Superfluid fraction in the inner crust of neutron stars

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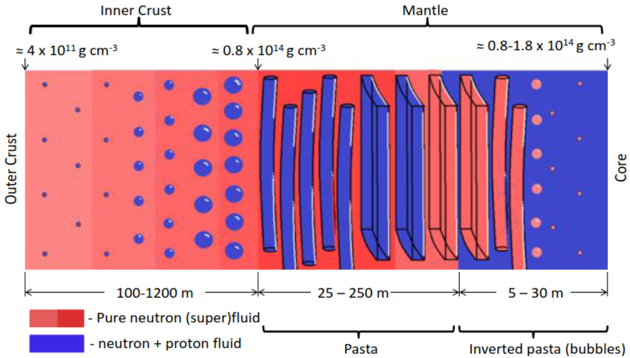
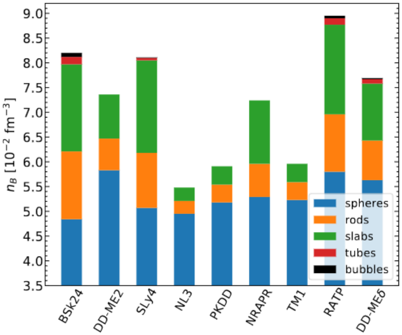
Outline

- Introduction: inner crust of neutron stars and its superfluid fraction
- Formalism: HFB with Bloch boundary conditions and Andreev-Bashkin two fluid model
- Results: fully self-consistent HFB and linear response in BCS approximation

Inner crust of neutron stars...

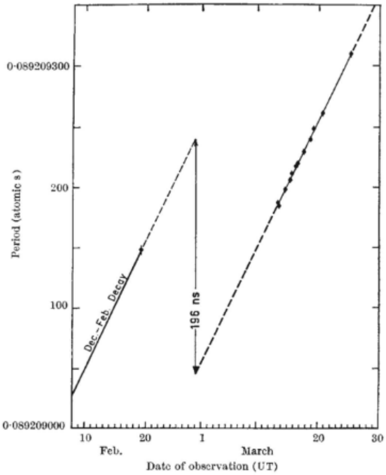


W.G. Newton et al,
Sym.En.,In.Crust,Gl.Mod. (2011)

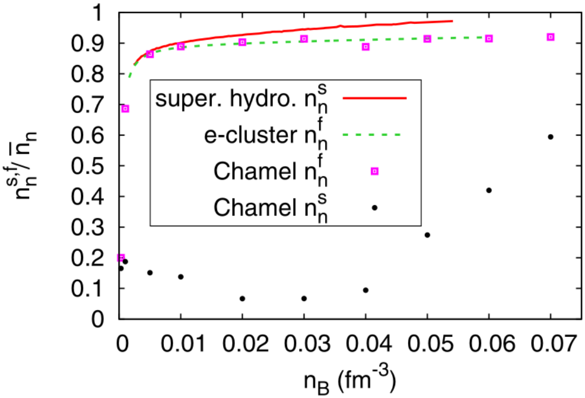


H. Dinh Thi et al,
A&A 654, A114 (2021)

...and its superfluid fraction



Radhakrishnan & Manchester,
Nature 222, 228-229 (1969)



Martin & Urban,
Phys. Rev. C 94, 065801 (2016)

Hartree-Fock-Bogoliubov

$$\begin{pmatrix} h-\mu & -\Delta \\ -\Delta^\dagger & -\bar{h}+\mu \end{pmatrix} \begin{pmatrix} U^*_\alpha \\ -V^*_\alpha \end{pmatrix} = E_\alpha \begin{pmatrix} U^*_\alpha \\ -V^*_\alpha \end{pmatrix}$$

the HFB equations in this form allow to compute the ground state properties of our system

$$h_{kk'} = \left(\frac{\hbar^2}{2m^*}\right)_{kk'} k \cdot k' + U_{kk'} - (k+k') \cdot J_{kk'} - \hbar\, k \cdot v\, \delta_{kk'} \tag{1}$$

$$U_{kk'} = -\sum_{pp'} V_{kpk'p'} \rho_{p'p} \rightarrow \text{Skyrme potential (SLy4 \& BSk24)} \tag{2}$$

$$\Delta_{kk'} = -\sum_{pp'} V_{kk'p'p} \kappa_{p'p} \rightarrow \text{separable interaction (fitted on } V_{\text{low-k}}) \tag{3}$$

Hartree-Fock-Bogoliubov in a lattice

Periodicity can be imposed trough the Bloch's theorem,
in momentum space this introduces a decomposition in integer multiples of $2\pi/L$ and a continuos momentum known as Bloch momentum

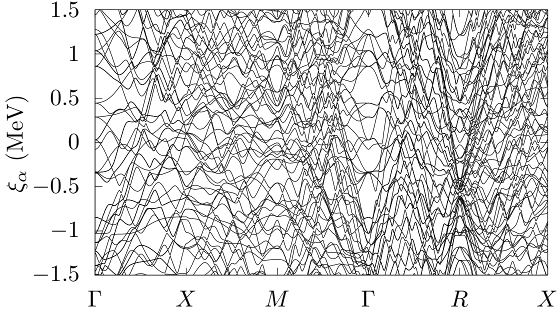
$$k = \frac{2\pi}{L} n + k_b \quad \text{with} \quad n \in \mathbb{Z} \;\; , \;\; k_b \in \left[-\frac{\pi}{L}, \frac{\pi}{L} \right)$$

as a consequence the HFB matrix has only integer indices, and it is diagonal in the Bloch and (if any) parallel momenta

$$\begin{pmatrix} h_{nn'}(k_b) - \mu & -\Delta_{nn'}(k_b) \\ -\Delta^*_{n'n}(k_b) & -h_{-n'-n}(k_b) + \mu \end{pmatrix} \begin{pmatrix} U^*_{n'\alpha}(k_b) \\ -V^*_{n'\alpha}(k_b) \end{pmatrix} = E_\alpha(k_b) \begin{pmatrix} U^*_{n\alpha}(k_b) \\ -V^*_{n\alpha}(k_b) \end{pmatrix}$$

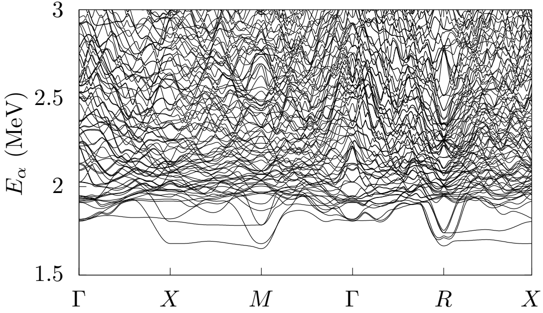
Hartree-Fock-Bogoliubov in a lattice, a taste

Solving the HFB equations with Bloch boundary conditions means computing the band structure of the inner crust



single-particle bands

$\xi_\alpha = \epsilon_\alpha - \mu$



quasi-particle bands

$E_\alpha > 0$

$\Gamma - X - M - \Gamma - R - X$ is the high symmetrical path in the positive $k_b \in$ BZ

Andreev-Bashkin formalism

Our setup gets us access to densities, currents and other relevant quantities.
Then with the formalism due to Andreev and Bashkin we can compute the superfluid fraction

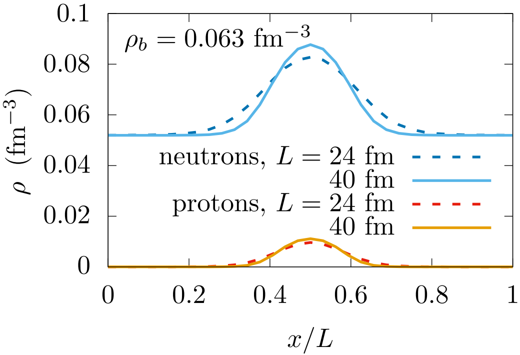
$$\vec{\rho}_n = (\rho_n - \rho_S)\vec{v} + \rho_S \vec{V}_n \quad ; \quad \vec{\rho}_p = \rho_p \vec{v}$$

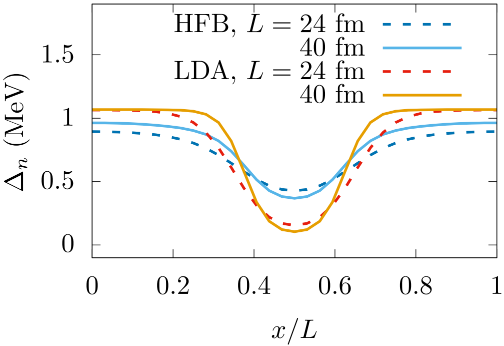
This relation has to be understood in an average sense (ϕ = phase of the gap: $\Delta = |\Delta| e^{i\phi}$)

$$\vec{V}_n = \int_V \frac{d^3x}{V} \frac{\hbar}{2m} \vec{\nabla}\phi$$

Since our quantities are periodic the average superfluid velocity is zero, thus we are working in the reference frame in which the superfluid component carries no momentum

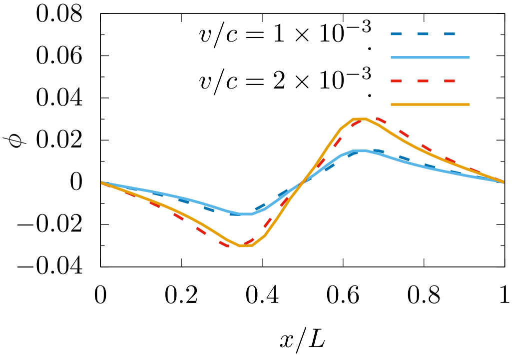
Lasagna statics

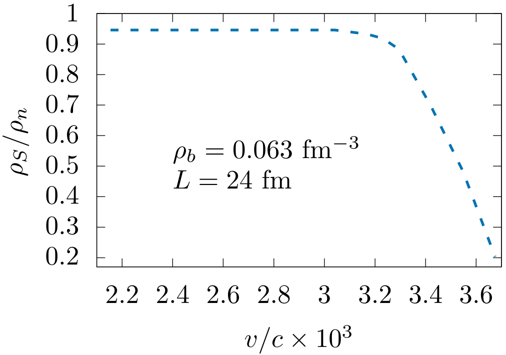




Almirante & Urban, Phys. Rev. C 109, 045805 (2024)

Lasagna dynamics

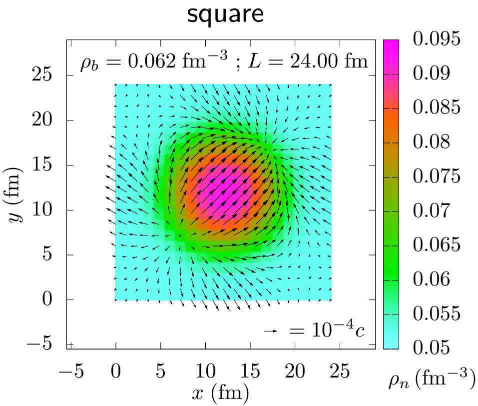


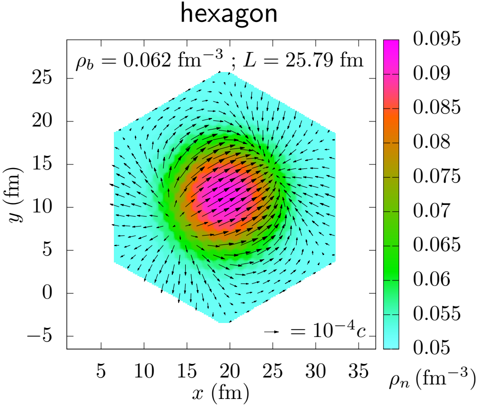


Almirante & Urban, Phys. Rev. C 109, 045805 (2024)

Spaghetti

$\rho_b = 0.062$ fm^{-3} ; $\rho_S/\rho_n = 95\%$



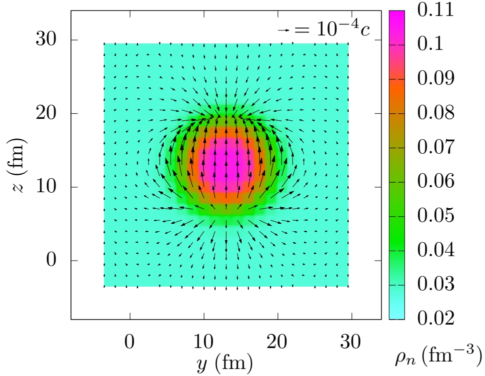


Almirante & Urban, Phys. Rev. C 110, 065802 (2024)

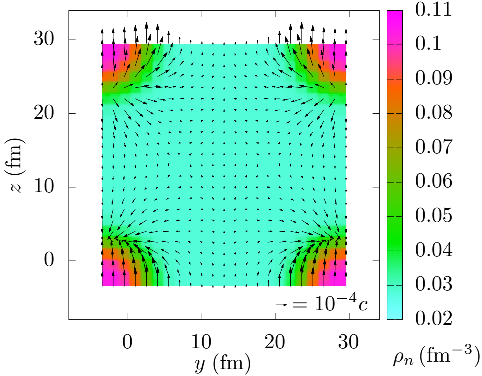
Crystal

Body-centered cubic lattice

$\rho_b = 0.033\ \mathrm{fm}^{-3}$; $L = 33$ fm ; $\rho_S/\rho_n = 92\%$



$x = x_0$



$x = x_0 + L/2$

Results and comparison

Crystal

μ_n (MeV)	L (fm)	ρ_b (fm^{-3})	ρ_S/ρ_n (%)	ρ_S/ρ_n (HF%)
9	33	0.0334	92.1	7
10	31	0.0425	92.8	9
11	29	0.0518	94.1	27

Spaghetti

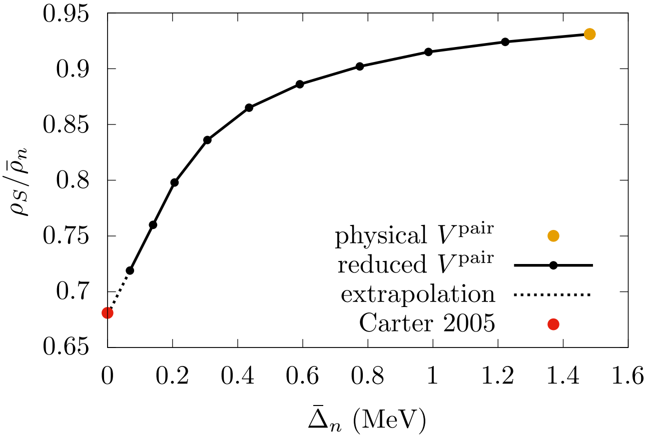
μ_n	L	ρ_b	ρ_S/ρ_n	ρ_S/ρ_n
12	24	0.0619	94.5	75
12.5	24	0.0670	95.4	82

Lasagna

μ_n	L	ρ_b	ρ_S/ρ_n	ρ_S/ρ_n
13	20	0.0723	96.3	93
13.5	20	0.0768	97.2	94

Bands effects VS pairing gap

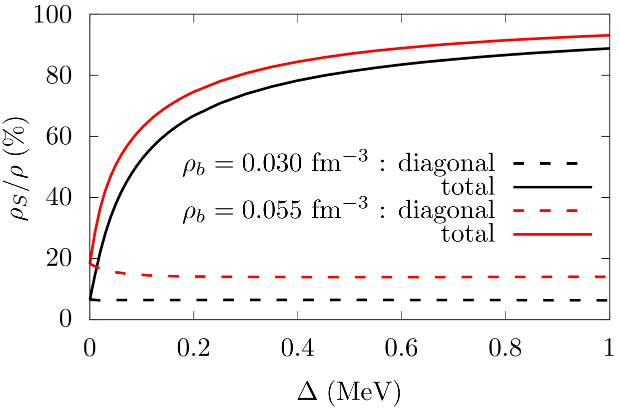
Normal band theory should be valid in the weak-coupling limit (pairing gap ≪ Fermi energy)



$\bar{\rho}_n = 0.059\ \mathrm{fm}^{-3}$; $L = 27.17$ fm

Almirante & Urban, Phys. Rev. C 110, 065802 (2024)

Geometric contribution in BCS approximation



$$\rho_S^{ij} = \int_{\mathrm{BZ}} \frac{d^3k}{(2\pi)^3} \left[m \sum_{\alpha} \frac{\Delta^2}{E_\alpha^3} \frac{\partial \xi_\alpha}{\partial k^i} \frac{\partial \xi_\alpha}{\partial k^j} + \frac{2}{m} \sum_{\alpha \neq \beta} \frac{\Delta^2}{E_\alpha E_\beta (E_\alpha + E_\beta)} p^i_{\alpha\beta} p^j_{\beta\alpha} \right]$$

Almirante & Urban, arXiv:2503.21635 (2025)

Conclusions

- 1D: $\rho_S/\bar{\rho}_n \simeq 97\%$ No surprise.
- 2D: $\rho_S/\bar{\rho}_n \simeq 95\%$ Normal band theory underestimates the superfluid fraction !
- 3D: $\rho_S/\bar{\rho}_n \simeq 92\%$ Entrainment is very small because of the geometric contribution !!

The superfluid reservoir in the inner crust of neutron stars is big enough to explain pulsar glitches with the crust only !!!

The geometric contribution could be non-zero also in the shallowest layers of the inner crust, and it is expected to appear in all the superfluid response functions

Thanks for your attention!