SOUND AND ANOMALOUS DOPPLER EFFECT IN SUPERSOLIDS

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research funded and supported also by:

PROVINCIA AUTONOMA DI TRENTO

Q@TN Quantum Science and Technology in Trento

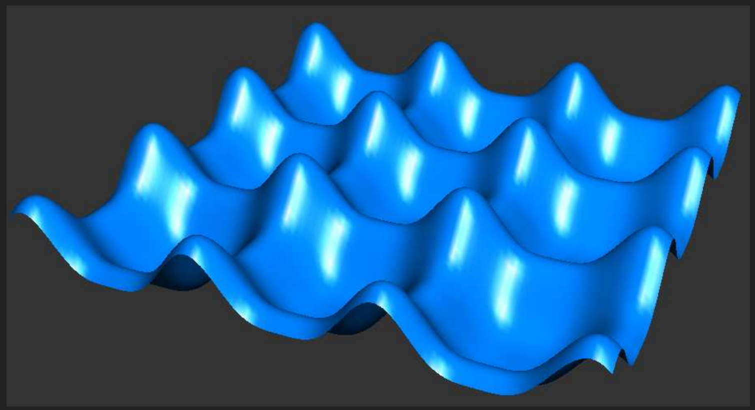
CINECA

Based on:
Marija Sindik, Tomasz Zawiślak, Sandro Stringari, AR
PRL 132, 146001 (2024) &
arXiv:2408.16489 (to appear in PRL)

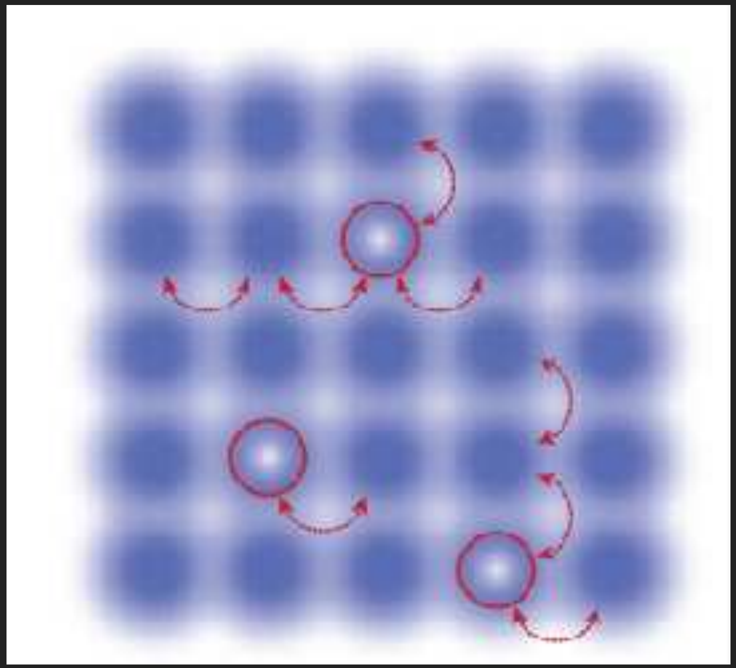
NONEQUILIBRIUM PHENOMENA IN SUPERFLUID SYSTEMS: ATOMIC NUCLEI, LIQUID HELIUM, ULTRACOLD GASES, AND NEUTRON STARS (12-16 May ECT*, Trento, I)

WHAT IS A SUPERSOLID? WHY IS RELEVANT?

A system which shows spatial order and superfluid properties *(spontaneously breaks continuous translational invariance and "particle conservation" - U(1))*



Seminal works [1] based on SUPERFLUID HELIUM



The hope/believe was that due to large enough zero-point motion the solid has vacancies, which move around and at low T Bose condense giving rise to finite superfluid density

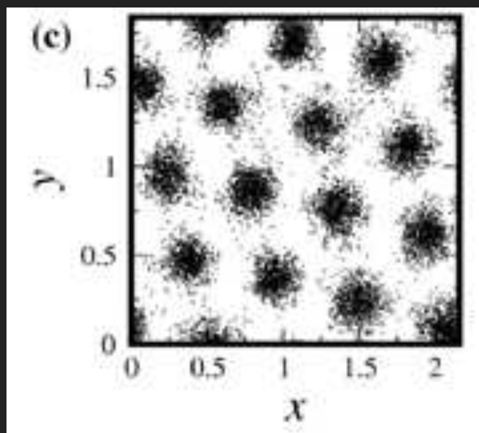
NO CLEAR EVIDENCE SO FAR…

it seems very hard if not impossible to have a supersolid with *"one atom per site"*
(QMC calculation for He-4 as well as dipolar gases)

[1] Gross (1957), Andreev & Lifshitz (1969), Chester (1970), Leggett (1970) -
excluded in Penrose & Onsanger (1956)

SUPERSOLID PHASE IN BOSE-EINTEIN CONDESATE GASES

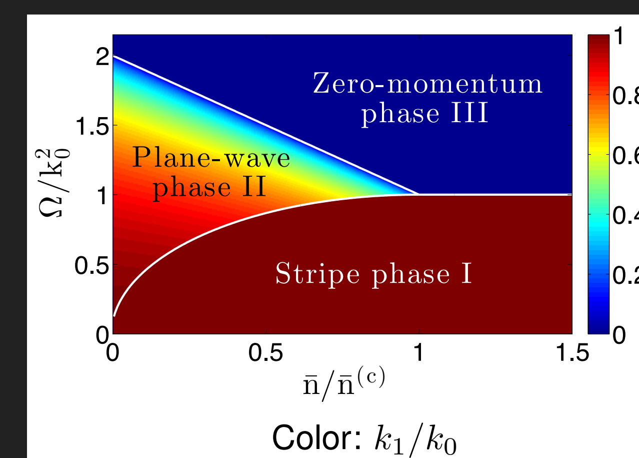
IDEA: change paradigm from *"one atom per site"* to a **modulated superfluid** or **coupled superfluid clusters** (easier to have coherence…) a.k.a. cluster supersolids (E. Gross, L. Pitaevskii…)



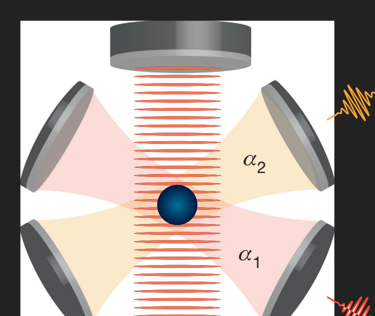
One of the first attempt in this direction was to use a gaseous BEC with soft sphere interaction. [Pomeau & Rica, PRL 1994]
Proposal with Rydberg atoms to engineer such a system in [Cinti et al. PRL (2010)]

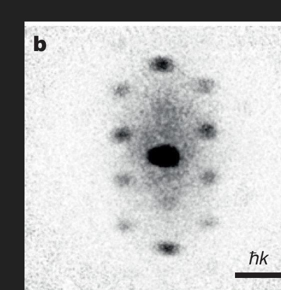
Since 2017 CLUSTER SUPERSOLIDITY have been observed in platforms using ultra-cold Bose gases

- spin-orbit coupling (stripe phase)
(exp. by [Ketterle's group, Nature ('17)] and more recently better evidence by [Tarruell's group, arXiv:2412.13861])



- in cavity QED
(exp. [Esslinger's group, Nature 2017…])





- **dipolar interaction (the platform used in my talk) - see also Poli's talk**
(Many exps. since 2019 in Florence, Innsbruck and Stuttgart…)

T=0 SUPERSOLID HYDRODYNAMIC

Supersolid: "a state of matter, which spontaneously breaks translational invariance and U(1) symmetry" , i.e., the state is characterised by crystalline and superfluid long range orders

T=0 SUPERSOLID HYDRODYNAMIC

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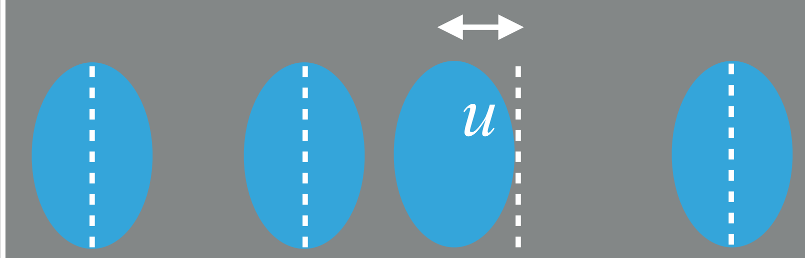
T=0 one dimensional hydrodynamic equation (2 conservation + 2 order parameters)

$$\partial_t \rho + \partial_x j = 0$$
$$\partial_t j + \partial_x \left(p + \rho_n v_n^2 + \rho_s v_s^2\right) = 0$$
$$\partial_t v_s + \partial_x (v_n v_s + \mu) = 0$$
$$\partial_t u_x + \partial_x (v_n u_x - v_n) = 0$$

$$\rho = \rho_s + \rho_n$$
$$j = \rho_n v_n + \rho_s v_s$$

crystal displacement



$$u_x = \partial_x u$$

Constituent (infinitesimal) relations

$$\delta\mu = \frac{1}{\rho\kappa}\delta\rho + \gamma u_x + \frac{\rho_s'}{2}\delta w^2 - \frac{1}{2}\delta v_n^2,$$
$$\delta p = \left(\frac{1}{\kappa} - \gamma\right)\delta\rho + (\gamma\rho - \lambda)\, u_x + \frac{\rho_n - \rho\rho_n'}{2}\delta w^2,$$
$$\delta\rho_n = \rho_n'\delta\rho + \rho_n^u u_x,$$

$$\rho_s' = \frac{\partial\rho_s}{\partial\rho} \qquad \rho_n^u = \frac{\partial\rho_n}{\partial u_x}$$

$$w = v_s - v_n$$

λ elastic constant

γ strain density coupling

T=0 SUPERSOLID HYDRODYNAMIC

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$$\partial_t j + \partial_x \left(p + \rho_n v_n^2 + \rho_s v_s^2\right) = 0$$
$$\partial_t v_s + \partial_x (v_n v_s + \mu) = 0$$
$$\partial_t u_x + \partial_x (v_n u_x - v_n) = 0$$

$$j = \rho_n v_n + \rho_s v_s$$

The "clusters" give rise to a normal component (Landau 2-fluid model)

$$\longrightarrow \quad \partial_t u = v_n$$

In absence of current the linearised hydrodynamic leads to the 2 sounds

[Yoo & Dorsey, PRB 81, 134518 (2010)]

$$c_\pm^2 = \frac{c_\kappa^2}{2}\left[1 + \beta\kappa - 2\gamma \pm \sqrt{(1+\beta\kappa)^2 - 4 f_s(\beta\kappa - \gamma^2)}\right]$$

$$c_\kappa = \sqrt{\kappa^{-1}}, \; \beta = \lambda/\rho_n, \; f_s = \rho_s/\rho$$

THE 2 SOUNDS ARE RELATED TO THE GOLDSTONE MODES DUE TO THE SPONTANEOUS BREAKING OF U(1) AND 1D GALILEAN CONTINUOUS SYMMETRY

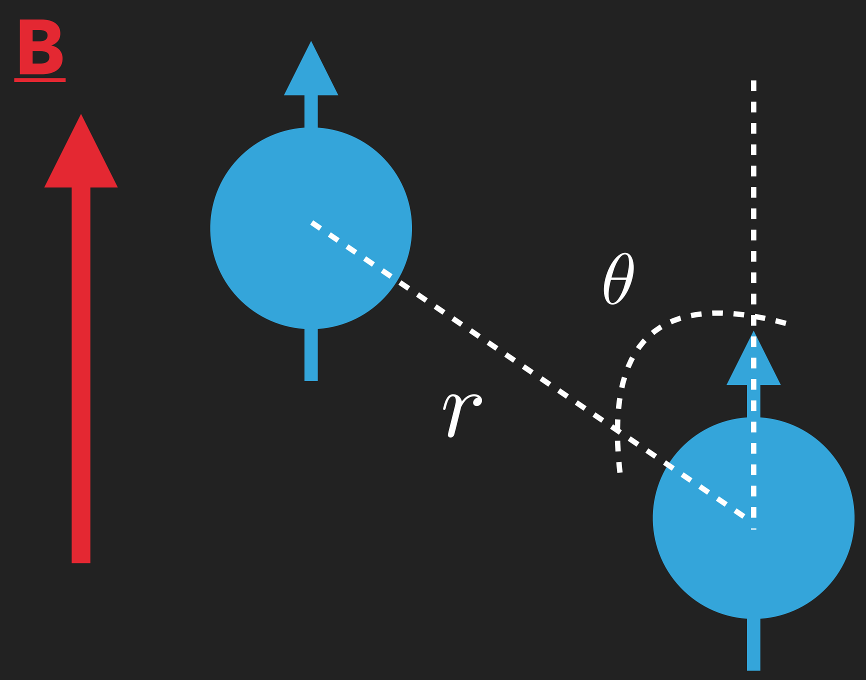
Close to the (incoherent) solid regime

$$\longrightarrow \quad f_S \to 0$$

$$c_+ \to \sqrt{1 + \beta\kappa - \frac{f_S \beta\kappa}{1+\beta\kappa}} c_\kappa, \quad c_- \to \sqrt{f_S \frac{\beta\kappa}{1+\beta\kappa}} c_\kappa$$

(MAGNETIC) DIPOLAR BOSE GASES

We consider magnetic dipolar gases, with dipole oriented by an external magnetic field.



The atoms interact via short range potential
which for the temperature and energy of the experiments can be replaced by the usual s-wave contact interaction

$$V_c(\mathbf{r}) = \frac{4\pi\hbar^2 a}{m}\delta^{(3)}(\mathbf{r}) = g\delta^{(3)}(\mathbf{r})$$

as well as via dipole-dipole interaction

$$V_{dd}(\mathbf{r}) = \frac{\mu_0\mu^2}{4\pi}\frac{1-3\cos^2\theta}{r^3} = \frac{3\hbar^2 a_{dd}}{m}\frac{1-3\cos^2\theta}{r^3}$$

$$\varepsilon_{dd} = \frac{a_{dd}}{a}$$

the relative strength which can be varied by changing the s-wave scattering through Feshbach resonances

At zero temperature the gas forms a Bose-Einstein condensate, whose mean-field equation of motion for the order parameter (Gross-Pitaevskii equation) reads:

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ho}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2 + \int d\mathbf{r}' V_{dd}(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}',t)|^2\right]\Psi(\mathbf{r},t)$$

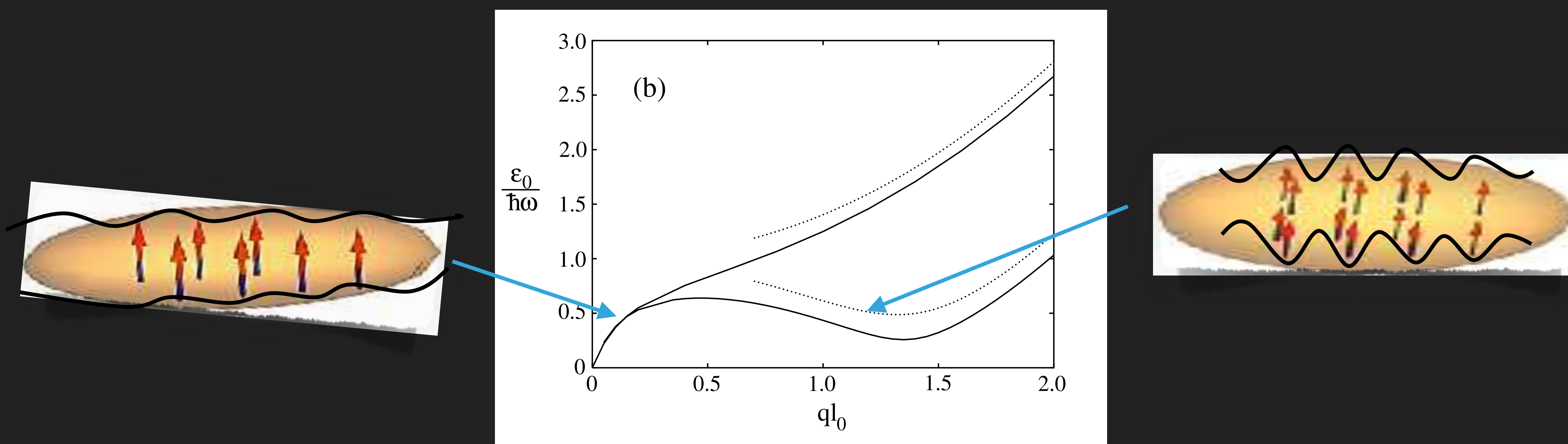
(MAGNETIC) DIPOLAR BOSE GASES

For an homogenous system solutions exists if $\varepsilon_{dd} = a_{dd}/a$ is <1.

There exist directions for which the speed of sound is imaginary

$$c(\theta)^2 = c_0^2 \left(1 + \varepsilon_{dd}(3\cos^2(\theta) - 1)\right)$$

By (harmonically) confining the gas in the z-direction the spectrum shows a roton-like minimum [Santos, Shlyapnikov, Lewenstein, PRL 2003]

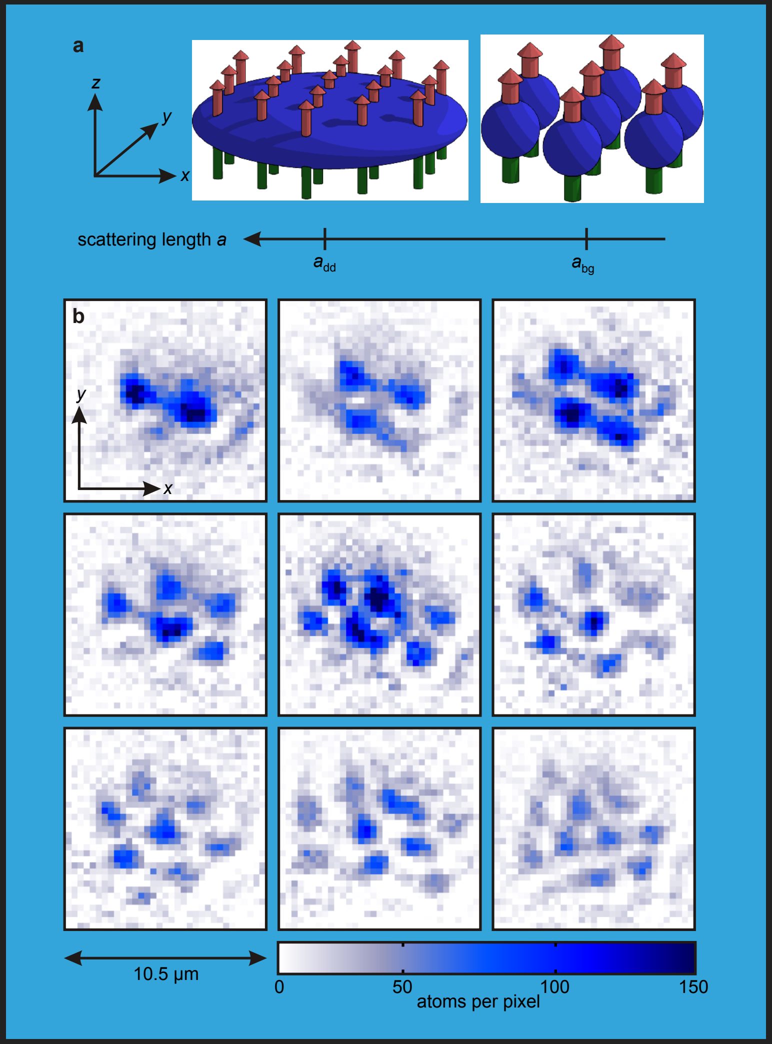


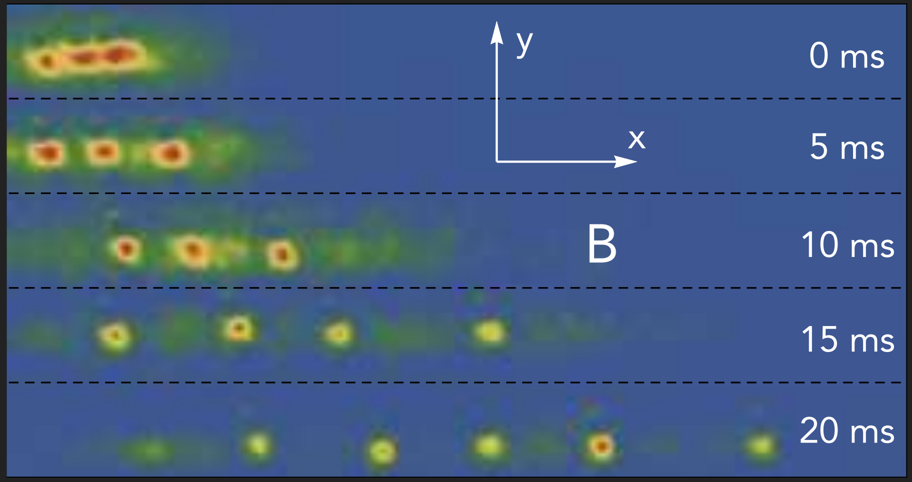
For large dipolar interaction the roton eventually should lead to collapse of the cloud… BUT it does not thanks to stabilisation beyond mean field: Lee-Huang-Yang correction (i.e., energy correction due to zero-point fluctuations)

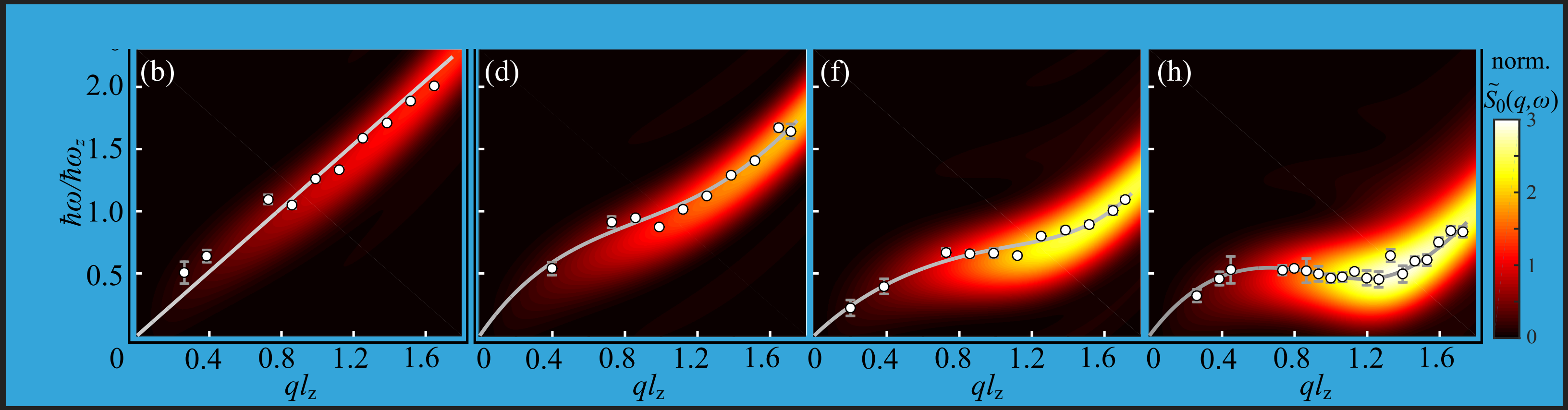
(MAGNETIC) DIPOLAR BOSE GASES: DROPLET

For large dipolar interaction the roton eventually should lead to collapse of the cloud… BUT it does not thanks to stabilisation beyond mean field: Lee-Huang-Yang correction (i.e., energy correction due to zero-point fluctuations): $n^2 \; vs. \; n^{5/2}$

Experimentalists have observed the droplets [Pfau's group, 2015-2018] and measured the roton spectrum [Ferlaino's group, PRL 2019]







DIPOLAR BOSE GASES: COHERENT DROPLETS

The phenomenology is very well capture by a time dependent density functional theory build on the GPE including the LHY term, a.k.a. extended GPE

(perfect analogy with the Bose-Bose mixtures as well as strong analogy with the liquid of light (non-linear optics), He-4 droplets, liquid droplet model for nuclei)

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ho}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2 + \int d\mathbf{r}' V_{dd}(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}',t)|^2 + \gamma(\varepsilon_{dd})|\Psi(\mathbf{r},t)|^3\right]\Psi(\mathbf{r},t)$$

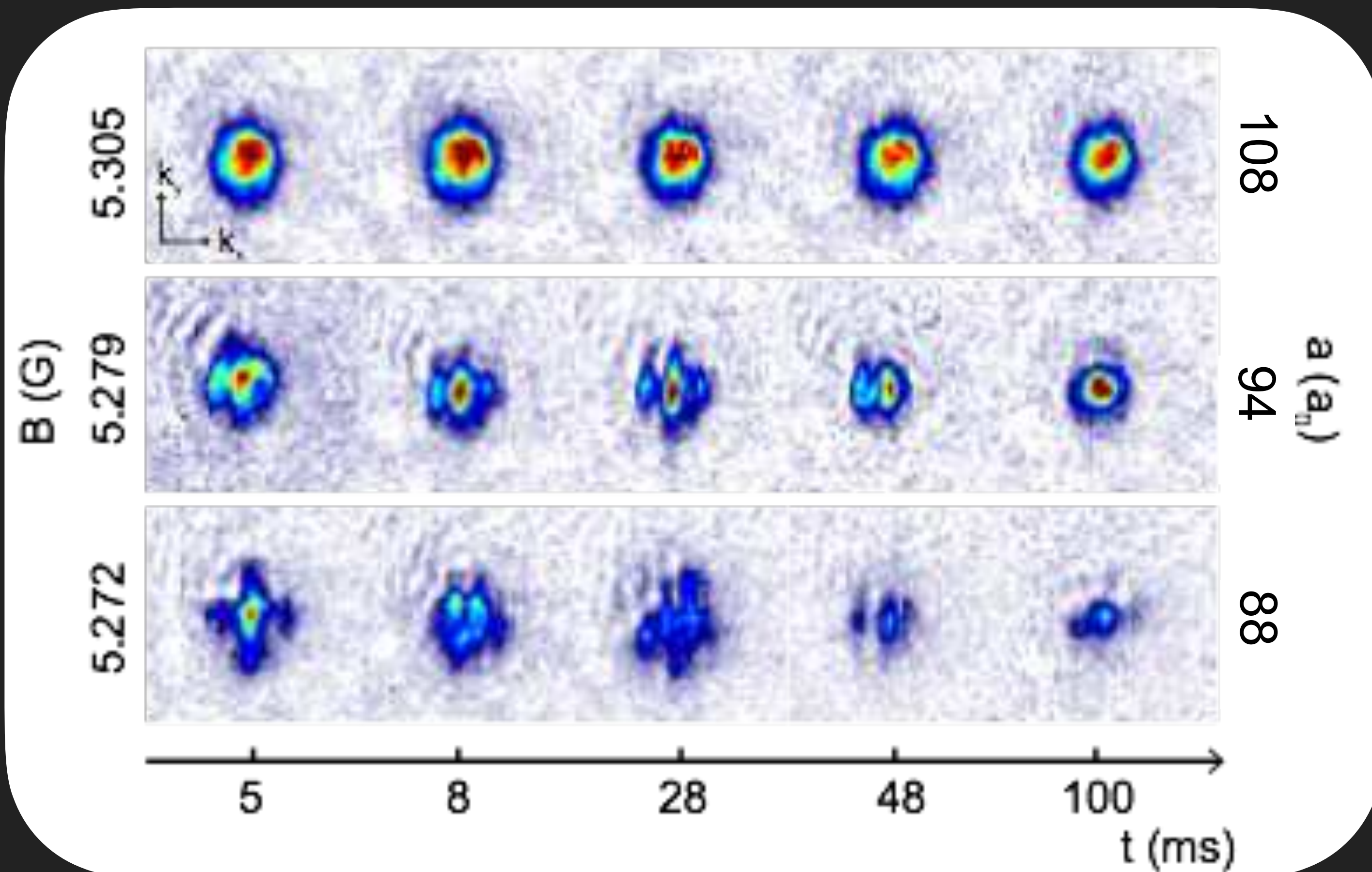
LHY term: $\gamma(\varepsilon_{dd}) = \frac{16}{3\sqrt{\pi}} g a^{\frac{3}{2}} \int_0^{\pi} d\theta \sin\theta [1+\varepsilon_{dd}(3\cos^2\theta - 1)]^{\frac{5}{2}}$

IS IT POSSIBLE TO HAVE COHERENCE AMONG THE DROPLETS? YES (ALTHOUGH IN A SMALL PARAMETER RANGE)

[Theory results by Roccuzzo and Ancilotto, PRA 2019]

The existence of a "supersolid" phase in dipolar BEC has been experimentally verified in a number of experiments where the coherence, the SF and crystal modes, Josephson-like effect and the existence of **vortices** have been observed.

DIPOLAR BOSE GASES: COHERENT DROPLETS

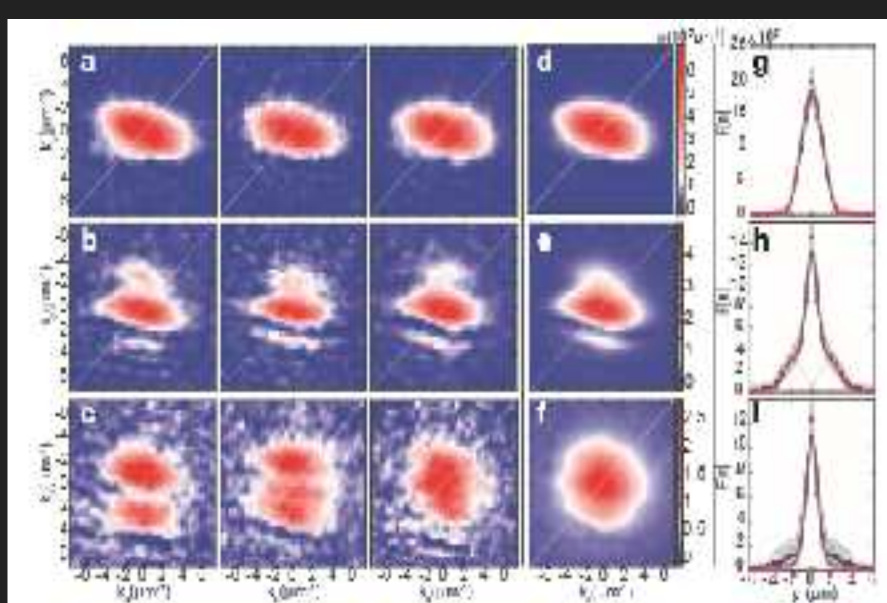


"BEC"

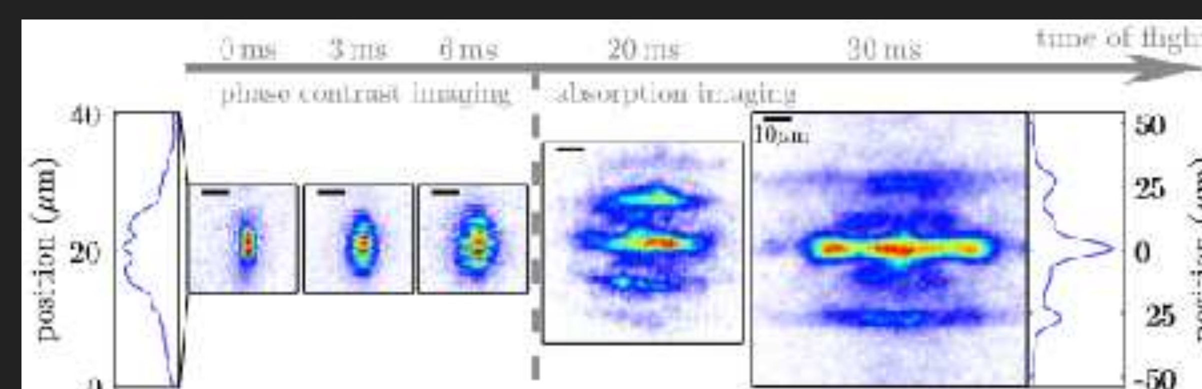
"Coherent Droplets"

"Incoherent Droplets"

Tanzi et al., ***Observation of a dipolar quantum gas with metastable supersolid properties,*** PRL 2019



L. Chomaz et al., ***Long-lived and transient supersolid behaviors in dipolar quantum*** *gases,* PRX 2019



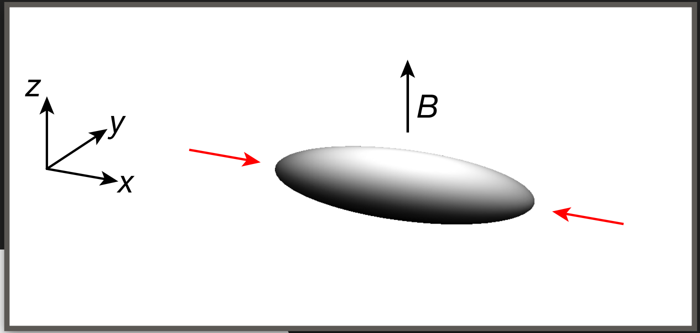
F. Böttcher et al, ***Transient supersolid properties in an array of dipolar quantum droplets***, PRX 2019

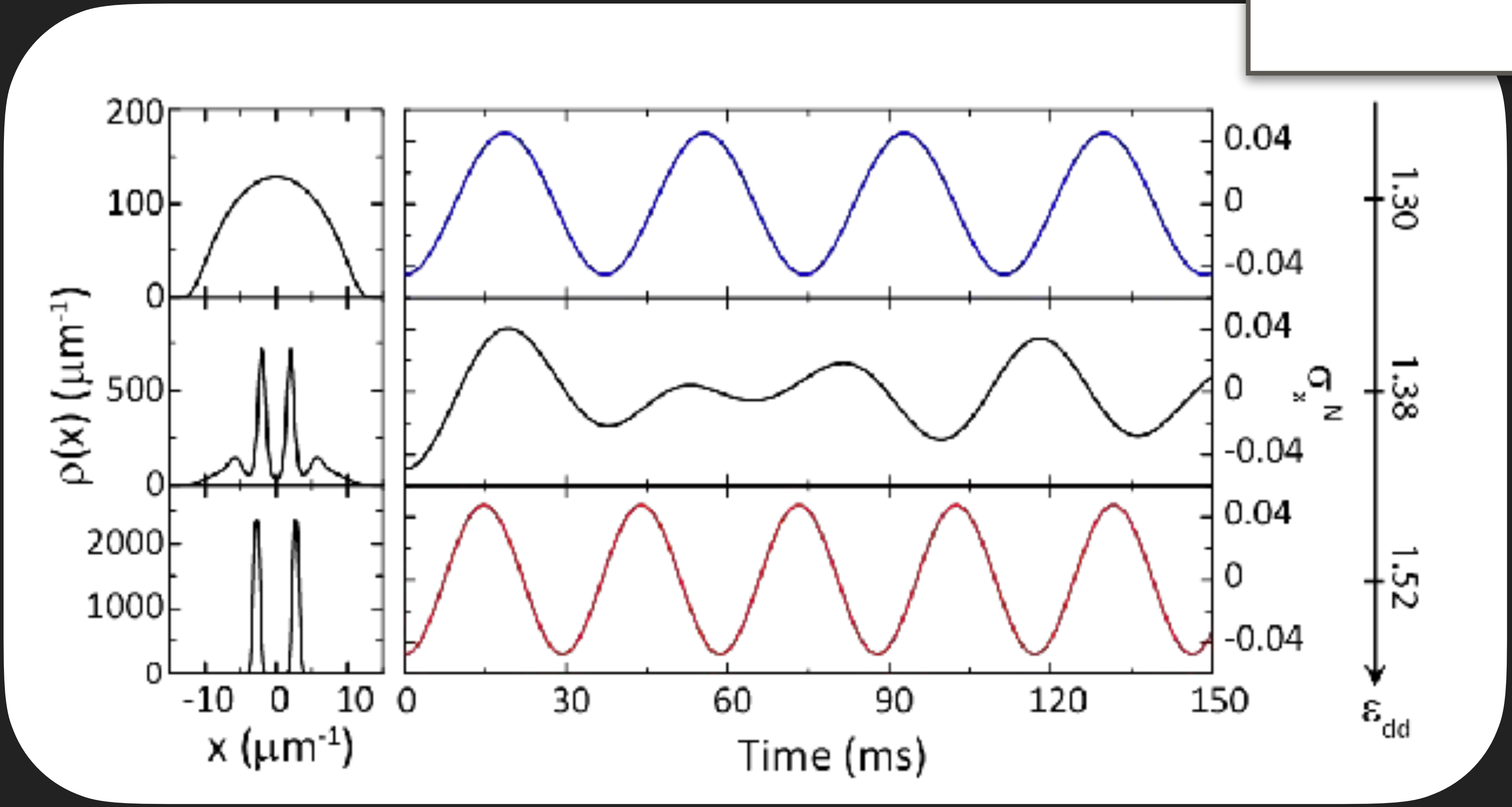
DIPOLAR BOSE GASES: COLLECTIVE MODES

What does it happen in a trapped gas?

- Collective (discrete) modes replace the "usual" dispersion relation
- We need to probe bulk properties -> breathing (compressional) mode

1. determine the ground state of the system
2. apply a small external perturbation along the longitudinal axes
3. evolve in time





WE OBSERVE A WELL-DEFINED SINGLE FREQUENCY IN THE SUPERFLUID AND INCOHERENT DROPLETS PHASE AND A CLEAR BEATING IN THE SUPERSOLID

DIPOLAR BOSE GASES: COLLECTIVE MODES

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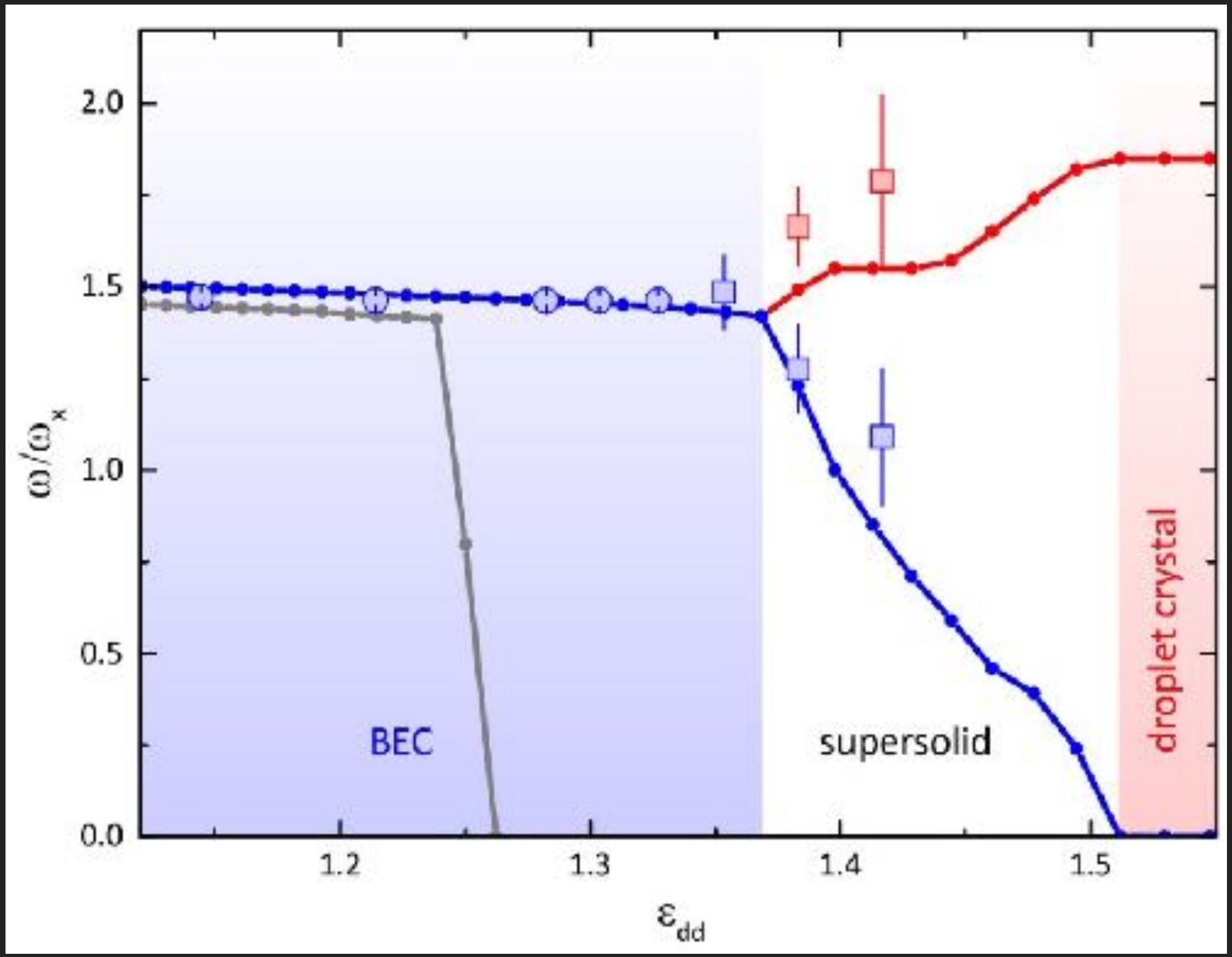

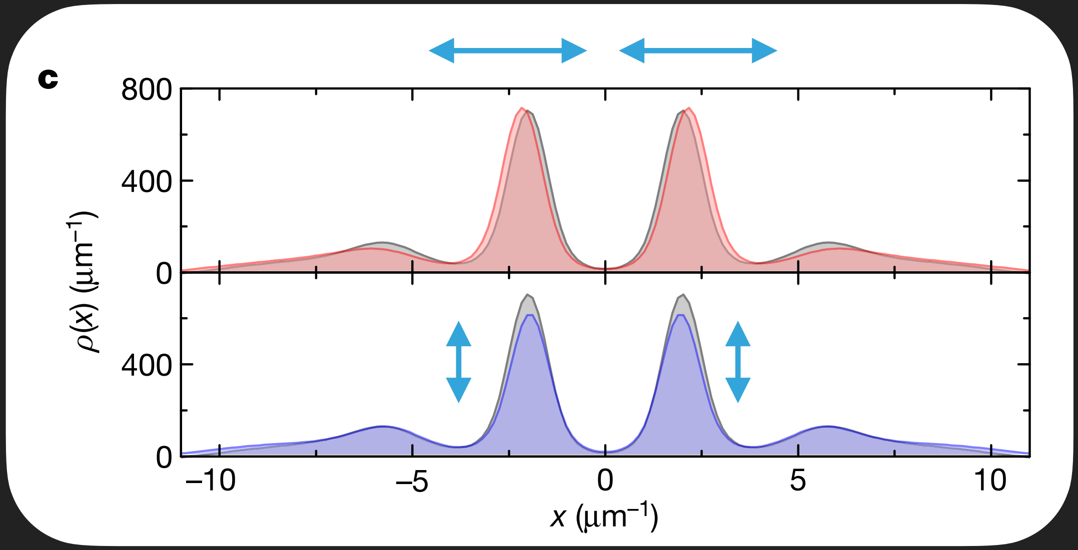

Fourier decomposition clearly show that the higher mode is a (mainly) **crystal like** (lattice deformation) and the lower one is (mainly) an **amplitude modulation**

[L. Tanzi, S. Roccuzzo et al., Nature 574, 382 (2019)]

DIPOLAR BOSE GASES: COLLECTIVE MODES

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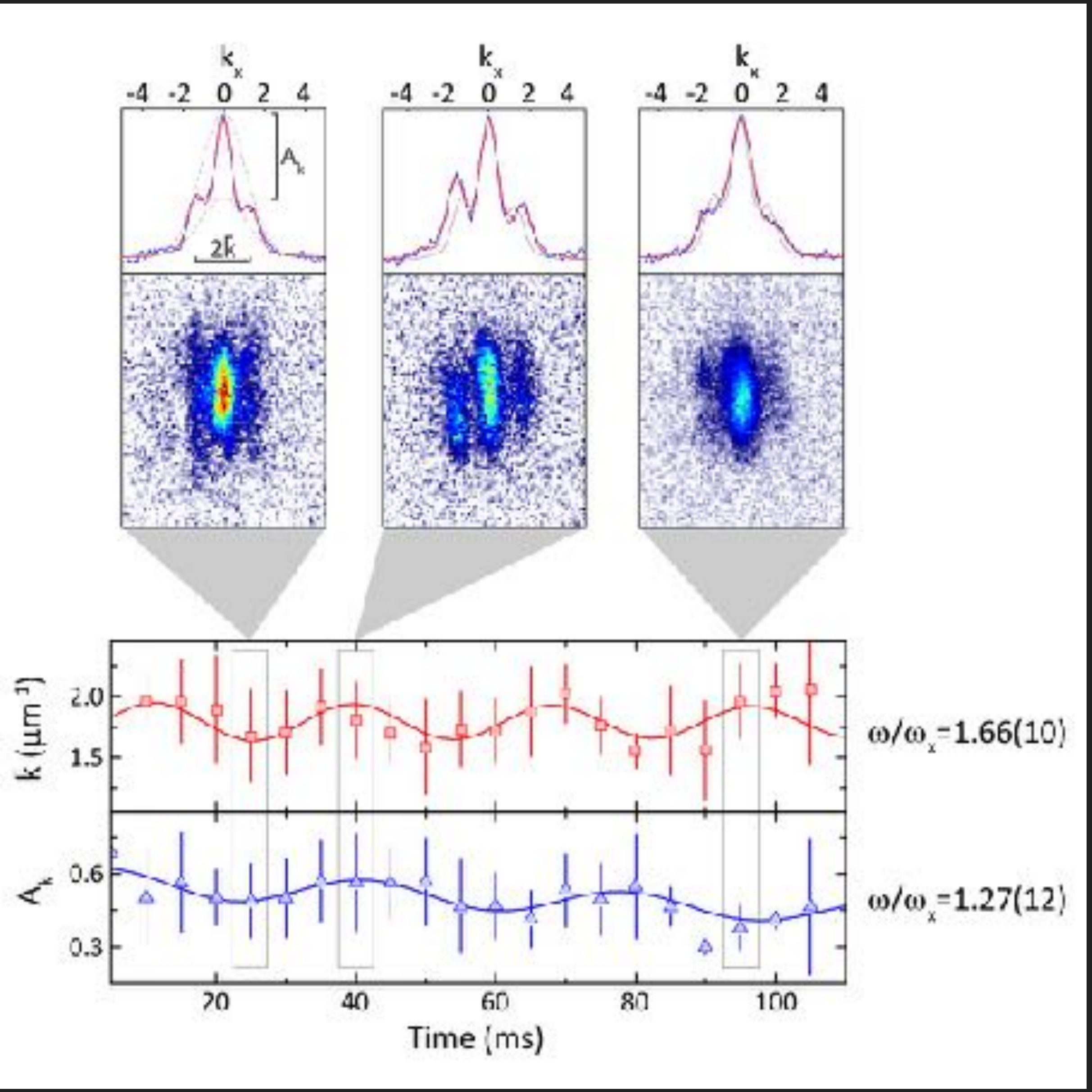
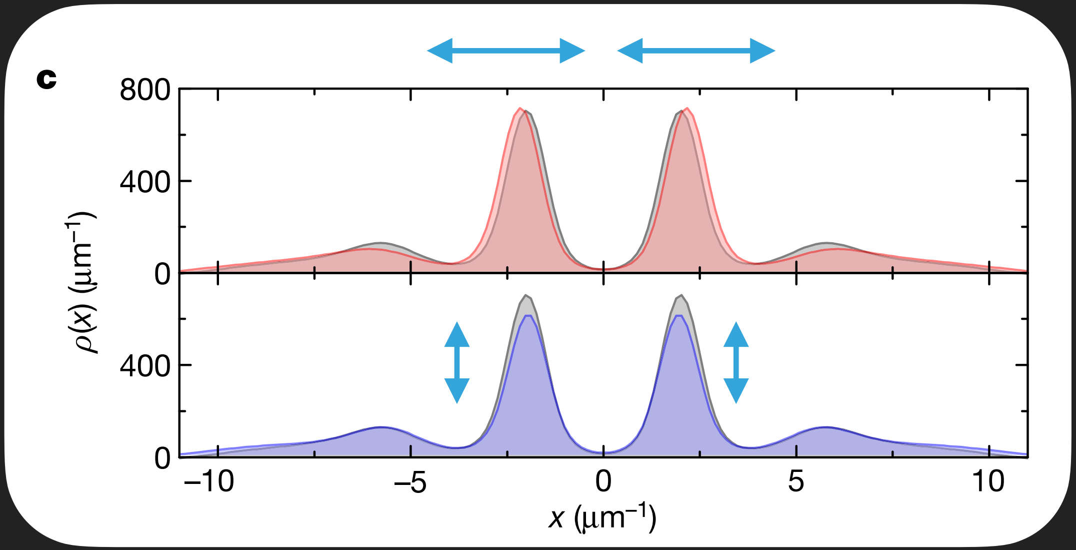




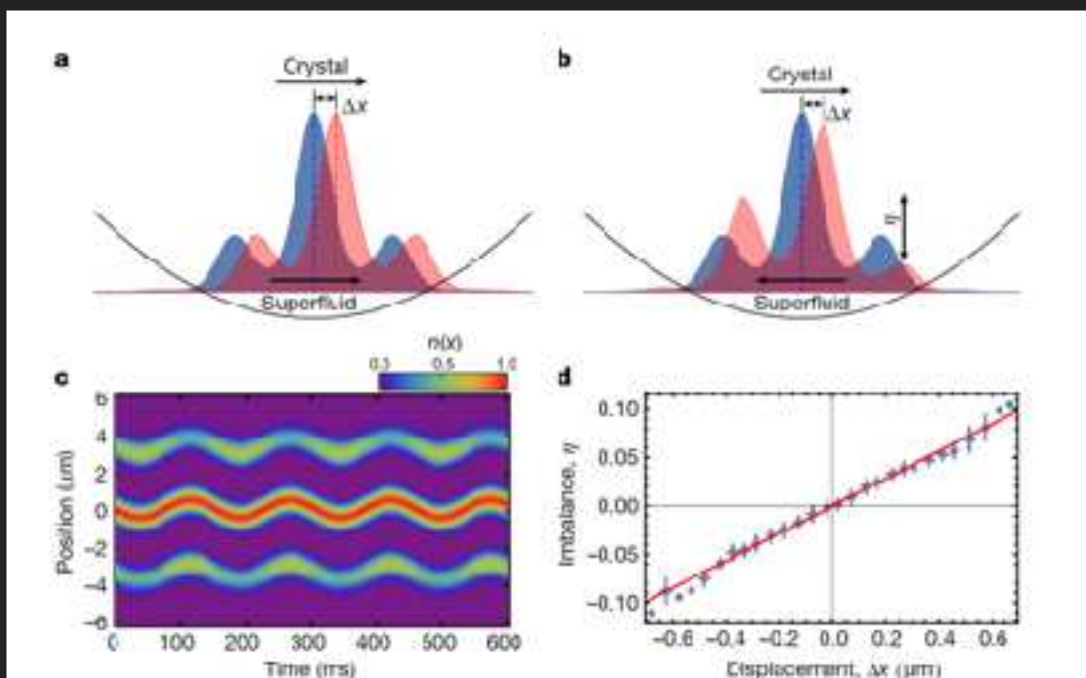
Fourier decomposition clearly show that the higher mode is a (mainly) **crystal like** (lattice deformation) and the lower one is (mainly) an **amplitude modulation**

Experimentally indeed in the SS phase the amplitude and the lattice oscillates with 2 different we found two frequencies

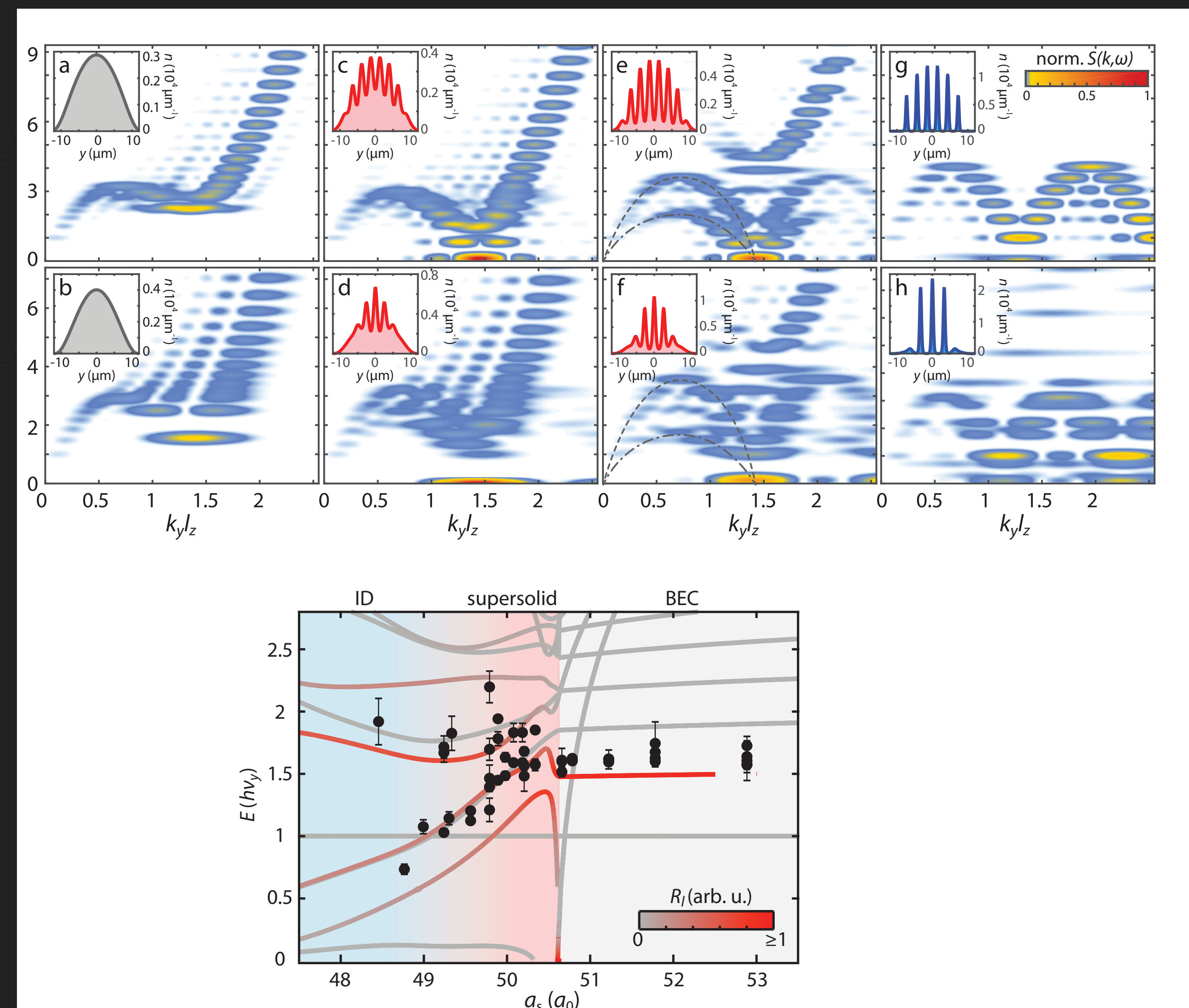
[L. Tanzi, S. Roccuzzo et al., Supersolid symmetry breaking from compressional oscillations in a dipolar quantum gas, Nature 574, 382 (2019)]

DIPOLAR BOSE GASES: COLLECTIVE MODES

Short after very analogous conclusions have been drawn by the Pfau's and Ferlaino's group in very nice and complementary experiments!



[M. Guo et al. The low energy Goldstone mode in a trapped dipolar supersolid, Nature 574, 386 (2019)]

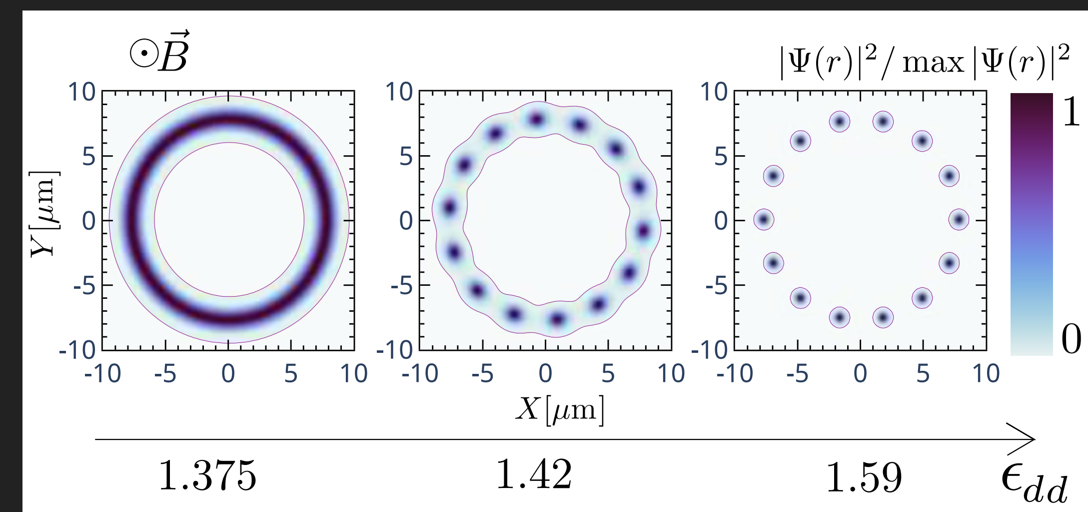


[G. Natale et al. The excitation spectrum of a trapped dipolar supersolid and its experimental evidence, Phys. Rev. Lett. 123, 050402 (2019)]

A PROTOCOL TO MEASURE THE SOUND

On the theory side the speed of sound can be extracted in an infinite system and calculating the (Bogloliubov) spectrum of the gas.

We however used a protocol based on solving the (e)GPE which should be applicable in a real experiment:



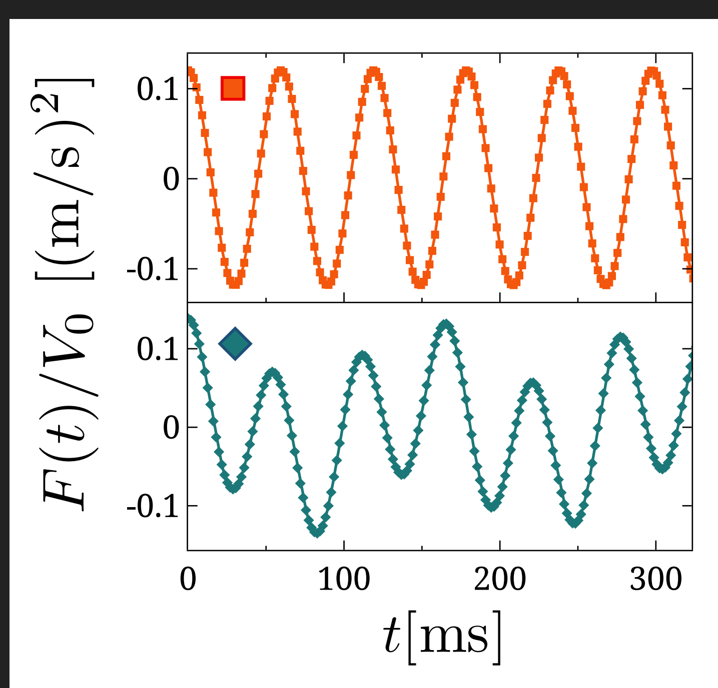
> Prepare the initial state of the system in a (large) ring geometry in presence of a small additional potential $\cos(\phi)$

> Remove the potential

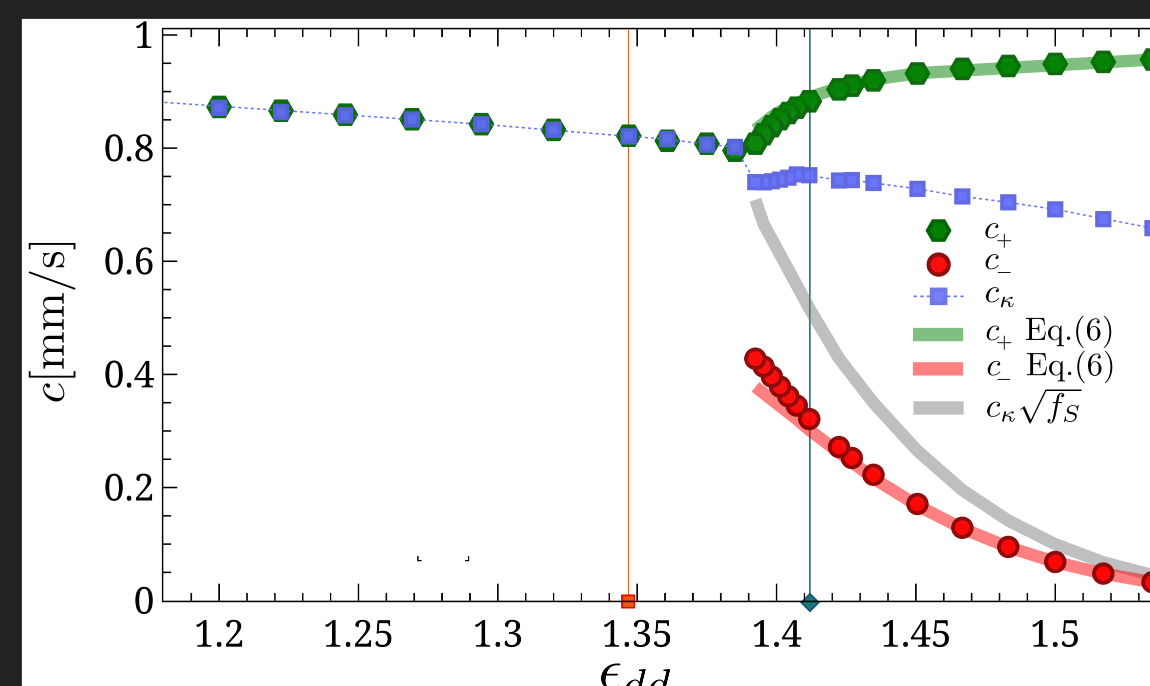
> Record the average value

$$F(t) = \langle\cos(\phi)\rangle(t)$$

Speed without current



$\omega_i \propto c_i$



SOUND SPEED FOR A DIPOLAR SUPERSOLID

HD parameter's estimation

$$c_\pm^2 = \frac{c_\kappa^2}{2}\left[1 + \beta\kappa - 2\gamma \pm \sqrt{(1+\beta\kappa)^2 - 4f_s(\beta\kappa - \gamma^2)}\right]$$

a) $c_\pm^2 \rightarrow_{f_s \simeq 1} 1/\kappa,\ \beta$

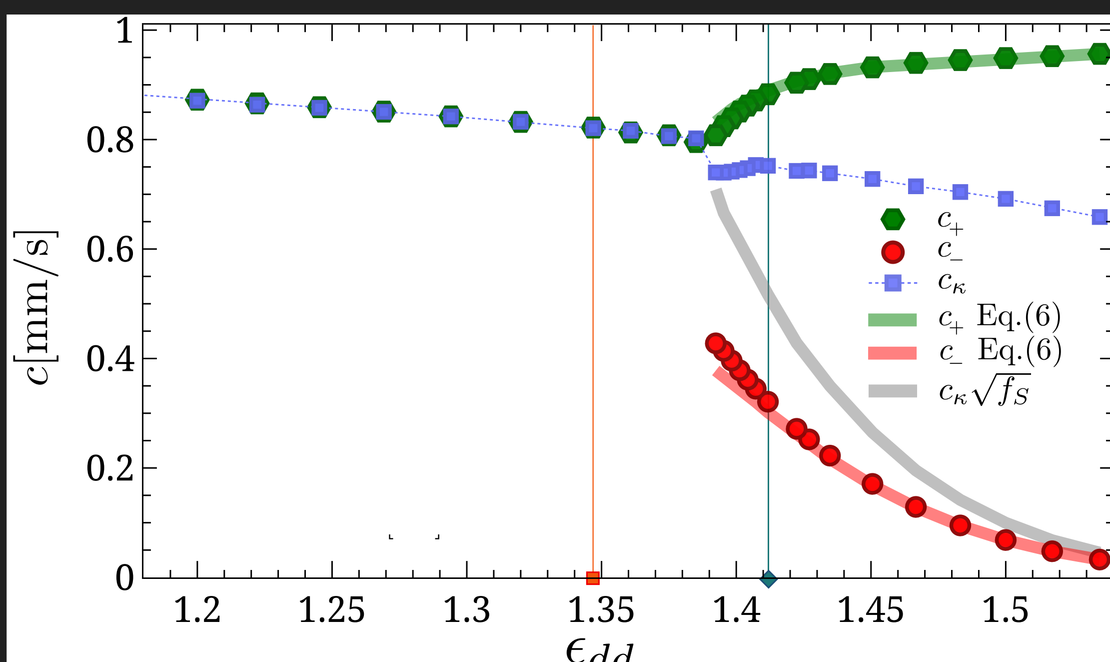
b) incompressible solid

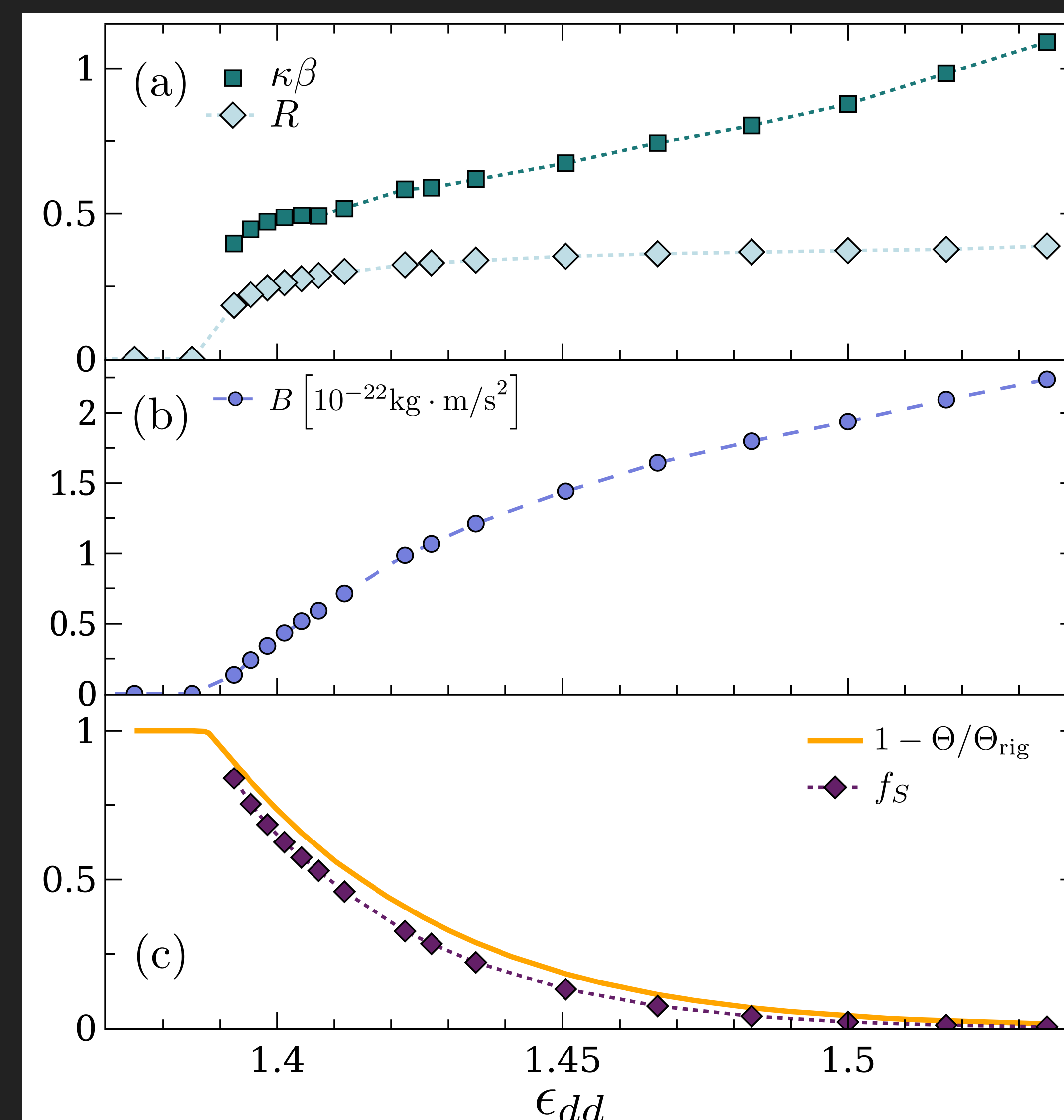
$$c_-^2 \rightarrow_{\beta\rightarrow\infty} \sqrt{f_s} c_\kappa$$

$\gamma \simeq 0$ Dipolar supersolid

$$\beta\kappa = (c_+^2 + c_-^2)/c_\kappa^2 - 1$$

$$f_s\beta\kappa = c_+^2 c_-^2 / c_\kappa^4$$





Speeds of sound for a dipolar gas

80,000 ^{164}Dy in a tube

with trapping frequency 100 Hz

T=0 SUPERSOLID HYDRODYNAMIC

$$\partial_t \rho + \partial_x j = 0$$

$$\partial_t j + \partial_x \left(p + \rho_n v_n^2 + \rho_s v_s^2\right) = 0$$

$$\partial_t v_s + \partial_x (v_n v_s + \mu) = 0$$

$$\partial_t u_x + \partial_x (v_n u_x - v_n) = 0$$

$$j = \rho_n v_n + \rho_s v_s$$

The "clusters" give rise to a normal component (Landau 2-fluid model)

$$\longrightarrow \quad \partial_t u = v_n$$

In <u>absence of current</u> the linearised hydrodynamic leads to the 2 sounds

$$c_\pm^2 = \frac{c_\kappa^2}{2}\left[1 + \beta\kappa - 2\gamma \pm \sqrt{(1+\beta\kappa)^2 - 4 f_s(\beta\kappa - \gamma^2)}\right]$$

$$c_\kappa = \sqrt{\kappa^{-1}},\ \beta = \lambda/\rho_n,\ f_s = \rho_s/\rho$$

IN <u>PRESENCE OF CURRENT</u> SPEEDS OF SOUND ARE MODIFIED BY A DOPPLER CORRECTION, BUT HOW?

(STANDARD) DOPPLER EFFECT IN FLUIDDYNAMICS

"In presence of a current the speed of sound of a fluid is modified w.r.t. to its value at rest"

$$j = \rho v_f \quad \Longrightarrow \quad c = c_0 \pm v_f$$

Hydrodynamic equation for a Galilean system (one dimensional):

Number conservation (continuity equation)

$$\partial_t \rho + \partial_x j = 0$$

Momentum conservation

$$\partial_t j + \partial_x p = 0$$

Constituent relations

$$j = \rho v$$

$$p = p_0 + \rho v^2$$

Sound: linearise the hydrodynamic equations around a state with velocity v_f

$$c = \sqrt{\kappa^{-1}} \pm v_f$$

(ANOMALOUS) DOPPLER EFFECT IN SUPERFLUID DYNAMICS

In presence of a **superfluid** and a **normal** component which can have a **relative motion** , Khalatnikov realised that the situation can be less trivial.

The Propagation of Sound in Moving Helium II and the Effect of a Thermal Current upon the Propagation of Second Sound

I. M. KHALATNIKOV

Institute for Physical Problems, Academy of Sciences, USSR

(Submitted to JETP editor December 15, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 617-619 (March, 1956)

Two fluid model

$$j = \rho_n v_n + \rho_s v_s \qquad \rho = \rho_n + \rho_s$$

1. Not only first (standard) sound: different sound has a different Doppler shift
2. Non trivial Doppler due to the dependence of the superfluid fraction on the thermodynamic quantities

A series of works by **Y. Nepomnyashchy**, explored the effect in details,
finding interesting behaviours of the Doppler for the second and fourth sound.

ANOMALOUS DOPPLER EFFECT - NO EXPERIMENTAL EVIDENCE YET

finite temperature weakly interacting Bose gas
[Physics LettersA 161 (1991)]

Curious Doppler shift of fourth sound in the low temperature limit

Yu.A. Nepomnyashchy and M. Revzen
Physics Department, Technion – Israel Institute of Technology, Haifa 32000, Israel

Unusual Doppler effect in He II

Y. A. Nepomnyashchy
Physics Department, Technion-Israel Institute of Technology, Haifa 32 000, Israel

Unusual Doppler Effect in Superfluid and Nonanalyticity of ^{4}He-^{3}He Hydrodynamics

Y. A. Nepomnyashchy, N. Gov, A. Mann and M. Revzen

(ANOMALOUS) DOPPLER EFFECT IN SUPERFLUID DYNAMICS

Q: Does a system with a finite normal component **at zero temperature** show an anomalous Doppler effect?

DOPPLER EFFECT FOR A BEC IN AN OPTICAL LATTICE

In an optical lattice the superfluid of an atomic BEC is not 1 even at T=0, the normal fraction is locked to the lattice.

T=0 Hydrodynamic equation (one dimensional):

Number conservation (continuity equation)

$$\partial_t \rho + \partial_x j = 0$$

Order parameter (Josephson equation)

$$\partial_t v_s + \partial_x \mu = 0$$

Constituent relations

$$j = \rho_s v_s$$

$$\mu = \partial_n \varepsilon = \mu_0 + \frac{1}{2}\frac{\partial \rho_s}{\partial \rho} v_s^2$$

(Fourth) Sound: linearise the hydrodynamic equations around a state with velocity v_s^0

$$c = \sqrt{f_s \kappa^{-1}} + \frac{\partial \rho_s}{\partial \rho} v_s^0 = \sqrt{f_s \kappa^{-1}} + \left(f_s + \rho \frac{\partial f_s}{\partial \rho} \right) v_s^0$$

Recently measured
[Phys. Rev. Lett. 130, 226003 (2023)]

Kinematic Doppler

Anomalous Doppler

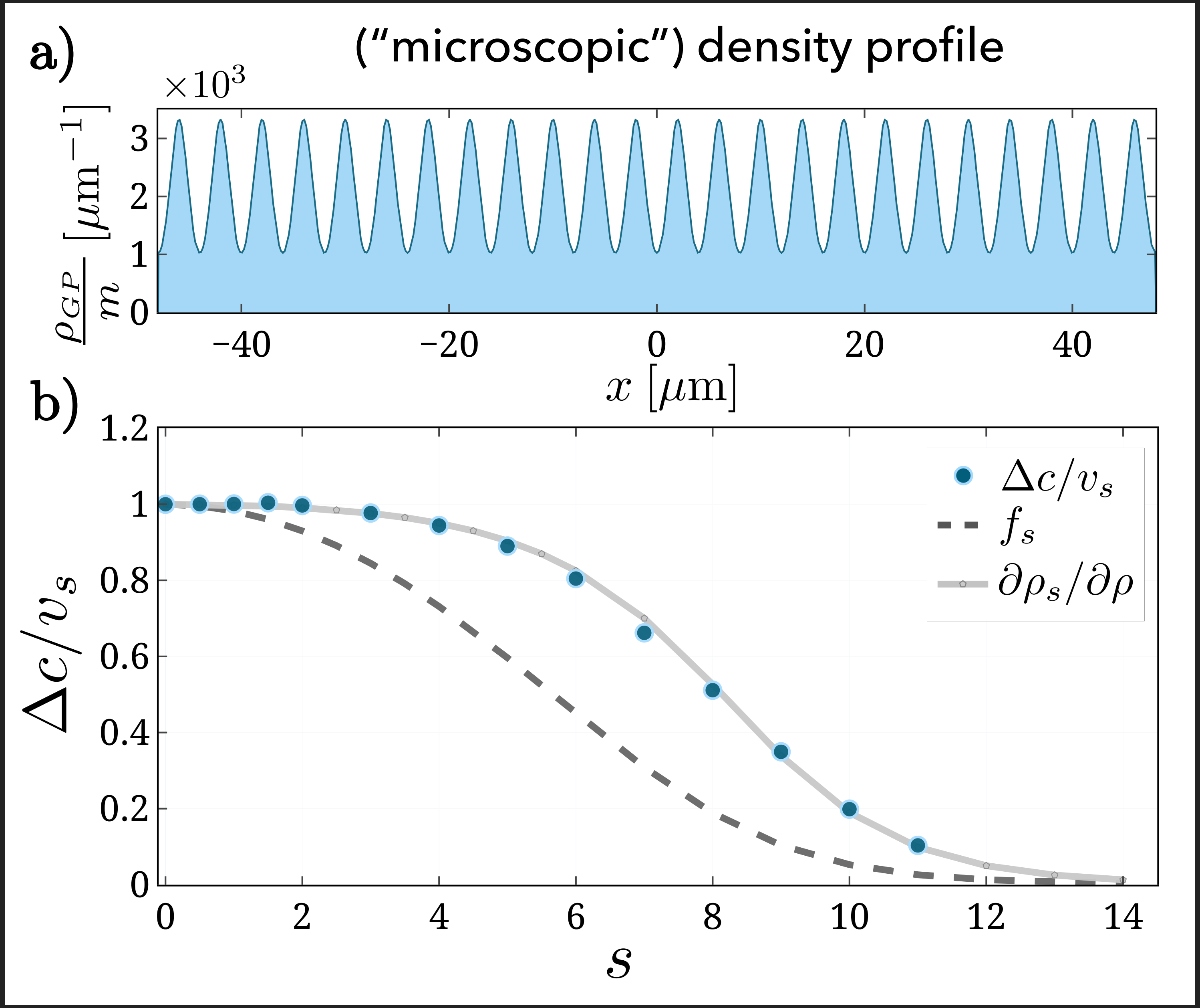
[see also: Bergman, Halperin, &Hohenberg (1975), Smerzi & Trombettoni (2003), Taylor & Zaremba (2003)]

GROSS-PITAEVSKII ANALYSIS

The T=0 atomic BECs can be microscopically described by means of the Gross-Pitaevskii equation

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2\Delta}{2m} + V_{opt} + g|\psi|^2\right)\psi$$

- Prepare the stationary states (with PBC)
- Extract speed of sound
- Calculate superfluid fraction for different densities



On the Doppler shift:

To get an anomalous Doppler effect one needs that the: ***healing length is SMALLER that the lattice period***

$$\xi \ll d$$

Parameters:
N=200.000 Rb-87 atoms,
transverse trapping: 150 Hz,
Length = 96 micron
$d=16.7\,\xi$

DOPPLER EFFECT FOR A SUPERSOLID

The linearised hydrodynamic leads to 2 sound (Goldstone) modes with speeds:

$$c^{\pm}_{1,2} = c^0_{1,2} \pm [v^0_s + (v^0_n - v^0_s)\delta_{1,2}]$$

The Doppler shift due to the relative motion of the normal and superfluid is $(\gamma = 0)$

$$\delta_{1,2} = \frac{\frac{\rho^u_n}{2\rho_n}[(c^0_{1,2})^2 - c^2_\kappa] + 2(c^0_{1,2})^2 - (1+f_s)c^2_\kappa - \rho'_n\beta}{2(c^0_{1,2})^2 - c^2_\kappa - \beta}$$

$$\rho'_s = \frac{\partial \rho_s}{\partial \rho} \qquad \rho^u_n = \frac{\partial \rho_n}{\partial u_x}$$

Some limiting cases:

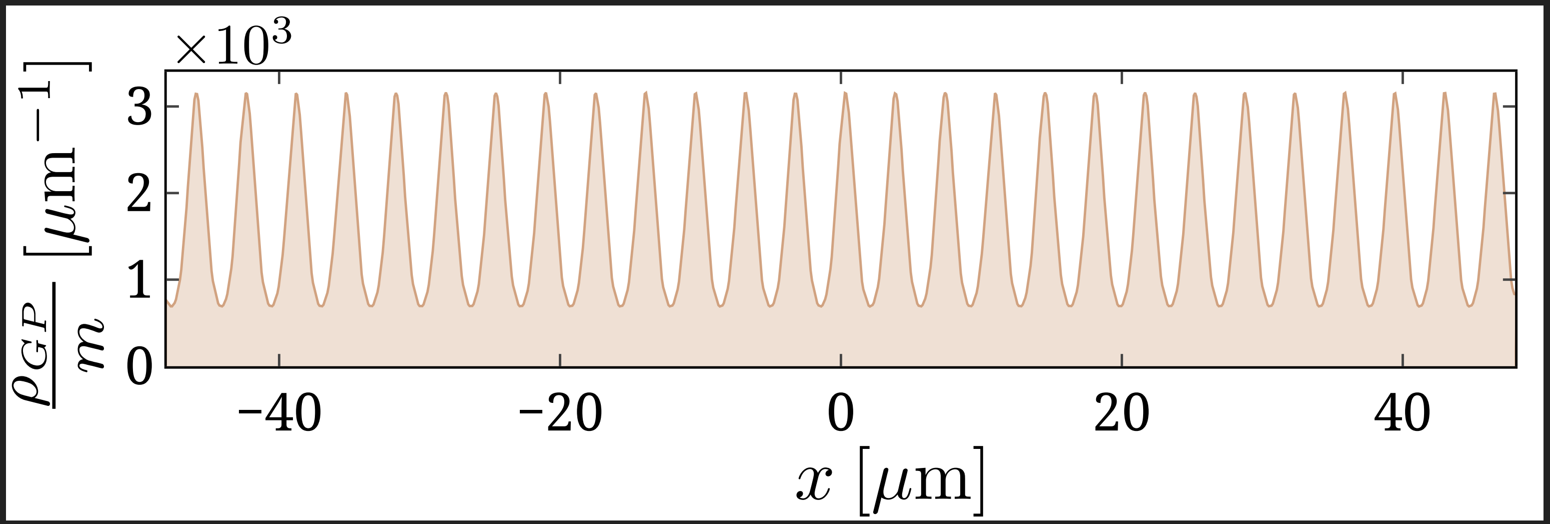
$v^0_n = v^0_s$: kinematic Doppler effect

$\lambda \to \infty$ (incompressible lattice): $v^0_n + \rho'_s(v^0_s - v^0_n)$ 4th sound Doppler as in an ext. lattice

$\rho_n \to \rho$ (Normal solid phase): $\delta_{1,2} \to 1$ kinematic Doppler effect v^0_n

DOPPLER EFFECT IN THE SS PHASE OF DIPOLAR BOSE GASES

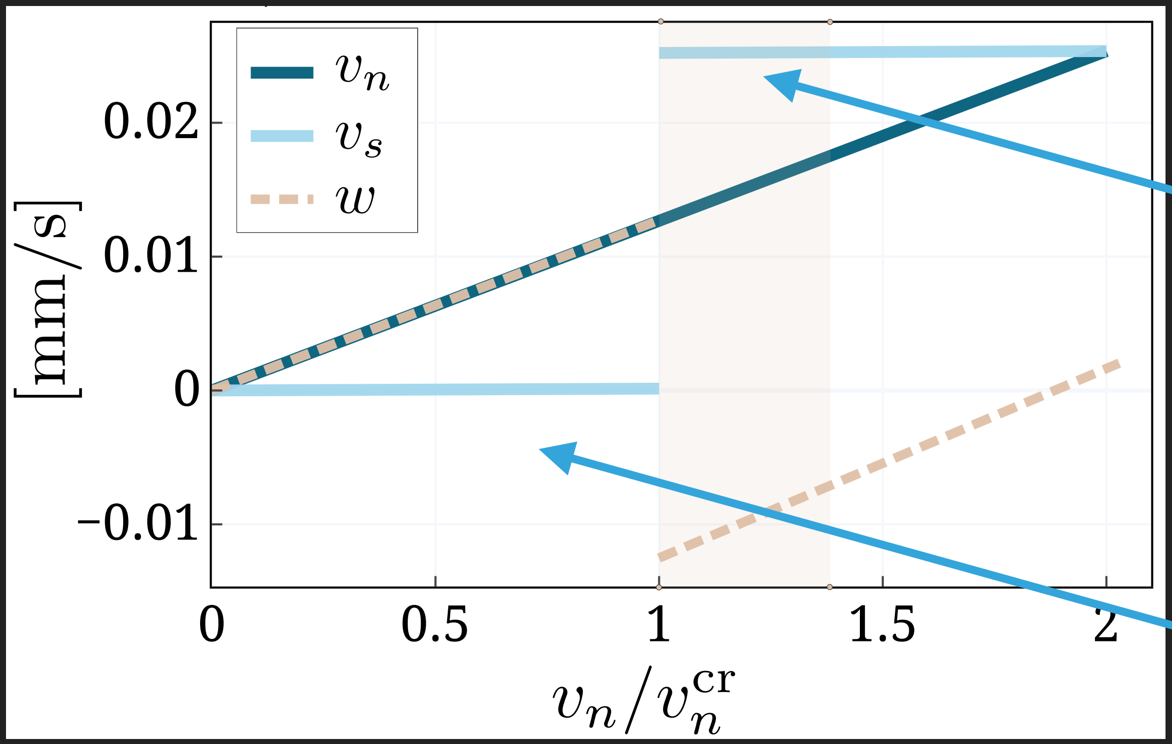
$$i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ho}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2 + \int d\mathbf{r}' V_{dd}(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}',t)|^2 + \gamma(\varepsilon_{dd})|\Psi(\mathbf{r},t)|^3\right]\Psi(\mathbf{r},t)$$



Integrated density profile for N=160.000 Dy-164 atoms, transverse trapping: 100 Hz, Length = 96 micron (with PBC)

The current is due in this case by **both the normal and the SF component** and we prepare the stationary state via imaginary time evolution in presence of the boost: $-v_n P_x$

leading to $\langle P_x \rangle = m(f_n v_n + f_s v_s)$



Quantum of "circulation":

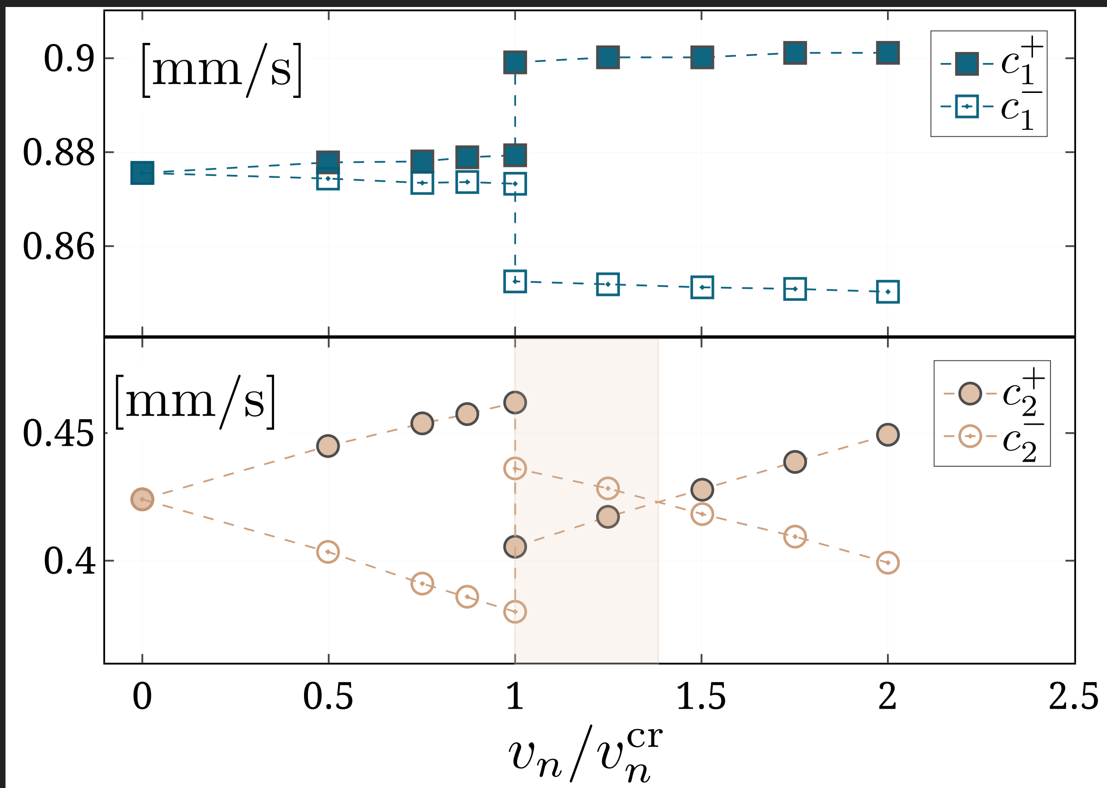
Non-zero SF velocity

$$v_n^{cr} = \pi\hbar/(mL)$$

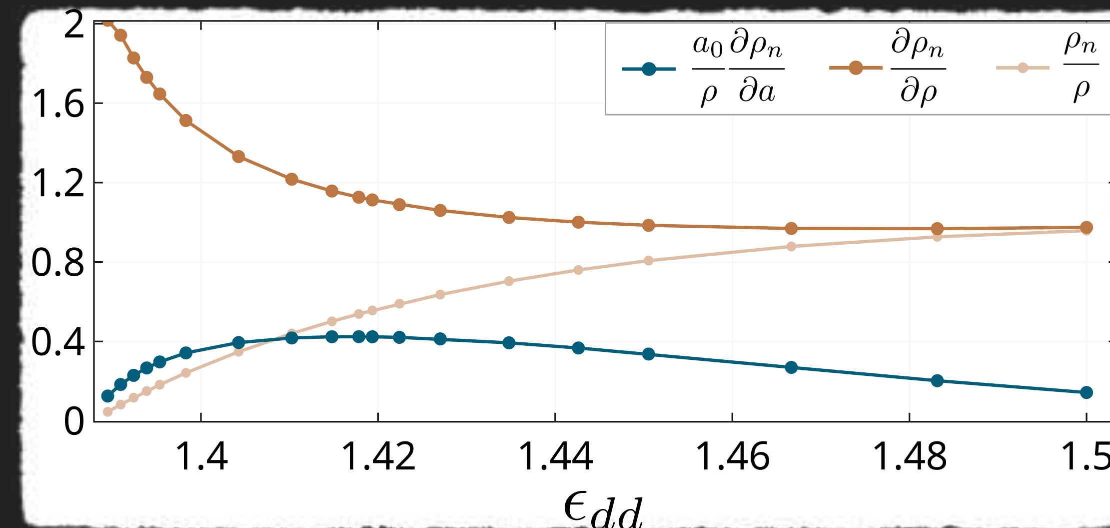
Only normal velocity

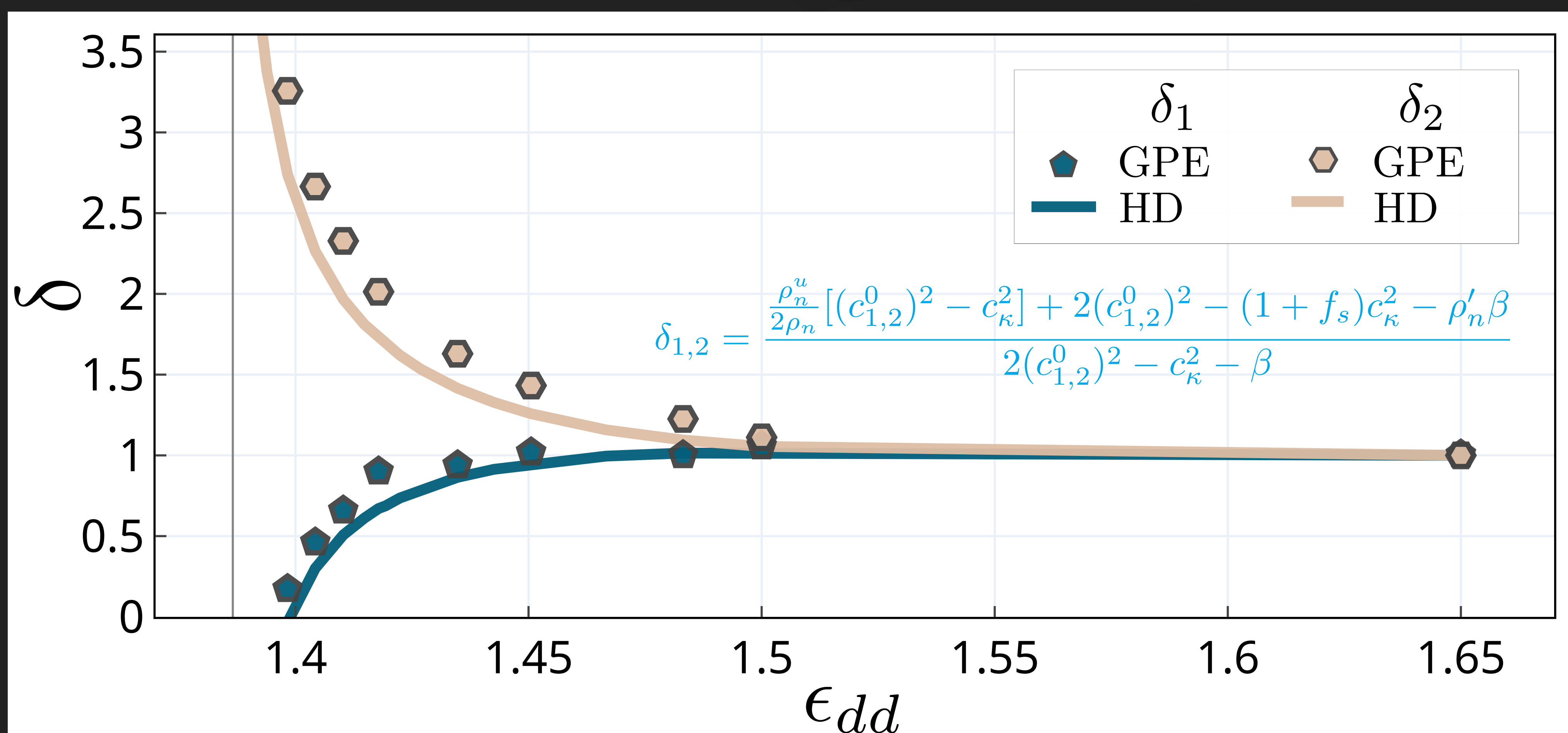
DOPPLER EFFECT IN THE SS PHASE OF DIPOLAR BOSE GASES

The two sound speeds



For the comparison we need the SS parameters as well as the derivatives of the normal density w.r.t. density and displacement. The GPE allows to determine all of them.





DOPPLER EFFECT IN A BEC MIXTURE

T=0 Hydrodynamic equation (4 conservation + 2 order parameters):

$$\partial_t n_1 + \partial_x j_1 = 0$$
$$\partial_t n_2 + \partial_x j_2 = 0$$
$$\partial_t v_1 + \partial_x(\mu_1) = 0$$
$$\partial_t v_2 + \partial_x(\mu_2) = 0$$

with

$$j_i = n_i v_i$$

$$\mu_i = g_i n_i + g_{12} n_{3-i} + \frac{v_i^2}{2} \qquad i = 1, 2$$

(Momentum conservation is redundant for zero temperature homogeneous SF)

A mixture one can have different velocities for the two component: $w^0 = v_1^0 - v_2^0 \neq 0$ → $c_{1,2} = c_{1,2}^0 + v^0 + w^0 \delta_{1,2}$

$$c_{1,2}^0 = \left[g_1 n_1 + g_2 n_2 \pm \sqrt{(g_1 n_1 + g_2 n_2)^2 - 4(g_1 g_2 - g_{12}^2) n_1 n_2} \right]^{1/2}$$

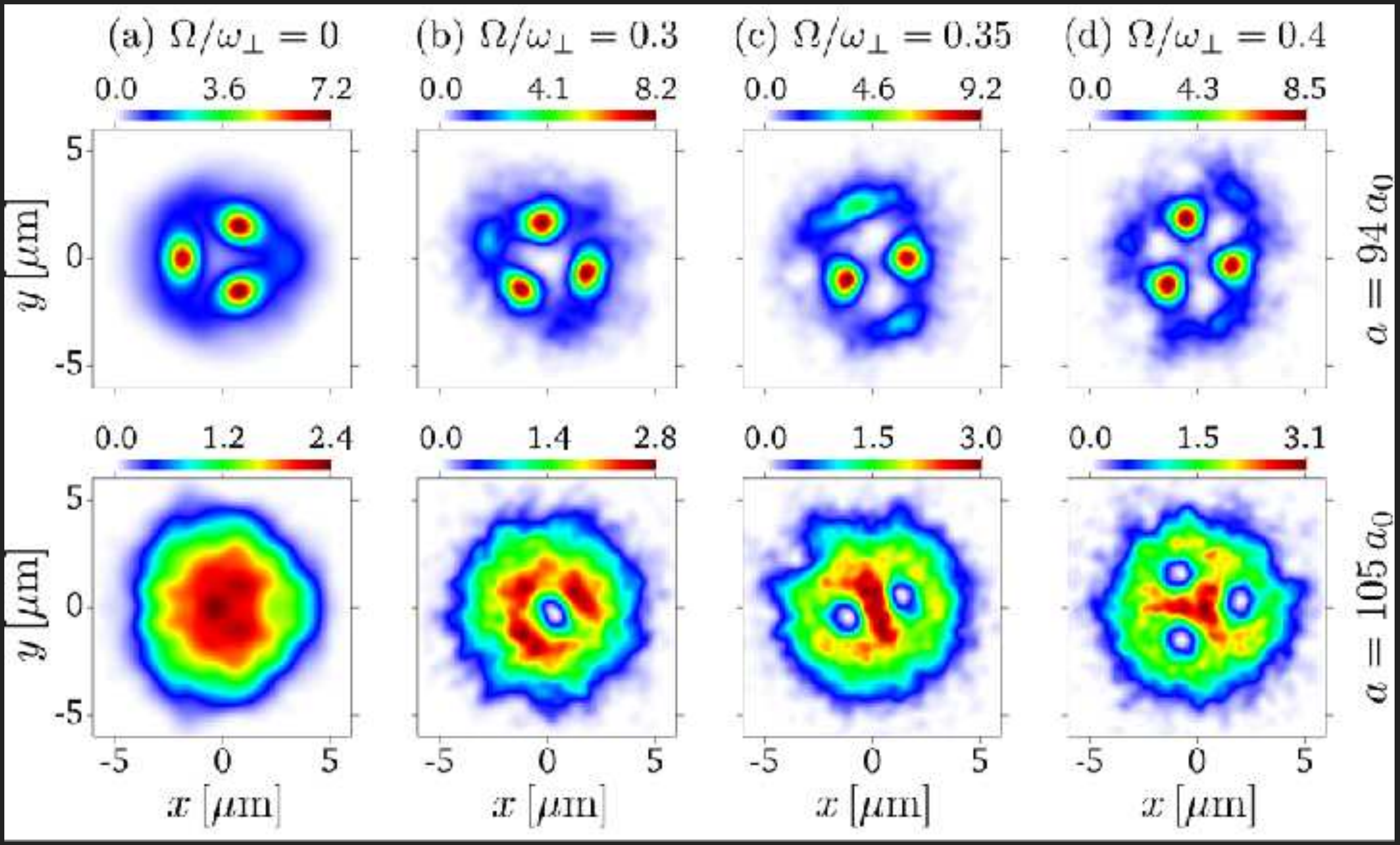
[Pethick & Smith Book]

$$\delta_{1,2} = \pm \frac{1}{2} \frac{g_1 n_1 - g_2 n_2}{\sqrt{(g_1 n_1 + g_2 n_2)^2 - 4(g_1 g_2 - g_{12}^2) n_1 n_2}}$$

$$\begin{cases} g_{12} = 0 : \; c_i = c_i^0 + v_i^0 \\ g_1 n_1 = g_2 n_2 : \; \delta_{1,2} = 0 \end{cases}$$

In this case is clear how the anomalous Doppler shift is due to Z_2 broken symmetry, i.e., to the **hybridisation** of the density and magnetisation (Goldstone) modes, as it occurs in a supersolid.

THANKS



(a) $\Omega/\omega_\perp = 0$
(b) $\Omega/\omega_\perp = 0.3$
(c) $\Omega/\omega_\perp = 0.35$
(d) $\Omega/\omega_\perp = 0.4$
0.0 3.6 7.2
0.0 4.1 8.2
0.0 4.6 9.2
0.0 4.3 8.5
0.0 1.2 2.4
0.0 1.4 2.8
0.0 1.5 3.0
0.0 1.5 3.1
y [μm]
x [μm]
$a = 94\,a_0$
$a = 105\,a_0$

$$\partial_x[\rho(x)(v(x) - V)] = 0 \text{ i.e. } v(x) = V + C/\rho(x)$$

$$\int v(x)dx = s \cdot 2\pi\hbar/m$$

$$v(x) = V\left(1 - \frac{\rho_s^{\mathcal{L}}}{\rho(x)}\right) + s\frac{\rho_s^{\mathcal{L}}}{\rho(x)}\frac{2\pi\hbar}{mL}$$

$$j = L^{-1}\int dx \rho(x)v(x)$$

$$j = \rho_n V + \rho_s \frac{2\pi\hbar}{mL}s$$