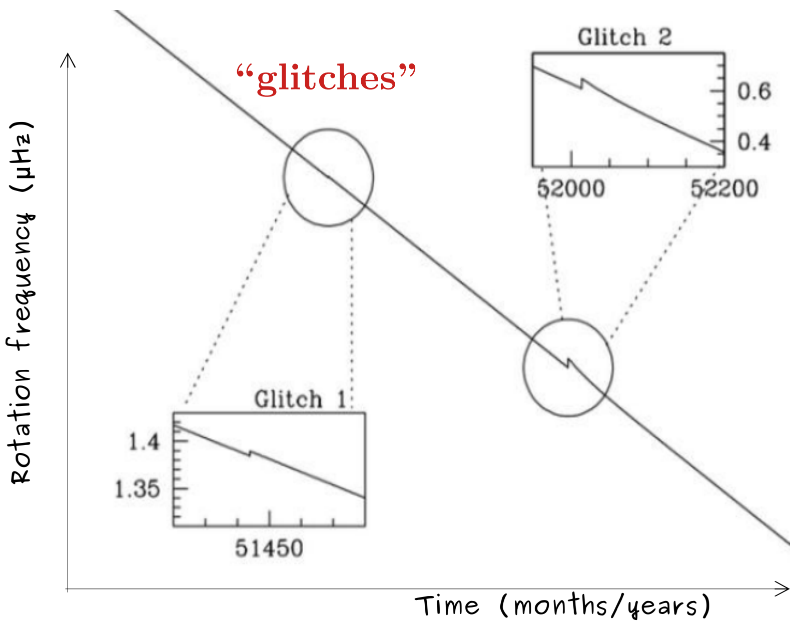
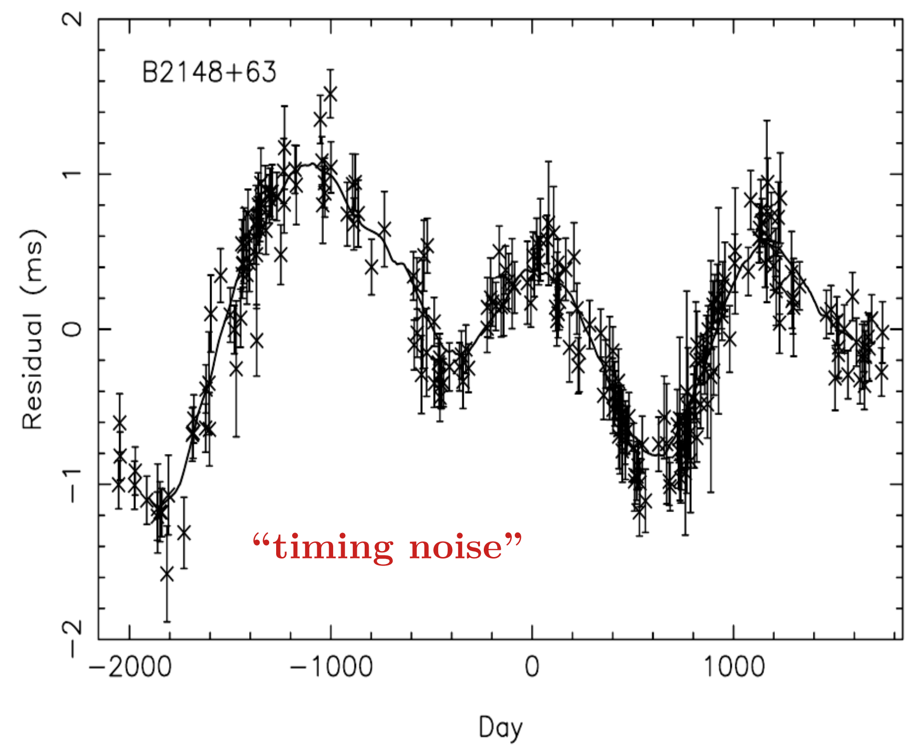
Restless Neutron Stars:
Timing Noise and Pulsar Glitches





Marco Antonelli

CNRS, LPC-Caen

antonelli@lpccaen.in2p3.fr

Wednesday, May 14, 2025 – ECT*, Trento

Pulsar glitches

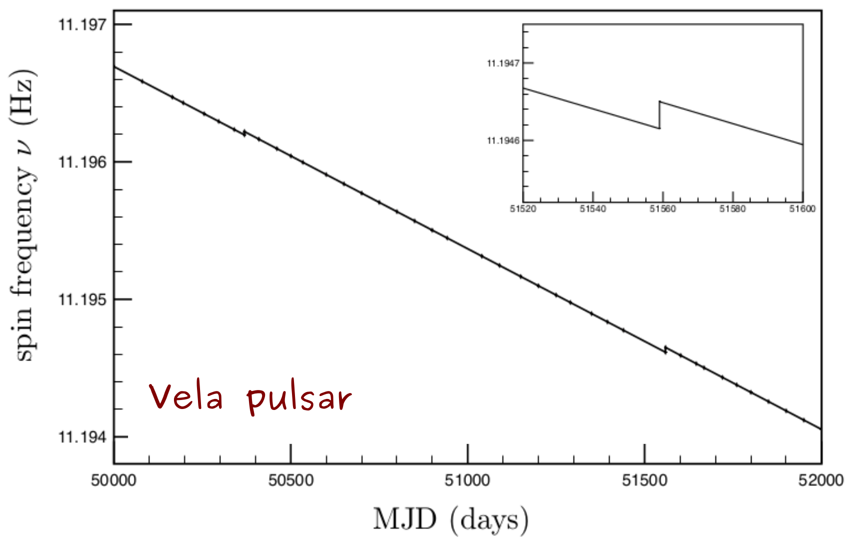
- First detected in the Vela and Crab in 1969

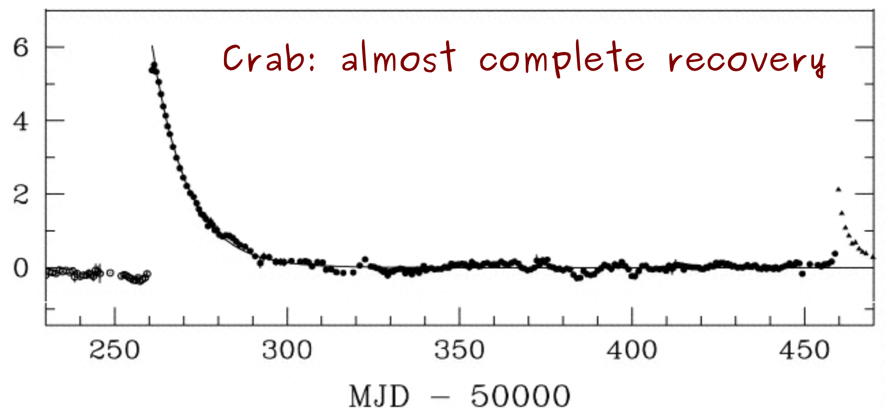
- Diverse phenomenology: probably due to different age (temperature), mass, rotational parameters, magnetic field

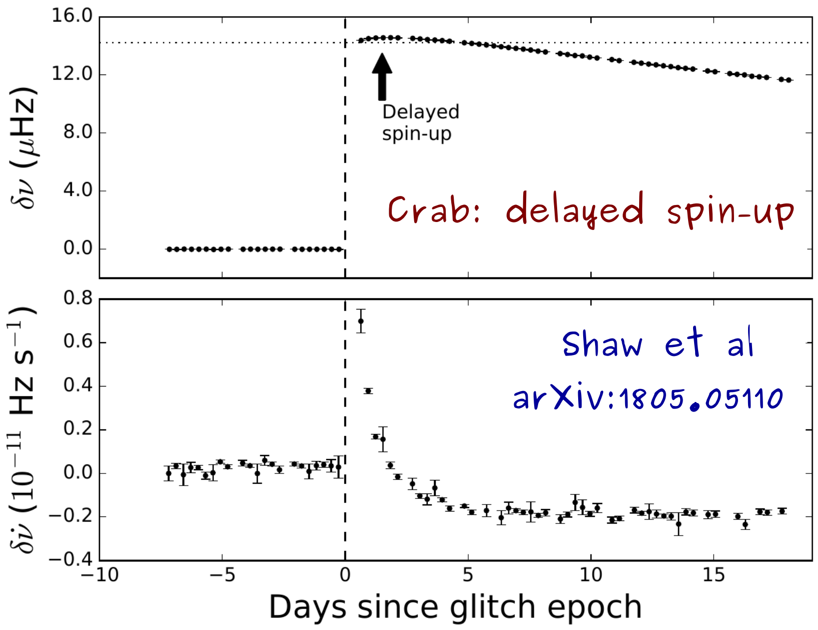
- Detected in isolated objects: conservation of the total angular momentum must be satisfied

- Typical amplitudes: $\Delta\Omega \sim 10^{-8} - 10^{-4}$ rad/s

Is there a unified explanation?
Different causes for glitch subpopulations?

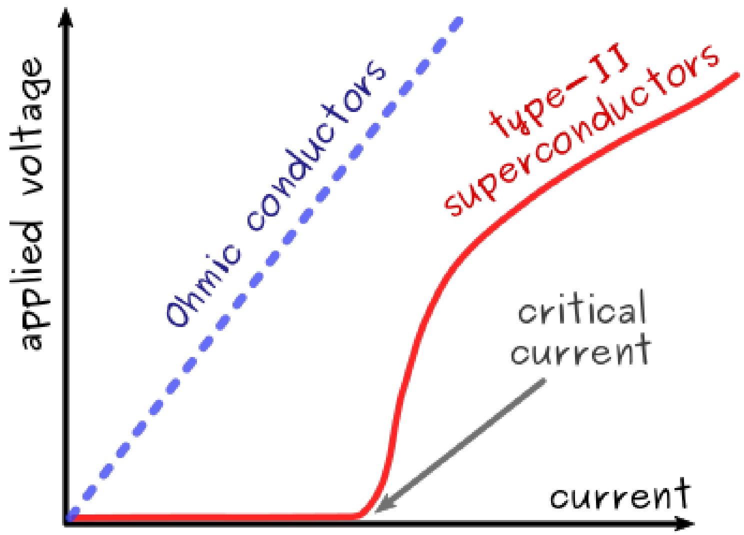


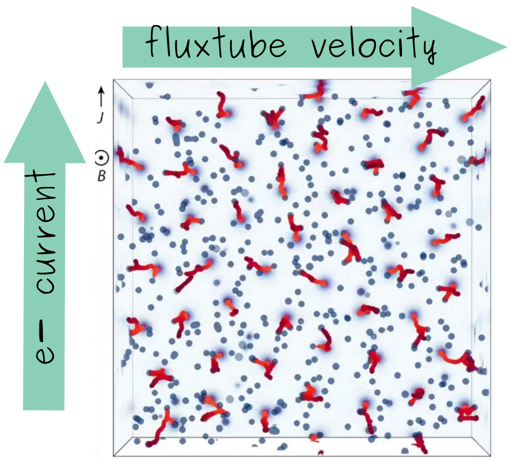


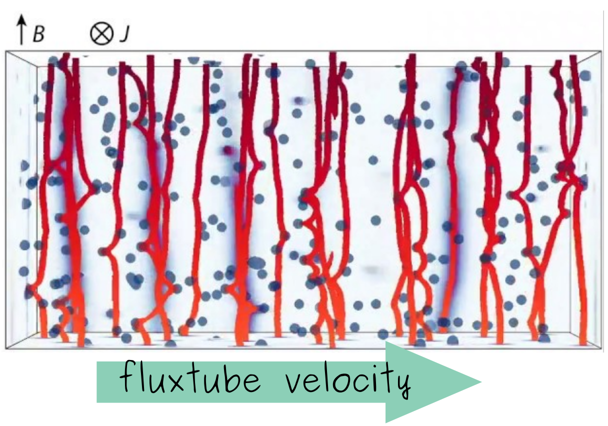


Analogy with type-II superconductors

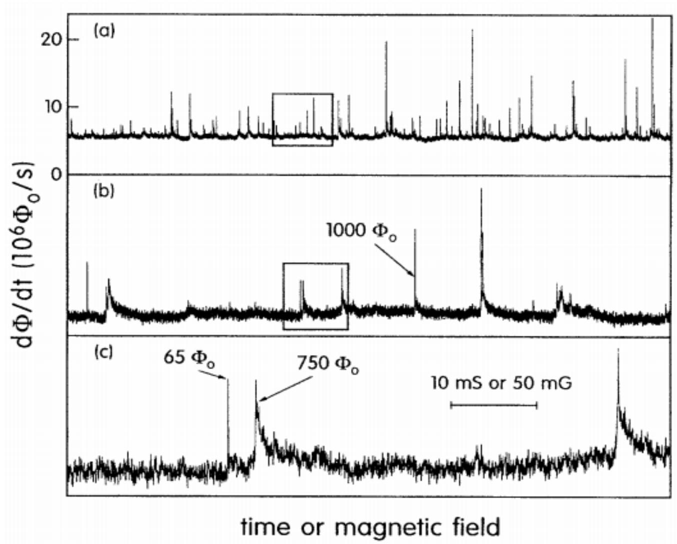
The interpretation of **glitches** as events associated with **dissipation** of **persistent currents** derives from the phenomenology of type-II superconductors







Willa et al. (2018)

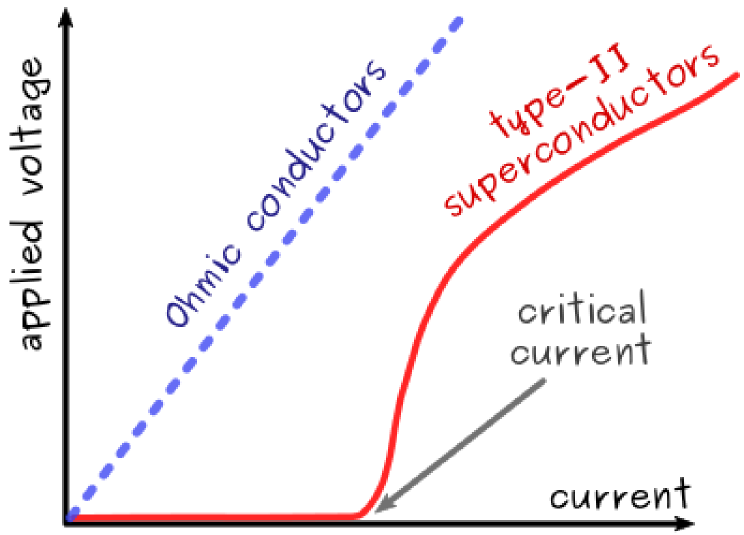


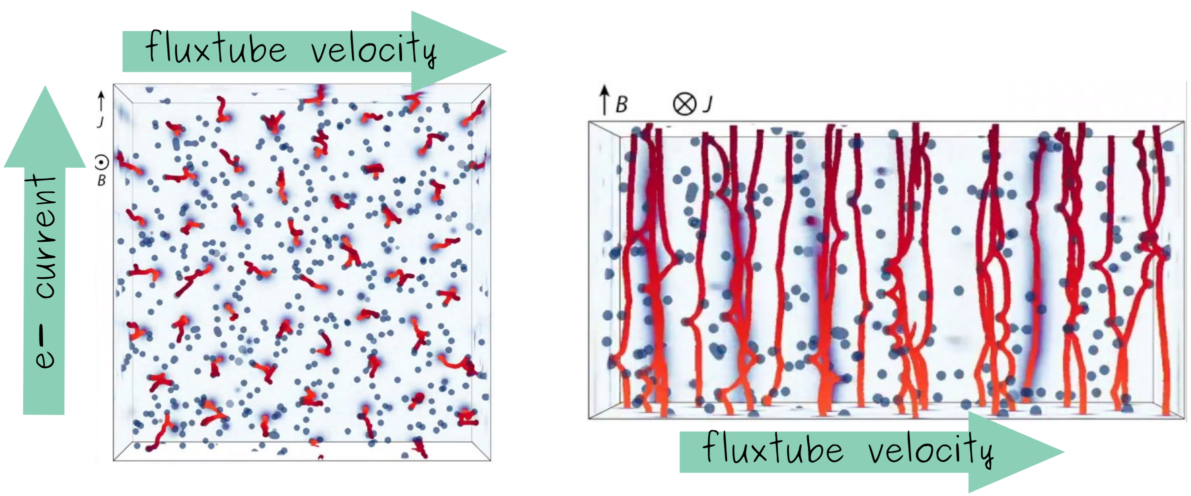
Field et al. *Superconducting Vortex Avalanches* (1995)

Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769 (2022)

Analogy with type-II superconductors

The interpretation of **glitches** as events associated with **dissipation** of **persistent currents** derives from the phenomenology of type-II superconductors





Willa et al. (2018)

Nutron Stars – Type-II Superconductor

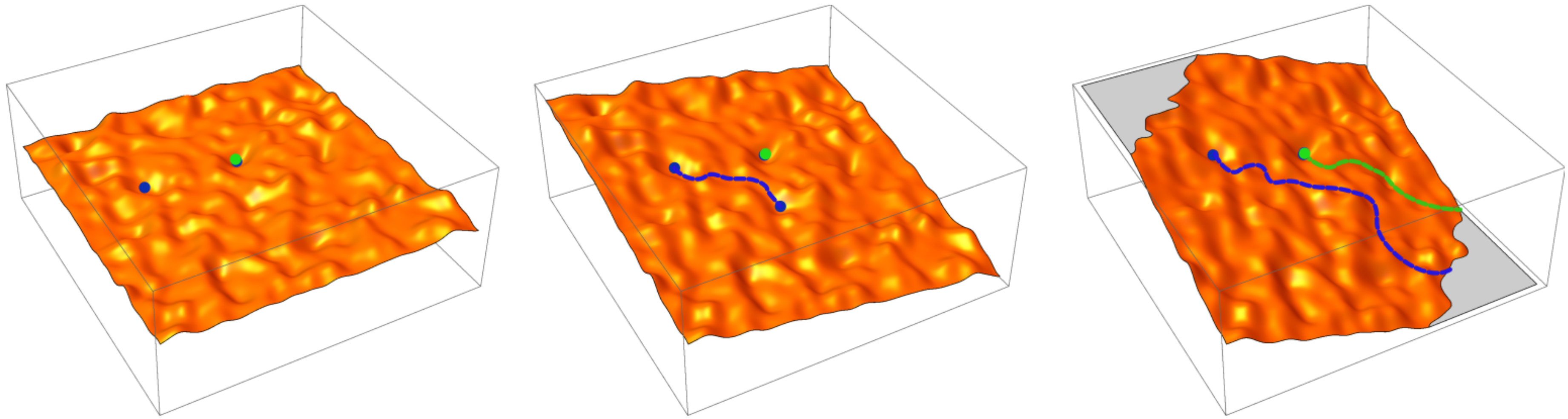
Crust's frame of reference – Frame of the lattice
Delocalized neutrons – Electron sea
Vorticity – magnetic field
Vortices – Flux-tubes
Neutron current – electric current
Magnus force – Lorentz force

Conservative regime:
Pinned vortices – pinned flux-tubes

Dissipative regime:
Moving vortices – Moving flux-tubes

Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769 (2022)

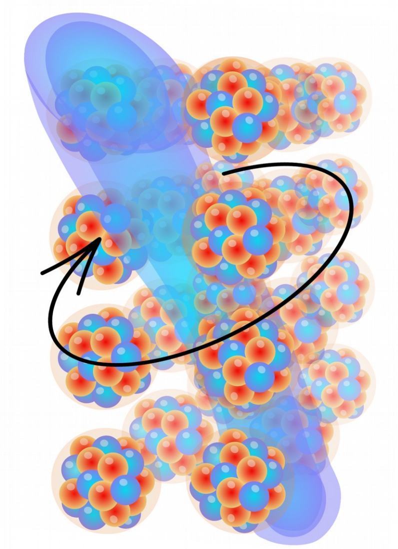
Vortex motion in a pinning landscape



Vortices in the inner-crust are immersed in a complex **pinning landscape**
→ pinned vortices: superfluid can not spin down
→ the normal component spins down
→ **persistent superfluid current** in the frame of the pinning landscape

The "lag" slowly increases in time (because of the steady spin-down)
→ the **pinning landscape** is continuously **"tilted"**
→ possible to trigger a **catastrophic unpinning** event?

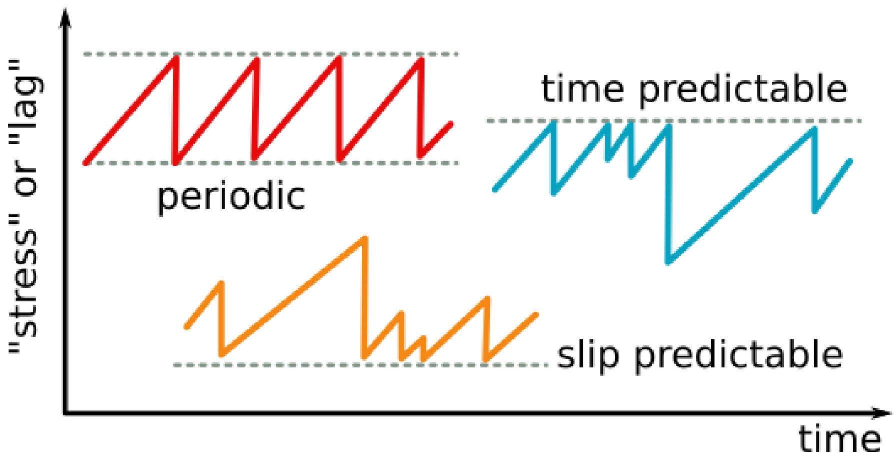
Interesting features: complex evolution with possible **avalanche** dynamics and **self-organized-criticality**

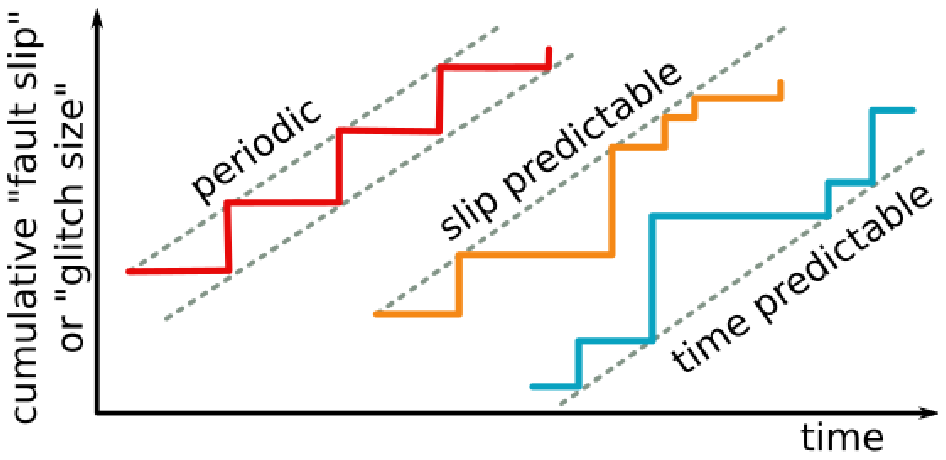


$\xi \sim$ 10–100 fm
$E_{pin} \sim$ 3–0.02 Mev
Lattice spacing: 50-10 fm

Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769 (2022)

Glitch activity: time & slip predictability





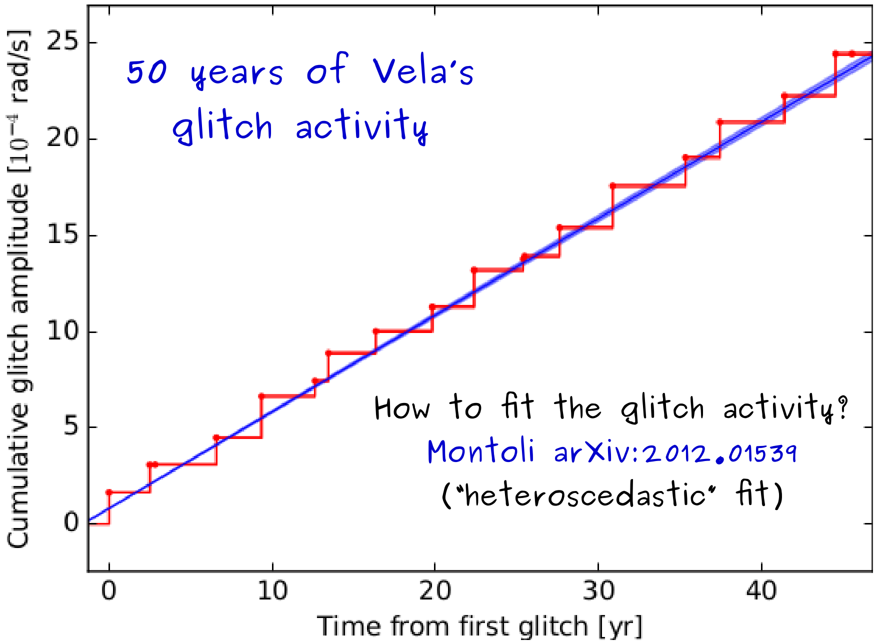
Assume a nearly ***constant driving force*** and 2 "global" thresholds:

Upper threshold: the "stress release" starts

Lower threshold: the "stress release" stops

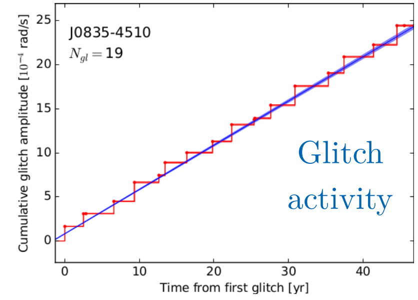
- ***Upper threshold constant***:
  observe the "slip" → predict the ***"time to the next slip"***

- ***Lower threshold constant***:
  measure time from last glitch → predict the ***"slip"***



Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769 (2022)

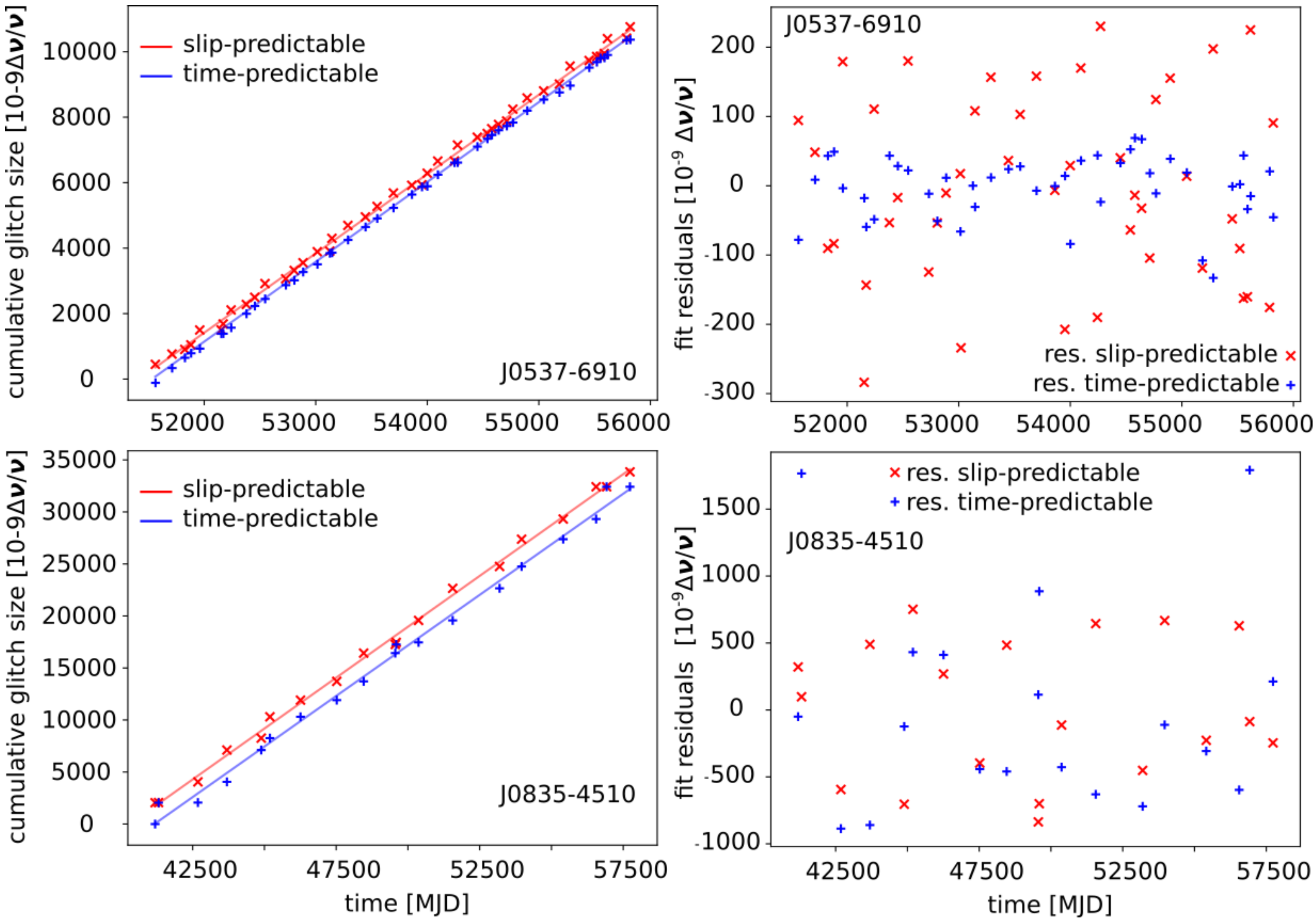
Glitch activity: time & slip predictability



There is a quite stable unpinning threshold in Vela and J0537-6910:
→ in both cases the “upper” thresholds is the most constant in time

0537-6910

some degree of preference for “time predictable”



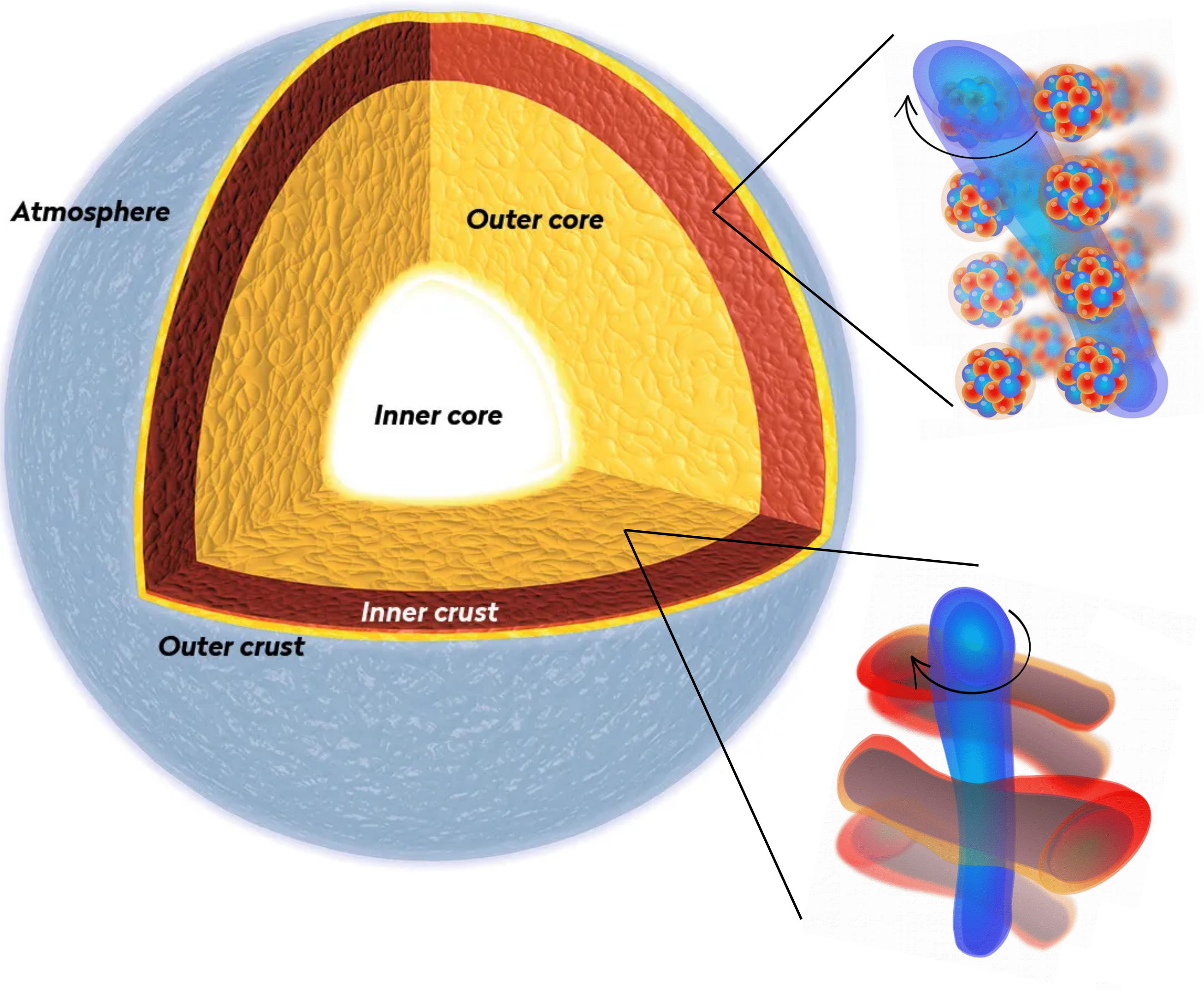
Vela pulsar

“quasi-periodic”

Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769 (2022)

Continuum description?

Not a simple fluid but many layers with different inhomogeneities, defects, currents



Inner crust

(Visco-)elastic lattice
Neutron "scalar" superfluid
Excitations (entropy/heat)
Ideal gas of electrons
B field
Frame of the lattice: 3 currents

Outer core

Neutron superfluid
Proton superconductor
Excitations (entropy/heat)
Ideal gas of electrons
B field in fluxtubes (type II?)

Different phenomena may be well described with a smaller number of "fluids"

e.g. cooling, glitches, oscillation modes, "mountains", B evolution...

...it is however true that the "mageto-thermo-rotational" evolution is coupled

Superfluid hydrodynamics

Tree-forest analogy: Barenghi's talk @INT, Seattle 2018

Vortex core scale:
~ 10 fm in a NS

Microscopic models needed:

Helium: stochastic GPE, mean field…

NS: effective classical fields (Ginzburg-Landau+GPE), TDLDA, HFB…

Conserved baryon charge:

Baryons ~ $n_0\,(10\ \mathrm{fm})^3 \sim 10^3$

Inter-vortex scale:
~ 10^{-3} cm in a NS

Mescoscopic models needed:

Helium: Vortex filament model
(K. Schwarz's 80s papers)

NS: the same but needs extension to non-homogeneous environment

Conserved baryon charge:

Baryons ~ $n_0\,(10^{-3}\ \mathrm{cm})^3 \sim 10^{29}$

Fluid element:
from ~*mm* in a NS

Macroscopic hydro:

Helium: HVBK hydro
(Hall & Vinen 1956)

NS: extensions/decorations of HVBK hydro (more species, GR)

Conserved baryon charge:
Not so relevant, need to include many vortices in the fluid element

Superfluid hydrodynamics

Tree-forest analogy: C. Barenghi talk @INT, Seattle 2018

Vortex core scale:
$\sim$ 10 fm in a NS

Inter-vortex scale:
$\sim 10^{-3}$ cm in a NS

Fluid element:
from $\sim mm$ in a NS

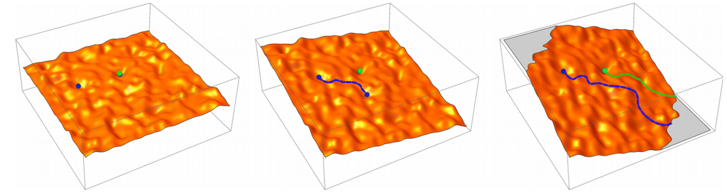
We can not take into account each vortex ($\sim 10^{16}$ in a pulsar) $\rightarrow$ HVBK-like hydrodynamics

2 Euler-like equations + **entrainment** + **mutual friction**

$$\partial_t \rho_{\mathrm{x}} + \nabla_i (\rho_{\mathrm{x}} v^i_{\mathrm{x}}) = 0$$

$$(\partial_t + v^j_{\mathrm{x}} \nabla_j)(v^{\mathrm{x}}_i + \varepsilon_{\mathrm{x}} w^{\mathrm{yx}}_i) + \nabla_i (\tilde{\mu}_{\mathrm{x}} + \Phi) + \varepsilon_{\mathrm{x}} w^j_{\mathrm{yx}} \nabla_i v^{\mathrm{x}}_j = f^{\mathrm{x}}_i / \rho_{\mathrm{x}}$$

The **dynamics of vortices** in a fluid element gives the form and strength of the macroscopic **"mutual friction"**



x,y=n $\rightarrow$ superfluid neutrons $\quad$ x,y=p $\rightarrow$ normal component (electrons, excitations, protons, lattice...)

Superfluid hydrodynamics

Tree-forest analogy: C. Barenghi talk @INT, Seattle 2018

Vortex core scale:
~ 10 fm in a NS

Inter-vortex scale:
~ 10^{-3} cm in a NS

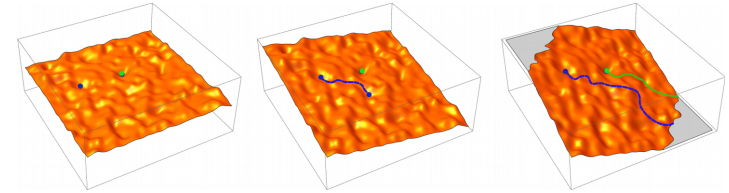
Fluid element:
from ~*mm* in a NS

We can not take into account each vortex (~10^{16} in a pulsar) → HVBK-like hydrodynamics

2 Euler-like equations + **entrainment** + **mutual friction**

$$\rho_n D_t \mathbf{v}_n + \ldots = \mathbf{F}_{MF}$$
$$\rho_p D_t \mathbf{v}_p + \ldots = -\mathbf{F}_{MF}$$

The **dynamics of vortices** in a fluid element gives the form and strength of the macroscopic **"mutual friction"**



$x,y=n$ → superfluid neutrons $x,y=p$ → normal component (electrons, excitations, protons, lattice...)

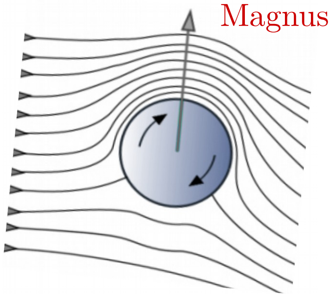
Vortex motion → HVBK friction (no pinning)

The mutual friction follows by balancing the Magnus force that acts on vortices with a resistive force:

Magnus force Drag

$$\mathbf{f}_M + \mathbf{f}_D = 0 \qquad \hat{\boldsymbol{\kappa}} \times (\mathbf{v}_L - \mathbf{v}_n) - \mathcal{R}(\mathbf{v}_L - \mathbf{v}_p) = 0$$

$$\mathbf{v}_L = -\frac{1}{1+\mathcal{R}^2}\hat{\boldsymbol{\kappa}} \times (\boldsymbol{\kappa} \times \mathbf{v}_{np}) - \frac{\mathcal{R}}{1+\mathcal{R}^2}\boldsymbol{\kappa} \times \mathbf{v}_{np}$$



Crust: drag force due to excitations of lattice phonons (Jones 1990) and Kelvin waves (Epstein & Baym 1992, Jones 1992, Graber+ 2018), $\mathcal{R}$=10^{-5} – 10^{-1}

Core: drag force due to the scattering of electrons off of the magnetic field with which each vortex core is endowed (Alpar+ 1984, Mendell 1991, Andersson+ 2006), $\mathcal{R}$=10^{-3} – 10^{-4}

$$\boldsymbol{F}_n = -\kappa\, n_v\, \hat{\boldsymbol{\kappa}} \times (\langle \dot{\boldsymbol{x}} \rangle - \boldsymbol{v}_{np}) = \rho_n n_v \left[\frac{\mathcal{R}}{1+\mathcal{R}^2}\hat{\boldsymbol{\kappa}} \times (\boldsymbol{\kappa} \times \mathbf{v}_{np}) + \frac{\mathcal{R}^2}{1+\mathcal{R}^2}\boldsymbol{\kappa} \times \mathbf{v}_{np}\right]$$

Message: how to extract mutual friction?

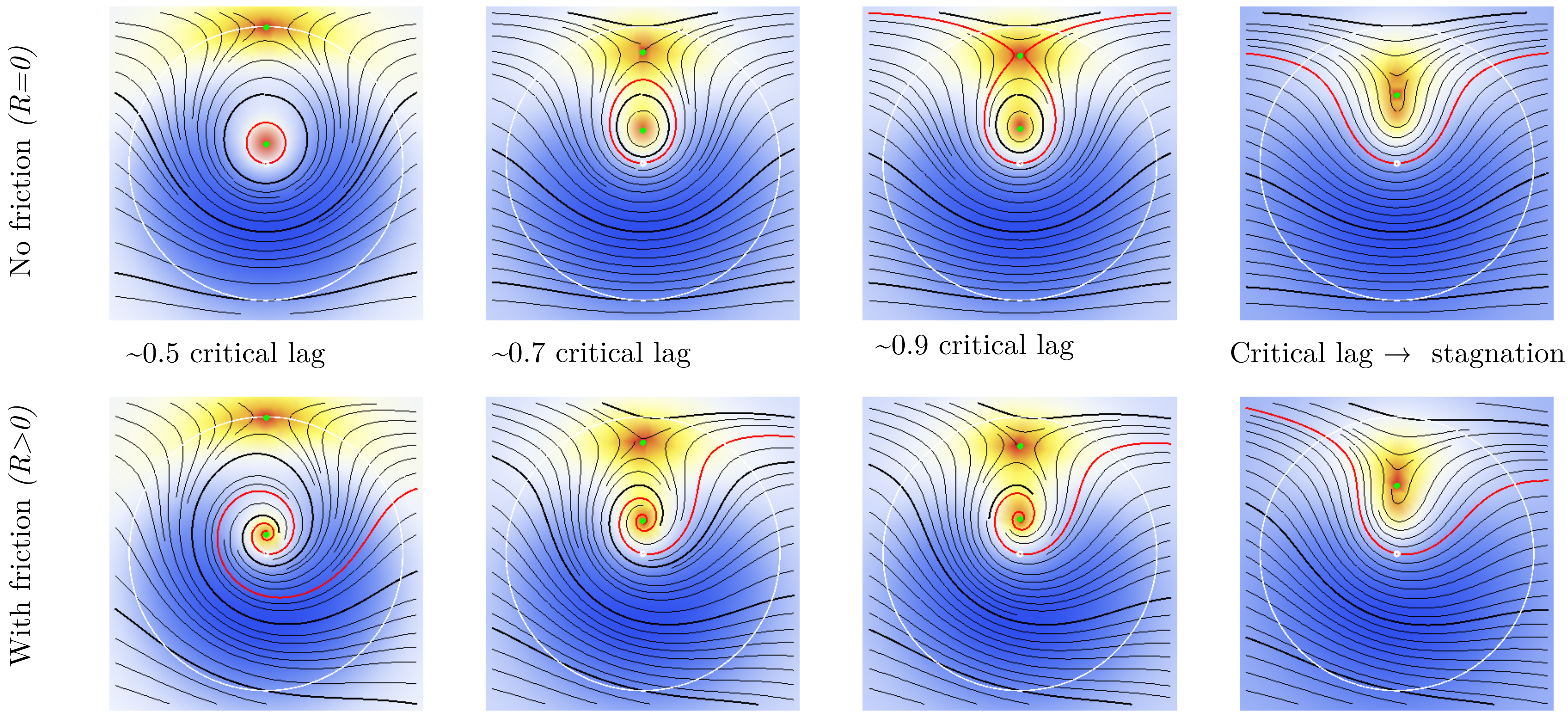
- Solve for the vortex dynamics for a given "lag"
- Take an average over many vortices
  - → Standard HVBK: simple algebraic EOM
  - → With pinning: simulations needed

Point vortex orbits in a “repulsive” potential

$$\hat{\boldsymbol{\kappa}} \times (\dot{\boldsymbol{x}}(t) - \boldsymbol{v}_{np}) - \mathcal{R}\,\dot{\boldsymbol{x}}(t) + \boldsymbol{f} \;=\; 0$$

Just to build some intuition: streamlines for a “repulsive” (positive) potential

$$\dot{\boldsymbol{x}} \;=\; \cos\theta_d \, R_{\theta_d}^{-1} \left(\boldsymbol{v}_{np} + R_{\pi/2}\boldsymbol{f}\right) \qquad \boldsymbol{f} \;=\; -\nabla\Phi \qquad \Phi(\boldsymbol{x}) = \Phi_0 \; e^{-|\boldsymbol{x}-\boldsymbol{r}_a|^2/2\sigma^2}$$

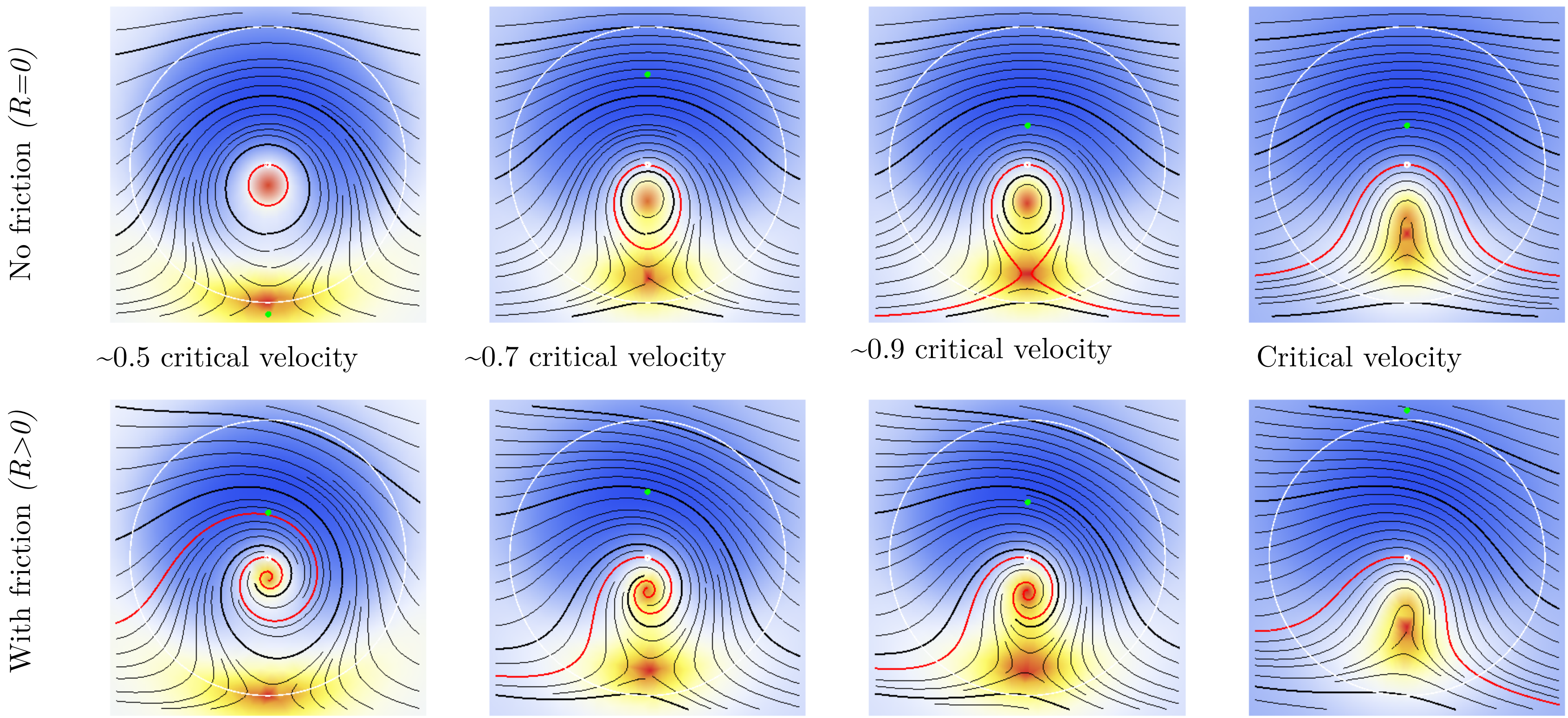


Point vortex orbits in an “attractive” potential

$$\hat{\boldsymbol{\kappa}} \times (\dot{\boldsymbol{x}}(t) - \boldsymbol{v}_{np}) - \mathcal{R}\,\dot{\boldsymbol{x}}(t) + \boldsymbol{f} \;=\; 0$$

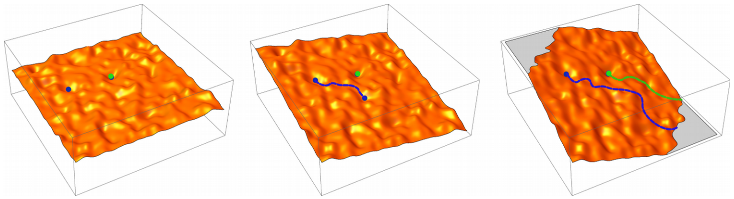
Just to build some intuition: streamlines for an “attractive” (negative) potential

$$\dot{\boldsymbol{x}} \;=\; \cos\theta_d\, R_{\theta_d}^{-1}\,(\boldsymbol{v}_{np} + R_{\pi/2}\boldsymbol{f}) \qquad \boldsymbol{f} \;=\; -\nabla\Phi \qquad \Phi(\boldsymbol{x}) = \Phi_0\; e^{-|\boldsymbol{x}-\boldsymbol{r}_a|^2/2\sigma^2}$$



Vortex motion → Mutual friction (with pinning)

Antonelli & Haskell, arXiv:2007.11720 (2020)



It's a "kinetic approach" but with point vortices instead of particles

- Fix a background "lag" (background current of superfluid neutrons)
- Assign random position of a vortex in the pinning landscape and solve the trajectory

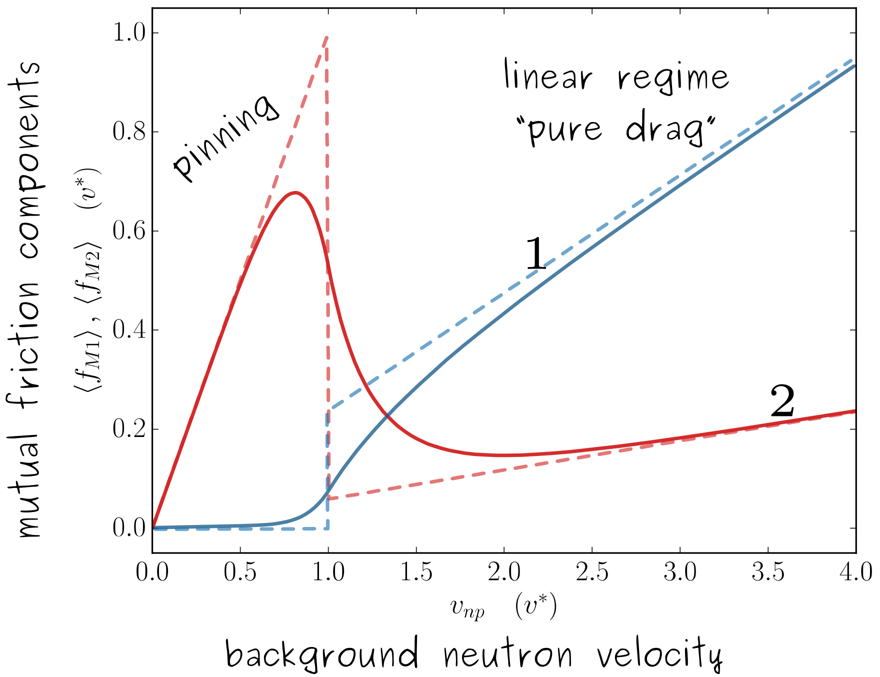
disordered pinning force field

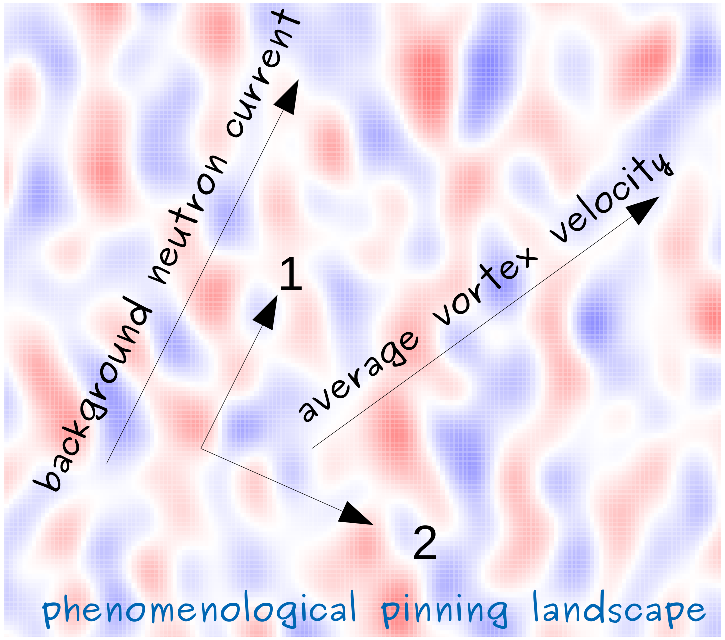
$$\hat{\boldsymbol{\kappa}} \times (\dot{\boldsymbol{x}}(t) - \boldsymbol{v}_{np}) - \mathcal{R}\,\dot{\boldsymbol{x}}(t) + \boldsymbol{f} = 0$$

- Repeat many times and find the average vortex velocity for the given "lag"
- The mutual friction is given by $$\boldsymbol{F}_n = -\kappa\, n_v\, \hat{\boldsymbol{\kappa}} \times (\langle \dot{\boldsymbol{x}} \rangle - \boldsymbol{v}_{np})$$

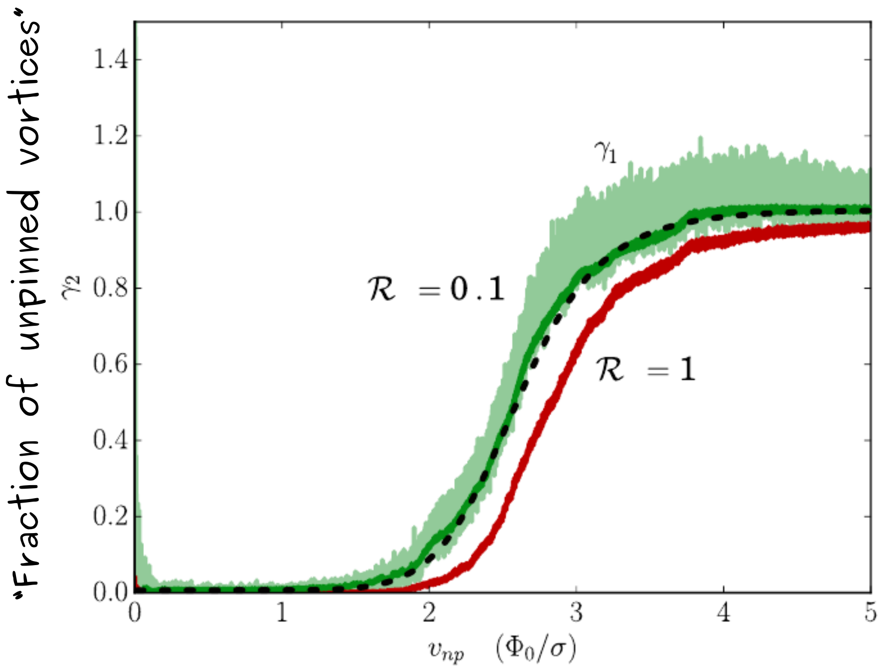
average vortex velocity

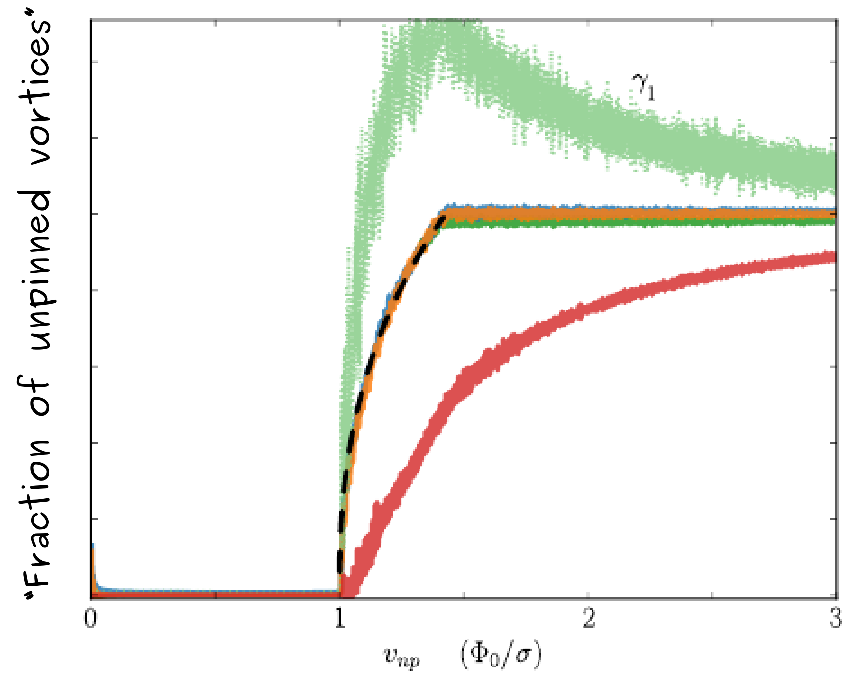
assigned "lag"

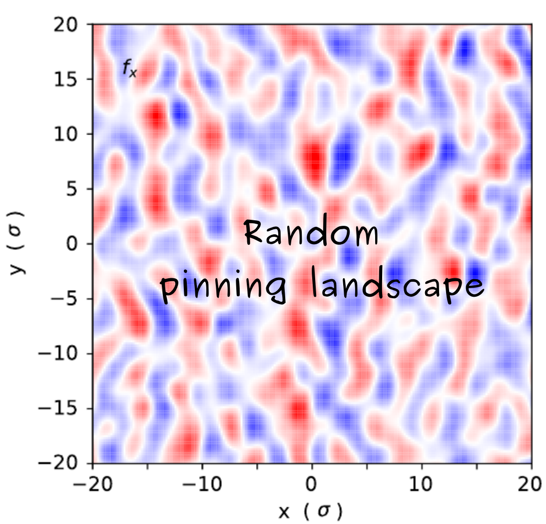


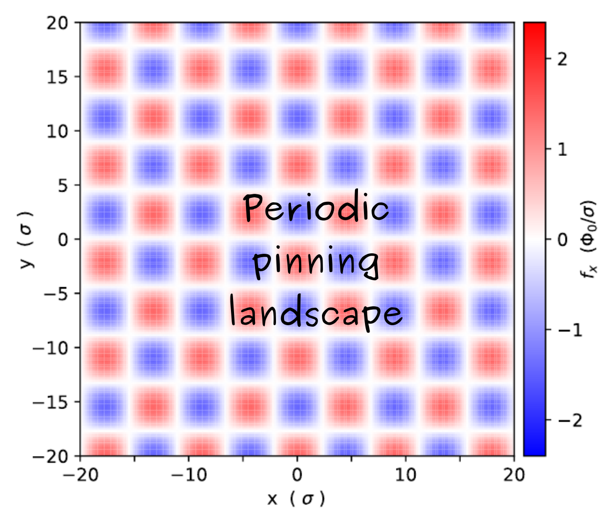


Vortex simulations: effect of disorder







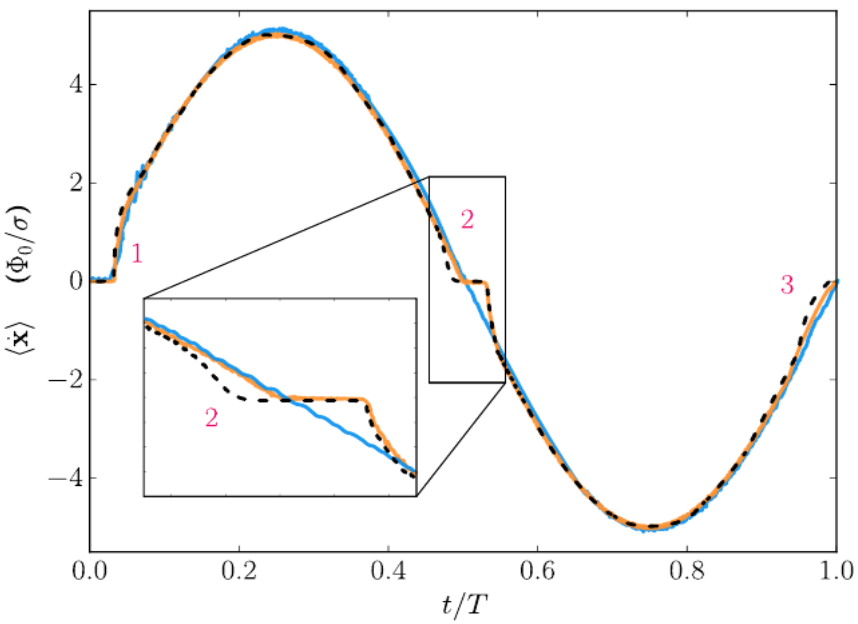


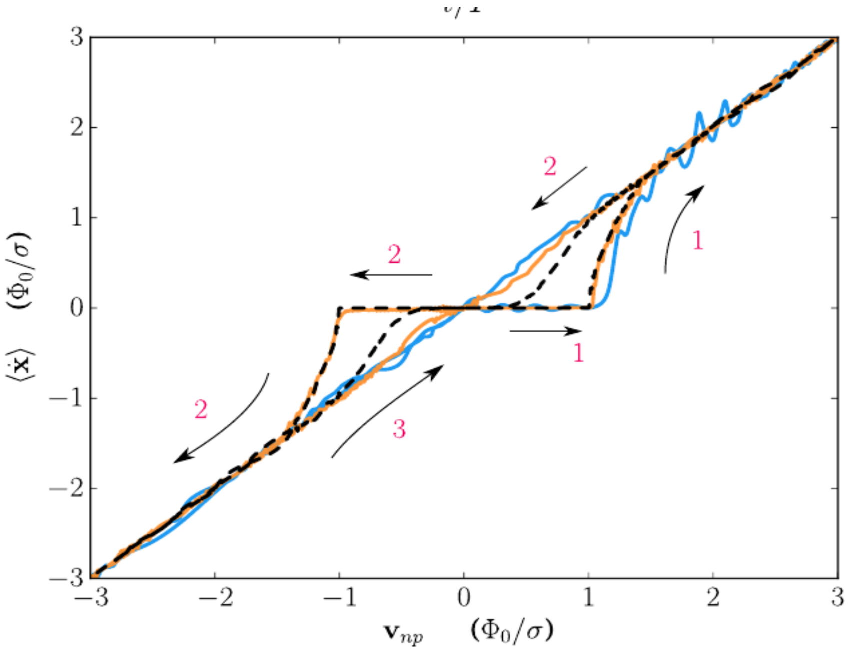
Beyond hydrodynamics: hysteresis

Rate-dependent hysteresis: lag between an input and an output that disappears if the input is varied more slowly. If the input is reduced to zero, the output continues to respond for a finite time.

Instantaneous drop to null lag → vortex velocity drops to zero immediately (if NO pinning forces)
Instantaneous drop to null lag → vortex velocity relaxes to zero (with pinning forces)

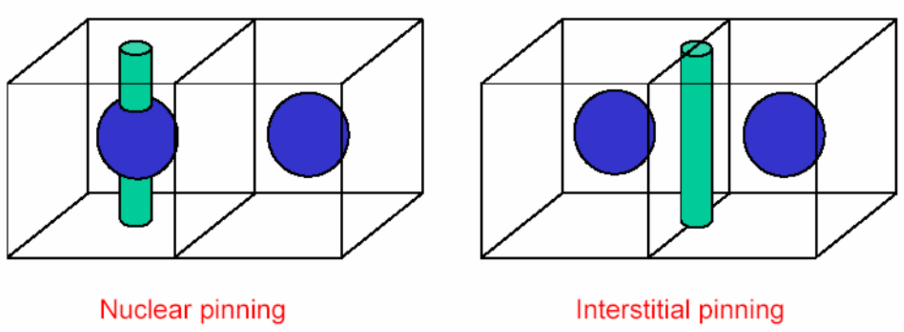
Rate-independent hysteresis → vortex-vortex interactions



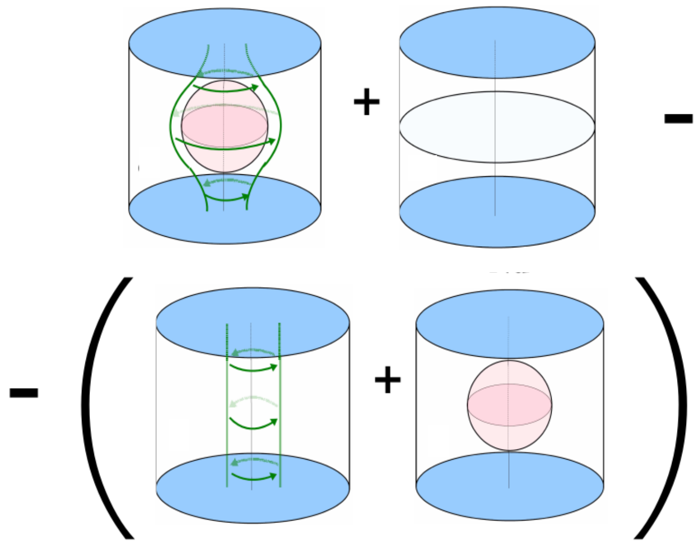


MA & Haskell 2020 arXiv:2007.11720

Pinning energy (vortex – single nucleus)



Donati & Pizzochero, Phys Lett B, 640 (2006)
Semiclassical approach: static LDA calculation (local Fermi momentum is a function of the neutron number density)



Klausner et al, arXiv:2303.18151 (2023)
Hartree-Fock-Bogolyubov

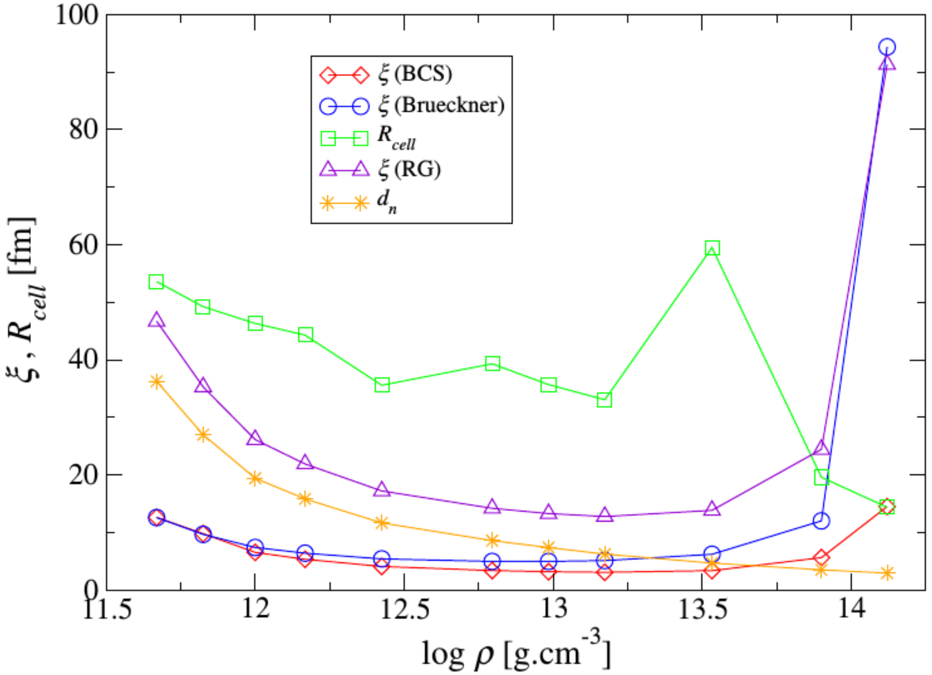
Energy contributions to pinning:
→ negative condensation energy of the order of Δ^2 / E_F
→ kinetic energy of the irrotational vortex-induced flow
→ Fermi energy E_F of neutrons
→ nuclear cluster energy (Woods-Saxon potential)

Uncertain pairing gap Δ: modifies the strength and location of the pinning energies (in-medium effects!)
Maximum pinning energies < 3.5 MeV
Significant pinning occurs only in a restricted range: $0.07\ n_0 < n_B < 0.2\ n_0$

Improvements: TDLDA, Wlazłowski+ (2016), Hartree-Fock-Bogolyubov Klausner+ (2023)

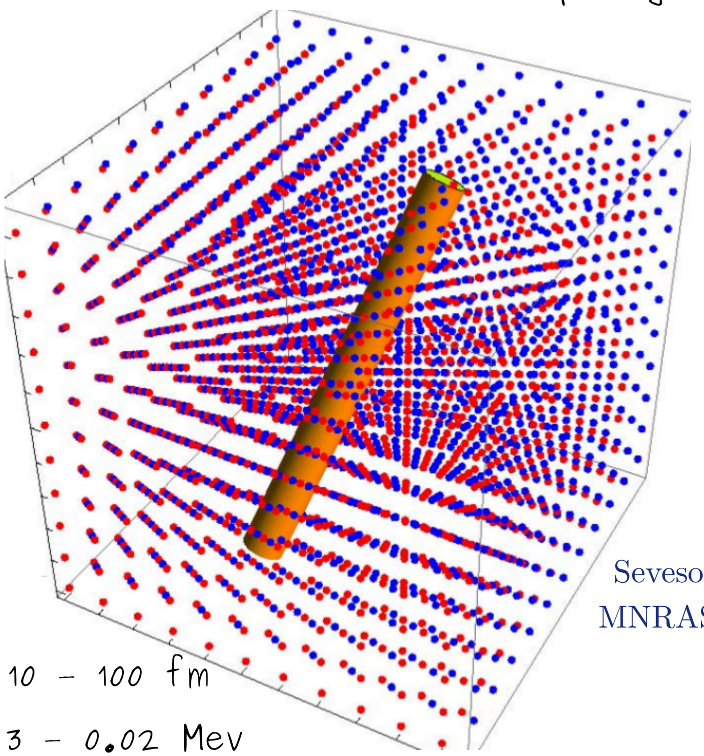
Pinning forces (inner crust)

Chamel & Haensel, Living Rev.Rel. 11 (2008)



Lattice spacing: 50-10 fm

Lattice spacing: 50—10 fm



Seveso et al MNRAS 2016

ξ = 10 – 100 fm

E_{pin} = 3 – 0.02 Mev

Coherence length ξ ~ vortex core radius

Strong pinning when ξ < lattice spacing

Rigid (straight) vortices are "less pinned"

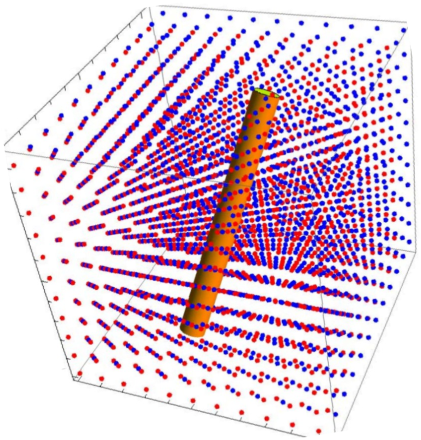
Inner crust:

Problem: how to calculate the "vortex-lattice" interaction from the "vortex-nucleus" interaction ?

IDEA: consider a segment of vortex line (the length L is given by the tension) and average over translations and rotations of the total pinning force divided by L

Coherence length ξ estimates: Mendell, ApJ 38 1991

Stationary test (glitch amplitude)

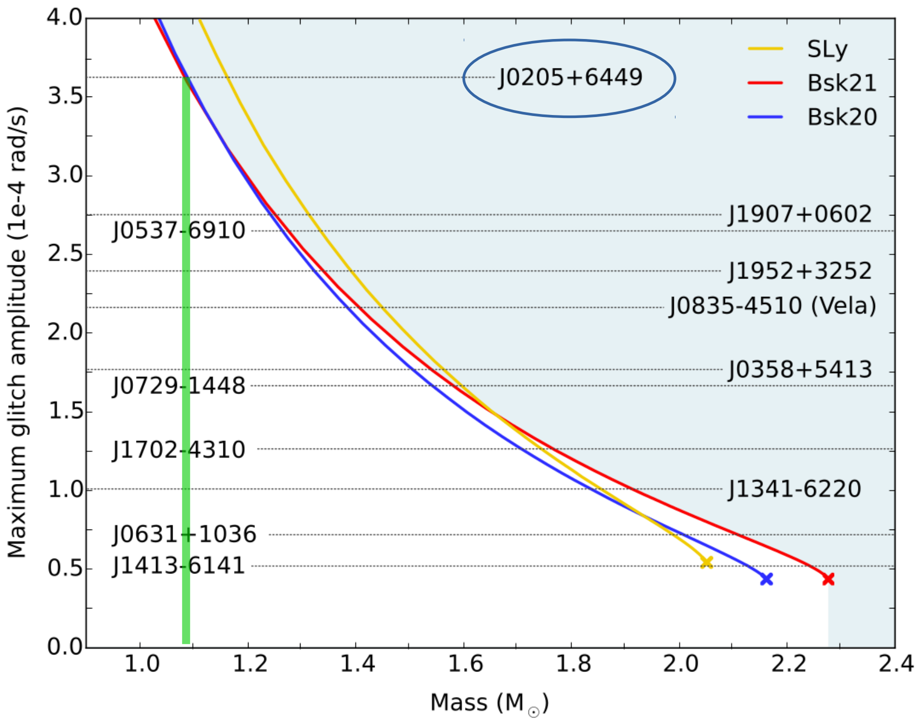


We can **test** theoretical single-vortex pinning forces with glitches of large amplitude:

- Consider the **critical current** everywhere in the crust and its associated angular momentum
- Solve the hydrostatic equilibrium in slow rotation and calculate:

$$\Delta\Omega_{\rm abs} = \frac{\pi^2}{I\kappa}\int_0^{R_d} dr\, r^3\, e^{\Lambda(r)}\, \frac{\mathcal{E}(r)+P(r)}{m_n\, n_B(r)\, c^2}\, f_P(r)$$

$\rightarrow$ Theoretical **upper bound** based only on angular momentum balance
$\rightarrow$ No superfluid fraction and no entrainment
$\rightarrow$ No need to solve the internal dynamics
$\rightarrow$ No dependence on the assumed vortex configuration
$\rightarrow$ Pinning forces are just a "proxy" for the critical current



Compare with bounds on the **minimum** mass:

Observed: M=1.174 M⊙ Martinez+2015 arXiv:1509.08805

CCS simulations: M≈1.15 M⊙ Suwa+2017 arXiv:1808.02328

Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769 (2022)

Vela's 2016 glitch

Palfreyman et al. Nature 556, 2018

→ Glitch detected "in the act"

→ TOA of single pulses

→ Arrival time residual: tells us if the pulse arrives before or after the expected arrival time predicted by a spin-down model.

What can we learn from this observation?

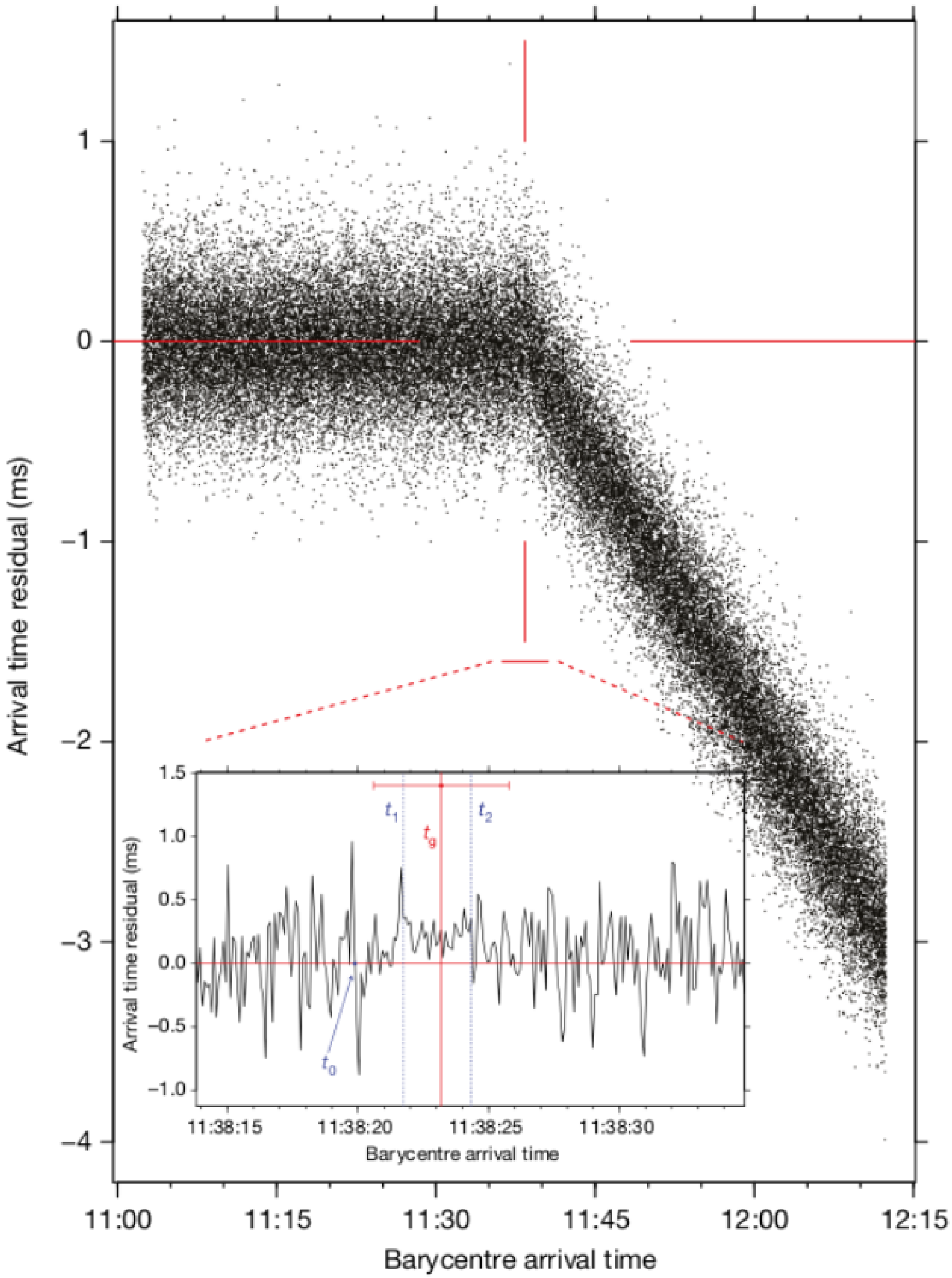
Bayesian fit: spin-up timescale of ~10s

Ashton+2019 arXiv:1907.01124

Montoli+2020 arXiv:2005.01594

Test of the friction parameters

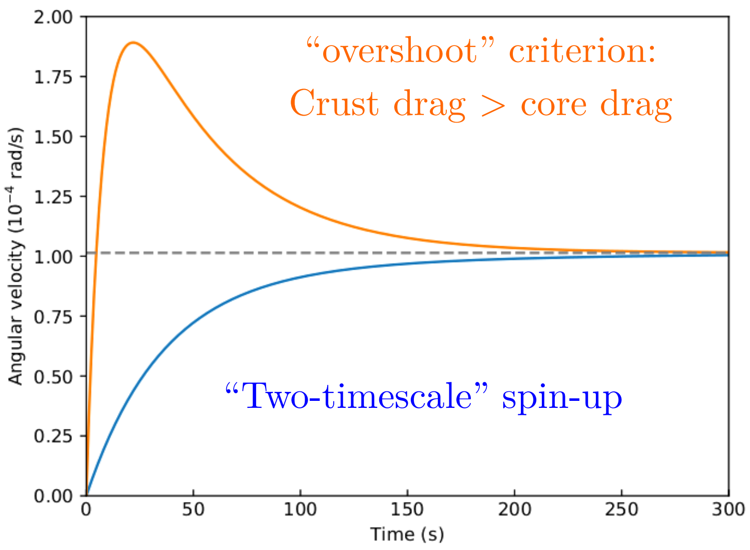
Graber+2018 arXiv:1804.02706



Minimal model for the "spin-up"

Model with **3 components**:

- **Minimal model** to go beyond pure exponential relaxation
- Natural idea because there is superfluid in the **crust** but also in the **core**
- Allows us to resolve a **possible overshoot** during the spin-up



Equations of motion: "**physical**" parameters

$$\dot{\Omega}_p = -\frac{1}{x_p}\left(x_1\dot{\Omega}_1 + x_2\dot{\Omega}_2 + |\dot{\Omega}_\infty|\right)$$

$$\dot{\Omega}_1 = -b_1\left(\Omega_1 - \Omega_p\right)$$

$$\dot{\Omega}_2 = -b_2\left(\Omega_2 - \Omega_p\right)$$

It is possible to solve analytically the system, in order to obtain the angular velocity of the normal component with respect to the spin down of the star.

$$\Delta\Omega_p(t) \;=\; \Delta\Omega_p^\infty\left[1 - \omega\, e^{-t\lambda_+} - (1-\omega)\, e^{-t\lambda_-}\right]$$

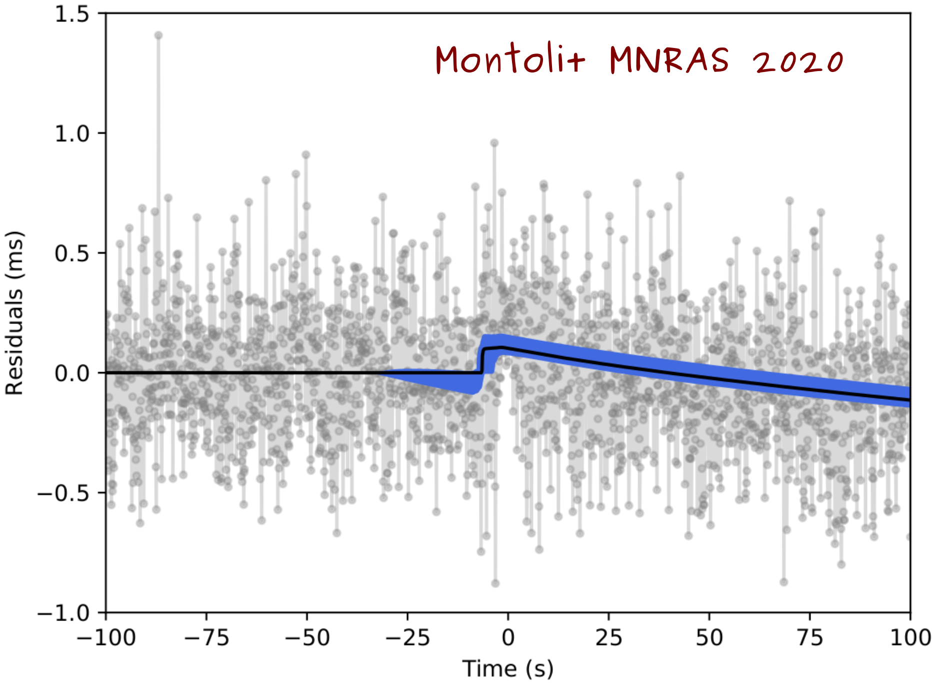
General exact solution: "**agnostic**" parameters

Montoli+2020 arXiv:2005.01594

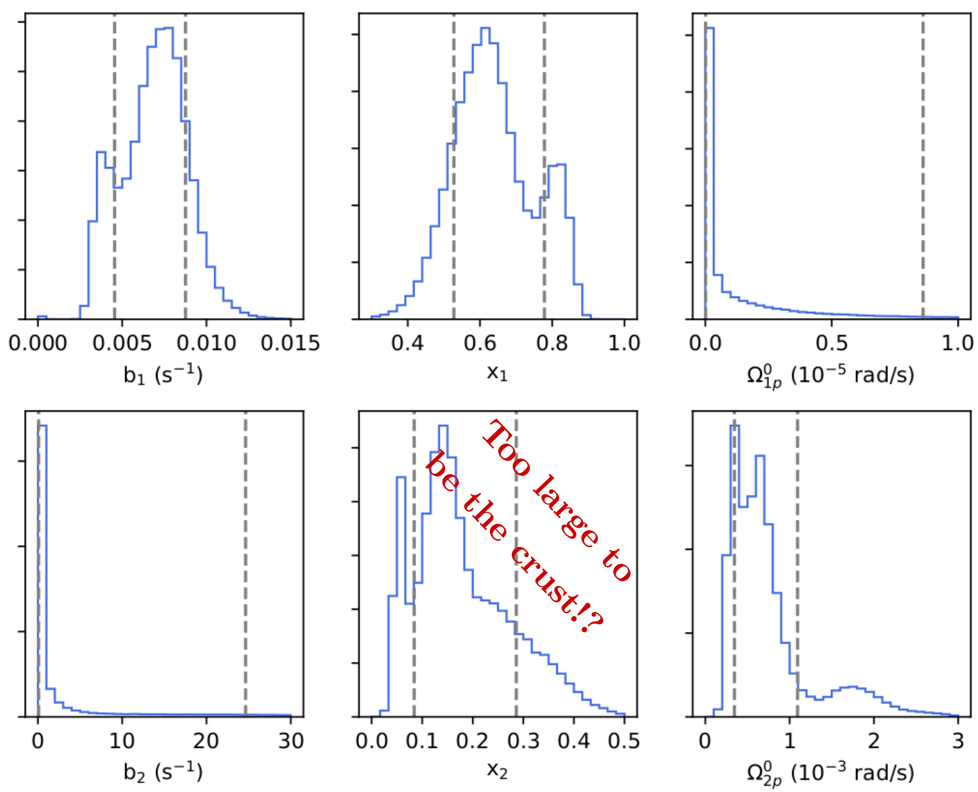
Bayesian fit of Vela 2016

$$x_p \dot{\Omega}_p = -\mathcal{T}_1 - \mathcal{T}_2 - |\dot{\Omega}_\infty|$$
$$x_1 \dot{\Omega}_1 = \mathcal{T}_1$$
$$x_2 \dot{\Omega}_2 = \mathcal{T}_2$$

$$\mathcal{T}_i = -x_i b_i (\Omega_i - \Omega_p) \qquad i = 1, 2$$



Posteriors for the "**physical**" parameters



Fit of the TOA residuals of Palfreyman+2018:

→ Estimated **moment of inertia fractions**:

"active" $x_2 \sim 0.1 - 0.3$, "passive" $x_1 \sim 0.5 - 0.7$

→ Confirmed **overshoot** found by Ashton+2019, Pizzochero+2020

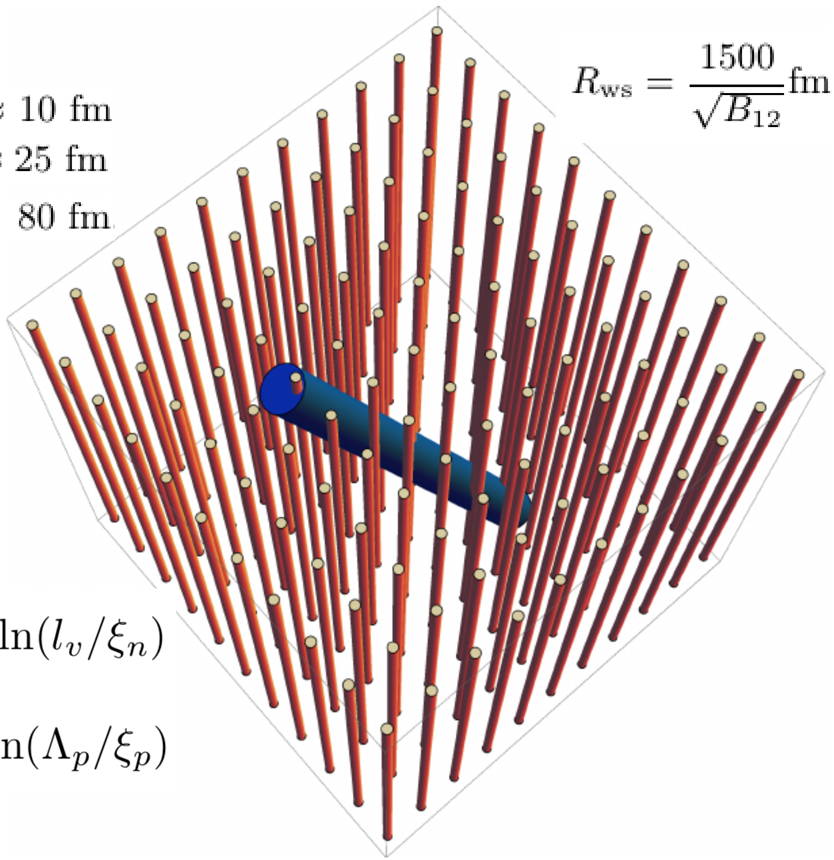
→ Posteriors for **friction parameters** in agreement with the revised estimates of friction by Graber+2018

→ First "entrainment-independent" clue that the **superfluid** in the **outer core** is actively involved. Maybe a sign of **pinning with fluxtubes?**

Vortex – flux tubes pinning

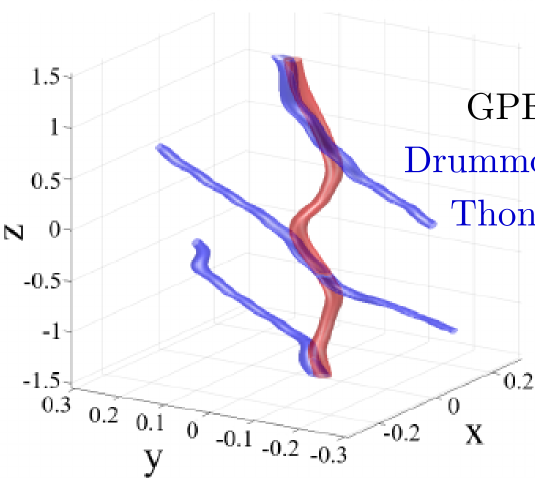
$\xi_p \approx 10$ fm
$\xi_n \approx 25$ fm
$\Lambda_p \approx 80$ fm

$$R_{\rm ws} = \frac{1500}{\sqrt{B_{12}}}\,\text{fm}$$



$$T_n \approx \frac{\kappa\rho_n}{4\pi}\ln(l_v/\xi_n)$$

$$T_p \approx \frac{\kappa\rho_p}{4\pi}\ln(\Lambda_p/\xi_p)$$



GPE+GL simulations
Drummond+ MNRAS (2017)
Thong+ MNRAS (2022)

Not vortex–flux tube interaction...
...but vortex–array interaction
Result: pinning to flux-tubes greatly reduced

$$\xi_p \approx 16\, x_p^{1/3}\rho_{14}^{1/3}\Delta_p(\text{MeV})^{-1}\text{fm}$$

$$\xi_n \approx 16\, x_n^{1/3}\rho_{14}^{1/3}\Delta_n(\text{MeV})^{-1}\text{fm}$$

Coherence length estimates: Mendell, ApJ, 380 (1991)

Overlap of vortex line and flux tube is energetically favored because the volume of non-condensed fluid is **minimized** by such overlap (Srinivasan et al. 1990)

$$E_{\rm int} \sim n_n \frac{\Delta_p^2}{E_{F_p}^2}\frac{\Delta_n^2}{E_{F_n}}(\xi_n^2\,\xi_p) \simeq 0.1\ \text{MeV}$$

A larger contribution to the interaction energy is the **magnetic interaction** between the vortex and a flux-tube. The magnetic field in a flux tube is B ~ 10^{15} G

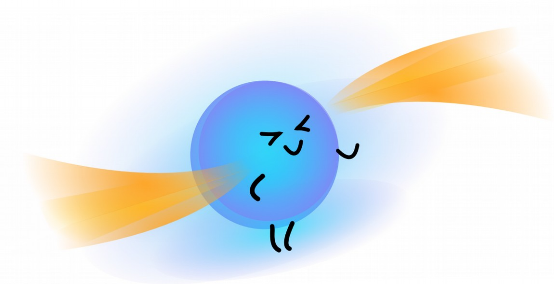
$$E_p(\theta) \simeq l\,\frac{E_0}{L} = \frac{\pi}{8}\,\boldsymbol{B}_v\cdot\boldsymbol{B}_\Phi\,(\Lambda_*^2 l)\ln\left(\frac{\Lambda_*}{\xi_n}\right)$$

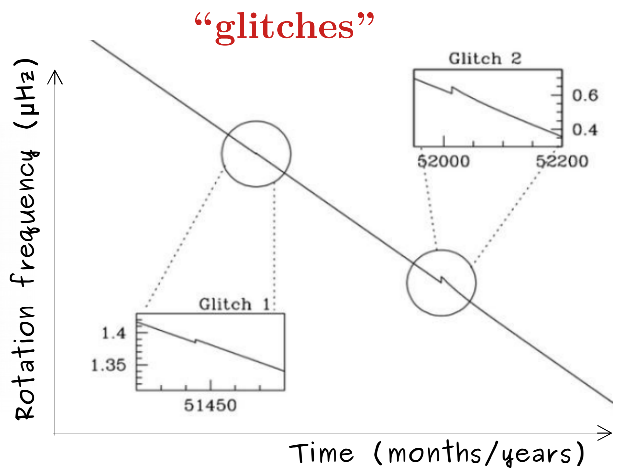
$$E_p \approx 5\,\text{MeV},$$

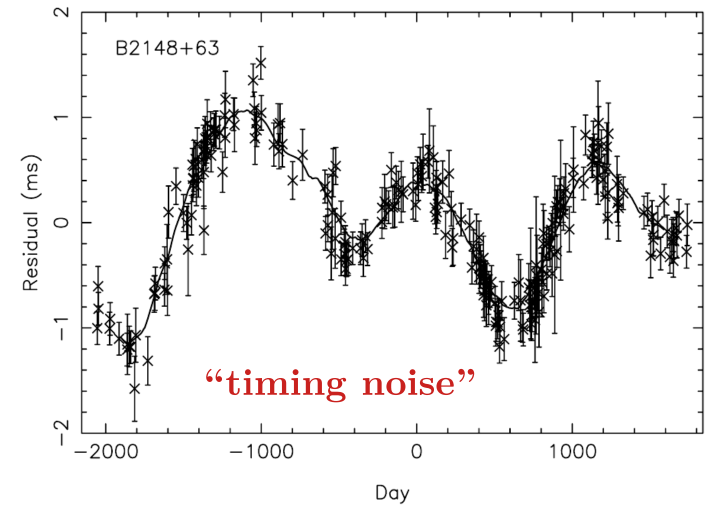
$$f = \frac{F_p}{l_\phi} = \frac{E_p}{\Lambda_p\, l_\phi}$$

(Alpar et al 1984, Jones 1991, Link 2012)

Pulsar "timing noise"







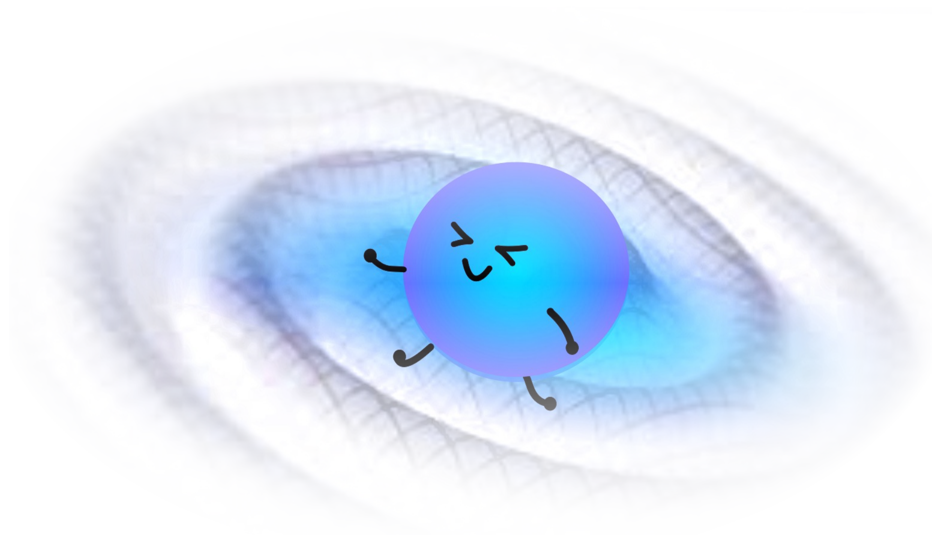
Antonelli, Basu, Haskell arXiv:2206.10416

→ we import physical ideas from "glitch theory" to model "timing noise"

→ understanding the properties of timing noise is important also for Virgo/Ligo

NS emitting *continuous* GW is a "gravitational pulsar"

→ need to keep track of pulsar timing (including glitches and noise) for "targeted" and "narrow-band" searches of continuous GW by Ligo/Virgo



Antonelli, Basu, Haskell, MNRAS (2023)
arXiv:2206.10416

General scheme for timing noise

State vector: **normal** component + m **superfluid** layers

$$\boldsymbol{\Omega}_t = \big(\underbrace{\Omega_p(t)}_{\text{normal}}, \underbrace{\Omega_1(t), \ldots, \Omega_m(t)}_{\text{superfluid}}\big)$$

General *m+1* components **Ito stochastic process** with linear friction:

$$\dot{\boldsymbol{\Omega}}_t = B\,\boldsymbol{\Omega}_t + \mathbf{A}_t + M\dot{\mathbf{W}}_t$$

$$\langle \dot{\mathbf{W}}_t \rangle = 0$$

$$\langle \dot{W}_t^i \dot{W}_s^j \rangle = \delta_{ij}\delta(t-s)$$

Total angular momentum

$$L(t) = \mathbf{x}^\top \cdot \boldsymbol{\Omega}$$

Moment of inertia fractions

$$\sum_{i=1\ldots m} x_i = 1 - x_p$$

Angular velocity residuals

$$\delta\boldsymbol{\Omega}_t = \boldsymbol{\Omega}_t - \langle \boldsymbol{\Omega}_t \rangle = \int_0^t e^{B(t-z)} M\, d\mathbf{W}_z$$

There are 3 fundamental constraints:

→ evolution of the **total angular momentum**: $\dot{L} = \mathbf{x}^\top \cdot \dot{\boldsymbol{\Omega}} = \mathcal{T}_\infty + \eta_t^\infty \;\rightarrow\; \mathbf{x}^\top \cdot \mathbf{A}_t = \mathcal{T}_\infty \quad \mathbf{x}^\top M\dot{\mathbf{W}}_t = \eta_t^\infty$

→ "thermodynamic" **equilibrium state** at corotation: $B(\Omega, \ldots, \Omega) = 0$

→ **action-reaction** principle for the **internal** torques: $B^\top \mathbf{x} = 0$

Power Spectral Density of the **angular velocity residuals**: $P_p(\omega) \propto \langle\, |\delta\Omega_p(\omega)|^2 \,\rangle$ ← PSD definition

$$P_p(\omega) = \left[(i\omega\,\mathbb{I} - B)^{-1} M M^\top (-i\omega\,\mathbb{I} - B^\top)^{-1} \right]_{pp}$$ ← PSD general result

Antonelli, Basu, Haskell
arXiv:2206.10416

The simplest model: 2 components

Independent noise processes for internal and external fluctuations

fluctuating external torque

Non-superfluid component (coupled to magnetosphere)

$$x_p \dot{\Omega}_p = -\mathcal{T} - \eta_t^{\mathcal{T}} + \mathcal{T}_\infty + \eta_t^\infty$$

fluctuating internal torque

Superfluid component (not directly observable)

$$x_1 \dot{\Omega}_1 = \mathcal{T} + \eta_t^{\mathcal{T}}$$

$$\eta_t^\infty = \sigma_\infty \dot{W}_t^\infty$$

$$\eta_t^{\mathcal{T}} = \sigma_{\mathcal{T}} \dot{W}_t^{\mathcal{T}}$$

$$\langle \dot{W}_t^i \dot{W}_s^j \rangle = \delta_{ij} \delta(t-s)$$

Why is it **important to distinguish** between ***internal*** and ***external*** fluctuations?

→ Different physical processes (*external:* magnetosphere, emission – *internal:* vortex motion, turbulence)
→ Practical: fluctuations in the torques as **independent** noise processes
→ Automatic implementation of **action-reaction** principle for the fluctuating mutual friction
→ Clear physical interpretation of the **corner frequencies** in the Power Spectral Density (**PSD**)

$$P_p(\omega) \propto \frac{1}{\omega^2} \cdot \frac{\mu^2 + \omega^2}{\xi^2 + \omega^2} \propto \begin{cases} \mu^2/(\xi^2 \omega^2) & \omega \ll \mu \\ 1/\xi^2 & \mu \ll \omega \ll \xi \\ 1/\omega^2 & \xi \ll \omega \end{cases}$$

PSD of the observable component's angular velocity

Corner frequencies: $0 < \mu < \xi$

$$\mu^2 = \frac{b^2 \sigma_\infty^2}{\sigma_\infty^2 + \sigma_{\mathcal{T}}^2} \qquad \xi^2 = \frac{b^2}{x_p^2}$$

$$b = 2 \Omega \mathcal{B} = x_p / \tau$$

Antonelli, Basu, Haskell
arXiv:2206.10416

The simplest model: 2 components

Independent noise processes for internal and external fluctuations

fluctuating external torque

Non-superfluid component (coupled to magnetosphere)

$$x_p \dot{\Omega}_p = -\mathcal{T} - \eta_t^{\mathcal{T}} + \mathcal{T}_\infty + \eta_t^\infty$$

fluctuating internal torque

Superfluid component (not directly observable)

$$x_1 \dot{\Omega}_1 = \mathcal{T} + \eta_t^{\mathcal{T}}$$

$$\eta_t^\infty = \sigma_\infty \dot{W}_t^\infty$$

$$\eta_t^{\mathcal{T}} = \sigma_{\mathcal{T}} \dot{W}_t^{\mathcal{T}}$$

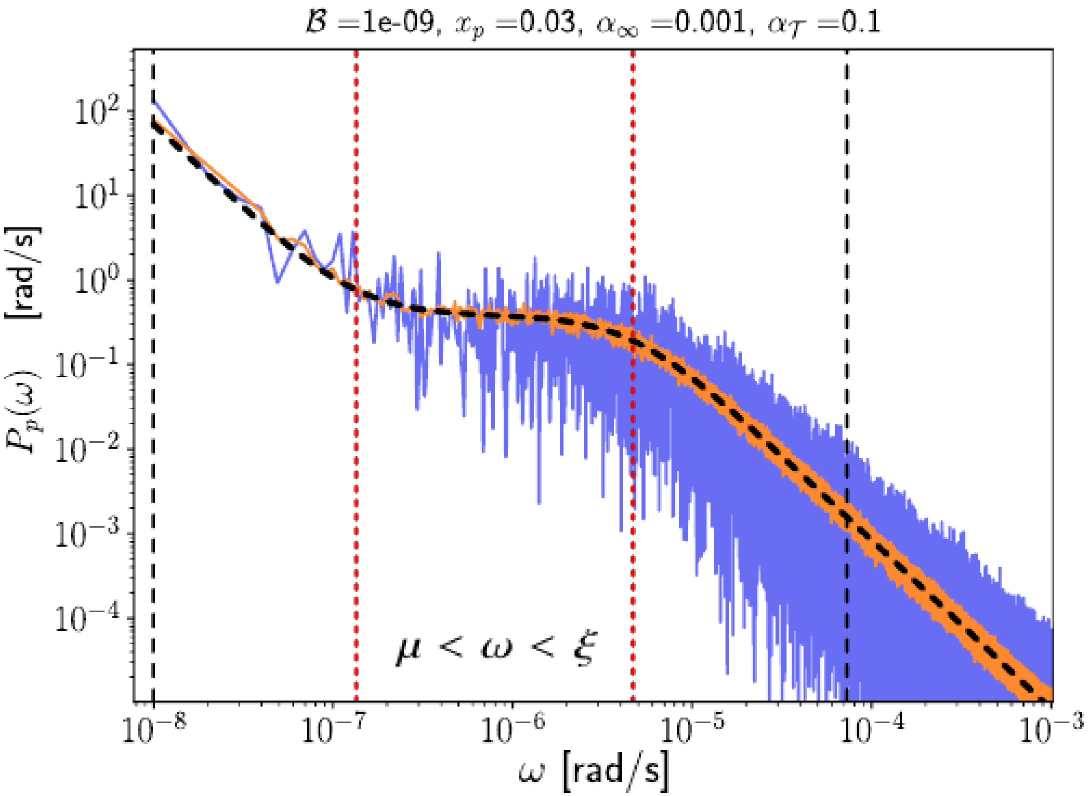
$$\langle \dot{W}_t^i \dot{W}_s^j \rangle = \delta_{ij} \delta(t-s)$$

Only ***external*** noise: PSD plateau shrinks
→ pure Wiener process + "inertial" corrections

$$P_p(\omega) = \frac{\alpha_\infty^2 \, \Omega \, \dot{\Omega}}{x_p^2 \, \omega^2} \cdot \frac{\omega^2 + x_p^2 \tau^{-2}}{\omega^2 + \tau^{-2}} \approx \frac{\alpha_\infty^2 \Omega \dot{\Omega}}{\omega^2}$$

Only ***internal*** noise: plateau extends to the origin
→ Lorentzian PSD (Ornstein-Uhlenbeck process)

$$P_p(\omega) = \frac{\alpha_{\mathcal{T}}^2 x_1^2 \, \dot{\Omega}^2}{2 \, x_p^2 \mathcal{B} \, \Omega} \cdot \frac{1}{\omega^2 + \tau^{-2}}$$

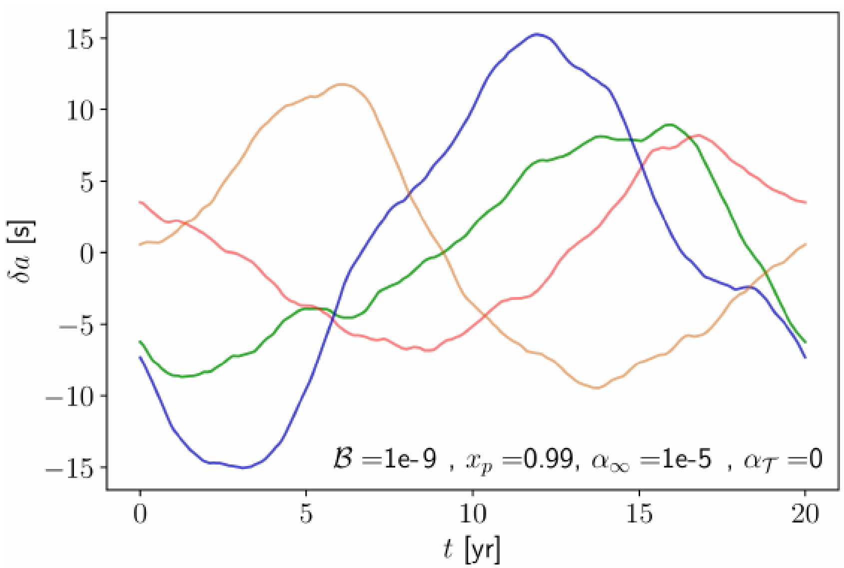


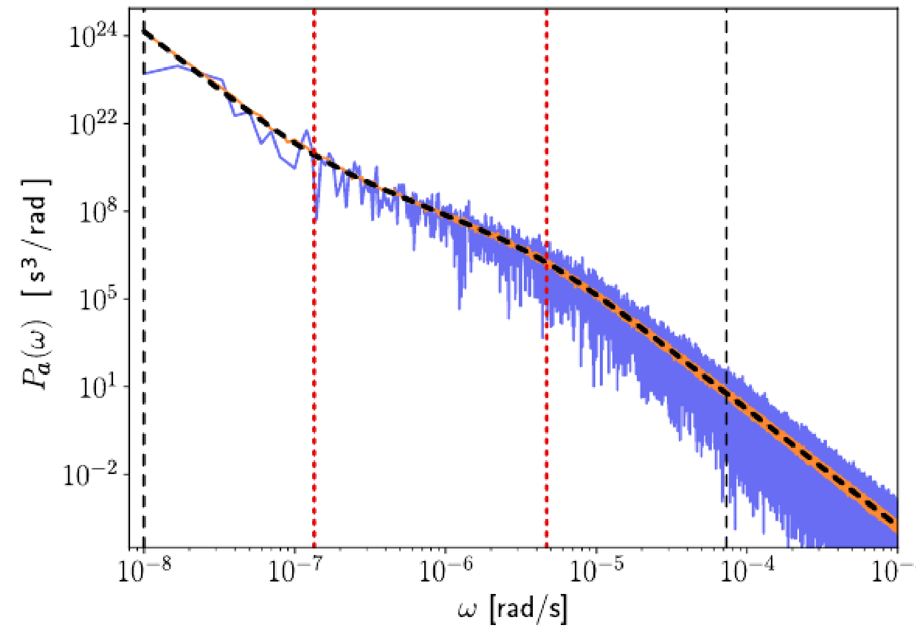
Antonelli, Basu, Haskell
arXiv:2206.10416

Times of arrival: PSD

Simulations used to validate the approximate analytical result

$$P_a(\omega) \approx (\omega\Omega)^{-2} P_p(\omega)$$





To obtain the TOA in computer simulations we have to solve the **implicit** equation

$$\phi(t + \delta a_t) = \phi^{\text{mod}}(t) \qquad \text{(root-finder needed, very slow!)}$$

Trick to simplify the problem: **implicit differentiation** of the above equation gives

$$\frac{d}{dt}\,\delta a_t = -\frac{\Omega_p^t - \dot{\phi}_t^{\text{mod}}}{\Omega_p^t} - \frac{\dot{\phi}_t^{\text{mod}}\dot{\Omega}_p^t}{{\Omega_p^t}^2}\,\delta a_t + O\left(\frac{\delta a_t^2}{T^2}\right)$$

Very good **approximation**:

$$\delta a_t \approx -\frac{\delta\phi(t)}{\Omega}$$

$$\delta\phi_t = \phi_t - \phi_t^{\text{mod}}$$

$$P_a(\omega) \approx (\omega\Omega)^{-2} P_p(\omega)$$

Timing noise across the pulsar population

Antonelli, Basu, Haskell, MNRAS (2023), arXiv:2206.10416

Time of arrival $a(t)$

$$\phi(t + \delta a_t) = \phi^{\mathrm{mod}}(t)$$

$$\sigma^2 = \frac{1}{N_o} \sum_{j=1}^{N_o} |\delta a_j|^2$$ TN strength (observations)

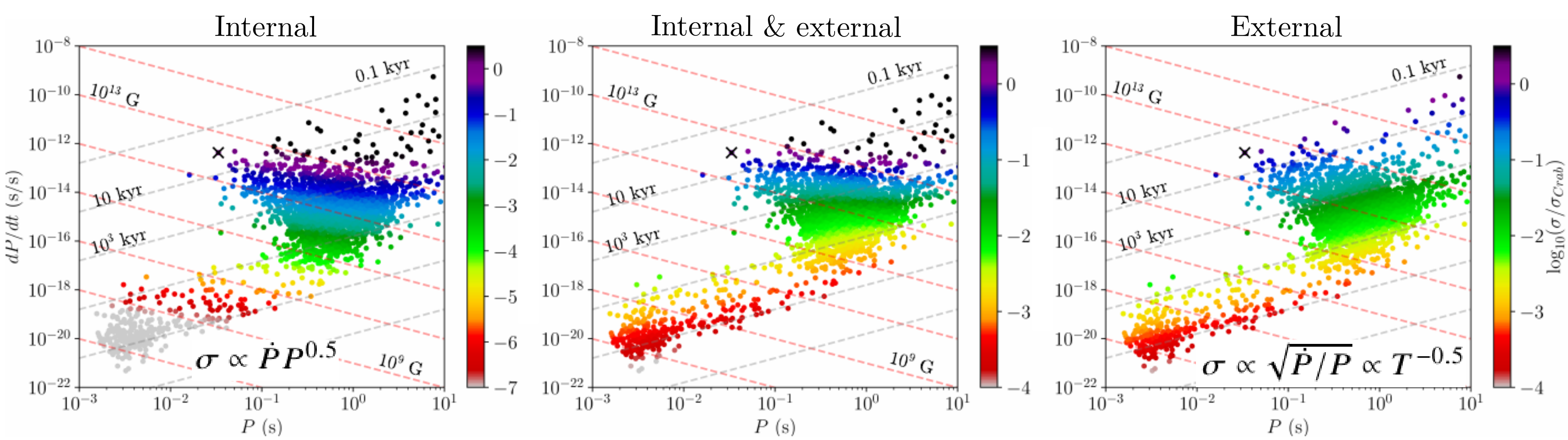
$$\sigma^2 = \frac{1}{T_o} \int_{t_0}^{t_0+T_o} dz \, |\delta a_z|^2$$ TN strength (analytical)

$\sigma \propto \Omega^a \dot{\Omega}^b$	a	b	b/a
"external" noise	-0.5	0.5	-1
"internal" noise,	-2.5	1	-0.4
Lower et al. (2021)	-0.84	0.97	-1.15
Parthasarathy et al. (2019)	1 (assumed)	-0.6	-0.6
Shannon & Cordes (2010) (MSP+CP)	-1.5	1.2	-0.8
Shannon & Cordes (2010) (CP)	-0.9	1	-1.1
Hobbs et al. (2010)	-0.39	0.75	-1.9

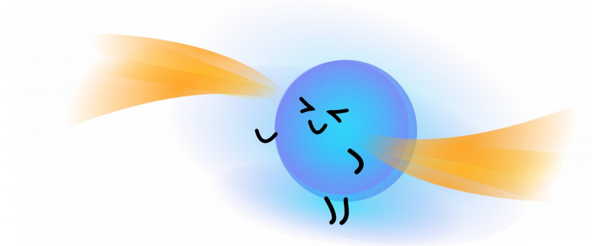
Analytical results (arbitrary number m of components)

$$\sigma^2 \approx a_{e0}\, \dot{\Omega}\Omega^{-1} T_0^3 + a_{e1}\, \dot{\Omega}\Omega^{-3} T_o \qquad \text{(pure external noise)}$$

$$\sigma^2 \approx a_{i0}\, \dot{\Omega}^2\Omega^{-5} T_o + a_{i1}\, \dot{\Omega}^2\Omega^{-6} \qquad \text{(pure internal noise)}$$



References



Review: Antonelli, Montoli, Pizzochero arXiv:2301.12769

Stationary "activity" test ("effective moment of inertia of pinned region">"observed activity") Link+1999 arXiv:9909146, Andersson+2012 arXiv:1207.0633, Chamel 2013 arXiv:1210.8177, Montoli+2020b arXiv:2012.01539

Stationary "glitch size" test ("glitch from maximum lag">"observed glitch amplitude") Antonelli+2018 arXiv:1710.05879

Beyond the stationary tests (need to solve temporal evolution → model dependent): Ho+2015 arXiv:1703.00932 (activity+cooling: need to integrate the thermal evolution), Pizzochero+2017 arXiv:1611.10223, Montoli+2020 arXiv1809.07834 (glitch size+activity: need to integrate the superfluid reservoir evolution)

GR corrections: Sourie+2017 arXiv:1607.08213 (2 rigid components), Antonelli+2018 arXiv:1710.05879 (fluid, corrections for the glitch size), Gavassino+2019 arXiv:2001.08951, arXiv:2012.10288 (fluid, corrections for the spin-up timescale)

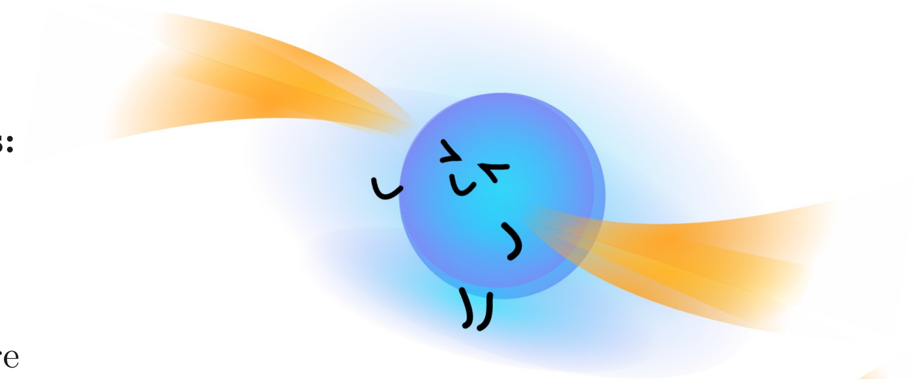
Revisiting the starquake paradigm (Ruderman, Nature 223 1969): Giliberti+2019 arXiv:1902.06345, arXiv:1809.08542 (continuously stratified), Reconret+2021 arXiv:2106.12604 ("starquake is not enough" argument), Bransgrove+2020 arXiv:2001.08658 (quake and null pulses in Vela 2016)

Glitch overshoot: virtually any model with more degrees of freedom than **Baym+ Nature 224 1969**, namely all fluid models that are in the same class of **Alpar+ ApJ 273 1984**, e.g., Haskell+2012 arXiv:1107.5295, Antonelli+2017 arXiv:1603.02838, Graber+2018 arXiv:1804.02706, Pizzochero+2020 arXiv:1910.00066 (mathematical condition for the overshoot)

Vela's 2016 glitch (Palfreyman+ Nature 556 2018), Montoli+2020a arXiv:2005.01594 (theory of 3-component model, Bayesian fit of physical parameters), Ashton+2019 arXiv:1907.01124 (agnostic Bayesian fit), Sourie+2020 arXiv:2001.09668 (pinning in the core), Gügercinoğlu+2020 arXiv:2003.08724 (fit with the vortex creep model)

Beyond hydrodynamics & SOC: we are still "stealing" from **Anderson&Itoh Nature 256 1975** (short but very dense paper!) Melatos+2018 arXiv:1809.03064 (correlations in glitches), Khomenko+2018 arXiv:1801.01413, Antonelli+2020 arXiv:2007.11720 (pinning/depinning transition, hysteresis loop), Howitt+2020 arXiv:2008.00365 (point vortex simulations), Haskell+2020 arXiv:2007.02748 (quantum turbulence), Carlin+2021 arXiv:2105.13588

Final considerations



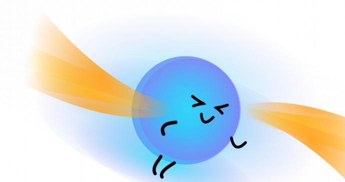
Glitches provide us with some interesting **theoretical challenges:**

→ **vortex dynamics** in non-homogeneous environments
→ collective **avalanche** dynamics
→ how to describe **pinning** at the microscopic scale?
→ physics in the core: **superfluid-superconductor** mixture

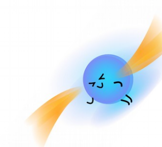
Cross contamination between different fields is necessary. **Some open questions:**

→ role of starquakes? (can we really have quakes in a NS?)
→ role of entrainment (strong/weak? affected by disorder?)
→ better understanding of dissipation at micro/meso scale
→ collective aspects of vortex dynamics (Hysteresis? Collective pinning?)

How to probe NS interiors?



→ 2 robust "tests":
  Glitch amplitude → pinning forces (marginally enough)
  Glitch activity → entrainment parameters ("crust not enough" if strong entrianment)
→ Vela 2016: first entrainment-independent clue that the outer core is involved in the glitch
→ Extend "glitch technology" to timing noise modeling: superfluid fingerprint in the PSD?



The most important thing: more and better observations

Improved timing techniques (and more observation time) → test current models

NS structure

Proton radius (e-p Rutherford scattering) ~ 0.85 fm
Neutron r.m.s. radius ~ 0.8 fm
Strong nuclear force range ~1 fm–3 fm (mesons)

Nuclear **saturation** density: $\mathbf{n_0 \sim 0.16\ fm^{-3}}$
→ WS radius per nucleon at saturation ~ 1.2 fm

Inside a neutron star:

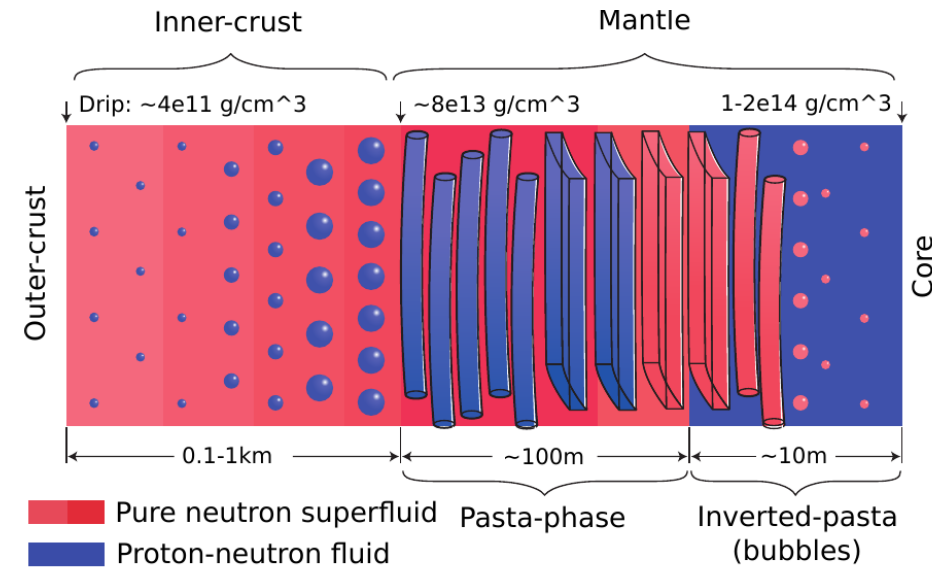
Neutron drip (outer crust → inner crust) ~$2.5 \cdot 10^{-4}\ n_0$

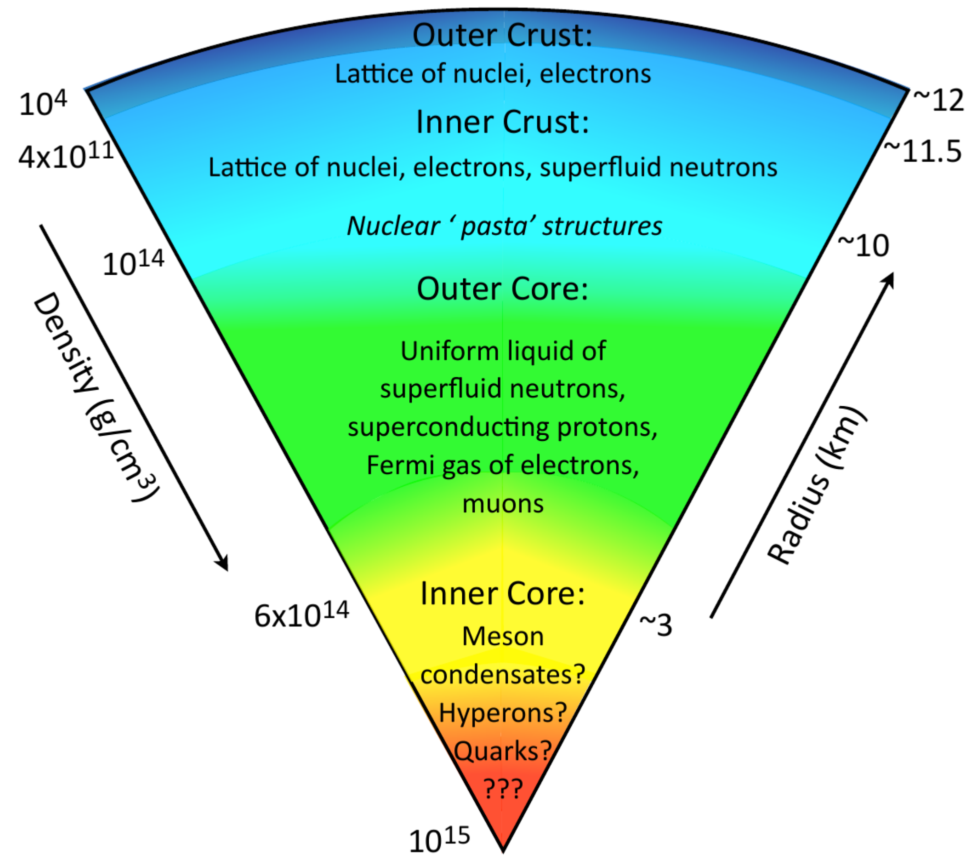
Pasta phase (inner crust → outer core) ~$0.3 - 0.5\ n_0$

Homogeneous npe$^-$ matter ~$0.5\ n_0 - 1\ n_0$

Muons appear at ~$1\ n_0$ (e$^-$ Fermi energy > 105.7 MeV)

Hyperons appear at ~$2\ n_0$ (inner core)





Reminder on notation: $^{2S+1}L_J$ (L=0,1,2,3… → S,P,D,F…)

Superfluidity in NS

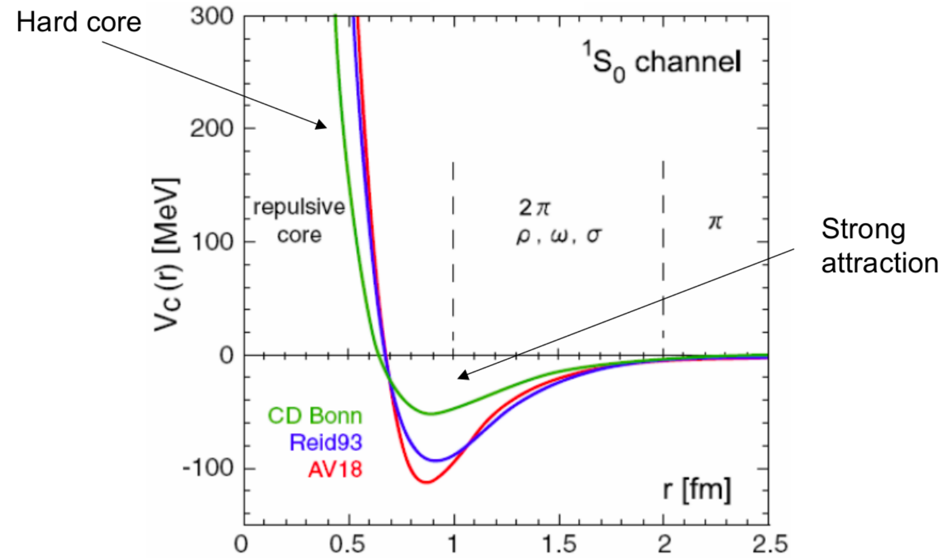
Mature neutron stars are "cold"
(T = 10^8 K = 0.01 MeV) << (E_F = 10 - 100 MeV)
Fermi surface is unstable against pairing:

- Neutrons in the **crust** feel **attractive** components of the NN potential in the $^1S_0$ channel

- **Core:** $^1S_0$ NN force is repulsive above ~0.16 fm^{-3}

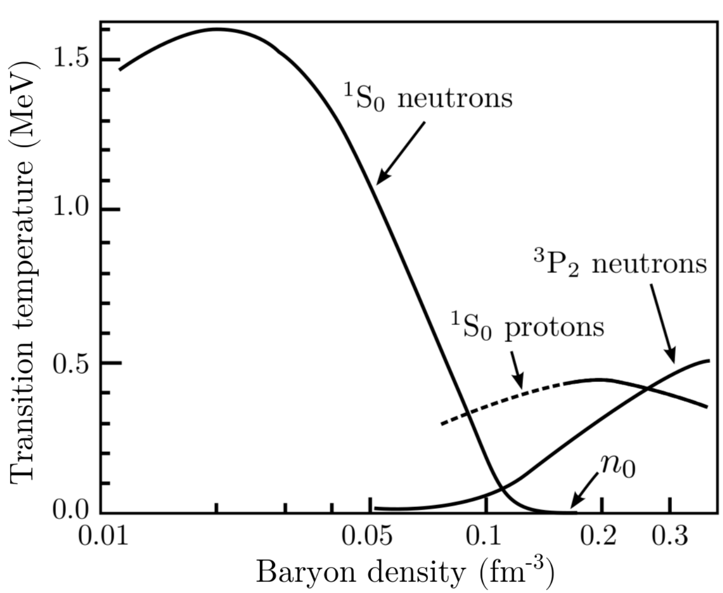
$^3S_1$-$^3D_1$ binds the deuteron: but in NS **n** and **p** have very different Fermi surfaces → **no n-p superfluid**

NN potential for S=0, L=0, J=0

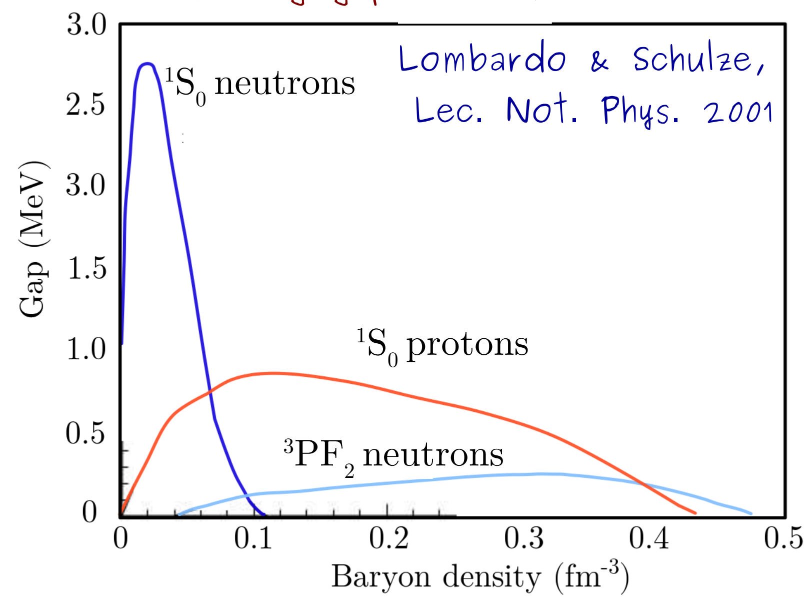


Ishii et al, PRL 2007

Transition temperature



Pairing gap Δ ~ 1 MeV ~ 10^{10} K



Pairing channels

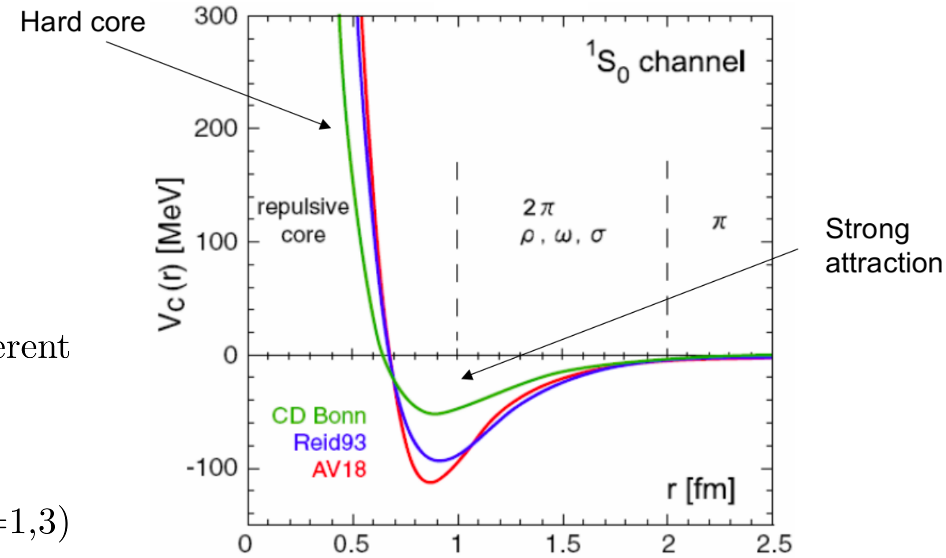
Total angular momentum operator: $\mathbf{J} = \mathbf{L} + \mathbf{S}$
Notation: $^{2S+1}L_J$ (L=0,1,2,3... → S,P,D,F...)

$^1S_0$ isotropic pairing: Δ = "energy gap" ~ 0.57 T_c

$^3S_1$–$^3D_1$ binds the deuteron: but in NS **n** and **p** have very different Fermi surfaces → **no n-p pairing**

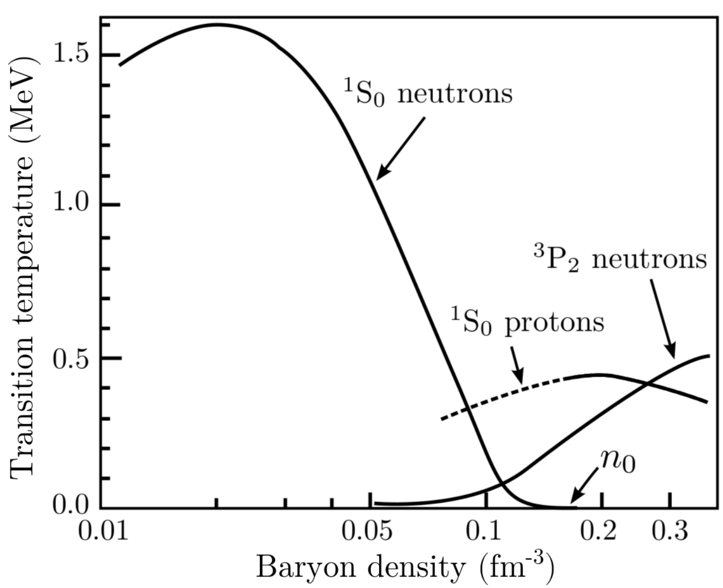
$^3PF_2$ partial–wave channel (Δ has contributions from both L=1,3) preferred at larger Fermi momenta, where $^1S_0$ is repulsive. Uncertain gap, usually treated as free **parameter** in cooling simulations.

NN potential for S=0, L=0, J=0

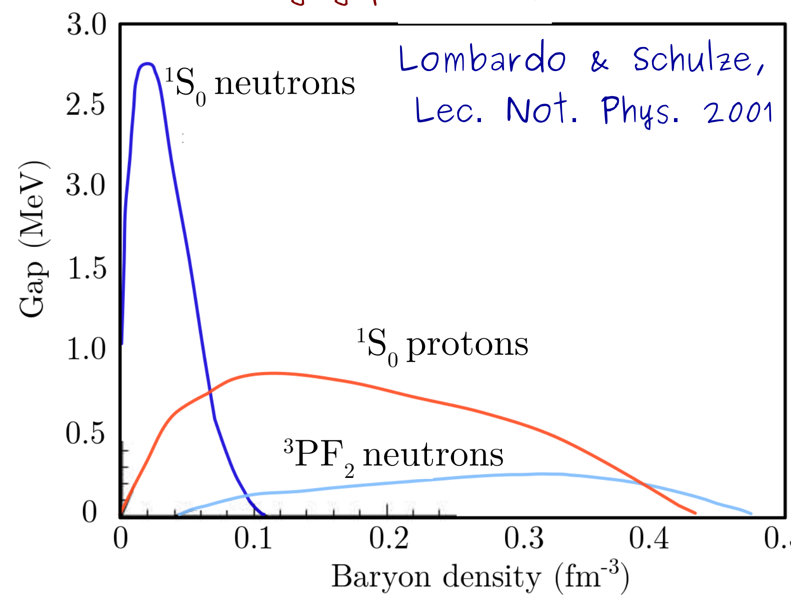


Ishii et al, PRL 2007

Transition temperature



Pairing gap Δ ~ 1 MeV ~ 10^{10} K

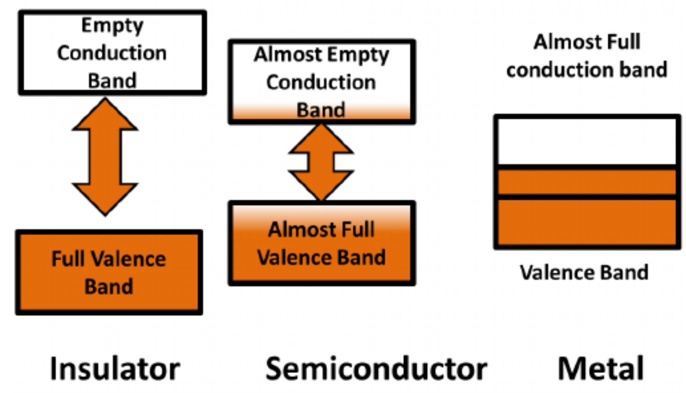


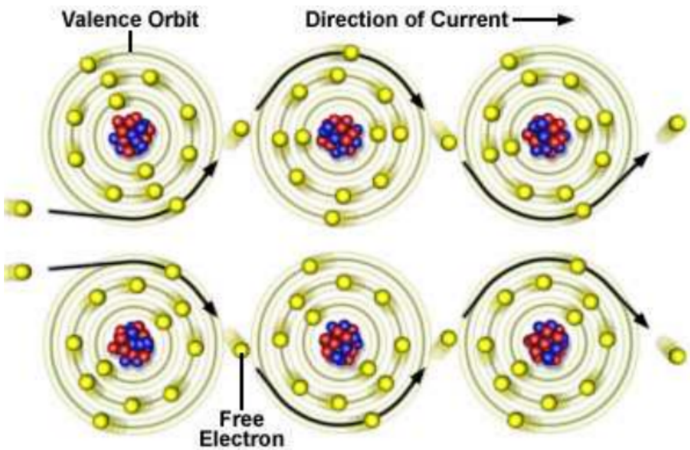
Band theory (inner crust VS “metal”)

Due to the interactions with the periodic lattice, neutrons move in the inner crust as if they had an effective mass m*.

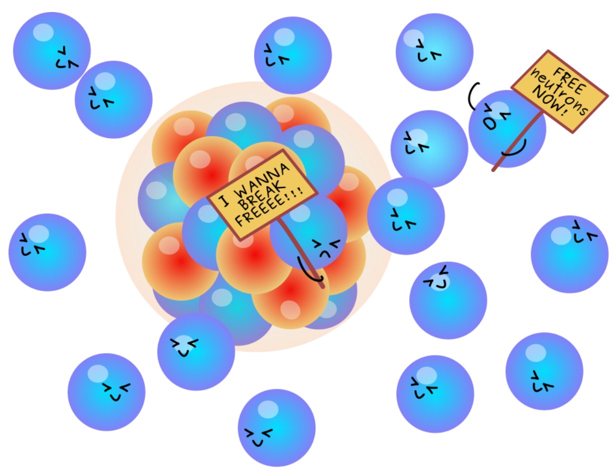
At the highest energies of the valence band (or at the lowest energies of the conduction band), the band structure E(K) of an electron can be approximated as a “free electron” but with an “effective mass”

$$m^* \leftrightarrow \text{crustal entrainment}$$





Usual metal: how to distinguish between a “conduction electron” and a “confined” one?



Neutron star inner crust: how to distinguish a “leaked neutron” from a “confined” one?

Entrainment coupling: crust and core

In the inner crust (lattice of ions & S-wave superfluid):

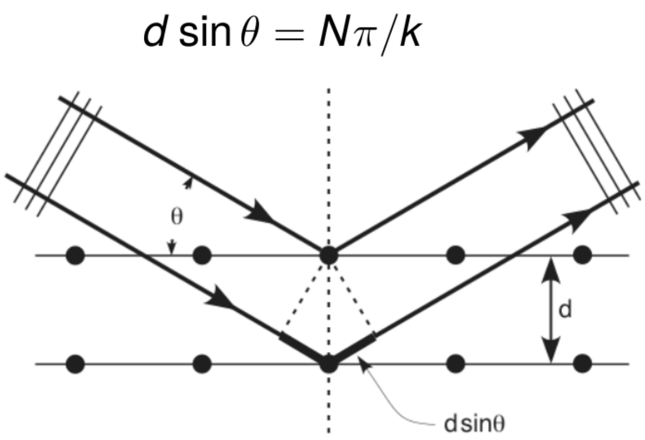
Chamel, PRC 2012

Bragg scattering by crustal lattice entrains the "free" neutrons.

Non-local effect: m* > 1

→ Consequence: the crustal superfluid is entrained by the normal component: **reduced mobility** of "free" neutrons is a potential problem for pulsar glitch theory.

Chamel PRL 2013, Montoli, Antonelli et al, Universe 2020



In the core (S-wave superconductor & P-wave superfluid):

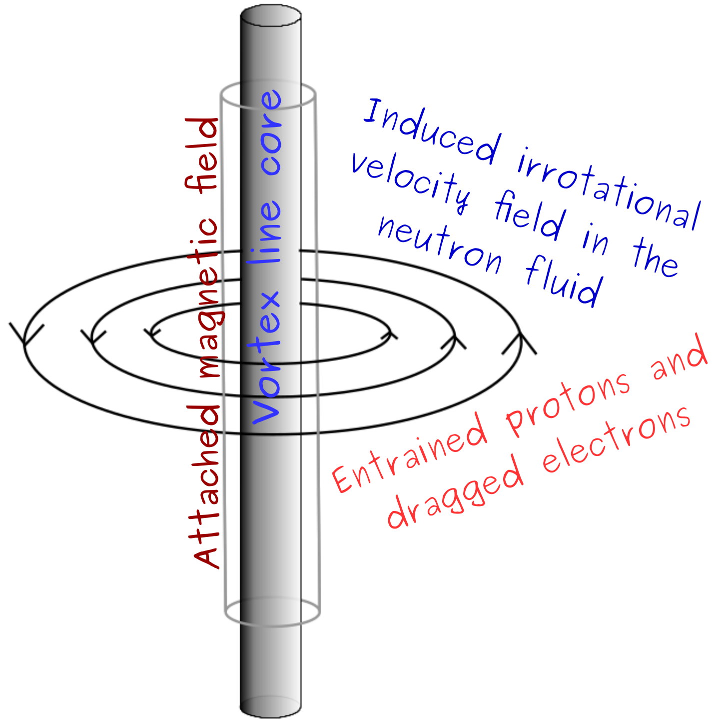
Chamel & Haensel PRC 2006

Entrainment is due to the strong interaction between protons and neutrons. Local effect: m*<1

–Consequence #1: Scattering of electrons off vortex cores: the core is coupled to the crust on the timescale of a second

Alpar et al, ApJ 1984

–Consequence #2: Dipole-dipole interaction with flux-tubes (core pinning?)



Pinning – Length scales

Core $\rightarrow$ "Abrikosov lattice" spacing between flux-tubes ~ 1000 fm

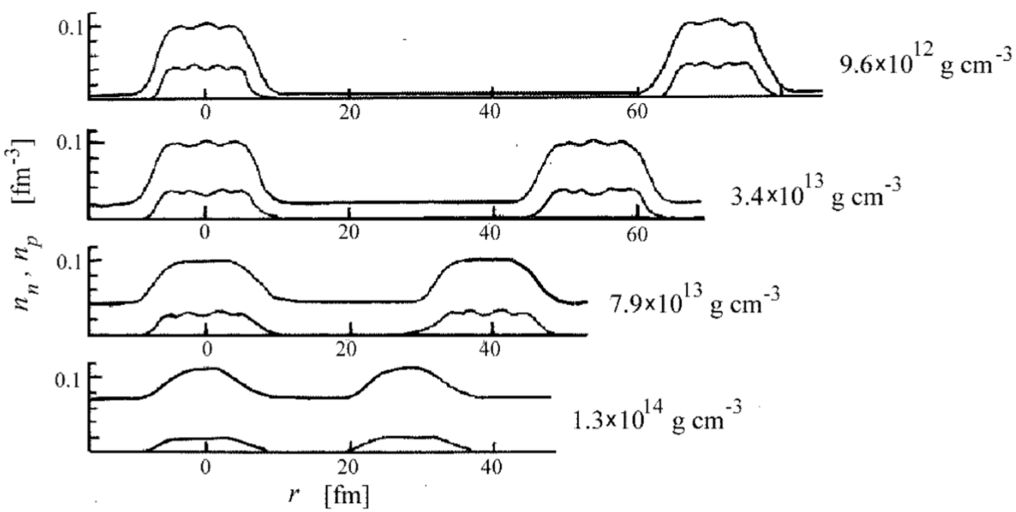
Crust $\rightarrow$ crustal lattice spacing ~ 100 – 20 fm

Inter-vortex spacing

$$l_v = \frac{\sqrt{\kappa P}}{2\pi} \approx 7 \times 10^{-3} \sqrt{P} \text{ cm}$$

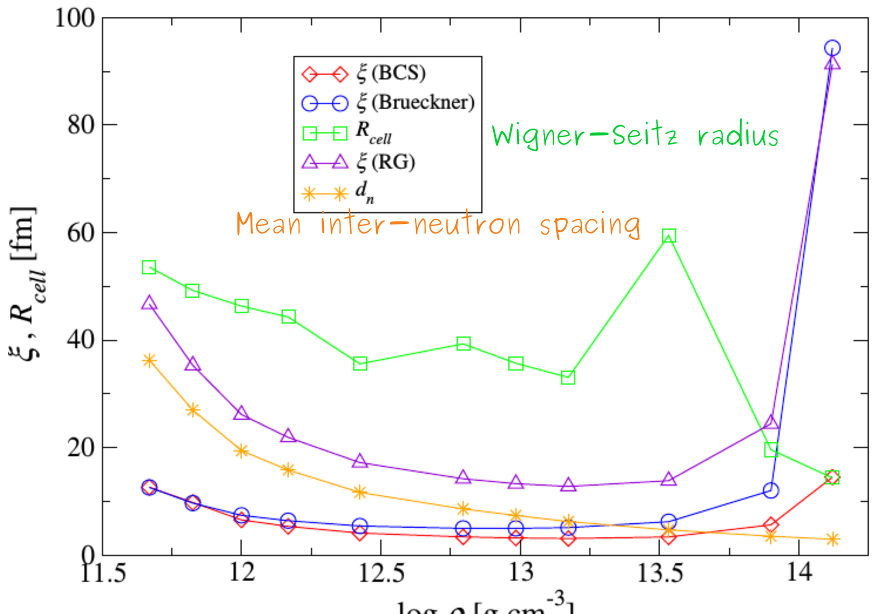
Vortex-*nucleus* interaction $\rightarrow$ coherence length ~ 10 – 100 fm

Vortex dynamics and vortex-*lattice* interaction $\rightarrow$ "mesoscale" (inter-vortex spacing)



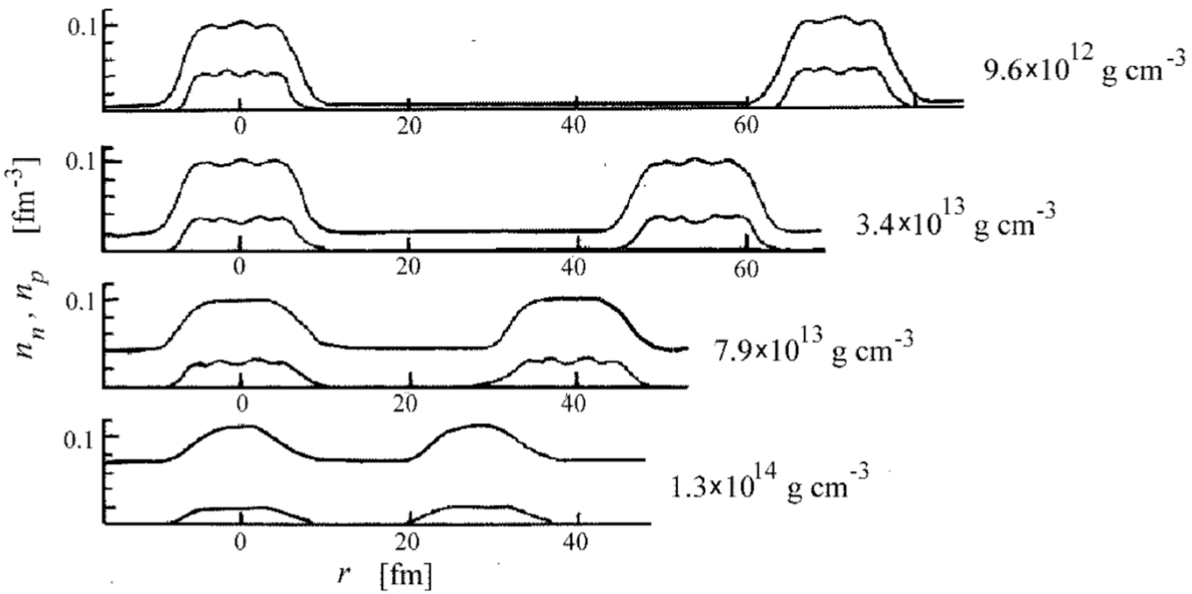
Negele & Vautherin (1973)

Neutron star matter at sub-nuclear densities



Chamel & Haensel, Living Rev.Rel. 11 (2008)

Inner crust structure



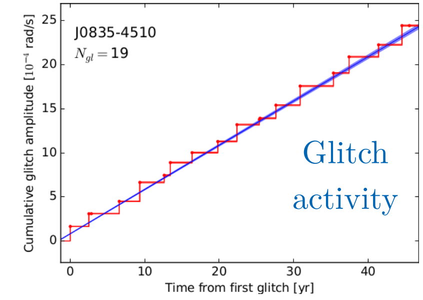
Density profiles of neutron and protons, at several average densities, along a line joining the centers of two adjacent unit cells (HF calculation of the GS in the **inner crust** with effective NN interaction, **no pairing correlations**)

Negele & Vautherin, *Neutron star matter at sub-nuclear densities* (1973)

Include **pairing correlations**: Baldo et al, *The role of superfluidity in the structure of the neutron star inner crust* (2005)

Band theory of solids: Carter et al, *Entrainment Coefficient and Effective Mass for Conduction Neutrons in Neutron Star Crust* (2006)

Stationary test (glitch activity)

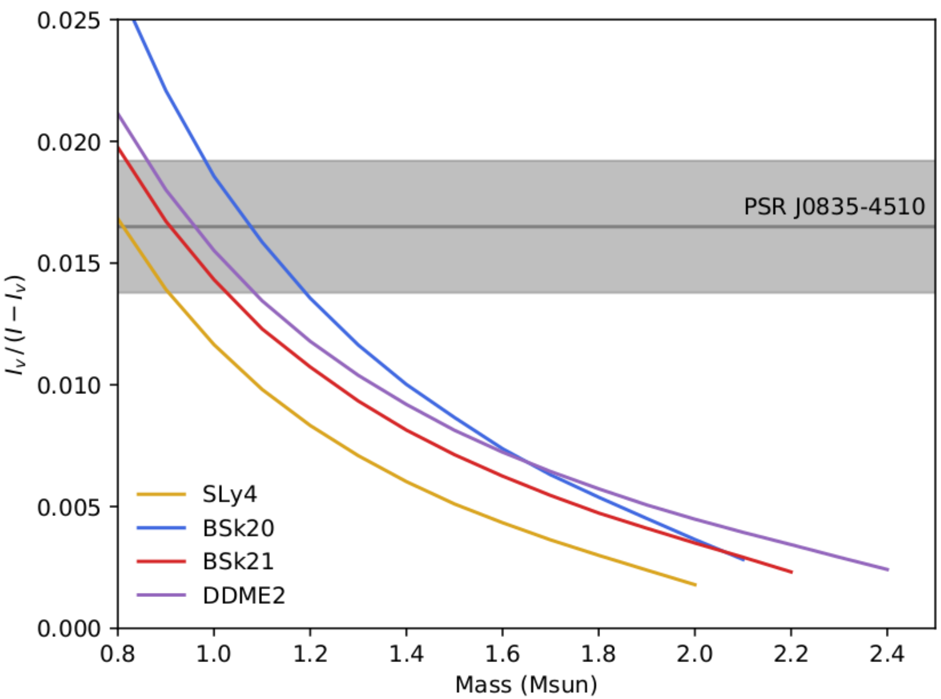


Result: the *superfluid in the crust is not enough* to explain Vela's activity when strong entrainment is accounted for Andersson+2012 arXiv:1207.0633, Chamel 2013 arXiv:1210.8177

Revised argument: Montoli+2020 arXiv:2012.01539

$$I_v = \frac{8\pi}{3}\int_0^{R_d} dr\, r^4\, \frac{\rho_n(r)}{1-\epsilon_n(r)} \qquad\qquad \frac{\mathcal{A}}{|\dot{\Omega}|} < \frac{I_v}{I - I_v}$$

Note: no need to solve the internal dynamics ("stationary")



"Heteroscedastic" linear regression: uncertainties larger by a factor ~10 for the Vela (Montoli+2020 arXiv:2012.01539)

$$\mathcal{A}_a = \frac{\sum_i \Delta\Omega_i}{\sum_i \Delta t_i} = \frac{\sum_i \Delta\Omega_i}{t_{N_{\rm gl}-1} - t_0} \qquad \mathrm{Var}(\mathcal{A}_a) = \frac{1}{(N_{\rm gl}-1)(t_{N_{\rm gl}-1}-t_0)} \sum_i \frac{(\Delta\Omega_i - \mathcal{A}_a \Delta t_i)^2}{\Delta t_i}$$

Unbiased estimators for cumulated data

Compare with bounds on the minimum mass of a NS:

Observed: M=1.174 M⊙ Martinez+2015 arXiv:1509.08805 CCS simulations: M≈1.15 M⊙ Suwa+2017 arXiv:1808.02328

Heteroscedastic linear regression

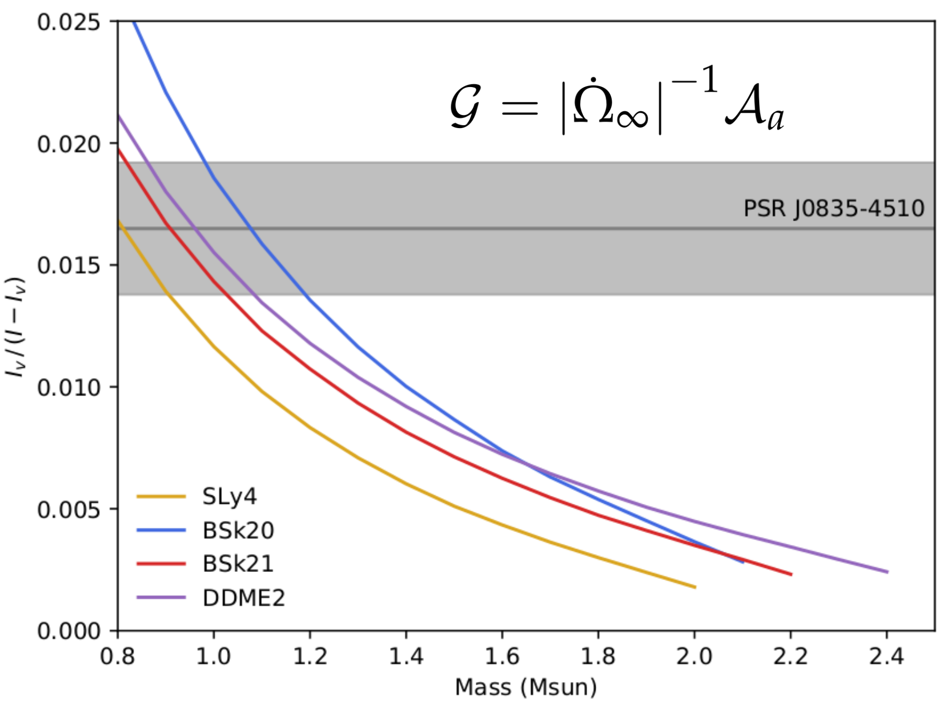
Montoli+2020 arXiv:2012.01539

$$\Delta\Omega_i = \mathcal{A}_a\, \Delta t_i + \varepsilon_i \qquad\qquad \Omega_i = \mathcal{A}_a\, t_i + \sum_{j=1}^{i} \varepsilon_j$$

Usual regression: deviations i.i.d.

Cumulated data: deviations are not i.i.d.

Important: careful inclusion of the "intercept" may lower the estimated uncertainty.



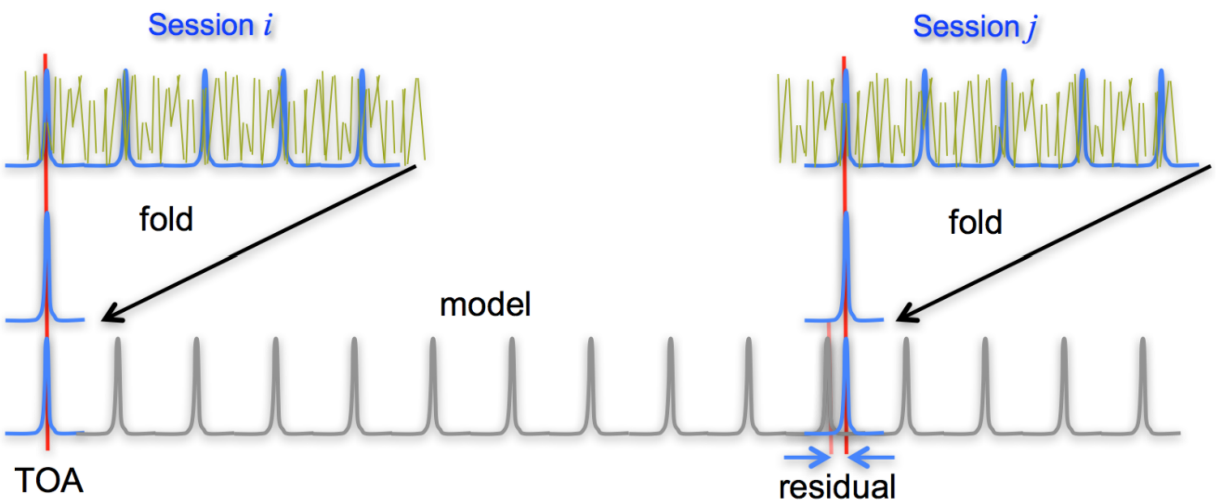
Pulsar	$\mathcal{G}_{\mathrm{hom}}$ (%)	$\mathcal{G}_{\mathrm{het}}$ (%)
0534+2200	0.0079 ± 0.0007	0.008 ± 0.006
0537-6910	0.874 ± 0.003	0.85 ± 0.15
0631+1036	1.77 ± 0.18	2.03 ± 1.95
0835-4510	1.62 ± 0.02	1.6 ± 0.2
1341-6220	1.52 ± 0.10	1.9 ± 0.6
1740-3015	1.22 ± 0.04	1.3 ± 0.7

$$\mathcal{A}_a = \frac{\sum_i \Delta\Omega_i}{\sum_i \Delta t_i} = \frac{\sum_i \Delta\Omega_i}{t_{N_{\mathrm{gl}}-1} - t_0} \qquad \mathrm{Var}(\mathcal{A}_a) = \frac{1}{(N_{\mathrm{gl}}-1)(t_{N_{\mathrm{gl}}-1} - t_0)} \sum_i \frac{(\Delta\Omega_i - \mathcal{A}_a \Delta t_i)^2}{\Delta t_i}$$

Unbiased estimators for cumulated data

Times of arrival

Timing irregularities: mismatch between **expected** (model) and **observed** TOA



TOA formal defintion
(observations)

$$\phi(t_j^{\text{mod}} + \delta a_j) = \phi^{\text{mod}}(t_j^{\text{mod}})$$

TOA defintion
(continuous model)

$$\phi(t + \delta a_t) = \phi^{\text{mod}}(t)$$

Phase of the
observable component

$$\phi_t = \int_0^t \Omega_p{}^z \, dz$$

To obtain the TOA in computer simulations we have to solve the **implicit** equation

$$\phi(t + \delta a_t) = \phi^{\text{mod}}(t) \qquad \text{(root-finder needed, very slow!)}$$

Trick to simplify the problem: **implicit differentiation** of the above equation gives

$$\frac{d}{dt}\delta a_t = -\frac{\Omega_p^t - \dot\phi_t^{\text{mod}}}{\Omega_p^t} - \frac{\dot\phi_t^{\text{mod}}\dot\Omega_p^t}{\Omega_p^t{}^2}\,\delta a_t + O\left(\frac{\delta a_t^2}{T^2}\right)$$

Very good **approximation**:

$$\delta a_t \approx -\frac{\delta\phi(t)}{\Omega}$$

$$\delta\phi_t = \phi_t - \phi_t^{\text{mod}}$$

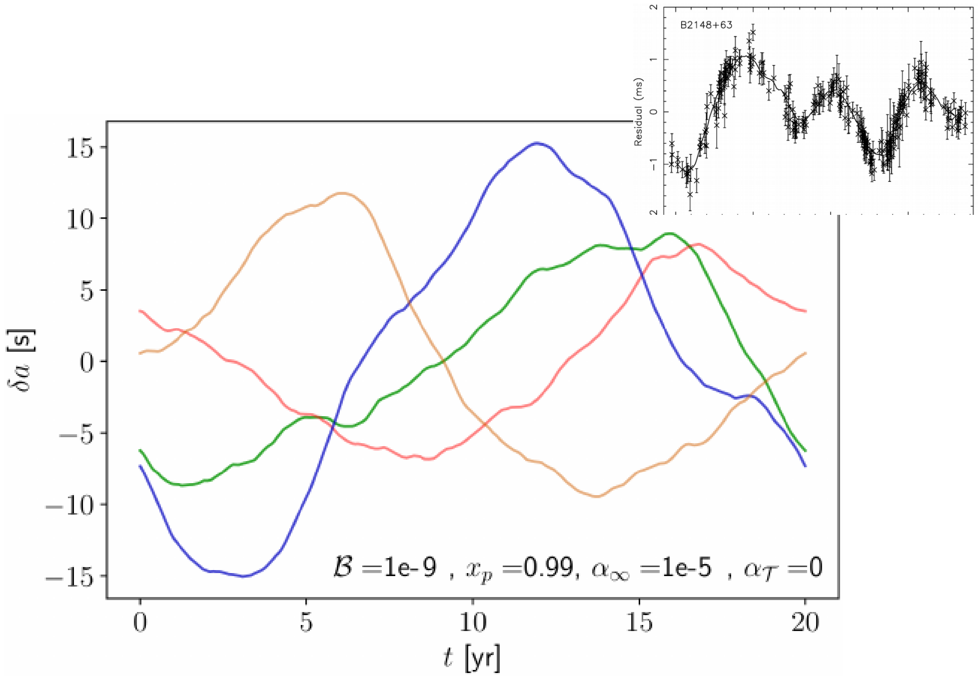
$$P_a(\omega) \approx (\omega\Omega)^{-2} P_p(\omega)$$

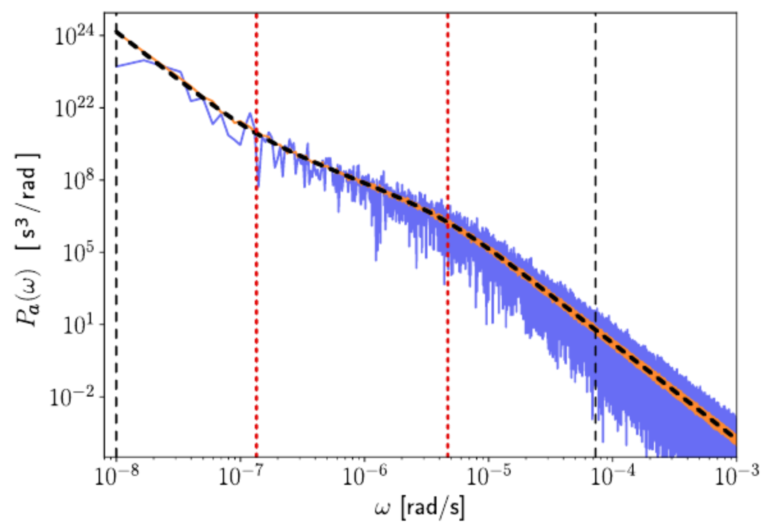
Times of arrival: PSD

Antonelli, Basu, Haskell, MNRAS (2023), arXiv:2206.10416

Simulations used to validate the approximate analytical result

$$P_a(\omega) \approx (\omega\Omega)^{-2}\, P_p(\omega)$$





To obtain the TOA in computer simulations we have to solve the **implicit** equation

$$\phi(t + \delta a_t) = \phi^{\rm mod}(t) \qquad \text{(root-finder needed, very slow!)}$$

Trick to simplify the problem: **implicit differentiation** of the above equation gives

$$\frac{d}{dt}\,\delta a_t = -\frac{\Omega_p^t - \dot\phi_t^{\rm mod}}{\Omega_p^t} - \frac{\dot\phi_t^{\rm mod}\dot\Omega_p^t}{{\Omega_p^t}^2}\,\delta a_t + O\left(\frac{\delta a_t^2}{T^2}\right)$$

Very good **approximation**:

$$\delta a_t \approx -\frac{\delta\phi(t)}{\Omega}$$

$$\delta\phi_t = \phi_t - \phi_t^{\rm mod}$$

$$P_a(\omega) \approx (\omega\Omega)^{-2}\, P_p(\omega)$$