Energy dissipation in the superfluid interior of neutron stars

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14^{th} May 2025, ECT*, Trento

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Nicolas Chamel

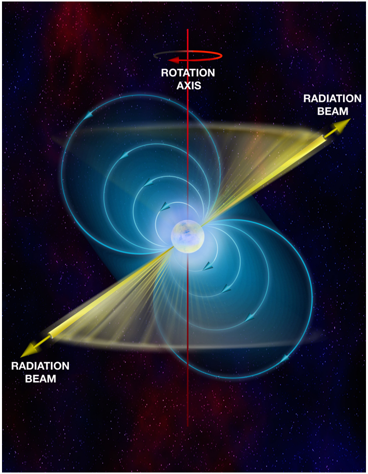
Physical Review X 14, 041054 (2024)

LUMI

PL GRID

NARODOWE CENTRUM NAUKI

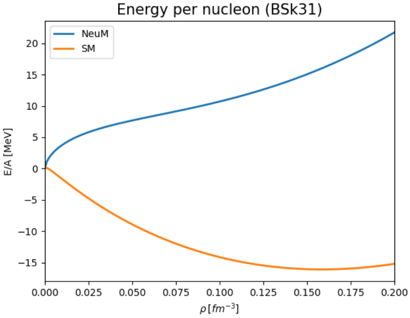
PLGrid LUMI PLL/2022/03/016433 ★ NCN 2021/40/C/ST2/00072

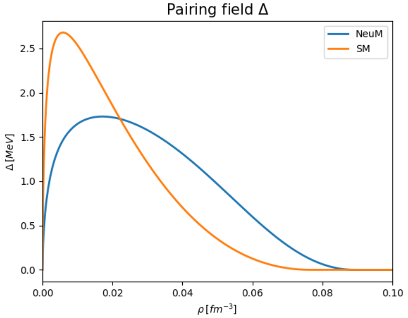


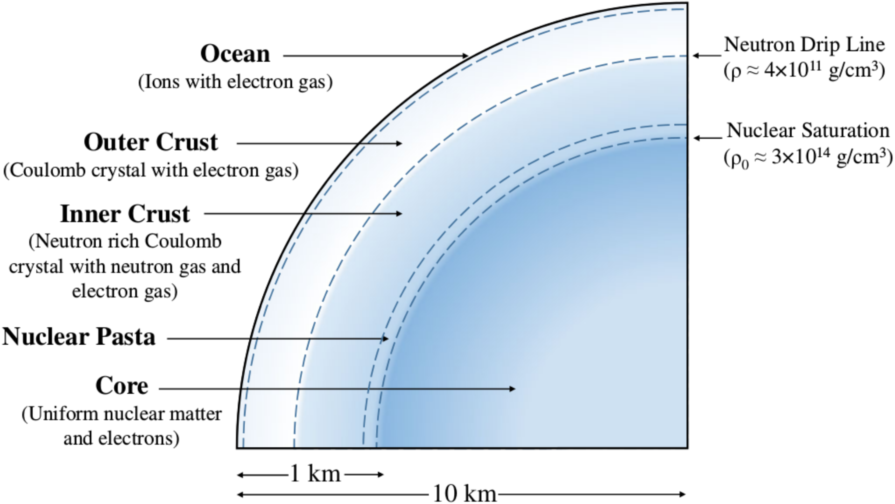
Credit: B. Saxton, NRAO/AUI/NSF

Thermal energy for $T = 10^{10}$ K:

$k_B T \approx 1$ MeV







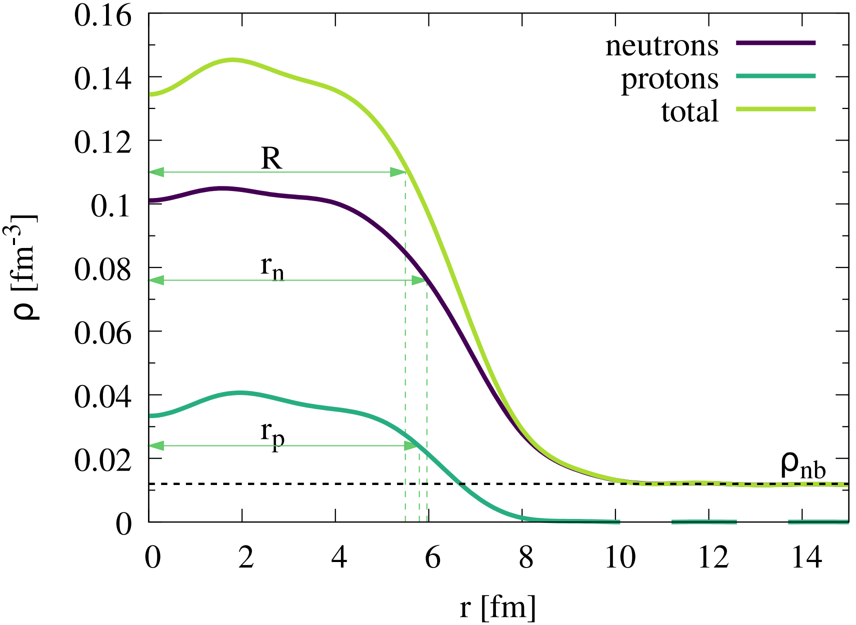
Caplan, M. E., and C. J. Horowitz, Reviews of Modern Physics 89, 041002 (2017)

Crust composition

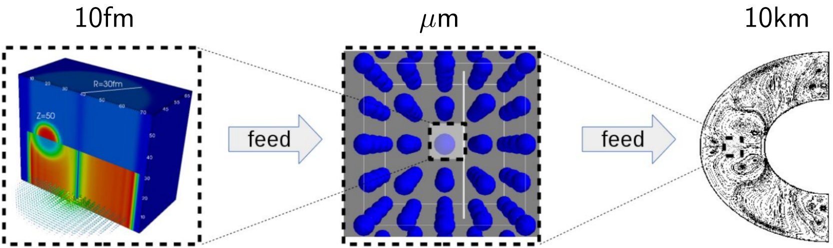
$\rho_{\max}$ [g cm^{-3}]	Element	Z	N	R_{cell} [fm]
8.02×10^{6}	^{56}Fe	26	30	1404.05
2.71×10^{8}	^{62}Ni	28	34	449.48
1.33×10^{9}	^{64}Ni	28	36	266.97
1.50×10^{9}	^{66}Ni	28	38	259.26
3.09×10^{9}	^{86}Kr	36	50	222.66
1.06×10^{10}	^{84}Se	34	50	146.56
2.79×10^{10}	^{82}Ge	32	50	105.23
6.07×10^{10}	^{80}Zn	30	50	80.58
8.46×10^{10}	^{82}Zn	30	52	72.77
9.67×10^{10}	^{128}Pd	46	82	80.77
1.47×10^{11}	^{126}Ru	44	82	69.81
2.11×10^{11}	^{124}Mo	42	82	61.71
2.89×10^{11}	^{122}Zr	40	82	55.22
3.97×10^{11}	^{120}Sr	38	82	49.37
4.27×10^{11}	^{118}Kr	36	82	47.92

ρ [g cm^{-3}]	Element	Z	N	R_{cell} [fm]
4.67×10^{11}	^{180}Zr	40	140	53.60
6.69×10^{11}	^{200}Zr	40	160	49.24
1.00×10^{12}	^{250}Zr	40	210	46.33
1.47×10^{12}	^{320}Zr	40	280	44.30
2.66×10^{12}	^{500}Zr	40	460	42.16
6.24×10^{12}	^{950}Sn	50	900	39.32
9.65×10^{12}	^{1100}Sn	50	1050	35.70
1.49×10^{13}	^{1350}Sn	50	1300	33.07
3.41×10^{13}	^{1800}Sn	50	1750	27.61
7.94×10^{13}	^{1500}Zr	40	1460	19.61
1.32×10^{14}	^{982}Ge	32	950	14.38

Chamel, Nicolas, and Pawel Haensel. "Physics of neutron star crusts." Living Reviews in relativity 11.1 (2008): 1-182.

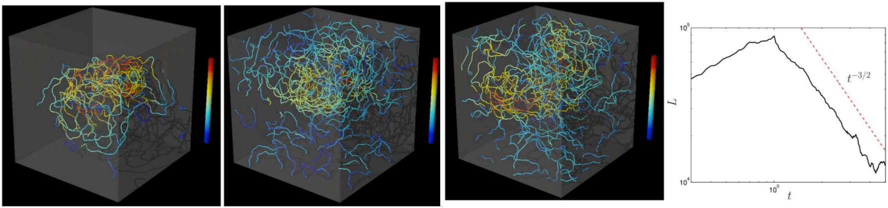


neutrons
protons
total
R
r_n
r_p
ρ_{nb}
ρ [fm^{-3}]
r [fm]
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
2
4
6
8
10
12
14



LUMI, Finland (#5 Top 500)

Piz Daint, Switzerland (#37 Top 500)



R. Hänninen, A. W. Baggaley, Proc. Nat. Acad. Sci. U.S.A. 111, 4667 (2014)

Warsaw University of Technology | W-SLDA Toolkit W-BSk Toolkit

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r})-\mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r})+\mu_b \end{pmatrix}\begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

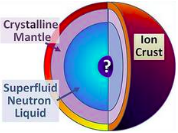
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t)-\mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t)+\mu_b \end{pmatrix}\begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

Extension to nuclear matter in neutron stars

Unified solvers for static and time-dependent problems

Dimensionalities of problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars



The W-SLDA Toolkit has been expanded to encompa available as the W-BSk Toolkit.

ALL FUNCTIONALITIES →

Integration with VisIt: visualization, animation and analysis tool

ce

W-BSk Toolkit

Getting the code

DOWNLOAD

The W-SLDA & W-BSk Toolkits are free to download. It is published as open source under GNU GPL License. In order to get W-SLDA or W-BSk Toolkit click "Read more" and follow instructions.

READ MORE →

Changelog

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» Contributing to W-SLDA

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

$$\rho(r) = \sum_k |v_k(r)|^2$$
$$\tau(r) = \sum_k |\nabla v_k(r)|^2$$
$$\nu(r) = \sum_k u_k(r) v_k^*(r)$$

$$h(r) = \frac{\delta\varepsilon}{\delta\rho} - \nabla\frac{\delta\varepsilon}{\delta\tau}\nabla - \frac{i}{2}\left\{\frac{\delta\varepsilon}{\delta\boldsymbol{j}}, \nabla\right\}$$
$$\Delta(r) = \frac{\delta\varepsilon}{\delta\nu}$$

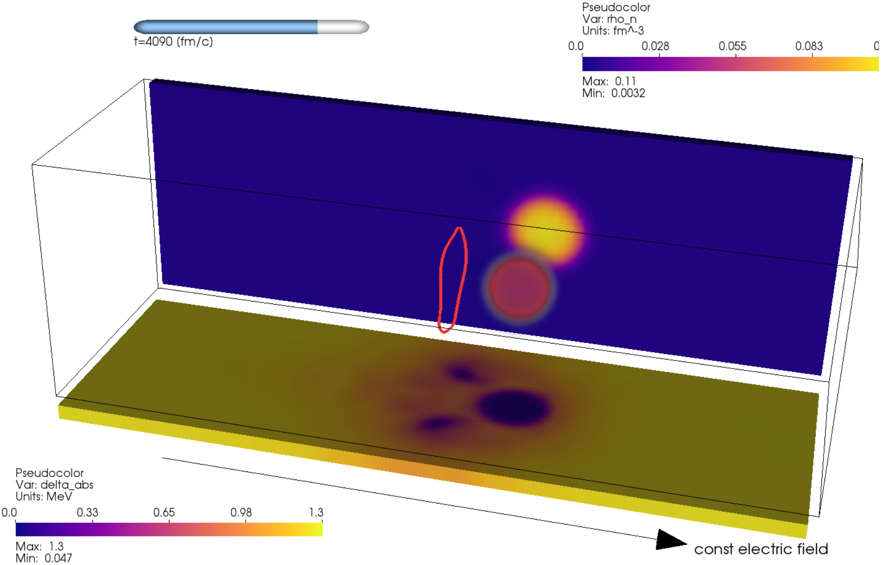
Superfluid **L**ocal **D**ensity **A**pproximation

A. Bulgac, Physical Review A **76**, 040502 (2007)

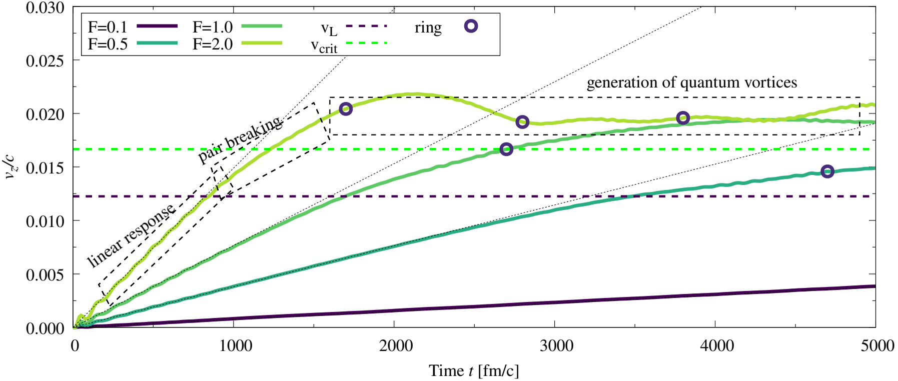
Hartree-Fock-Bogoliubov equations

$$\begin{pmatrix} h(r) & \Delta(r) \\ \Delta^*(r) & -h^*(r) \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = \epsilon_k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}$$

Quality of results highly depends on the quality of density functional!



https://www.youtube.com/watch?v=GS0yziDBZ68



$$v_z = a_z t$$

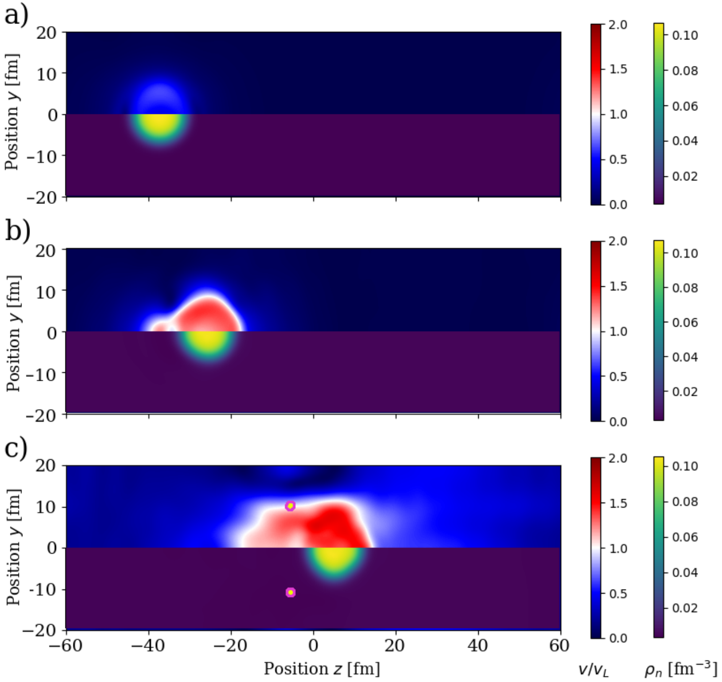
$$M_{eff}^{(d)} = \frac{F}{a_z}$$

$$v_L = \frac{\Delta}{\hbar k_F}$$

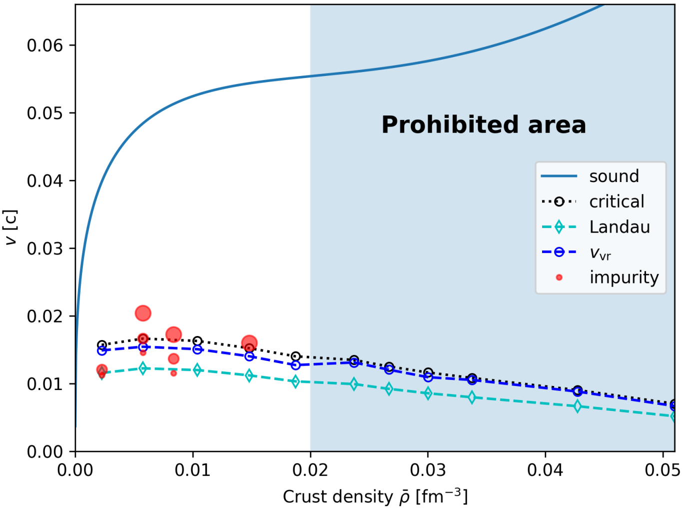
$$v_{\text{crit}} = \frac{e}{2} \frac{\Delta}{\hbar k_F}$$

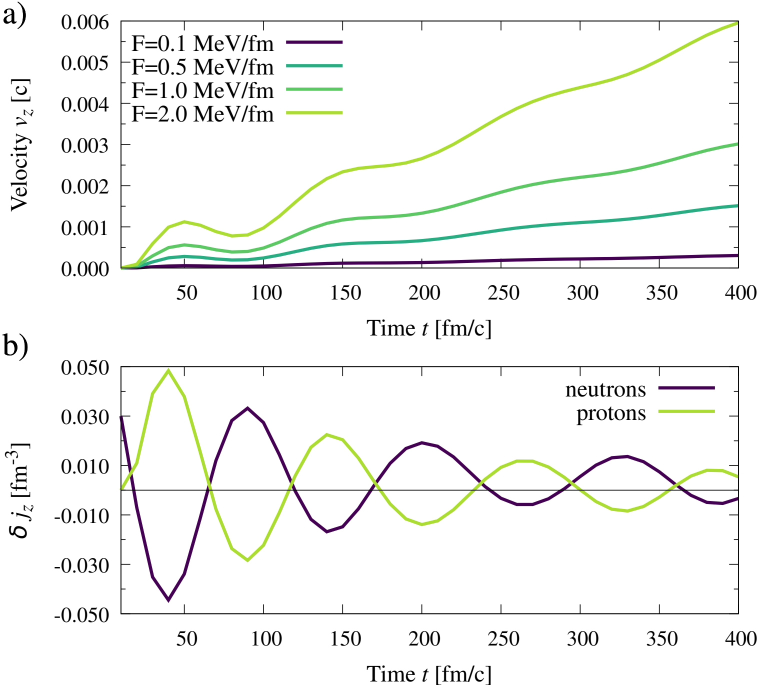
By Angelosalemi - Own work, CC BY-SA 3.0

By Robert D Anderson - Own work, CC BY-SA 3.0


a)
b)
c)
Position y [fm]
Position z [fm]
−60
−40
−20
0
20
40
60
20
10
0
−10
−20
2.0
1.5
1.0
0.5
0.0
0.10
0.08
0.06
0.04
0.02
v/v_L
ρ_n [fm^{-3}]

$$v_{vr} = \frac{1}{2\pi R}\frac{\hbar c}{M_n c^2}\left(\ln\frac{8R}{a_{core}} - \alpha\right)$$




a)
F=0.1 MeV/fm
F=0.5 MeV/fm
F=1.0 MeV/fm
F=2.0 MeV/fm
Velocity v_z [c]
0.006
0.005
0.004
0.003
0.002
0.001
0.000
50
100
150
200
250
300
350
400
Time t [fm/c]
b)
neutrons
protons
δj_z [fm^{-3}]
0.050
0.030
0.010
-0.010
-0.030
-0.050
50
100
150
200
250
300
350
400
Time t [fm/c]

Summary

- fully self-consistent 3D (TD)HFB calculations
- BSk31 Energy Density Functional
- effective parameters can be extracted
- effective mass
- dissipation channels
- creating vortex rings
- giant dipole resonance

More details

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Thank you!