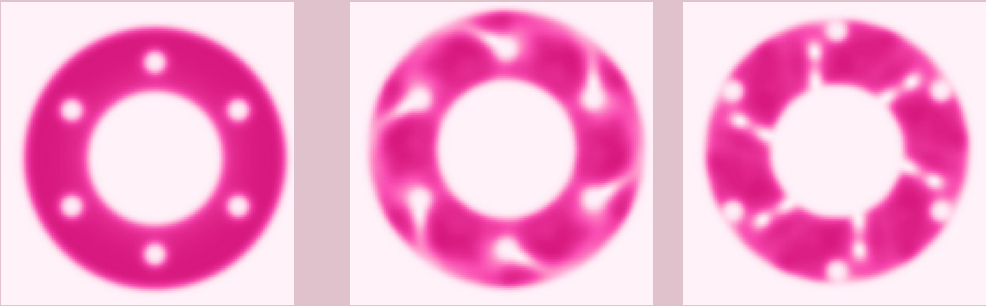
Exploring the role of impurities on the supercurrent stability in atomic superfluid rings



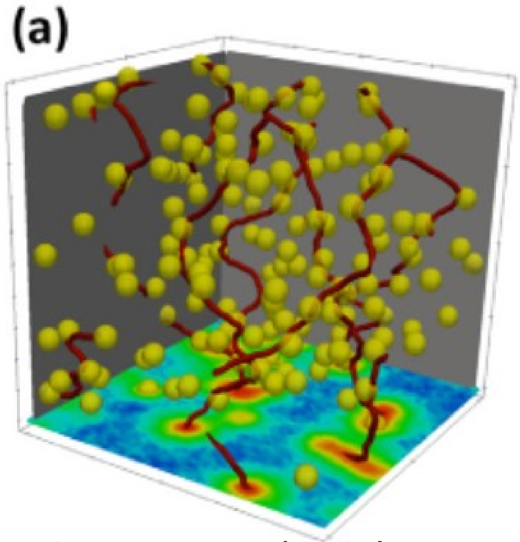
Klejdja Xhani
Politecnico of Torino (Italy)

Superflow stability in presence of impurities

Supercurrent stability ⟺ Vortex mobility

Vortex pinning by impurities

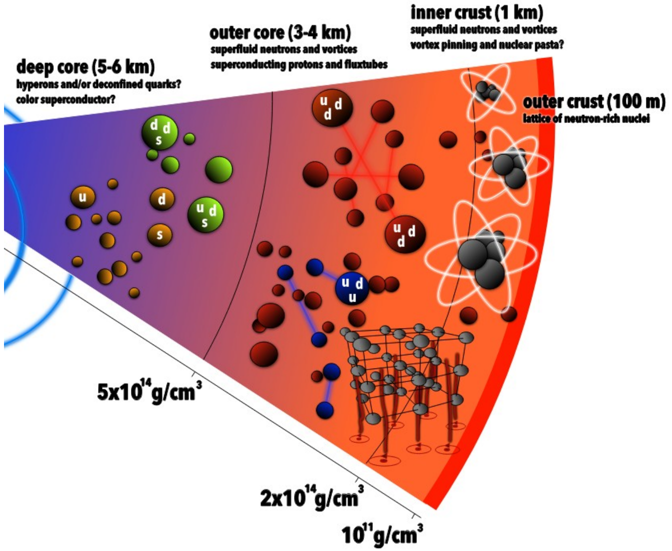
Superconductors



Strong and weak collective vortices pinning by impurities affect the critical current

Wai-Kwong Kwok et al 2016 Rep. Prog. Phys. 79 116501

Neutron stars



Macroscopic vortices unpinning and the angular momentum transfer to the outer crust → pulsar glitches

Vanessa Graber et al., https://doi.org/10.1142/S0218271817300154

Outline

Decay of persistent currents from BEC-UFG-BCS limit

Stabilizing persistent current in the presence of many impurities

Vortices pinning/unpinning and persistent current stability

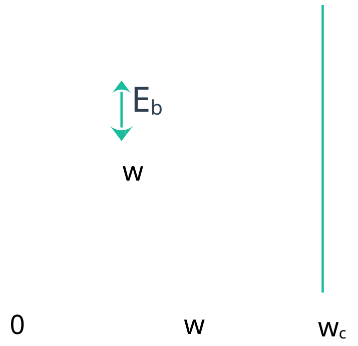
Spin imbalance effect on persistent current

Decay of persistent currents

Circulation Γ is quantized

$$\frac{m}{\hbar}\oint_\Gamma d\boldsymbol{r}\cdot\boldsymbol{v}(\boldsymbol{r}) = 2\pi w,$$

⇒ Persistent current states,
the minima of the washboard potential.

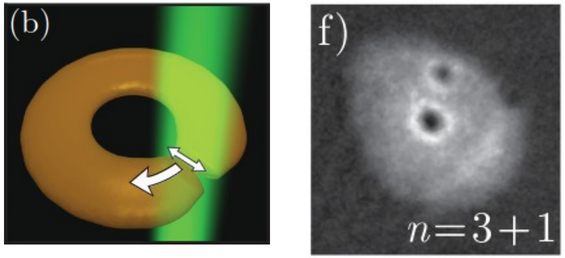


The energy barrier E_b goes to zero for $w \to w_c$

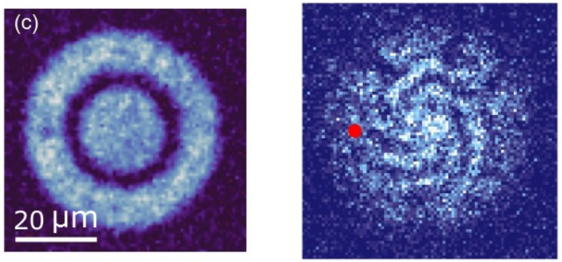
The presence of a defect decreases E_b and thus w_c

Thermal or quantum phase slippage → transition between metastable states

Moving a defect with $v > v_c$:Decay of w due to vortices emitted

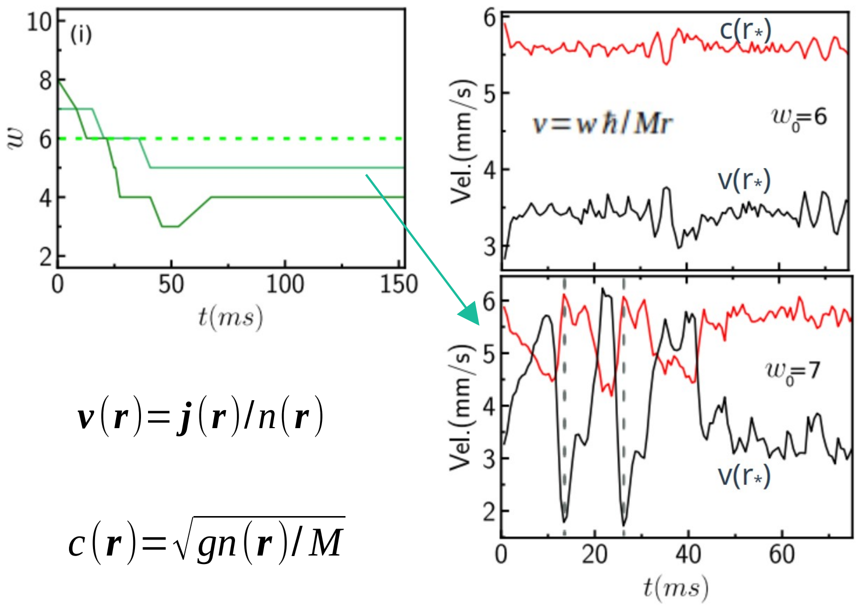


K. C. Wright, et al.,
Phys. Rev. Lett. 110, 025302 (2013)



G. Del Pace, K.Xhani et al.,
Phys. Rev. X 12, 041037 (2022)

Critical velocity: Single defect



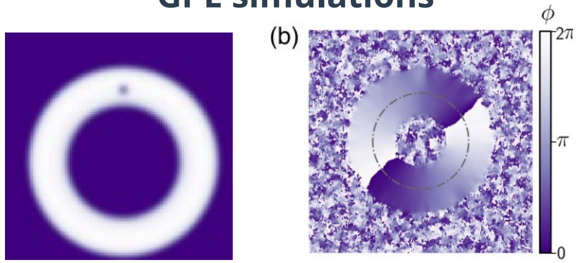
$$\mathbf{v}(\mathbf{r}) = \mathbf{j}(\mathbf{r})/n(\mathbf{r})$$

$$c(\mathbf{r}) = \sqrt{gn(\mathbf{r})/M}$$

$$c_{bulk}(\mathbf{r}) = \sqrt{\mu/M}$$

$V_0/\mu = 0.96$

GPE simulations



No defect: current is persistent

In the presence of the defect:
critical winding number for vortices emission

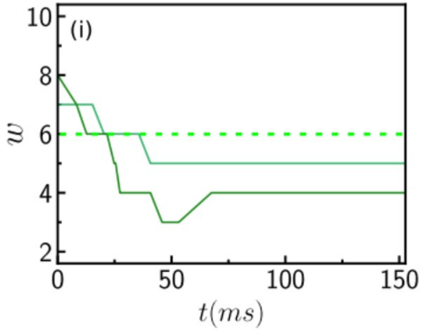
For $w_0 > w_c$, the winding number decays in time due to vortices emission

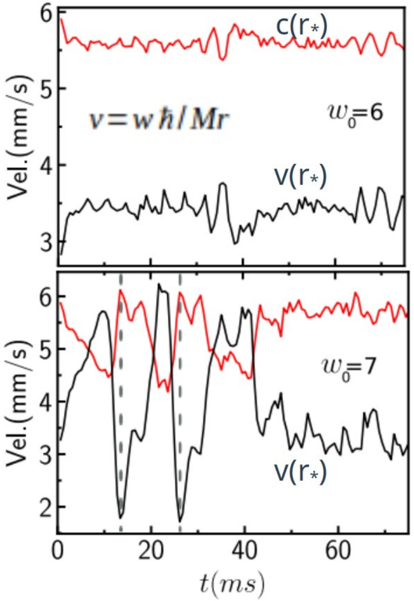
Locally a chemical potential gradient formed→ acceleration of superfluid
Josephson-Anderson relation

$$M\dot{\mathbf{v}} = -\nabla\mu$$

G. Del Pace, **K. Xhani** et al., Phys. Rev. X 12, 041037 (2022).

Critical velocity



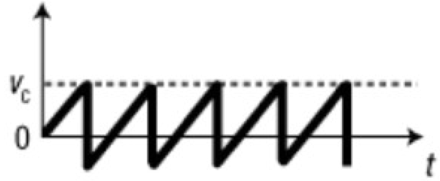


$$\boldsymbol{v}(\boldsymbol{r}) = \boldsymbol{j}(\boldsymbol{r}) / n(\boldsymbol{r})$$

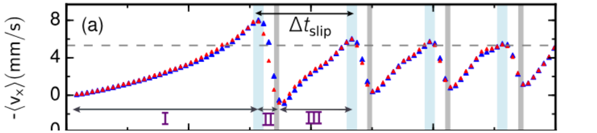
$$c(\boldsymbol{r}) = \sqrt{g n(\boldsymbol{r}) / M}$$

Vortices are emitted at v_{max} causing phase-slippage

The velocity v shows discrete jumps similar to findings in superfluid helium and atomic Josephson junction.



E.Hoskinson et al., Nature Phys. 2, 23 (2006).

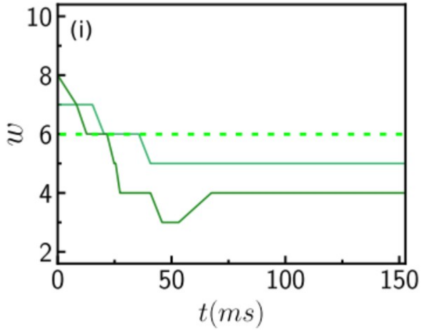


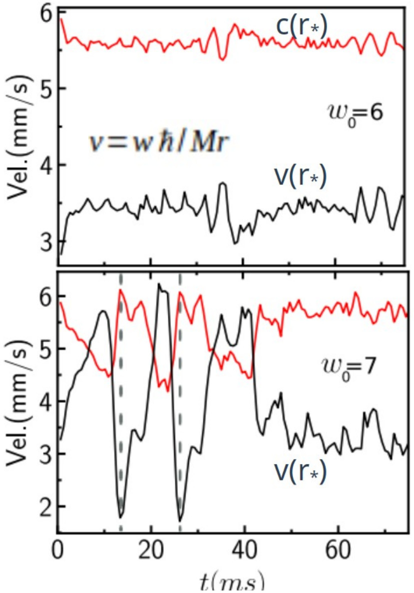
K.Xhani et al., Phys. Rev. Lett. 124, 045301 (2020).

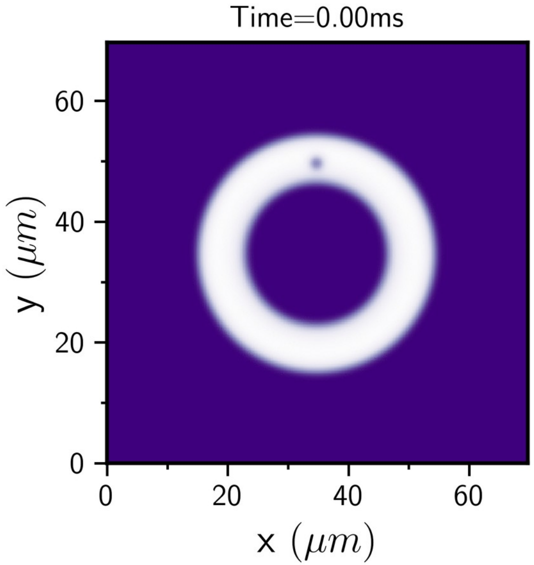
$V_0/\mu = 0.96$

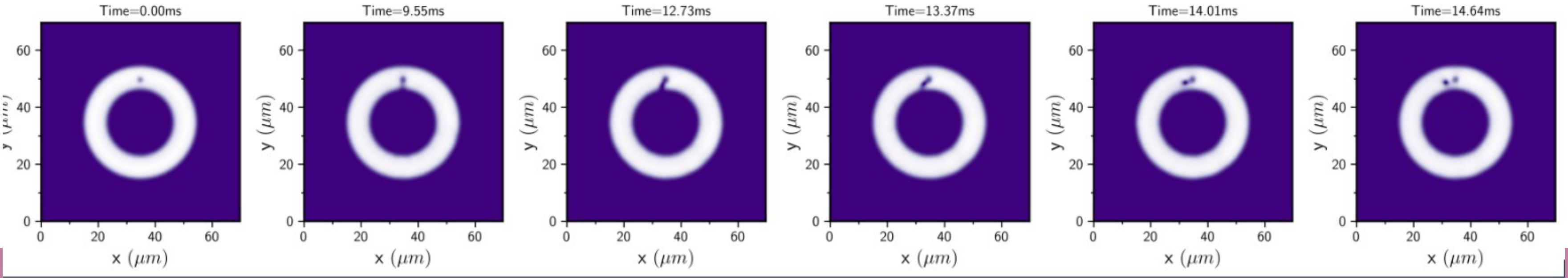
G. Del Pace, **K. Xhani** et al., Phys. Rev. X 12, 041037 (2022).

Dissipation mechanism: vortices emission









Fermionic persistent current: Theoretical model

BCS limit $|ak_F|\leq 1$: Time-dependent Bogoliubov de Gennes (BdG) equations

$$i\frac{\partial}{\partial t}\begin{pmatrix} u_n(\mathbf{r},t) \\ v_n(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r},t) & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -h^*(\mathbf{r},t) \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r},t) \\ v_n(\mathbf{r},t) \end{pmatrix}$$

Pauli principle: $\int \varphi_n^\dagger(\mathbf{r},t)\varphi_m(\mathbf{r},t)d\mathbf{r} = \delta_{nm}$

$h(\mathbf{r},t) = -\frac{\nabla^2}{2} + U(\mathbf{r},t) + V_{\text{ext}}(\mathbf{r},t) - \mu$ $U_{BCS}\approx 0$

Pairing potential: $\Delta(\mathbf{r},t) = -g\nu(\mathbf{r},t)$

Anomalous density: $\nu = \sum_{E_n \geq 0} v_n^* u_n$

UFG limit: Superfluid Local Density Approximation (SLDA) , Density Functional theory
Additional coupling between the pairing and density modes

$$U^{(\mathrm{UFG})} = \frac{\beta(3\pi^2 n)^{2/3}}{2} - \frac{|\Delta|^2}{3\gamma n^{2/3}}$$

$$\Delta^{(\mathrm{UFG})} = -\frac{\gamma}{n^{1/3}}\nu$$

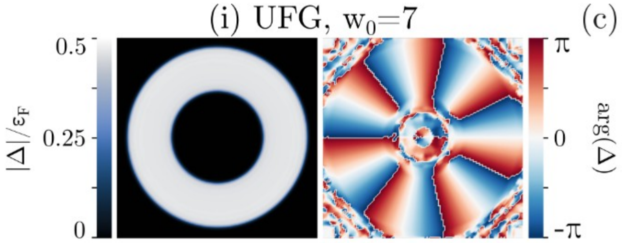
β and γ such that $\xi_0\approx 0.4$ and the energy gap $\Delta/\epsilon_F\approx 0.5$

Evolved quasiparticles $5\text{x}10^5$

1million coupled equations to be solved→ High performance supercomputer

Fermionic persistent current: Theoretical model

1) Find the static solution in a ring, without the defect and imprint a phase in the order paramater



2) Solve the dynamics $i\frac{\partial}{\partial t} \rightarrow E_n$

TDSLDA

$$i\frac{\partial}{\partial t}\begin{pmatrix} u_n(\mathbf{r},t) \\ v_n(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r},t) & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -h^*(\mathbf{r},t) \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r},t) \\ v_n(\mathbf{r},t) \end{pmatrix}$$

Two-dimensional geometry: The quasi-particle wave function $\varphi_n(\mathbf{r},t) \equiv \varphi_n(x,y,t)e^{ik_z z}$

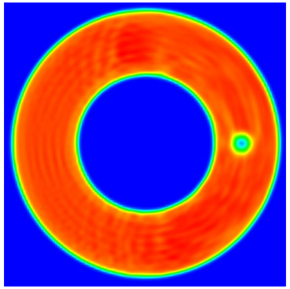
Dynamics: Without and with defect added in 25 te_F

$$V = V_0 e^{-2[(x-x_0)^2+(y-y_0)^2]/w^2}$$

$$V_0/\mu = 2 \qquad wk_F = 10$$

Fixed defect's parameters

$$N_x \, x \, N_y \, x \, N_z = 128 \, x \, 128 \, x \, 16$$



$\lambda^{-1} \simeq 0$, UFG limit

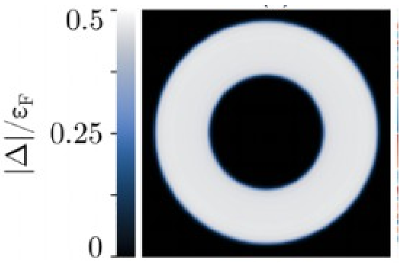
$\lambda^{-1} \simeq -0.4$, BCS limit

$\lambda^{-1} \simeq -1$, dBCS limit

Static calculations: no defect

In fermionic superfluids current dissipation does not necessarily mean winding number decay

$$\vec{j}(\vec{r}) = 2 \sum_{E_n>0} \mathrm{Im}\left(v_n \vec{\nabla} v_n^*\right) = \rho(\vec{r})\vec{v}(\vec{r})$$



Flow energy

$$E_{flow} = \int j^2(\mathbf{r})/2\rho(\mathbf{r})\, d\mathbf{r}$$

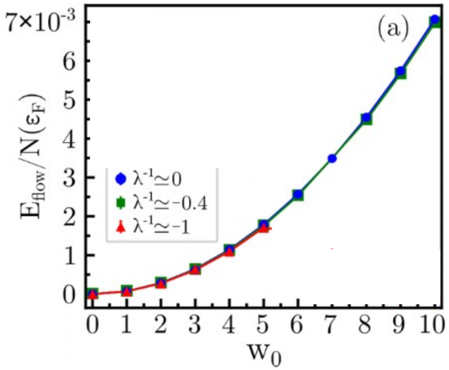
If pure superfluid: $\rho=\rho_s$

$v_s = w_0/2r$

$$E_{flow} = \frac{\pi}{4} L_z \rho_s^{bulk} w_0^2 \ln(R_2/R_1)$$

⇩

The flow energy scaling with w^2



$\lambda^{-1} \simeq 0$, UFG limit

$\lambda^{-1} \simeq -0.4$, BCS limit

$\lambda^{-1} \simeq -1$, dBCS limit

$\lambda^{-1} = 1/k_F a$

K. Xhani, A. Barresi, M. Tylutki, G. Wlazłowski and P. Magierski,
Phys. Rev. Research 7, 013225 (2025)

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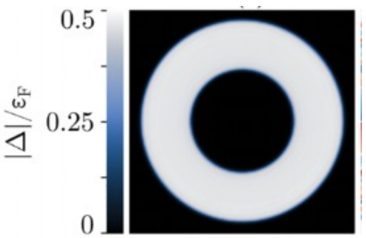
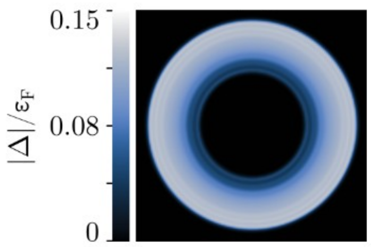


Increasing w_0



Threshold: Pair-breaking for $w_0 = w_{pb}$

Flow energy

$$E_{flow} = \int j^2(\mathbf{r})/2\rho(\mathbf{r})\, d\mathbf{r}$$

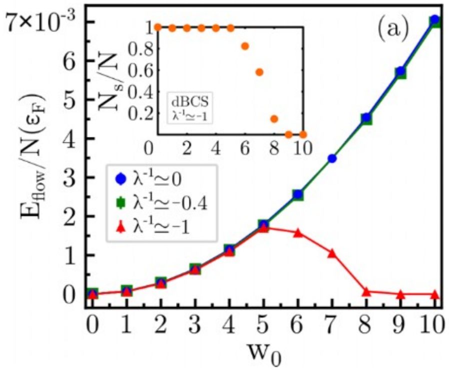
In two-fluid model: $\rho = \rho_s + \rho_n$

Suppose $\mathbf{v_n = 0}$ then $\mathbf{j = j_s = \rho_s * v_s}$

$$E_{flow} \propto \frac{N_s}{N} w_0^2$$

⇓

The flow energy decrease with w^2 for $w_0 > w_{pb}$



$\lambda^{-1} \simeq 0$, UFG limit

$\lambda^{-1} \simeq -0.4$, BCS limit

$\lambda^{-1} \simeq -1$, dBCS limit

$$N_s = \int \rho_s(\vec{r})d^3\vec{r} = \int \frac{2r}{w_0} j(\vec{r})d^3\vec{r},$$

$$N = \int \rho(\vec{r})d^3\vec{r},$$

K. Xhani, A. Barresi, M. Tylutki, G. Wlazłowski and P. Magierski, Phys. Rev. Research 7, 013225 (2025)

T=0 Static calculations: no defect

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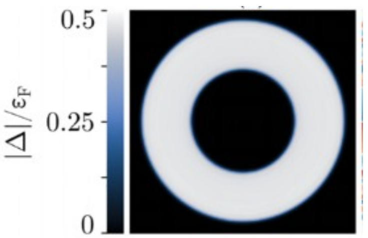
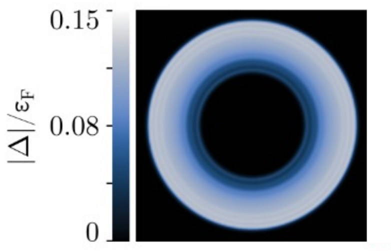


Increasing w_0



Flow energy

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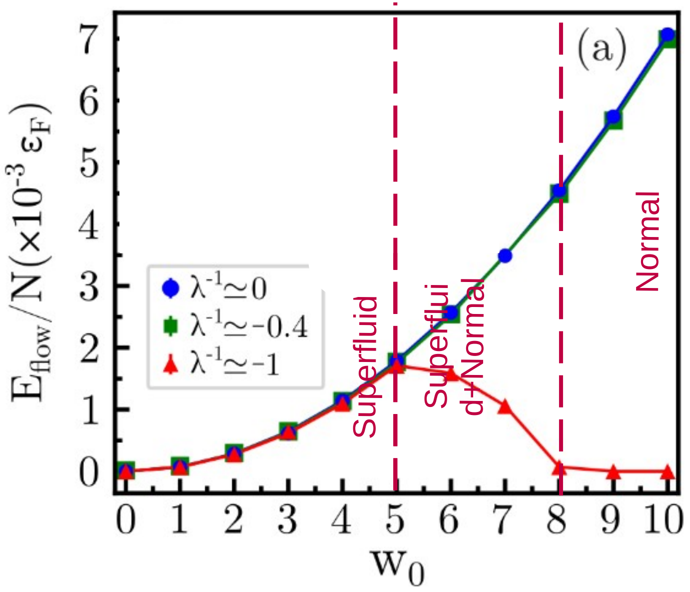
In two-fluid model: $\rho=\rho_s+\rho_n$

Suppose $\mathbf{v_n=0}$ then $\mathbf{j=j_s}=\rho_s{}^*\mathbf{v_s}$

$$E_{flow} \propto \frac{N_s}{N} w_0^2$$

⇓

The flow energy decrease with w^2 for $w_0>w_{pb}$



For $w_0>8$ unable to imprint a flow; normal state

K. Xhani, A. Barresi, M. Tylutki, G. Wlazłowski and P. Magierski,
Phys. Rev. Research 7, 013225 (2025)

Static calculations: no defect

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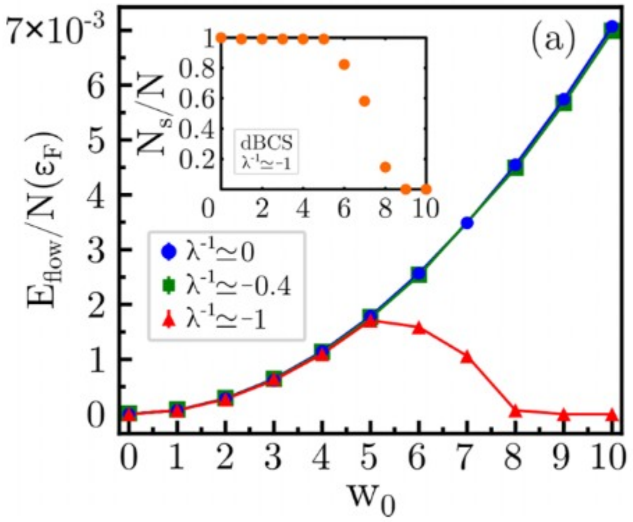

$\lambda^{-1} \simeq 0$, UFG limit $\lambda^{-1} \simeq -0.4$, BCS limit $\lambda^{-1} \simeq -1$, dBCS limit

Revealing pair-breaking mechanism through:

1-Drop of condensation energy
2-Local velocity exceeding pair-breaking threshold

Condensation energy

$$E_{\rm cond}(t) = \frac{3}{8} \int \frac{|\Delta(\vec{r},t)|^2}{\varepsilon_F(\vec{r},t)} \rho(\vec{r},t)\, d^3\vec{r}$$

Pair-breaking velocity

$$\tilde{v}_{pb} = \sqrt{\sqrt{\mu^2 + \Delta^2} - \mu^2},$$

$$\tilde{\Delta} E_{\rm cond}(w_0) = \frac{|E_{\rm cond}(w_0) - E_{\rm cond}(w_0{=}0)|}{E_{\rm cond}(w_0{=}0)}$$

$2rv_{pb} = w_{pb}$

K. Xhani, A. Barresi, M. Tylutki, G. Wlazłowski and P. Magierski, Phys. Rev. Research 7, 013225 (2025)

Static calculations: no external defect

In fermionic superfluids current dissipation does not necessarily mean winding number decay

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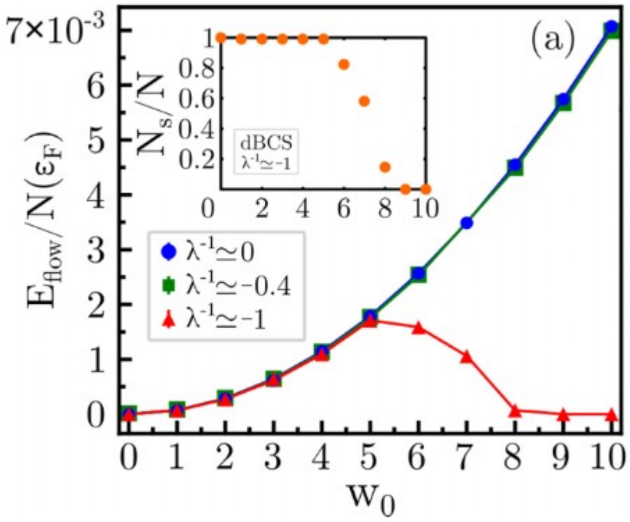

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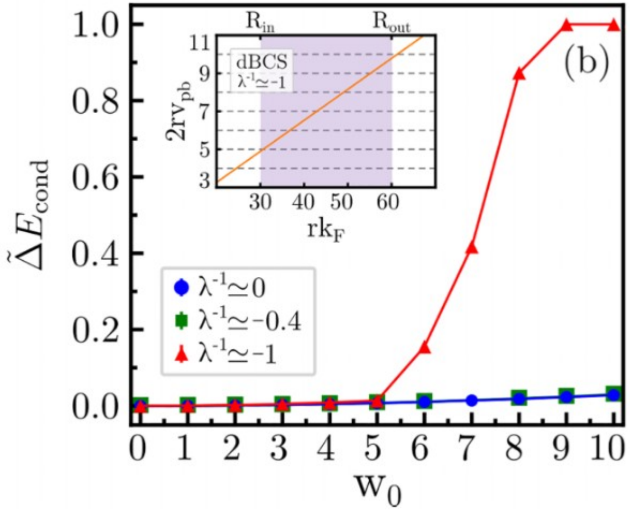
Condensation energy

$$E_{\text{cond}}(t) = \frac{3}{8}\int \frac{|\Delta(\vec{r},t)|^2}{\varepsilon_F(\vec{r},t)}\rho(\vec{r},t)\, d^3\vec{r}$$

Pair-breaking velocity

$$\tilde{v}_{pb} = \sqrt{\sqrt{\mu^2+\Delta^2}-\mu^2},$$

$$\tilde{\Delta}E_{\text{cond}}(w_0) = \frac{|E_{\text{cond}}(w_0)-E_{\text{cond}}(w_0=0)|}{E_{\text{cond}}(w_0=0)}$$

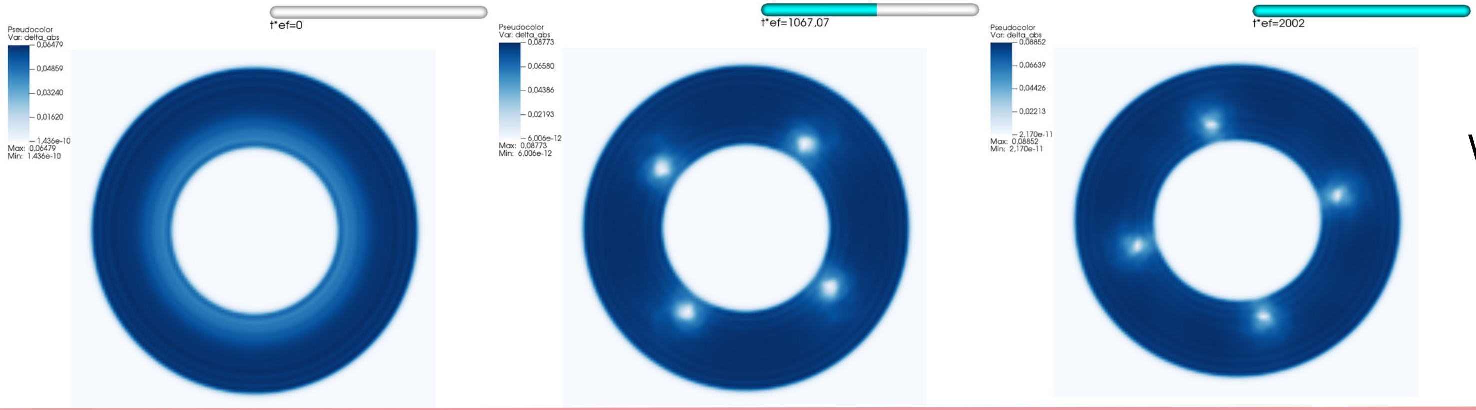


$2rv_{pb}=w_{pb}$

$\lambda^{-1}\simeq0$, UFG limit $\quad \lambda^{-1}\simeq-0.4$, BCS limit $\quad \lambda^{-1}\simeq-1$, dBCS limit

K. Xhani, A. Barresi, M. Tylutki, G. Wlazłowski and P. Magierski, Phys. Rev. Research 7, 013225 (2025)

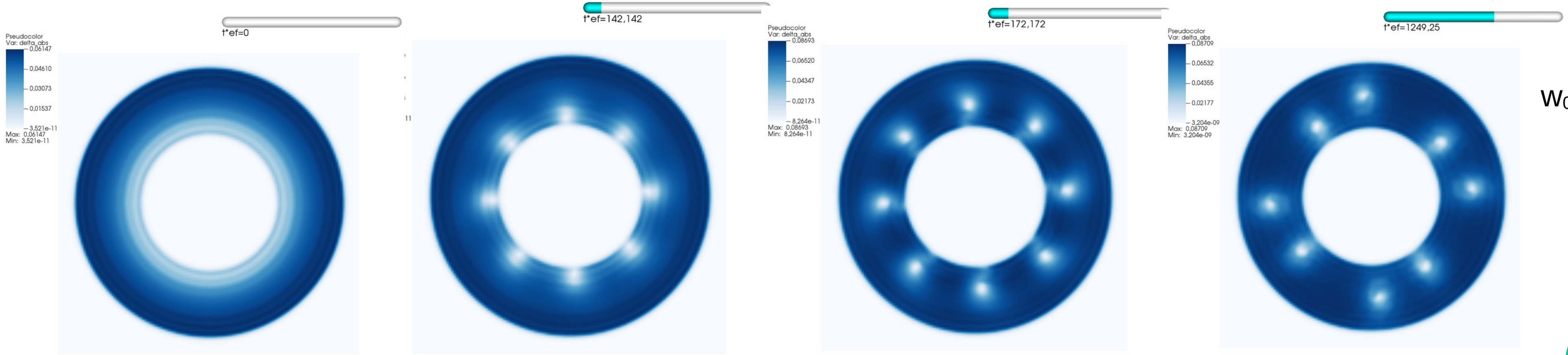
Dynamical simulations: no external defect



$W_0=6>w_{pb}$

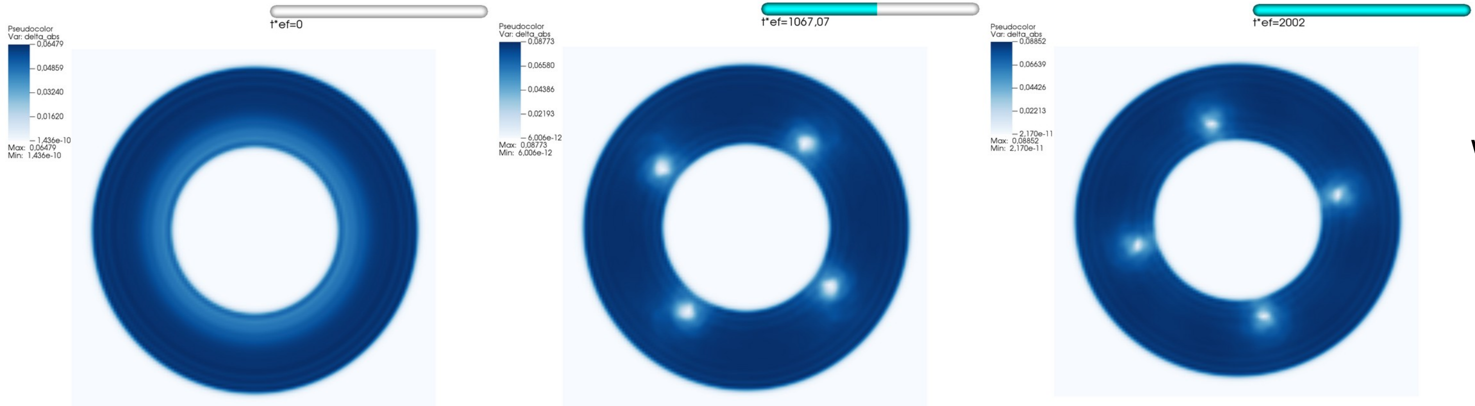
Vortex array due to superfluid-normal component interface Kelvin-Helmholtz instability?

Pair-breaking favours vortices emission



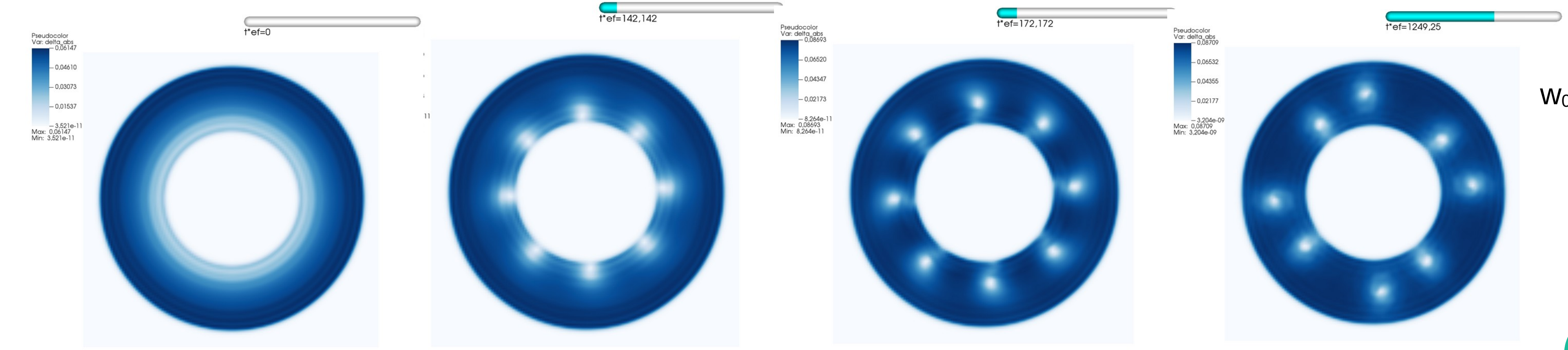
$w_0=7$

Dynamical simulations: no external defect



$w_0=6$

More on KHI→
Diego H. Rajkov's talk



$w_0=7$

BEC: Many impurities

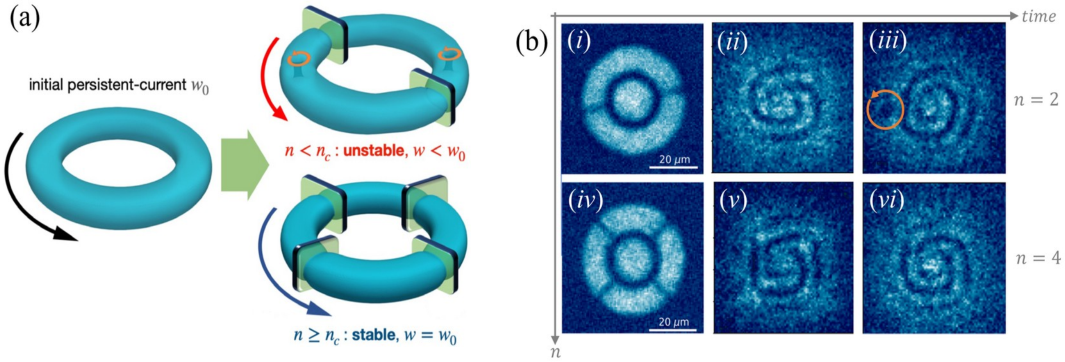
$$V_{defect}=\sum_{i=1}^{i=n/2} V_0 \exp\left[-2\left(\frac{x\cos(i\pi/n)-y\sin(i\pi/n)}{w}\right)^2\right]$$

$V_0/\mu=3.7$
$w=2\,\xi$

Num.

^{6}Li experiment at LENS

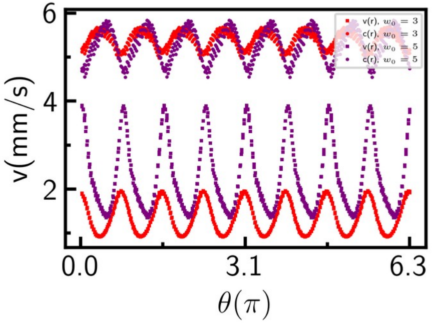
BEC limit GPE



L. Pezzè*, **K. Xhani***, C. Daix* et al., Nat. Commun 15, 4831 (2024).

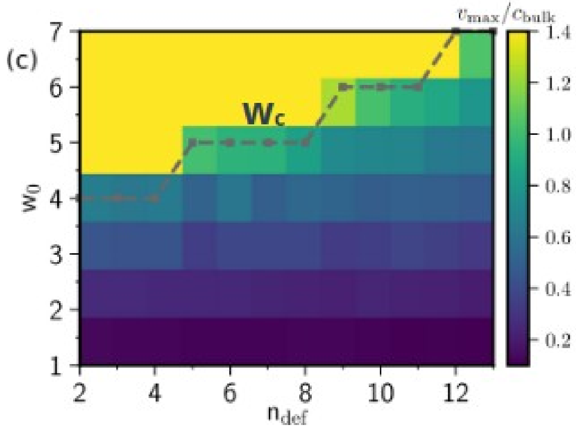
BEC: Many impurities

$$V_{defect} = \sum_{i=1}^{i=n/2} V_0 \exp\left[-2\left(\frac{x\cos(i\pi/n) - y\sin(i\pi/n)}{w}\right)^2\right] \qquad V_0/\mu = 3.7 \qquad w = 2\,\xi$$



Extract velocity peak values v_{max}

Extract bulk speed of sound



w_c increases with n_{def}

I_c increases with n_{def}

Similar to the Josephson junction necklace

L. Pezzè*, **K. Xhani***, C. Daix* et al., Nat. Commun 15, 4831 (2024).

For each number of defects, extract w_c and v_{max}/c_{bulk}

$$\boldsymbol{v}(\boldsymbol{r}) = \boldsymbol{j}(\boldsymbol{r})/n(\boldsymbol{r}) \qquad c_{bulk} = \sqrt{\mu/M}$$

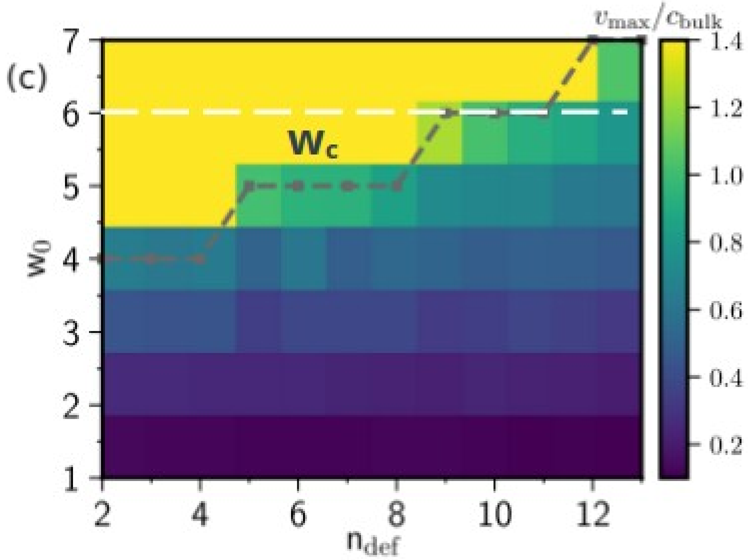
$$c(\boldsymbol{r}) = \sqrt{gn(\boldsymbol{r})/M}$$

$$\frac{m}{\hbar}\oint_\Gamma d\boldsymbol{r}\cdot v(\boldsymbol{r}) = 2\pi w,$$

$$v = v_{buk} + n\,v_{peaks}$$

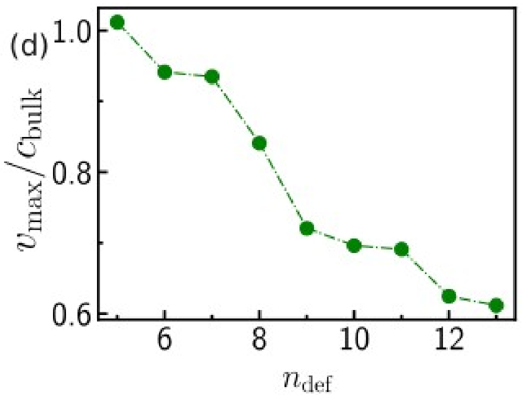
K. Xhani et al., in preparation

Critical current vs number of impurities

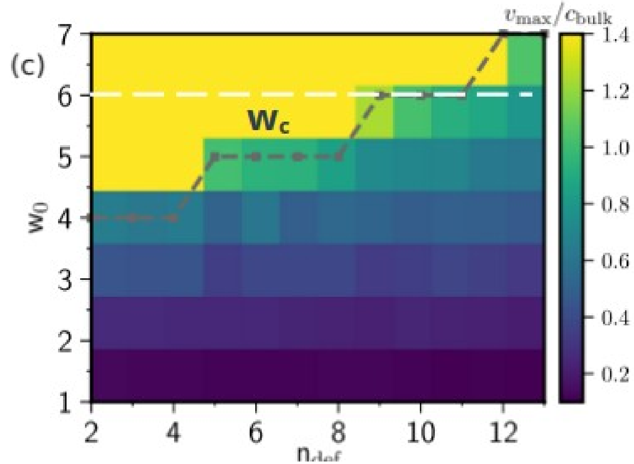


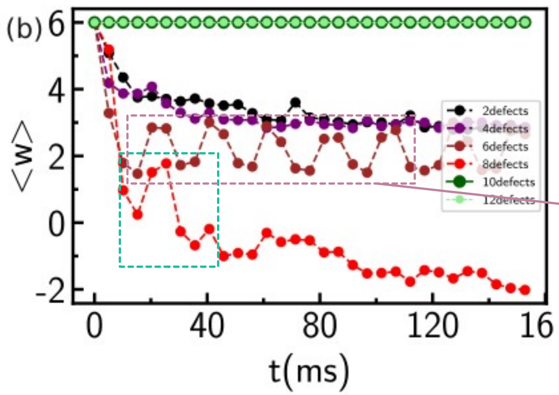
v_{max}/c_{bulk} almost indipendent on n_{def} at the critical point

v_{max}/c_{bulk} decreases at fixed w_0



Role of impurities density on critical current

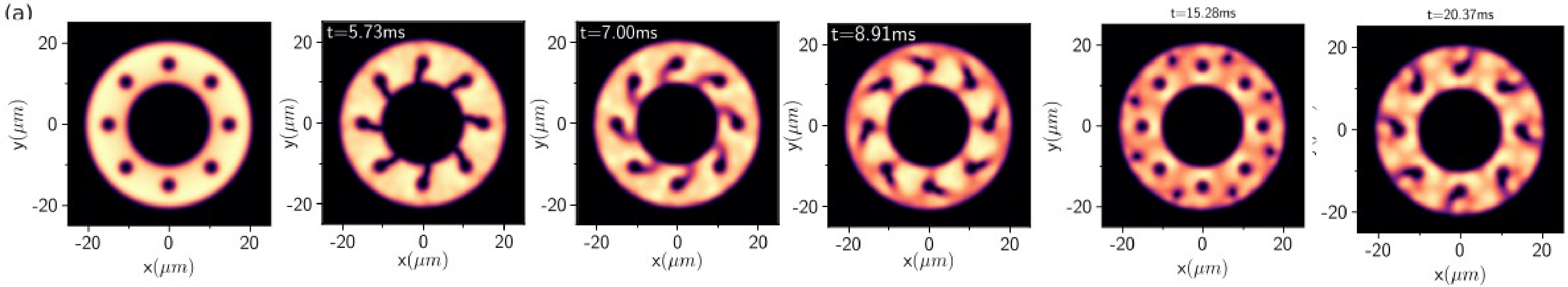




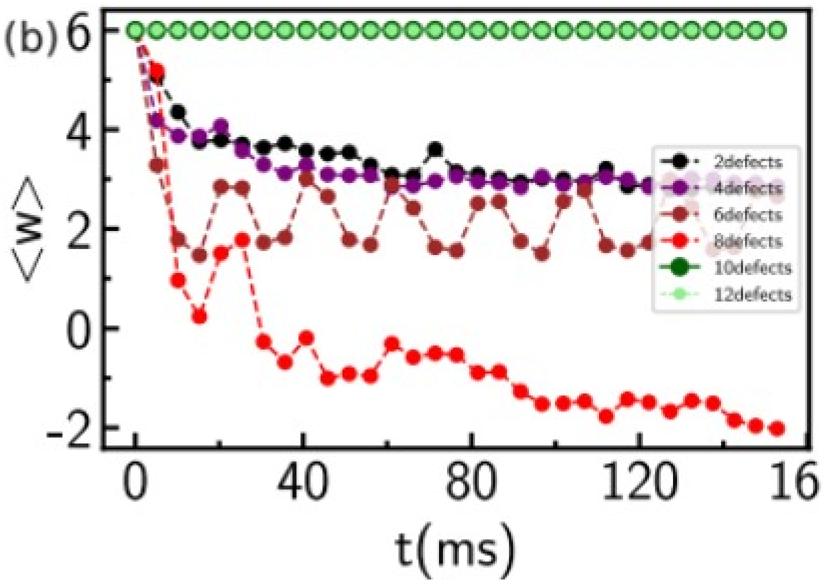
By increasing the number or impurities the supercurrent is stabilized

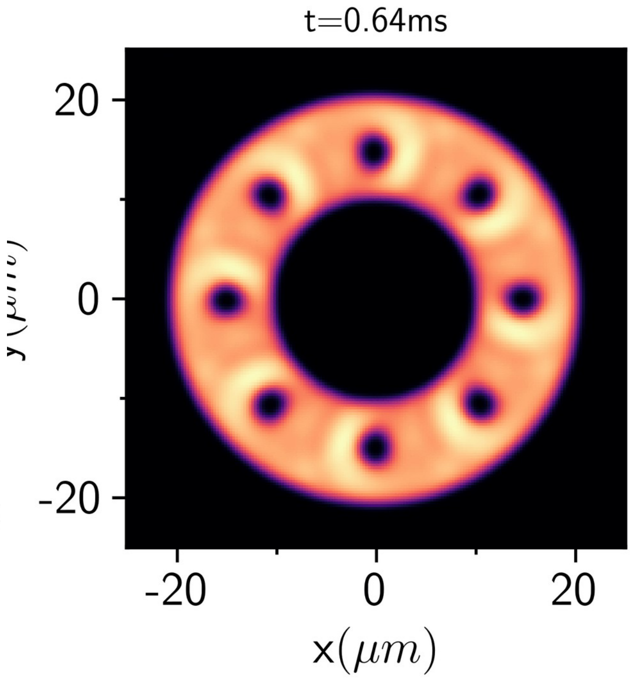
The reason behing the average w oscillations?

Vortex weakly and temporarily pinned to defects



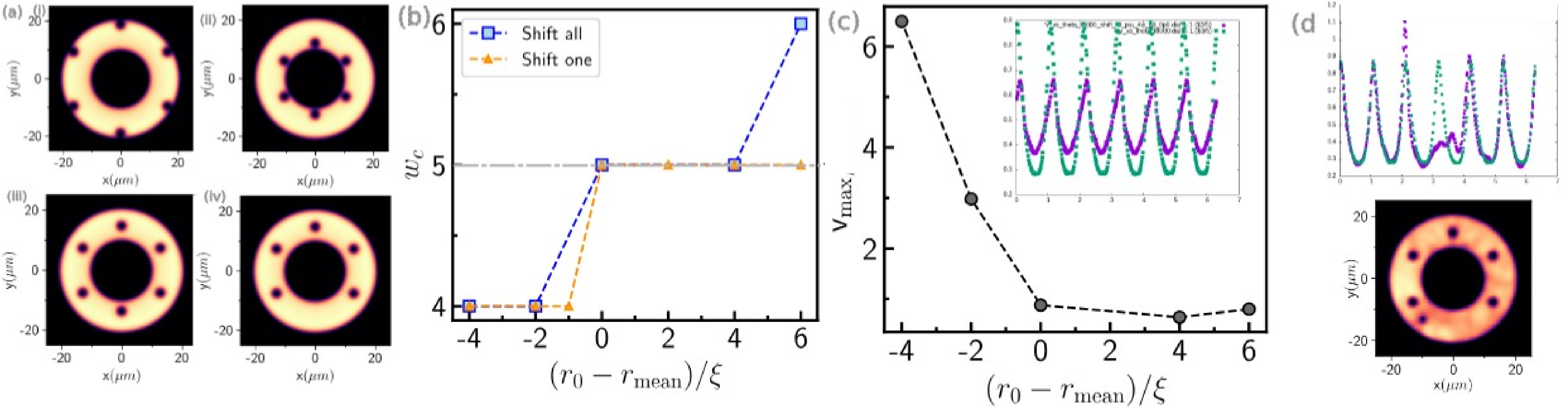
Stability of persistent currents in the presence of impurities





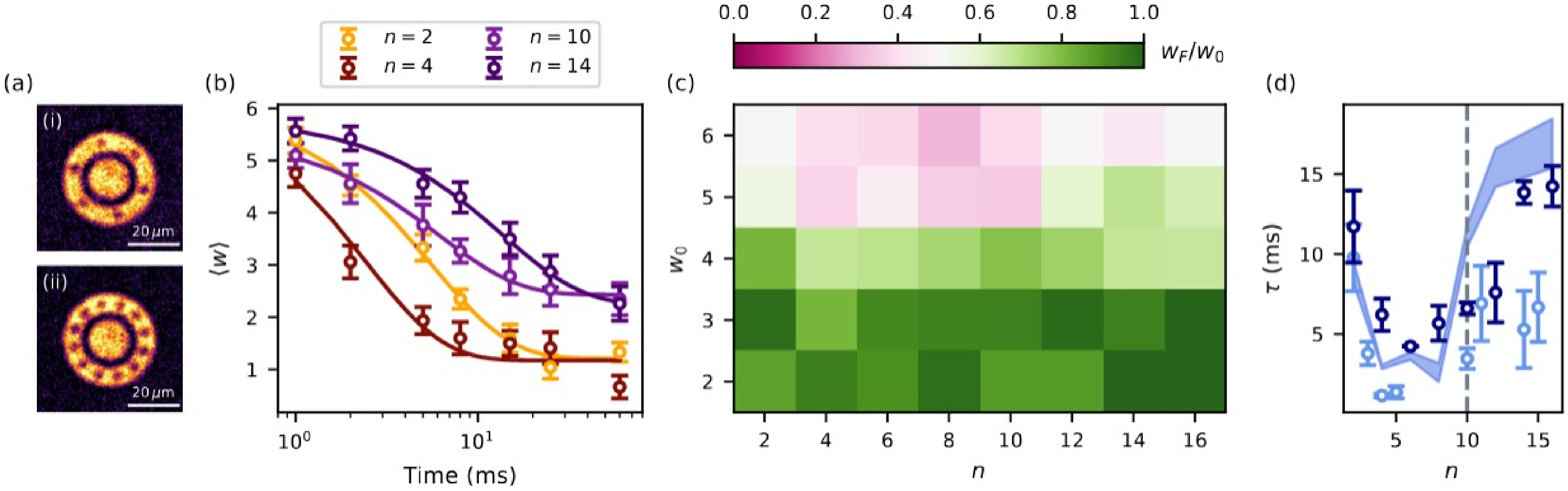
Periodic vortex pinning/unpinning affects supercurrent

Role of impurities distribution on current stability



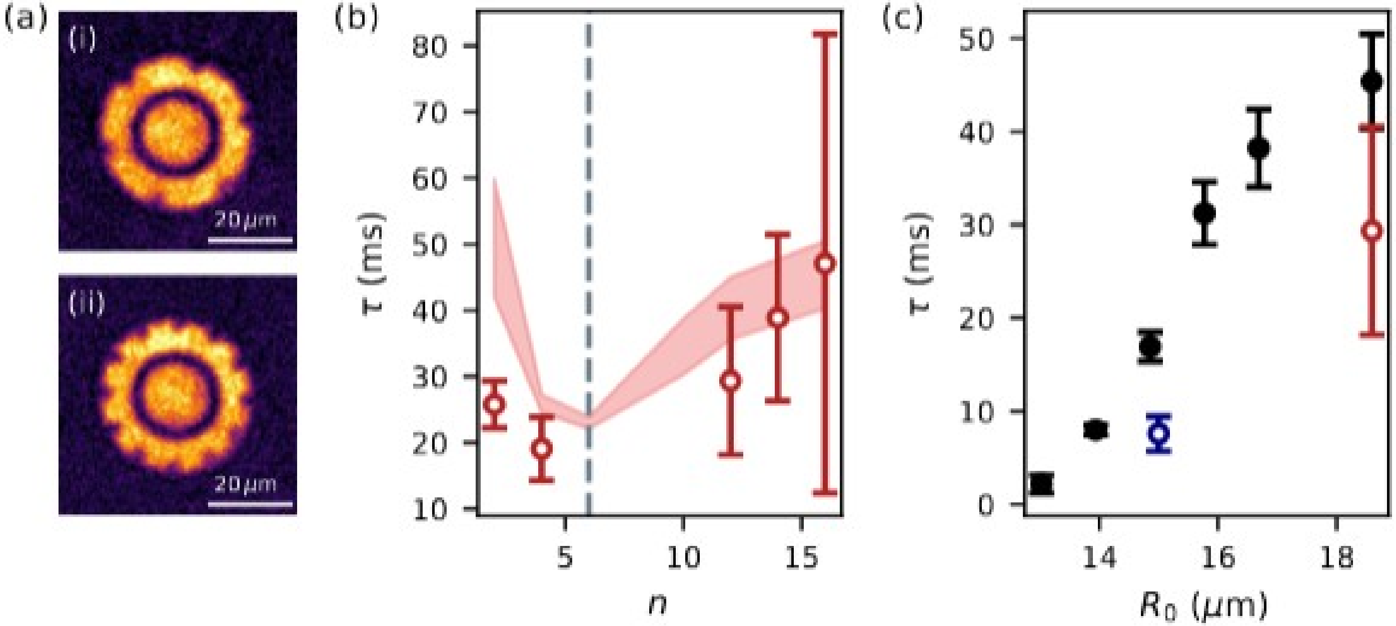
Supercurrent stability increases as defects are moved towards the outer edge and for periodic configuration

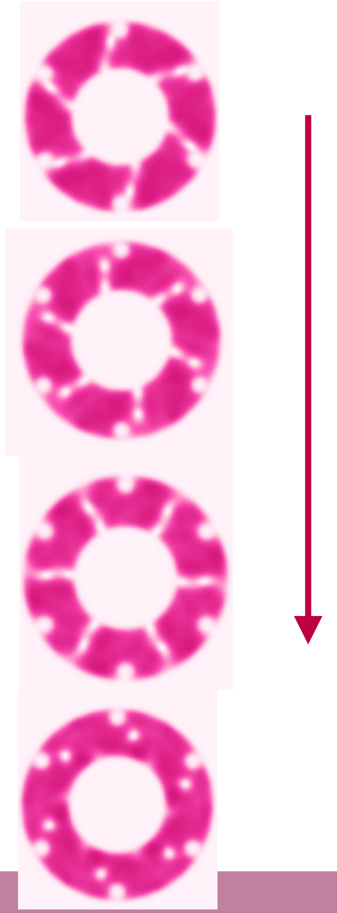
Comparison with the experiment: disordered configuration



Supercurrent stability strongly dependent on defect's parameters error bars

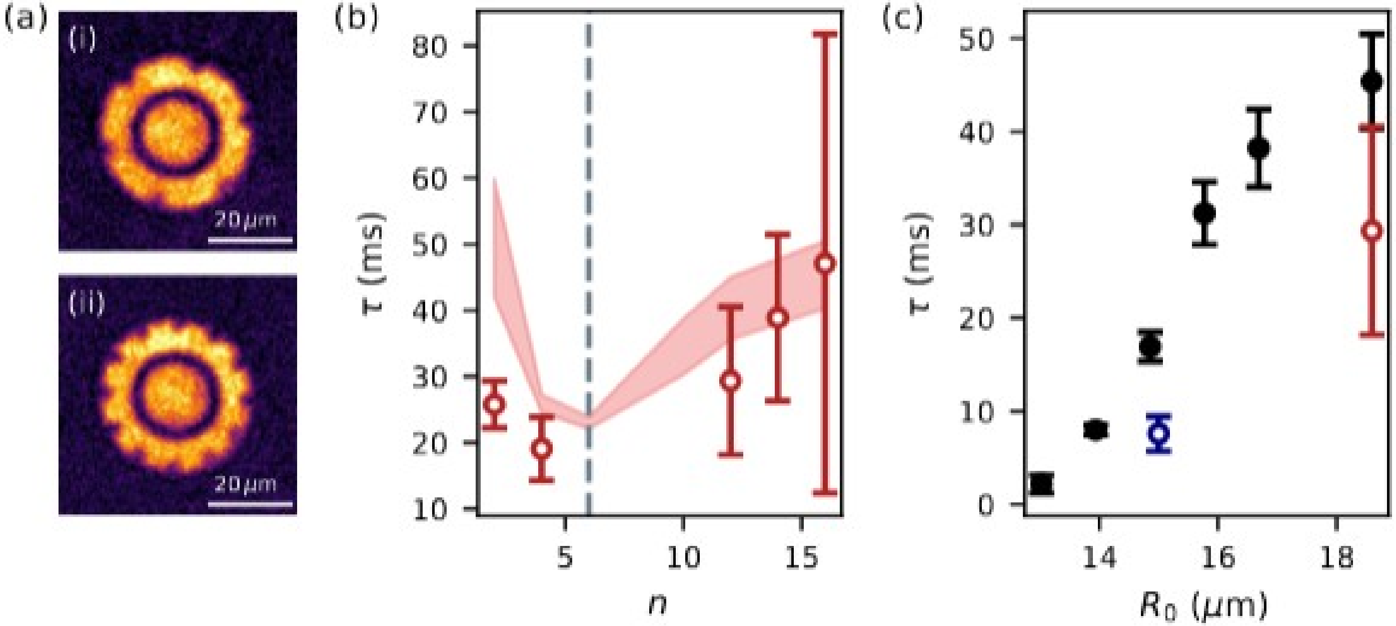
Comparison with the experiment: disordered configuration





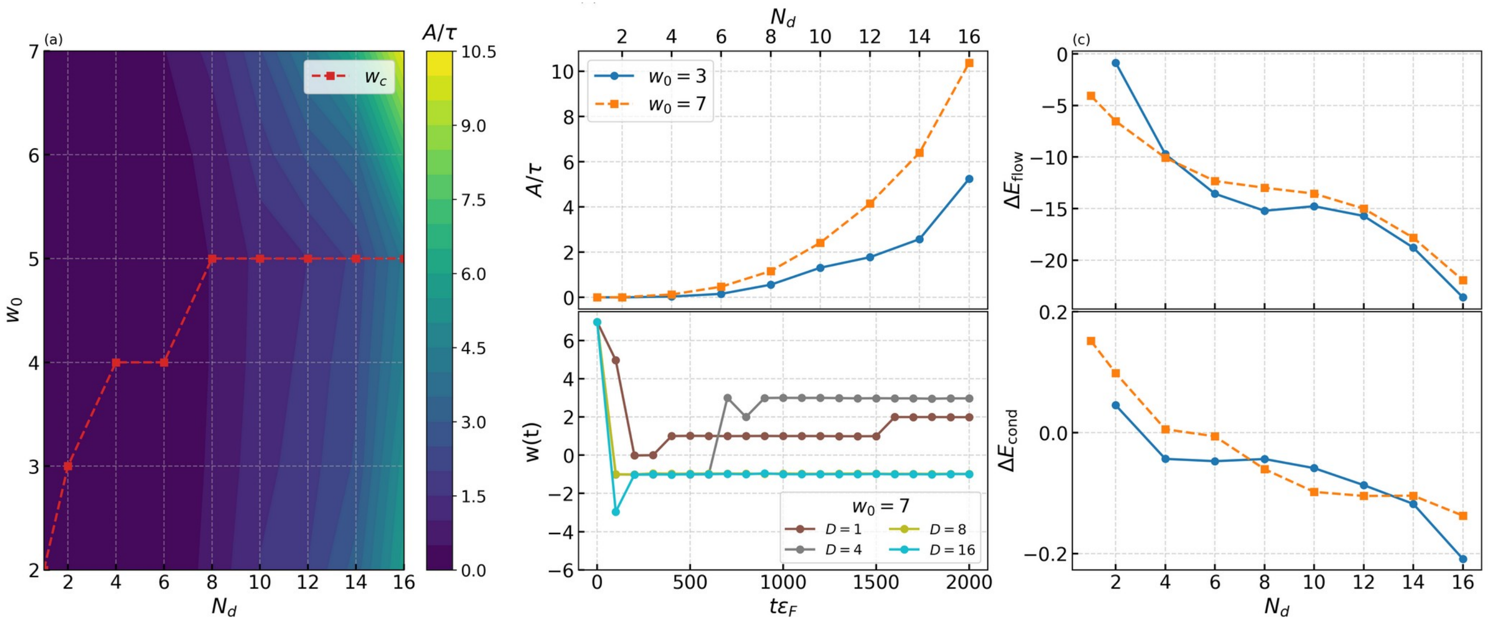
Moving defects towards outer edge stabilizing the current

Comparison with the experiment: disordered configuration

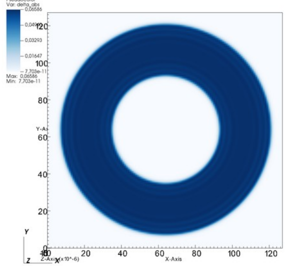


Moving defects towards outer edge stabilizing the current

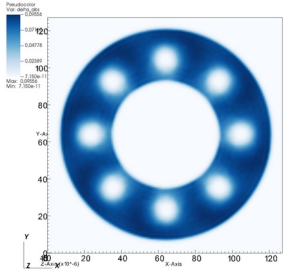
BCS limit: Multidefects



Static



Dynamics



Critical winding number increases with n defects until $w_c = w_{pb}$

Dissipation is larger at higher w_0 and n defects

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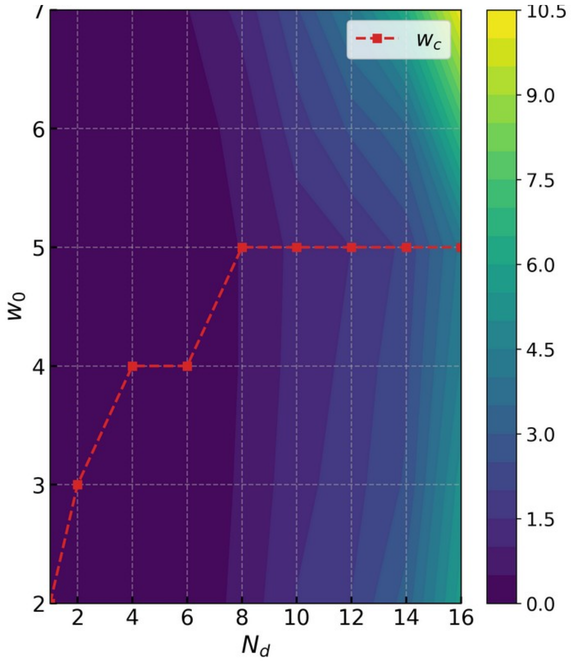
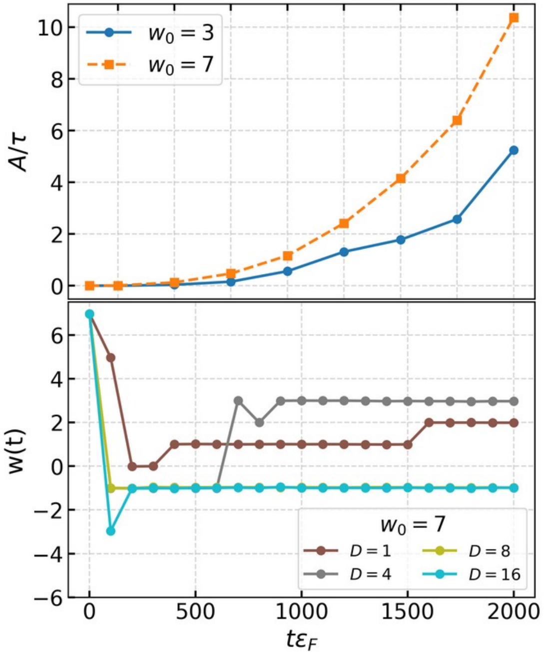
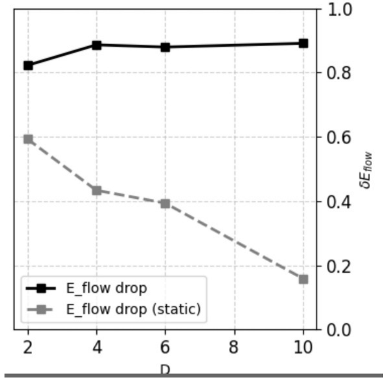
No stabilization affects as we increase n defects, why?

BCS limit: Multidefects





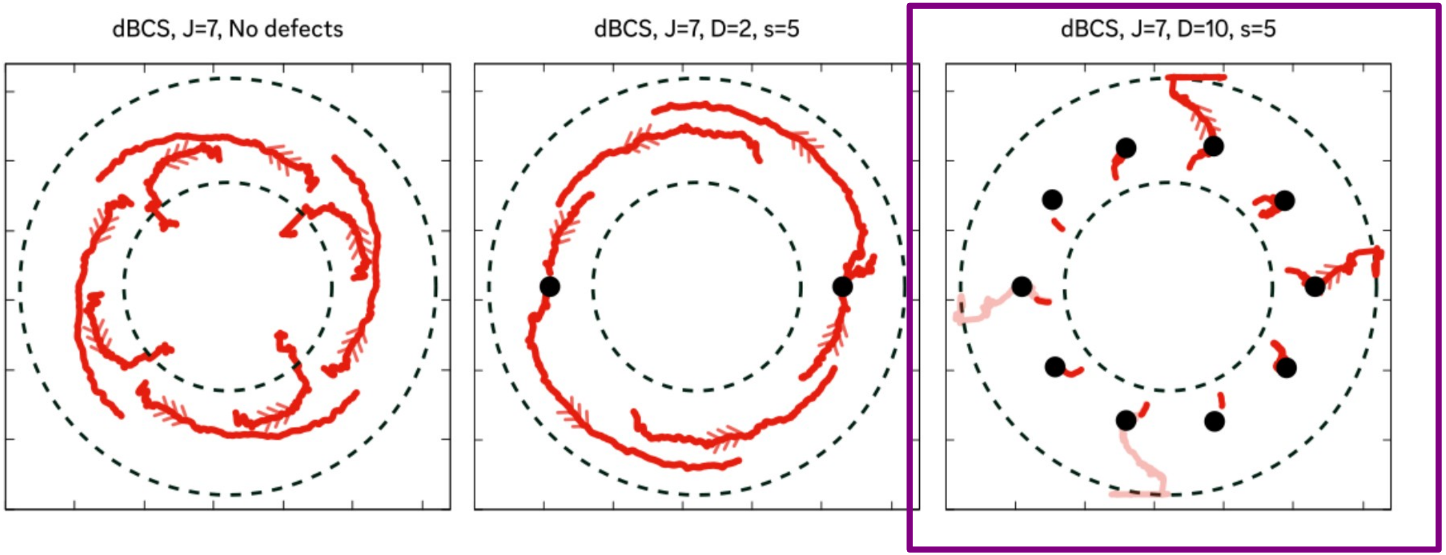


If defects added at static calculations, stabilization of Eflow with n defects

Switching on the defect causes significant dissipation

Critical winding number increases with n defects until w_c=w_{pb}

Vortex mobility in the presence of impurities



Impurities affect vortices mobility by pinning the vortices → it affects the local winding number and current

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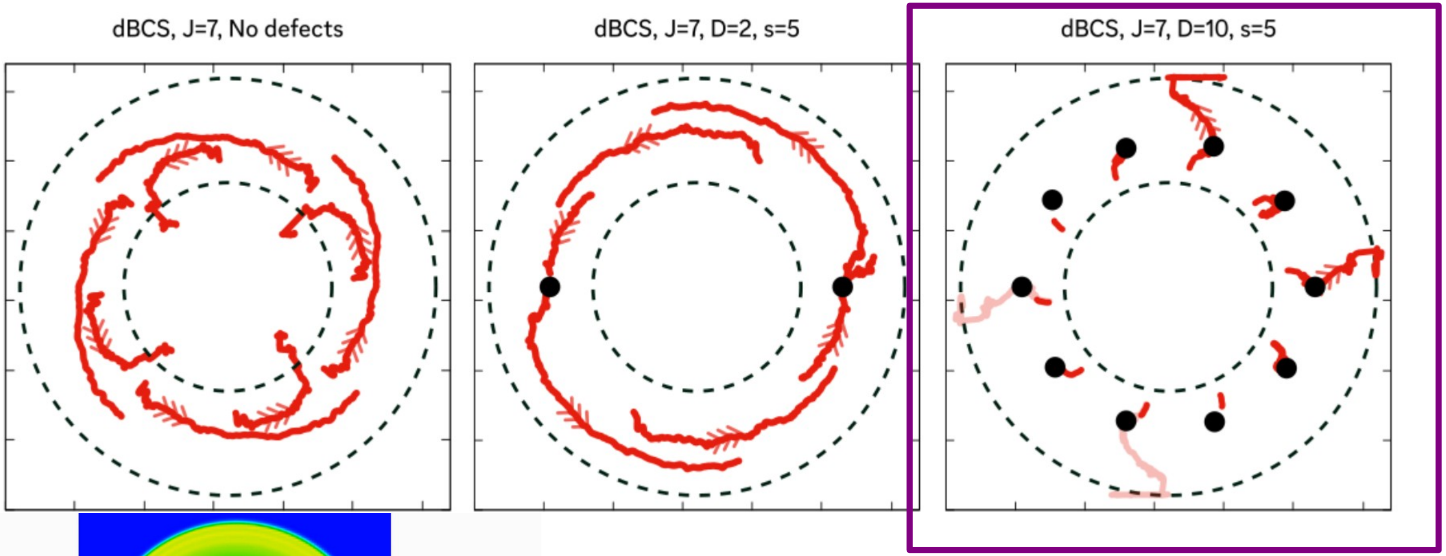
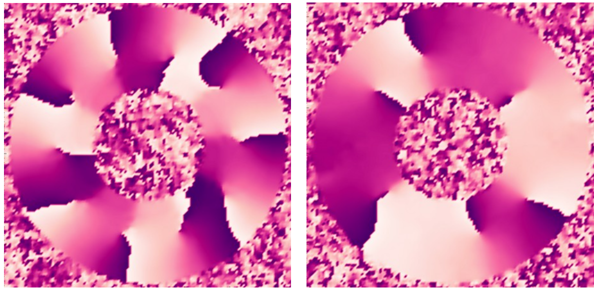
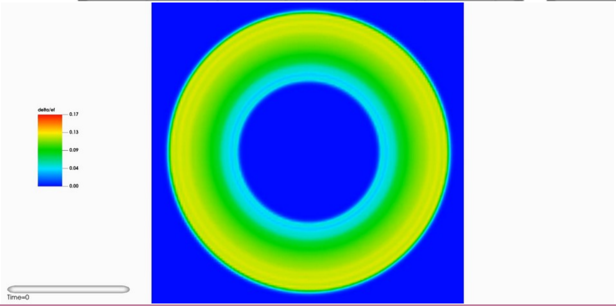
Vortex mobility in the presence of impurities



Pinned vortices **Unpinned vortices**



Investigate superfluid to normal angular momentum transfer



Impurities affect vortices mobility by pinning the vortices → it affects the local winding number and current

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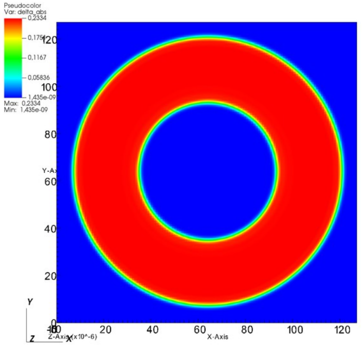
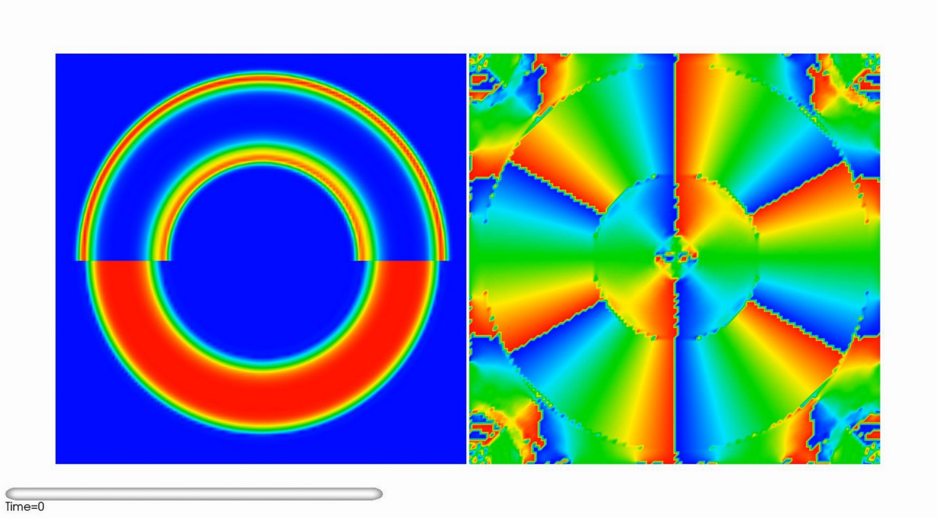
Spin imbalance: New probe of FFLO state?



No spin imbalance p
w0=6

P=20%



Bugra Tüzemen
talk on friday

Static: phase separation
Dynamics: FFLO state?
How does it affect
interference patterns

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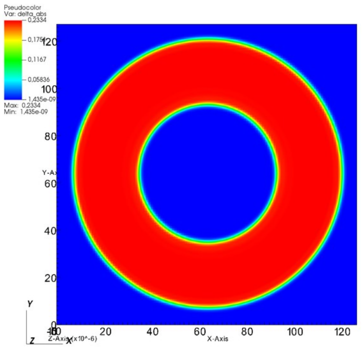
Spin imbalance: New probe of FFLO state?



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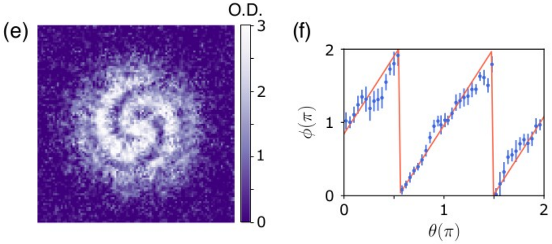

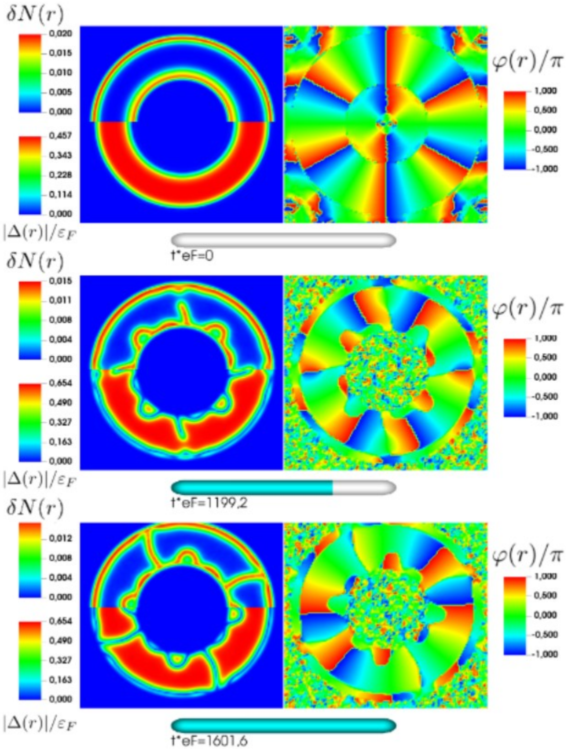

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Special Issue: Quantum Technologies with Ultracold Atoms

Guest Editors

Dr. Klejdja Xhani

Dr. Giulia Del Pace

Dr. Chiara Mazzinghi

Key research areas:

- Quantum simulation: Ultracold atoms act as sophisticated quantum simulators, enabling scientists to model complex systems from condensed matter physics to cosmological mysteries. With unprecedented control over trapping potentials and interaction strengths at ultra-low temperatures, researchers can address fundamental questions on superfluidity and phase transitions.
- Quantum information and computing: the remarkable ability to maintain coherence and to be manipulated into qubits—through various atomic states or species—make ultracold atoms suitable for implementing quantum gates and algorithms.
- Atomtronic circuits: this new emerging field utilizes ultracold atoms to create circuits analogous to electronic circuits, both for demonstrating fundamental physics phenomena and for implementing new quantum devices and sensors.
- Metrology and sensing: Thanks to their low thermal noise characteristics, ultracold atoms enable highly precise measurements in metrology. Applications range from atomic clocks—essential for GPS and telecommunications due to their unparalleled accuracy—to high-resolution spectroscopy that allows researchers to study atomic transitions with minimal disturbances by confining ultracold atoms in engineered traps.

Special Issue website:
https://www.mdpi.com/journal/atoms/special_issues/344W57T512

Thank you

Collaborators

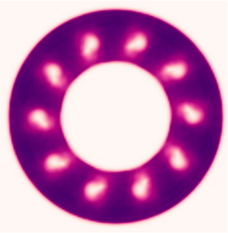
Persistent current BEC

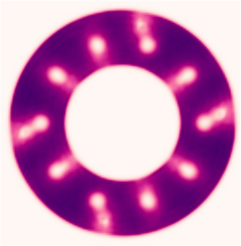
Theory:
Luca Pezzè,
Beatrice Donelli

Experiment:
Giacomo Roati,
Giulia Del Pace,
Cyprien Daix,
Diego Hernandez-Rajkov,
Nicola Grani,
Francesco Scazza

Persistent current UFG-BCS limit

Gabriel Wlazlowski
Piotr Magierski
Andrea Barresi,
Bugra Tüzemen
Marek Tylutki





Persistent current spin imbalance

Piotr Magierski
Gabriel Wlazlowski
Andrea Barresi,
Bugra Tüzemen