Andrea Richaud

Inertial effects in superfluid vortex dynamics

"la Caixa" Foundation

MARIE CURIE ACTIONS

LUMI

W-SLDA Toolkit

Trento, May 13th, 2025

Vortices as a manifestation of coherence

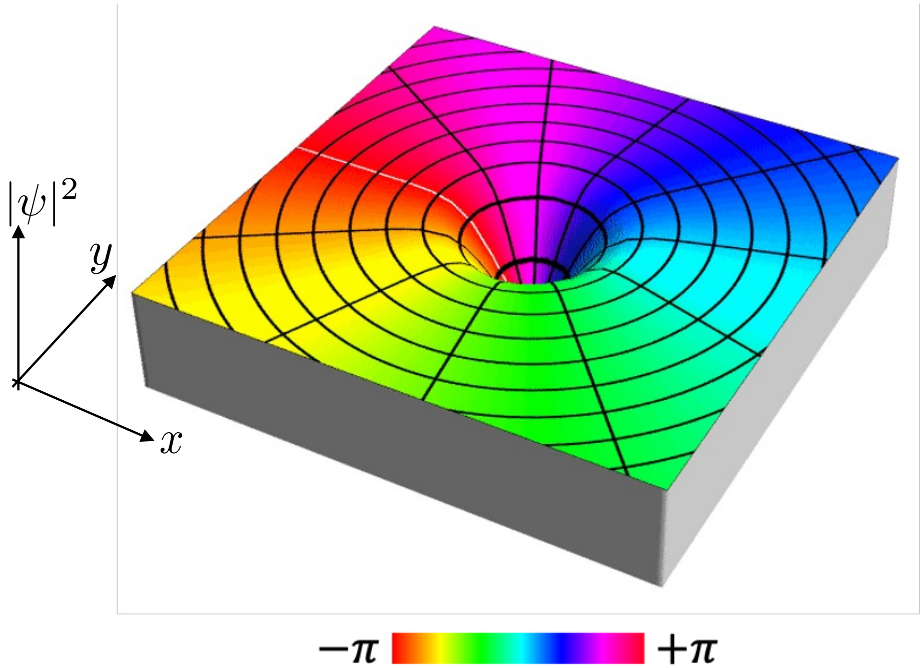
Dog puppies can be coherent too…

Outline

- Superfluid Vortex Dynamics
- From massless to massive vortices
- Intrinsic mass of a vortex in Fermi superfluids

SUPERFLUID VORTEX DYNAMICS

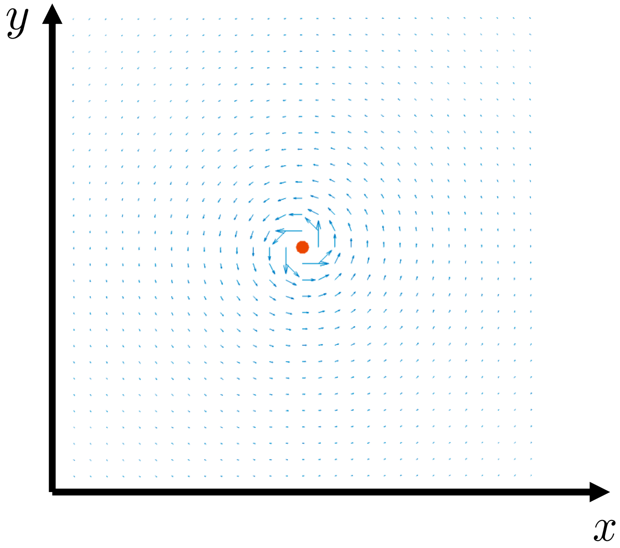
The ID of a quantum vortex



- The phase rolls up from 0 to 2π.
- The superfluid density goes to zero at the vortex centre.

A vortex in a plane

Each vortex generates a velocity vector field of the type:

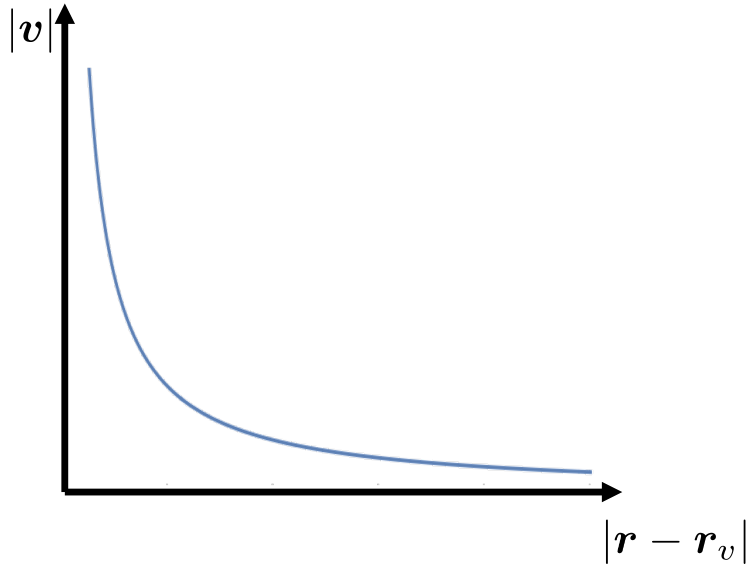


$$\boldsymbol{v}(\boldsymbol{r}) = \frac{\kappa}{2\pi}\hat{z} \times \frac{\boldsymbol{r} - \boldsymbol{r}_v}{|\boldsymbol{r} - \boldsymbol{r}_v|^2}$$

where

$$\kappa = \pm q \frac{h}{m}, \quad q \in \mathbb{N}$$

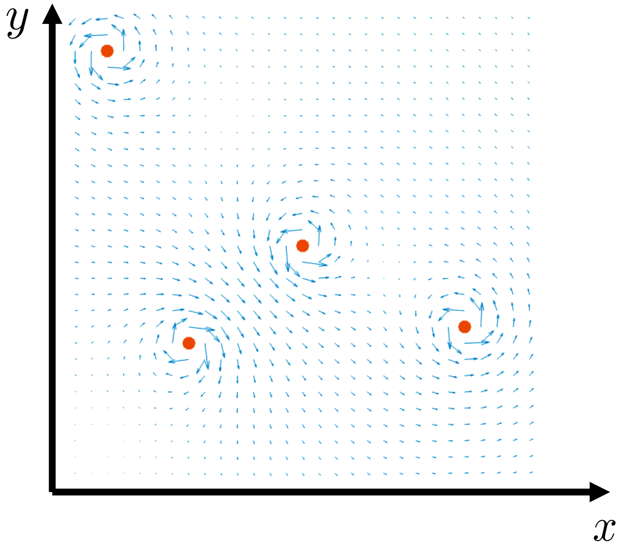
is the vortex strength.



Velocity diverges at the core!

Principle of superposition of the velocity fields

In a many-vortex system the total velocity field reads



$$\boldsymbol{v}(\boldsymbol{r}) = \sum_{i=1}^{N_v} \left[\frac{\kappa_i}{2\pi} \hat{z} \times \frac{\boldsymbol{r} - \boldsymbol{r}_i}{|\boldsymbol{r} - \boldsymbol{r}_i|^2} \right]$$

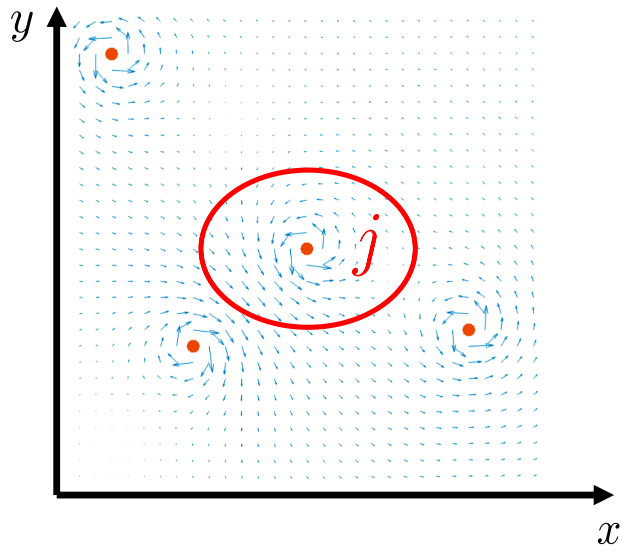
where

$$\kappa_i = \pm q \frac{h}{m}, \quad q \in \mathbb{N}$$

is the strength of the i-th vortex.

Principle of superposition of the velocity fields

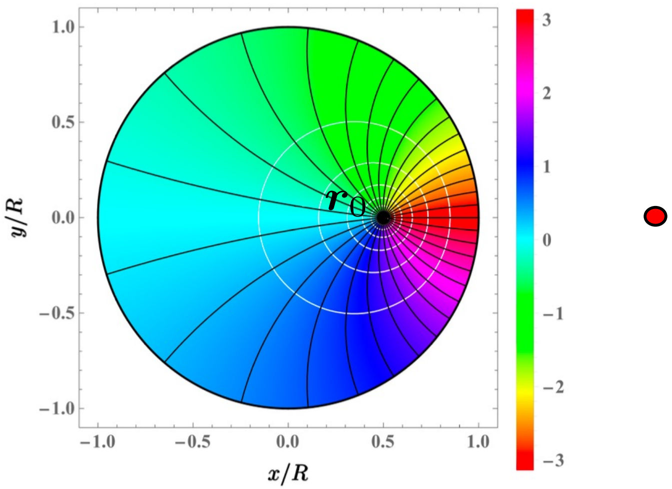
The j-th vortex moves under the influence of the remaining $N_v - 1$ vortices



$$\boldsymbol{v}(\boldsymbol{r}_j) = \sum_{i \neq j} \left[\frac{\kappa_i}{2\pi} \hat{z} \times \frac{\boldsymbol{r}_j - \boldsymbol{r}_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2} \right]$$

where the self-contribution $i = j$ has been removed from the summation.

Boundaries and image vortices



In the case of a disk-like domain, if a vortex is present at

$$\boldsymbol{r}_0 = (x_0,\, y_0)$$

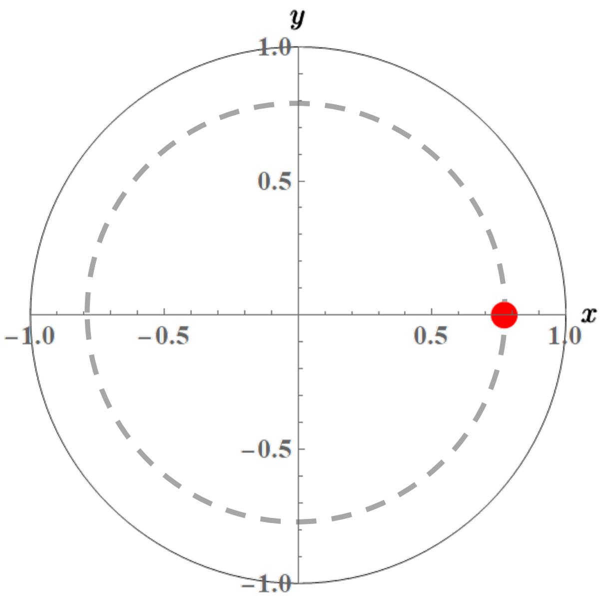
an image vortex is automatically present at

$$x_0' = \frac{R^2}{x_0^2 + y_0^2} x_0, \qquad y_0' = \frac{R^2}{x_0^2 + y_0^2} y_0$$

and has **opposite charge**.

This ensures that **streamlines are tangent** to the circular boundary and **constant-phase lines are perpendicular** to it.

Equation of motion



$$\dot{\boldsymbol{r}}_0 = \hat{z} \times \frac{\hbar}{m} \frac{\boldsymbol{r}_0}{R^2 - r_0^2}$$

The equation of motion is a **first-orde**r differential equation, and its solutions are (trivial) **uniform circular orbits**.

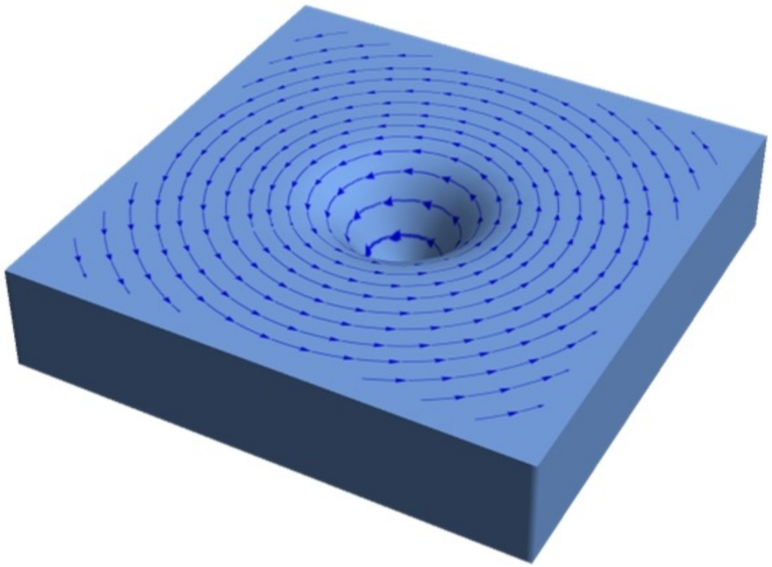
[J.-K. Kim, A. L. Fetter, Phys. Rev. A **70**, 043624 (2004)]

FROM MASSLESS TO MASSIVE VORTICES

[A. Richaud, V. Penna, R. Mayol, M. Guilleumas, Phys. Rev. A **101**, 013630 (2020)]

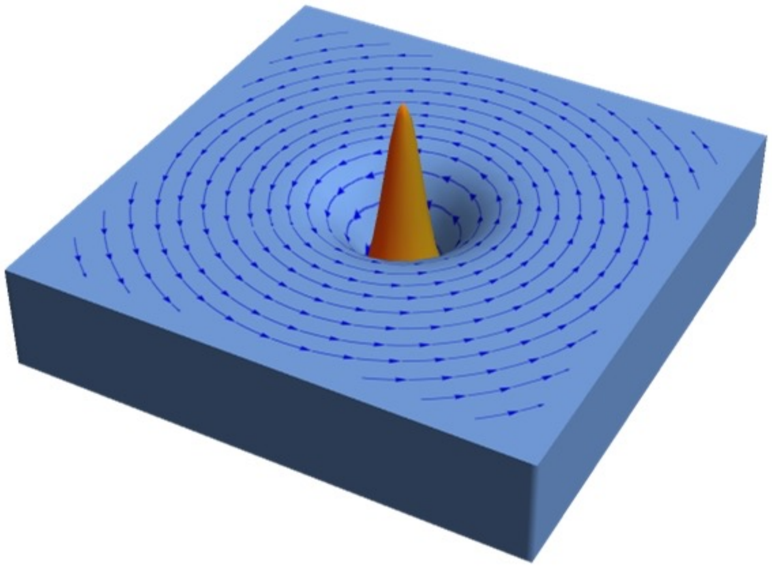
[A. Richaud, V. Penna, A. L. Fetter, Phys. Rev. A **103**, 023311 (2021)]

Vortices: just empty holes?



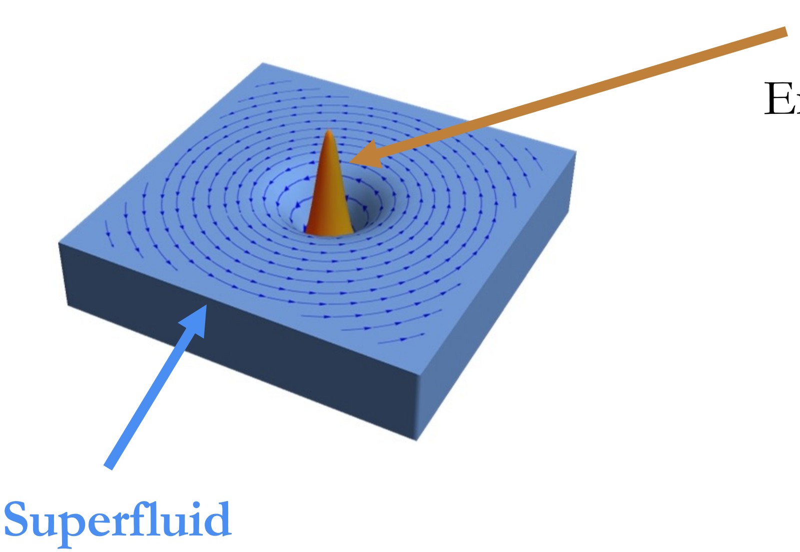
Traditionally, the core is represented as a funnel-like **<u>hole</u>** around which the superfluid exhibits a swirling flow, a sort of *tornado* in the corresponding wavefunction.

Vortices with filled massive cores

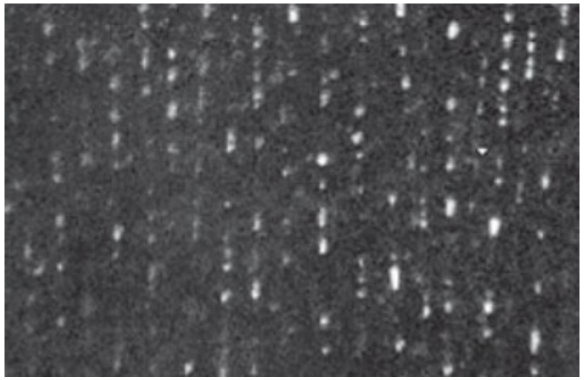


Actually, the vortex core turns out to be commonly filled by particles!

Vortices with filled massive cores

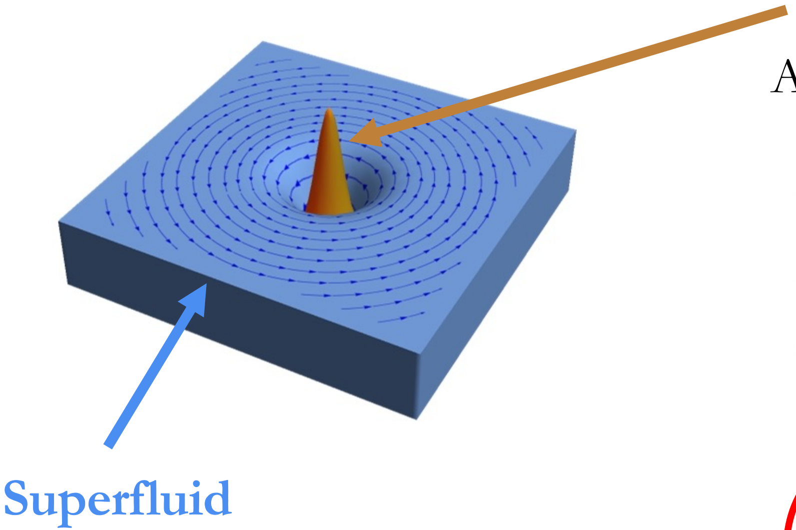


Experimentalists use particles as "vorticity tracers", e.g. in liquid helium.



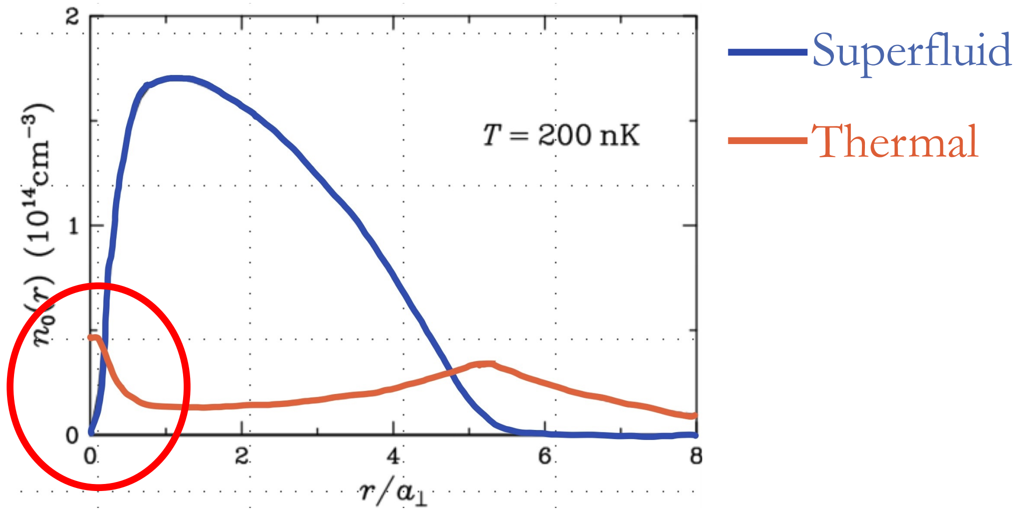
[G. P. Bewley et al., Nature **441**, 588 (2006)]

Vortices with filled massive cores



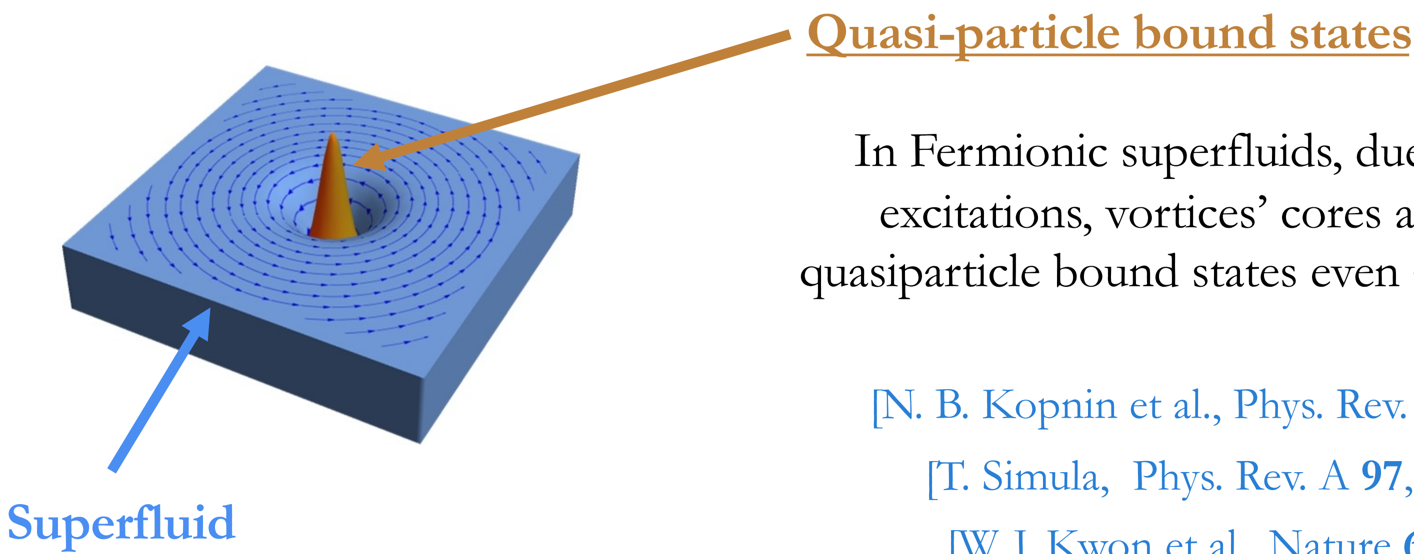
Thermal atoms

Atoms which do not belong to the superfluid fraction.



[A. Griffin, T. Nikuni, E. Zaremba, *Bose-Condensed Gases at Finite Temperature*, Chap. 9, Cambridge University Press (2009)]

Vortices with filled massive cores



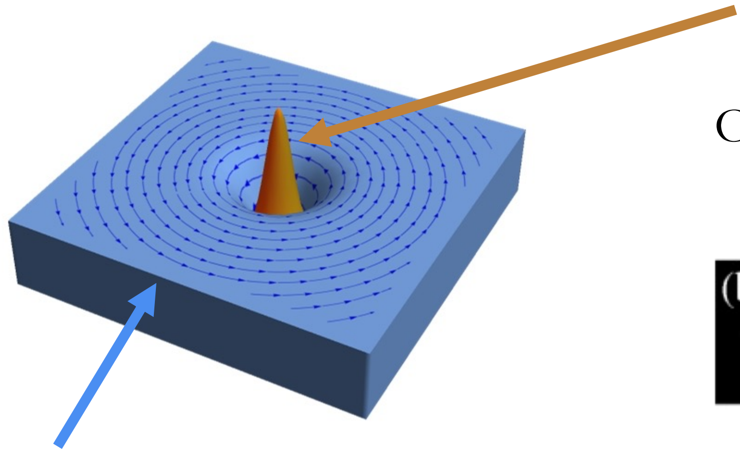
In Fermionic superfluids, due to pair-breaking excitations, vortices' cores are filled up with quasiparticle bound states even at zero temperature.

[N. B. Kopnin et al., Phys. Rev. B **44**, 9667 (1991)]

[T. Simula, Phys. Rev. A **97**, 023609 (2018)]

[W. J. Kwon et al., Nature **600**, 64 (2021)]

Vortices with filled massive cores

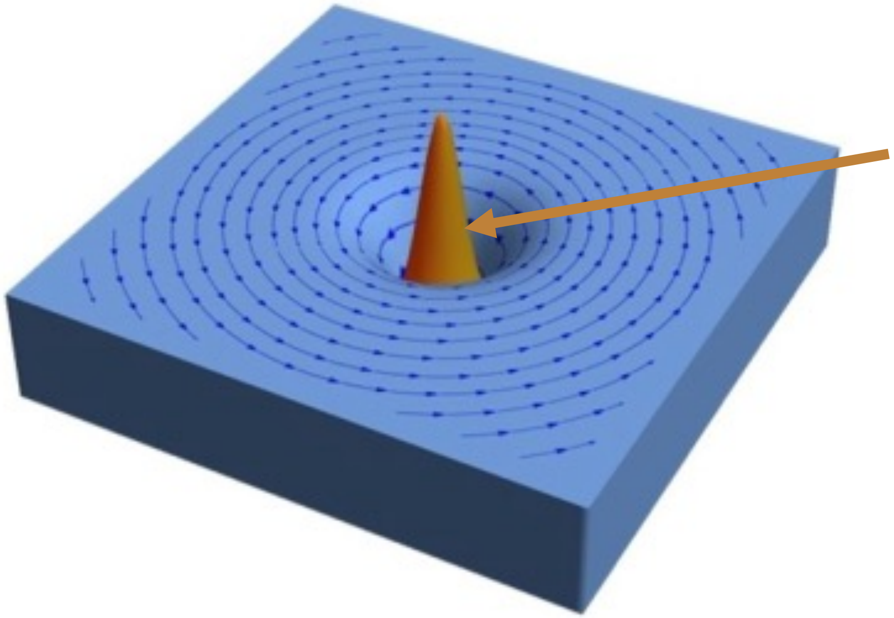


One of the first vortices ever observed in a BEC had a core filled by another component!

The two components were two different internal states of ^{87}Rb.

[B. P. Anderson et al., Phys. Rev. Lett. **85**, 2857 (2000)]

Vortices with filled massive cores

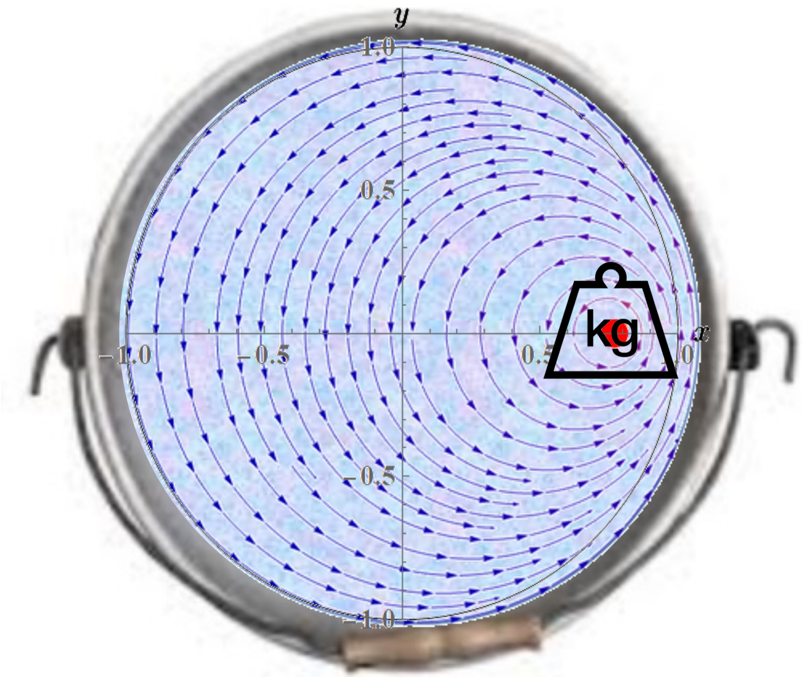


- **Tracer particles**
- **Thermal atoms**
- **Quasiparticle bound states**
- **Another (minority) BEC**

Superfluid vortices are often filled by massive cores (deliberately or accidentally!)

Massive Point Vortex Model

The Lagrangian of massive vortex in a disk can be derived in a rigorous way:



Start from the Lagrangian of a massless vortex in a disk:

$$L_a = \hbar n_a \pi (\dot{\boldsymbol{r}}_0 \times \boldsymbol{r}_0 \cdot \hat{z}) \frac{r_0^2 - R^2}{r_0^2} - \frac{\hbar^2 n_a \pi}{m_a} \log\left(1 - \frac{r_0^2}{R^2}\right)$$

Write the Lagrangian ensuing from the inertial contribution of the core:

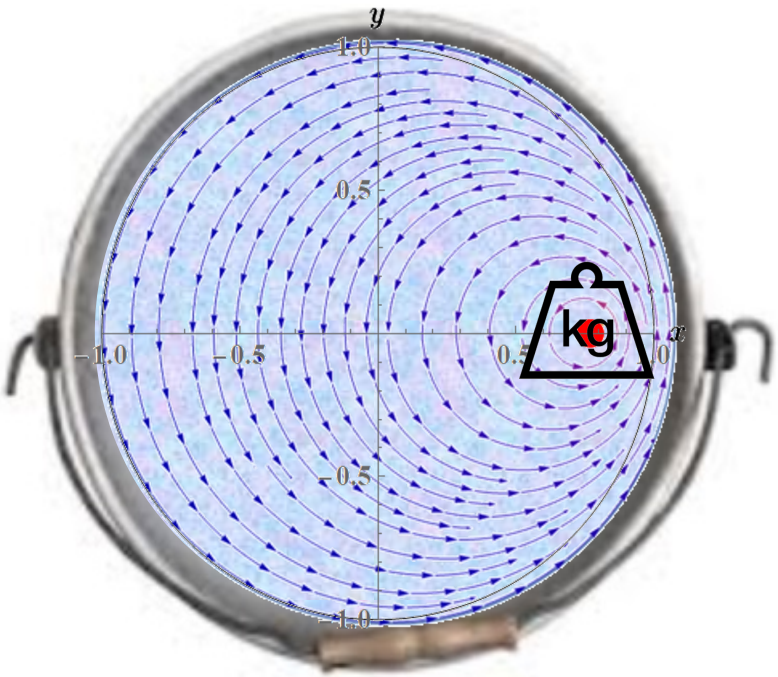
$$L_b = \frac{1}{2} M_b \dot{\boldsymbol{r}}_0^2$$

Recognize that the total Lagrangian of the system is:

$$L = L_a + L_b$$

[A. Richaud, V. Penna, A. L. Fetter, Phys. Rev. A **103**, 023311 (2021)]

Massive Point Vortex Model



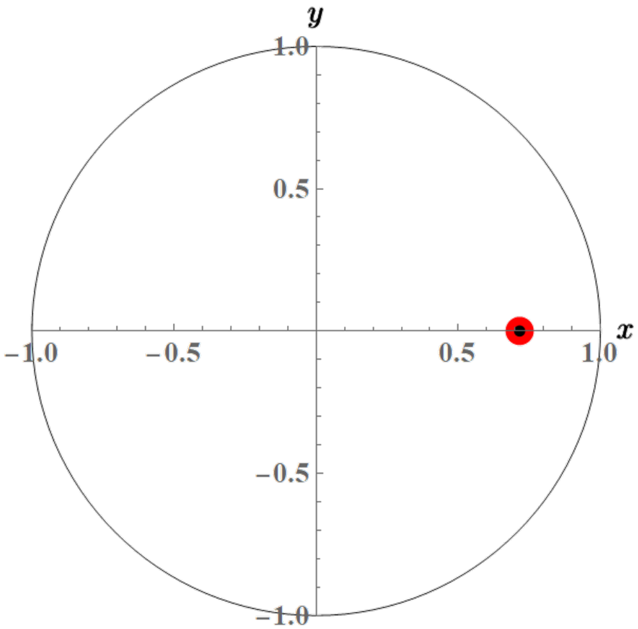
$$L = L_a + L_b$$

Compute the associated Euler-Lagrange equations:

$$M_b \ddot{\boldsymbol{r}}_0 = 2 n_a \pi \hbar \left[\hat{z} \times \dot{\boldsymbol{r}}_0 + \frac{\hbar}{m_a} \frac{\boldsymbol{r}_0}{R^2 - r_0^2} \right]$$

- **This is a second-order equation of motion: the introduction of mass is a singular perturbation.**
- **The number of dynamical variables associated to each vortex doubles!**

Massive Point Vortex Model



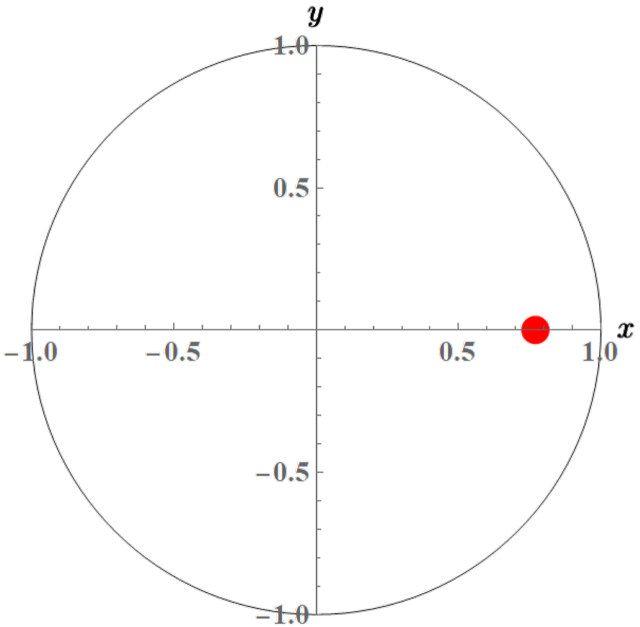
$$L = L_a + L_b$$

Compute the associated Euler-Lagrange equations:

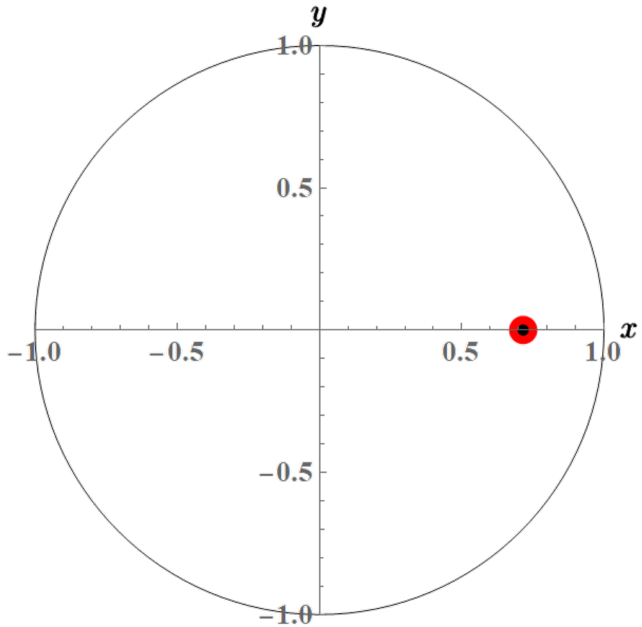
$$M_b \ddot{\boldsymbol{r}}_0 = 2 n_a \pi \hbar \left[\hat{z} \times \dot{\boldsymbol{r}}_0 + \frac{\hbar}{m_a} \frac{\boldsymbol{r}_0}{R^2 - r_0^2} \right]$$

These equations tell us that the motion is not simply a uniform circular one!

Massless vs Massive Vortices



Massless → Only uniform circular orbits



Massive → **Radial oscillations** superimposed to circular orbits.

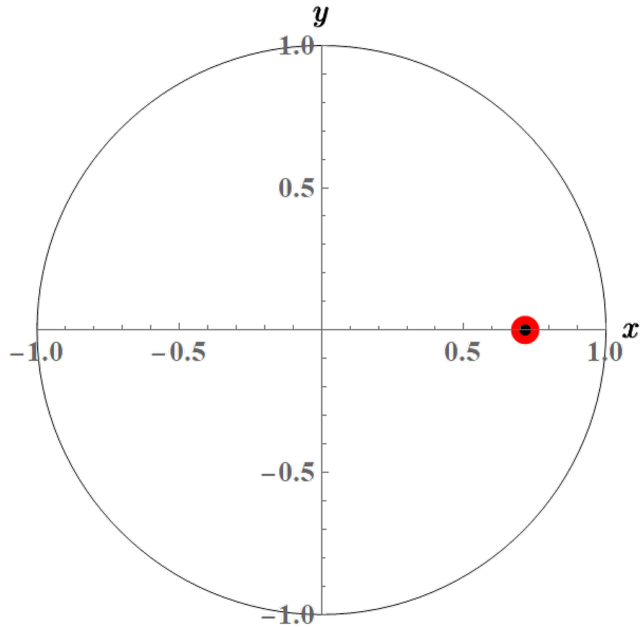
Transverse oscillation frequency as mass signature

The frequency ω of radial oscillations is **inversely proportional** to the core mass:

$$\omega = \frac{\hbar}{m_a R^2} \frac{2}{\mu} \sqrt{1 - \mu \frac{2 - \tilde{r}_0^2}{(1 - \tilde{r}_0^2)^2}}.$$

where $\tilde{r}_0 = r_0/R$ and $\mu = M_b/M_a$.

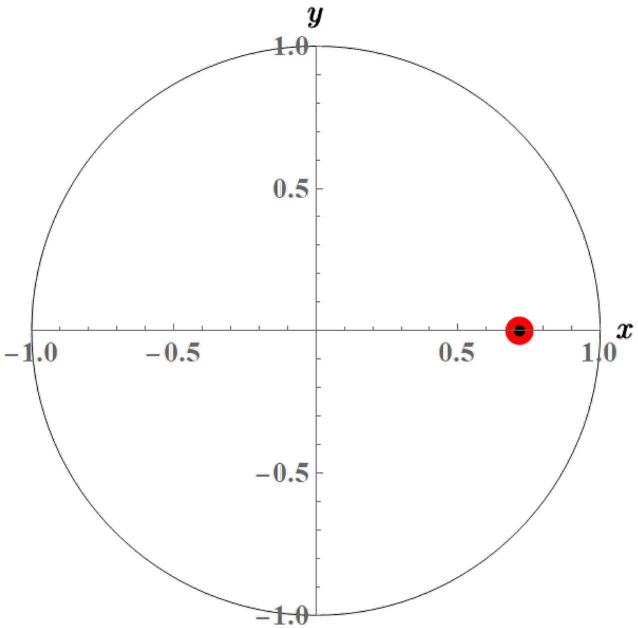
Typical signature of a **singular perturbation**.



[A. Richaud, V. Penna, A. L. Fetter, Phys. Rev. A **103**, 023311 (2021)]

Massive → **Radial oscillations** superimposed to circular orbits.

Magnus effect and Magnus force



[A. Richaud, P. Massignan, V. Penna, and A. L. Fetter, Phys. Rev. A **106**, 063307 (2022)]

The equation of motion of a massive vortex

$$M_b \ddot{\boldsymbol{r}}_0 = 2n_a \pi \hbar \left[\hat{z} \times \dot{\boldsymbol{r}}_0 + \frac{\hbar}{m_a} \frac{\boldsymbol{r}_0}{R^2 - r_0^2} \right]$$

can be rewritten as

$$M_b \ddot{\boldsymbol{r}}_0 = \boldsymbol{F}^M$$

where

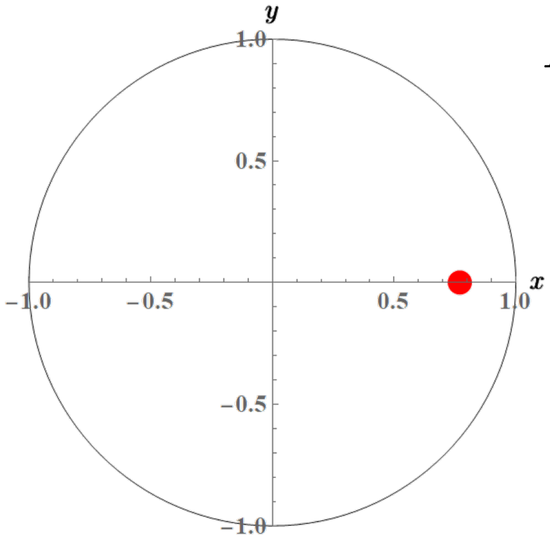
$$\boldsymbol{F}^M = 2n_a \pi \hbar (\boldsymbol{v}_s - \dot{\boldsymbol{r}}_0) \times \hat{\boldsymbol{z}},$$

is the **Magnus force**, proportional to the difference between the actual vortex velocity, $\dot{\boldsymbol{r}}_0$ and the local superfluid velocity $\boldsymbol{v}_s$ (simply induced by the image vortex).

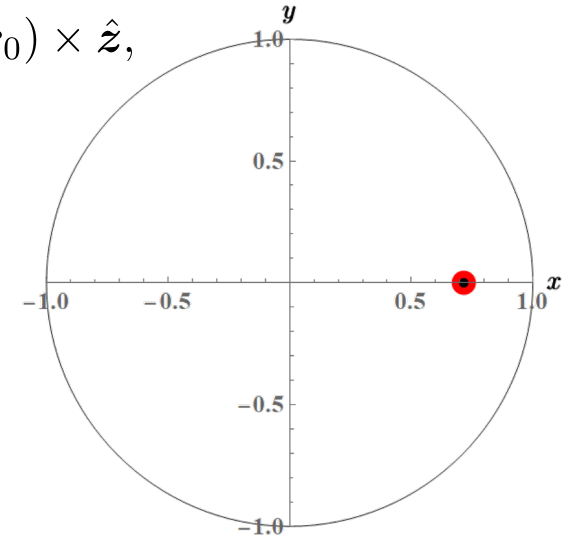
Magnus effect and Magnus force

$$M_b \ddot{\boldsymbol{r}}_0 = \boldsymbol{F}^M$$

$$\boldsymbol{F}^M = 2 n_a \pi \hbar (\boldsymbol{v}_s - \dot{\boldsymbol{r}}_0) \times \hat{\boldsymbol{z}},$$

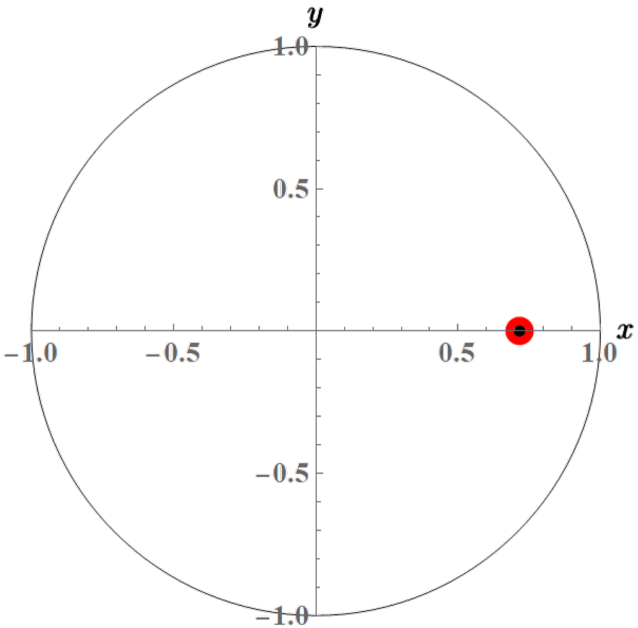


A **massless** vortex moves with the local superfluid velocity not to be subject to any net force. ← **Magnus effect.**



A **massive** vortex moves according to Newton's second law, where **F** is the Magnus force.

Vortices as interacting point charges subject to a transverse magnetic field



The equation of motion of a single massive vortex

$$M_b \ddot{\boldsymbol{r}}_0 = 2n_a \pi \hbar \left[\hat{z} \times \dot{\boldsymbol{r}}_0 + \frac{\hbar}{m_a} \frac{\boldsymbol{r}_0}{R^2 - r_0^2} \right]$$

can be also rewritten as

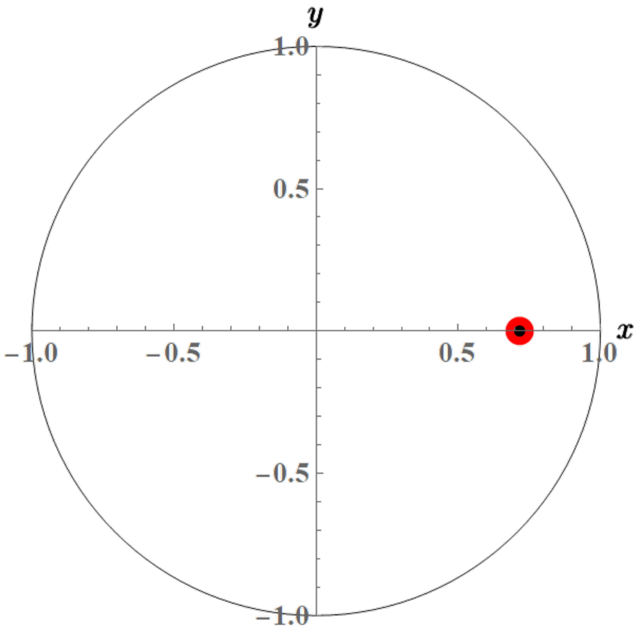
$$M_b \ddot{\boldsymbol{r}}_0 = \underbrace{\kappa \dot{\boldsymbol{r}}_0 \times (-m_a n_a \hat{z})}_{\text{Lorentz-like term}} + \underbrace{\frac{m_a n_a}{2\pi} \kappa \kappa' \frac{\boldsymbol{r}_0 - \boldsymbol{r}_0'}{|\boldsymbol{r}_0 - \boldsymbol{r}_0'|^2}}_{\text{Coulomb (2D) – like term}}$$

where $\boldsymbol{r}_0' = \dfrac{R^2}{r_0^2} \boldsymbol{r}_0$ and $\kappa' = -\kappa$

Lorentz-like term   Coulomb (2D) – like term

[A. Richaud, P. Massignan, V. Penna, and A. L. Fetter, Phys. Rev. A **106**, 063307 (2022)]

Vortices as interacting point charges subject to a transverse magnetic field



$$M_b \ddot{\boldsymbol{r}}_0 = \underbrace{\kappa \dot{\boldsymbol{r}}_0 \times (-m_a n_a \hat{z})}_{\text{Lorentz-like term}} + \underbrace{\frac{m_a n_a}{2\pi} \kappa \kappa' \frac{\boldsymbol{r}_0 - \boldsymbol{r}_0'}{|\boldsymbol{r}_0 - \boldsymbol{r}_0'|^2}}_{\text{Coulomb (2D) – like term}}$$

where $\boldsymbol{r}_0' = \dfrac{R^2}{r_0^2}\boldsymbol{r}_0$ and $\kappa' = -\kappa$

Lorentz-like term

Coulomb (2D) – like term

A massive vortex is formally equivalent to a massive particle of charge κ subject to an electric field (generated by all the other vortices, be them real or virtual) and a transverse magnetic field $\boldsymbol{B} = -m_a n_a \hat{z}$.

[A. Richaud, P. Massignan, V. Penna, and A. L. Fetter, Phys. Rev. A **106**, 063307 (2022)]

INTRINSIC MASS OF A VORTEX IN FERMI SUPERFLUID

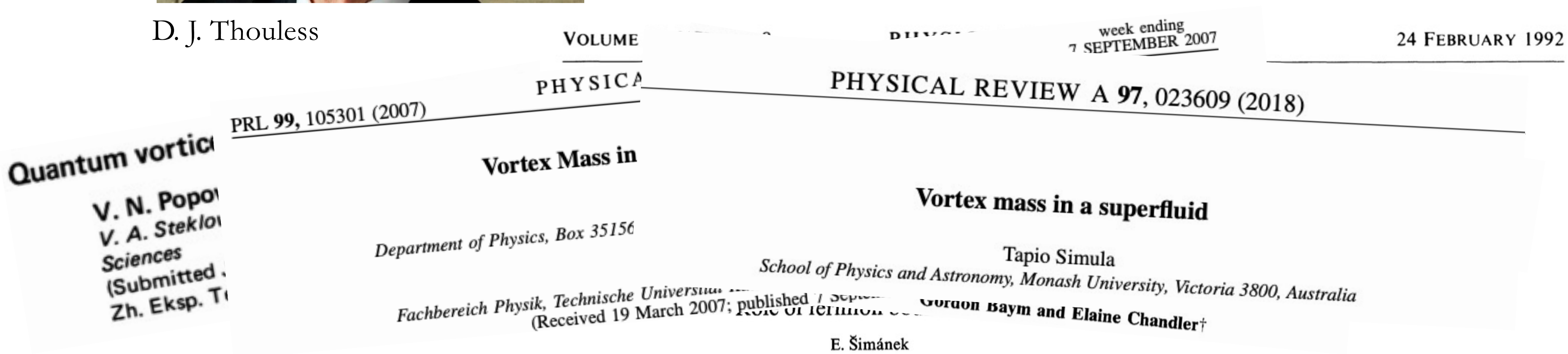
[A. Richaud, M. Caldara, M. Capone, P. Massignan, G. Wlazłowski, arXiv:2410.12417]

A long-standing theoretical puzzle…

D. J. Thouless

"Conflicting results on the mass of a quantized vortex in a neutral superfluid can be found in the literature. Popov and Duan argue that the mass per unit length is infinite, while Baym and Chandler argue that it is negligible".

[D. J. Thouless, J. R. Anglin, Phys. Rev. Lett. 99, 105301 (2007)]



…and experimental milestones

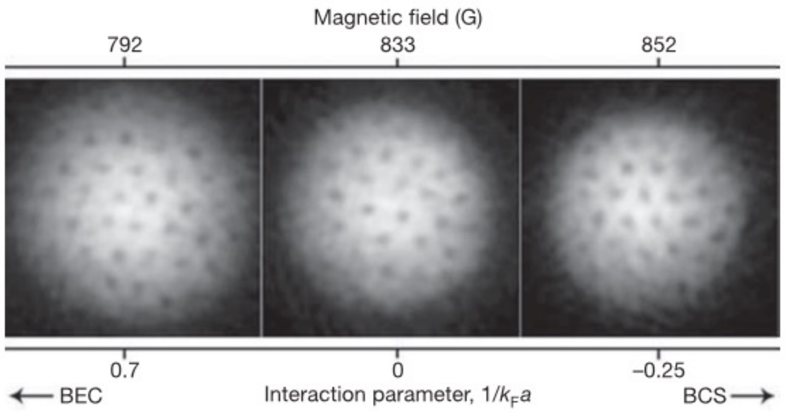
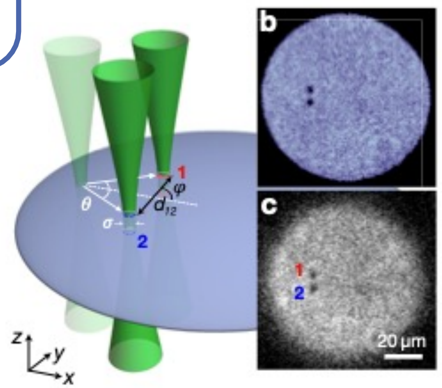


Figure 3 | Optimized vortex lattices in the BEC-BCS crossover. After a

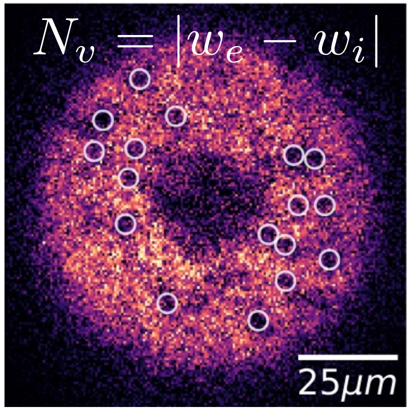
[M. W. Zwierlein, J. R. Abo-Shaeer, A. Schirotzek, C. H. Schunck & W. Ketterle, Nature **435**, 1047 (2005)]

…and experimental milestones (LENS)

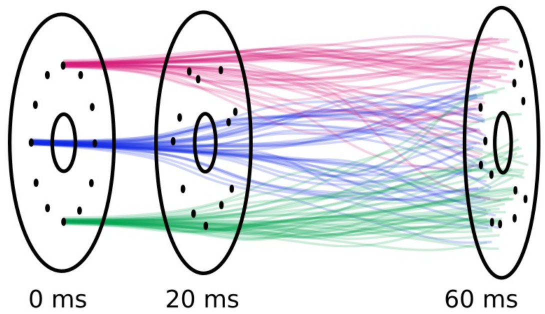
^{6}Li



Generation of vortices



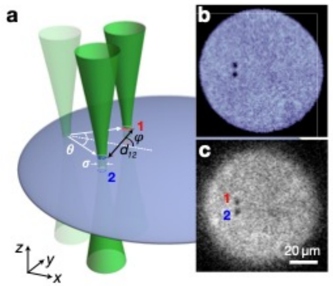
In-situ imaging and tracking of each vortex.



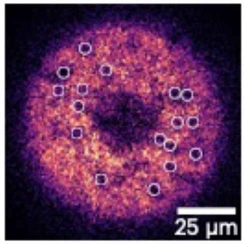
Post-processing of the extracted trajectories.

…and experimental milestones

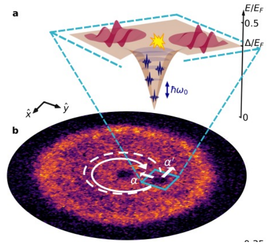
Possibility to do high-precision experiments on vortex dynamics in strongly interacting Fermi superfluids [as shown by Giulia del Pace]



[W. J. Kwon, G. Del Pace, K. Xhani, L. Galantucci, A. Muzi Falconi, M. Inguscio, F. Scazza & G. Roati, Nature **600**, 64 (2021)]



[D. Hernández-Rajkov, N. Grani, F. Scazza, G. Del Pace, W. J. Kwon, M. Inguscio, K. Xhani, C. Fort, M. Modugno, F. Marino & G. Roati, Nat. Phys. 20, **939** (2024)].

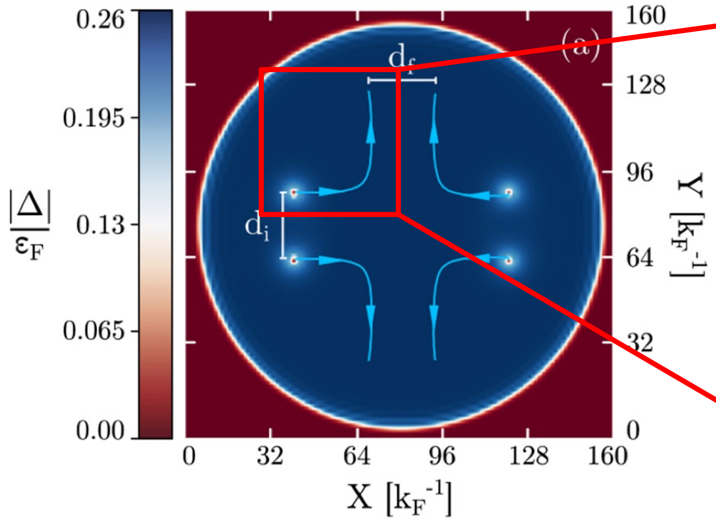


[N. Grani, D. Hernández-Rajkov, C. Daix, P. Pieri, M. Pini, P. Magierski, G. Wlazłowski, M. Frómeta Fernández, F. Scazza, G. Del Pace, G. Roati, arXiv:2503.21628].

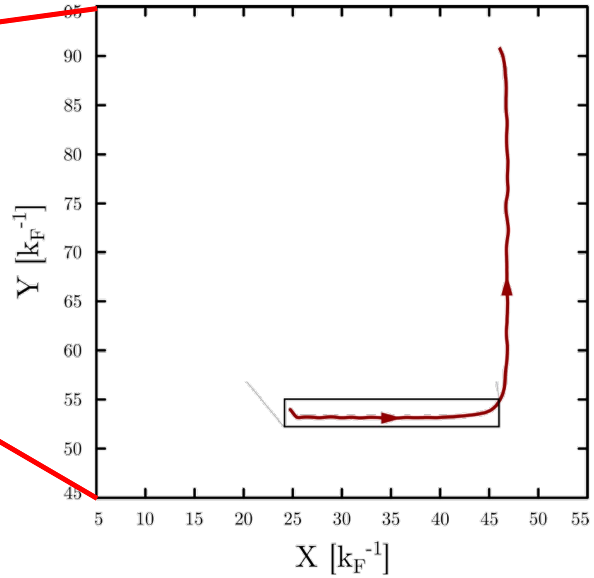
→ See talk by Diego Hernandez Rajkov (Thursday)

A pleasant discovery

Gabriel Wlazłowski
Warsaw University



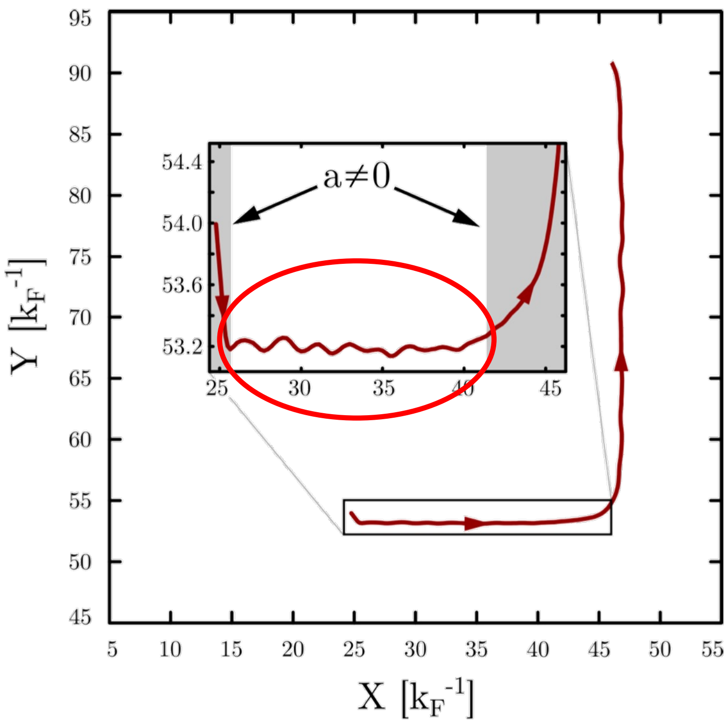
An head-on collision of vortex dipoles. Compare d_i and d_f to quantify the dissipated energy.



Trajectory of the top left vortex. Magnification showed **oscillations**!

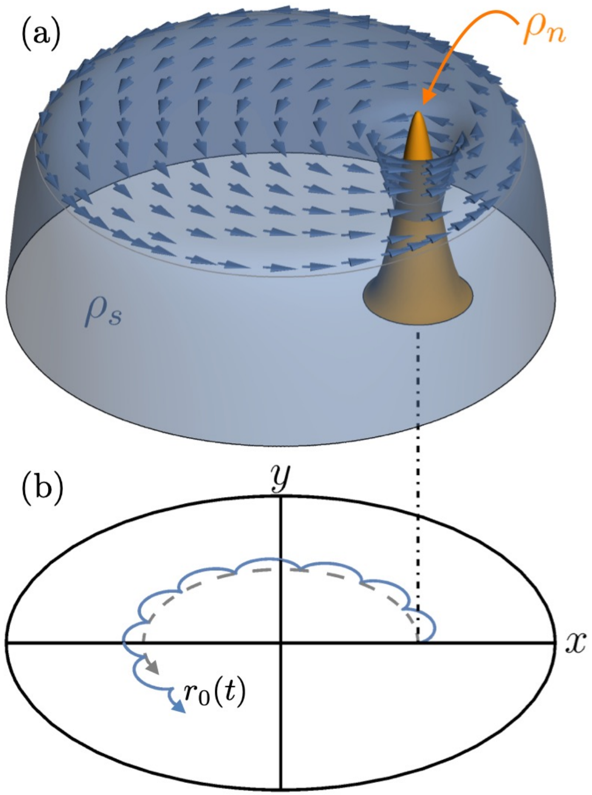
[A. Barresi, A. Boulet, P. Magierski, G. Wlazłowski, Phys. Rev. Lett. **130**, 043001 (2023)]

A pleasant discovery



Are these **transverse oscillations** the dynamical manifestation of the vortex **intrinsic inertia**?

Detailed analysis in a controlled setup

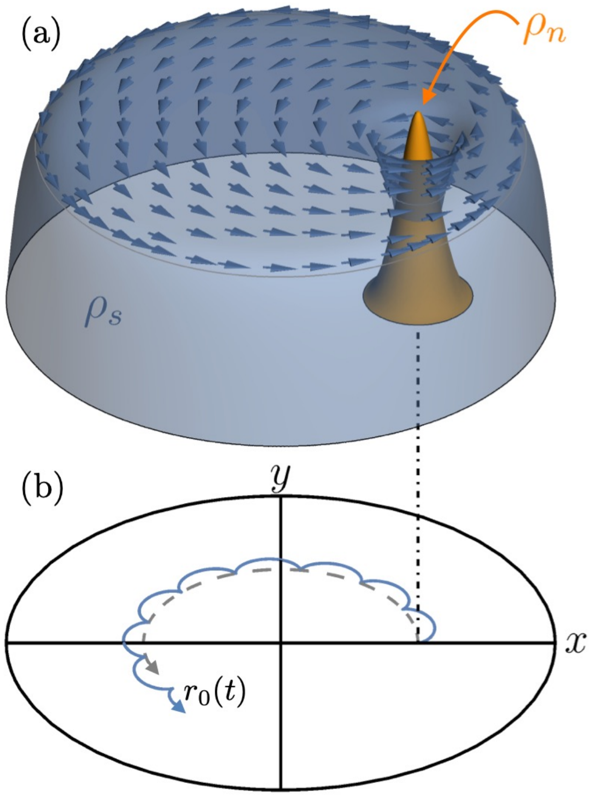


An effectively 2D superfluid (blue) confined in a disk-like trap and hosting an off-centered vortex whose core is filled by some normal component (orange).

Massless vortex → uniform circular orbit.

Massive vortex → superimposed radial oscillations leading to a characteristic "cyclotron" motion.

Time-dependent numerical simulations



The Fermi gas is modelled using a Density Functional Theory approach.

Time-dependent Superfluid Local Density Approximation (SLDA)
[A. Boulet, G. Wlazlowski, P. Magierski, Phys. Rev. A **106**, 013306 (2022)]

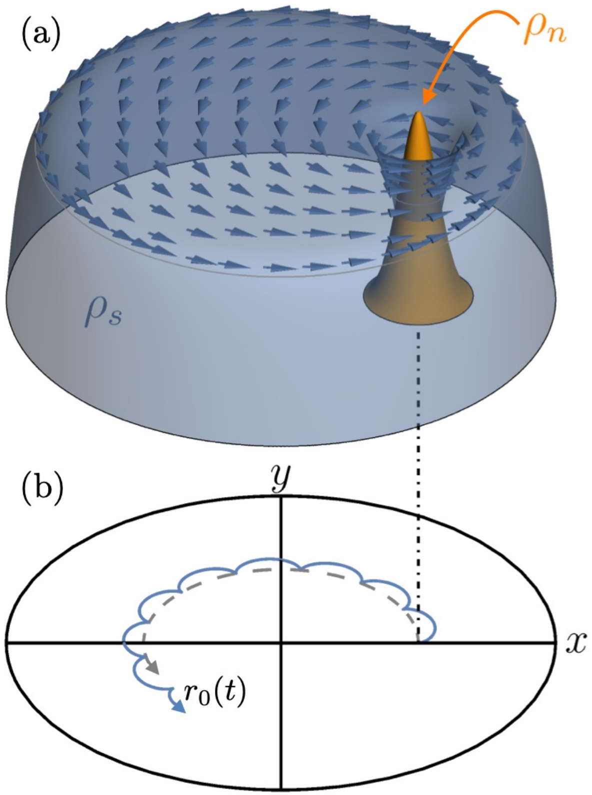
The energy of the system is computed as an integral over the energy density functional

$$E(t) = \int \mathcal{E}[\rho(\boldsymbol{r},t), \tau(\boldsymbol{r},t), \boldsymbol{j}(\boldsymbol{r},t), \nu(\boldsymbol{r},t)]\, \mathrm{d}\boldsymbol{r}$$

that depends on the local densities.

total kinetic current anomalous

Time-dependent numerical simulations



The energy of the system is computed as an integral over the energy density functional

$$E(t) = \int \mathcal{E}[\rho(\boldsymbol{r},t), \tau(\boldsymbol{r},t), \boldsymbol{j}(\boldsymbol{r},t), \nu(\boldsymbol{r},t)]\, \mathrm{d}\boldsymbol{r}$$

that depends on the local densities.

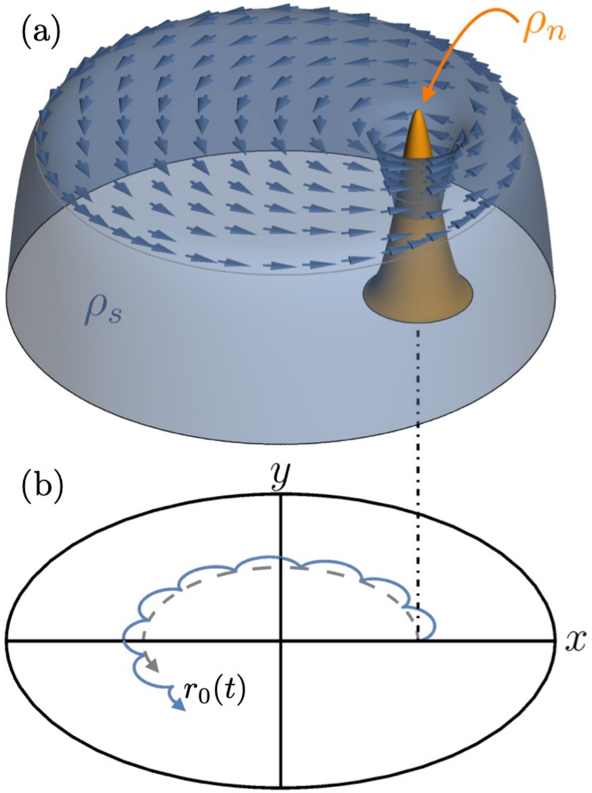
total kinetic current anomalous

These local densities are expressed in terms of Bogoliubov amplitudes: $\{u_n(\boldsymbol{r},t), v_n(\boldsymbol{r},t)\}$

$$\rho = 2 \sum_{0<E_n<E_c} \left[|v_n|^2 f(-E_n) + |u_n|^2 f(E_n) \right],$$

$$\boldsymbol{j} = \frac{2}{m} \sum_{0<E_n<E_c} \mathrm{Im}\left[(v_n \boldsymbol{\nabla} v_n^*) f(-E_n) - (u_n \boldsymbol{\nabla} u_n^*) f(E_n) \right].$$

Time-dependent numerical simulations



The equations of motion are formally equivalent to time-dependent Bogoliubov-de Gennes (BdG) equations:

$$i\frac{\partial}{\partial t}\begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} \hat{h}(\boldsymbol{r},t) & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -\hat{h}^*(\boldsymbol{r},t) \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

where

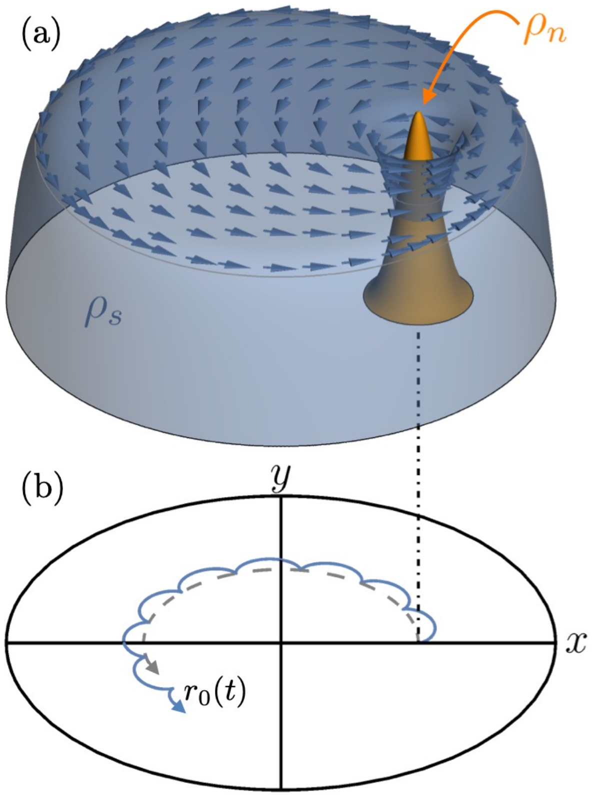
$$\hat{h} = -\frac{\nabla^2}{2m} + U(\boldsymbol{r}) + V_{\text{ext}}(\boldsymbol{r}), \qquad U = \frac{\delta \mathcal{E}}{\delta \rho}$$

$$\Delta = -\frac{\delta \mathcal{E}}{\delta \nu^*}$$

are related to our energy density functional via suitable functional derivatives.

They also depend on Bogoliubov amplitudes and include beyond-mean-field corrections (that depend on k_Fa).

Time-dependent numerical simulations



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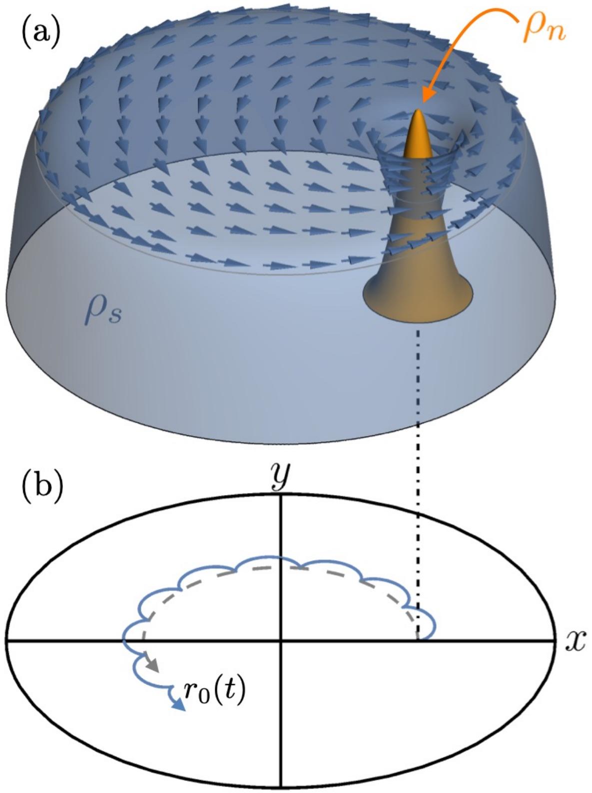
$\Rightarrow$ The resulting equations of motion are highly nonlinear!

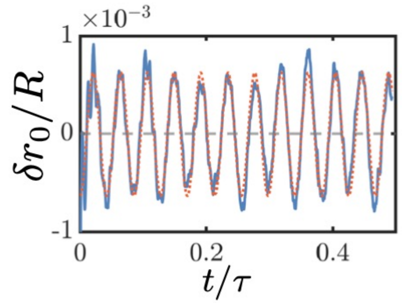
https://wslda.fizyka.pw.edu.pl

LUMI supercomputer

Time-dependent numerical simulations



Measurement of regular radial oscillations:



$k_F a = -0.70$

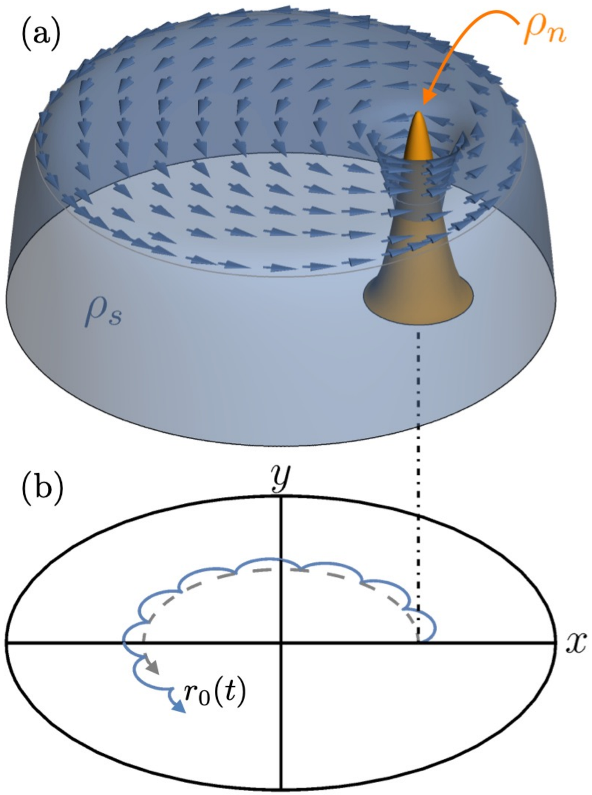
← BCS

UFG →

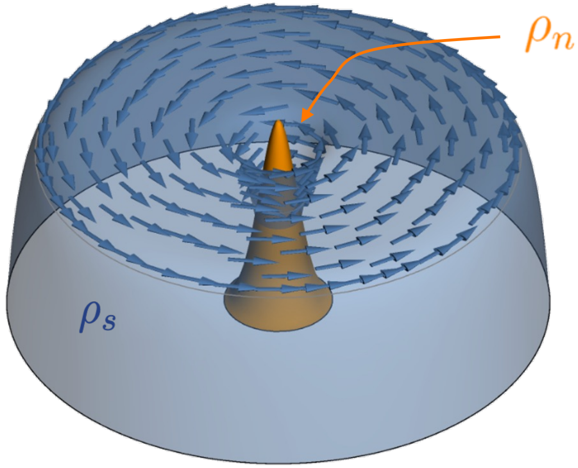
Interaction strength

Dynamics vs Statics

Off-centered vortex



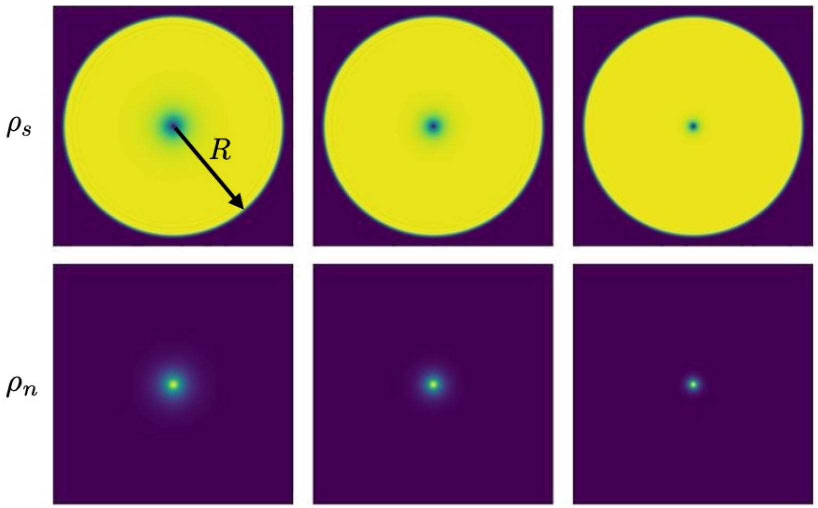
$$\omega(ak_F)$$



Centered vortex

$$\mathfrak{m}(ak_F) = \frac{N_n}{N_s} = \frac{\int \rho_n(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}}{\int \rho_s(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}}$$

Statics

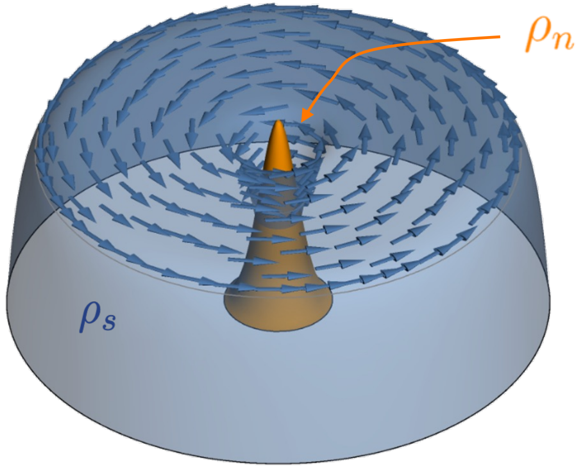


$k_F a = -0.70$ $k_F a = -0.85$ $k_F a = -1.50$

← BCS

Interaction strength

UFG →



Centered vortex

$$\mathfrak{m}(ak_F) = \frac{N_n}{N_s} = \frac{\int \rho_n(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}}{\int \rho_s(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}}$$

Extracting ρ_s and ρ_n

Start from the known total density $\rho(\boldsymbol{r})$, current vector field $\boldsymbol{j}(\boldsymbol{r})$, and superfluid gap $\Delta(\boldsymbol{r})$

↓

Decompose the current as:

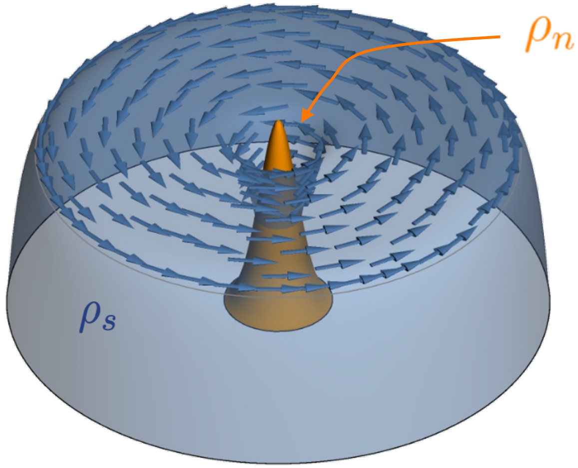
$\boldsymbol{j}(\boldsymbol{r}) = \rho_s \boldsymbol{v}_s + \rho_n \boldsymbol{v}_n$

↓

Assume that $\boldsymbol{v}_n = \boldsymbol{0}$ and recall that

$$\boldsymbol{v}_s = \frac{\hbar}{m^*} \nabla \varphi$$

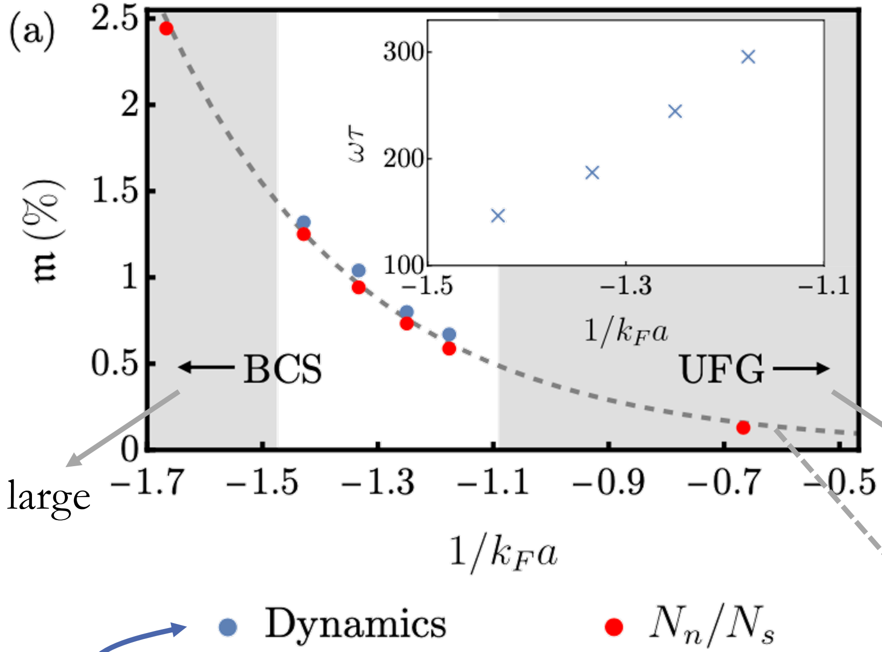
where $\Delta = |\Delta| e^{i\varphi}$ → Compute ρ_s and then $\rho_n = \rho - \rho_s$



Centered vortex

$$\mathfrak{m}(ak_F) = \frac{N_n}{N_s} = \frac{\int \rho_n(\boldsymbol{r})\, \mathrm{d}\boldsymbol{r}}{\int \rho_s(\boldsymbol{r})\, \mathrm{d}\boldsymbol{r}}$$

Comparing Dynamics and Statics



Vortex too large

$$\omega = \frac{1}{\tau}\frac{2}{\mathfrak{m}}\sqrt{1-\mathfrak{m}\frac{2-\tilde{r}_0^2}{(1-\tilde{r}_0^2)^2}}$$

The value of the **vortex mass** as extracted from the **dynamics** is in excellent agreement with the **mass of the normal component** localized in the vortex core.

Vortex too light to be weighted

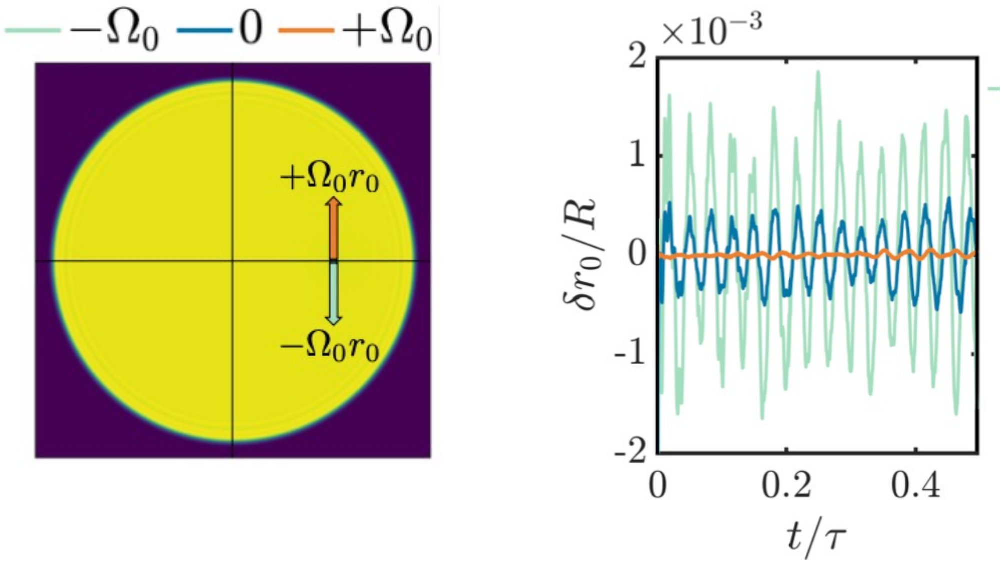
Mass proportional to the vortex core area

$$\mathfrak{m} = \alpha \times (\xi/R)^2$$

$$\xi = \frac{\hbar^2 k_F}{m\pi\bar{\Delta}}$$

Playing with the initial conditions

It is possible to set the **initial velocity** of the vortex.
This allows to **suppress** or to amplify the transverse oscillations.



According to the massive point-vortex model, there exists a specific angular velocity

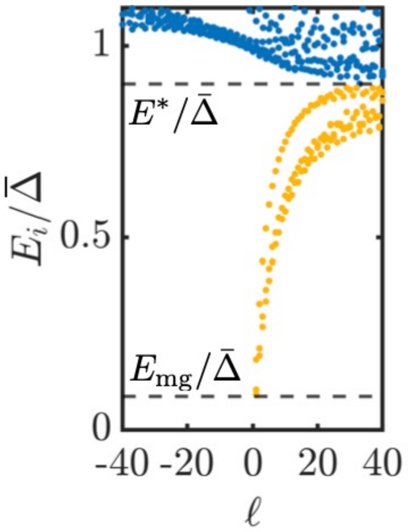
$$\Omega_0(r_0) = \frac{1}{\tau} \frac{2/(1-\tilde{r}_0^2)}{1+\sqrt{1-2\mathfrak{m}/(1-\tilde{r}_0^2)}}$$

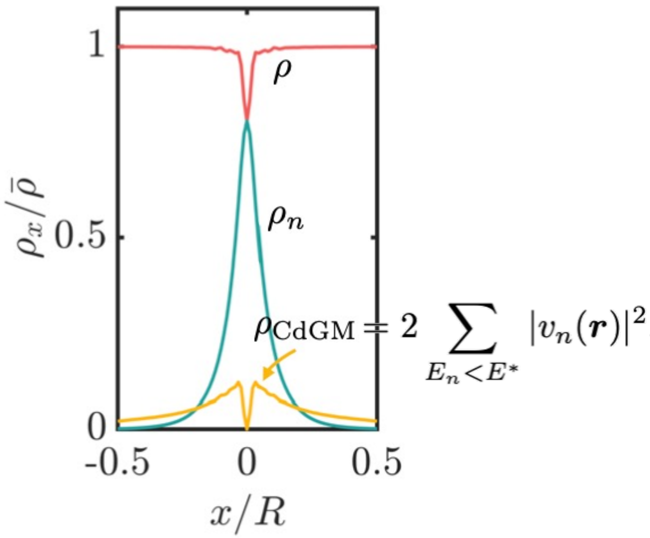
and thus an initial velocity $\dot{\boldsymbol{r}}_0 = \Omega_0 r_0 \,\hat{\phi}$ for which the massive vortex exhibits a smooth, uniform circular motion.

Varying the initial velocity, oscillations share the **same frequency**, but have **different amplitudes**, as expected!

Bridging micro and macro?

What is the microscopic origin of the vortex core mass?
Does it originate from the Caroli-de Gennes-Matricon states localized within the superfluid gap?





ρ_n is not simply related to the density from CdGM core states!

Bridging micro and macro?

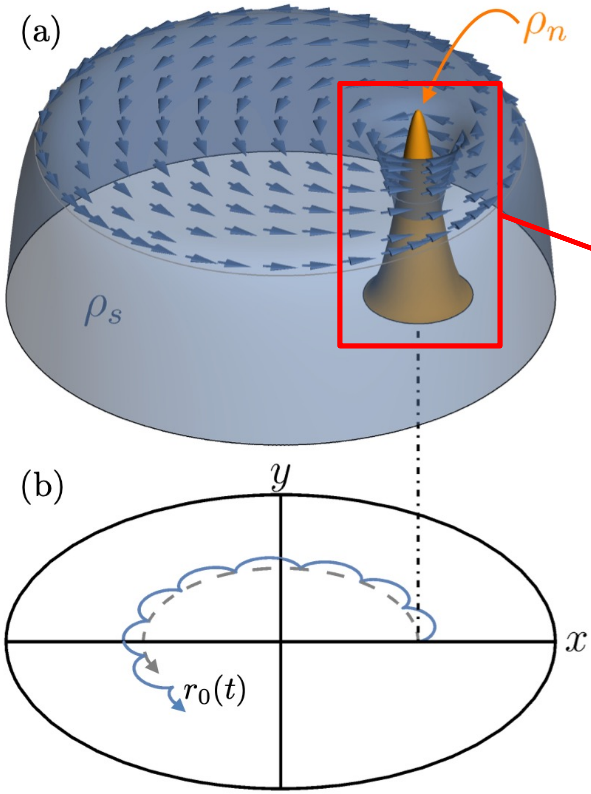
Robert Laughlin (1998)

Nobel Prize in Physics 1998

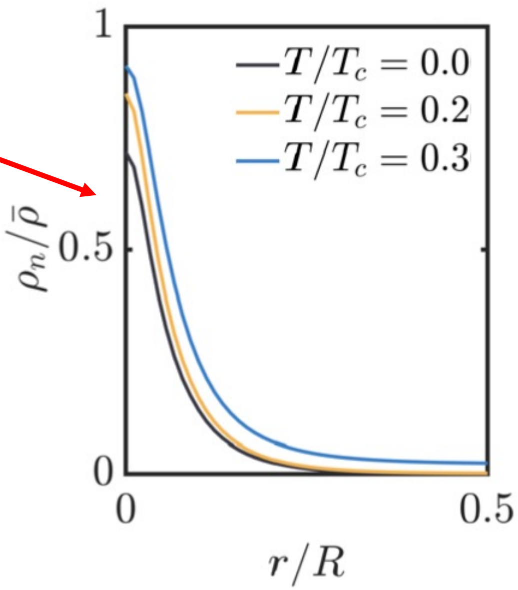
__Nobel Lecture, December 8, 1998:__

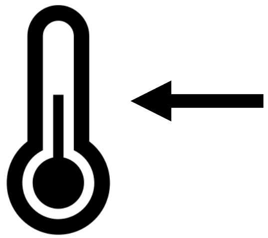
" One of my favorite times in the academic year occurs [..] when I give my class of extremely bright graduate students [..] a take home exam in which they are asked TO DEDUCE SUPERFLUIDITY FROM FIRST PRINCIPLES. There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is IMPOSSIBLE. Superfluidity [..] is an EMERGENT phenomenon – a low energy collective effect of huge number of particles that CANNOT be deduced from the microscopic equations of motion in a RIGOROUS WAY and that DISAPPEARS completely when the system is taken apart. [..]students who stay in physics long enough [..] eventually come to understand that the REDUCTIONIST IDEA IS WRONG. "

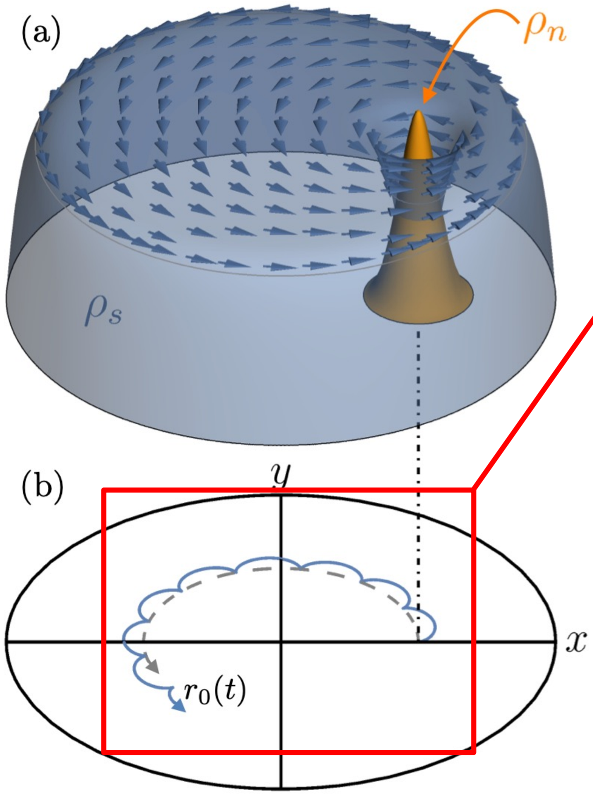
Switching on the temperature



Switching on the temperature, the vortex mass increases, as the amount of ρ_n localized in the vortex core increases.

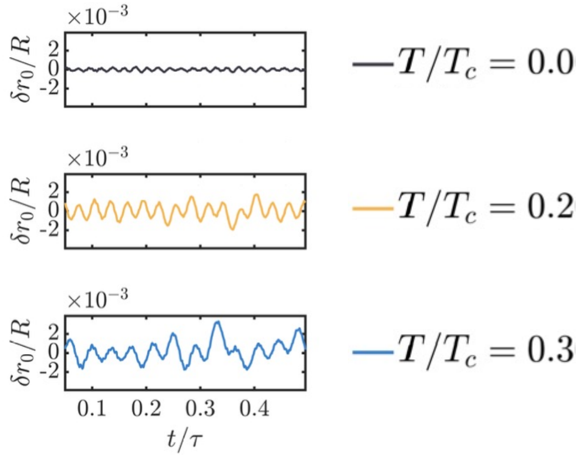


Switching on the temperature 

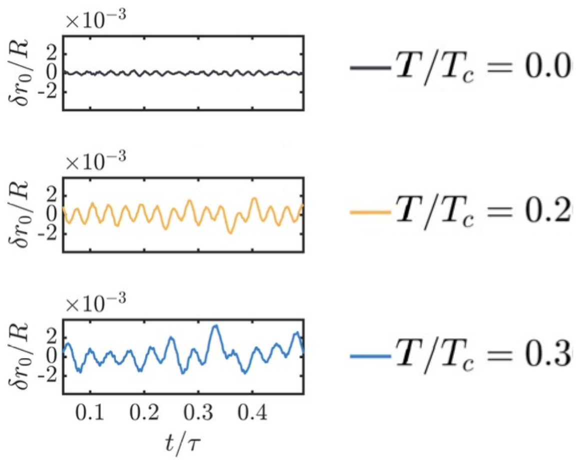


Switching on the temperature, the vortex mass increases, as the amount of ρ_n localized in the vortex core increases.

Accordingly, the frequency of transverse oscillations decreases:



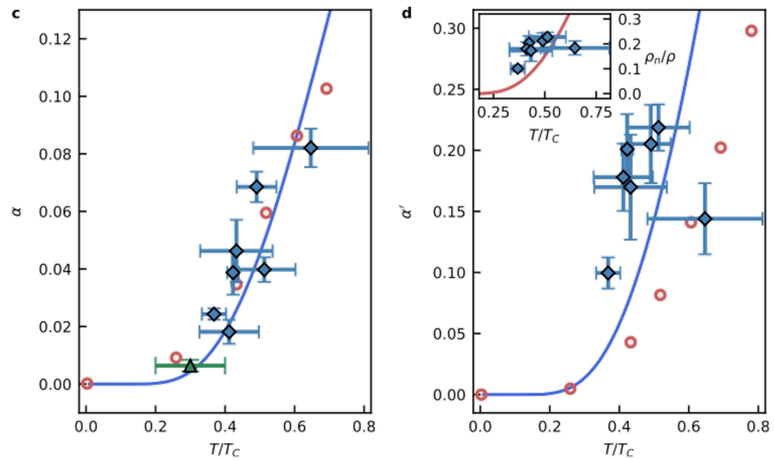
Switching on the temperature



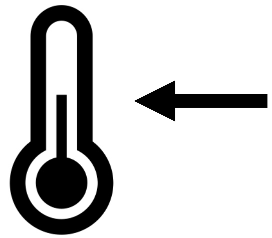
Undamped oscillations, even at finite but small temperatures, indicating that dissipation is not active for $T/T_c < 0.3$.

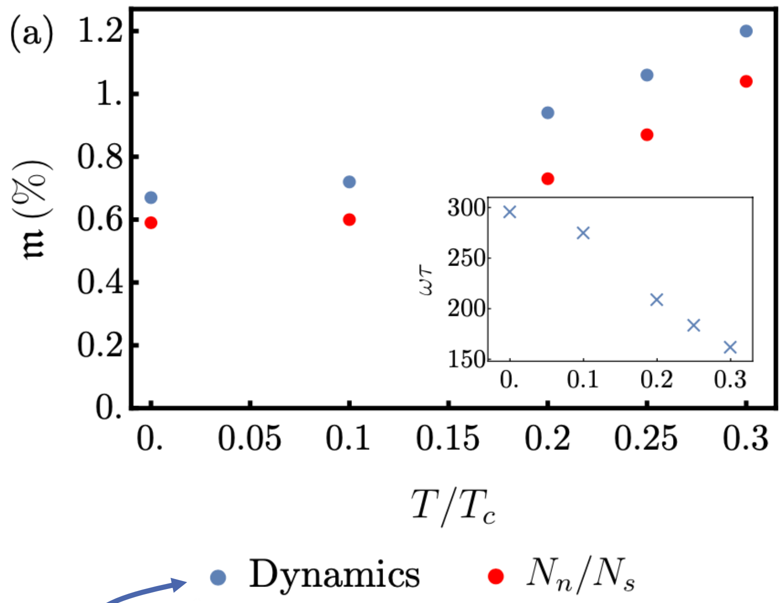
↓

Consistent with the recent results by the LENS group.



[N. Grani, D. Hernández-Rajkov, C. Daix, P. Pieri, M. Pini, P. Magierski, G. Wlazłowski, M. Frómeta Fernández, F. Scazza, G. Del Pace, G. Roati, arXiv:2503.21628].

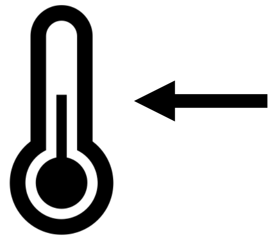
Switching on the temperature 

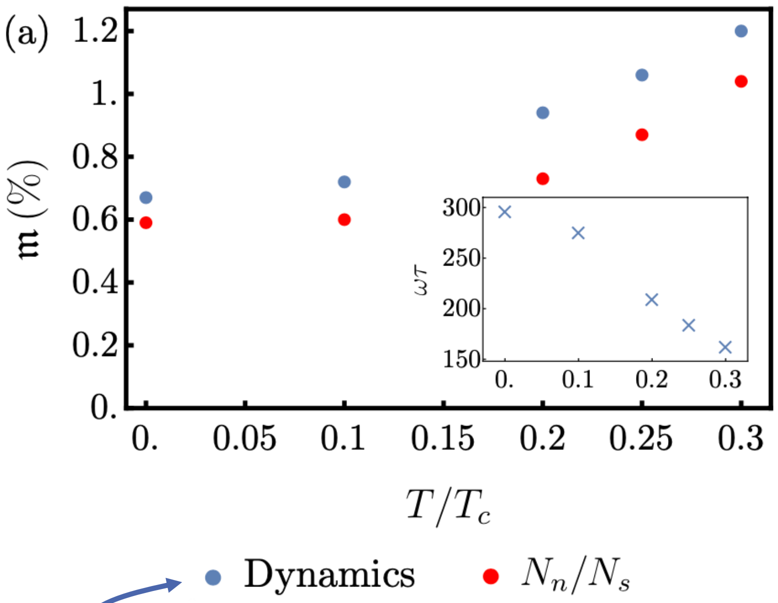


$$\omega = \frac{1}{\tau}\frac{2}{\mathfrak{m}}\sqrt{1-\mathfrak{m}\frac{2-\tilde{r}_0^2}{(1-\tilde{r}_0^2)^2}}$$

The value of the **vortex mass** as extracted from the **dynamics** is in excellent agreement with the **mass of the normal component** localized in the vortex core.

One of the rare cases in which temperature enhances the visibility of the described phenomenon.

Switching on the temperature 

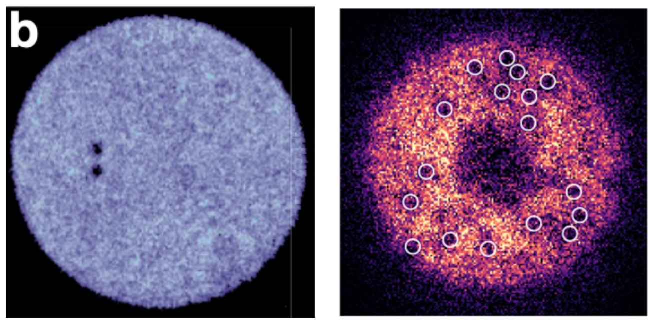


$$\omega = \frac{1}{\tau}\frac{2}{\mathfrak{m}}\sqrt{1-\mathfrak{m}\frac{2-\tilde{r}_0^2}{(1-\tilde{r}_0^2)^2}}$$

Possibility to observe it in precision experiments investigating vortex dynamics in uniform superfluids across the BCS-BEC crossover:

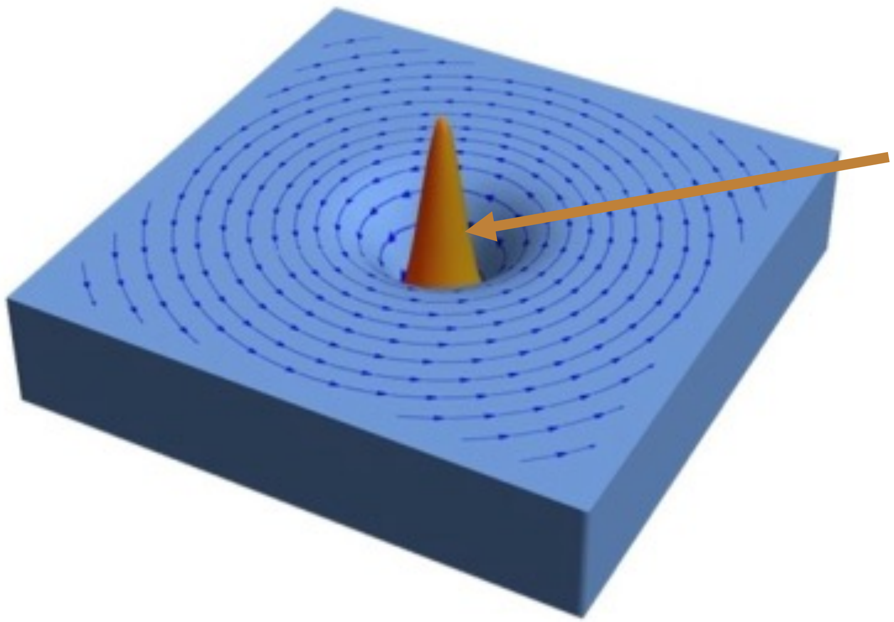
[W. J. Kwon, G. Del Pace, K. Xhani, L. Galantucci, A. Muzi Falconi, M. Inguscio, F. Scazza & G. Roati, Nature **600**, 64 (2021)]

[D. Hernandez-Rajkov et al., Nat. Phys. **20**, 939 (2024)]



CONCLUSIONS

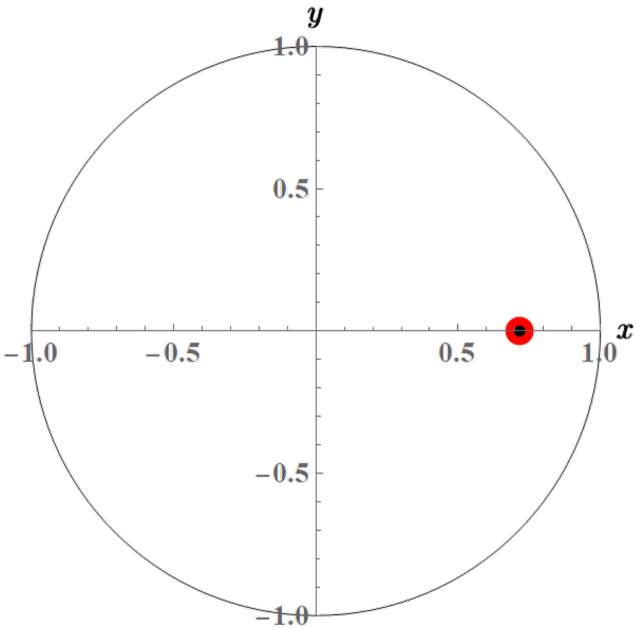
Most real superfluid vortices are massive



- **Tracer particles**
- **Thermal atoms**
- **Normal component**
- **Another (minority) BEC**

Superfluid vortices are often filled by massive cores (deliberately or accidentally!)

Massive Point Vortex Model



The massive point vortex model, unlike its massless counterpart, leads to second-order equations of motion:

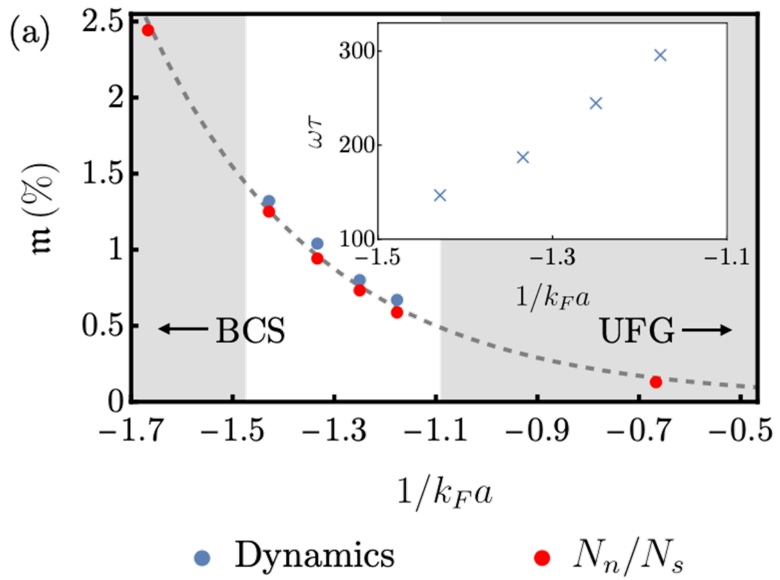
$$M_b \ddot{\boldsymbol{r}}_0 = 2 n_a \pi \hbar \left[\hat{z} \times \dot{\boldsymbol{r}}_0 + \frac{\hbar}{m_a} \frac{\boldsymbol{r}_0}{R^2 - r_0^2} \right]$$

The dynamical signature of vortex mass is represented by small-amplitude transverse oscillations.

[A. Richaud, V. Penna, A. L. Fetter, Phys. Rev. A **103**, 023311 (2021)]

Evidence of intrinsic vortex mass in Fermi superfluids

We have characterized the **intrinsic** mass of a vortex in Fermi superfluids, showing that it originates from the normal component localized in the vortex core.



[A. Richaud, M. Caldara, M. Capone, P. Massignan, G. Wlazłowski, arXiv:2410.12417]