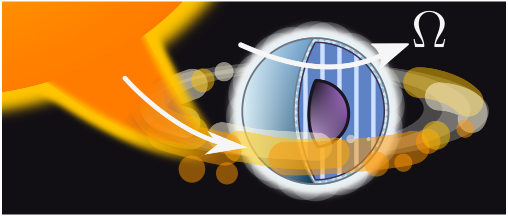
Superfluid dynamics in neutron stars

Valentin Allard



Faculty of Physics
WARSAW UNIVERSITY OF TECHNOLOGY

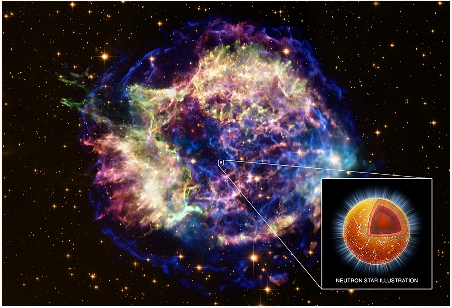
(ECT*, Trento, 14 May 2025)

Table of contents

Neutron stars

Formed in gravitational core-collapse supernova explosions. **Predicted in 1933** (Baade and Zwicky) and **observed in 1967** (Bell and Hewish)

- **Radius** $\sim$10 km,
- **Mass** $\sim 1.4\ M_{\odot}$,
- **Density** $\sim 10^{15}$ g/cm^3,
- **Energy scale**: 1 MeV = 10^{10} K.
- Initially very hot ($T \sim 100$ MeV) but **cool down** to $T \sim 0.1$ MeV, within days.

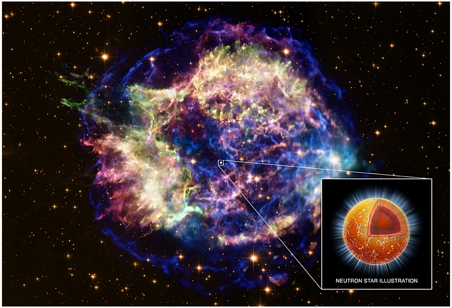


Cassiopeia A, credits: NASA/CXC/SAO

Neutron stars

Formed in gravitational core-collapse supernova explosions. **Predicted in 1933** (Baade and Zwicky) and **observed in 1967** (Bell and Hewish)

- **Radius** $\sim$10 km,
- **Mass** $\sim 1.4\ M_\odot$,
- **Density** $\sim 10^{15}$ g/cm^3,
- **Energy scale**: 1 MeV = 10^{10} K.
- Initially very hot ($T \sim 100$ MeV) but **cool down** to $T \sim 0.1$ MeV, within days.



Cassiopeia A, credits: NASA/CXC/SAO

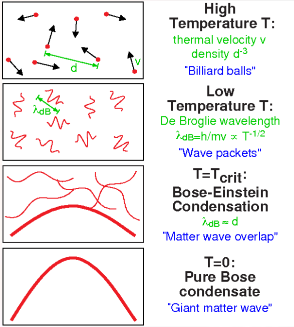
The **dense matter** in neutron stars is expected to undergo various **phase transitions** such as **superfluidity**.

Bose Einstein Condensation (BEC) and superfluidity

In 1925, Bose and Einstein predicted that **below a critical temperature**, an ideal gas of **bosons** can **condense into a macroscopic quantum state**.

In 1930's, liquid ^{4}He was found to not behave like an ordinary liquid below $T_c = 2.17$ K : **superfluidity**.

In 1938, London **associated the BEC with ^{4}He superfluidity**.



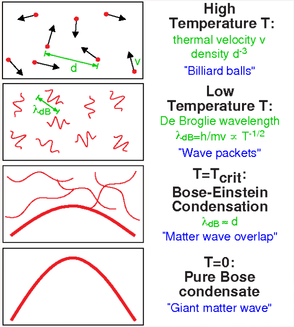
Bose Einstein Condensate from MIT group

Bose Einstein Condensation (BEC) and superfluidity

In 1925, Bose and Einstein predicted that **below a critical temperature**, an ideal gas of **bosons** can **condense into a macroscopic quantum state**.

In 1930's, liquid ^{4}He was found to not behave like an ordinary liquid below $T_c = 2.17$ K : **superfluidity**.

In 1938, London **associated the BEC with ^{4}He superfluidity**.



Bose Einstein Condensate from MIT group

Fermionic superfluidity ?

^{3}He superfluidity was observed at far lower temperatures, in 1971 !

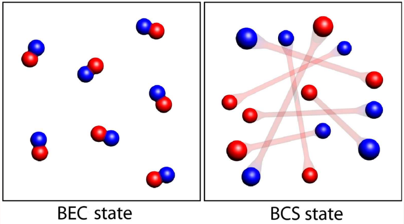
Osheroff, Richardson and Lee, PRL 28, 885 (1972)

Bardeen Cooper Schrieffer theory

The **microscopic theory** of Bardeen, Cooper and Schrieffer (originally for electronic systems) was published in 1957
Bardeen, Cooper and Schrieffer, Phys. Rev., 108, 1175 (1957).

Pair formation

A **small attractive interaction** leads to formation of **bosonic pairs** which condense below T_c.



Ketterle and Zwierlein, Riv. Nuovo Cimento., Vol. 31, Issue 5-6, p.247-422 (2018)

Pairing and superfluidity

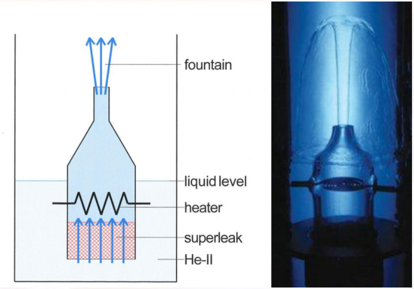
Fermionic superfluidity relies on the **pairing phenomena** and is described by the **binding energy of the pairs, Δ**.

Superfluid hydrodynamics

Tisza (1938) suggested that superfluid helium can be described by two inter-penetrating “fluids".

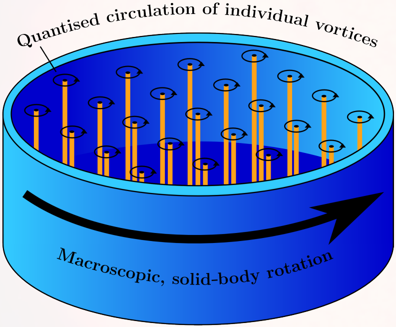
Tisza, Nature, 141: 913 (1938)

- A **superfluid**, carrying no entropy, with mass density $\rho^{(S)}$ and “Superfluid velocity" (momentum) $\boldsymbol{V_S}$.
- A **normal** viscous **fluid**, carrying heat, with mass density $\rho^{(N)}$ and normal fluid velocity $\boldsymbol{v_N}$.

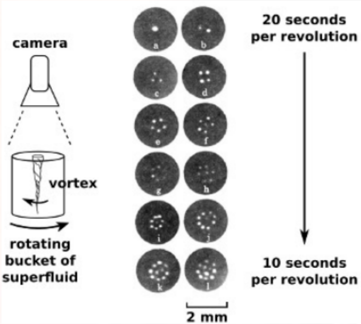


Allen and Jones, Nature, 141:243 (1938)

A rotating superfluid is threaded by an **array of quantum vortices**.



Graber and Andersson, International Journal of Modern Physics D, Vol. 26, No. 08, 1730015 (2017)



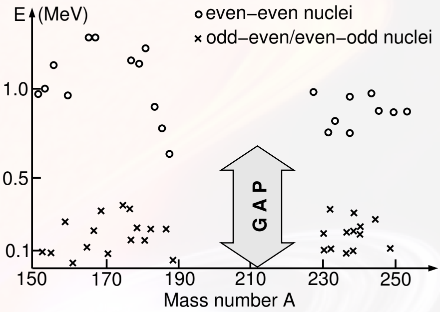
Yarmchuk et al., PRL 43:214–217 (1979)

Spinning-up (spinning-down) the superfluid is achieved by creating (destroying) vortices.

Nuclear superfluidity

Nuclear superfluidity was predicted **well before the first discovery of neutron stars** in 1967.

In 1957, Bohr, Mottelson, and Pines invoked the **nuclear pairing** to explain the **energy gap in the excitation spectra of nuclei** (first implication of superfluidity in the nuclear context).



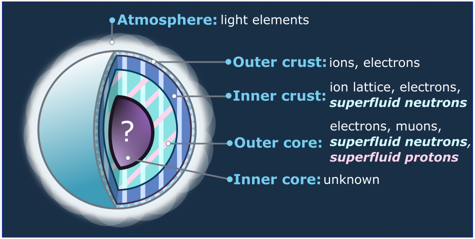
Bohr et al., Phys. Rev. 110, 936 (1958)

They also anticipated that pairing phenomenon was the key to explain the odd-even mass staggering !

Nuclear superfluidity in neutron stars

Neutron star superfluidity was predicted by Migdal (1959) and studied by Ginzburg and Kirzhnits (1964) !

Migdal, Nucl. Phys. 13, 655 (1959); Ginzburg and Kirzhnits, Zh. Eksp. Teor. Fiz. 47, 2006 (1964)

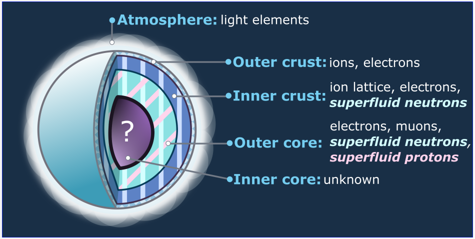


- **Neutron superfluidity** in the inner crust and **neutron-proton superfluid mixture** in the core.
- Impact on **transport and thermal properties**.
- **Superfluid neutrons weakly coupled** to the rest of the star $\Longrightarrow$ **Superfluid currents**.

Nuclear superfluidity in neutron stars

Neutron star superfluidity was predicted by Migdal (1959) and studied by Ginzburg and Kirzhnits (1964) !

Migdal, Nucl. Phys. 13, 655 (1959); Ginzburg and Kirzhnits, Zh. Eksp. Teor. Fiz. 47, 2006 (1964)



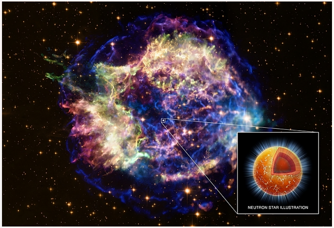
- **Neutron superfluidity** in the inner crust and **neutron-proton superfluid mixture** in the core.
- Impact on **transport and thermal properties**.
- **Superfluid neutrons weakly coupled** to the rest of the star $\implies$ **Superfluid currents**.

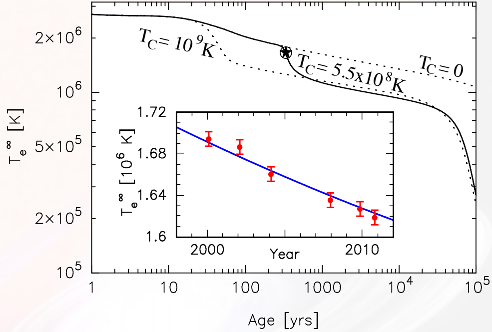
Most microscopic studies tacitly consider a **neutron superfluid co-moving with the rest of the star**.

Cooling of Cassiopeia A

The fast cooling of the neutron star in the Cassiopeia A remnant suggests a recent transition to **neutron superfluidity in its core**.

Page et al., PRL 106, 081101; Shternin et al., MNRAS 412, L108; Ho et al., MNRAS 506, 5015; Posselt et al., ApJ. 932, 83

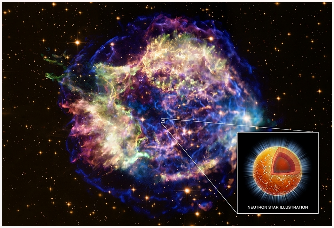


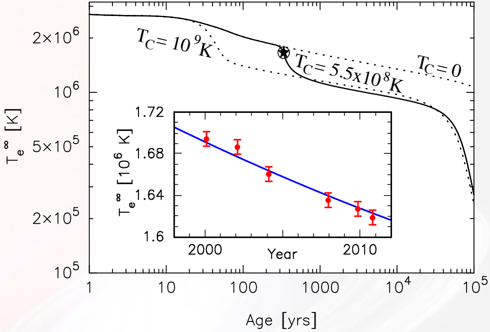


Cooling of Cassiopeia A

The fast cooling of the neutron star in the Cassiopeia A remnant suggests a recent transition to **neutron superfluidity in its core**.

Page et al., PRL 106, 081101; Shternin et al., MNRAS 412, L108; Ho et al., MNRAS 506, 5015; Posselt et al., ApJ. 932, 83





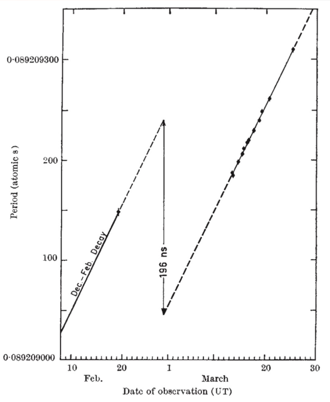
The neutron superfluid is co-moving with the rest of the star.

Pulsars are rotating neutron stars spinning with stable periods ($\dot{P} \gtrsim 10^{-12}$) BUT.

Pulsar glitches

Sudden **decrease of spin period** of pulsars interpreted as the **manifestation of superfluid dynamics**.

Antonopoulou et al., Rep. Prog. Phys., 85(12), 126901 (2022)



Radhakrishnan & Manchester, Nature, 222:228–229 (1969)

Pulsar frequency glitches

Pulsars are rotating neutron stars spinning with stable periods ($\dot{P} \gtrsim 10^{-12}$) BUT.

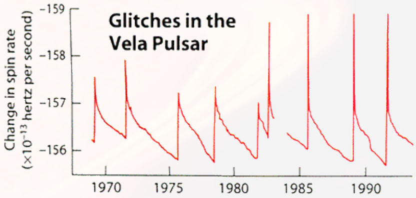
Pulsar glitches

Sudden **decrease of spin period** of pulsars interpreted as the **manifestation of superfluid dynamics**.

Antonopoulou et al., Rep. Prog. Phys., 85(12), 126901 (2022)

So far, 707 glitches (in 237 pulsars) have been detected.

(`http://www.jb.man.ac.uk/pulsar/glitches.html`)



A. Lyne & F. Graham-Smith, Pulsar astronomy 48, Cambridge University Press (2012)

Pulsar frequency glitches

Pulsars are rotating neutron stars spinning with stable periods ($\dot{P} \gtrsim 10^{-12}$) BUT.

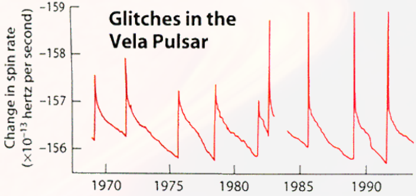
Pulsar glitches

Sudden **decrease of spin period** of pulsars interpreted as the **manifestation of superfluid dynamics**.

Antonopoulou et al., Rep. Prog. Phys., 85(12), 126901 (2022)

So far, 707 glitches (in 237 pulsars) have been detected.

(http://www.jb.man.ac.uk/pulsar/glitches.html)



A. Lyne & F. Graham-Smith, Pulsar astronomy 48, Cambridge University Press (2012)

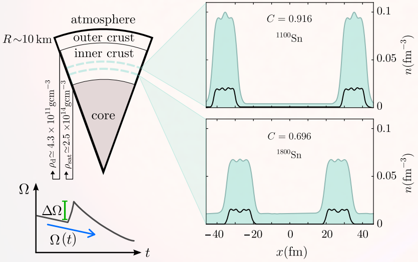
Glitches are interpreted as a global redistribution of angular momentum between a superfluid component and a “normal" component.

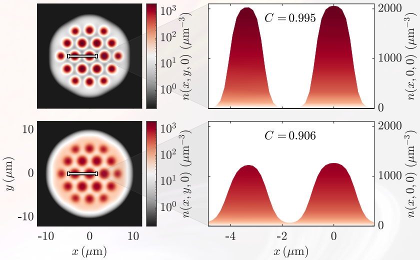
Anderson & Itoh, Nature, 256:25–27 (1975)

Pulsar frequency glitches

Glitches were also recently simulated using ultracold atoms (see Elena Poli's talk).

Poli et al., PRL 131, 223401 (2023)





Time-dependent Hartree-Fock Bogoliubov theory

The dynamics of a nuclear superfluid mixture ($q = n, p$) is governed by the **time-dependent Hartree-Fock Bogoliubov (TDHFB) equations**

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_1^{(q)}(\boldsymbol{r},t)\\ \psi_2^{(q)}(\boldsymbol{r},t)\end{pmatrix} = \begin{pmatrix}h_q(\boldsymbol{r},t)-\lambda_q & \Delta_q(\boldsymbol{r},t)\\ \Delta_q(\boldsymbol{r},t)^\star & -h_q(\boldsymbol{r},t)^\star+\lambda_q\end{pmatrix}\begin{pmatrix}\psi_1^{(q)}(\boldsymbol{r},t)\\ \psi_2^{(q)}(\boldsymbol{r},t)\end{pmatrix},$$

Time-dependent Hartree-Fock Bogoliubov theory

The dynamics of a nuclear superfluid mixture ($q = n, p$) is governed by the **time-dependent Hartree-Fock Bogoliubov (TDHFB) equations**

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r}, t) \\ \psi_2^{(q)}(\boldsymbol{r}, t) \end{pmatrix} = \begin{pmatrix} h_q(\boldsymbol{r}, t) - \lambda_q & \Delta_q(\boldsymbol{r}, t) \\ \Delta_q(\boldsymbol{r}, t)^\star & -h_q(\boldsymbol{r}, t)^\star + \lambda_q \end{pmatrix} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r}, t) \\ \psi_2^{(q)}(\boldsymbol{r}, t) \end{pmatrix},$$

Similar to **Bogoliubov-de Gennes** equations **BUT** :

- Equations for neutrons ($q = n$) and for protons ($q = p$) are coupled,

Time-dependent Hartree-Fock Bogoliubov theory

The dynamics of a nuclear superfluid mixture ($q = n, p$) is governed by the **time-dependent Hartree-Fock Bogoliubov (TDHFB) equations**

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r}, t) \\ \psi_2^{(q)}(\boldsymbol{r}, t) \end{pmatrix} = \begin{pmatrix} h_q(\boldsymbol{r}, t) - \lambda_q & \Delta_q(\boldsymbol{r}, t) \\ \Delta_q(\boldsymbol{r}, t)^\star & -h_q(\boldsymbol{r}, t)^\star + \lambda_q \end{pmatrix} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r}, t) \\ \psi_2^{(q)}(\boldsymbol{r}, t) \end{pmatrix},$$

Similar to **Bogoliubov-de Gennes** equations **BUT** :

- Equations for neutrons and for protons are coupled,
- The single-particle hamiltonian takes a much more complicated form and depends on densities, **effective mass** and **currents**:

$$h_q(\mathbf{r}, t) = -\boldsymbol{\nabla} . \frac{\hbar^2}{2 m_q^\oplus(\boldsymbol{r}, t)} \boldsymbol{\nabla} + \frac{1}{2i} \left[\boldsymbol{I}_{\boldsymbol{q}}(\boldsymbol{r}, t) . \boldsymbol{\nabla} + \boldsymbol{\nabla} . \boldsymbol{I}_{\boldsymbol{q}}(\boldsymbol{r}, t) \right] + U_q(\boldsymbol{r}, t) + \dots .$$

Time-dependent Hartree-Fock Bogoliubov theory

The dynamics of a nuclear superfluid mixture ($q = n, p$) is governed by the **time-dependent Hartree-Fock Bogoliubov (TDHFB) equations**

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_q(\boldsymbol{r},t) - \lambda_q & \Delta_q(\boldsymbol{r},t) \\ \Delta_q(\boldsymbol{r},t)^\star & -h_q(\boldsymbol{r},t)^\star + \lambda_q \end{pmatrix} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix},$$

Similar to **Bogoliubov-de Gennes** equations **BUT** :

- Equations for neutrons and for protons are coupled,
- The single-particle hamiltonian takes a much more complicated form and depends on densities, **effective mass** and **currents**:

$$\frac{\hbar^2}{2m_q^\oplus} = \frac{\delta E}{\delta \tau_q}, \quad \boldsymbol{I_q} = \frac{\delta E}{\delta \boldsymbol{j_q}}, \quad U_q = \frac{\delta E}{\delta n_q}, \quad \Delta_q = 2\frac{\delta E}{\delta \tilde{n}_q^\star}.$$

These mean fields are expressible in terms of $\psi_1^{(q)}(\boldsymbol{r},t)$ and $\psi_2^{(q)}(\boldsymbol{r},t)$!

Superfluidity velocity is not the true velocity!

Superfluid "velocity" is defined through the phase of the pairing field:

$$\boldsymbol{V_q}(\boldsymbol{r},t) = \frac{\hbar}{2m_q}\boldsymbol{\nabla}\left(\arg\Delta_q(\boldsymbol{r},t)\right),$$

Superfluidity velocity is not the true velocity!

Superfluid "velocity" is defined through the phase of the pairing field:

$$\boldsymbol{V_q}(\boldsymbol{r},t) = \frac{\hbar}{2m_q}\nabla\left(\arg\Delta_q(\boldsymbol{r},t)\right),$$

and represents the **momentum per unit mass carried by Cooper pairs**. It does not correspond to the **"true" velocity**:

$$\boldsymbol{v_q}(\boldsymbol{r},t) = \frac{\hbar \boldsymbol{j_q}(\mathbf{r},t)}{m_q^{\oplus}(\mathbf{r},t)n_q(\boldsymbol{r},t)} + \frac{\boldsymbol{I_q}(\mathbf{r},t)}{\hbar},$$

which is directly related to **mass transport**:

$$\partial_t\left(m_q n_q(\boldsymbol{r},t)\right) + \nabla.\left(m_q n_q(\boldsymbol{r},t)\boldsymbol{v_q}(\boldsymbol{r},t)\right) = 0$$

Chamel & Allard, PRC100, 065801 (2019); Allard & Chamel, PRC103, 025804 (2021).

TDHFB theory with superfluid currents

In the limit of uniform nuclear superfluids, the TDHFB equations can be solved analytically. An **effective superfluid velocity** naturally emerges:

$$\mathbb{V}_q = \frac{m_q}{m_q^{\oplus}} \boldsymbol{V_q} + \frac{\boldsymbol{I_q}}{\hbar} .$$

The order parameter for superfluidity Δ_q remains unchanged for $\mathbb{V}_q < \mathbb{V}_{Lq}$: this corresponds to the **Landau's criterion** !

Landau's velocity

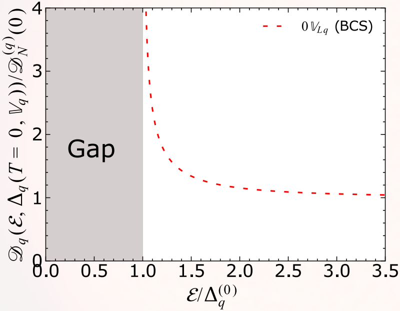
An exact expression can be derived (*Allard & Chamel, PRC 108, 015801 (2023)*) :

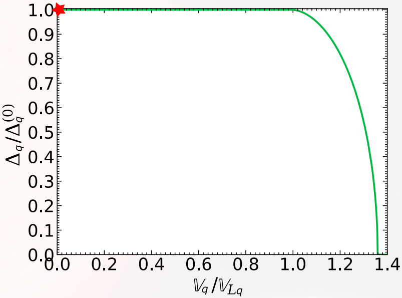
$$\mathbb{V}_{Lq} = \sqrt{\left[\frac{\mu_q}{2\varepsilon_{Fq}} \sqrt{1 + \left(\frac{\Delta_q}{\mu_q}\right)^2} - 1\right]} V_{Fq} \approx \frac{\Delta_q}{\hbar k_{Fq}} ,$$

and constitutes a **generalization to nuclear superfluids** of the expression obtained in a single cold Fermi gas (*Combescot et al., PRA 74, 042717 (2006)*).

Gapless superfluidity

Finite currents ($\mathbb{V}_q \neq 0$) influence the quasiparticle density of states $\mathscr{D}_q(\mathcal{E}, \Delta_q)$.

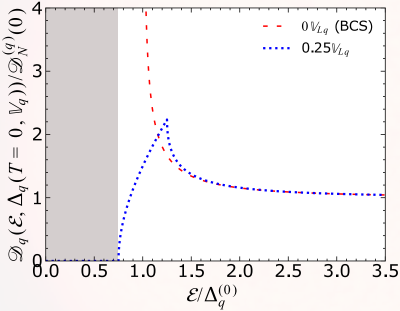


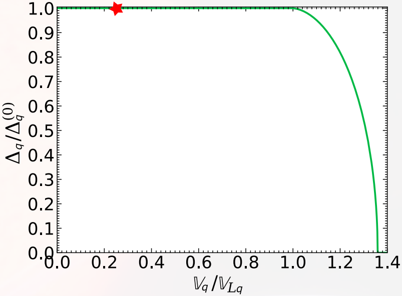


Energy **gap** $\neq \Delta_q$ order parameter for superfluidity ("Pairing *gap*" in litterature) !

Gapless superfluidity

Finite currents influence the quasiparticle density of states $\mathscr{D}_q(\mathcal{E}, \Delta_q)$.

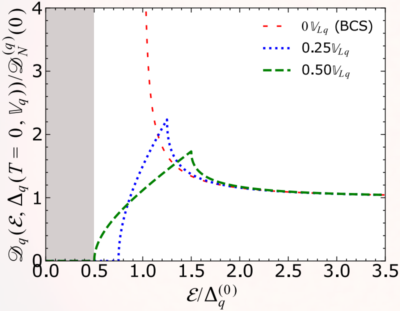


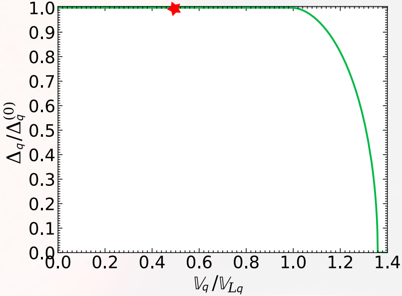


The energy gap shrinks with increasing $\mathbb{V}_q$.

Gapless superfluidity

Finite currents influence the quasiparticle density of states $\mathscr{D}_q(\mathcal{E}, \Delta_q)$.

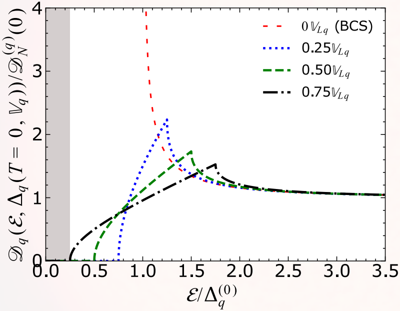


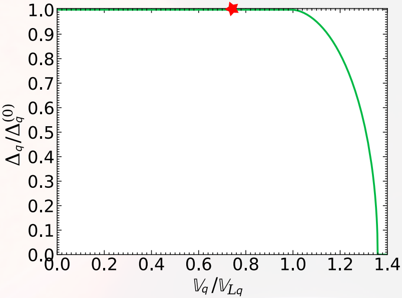


The energy gap shrinks with increasing $\mathbb{V}_q$.

Gapless superfluidity

Finite currents influence the quasiparticle density of states $\mathscr{D}_q(\mathcal{E}, \Delta_q)$.





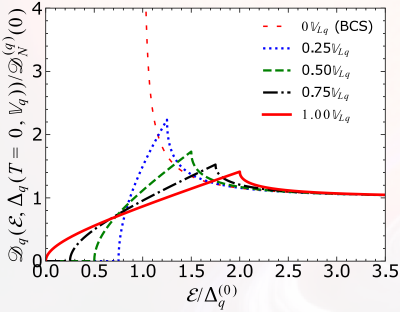
The energy gap shrinks with increasing $\mathbb{V}_q$.

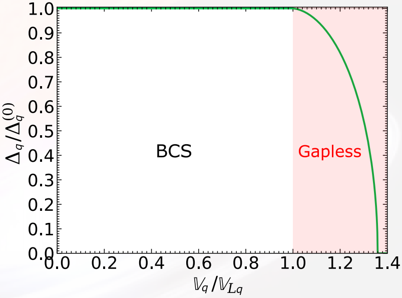
Gapless superfluidity

Gapless regime

The energy gap disappears at Landau's velocity $\mathbb{V}_{Lq}$ but superfluidity is only destroyed at $\mathbb{V}_{cq}^{(0)} \approx 1.36\mathbb{V}_{Lq}$. The intermediate regime corresponds to **gapless superfluidity**.

Allard & Chamel, PRC 108, 015801 (2023)

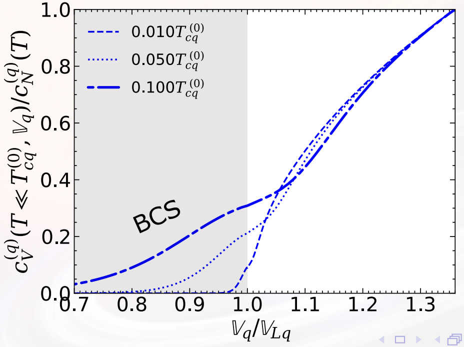




Gapless superfluidity and specific heat

Low velocities ($\mathbb{V}_q < \mathbb{V}_{Lq}$)

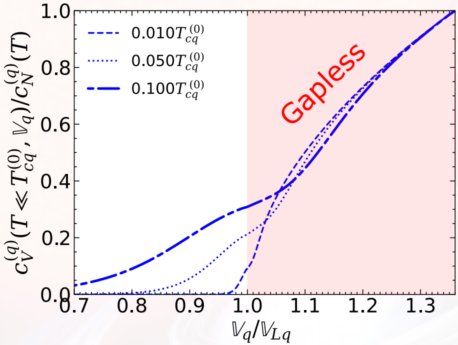
The specific heat $c_V^{(q)}(T, \mathbb{V}_q)$ is **exponentially suppressed** (compared to associated specific heat in non-superfluid phase $c_N^{(q)}(T)$).



Gapless superfluidity and specific heat

Gapless regime ($\mathbb{V}_{Lq} \leq \mathbb{V}_q \lesssim 1.36\mathbb{V}_{Lq}$)

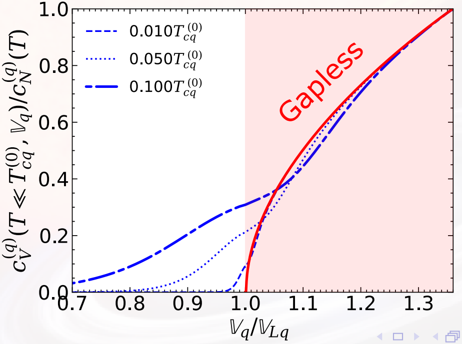
- The specific heat $c_V^{(q)}(T, \mathbb{V}_q)$ becomes comparable to $c_N^{(q)}(T)$.



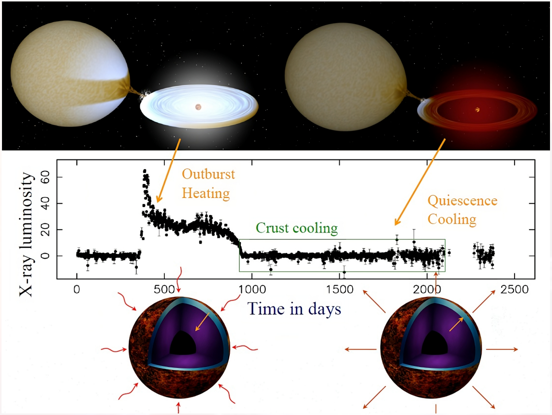
Gapless superfluidity and specific heat

Gapless regime ($\mathbb{V}_{Lq} \leq \mathbb{V}_q \lesssim 1.36\mathbb{V}_{Lq}$)

- The specific heat $c_V^{(q)}(T, \mathbb{V}_q)$ becomes comparable to $c_N^{(q)}(T)$.
- **Universal expression** for $c_V^{(q)}(T, \mathbb{V}_n)/c_N^{(q)}(T)$ as a function of $\mathbb{V}_q/\mathbb{V}_{Lq}$.
  Allard & Chamel, PRC 108, 015801 (2023)



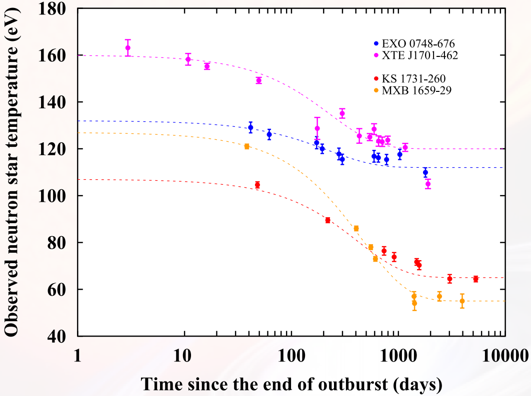
Neutron star crust heated during **accretion regime** (for ∼ 1-10 years) before **cooling phase**.



Wijnands, Degenaar & Page, J. Astrophys. Astr. **38**: 49 (2017)

Thermal relaxation **observed for several sources** up to 10^4 days.

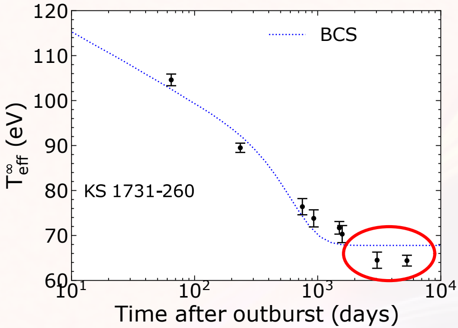
Wijnands, Degenaar & Page, J. Astrophys. Astr. ***38****: 49 (2017)*



Observational puzzle: KS 1731−260

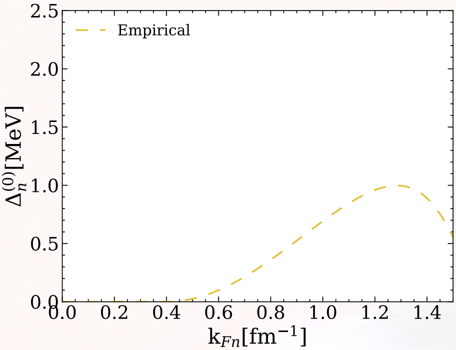
KS 1731−260 was found **colder than expected** after ∼ 3000 days.

Cackett et al., ApJL, 722: L137 (2010)



Observational puzzle: KS 1731−260

KS 1731−260 was found **colder than expected** after ∼ 3000 days.
Cackett et al., ApJL, 722: L137 (2010)

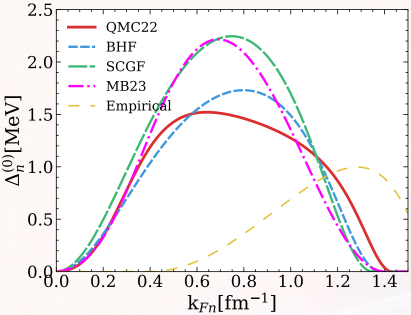


An **Empirical** neutron pairing gap $\Delta_n^{(0)}$ can be inferred from its cooling.
Turlione, Aguilera & Pons, A&A, 577: A5 (2015)

Observational puzzle: KS 1731−260

KS 1731−260 was found **colder than expected** after ~ 3000 days.

Cackett et al., ApJL, 722: L137 (2010)



An **Empirical** neutron pairing gap $\Delta_n^{(0)}$ can be inferred from its cooling.

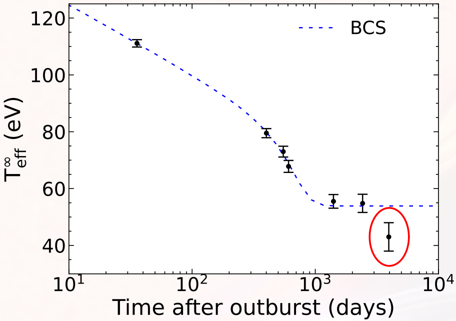
Turlione, Aguilera & Pons, A&A, 577: A5 (2015)

BUT this empirical gap is **not compatible with latest microscopic calculations** based on different many-body approaches !

Observational puzzle: MXB 1659–29

MXB 1659–29 showed an **unexpected** late-time temperature drop.

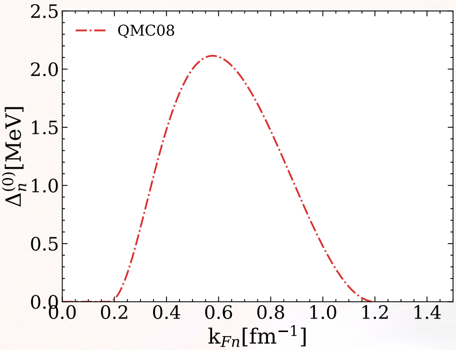
Cackett et al., ApJ, 774: 131 (2013)



Observational puzzle: MXB 1659–29

MXB 1659–29 showed an **unexpected** late-time temperature drop.

Cackett et al., ApJ, 774: 131 (2013)



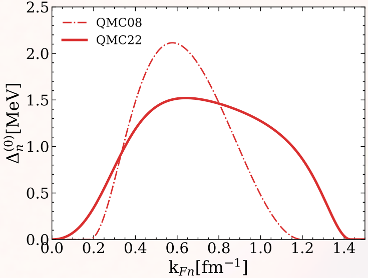
Data can be fitted considering a vanishing $\Delta_n^{(0)}$ at high densities.

Deibel et al., ApJ, 839: 95 (2017); Gandolfi et al., PRL 101: 132501 (2008)

Observational puzzle: MXB 1659–29

MXB 1659–29 showed an **unexpected** late-time temperature drop.

Cackett et al., ApJ, 774: 131 (2013)



Data can be fitted considering a vanishing $\Delta_n^{(0)}$ at high densities.

Deibel et al., ApJ, 839: 95 (2017); Gandolfi et al., PRL 101: 132501 (2008)

BUT this $\Delta_n^{(0)}$ is **contradicted by recent results from the same group** !

Gandolfi et al., Condensed Matter, 7 (2022)

Specific heat and thermal timescale

The heat diffuses over the **(thermal) timescale**:

$$\tau_{\text{th}} \propto C_V/\kappa_e\,,$$

where κ_e is the electronic thermal conductivity and C_V is the specific heat of the crust.

Page & Reddy, PRL 111, 241102 (2013)

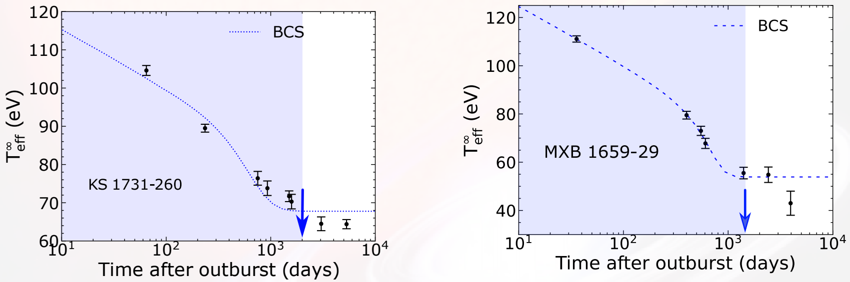
Specific heat and thermal timescale

The heat diffuses over the **(thermal) timescale**:

$$\tau_{\mathrm{th}} \propto C_V/\kappa_e\,,$$

where κ_e is the electronic thermal conductivity and C_V is the specific heat of the crust.

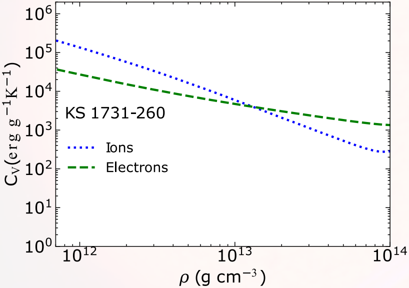
Page & Reddy, PRL 111, 241102 (2013)

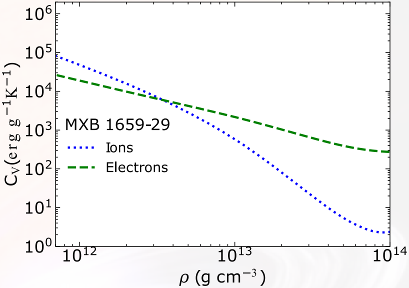


Within standard cooling models (when $\mathbb{V}_n < \mathbb{V}_{Ln}$), τ_{th} is **too low** !

Specific heat and thermal timescale

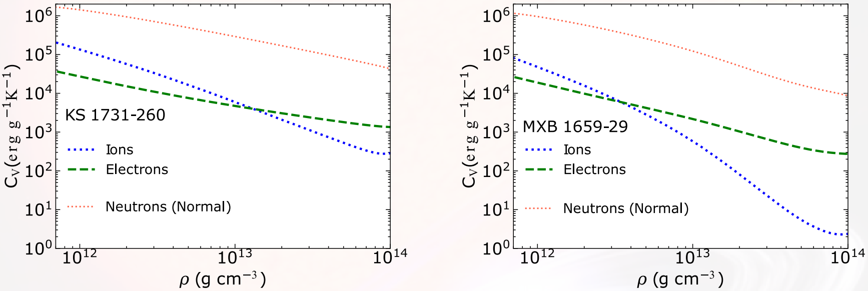
Within standard cooling models ($\mathbb{V}_n < \mathbb{V}_{Ln}$), the neutron contribution to the specific heat is exponentially suppressed !





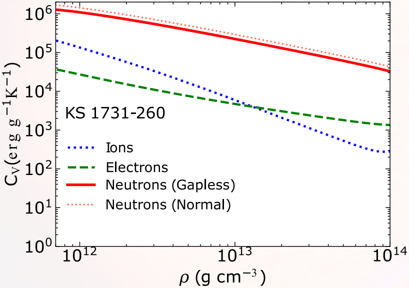
Specific heat and thermal timescale

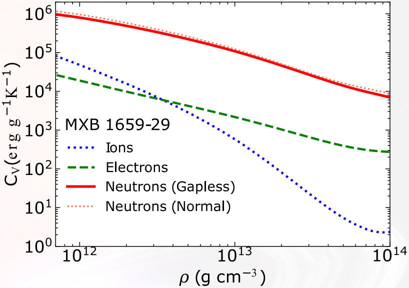
When superfluidity is destroyed ($\mathbb{V}_n \geq \mathbb{V}_{cq}$), neutrons give the major contribution to the crustal specific heat !



Specific heat and thermal timescale

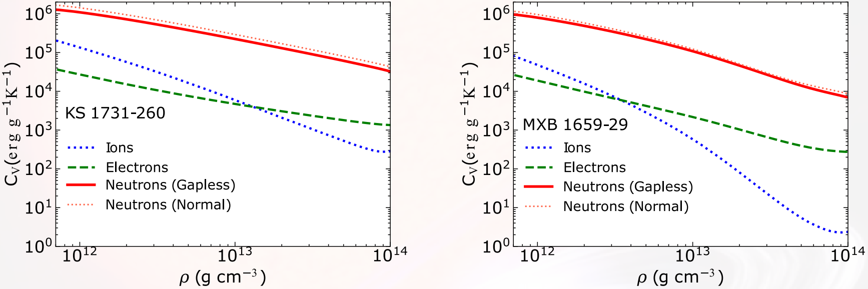
Neutrons in gapless superfluidity ($\mathbb{V}_{Ln} \leq \mathbb{V}_n < \mathbb{V}_{cn}$) also give the **major contribution** to the crustal specific heat !





Specific heat and thermal timescale

Neutrons in gapless superfluidity ($\mathbb{V}_{Ln} \leq \mathbb{V}_n < \mathbb{V}_{cn}$) also give the **major contribution** to the crustal specific heat !

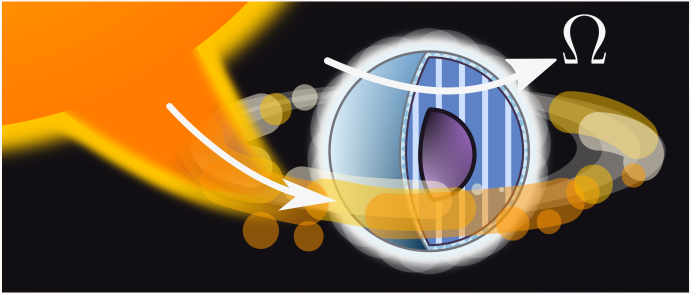


Question

How do we obtain finite $\mathbb{V}_n$?

Origin of the relative flow in neutron stars

Standard cooling models ignore the influence of superfluid currents…



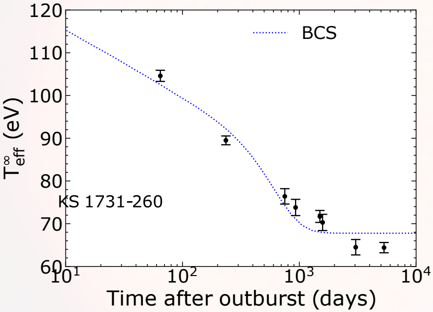
Recycling scenario

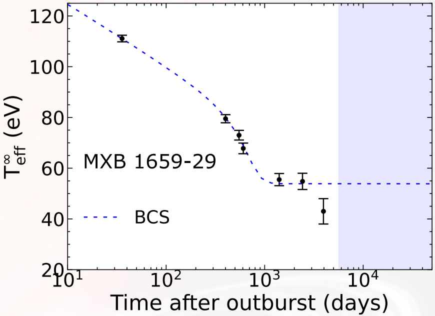
Accretion spins up the neutron star crust ⇒ Increase of $\mathbb{V}_n$.

Alpar, Cheng, Ruderman & Shaham, Nature, 300:728 (1982)

Observational evidence of gapless superfluidity

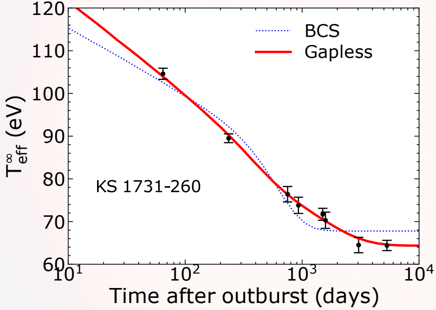
Standard cooling models lead to a too fast the thermal relaxation of the crust.

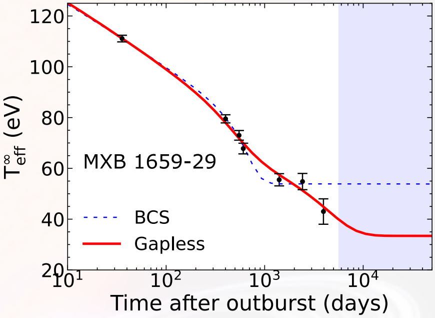




Observational evidence of gapless superfluidity

Standard cooling models lead to a too fast the thermal relaxation of the crust.



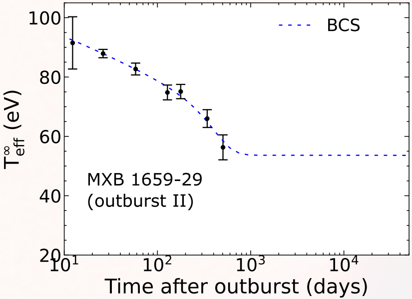


Gapless superfluidity can naturally explain the observed late-time cooling due to the delayed thermal relaxation of the crust.

Allard & Chamel, PRL 132, 181001 (2024); Allard & Chamel, EPJA 60, 116 (2024); Allard & Chamel, Universe 11(5), 140 (2025)

Consistency check: second outburst of MXB 1659–29

In 2015, MXB 1659-29 entered its **second outburst**.

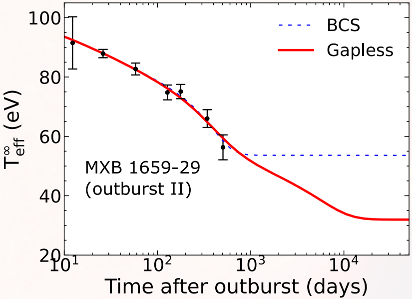


The subsequent cooling phase has been studied within the traditional cooling model.

Parikh et al., A&A 624, A84 (2019)

Consistency check: second outburst of MXB 1659–29

In 2015, MXB 1659-29 entered its **second outburst**.



The subsequent cooling phase has been studied within the traditional cooling model.

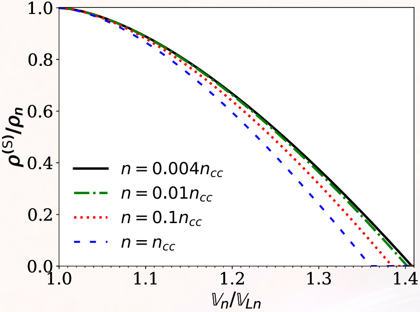
Parikh et al., A&A 624, A84 (2019)

Gapless superfluidity is also consistent with this cooling phase BUT different prediction at late time.

Other manifestations of gapless superfluidity

The superfluid density is reduced in gapless regime (even at T = 0) !

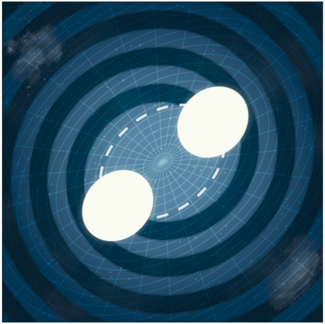
Allard & Chamel, PRC 108, 045801 (2023)



Anti-glitch in PSR B0540–69 interpreted as a *"Partial 'evaporation' of the superfluid component"* $\Longrightarrow$ Signature of gapless superfluidity ?

Tuo et al., ApJL 967, L13 (2024)

Other manifestations of gapless superfluidity



Tidal deformabilities and gravitational wave emission

- Tidal effects can heat the inner crust, inducing a phase shift $\Delta\varphi$ in the gravitational waveform.
- This $\Delta\varphi$ could be detected in the new generation of gravitational wave detectors !

Pan et al., PRL 125, 201102 (2020)

Conclusions

Neutron star cooling

- Order parameter ($\propto \Delta_q$) $\neq$ Quasiparticle energy gap $\Longrightarrow$ Impact on the specific heat and appearance of a normal fluid (at $T = 0$).

  Allard & Chamel, PRC 108, 015801 (2023); Allard & Chamel, PRC 108, 045801 (2023)
- Gapless superfluidity is compatible with the late-time cooling of KS 1731–260 and MXB 1659–29.

  Allard & Chamel, PRL 132, 181001 (2024); Allard & Chamel, EPJA 60, 116 (2024); Allard & Chamel, Universe 11(5), 140 (2025)
- Link between thermal and dynamical aspects of nuclear superfluidity !

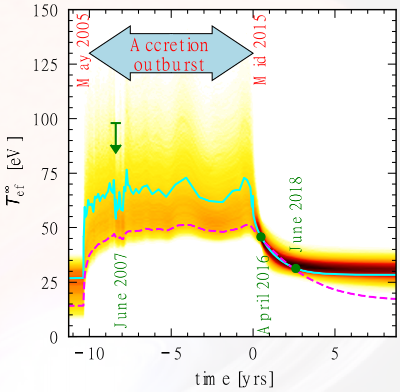
Neutron vortices

- Astrophysical manifestations of gapless superfluidity call for further studies of vortex dynamics in neutron star crusts and cores.

The accreting millisecond pulsar HETE J1900.1−2455

A similar **suppression of superfluidity** has also been suggested to explain the cooling of HETE J1900.1−2455 after its 10 year-long outburst:

"[…] *a significant fraction of the dense core is not superfluid/superconductor.*"
Degenaar et al., MNRAS 508 (2021)

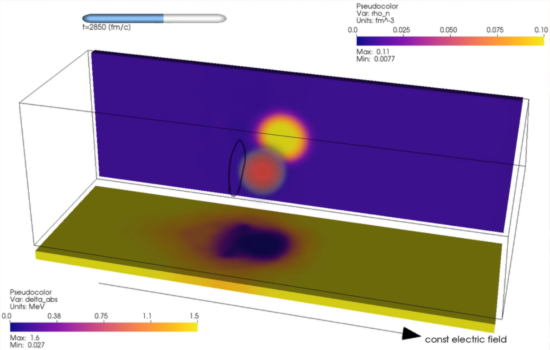


This constitutes a very promising source for testing gapless superfludity BUT **further observations are expected** !

Stability of gapless superfluidity

Fully self-consistent TDHFB simulations of the motion of a single cluster through the neutron superfluid.

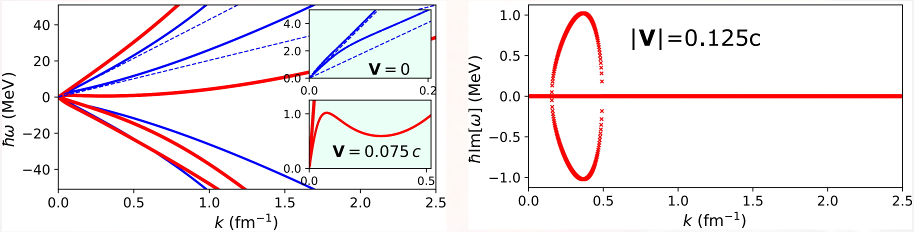
Pęcak, Chamel, Magierski & Wlazłowski, PRC 104, 055801 (2021)



Formation of vortex rings: onset of quantum turbulence? Glitch triggering mechanism?

Roton instabilities in neutron star cores ?

Dynamical instabilities triggered by the relative motion of the neutron-proton superfluid mixture **in the outer core** of neutron stars.
Gil Granados, Muñoz Mateo & Viñas, PRC 103, 065803 (2021)



BUT…

The threshold relative velocity is too high and **the influence of currents on superfluid densities is ignored**...

Stable or not ?

Stable

Gapless regime was found to be stable in the polar phase of p-wave superfluid ^{3}He.

Autti et al., Phys. Rev. Research 2, 033013 (2020)

Unstable

Quasiparticle excitations (in the inner crust of neutron stars) could be scattered with the vortex core normal neutrons, unpinning a large number of vortices and giving rise to a pulsar glitch !

Layek, Godaba Venkata & Yadav, PRD 107, 023004 (2023)

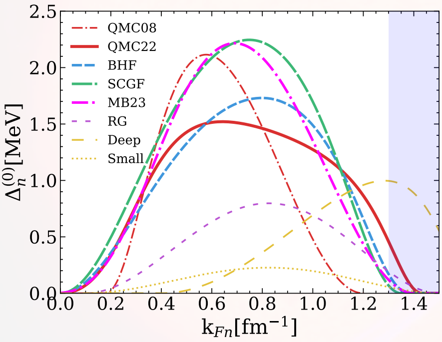
If there is a dynamical instability, what could its astrophysical manifestations be ?

Thank you !

Backup slides

$^1S_0$ neutron pairing

Neutron pairing gap computed using various N-body methods:



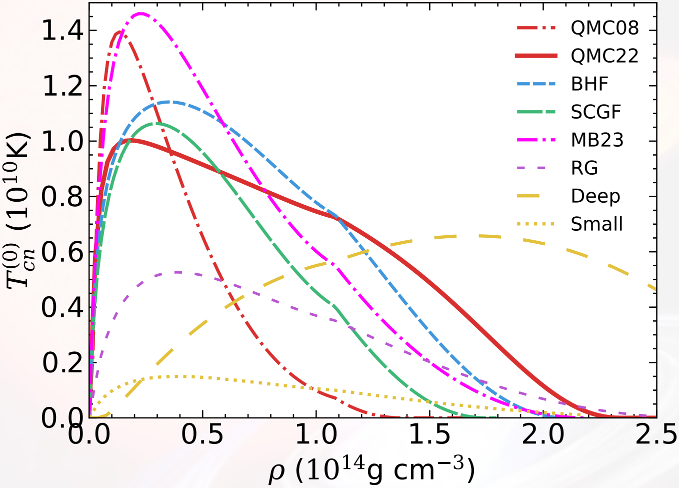
(Dashed box = Fermi wavevectors prevailing in neutron star core)

- **QMC08**: S. Gandolfi et al, Phys. Rev. Lett. **101** (2008).
- **QMC22**: S. Gandolfi et al, Condens. Matter, **7(1)** (2022).
- **BHF**: L. G. Cao et al, Phys. Rev. C **74** (2006).
- **SCGF**: M. Drissi and A. Rios, Eur. Phys. J. A **58** (2022).
- **MB23**: E. Krotscheck et al, arXiv.2305.07096 (2023)
- **Deep** and **Small**: A. Turlione et al, A&A **577** (2015).

$^1S_0$ neutron pairing

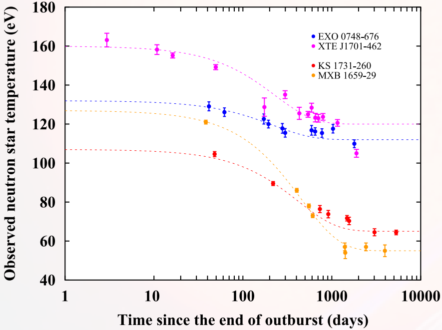
- **RG (Schwenk, Brown et al. 2003):** (One-loop) Renormalization Group equations for PNM. Medium polarization taken into account and self-energy contributions included (in a simple approximation).
- **QMC08 (Gandolfi et al. 2008):** Monte-Carlo computation by solving the many-body problem with a realistic interaction (containing Argonne v_8' (AV8') the two-nucleon interaction and the Urbana IX (UIX) three-nucleon interaction). Medium polarization effects included.
- **QMC22 (Gandolfi et al. 2022):** Most recent Monte-Carlo computation by solving the many-body problem using AV8'+UIX interaction BUT using a better starting trial wavefunction (taking more essential superfluid ground-state correlations into account than it does for QMC08).
- **BHF (Cao et al. 2006):** Brueckner Hartree-Fock computation, considering medium polarization and self-energy effects.
- **SCGF (Drissi et al. 2022):** Pairing gap computed beyond BCS+HF approximation + Three body-forces and medium effects (such as screening terms) and short-range correlations.
- **MB23 (Krotscheck et al.):** Inclusion of many-body effects through diagrammatic methods.

Critical temperatures



Cooling curve

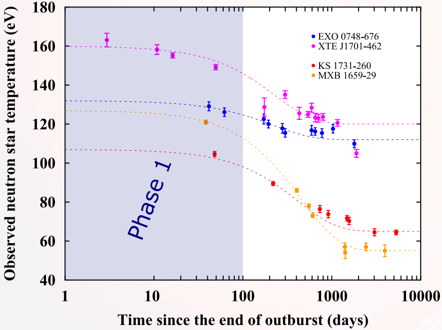
1-to-1 mapping between the **cooling curve** $T_{\text{eff}}^{\infty}(t)$ and the **NS interior**.



(Figure from R. Wijnands et al, J. Astrophys. Astr. **38** (2017))

Cooling curve

1-to-1 mapping between the **cooling curve** $T_{\text{eff}}^{\infty}(t)$ and the **NS interior**.

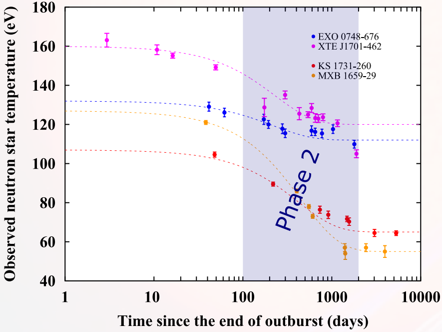


(Figure from R. Wijnands et al, J. Astrophys. Astr. **38** (2017))

- **Phase 1**: T_{eff}^{∞} sensitive to the **outer crust**.

Cooling curve

1-to-1 mapping between the **cooling curve** $T_{\text{eff}}^{\infty}(t)$ and the **NS interior**.

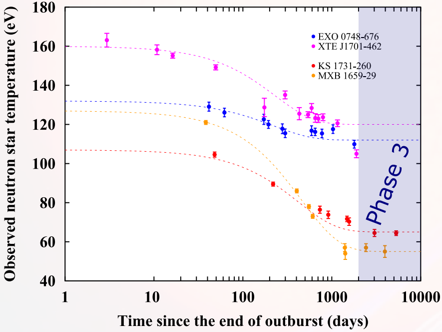


(Figure from R. Wijnands et al, J. Astrophys. Astr. **38** (2017))

- **Phase 1**: T_{eff}^{∞} sensitive to the **outer crust**.
- **Phase 2**: T_{eff}^{∞} sensitive to the **inner crust**.

Cooling curve

1-to-1 mapping between the **cooling curve** $T_{\mathrm{eff}}^{\infty}(t)$ and the **NS interior**.



(Figure from R. Wijnands et al, J. Astrophys. Astr. **38** (2017))

- **Phase 1**: $T_{\mathrm{eff}}^{\infty}$ sensitive to the **outer crust**.
- **Phase 2**: $T_{\mathrm{eff}}^{\infty}$ sensitive to the **inner crust**.
- **Phase 3**: $T_{\mathrm{eff}}^{\infty}$ sensitive to the **outer core** $\Longrightarrow$ Thermal equilibrium.

The **cooling curve** allows to **probe** the NS interiors.

Pinning forces

Pinning forces

Finite $\mathbb{V}_n$ can be sustained by the pinning of quantized vortices BUT pinning forces f_{pin} compete against Magnus forces $f_{\text{Magnus}} \Longrightarrow$ Existence of V_{cr}.

- f_{pin} differs by orders of magnitude.
- Averaging procedure over many vortices and pinning sites (model dependent).
- Vortices can pin to proton fluxons in the core (additional pinning sites: $f_{\text{pin}} \nearrow$).
- Landau's velocity can be suppressed significantly by the presence of clusters (Miller et al., Phys. Rev. Lett. 99, 070402 (2007)).

Astrophysical manifestations of gapless superfluidity call for further studies of vortex dynamics in neutron star crusts and cores.

Estimates of $V_{\rm cr}$

The lag $\mathbb{V}_n \simeq V_n$ is limited by the critical lag $V_{\rm cr}$ beyond which vortices are unpinned.

- Melatos & Millhouse, ApJ, 948(2), 106 (2023) (Statistical analysis of 541 glitches and 177 pulsars): $V_{\rm cr} \sim 10^5$ cm s^{-1} BUT no pinning in the core.
- Pizzochero, ApJL 743(1), 20 (2011) (straight parallel vortices pinned to the crust):

$$V_{\rm cr} \approx 10^7 (f_p/10^{18} \text{ dyn cm}^{-1}) \text{ cm s}^{-1} \,,$$

  where f_p is the maximum mesoscopically averaged pinning force per unit length.

The theoretical challenges to estimate this force are numerous (see, e.g., Antonopoulou et al., Rep. Prog. Phys. 85(12), 126901 (2022), for a recent review).

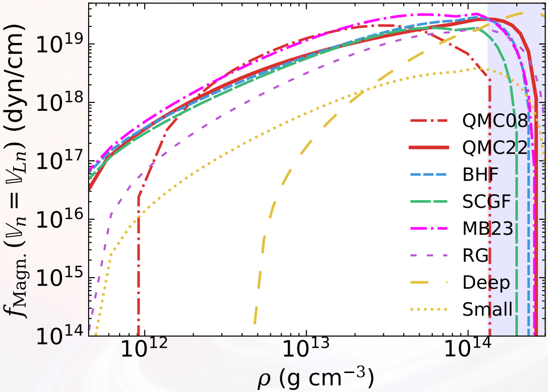
Computing f_p

- Pecak et al., Phys. Rev. C 104, 055801 (2021): internal quantum structure of a vortex locally modified in presence of nuclear clusters ⇒ a vortex should be described within a fully self-consistent quantum mechanical approach.
- Klausner et al., Phys. Rev. C 108, 035808 (2023): pinning force (for one single cluster) traditionally determined from static calculations of energy differences.
- Wlazlowski et al., Phys. Rev. Lett. 117(23), 232701 (2016): A more reliable approach consists in calculating the force dynamically BUT no such calculations have been systematically carried out so far !
- Calculations on a mesoscopic scale are more uncertain: results depend on the vortex tension and crustal structure (see, Seveso et al., MNRAS 455(4), 3952–3967 (2016) and Link and Levin, Astrophys. J. 941(2), 148 (2022)). **Estimates of f_p differ by orders of magnitude**.
- Vortices are also expected to pin to proton fluxoids in the deepest layers of the crust or, even, in the core ! (pinning to fluxoids is supported by observatons of Crab and Vela pulsar glitches, Sourie and Chamel, MNRAS 493(1), 98–102 (2020)).

Pinning force estimation

The pinning force can be roughly estimated from the Magnus force.

$$f_p(\rho) \approx 2.5 \times 10^{19} \text{ dyn cm}^{-1} \left(\frac{\Delta_n^{(0)}}{1 \text{ MeV}} \right) \left(\frac{\rho Y_{\rm nf}}{10^{14} \text{ g cm}^{-3}} \right)^{2/3} .$$



Inhomogeneities in the crust and V_{cr}

We can adopt the estimate $f_p \simeq 10^{18}$ dyn cm^{-1} (also given in Antonopoulou et al. ,Rep. Prog. Phys. 85(12), 126901 (2022)) which yields (using the *snowplow model*) $V_{\text{cr}} \sim 10^7$ cm s^{-1}.

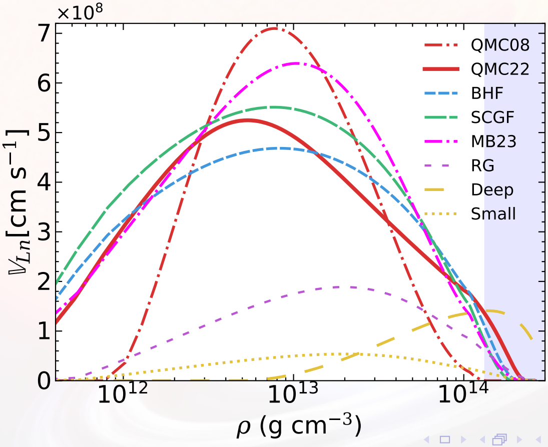
- Landau's velocity $\mathbb{V}_{Ln} \sim 10^8$ cm s^{-1} is one order of magnitude higher than V_{cr}.
- However, the estimates of V_{cr}, $\mathbb{V}_{Ln}$, and f_p were obtained ignoring the inhomogeneities in the crust.
- Antonelli et al., MNRAS 464(1), 721–733 (2017): curst inhomogeneities increase V_{cr} by a factor $(1-\varepsilon_n) = m_n^*/m_n$ (with being ε_n = entrainment parameter and m_n^*/m_n = dynamical effective mass).
- Chamel, Phys. Rev. C 85(3), 035801 (2012): $m_n^*/m_n \approx 1-14$ (depending on the crustal layer) $\Longrightarrow$ the maximum V_{cr} could be increased by an order of magnitude !

Having $\mathbb{V}_{Ln} \lesssim V_{\text{cr}}$ is not implausible !

Landau's velocity in neutron matter

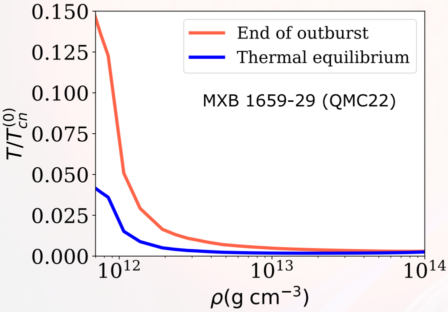
Landau's velocity is approximately given by

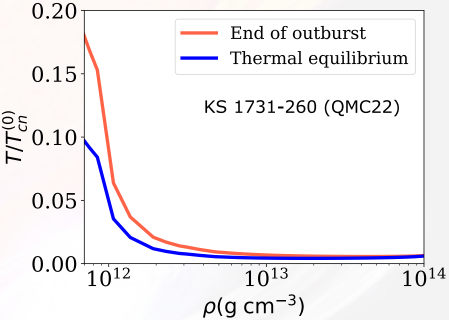
$$\mathbb{V}_{Ln} \approx 1.2 \times 10^8 \text{ cm s}^{-1} \left(\frac{\Delta_n^{(0)}}{1 \text{ MeV}} \right) \left(\frac{10^{14} \text{ g cm}^{-3}}{\rho Y_{\text{nf}}} \right)^{1/3} .$$



Low-temperature approximation ?

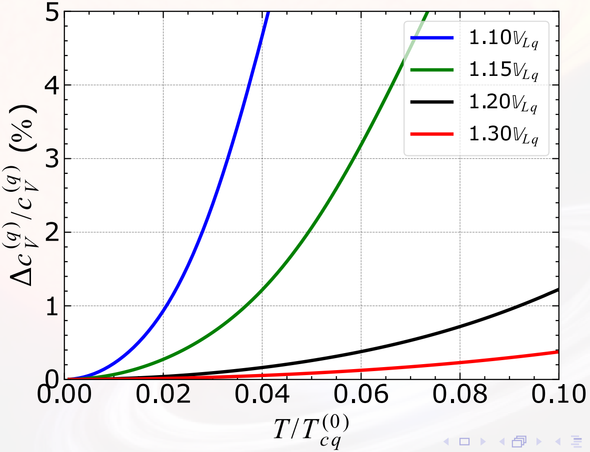
Highest temperatures reached at $\sim 0.15 - 0.20 T_{cn}^{(0)}$, at the end of outburst in the shallowest regions of the crust.





Low-temperature approximation ?

(Relative) errors not exceeding 2.9% (for MXB) and 0.028% (for KS) in shallowest regions. For the deeper layers, errors do not exceed 0.01% !



Superfluid hydrodynamics and entrainment effects

Similarly to superfluid ^{3}He-^{4}He mixture (Andreev and Bashkin, Sov. Phys. JETP 42, 164 (1975)), **superfluid neutrons (n) and protons (p) in a neutron star** are mutually coupled by non-dissipative **entrainment effects** (Gusakov and Haensel, Nucl. Phys. A, 761:333–348 (2005)).

Mass current and velocity fields (superfluid mixtures)

Mass currents $\boldsymbol{\rho_q}$ (with $q = n, p$) are not simply aligned to their associated superfluid velocities $\boldsymbol{V_q}$.

$$\boldsymbol{\rho_n} = \rho_n^{(\mathrm{N})}\boldsymbol{v_N} + \rho_{nn}\boldsymbol{V_n} + \rho_{np}\boldsymbol{V_p} \,,$$

$$\boldsymbol{\rho_p} = \rho_p^{(\mathrm{N})}\boldsymbol{v_N} + \rho_{pp}\boldsymbol{V_p} + \rho_{pn}\boldsymbol{V_n} \,,$$

$\rho_{qq'}$ = **Entrainment matrix** $\rho_q^{(N)}$ = **Normal density**

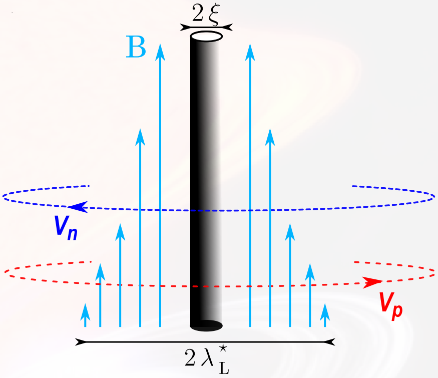
Superfluid hydrodynamics and entrainment effects

Entrainment effects induce a **circulation of protons around** neutron **vortices**.

Neutron vortices carry a magnetic flux $\Phi^\star$:

$$\Phi^\star = \frac{hc}{2|e|}\frac{\rho_{np}}{\rho_{pp}}\,.$$

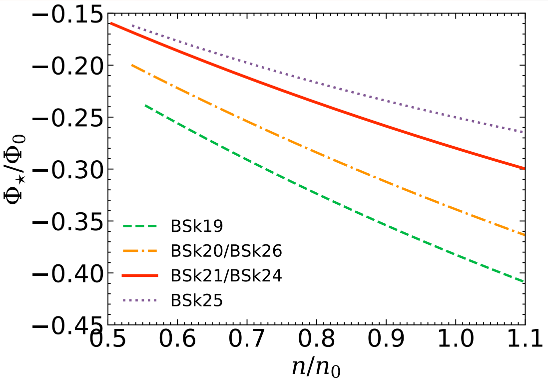
(Sedrakyan and Shakhabasyan, Astrofizika 8, 557 (1972); ibid. 16, 727 (1980))



Electrons scatter off the induced magnetic flux ⇒ strong coupling between the core superfluid and the crust (Alpar, Langer, Sauls, ApJ 282, 533 (1984)).

Neutron vortex magnetization

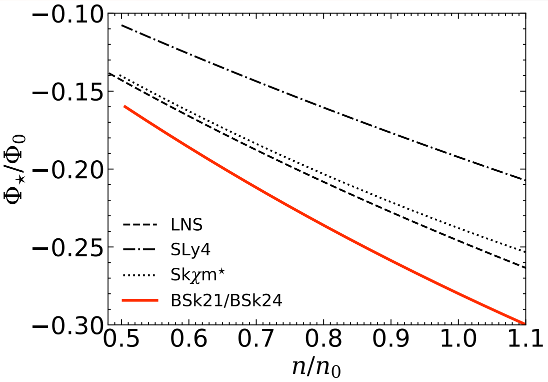
Exact solution for zero temperature and small currents ($\mathbb{V}_q \leq \mathbb{V}_{Lq}$).



For $T \neq 0$ or higher velocities (i.e. $\mathbb{V}_q > \mathbb{V}_{Lq}$), computing the vortex magnetic flux $\Phi^\star$ involves additional contributions coming from the normal fluid.

Neutron vortex magnetization

Exact solution for zero temperature and small currents ($\mathbb{V}_q \leq \mathbb{V}_{Lq}$).

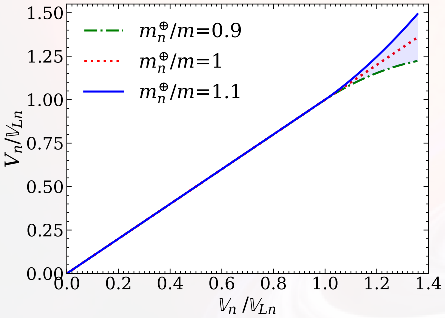


For $T \neq 0$ or higher velocities (i.e. $\mathbb{V}_q > \mathbb{V}_{Lq}$), computing the vortex magnetic flux $\Phi^\star$ involves additional contributions coming from the normal fluid.

Velocities

Three kind of velocities

- Superfluid velocity $\boldsymbol{V_q}$: Rescaled **momentum**.
- Effective superfluid velocity $\mathbb{V}_q$: **Dynamical decoupling** between neutrons and protons.
- True velocity $\boldsymbol{v_q}$: Velocity of **mass-transport** of nucleons.

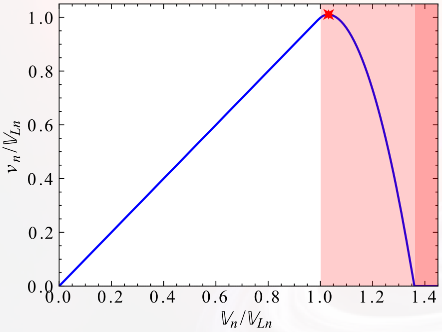


Example: Results obtained from neutron matter ($n_p = 0$) $\Longrightarrow$ **Non-linear universal relations** (beyond Landau's velocity)!

Velocities

Three kind of velocities

- Superfluid velocity $\boldsymbol{V_q}$: Rescaled **momentum**.
- Effective superfluid velocity $\mathbb{V}_q$: **Dynamical decoupling** between neutrons and protons.
- True velocity $\boldsymbol{v_q}$: Velocity of **mass-transport** of nucleons.



Example: neutron matter

Non-linear universal relations
(beyond Landau's velocity)!

Crustcool code

Study of the thermal evolution using `crustcool` (A. Cummings).

- Same microphysics as Brown et al., Astrophys. J., 698: 1020–1032 (2009) BUT
- Modified neutron specific heat & parameterized with $\mathbb{V}_n$ + Approximate formula for the critical velocities.
- Implementation of neutron diffusion.

Parameters

- Accretion duration $t_{\text{accretion}}$ and accretion rate $\dot{m}$.
- Neutron star mass M_{NS} and radius R_{NS}.
- Core temperature (at thermal equilibrium) T_{core} and temperature at the basis of the envelope T_{base}.
- Impurity parameter Q_{imp} entering the thermal conductivity.
- (Effective) superfluid velocity $\mathbb{V}_n$ entering the neutron specific heat.

Fixed $t_{\text{accretion}}$, $\dot{m}$, M_{NS} and R_{NS} and ($\mathbb{V}_n$, T_{core}, T_{base}, Q_{imp}) being free parameters.

Estimates

The thermal relaxation time is

$$\tau_{\rm th} \approx 3\times 10^4 Y_{\rm nf}^{1/3} \text{days} \left(\frac{Y_{\rm e}}{0.05}\right)^{-1/3} \left(\frac{Q_{\rm imp}\Lambda_{\rm eQ}}{\langle Z\rangle}\right)\left(\frac{\Delta R_{\rm crust}}{1\text{km}}\right)^2 \frac{c_V^{(n)}(T,\mathbb{V}_n)}{c_N^{(N)}(T)}.$$

During quiescence, the NS is electromagnetically braked (dipole model of Pacini 1967), the superfluid velocity evolves as:

$$-\frac{\dot{\mathbb{V}}_n}{\mathbb{V}_{Ln}} \approx 4.6\times 10^{-14}\text{s}^{-1} \left(\frac{B_{\rm PSR}}{10^9\text{G}}\right)^2 \left(\frac{1.4M_\odot}{M_{\rm NS}}\right)\left(\frac{R_{\rm NS}}{10\text{km}}\right)^5 \left(\frac{1\text{ms}}{P}\right)^3 \times \left(\frac{1\text{MeV}}{\Delta_n^{(0)}}\right)\left(\frac{\rho_n}{10^{14}\text{g cm}^{-3}}\right)^{1/3}.$$

If $\Delta t \sim 100$years, we find $\Delta(\mathbb{V}_n/\mathbb{V}_{Ln}) \sim 1.4\times 10^{-4}$: this justifies the use of a constant $\mathbb{V}_n/\mathbb{V}_{Ln}$ during the cooling phase.

Estimates

During accretion, the superfluid velocity evolves as (using *Patruno & Watts, Astrophysics and Space Science Library, vol 461. Springer, Berlin, Heidelberg (2021)*)

$$\frac{\dot{\mathbb{V}}_n}{\mathbb{V}_{Ln}} \approx 1.8\times 10^{-13}\xi^{1/2}\mathrm{s}^{-1}\left(\frac{\dot{m}}{\dot{m}_{\mathrm{Edd}}}\right)^{6/7}\left(\frac{M_{\mathrm{NS}}}{1.4M_\odot}\right)^{3/7}\left(\frac{B_{\mathrm{PSR}}}{10^9\mathrm{G}}\right)^{2/7}\left(\frac{R_{\mathrm{NS}}}{10\mathrm{km}}\right)^{13/7} \times\left(\frac{1\mathrm{MeV}}{\Delta_n^{(0)}}\right)\left(\frac{\rho_n}{10^{14}\mathrm{g\,cm^{-3}}}\right)^{1/3}.$$

- During accrection $\mathbb{V}_n/\mathbb{V}_{Ln}$ is weakly affected.
- Assuming $\mathbb{V}_n = 0$, a millisecond pulsar accreting with $\dot{m} \sim 10^7$ g/s reaches the Landau's Velocity after roughly $t \sim 10^6$ years. This is approximatly the age of globular clusters supposed to contain LMXBs !