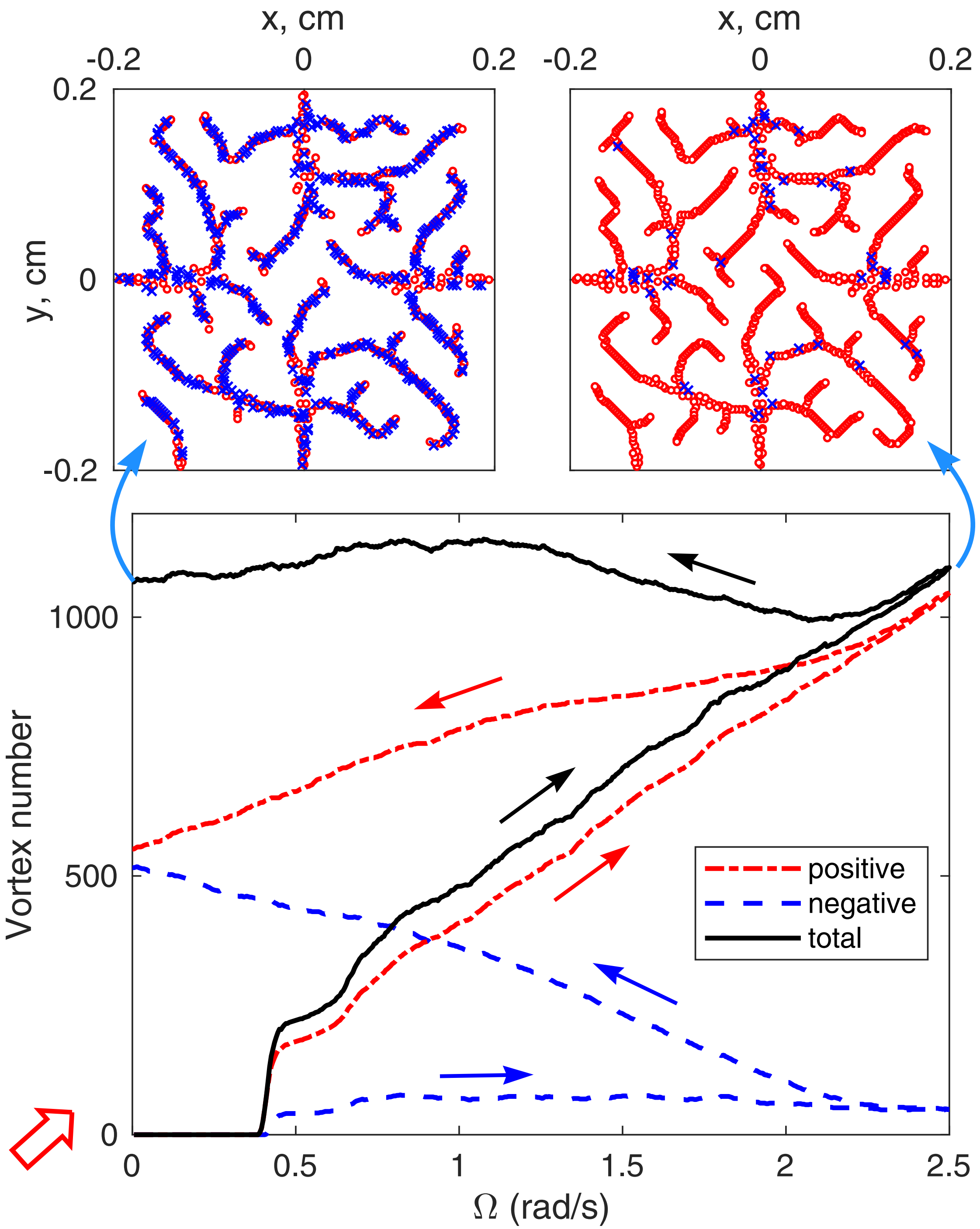


vortices do not decay due to pinning

Non-zero superflow

Vortices nucleated at $v_c \approx 0.2$ cm/s

Autti *et al*, Phys. Rev. Res. **2**, 033013 (2020)



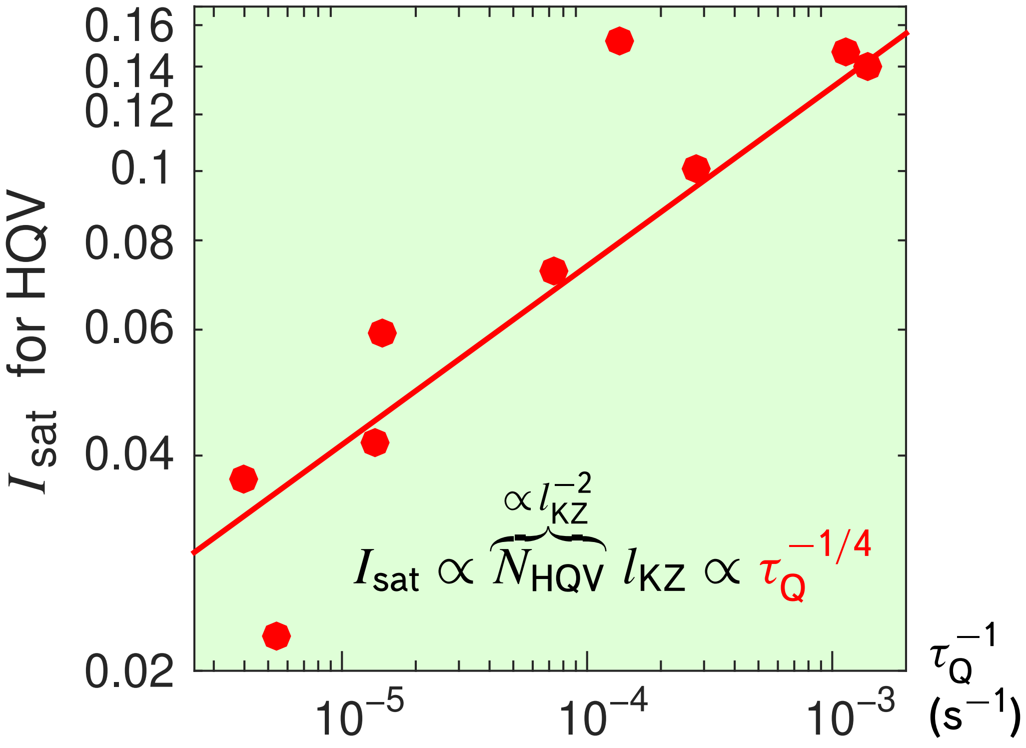
Simulations

ENABLED BY PINNING: KIBBLE-ZUREK MECHANISM (KZM) DENSITY AND KIBBLE WALLS

Non-equilibrium 2nd order phase transition: $T(t) = T_c\,(1 - t/\tau_Q) \Rightarrow$ defect formation.

In the polar phase KZM-created HQVs are pinned and can be counted.

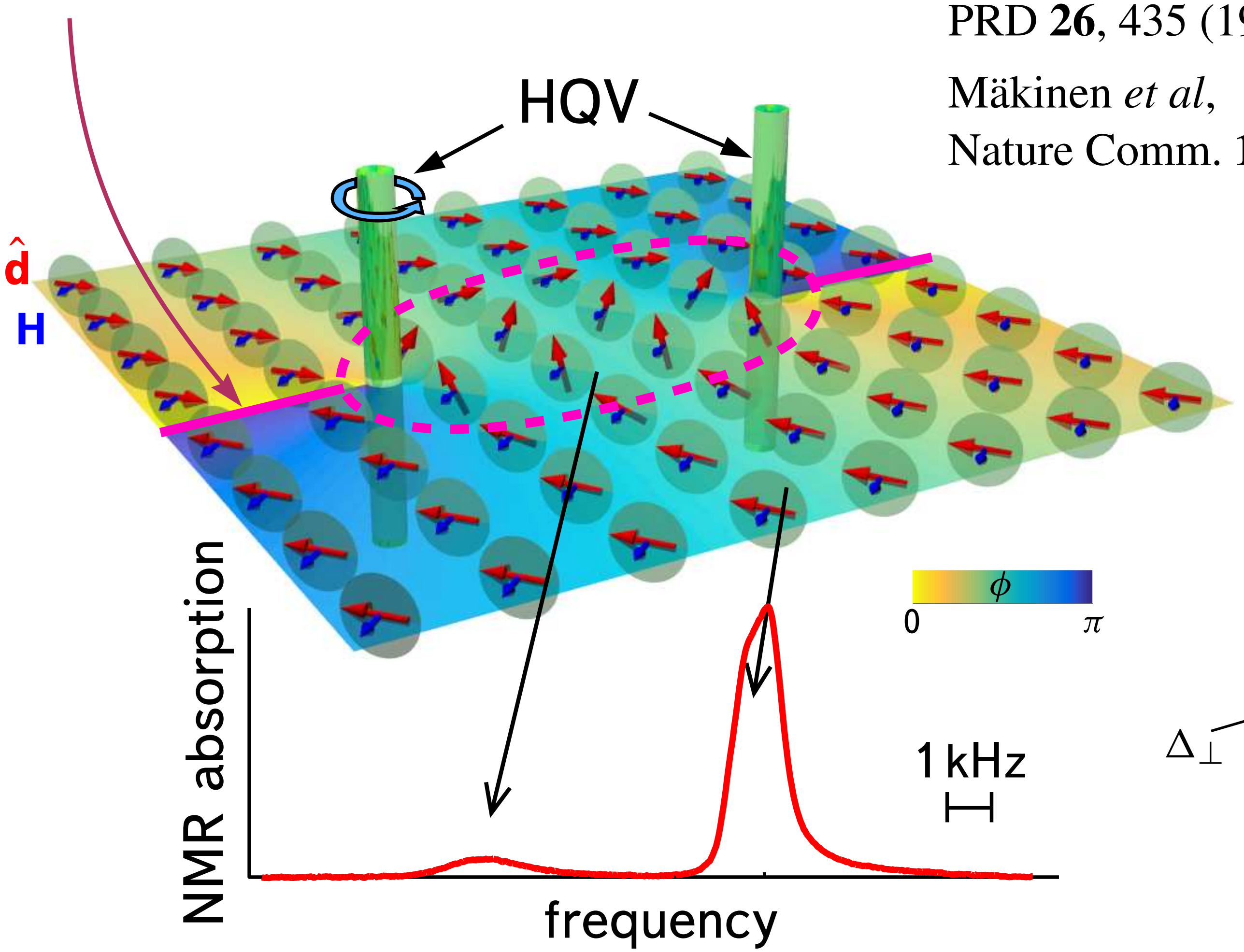
Autti *et al*, PRL **117**, 255301 (2016)
Rysti *et al*, PRL **127**, 115702 (2021)

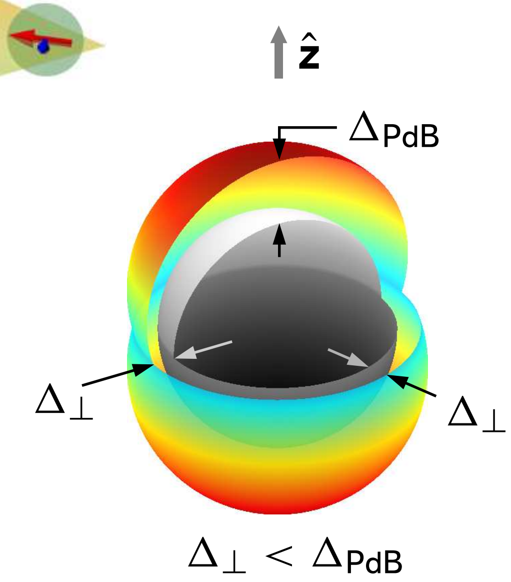


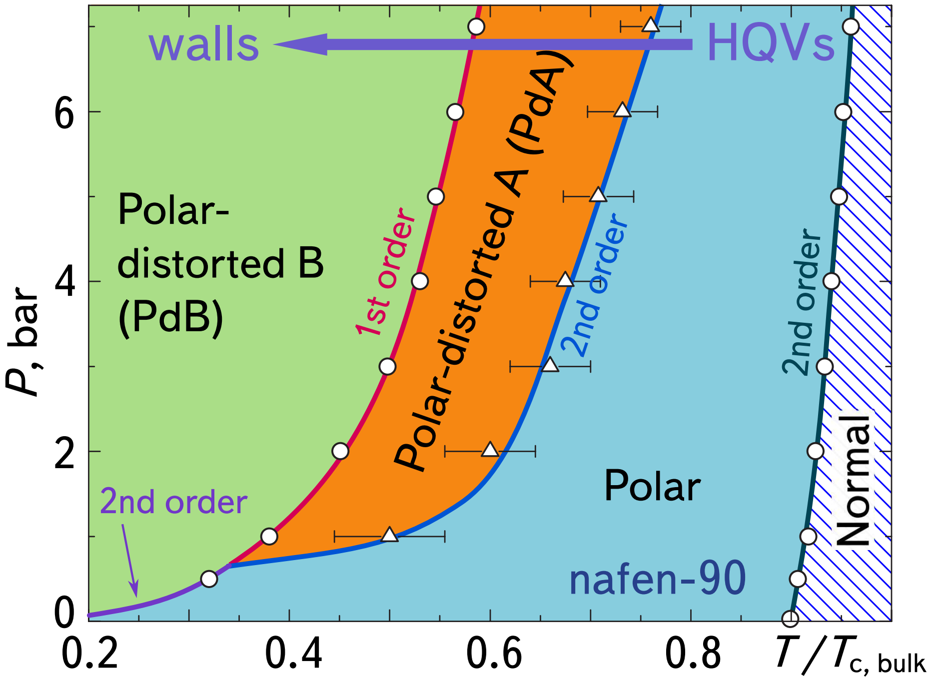
In the PdB phase HQV pairs form cosmological walls with suppression of $\Delta_\perp$.

Kibble, Lazarides & Shafi
PRD **26**, 435 (1982)

Mäkinen *et al*,
Nature Comm. **10**, 237 (2019)





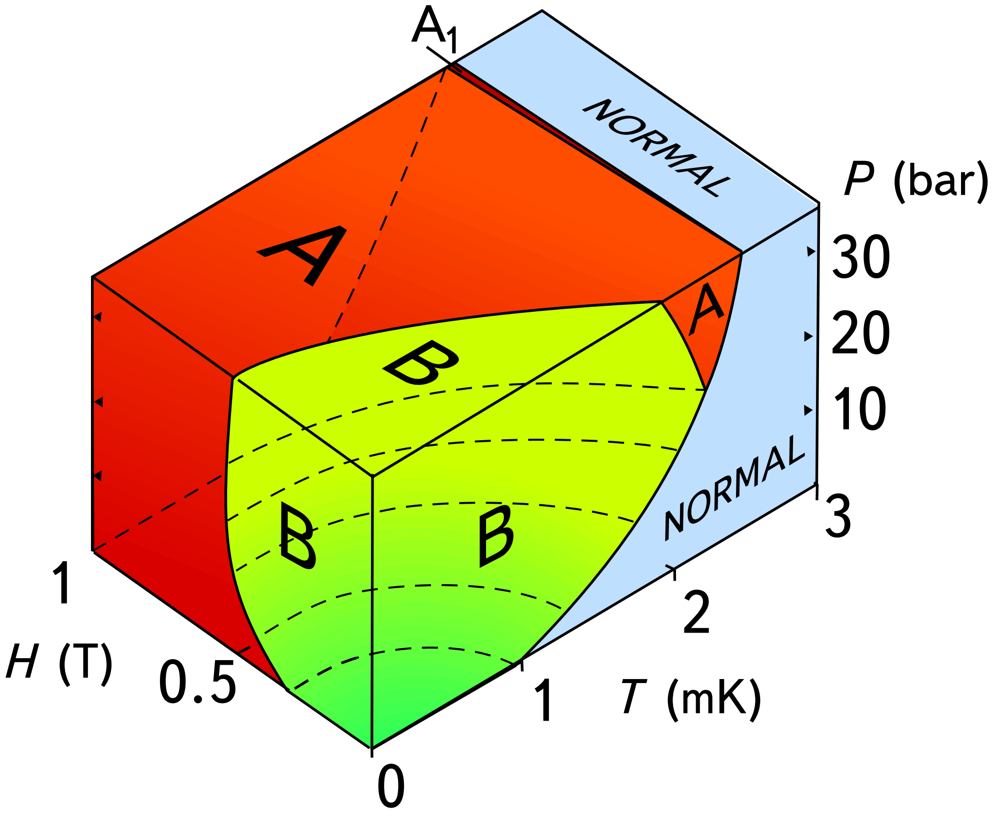


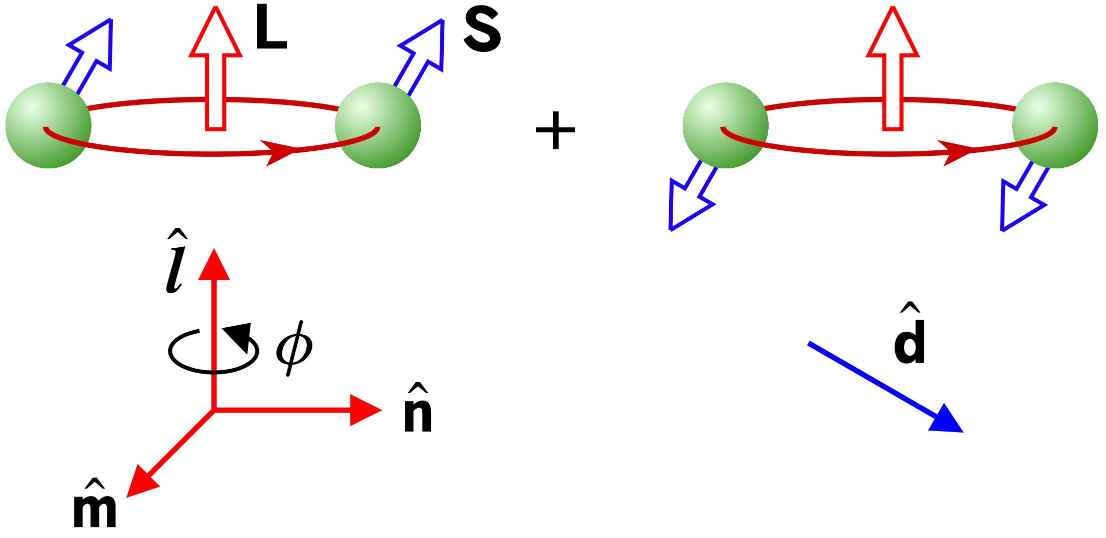
VORTEX SHEET IN SUPERFLUID ^{3}He-A

VORTICITY IN ^{3}He-A

Order parameter:

$$A_{\mu j} = \Delta\,\hat{\mathbf{d}}_\mu\,(\hat{\mathbf{m}}_j + i\,\hat{\mathbf{n}}_j)\,e^{i\phi}$$





$$\boxed{\nabla \times \mathbf{v}_s = \frac{\hbar}{2M}\hat{\mathbf{e}}_i \epsilon_{ijk} \hat{l} \cdot (\nabla_j \hat{l} \times \nabla_k \hat{l})}$$

(Mermin and Ho, 1976)

Continuous vorticity without suppressing superfluidity.

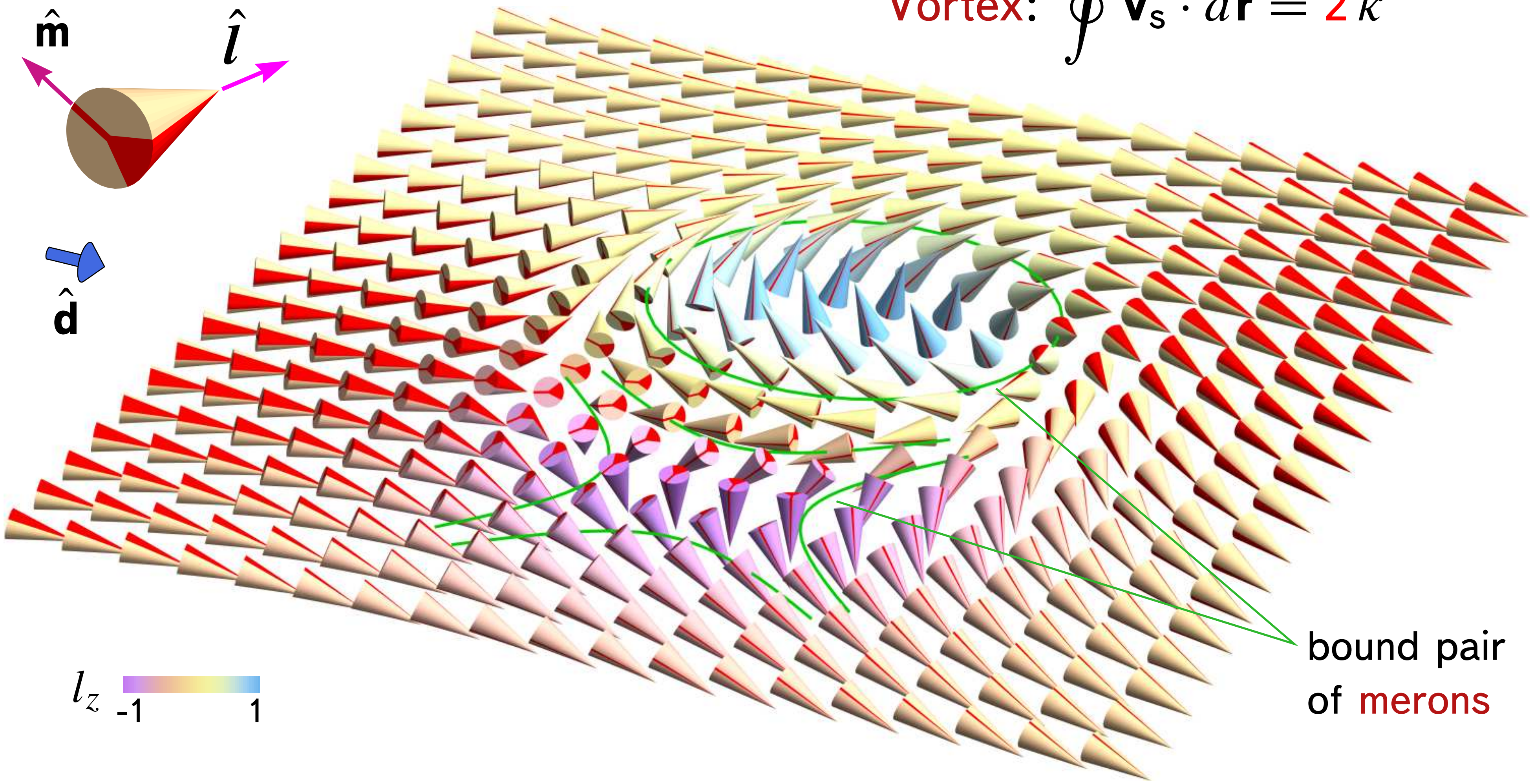
If $\hat{l}$ is in plane then $\nabla \times \mathbf{v}_s = 0$ and circulation is quantized.

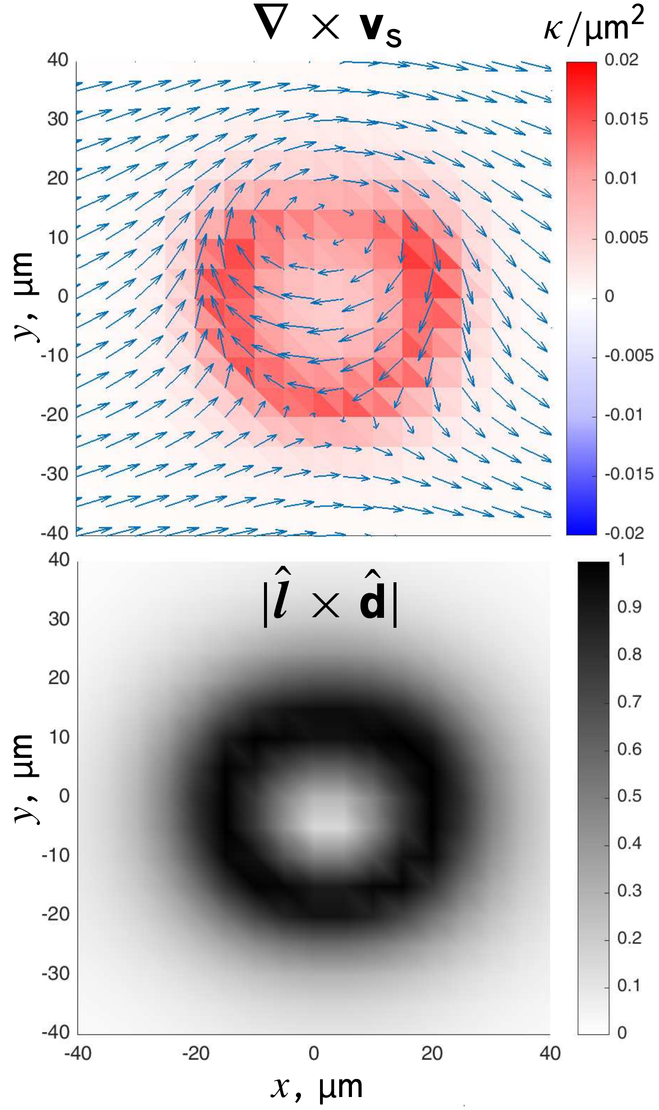
Zeeman energy $F_H \propto (\hat{\mathbf{d}} \cdot \mathbf{H})^2$

Spin-orbit interaction $F_D = -g_D\,(\hat{\mathbf{d}} \cdot \hat{l})^2 \;\sim\; 10^{-3}\,\Delta, \qquad \xi_D \sim 10^3\,\xi \sim 10\,\mu\mathrm{m}$

DOUBLE QUANTUM VORTEX SKYRMION

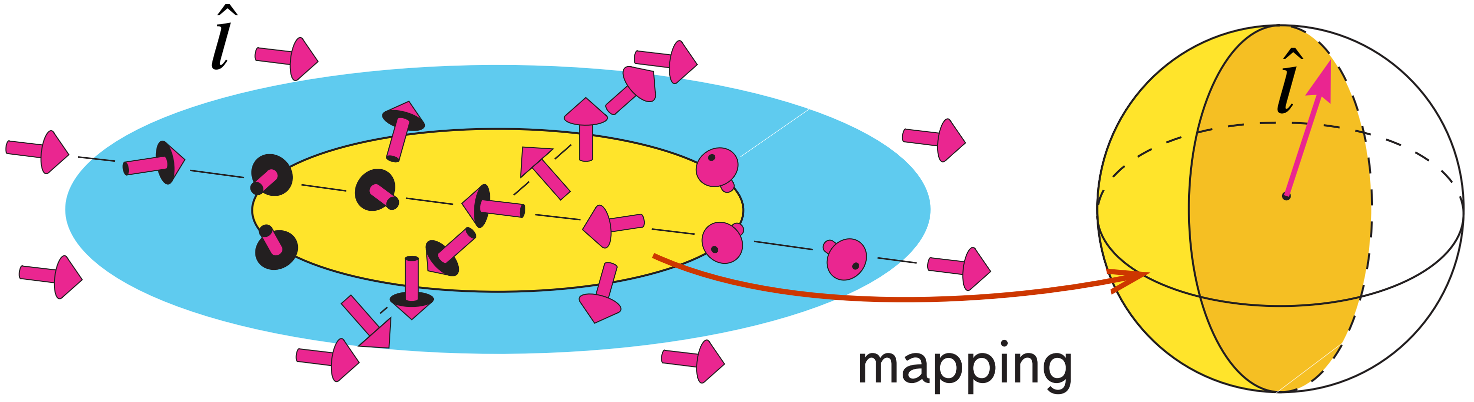
Vortex: $\oint \mathbf{v}_s \cdot d\mathbf{r} = 2\,\kappa$





Skyrmion: Topological invariant

$$m_l = \frac{1}{8\pi}\epsilon_{ijk}\int d\mathbf{S}_i\,\hat{\boldsymbol{l}}\cdot(\nabla_j\hat{\boldsymbol{l}}\times\nabla_k\hat{\boldsymbol{l}}) = 1$$



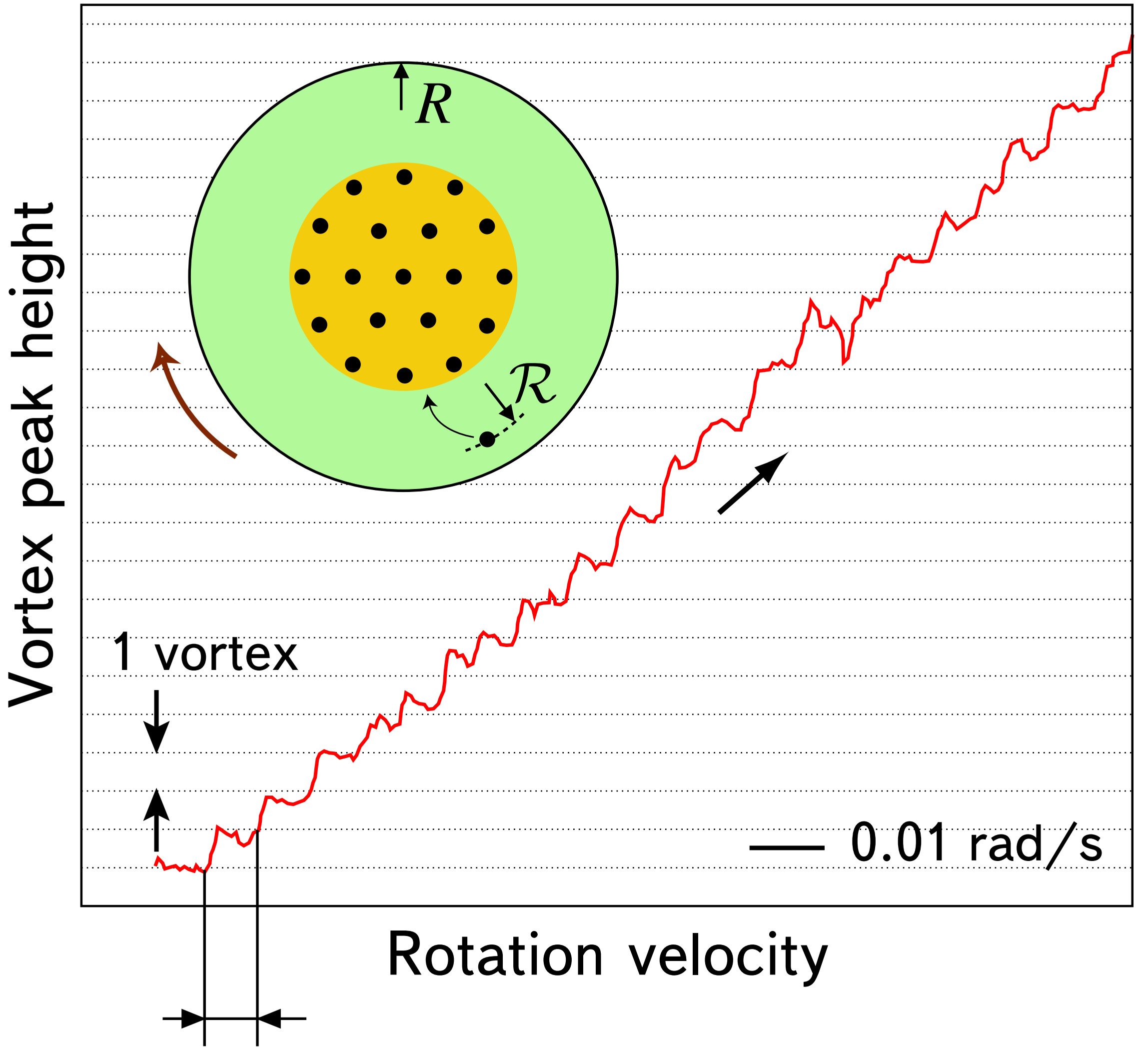
DOUBLE-QUANTUM VORTEX IN EXPERIMENTS

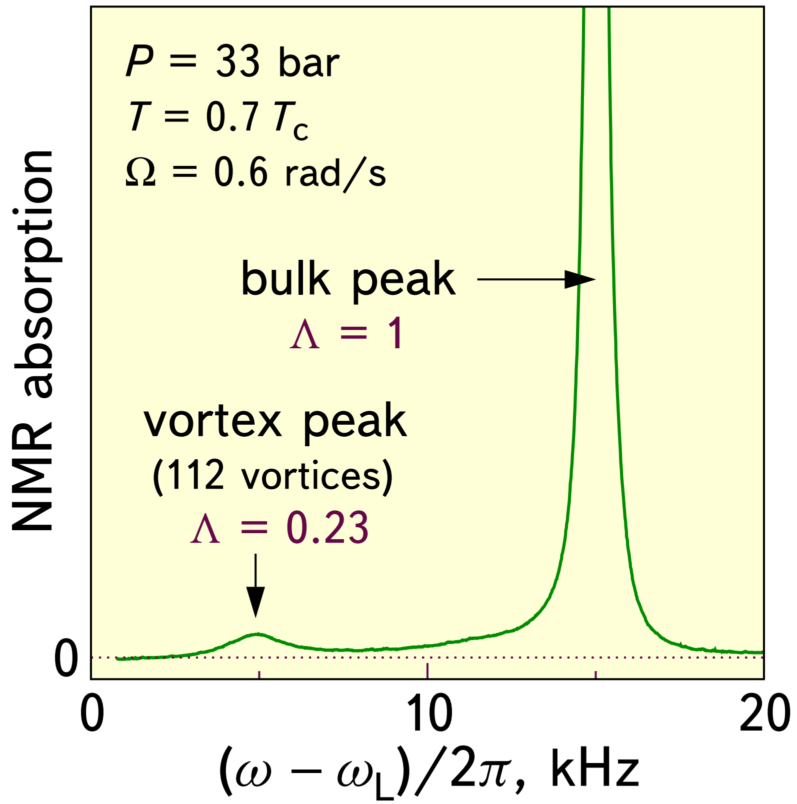
Satellite peak in the NMR spectrum with characteristic frequency shift.

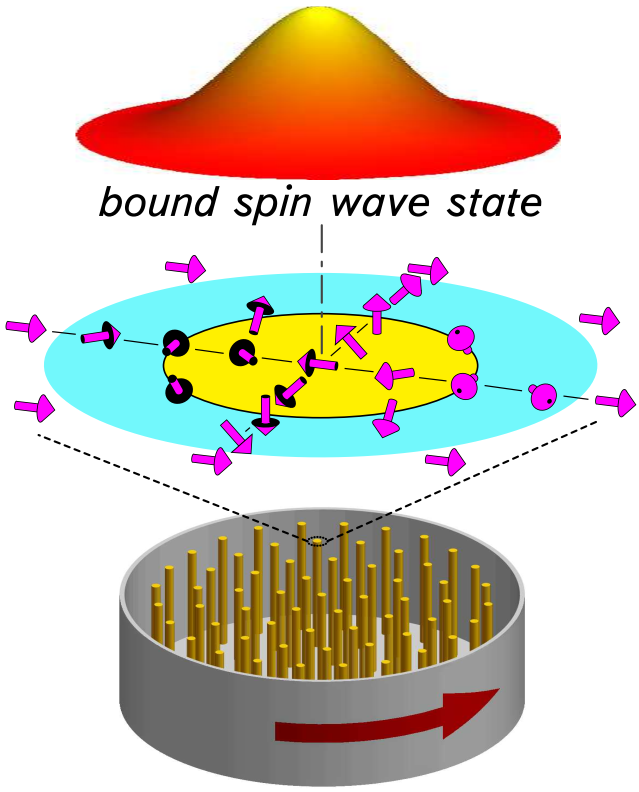
$$\omega^2 = \omega_\mathrm{L}^2 + \Lambda\,\Omega_\mathrm{A}^2$$

$\omega_\mathrm{L} = \gamma H$

Ω_A – Leggett frequency



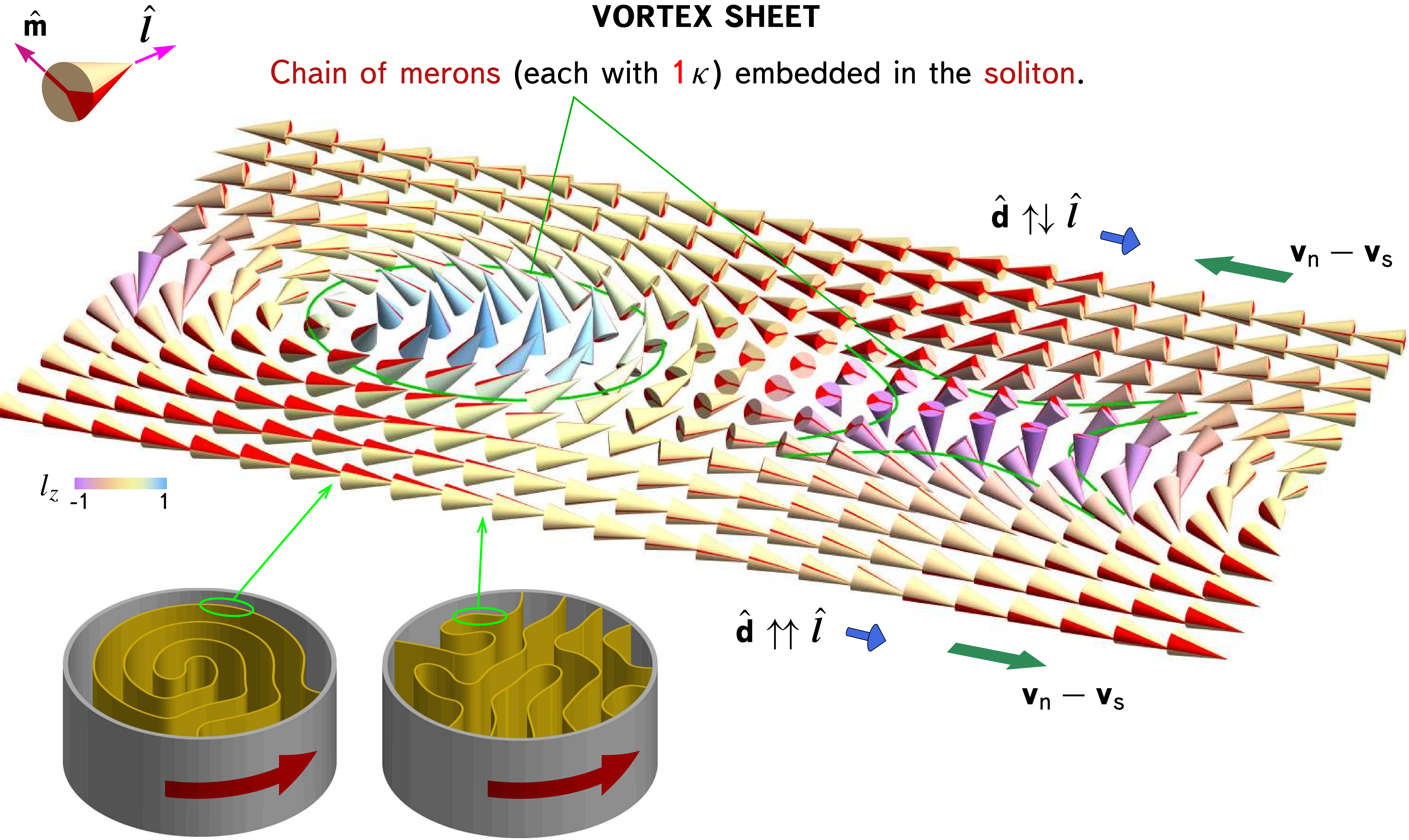




$$\Delta\Omega = \frac{n\kappa}{2\pi\mathcal{R}^2} = 6.24 \cdot 10^{-3}\,\mathrm{rad/s} \Rightarrow n = 2,\ R - \mathcal{R} \approx 0.1\,\mathrm{mm}$$

Blaauwgeers *et al*, Nature **404**, 471 (2000)

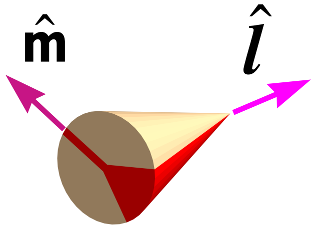
VORTEX SHEET



Chain of merons (each with $1\,\kappa$) embedded in the soliton.

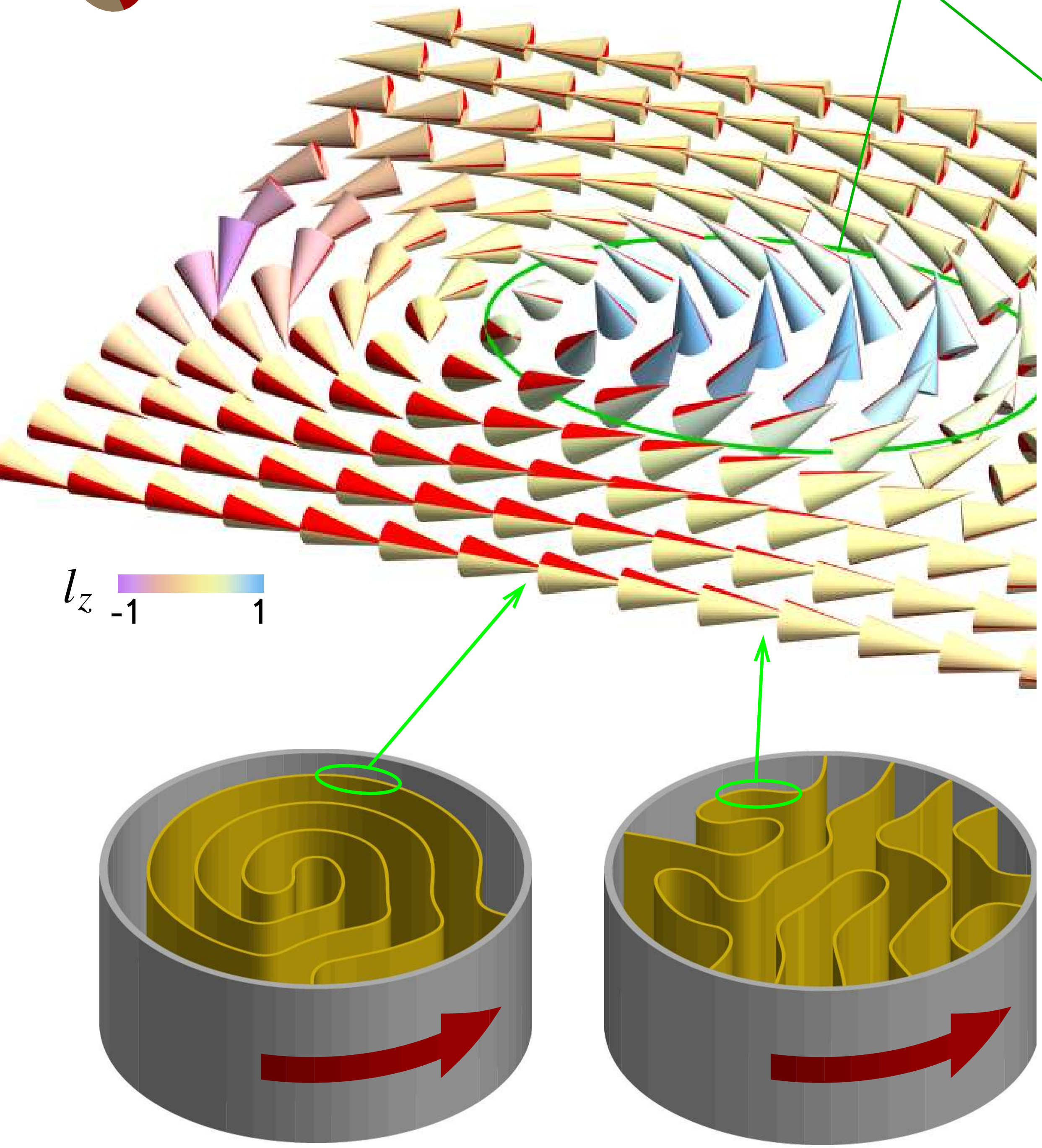
Parts *et al*, PRL **72**, 3839 (1994); Physica B **210**, 311 (1995)

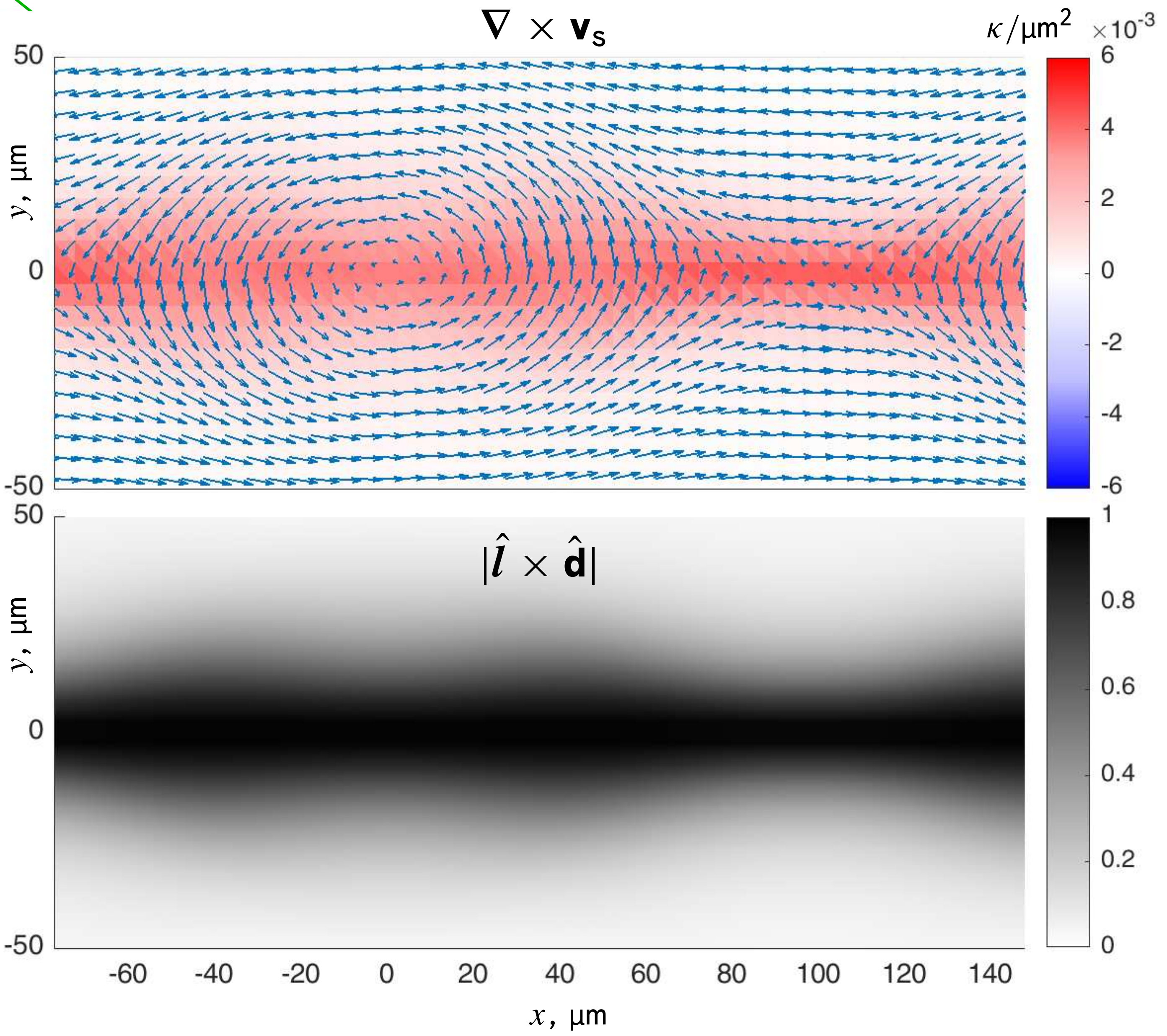
VORTEX SHEET



Chain of merons (each with 1κ) embedded in the soliton.

Vorticity is distributed along the sheet.

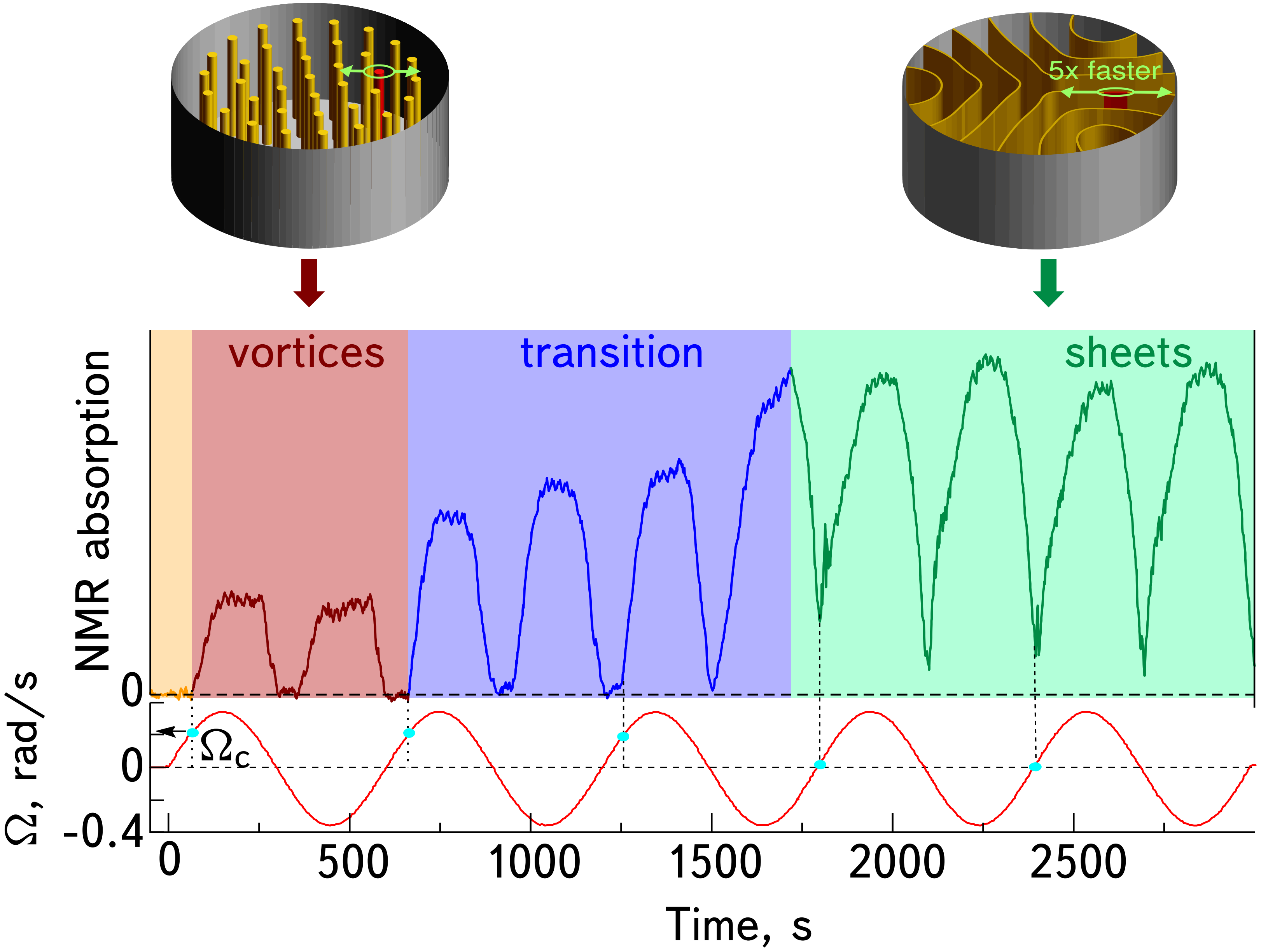




Parts *et al*, PRL **72**, 3839 (1994); Physica B **210**, 311 (1995)

TRANSITION FROM VORTEX LINES TO VORTEX SHEETS

Experiment: Sheets replace lines when rotation varies sufficiently fast. Dynamically driven topological transition.

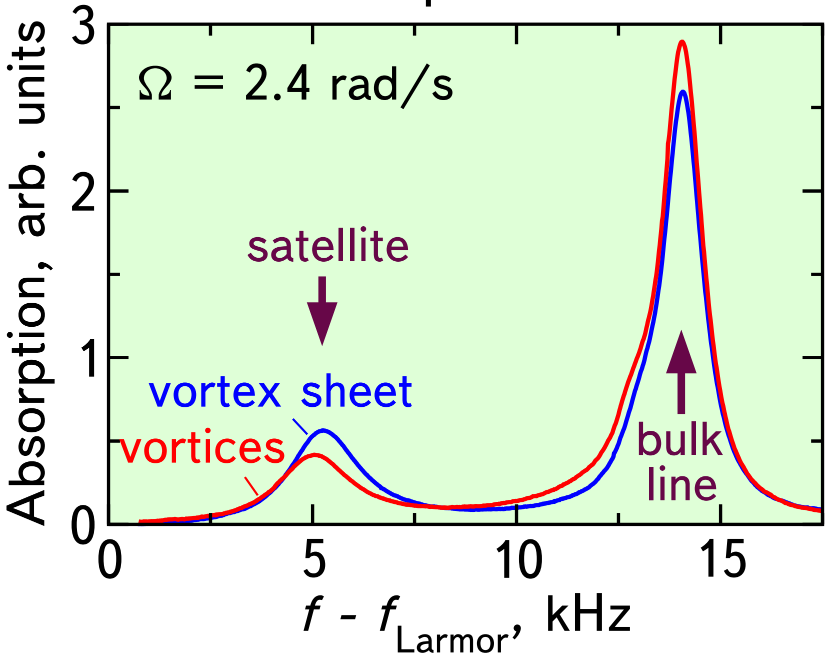


$P = 32.8\,\mathrm{bar},\ T = 0.7T_c,\ H = 9.6\,\mathrm{mT}$

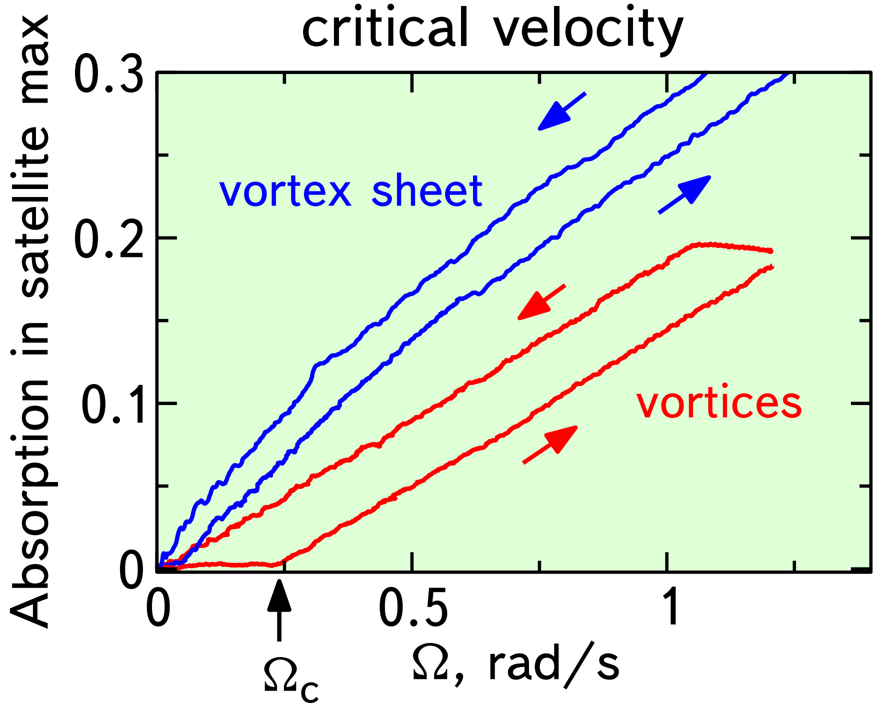
V.E. *et al*, PRL **88**, 065301 (2002)

lines *vs* sheets:

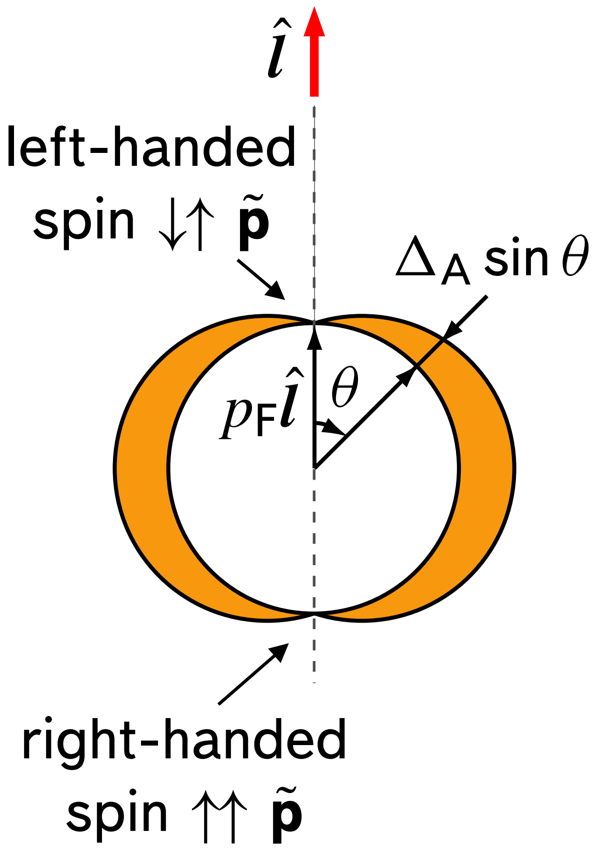
spectra



critical velocity



WEYL FERMIONS AND CHIRAL ANOMALY



Energy nodes in ^{3}He-A with linear dispersion for $\tilde{\mathbf{p}} = \mathbf{p} \mp p_F\hat{l}$

$\Rightarrow$ massless, spin-1/2, 3D particles

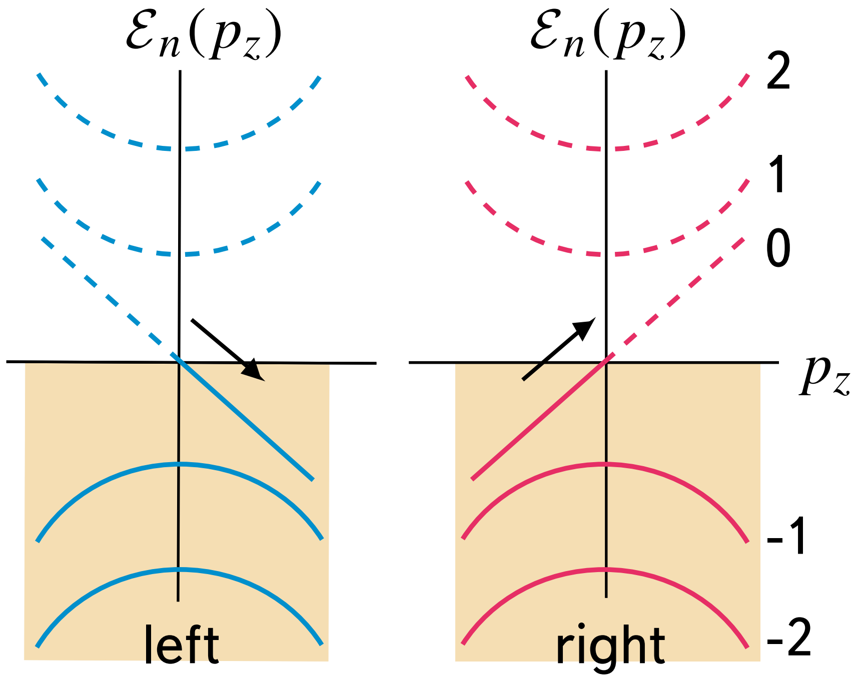
$\Rightarrow$ chiral, Weyl fermions

$$\mathcal{H} = \mp c\sigma \cdot (\mathbf{p} \mp \overset{\text{like vector potential}}{\boxed{p_F\hat{l}}})$$

(Volovik, Vachaspati 1996)

Synthetic gauge field: $\mathbf{B} = k_F\nabla \times \hat{l}$ and $\mathbf{E} = k_F\partial_t\hat{l}$.

Chiral anomaly: $\mathbf{B}$ $\Rightarrow$ Landau levels $\propto \underset{\text{orbital}}{n + \frac{1}{2}} \underset{\text{spin}}{- \frac{1}{2}} + \varepsilon(p_z)$.



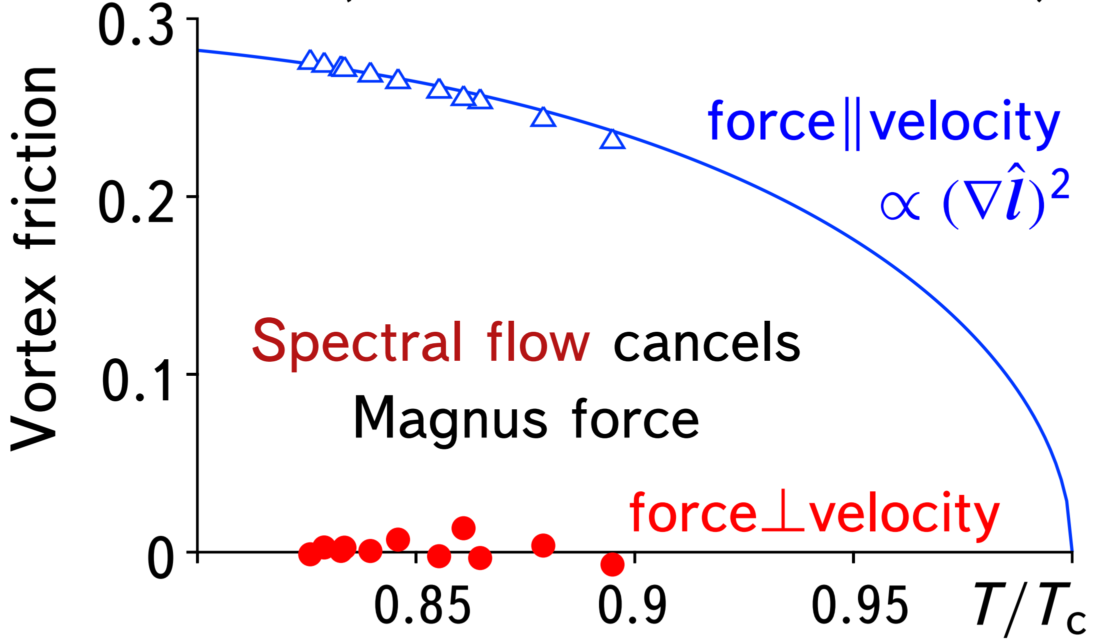
Vortex motion

$\Rightarrow$ $\mathbf{E} \parallel \mathbf{B}$

Adler-Bell-Jackiw equation:

$$\frac{dn_{\text{chiral}}}{dt} = -\frac{1}{4\pi^2}\mathbf{B}\cdot\mathbf{E}$$

(Bevan et al 1997, Nature 386:689)



force∥velocity $\propto (\nabla\hat{l})^2$

Spectral flow cancels Magnus force

force⊥velocity

TOPOLOGY DETERMINES DYNAMICS

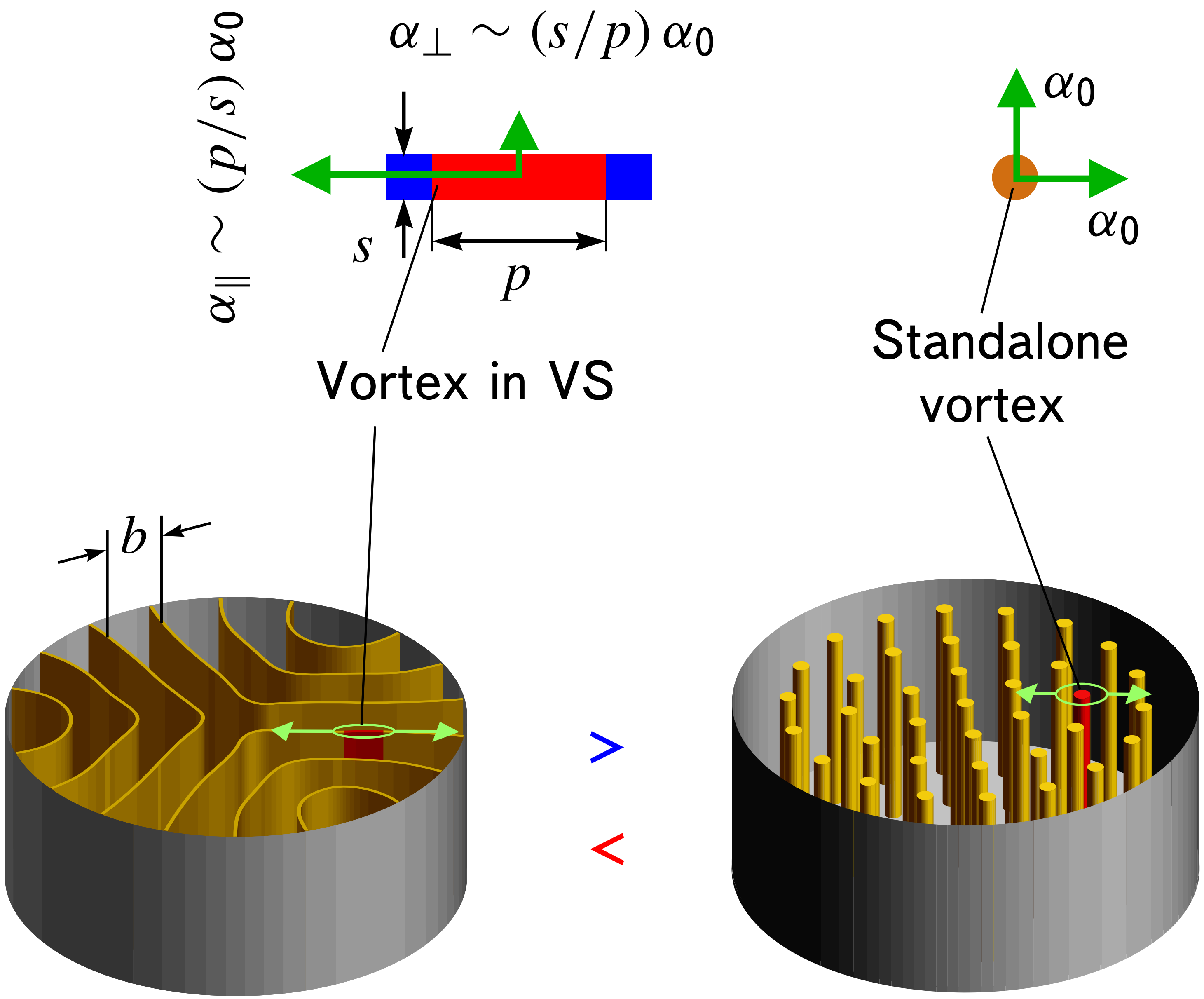
Mutual friction coefficient

$$\alpha = \kappa\rho_{\mathrm{s}}/D \propto \left[\int d^2r(\nabla\hat{\boldsymbol{l}})^2\right]^{-1}$$

$$s \sim 4\,\xi_{\mathrm{D}} \approx 40\,\mu\mathrm{m}$$

$$p = \kappa/(2\Omega b) \gg s$$

$$\alpha_\parallel : \alpha_0 : \alpha_\perp \sim 5 : 1 : 1/5$$



Energy: >

Response time: <

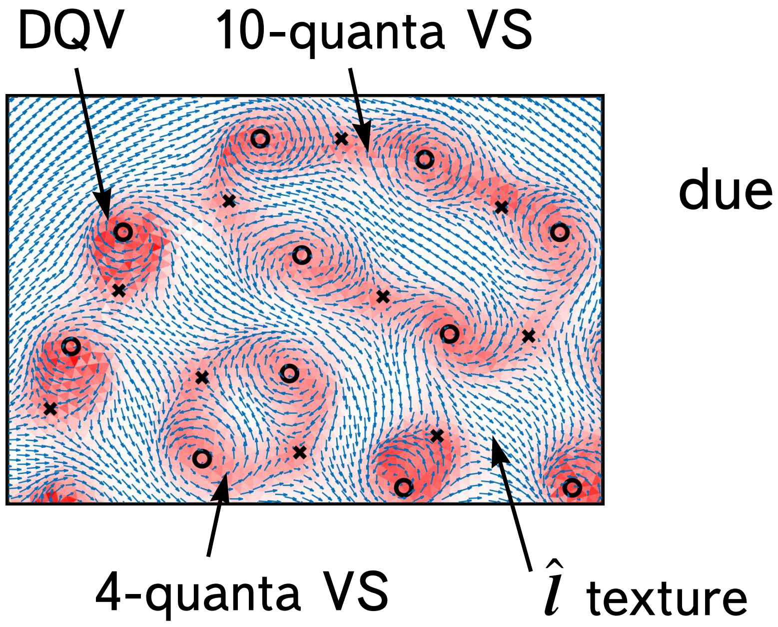
Multiple radially aligned sheets ⇒ fastest response to Ω change.

Dynamic drive $\Omega(t)$ selects the fastest response.

VE *et al*, PRL **88**,065301 (2002)

"ZERO-CHARGE" TRANSITION IN THE VORTEX SHEET STRUCTURE

In ^{3}He-A, synthetic gauge field possesses QED features. Running coupling constant:

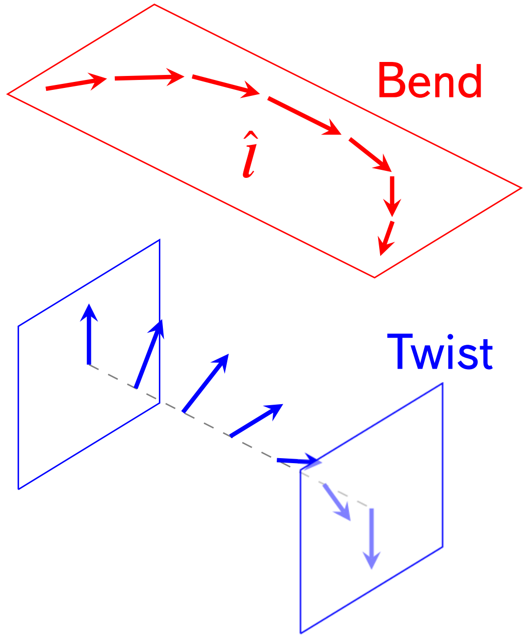


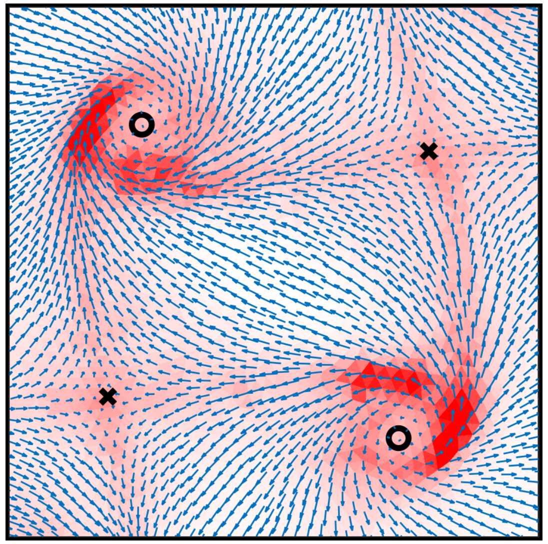
Action $S \propto \ln(\Delta_A^2/T^2)$

due to diverging coefficient in the bending energy

$$K_{\text{bend}} = K_{\text{b0}} + K_{\text{b1}} \ln(\Delta_A/T)$$

Volovik, JETP Lett **47**, 55 (1988)



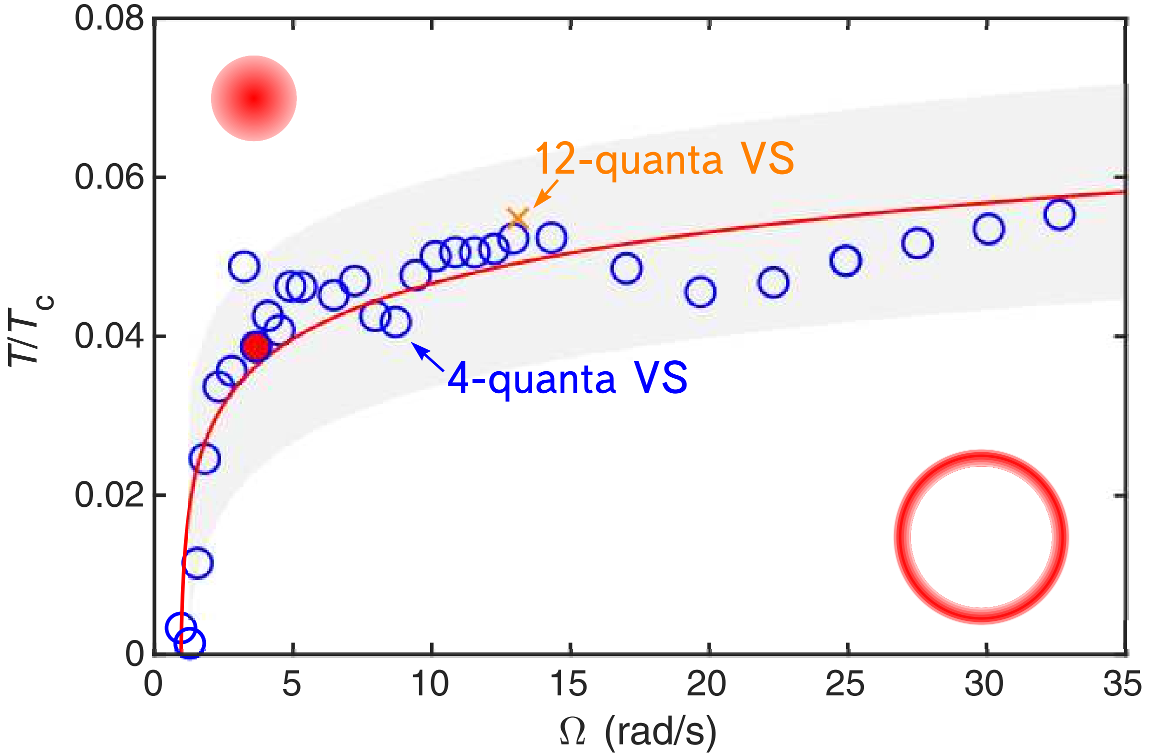


4-quanta VS at ultra-low T

Diverging K_{bend} when $T \to 0$ causes transition to tube-like vorticity (bend → twist).

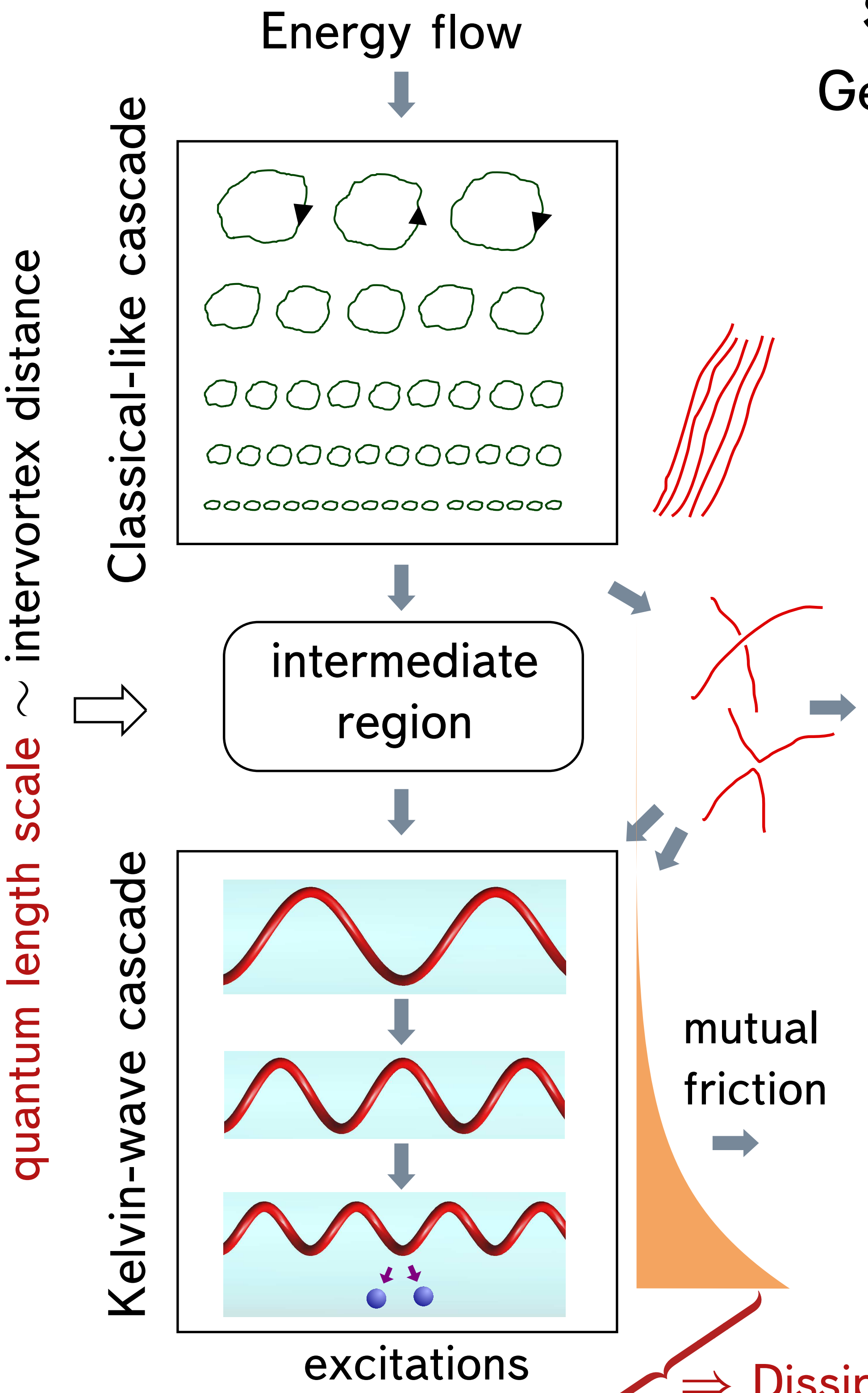
Rantanen & VE

PRB **107**, 104505 (2023)



DYNAMICS OF KELVIN WAVES IN SUPERFLUID ^{3}He-B

SCALES IN QUANTUM TURBULENCE


Energy flow

Classical-like cascade

intermediate region

Kelvin-wave cascade

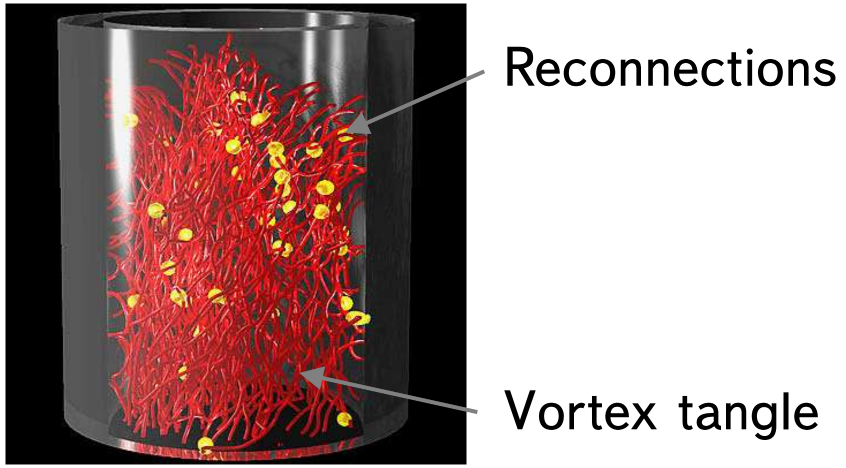
excitations

mutual friction

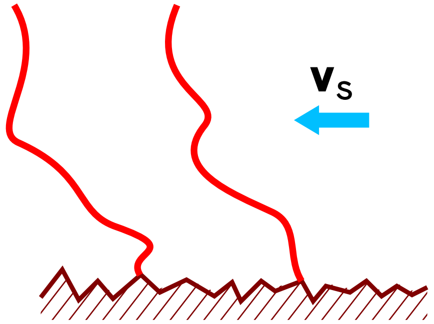
quantum length scale ∼ intervortex distance

Generally accepted structure of quantum turbulence:

- energy cascades of different nature,
- vortex reconnections,
- microscopic dissipation mechanisms.



Boundary conditions for quantum turbulence



How to address sub-quantum length scales in experiments?

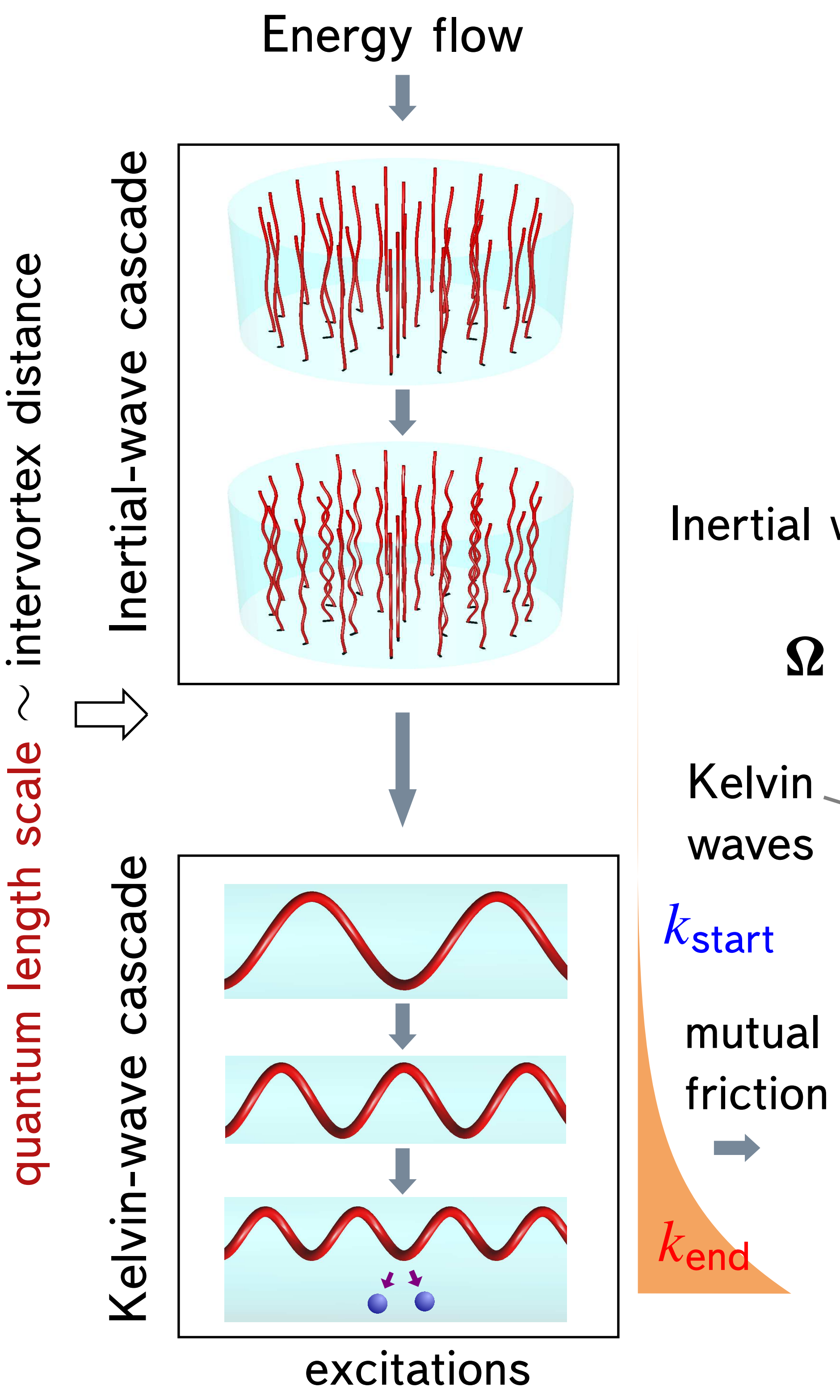
$\Rightarrow$ Dissipation anomaly: Finite damping in the $T \rightarrow 0$ limit

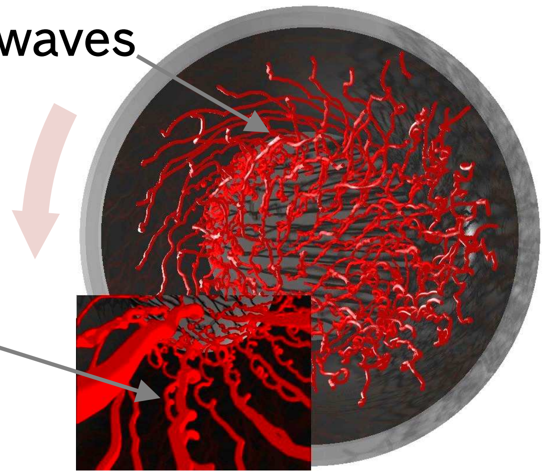
SCALES IN QUANTUM TURBULENCE

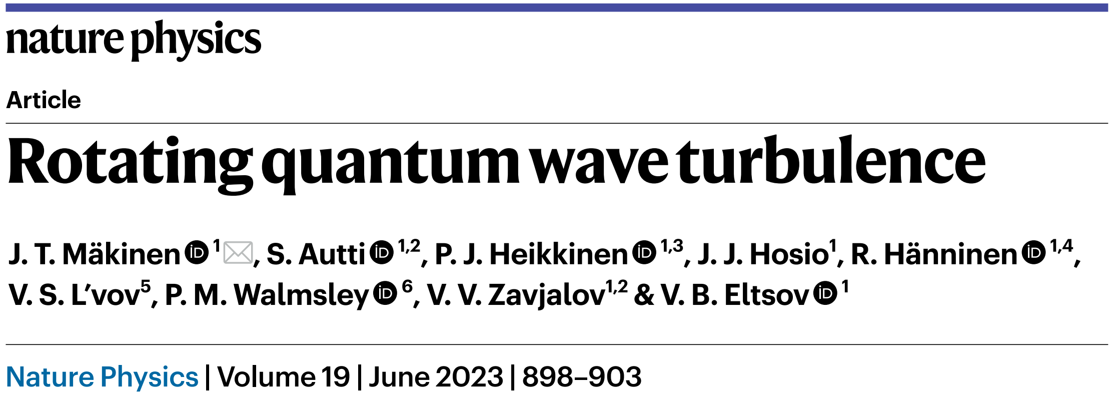
How to address sub-quantum length scales in experiments?

- Pure turbulence of vortex waves

  where rotation polarizes vortices and suppresses reconnections






nature physics

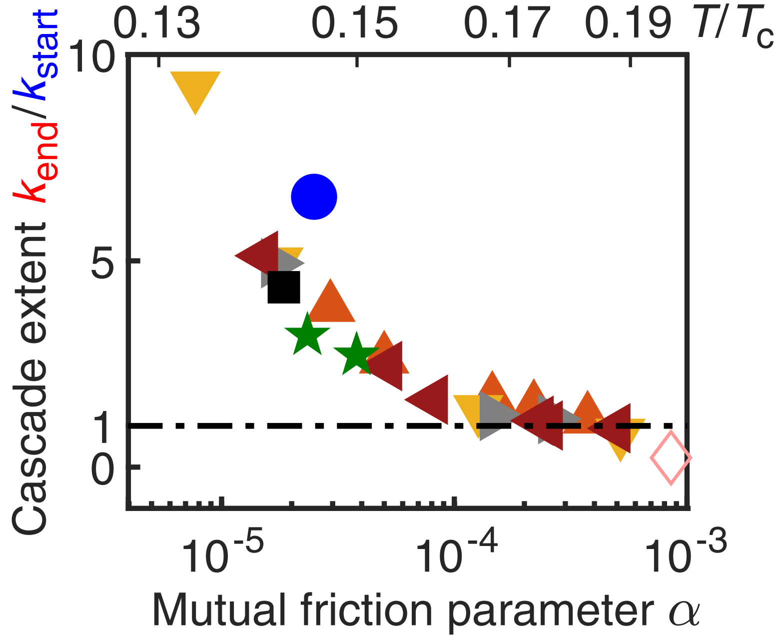
Article

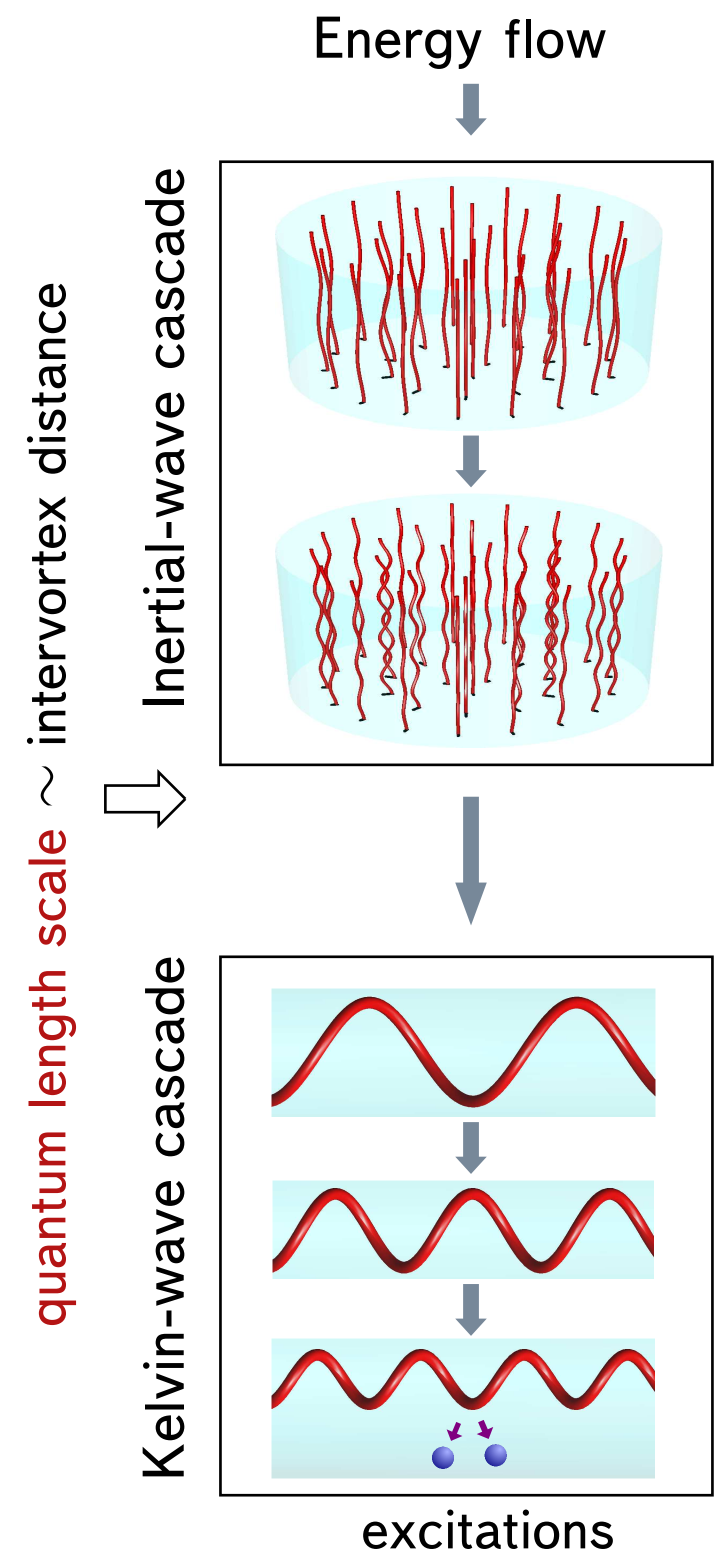
Rotating quantum wave turbulence

J. T. Mäkinen[1]✉, S. Autti[1,2], P. J. Heikkinen[1,3], J. J. Hosio[1], R. Hänninen[1,4], V. S. L'vov[5], P. M. Walmsley[6], V. V. Zavjalov[1,2] & V. B. Eltsov[1]

Nature Physics | Volume 19 | June 2023 | 898–903

Onset of the Kelvin-wave cascade at the lowest temperatures.



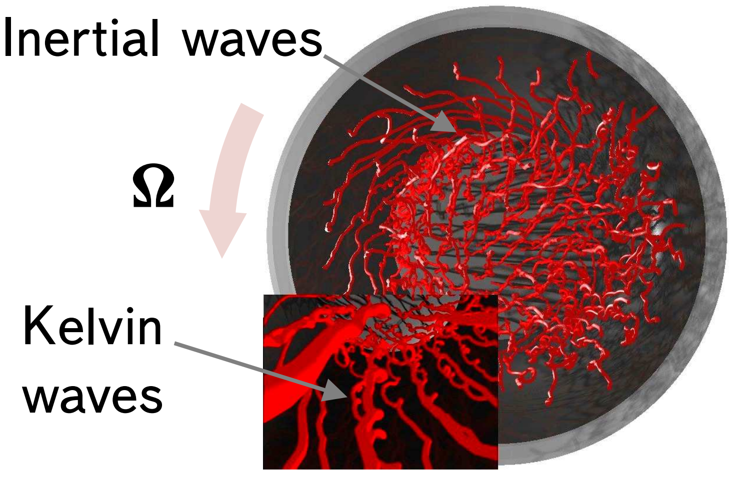


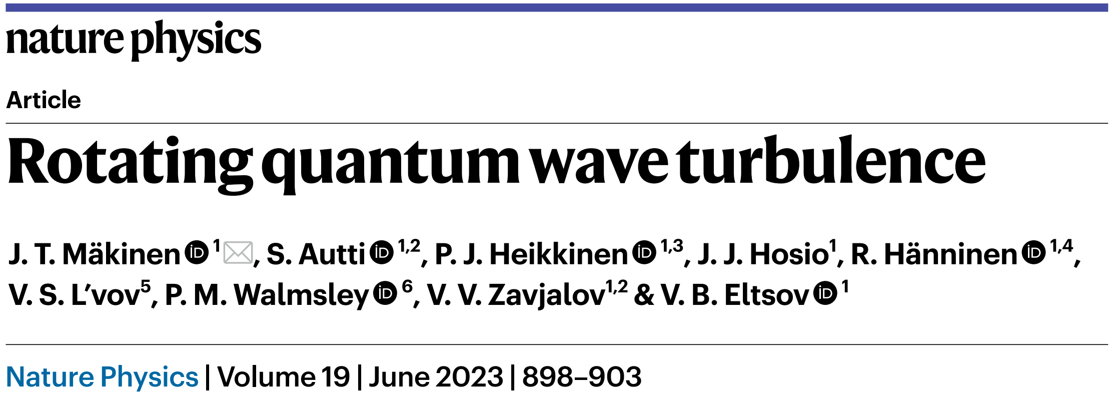
SCALES IN QUANTUM TURBULENCE

How to address sub-quantum length scales in experiments?

- Pure turbulence of vortex waves

  where rotation polarizes vortices and suppresses reconnections





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Rotating quantum wave turbulence

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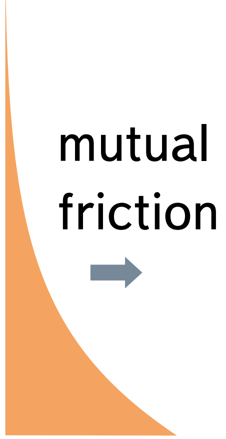
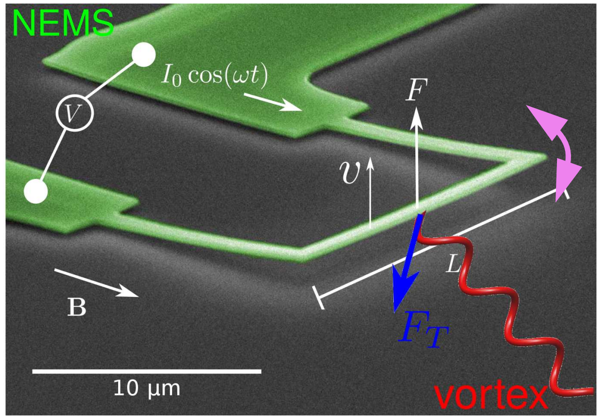


- MEMS/NEMS devices as sensors of individual vortices

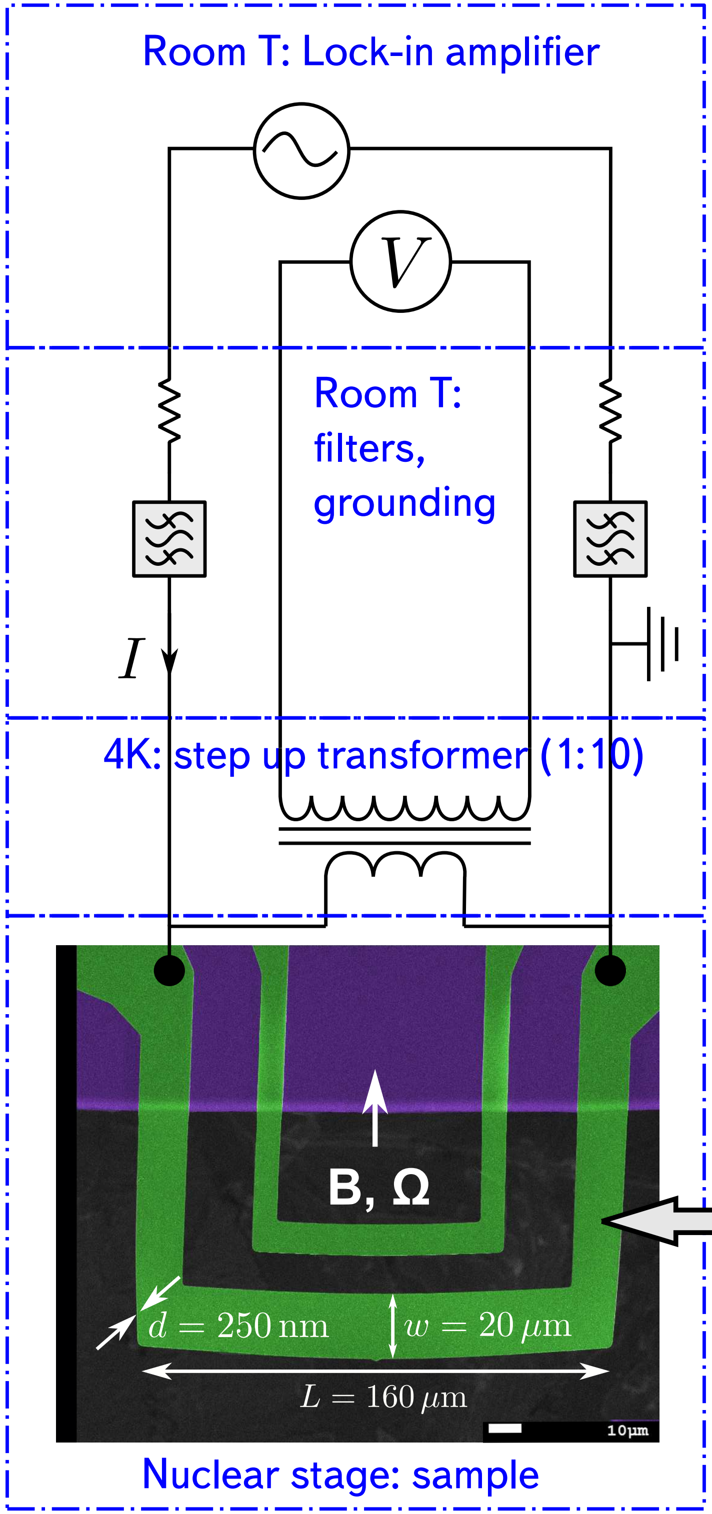
Largest size $<$ intervortex distance $\sim 100\,\mu\text{m}$.

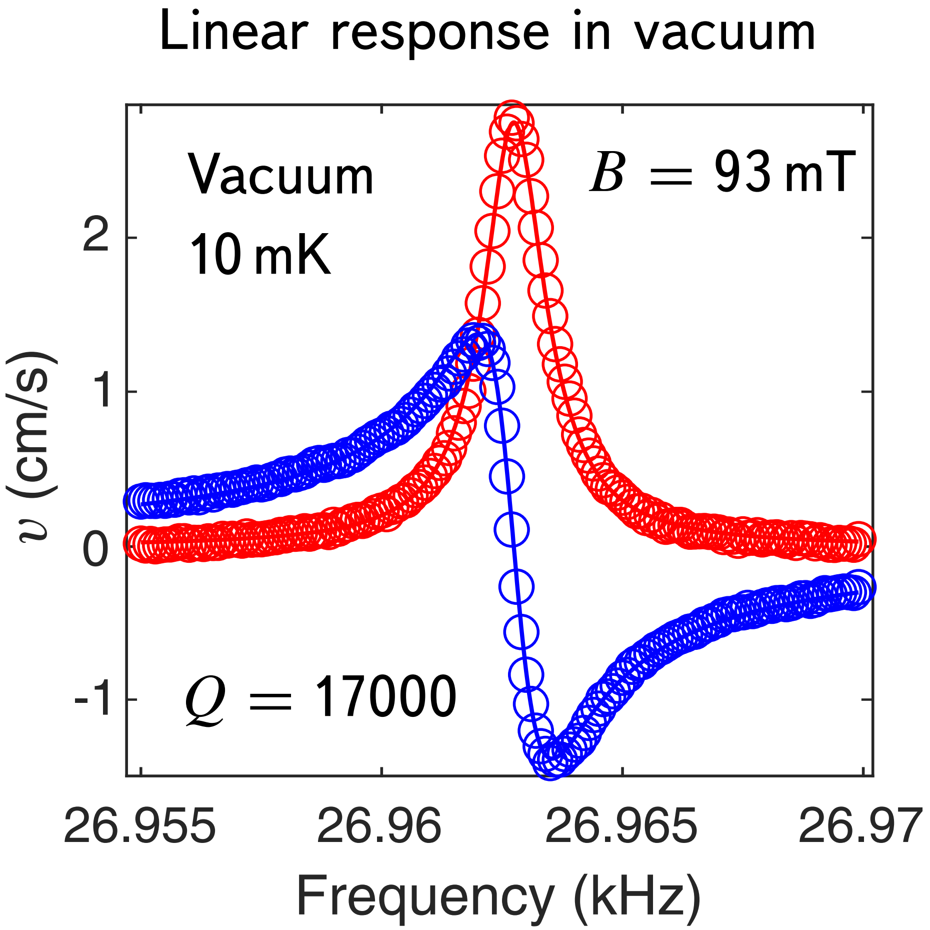
Smallest size $\sim$ 100 nm for force sensitivity better than vortex tension.



Barquist *et al*, PRB **106**, 094502 (2022); Guthrie *et al*, Nat. Commun. **12**, 2645 (2021)

NEMS IN THE ROTA CRYOSTAT





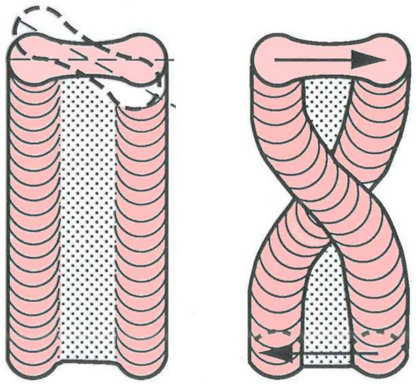
TTLS intrinsic damping

Kamppinen *et al*, PRB **105**, 035409 (2022)

Bare aluminum device suspended over window

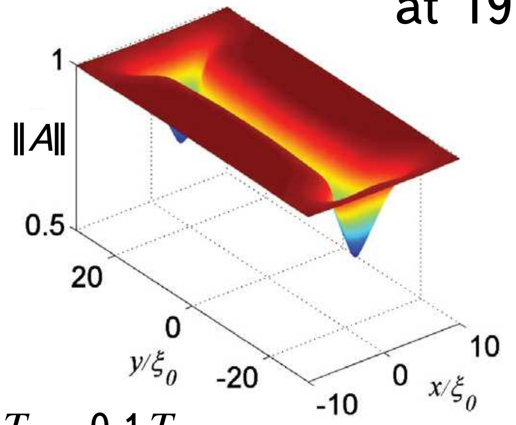
QUANTIZED VORTICES IN ^{3}He-B

New mode: twisting

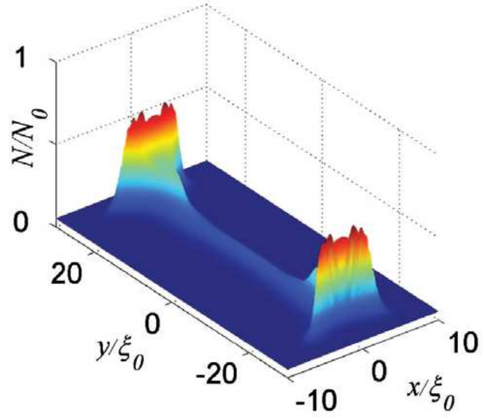


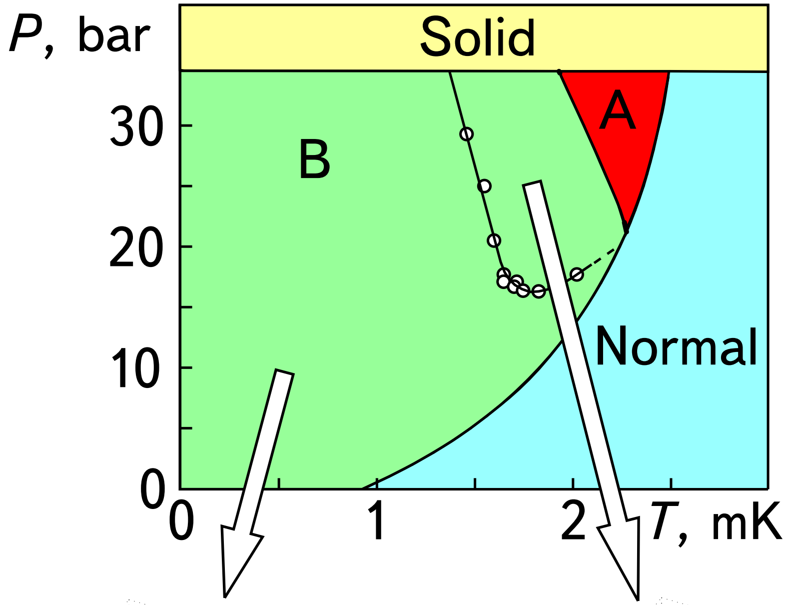
Fat: core $\approx 200 \times 900\,\mathrm{nm}^2$

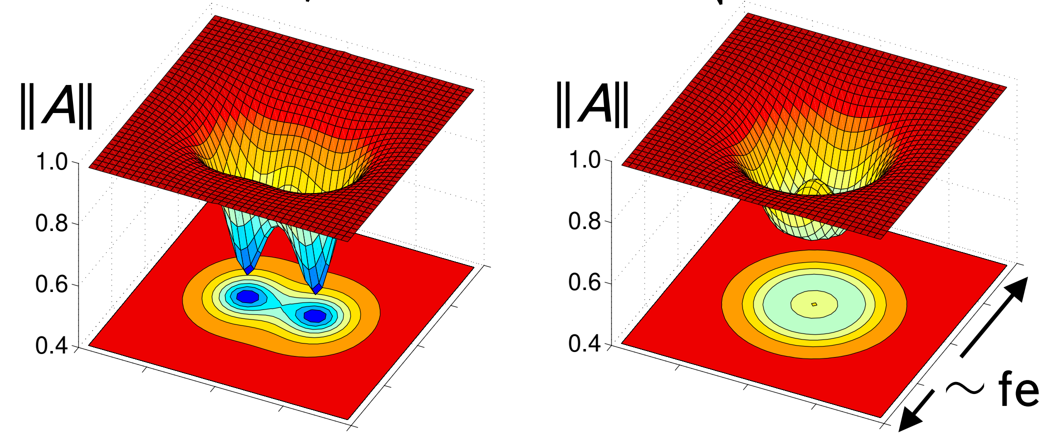
at 19 bar



$T = 0.1\,T_c$





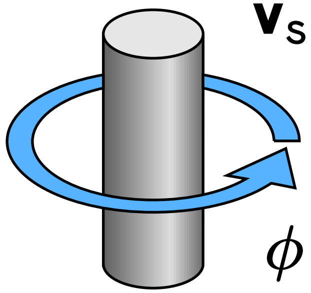


$\sim$ few $\xi \gtrsim 100\,\mathrm{nm}$

Broken symmetry core Axisymmetric core

Differ from usual ones

(like in ^{4}He and simple BEC):



$$\mathbf{v}_s = \frac{\hbar}{M}\nabla\phi.$$

$$\phi \rightarrow \phi + 2\pi$$

Not singular

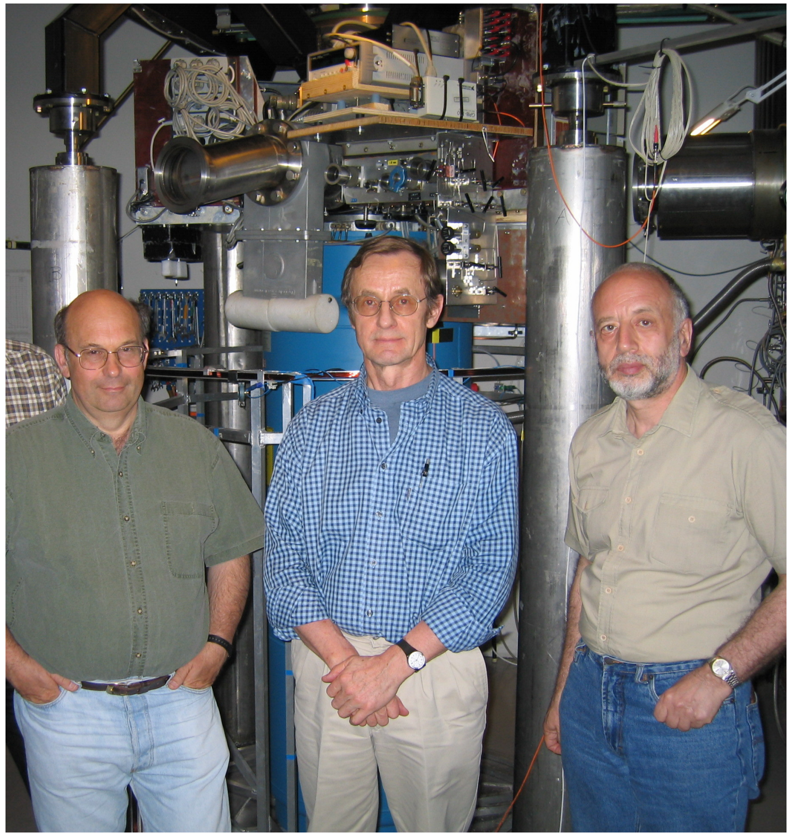
Not round

Core-bound fermions:

Dynamics — mass and dissipation

Minigap — $20 \div 100\,\mathrm{kHz}$ (P-dependent)

Thuneberg, PRL **56**, 359 (1986)
Volovik & Salomaa, JETP Lett **42**, 10 (1985)
Kondo *et al*, PRL **67**, 81 (1991)
Silaev *et al*, PRL **115**, 235301 (2015)
Regan *et al*, PRB **101**, 024517 (2020)
Rantanen & VE, PRR **6**, 043112 (2024)



Nikolai Kopnin   Matti Krusius   Grigori Volovik

Doctoral researchers (former and present):

Risto Hänninen
Jere Mäkinen
Vladislav Zavyalov
Lingju Meng
Juho Rysti
Saba Khan

Doctoral students (former and present):

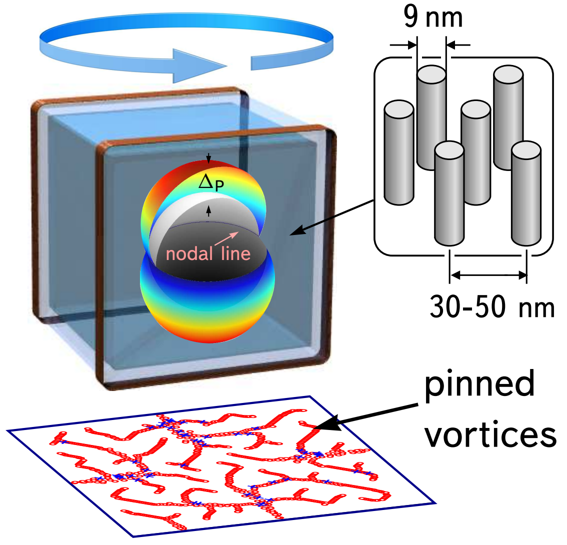
Antti Finne
Petri Heikkinen
Riku Rantanen
Rob de Graaf
Samuli Autti
Atso Ikäheimo
Jaakko Hosio
Timo Kamppinen

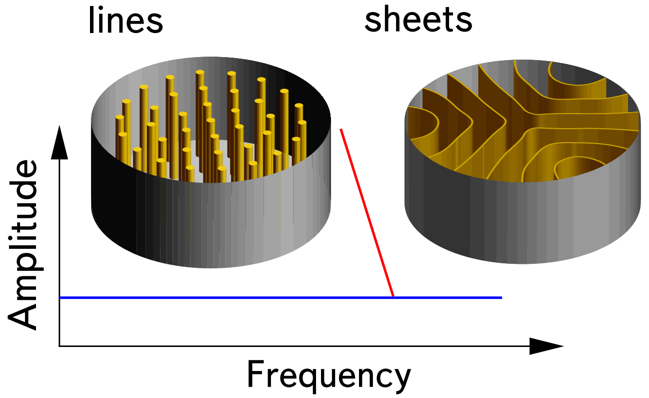
Collaborations:

Samuli Autti, *Lancaster University, UK*
Vladimir Dmitriev, Arkady Soldatov, Alexey Yudin, *Kapitza Institute*
Matthew Doyle, Andrei Golov, *University of Manchester, UK*
Victor L'vov, *Weizmann Institute, Israel*
Šimon Midlík, David Schmoranzer, *Charles University, Prague*

SUOMEN AKATEMIA
erc European Research Council
EMP

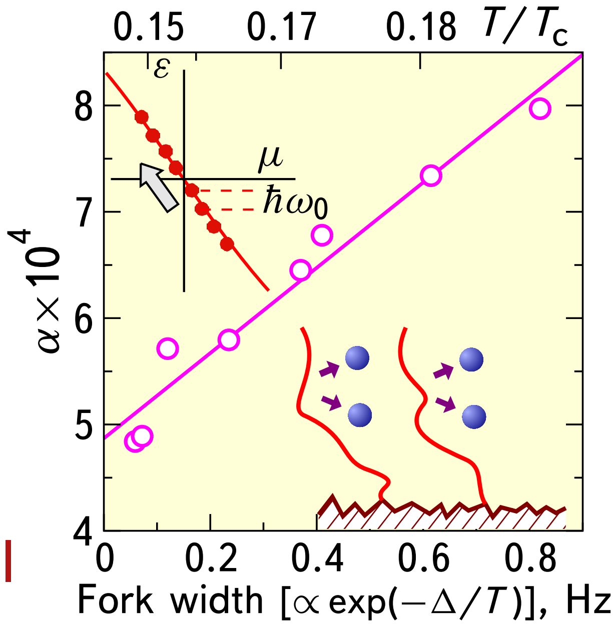
SUMMARY





- Vortex dynamics in superfluid ^{3}He is dominated by Kopnin force. Its temperature dependence leads to transitions in superfluid hydrodynamics.

  PNAS 111, 4711 (2014) **PRB 97, 014518 (2018)**



- Nanoconfinemet of ^{3}He stabilizes new topological phases and simultaneously provides strong pinning which drastically changes vortex dynamics.

PRR 2, 033013 (2020)
NatComm 10, 237 (2019)
NatComm 14, 4276 (2023)

- Anisotropic superfluid ^{3}He-A responds to dynamic drive with topological transition leading to faster response.

  PRL 88, 065301 (2002) **PRB 107, 104505 (2023)**

- Kelvin waves and KW-cascade are demonstrated in ^{3}He-B. KWs link to the vortex core physics including core overheating.

  NatPhys 19, 898 (2023) **PRB 107, 014502 (2023)**
  PRR 6, 043112 (2024)