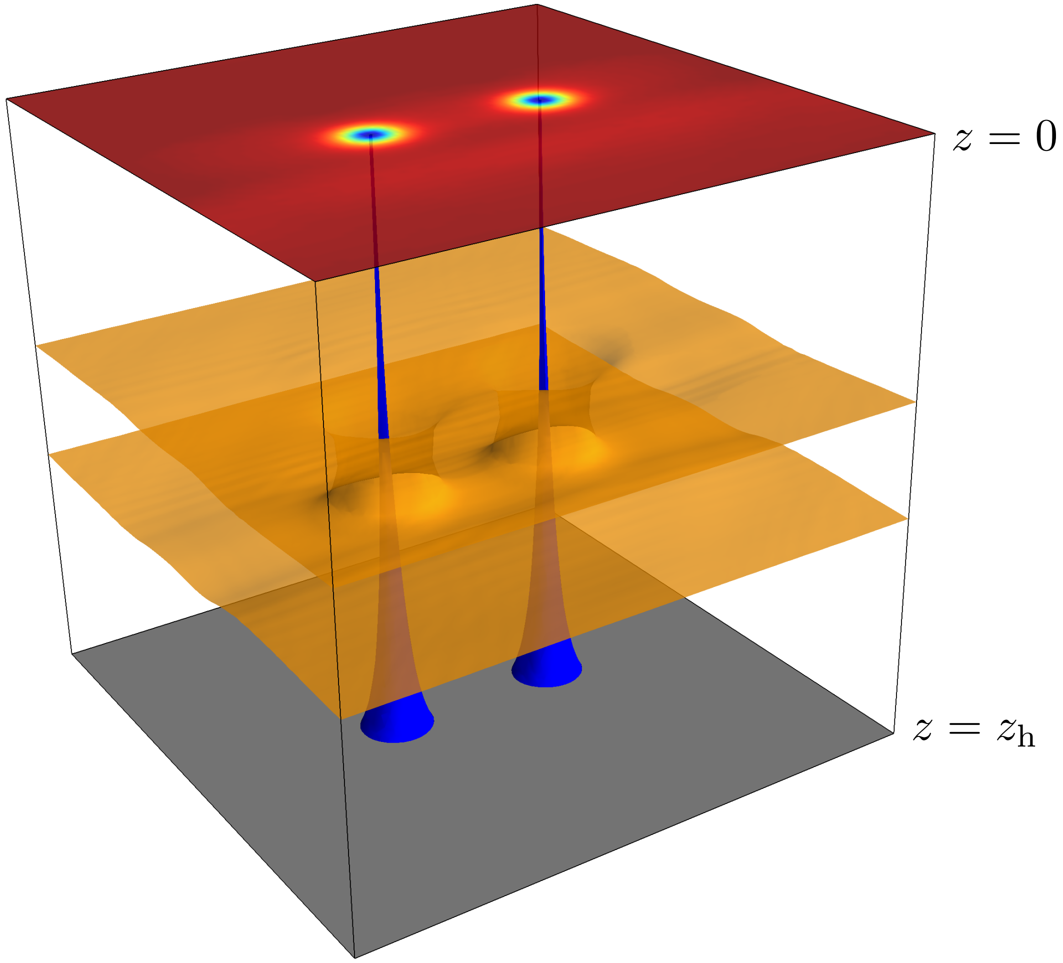
Strongly dissipative vortex dynamics in the holographic superfluid



Carlo Ewerz

Heidelberg University
& EMMI/GSI

Nonequilibrium phenomena
in superfluid systems
ECT*, May 13, 2025

Holographic Duality

AdS/CFT

AdS/CFT Correspondence

Juan Maldacena, 1997

Discovery made with help of string theory

Duality between AdS and CFT:
Anti-de-Sitter space and Conformal Field Theory

AdS/CFT Correspondence

quantum field theory without gravity
in 3+1 dimensions (CFT)

dual to

gravity theory in
4+1 dimensions (AdS)

- not yet strictly proven, but successfully tested in many special cases
- independent of question whether string theory is fundamental theory of nature

AdS/CFT - a Strong-Weak Duality

AdS/CFT correspondence maps strong coupling in one theory to weak coupling in the other, and vice versa.

Difficult problems are mapped to simple problems: weakly coupled gravity describes strongly coupled gauge theory!

AdS/CFT - the Full Conjecture

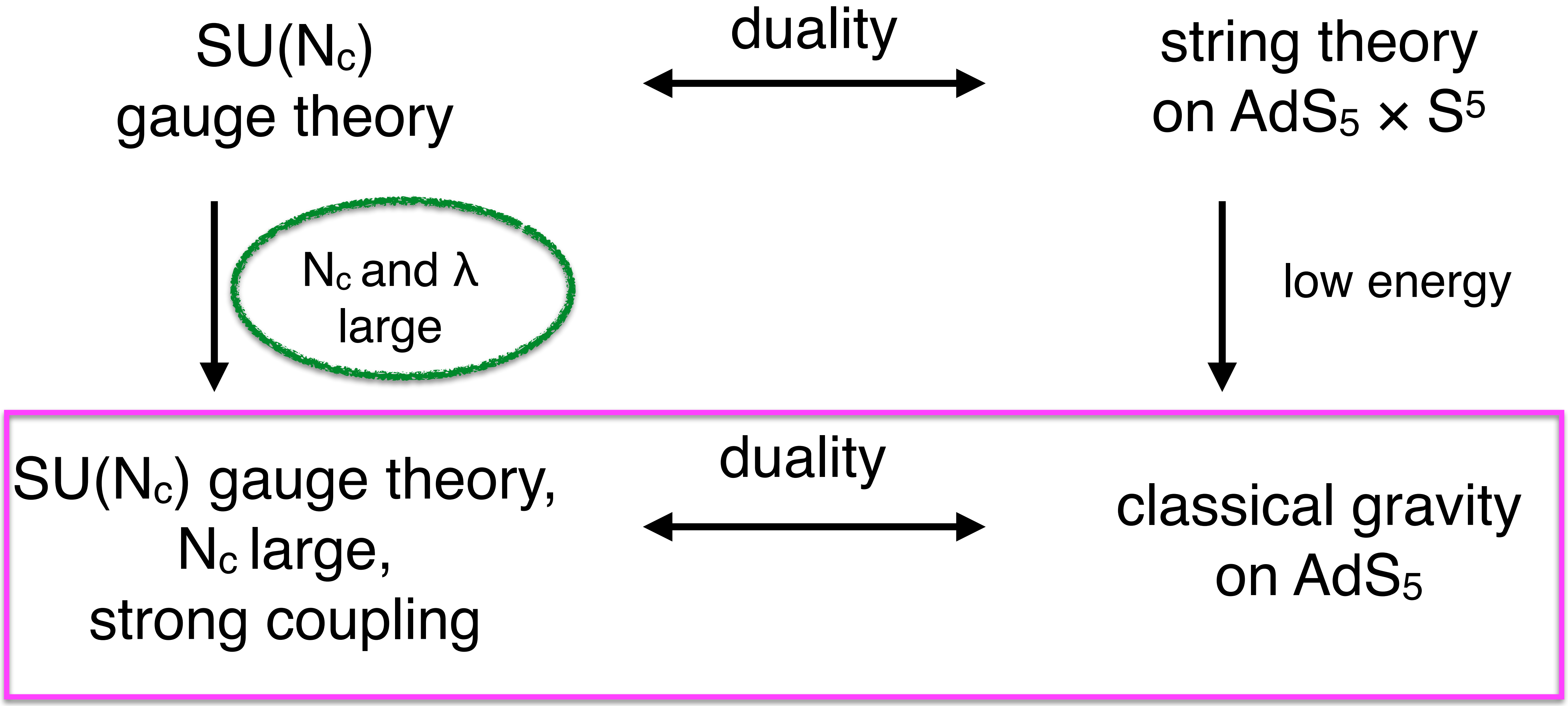
Maldacena

$\mathcal{N} = 4$ super Yang-Mills SU(N) theory
in 3+1 dimensions

$$\Longleftrightarrow$$

Typ IIB string theory on $AdS_5 \times S^5$

Simplification in Limits



SU(N_c)
gauge theory

duality

string theory
on AdS$_5$ × S^5

N_c and λ
large

low energy

SU(N_c) gauge theory,
N_c large,
strong coupling

duality

classical gravity
on AdS$_5$

number of colors N_c, 't Hooft coupling $\lambda = g^2 N_c$

Classical gravity describes quantized gauge theory!

AdS Space, Temperature and Black Holes

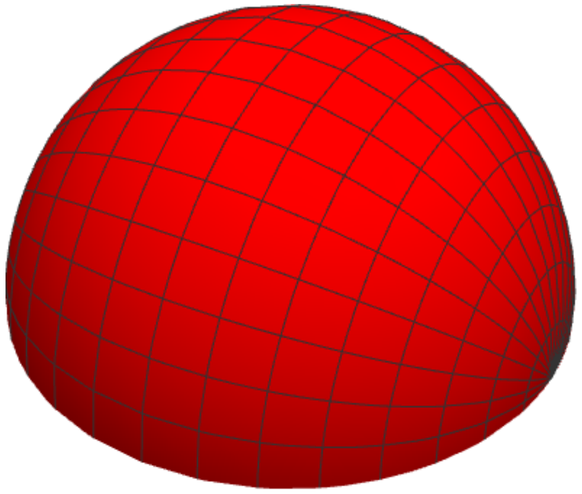
Anti-de-Sitter Space AdS$_5$

space-time with 4+1 dimensions with constant negative curvature

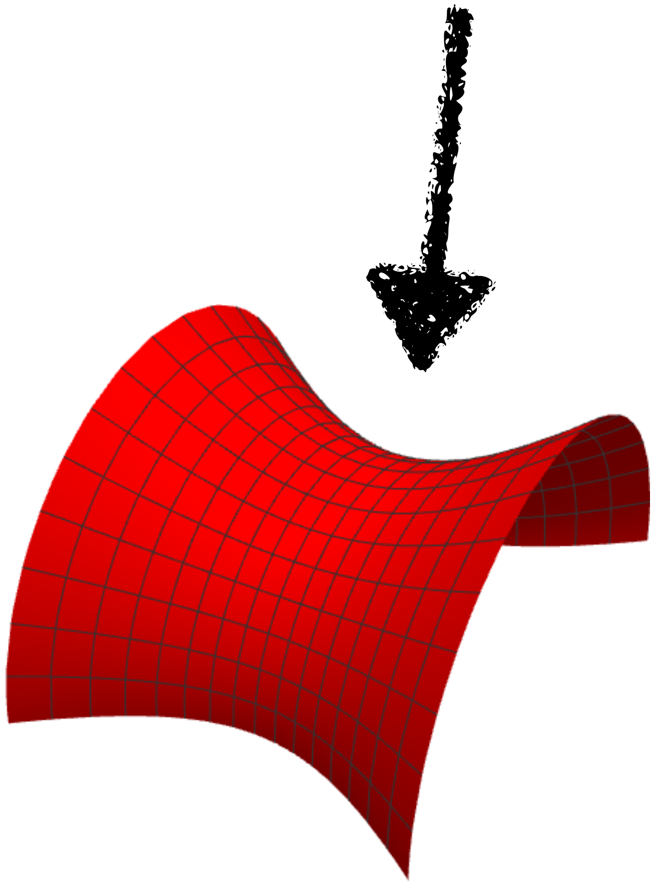
solution of Einstein equations of general relativity with negative cosmological constant

$$S = \frac{1}{16\pi G_{\mathrm{N}}^{(\mathrm{D})}} \int \mathrm{d}^{\mathrm{D}}x \, \sqrt{-g}(\mathcal{R} - 2\Lambda)$$

$$2\Lambda = -\frac{12}{L_{\mathrm{AdS}}}$$



positive curvature



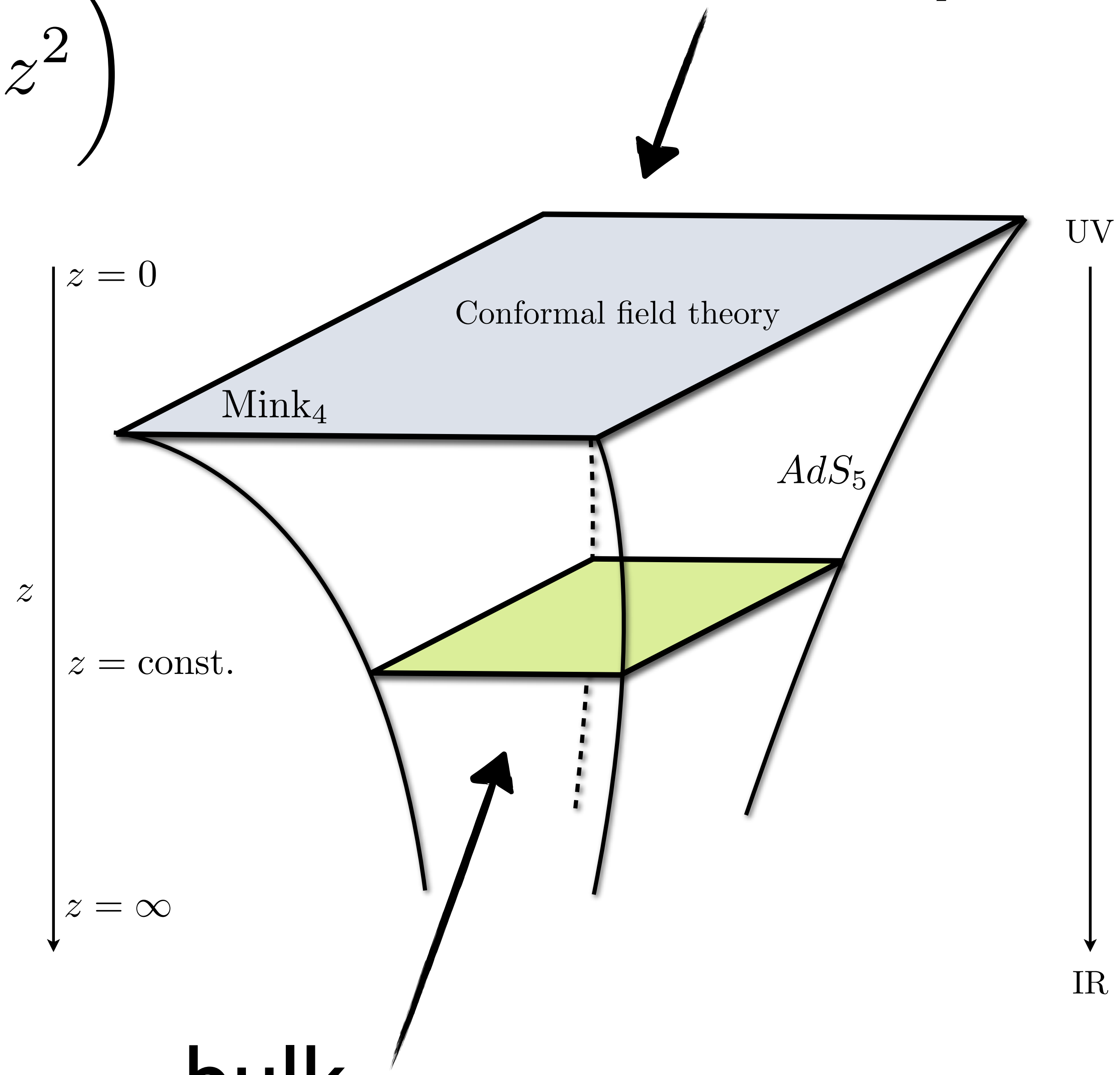
negative curvature

Anti-de-Sitter Space AdS_5

AdS_5 metric

$$ds^2 = \frac{L^2}{z^2}\Big(-dt^2 + d\vec{x}^2 + dz^2\Big)$$

Minkowski space
for any fixed z



boundary

bulk

Holographic Dictionary

Gubser, Klebanov, Polyakov
Witten

1-to-1 translation of AdS- to CFT quantities

examples:
observables in CFT as boundary values of fields in AdS space

→ "Gauge theory lives on boundary of AdS space"

Finite Temperature

CFT with temperature in AdS/CFT mapped
to AdS space with black hole,
more precisely: black brane

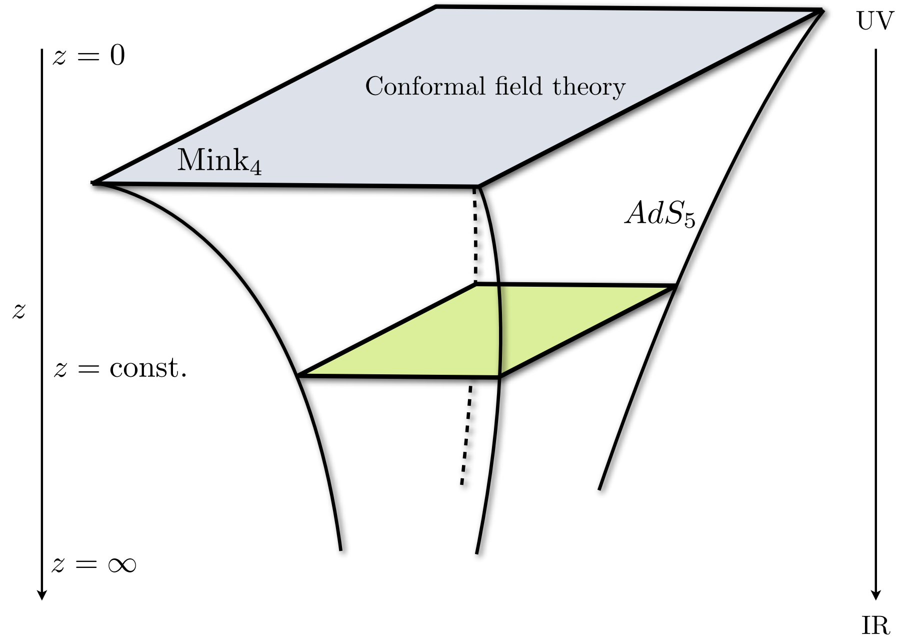
Physics of thermal systems completely encoded
in physics of black holes in higher dimensions!

Temperature of field theory equals
Hawking temperature of black hole.

Finite Temperature

zero temperature: AdS5

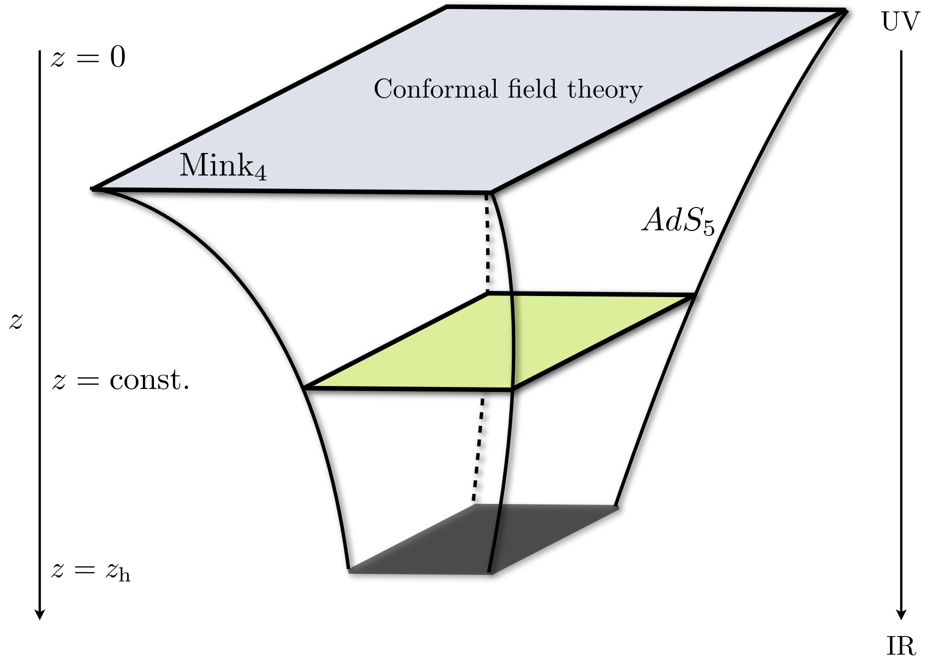
$$\mathrm{d}s^2 = \frac{L^2}{z^2}\Big(- \mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \mathrm{d}z^2 \Big)$$



finite temperature T:
AdS5 with black hole

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left(-h(z)\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \frac{\mathrm{d}z^2}{h(z)} \right)$$

$$h(z) = 1 - \frac{z^4}{z_\mathrm{h}^4} \qquad T = \frac{1}{\pi z_\mathrm{h}}$$



Gauge-/Gravity Duality
or
Holography

Extensions of the Correspondence

CFT in AdS/CFT is supersymmetric and not realistic.

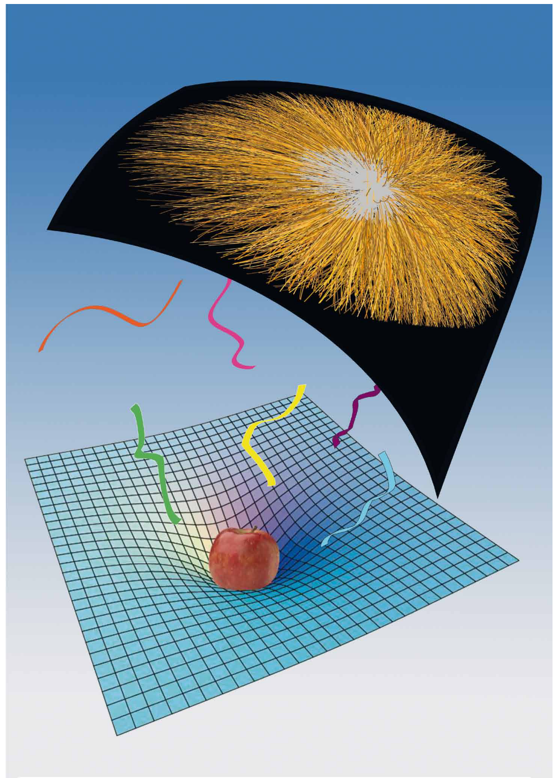
Many extensions of the correspondence:

- non-conformal
- spatially anisotropic
- with chemical potential
  → electrically charged black hole!
- non-relativistic
- other number of dimensions

→ Gauge-/Gravity Duality or Holography

Applications of Holography

Aspects of the Applications

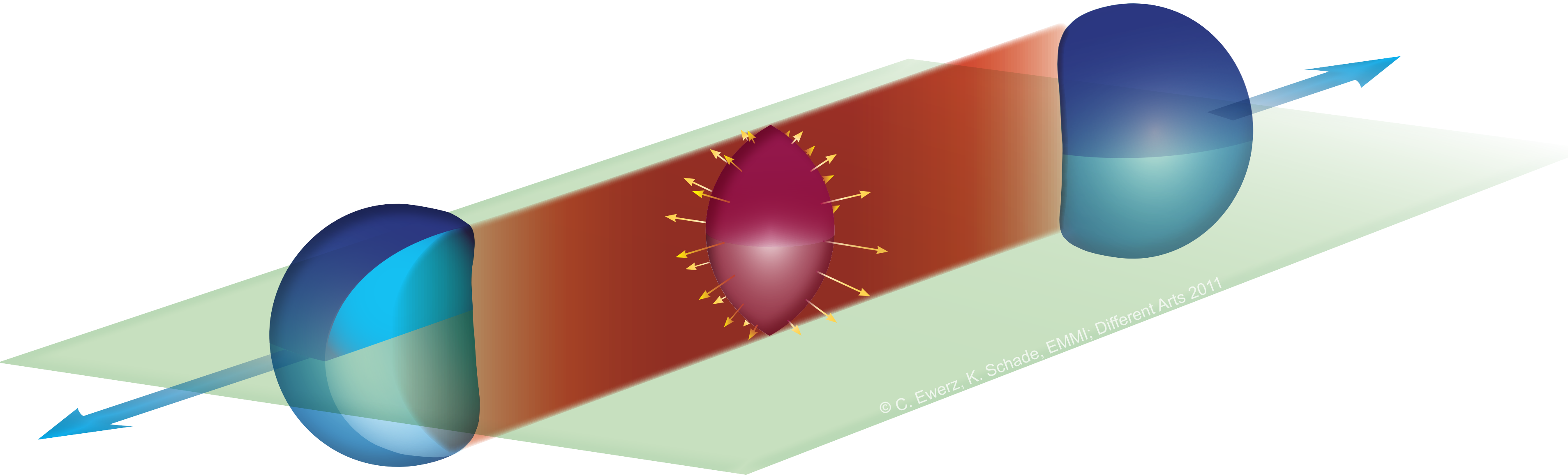


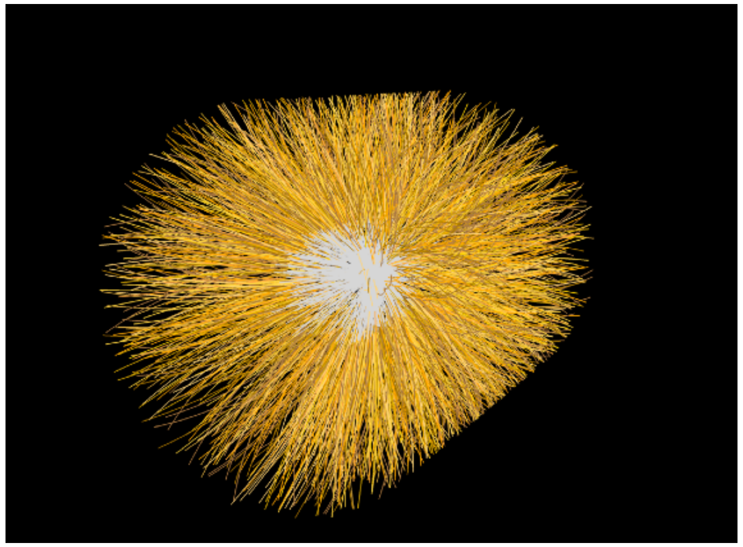
- model theories of strong coupling, search for new phenomena
- particularly successful for collective phenomena (microscopic theory often not essential)
- search for universal behavior in classes of holographic theories
- important observation:
  universality of strongly coupled systems

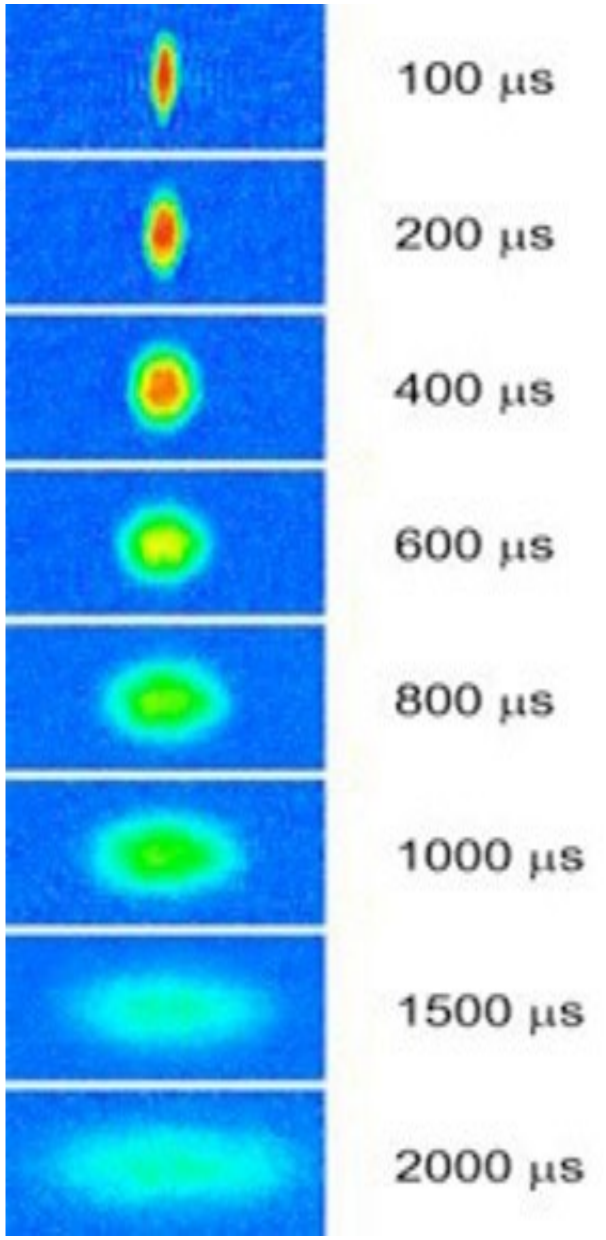
Viscosity and Expansion

quark-gluon plasma

cold quantum gas


© C. Ewerz, K. Schade, EMMI; Different Arts 2011





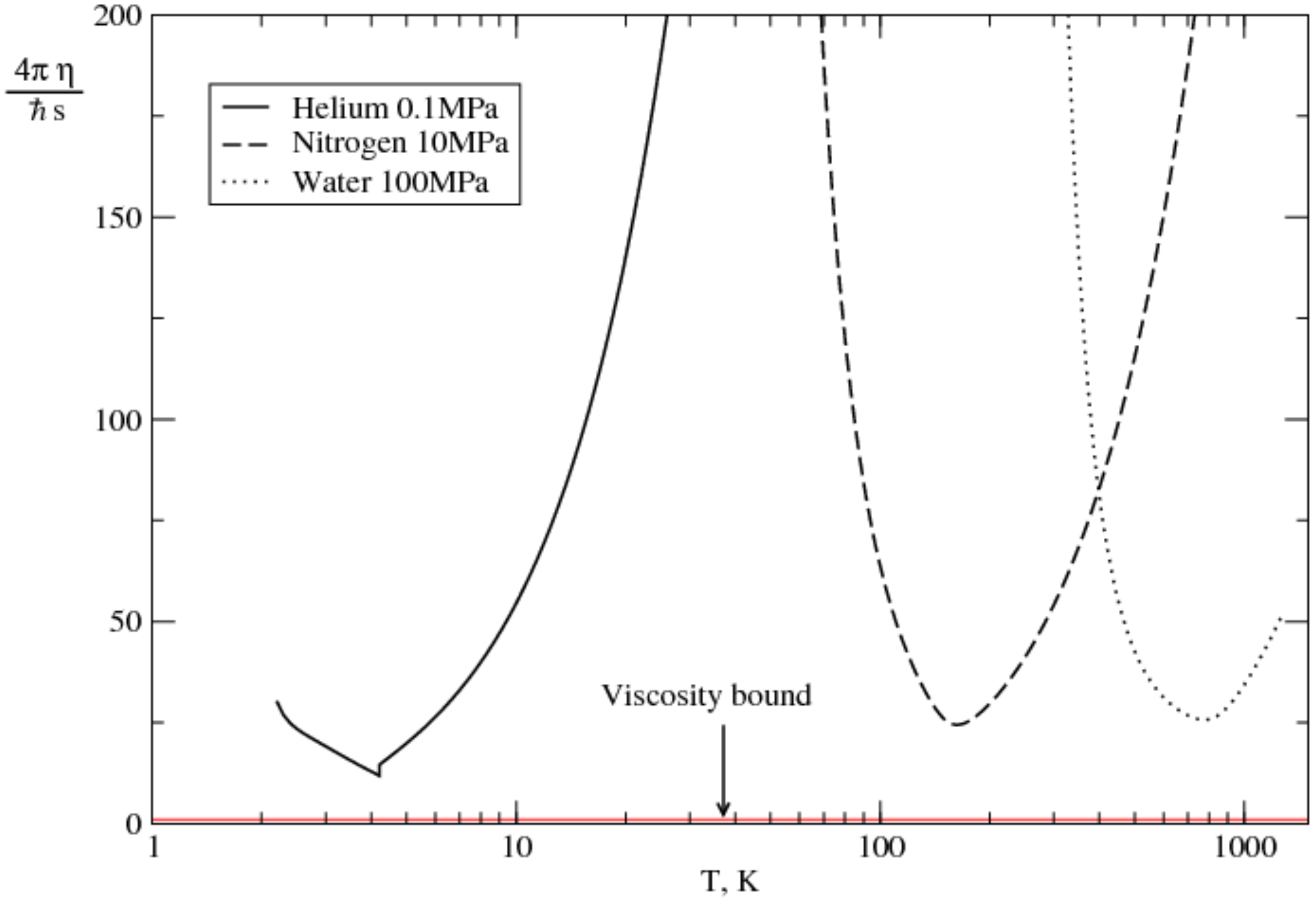
very small η/s, strongly coupled systems!

η/s and Holography

KSS bound: $$\boxed{\frac{\eta}{s} \geq \frac{1}{4\pi}}$$ Kovtun, Son, Starinets

quark-gluon plasma: about 0.2

for comparison:



Key Problem for Applications of Holography

- Lagrangian, field content and parameters of dual field theory in general unknown (in particular for bottom-up models)
- difficult to determine applicability to real-world materials
- We address this for the holographic superfluid.

Holographic Superfluid

Holographic Superfluid

Gubser
Hartnoll et al
Herzog et al

(d+1+1)-dimensional action:

- AdS-Schwarzschild space-time
- gauge field + complex scalar in bulk

D=d+1

$$S = \frac{1}{16\pi G_{\mathrm{N}}^{(D+1)}} \int \mathrm{d}^{D+1}x \sqrt{-g} \left(\mathcal{R} - 2\Lambda + \frac{1}{q^2} \mathcal{L}_{\text{gauge–matter}} \right)$$

$$\mathcal{L}_{\text{gauge–matter}} = -\frac{1}{4} F_{MN} F^{MN} - |(\nabla_M - \mathrm{i} A_M)\Phi|^2 - m^2 |\Phi|^2$$

- superfluid density $n = |\langle\psi\rangle|^2$ from (d=2)

$$\Phi(t, \vec{x}, z) = \eta(t, \vec{x})\, z + \langle\psi(t, \vec{x})\rangle\, z^2 + \mathcal{O}(z^3)$$

order parameter

η=0 chosen by boundary condition

Holographic Superfluid

Gubser
Hartnoll et al
Herzog et al

dictionary:

boundary: (d+1)-dimensional		bulk: (d+1+1)-dimensional
thermal background @ μ, T	holographic	charged black hole
complex scalar field operator Ψ	↔	complex scalar field Φ
$U(1)$ conserved current j^μ	dictionary	$U(1)$ gauge field A_M

bulk equations of motion:

$$\left(D^2 - m^2\right)\Phi = 0$$

$$\nabla_M F^{MN} = \mathcal{J}^N(\Phi)$$

$$G_{MN} - \Lambda g_{MN} = \frac{1}{q^2} T_{MN}$$

Holographic Superfluid

- Holographic superfluid captures dynamics on all relevant length scales.

- Relation to Tisza-Landau model:

  black hole corresponds to normal component, gauge-matter sector to superfluid component

Implementation

- probe approximation:
  solve gauge-matter sector on fixed AdS-BH background:

$$\mathrm{d}s^2 = \frac{L_{\mathrm{AdS}}^2}{z^2}\left(-h(z)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2 - \mathrm{d}t\mathrm{d}z\right)$$

$$h(z) = 1 - \left(\frac{z}{z_{\mathrm{h}}}\right)^D$$

Eddington-Finkelstein coordinates

- chem. potential via boundary condition:

$$A_t(z=0) = \mu$$

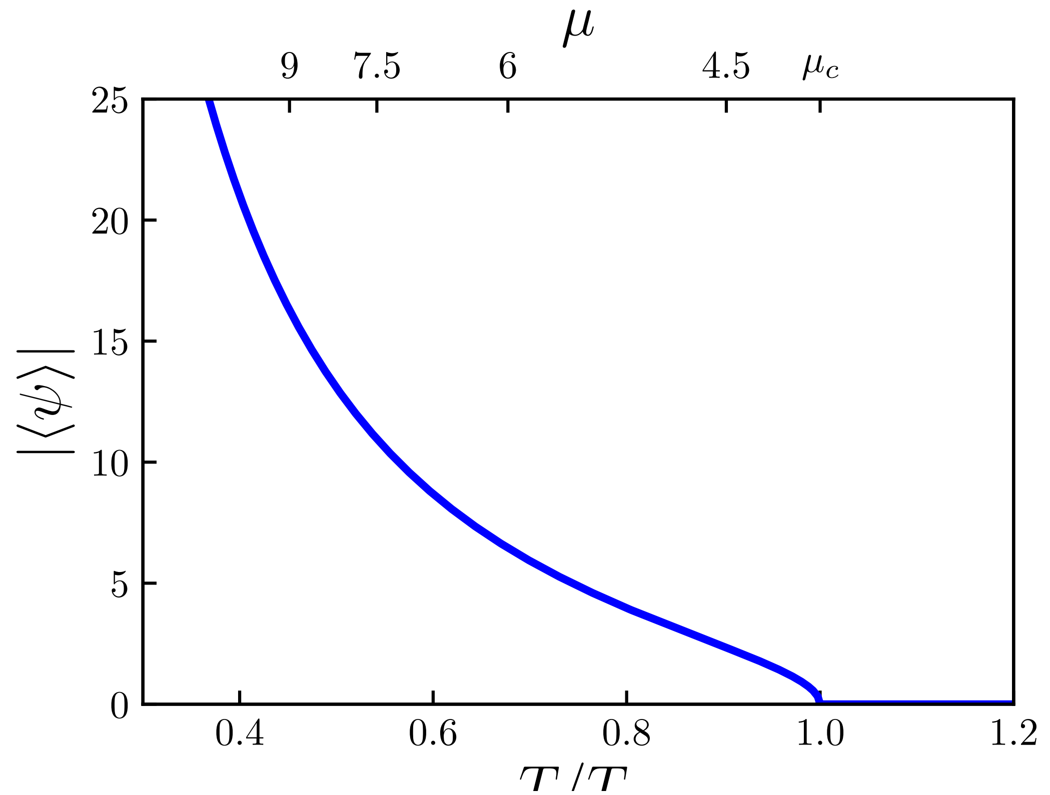
$$A_t(z=z_{\mathrm{h}}) = 0$$

canonical or grand-canonical ensemble possible

- superfluid coupled to heat bath with $T = \dfrac{D}{4\pi z_{\mathrm{h}}}$

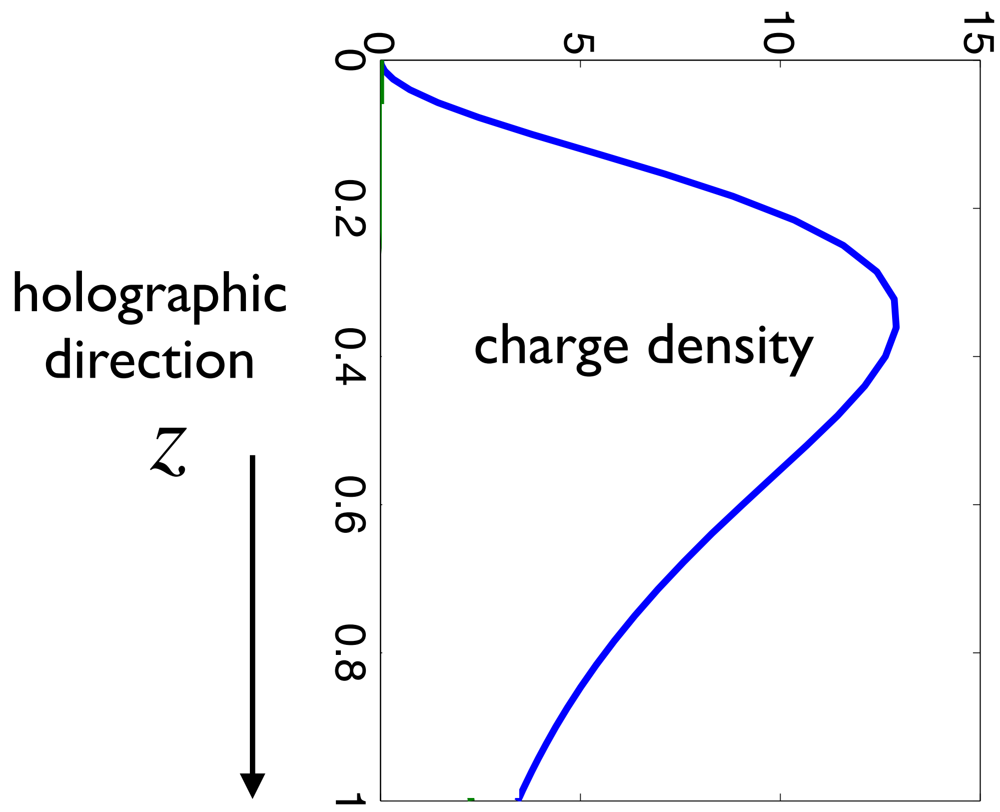
2D Equilibrium Configuration

- Choose temperature and chemical potential in the superfluid phase



- scalar condenses, $\Phi = \Phi(z)$

- cloud of charge in the bulk, $\sqrt{-g}|\mathcal{J}^0|$



Numerical Simulations of the Holographic Superfluid

Team

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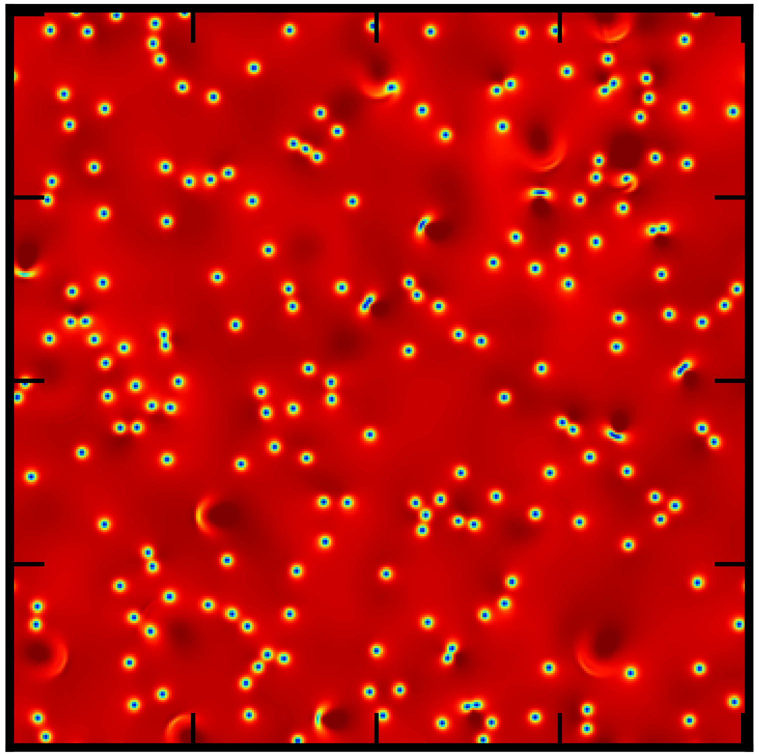
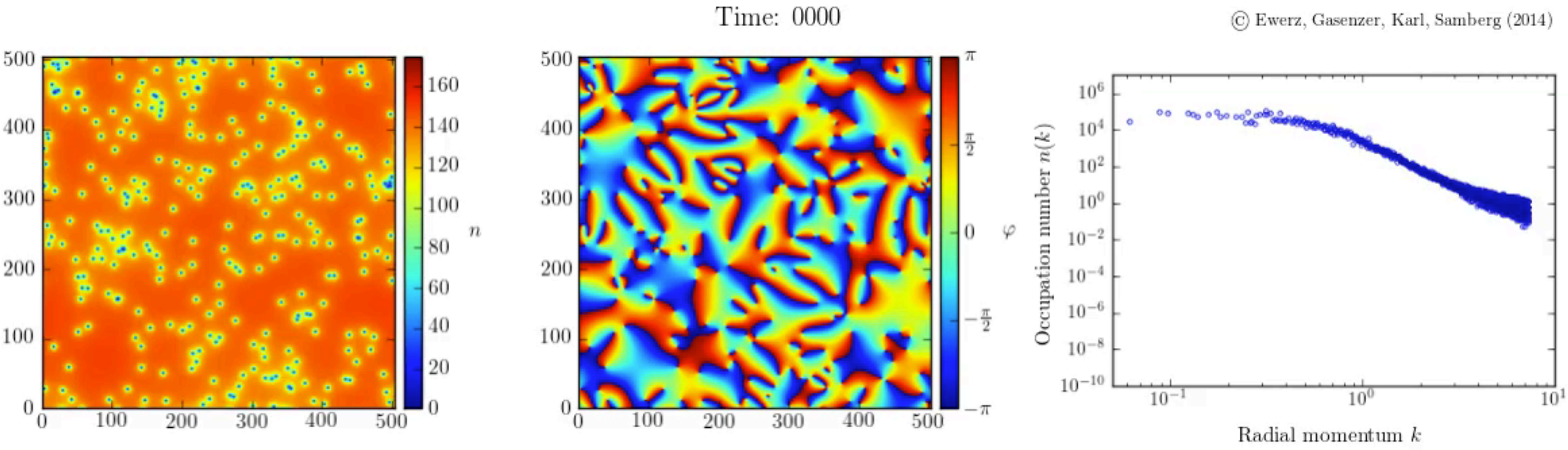
Markus Karl

Holographic Superfluid in 2 Dimensions



Turbulence: Simulation

CE, T. Gasenzer, M. Karl, A. Samberg
JHEP 05 (2015) 070 [arXiv:1410.3472]

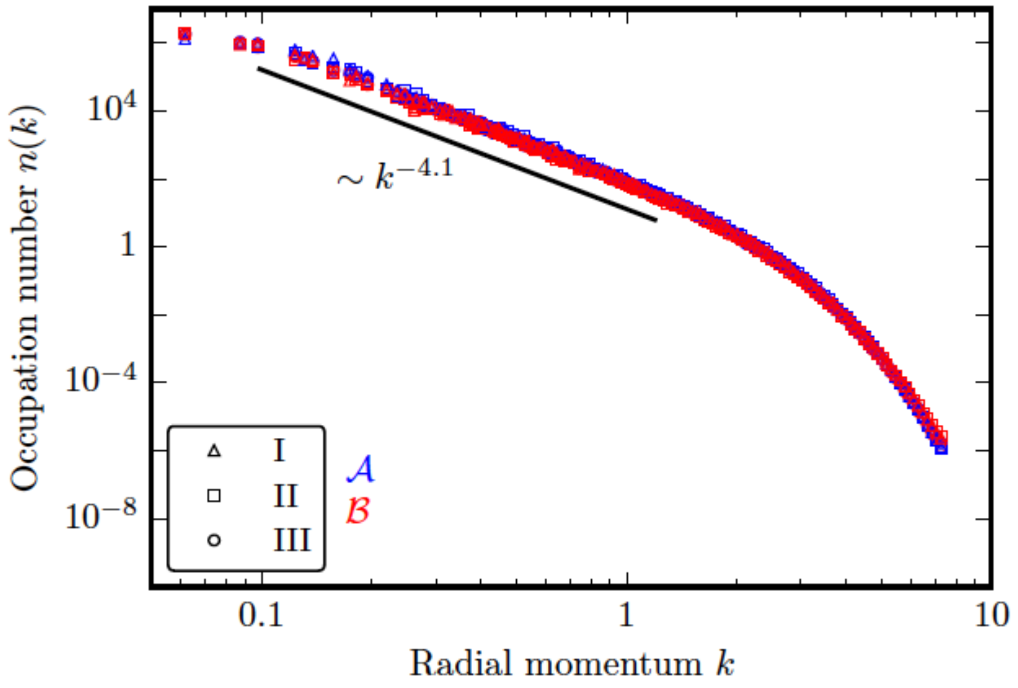


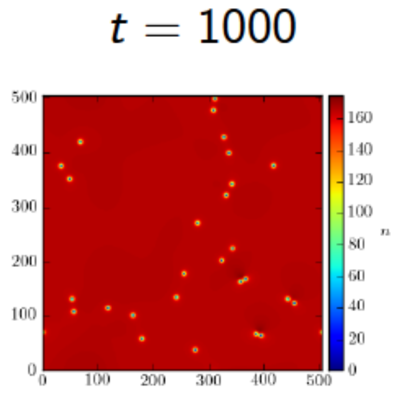
Adams, Chesler, Liu
Du, Niu, Tian, Zhang

Turbulence: Results

- Kolmogorov-like scaling (Adams et al) at intermediate times - only transient

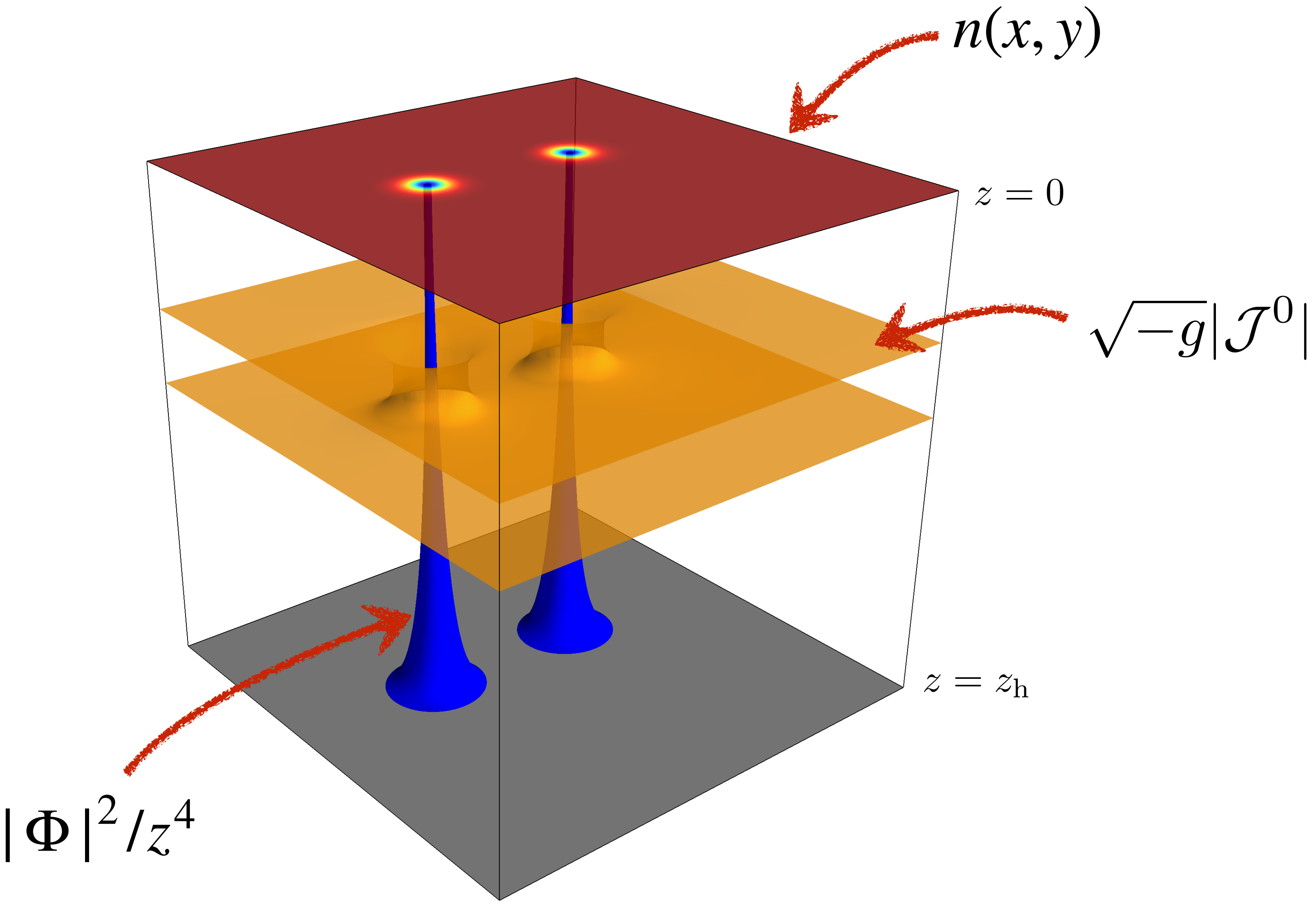
- universal turbulent behavior at late times

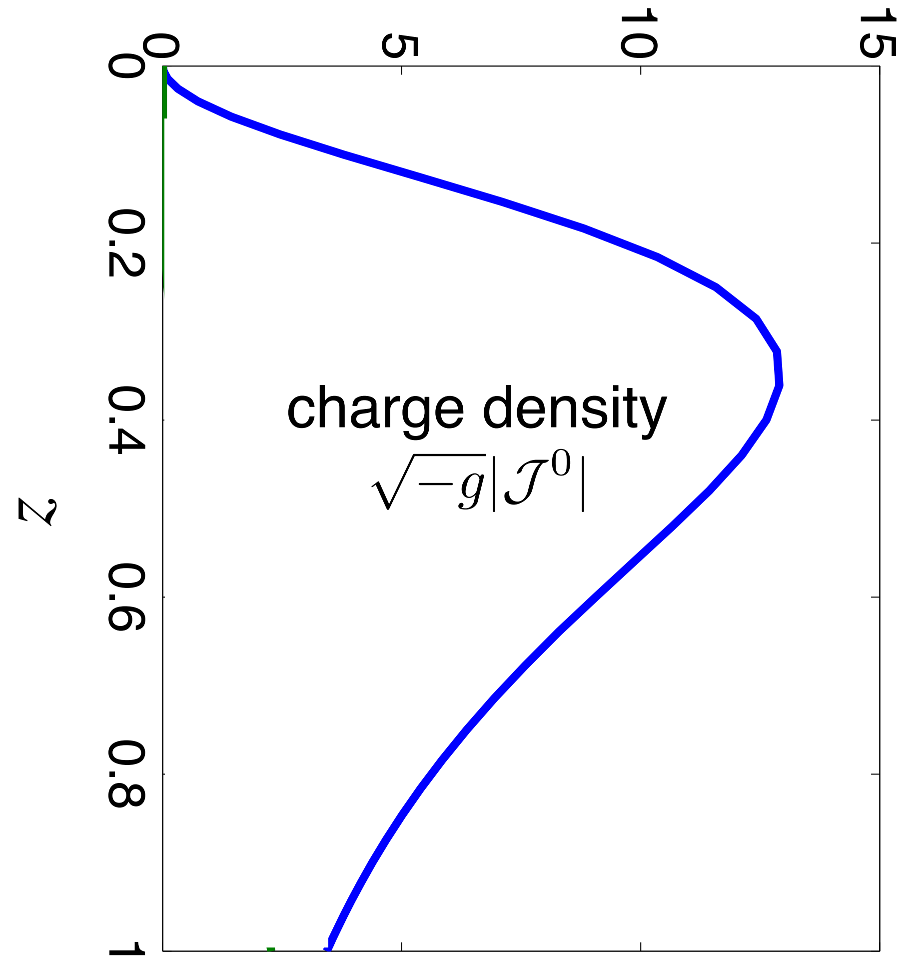




- first observations of non-thermal fixed point in strongly coupled system

Bulk View of Vortex Pair

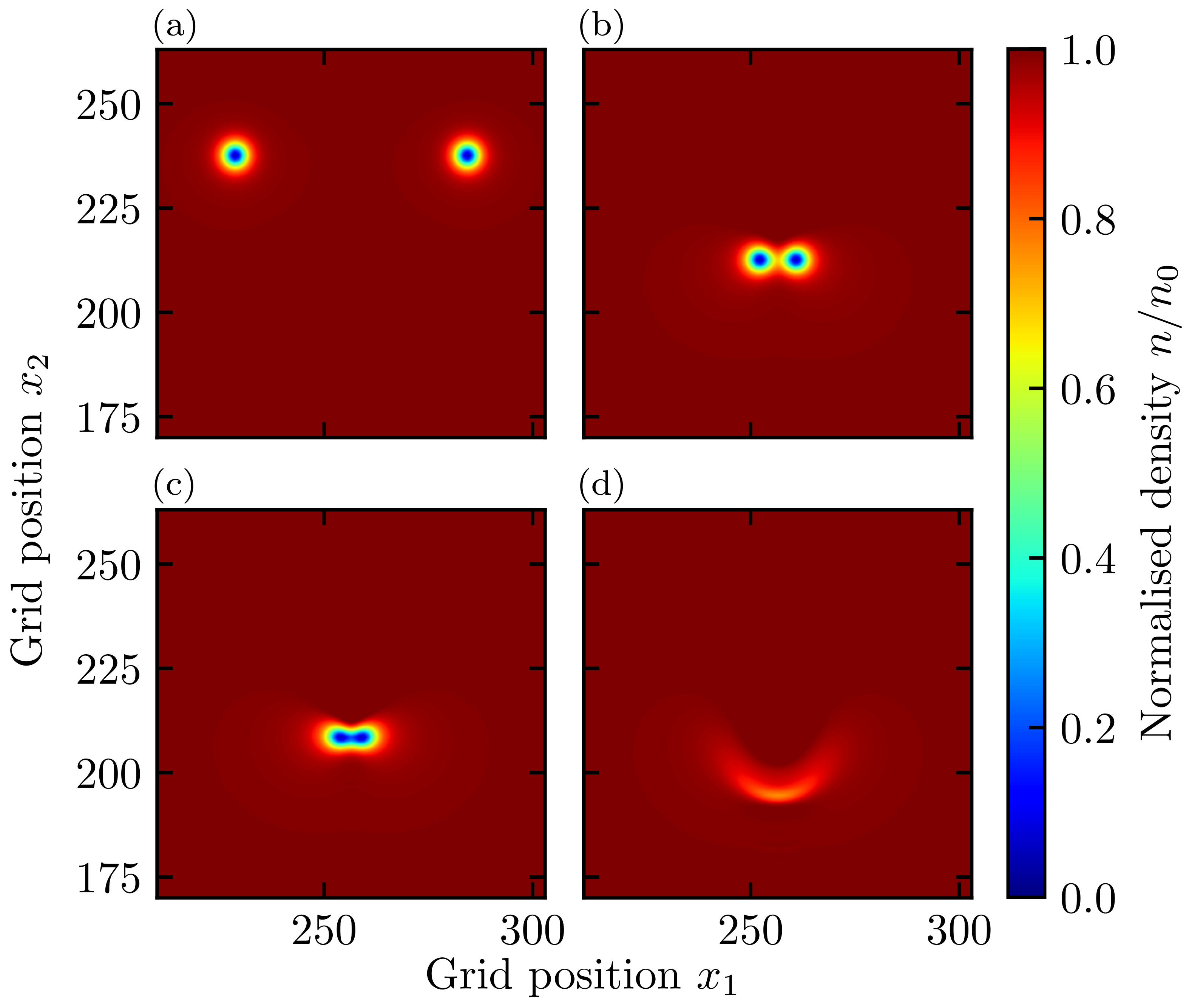




- momentum modes fall into black hole
- strong energy flux at vortex cores, mostly UV modes dissipated

Adams, Chesler, Liu

Vortex-Antivortex Pair



CE, A. Samberg, P. Wittmer

JHEP11 (2021) 199 [arXiv:2012.08716]

Quantifying Dissipation

P. Wittmer, C. Schmied, T. Gasenzer, CE
Phys.Rev.Lett. 127 (2021)101601 [arXiv: 2011.12968]

- idea:
  match solutions of dissipative GPE to holographic vortex trajectories

- DGPE:

$$\partial_t \psi(\mathbf{x},t) = -\left(\mathrm{i}+\gamma\right)\left[-\frac{1}{2M}\nabla^2 + g\left|\psi(\mathbf{x},t)\right|^2 - \mu\right]\psi(\mathbf{x},t)$$

$$\psi(\mathbf{x},t) = \sqrt{n_0}\, u(\mathbf{x},t)$$

or after some rescaling …

$$\partial_t u(\mathbf{x},t) = \frac{\mathrm{i}+\gamma}{2\tau}\left[\nabla^2 + 2\mu\left(1-\left|u(\mathbf{x},t)\right|^2\right)\right]u(\mathbf{x},t)$$

- We tune only DGPE parameters (not holographic ones).

Quantifying Dissipation

- DGPE has three free parameters for vortex-pair solution

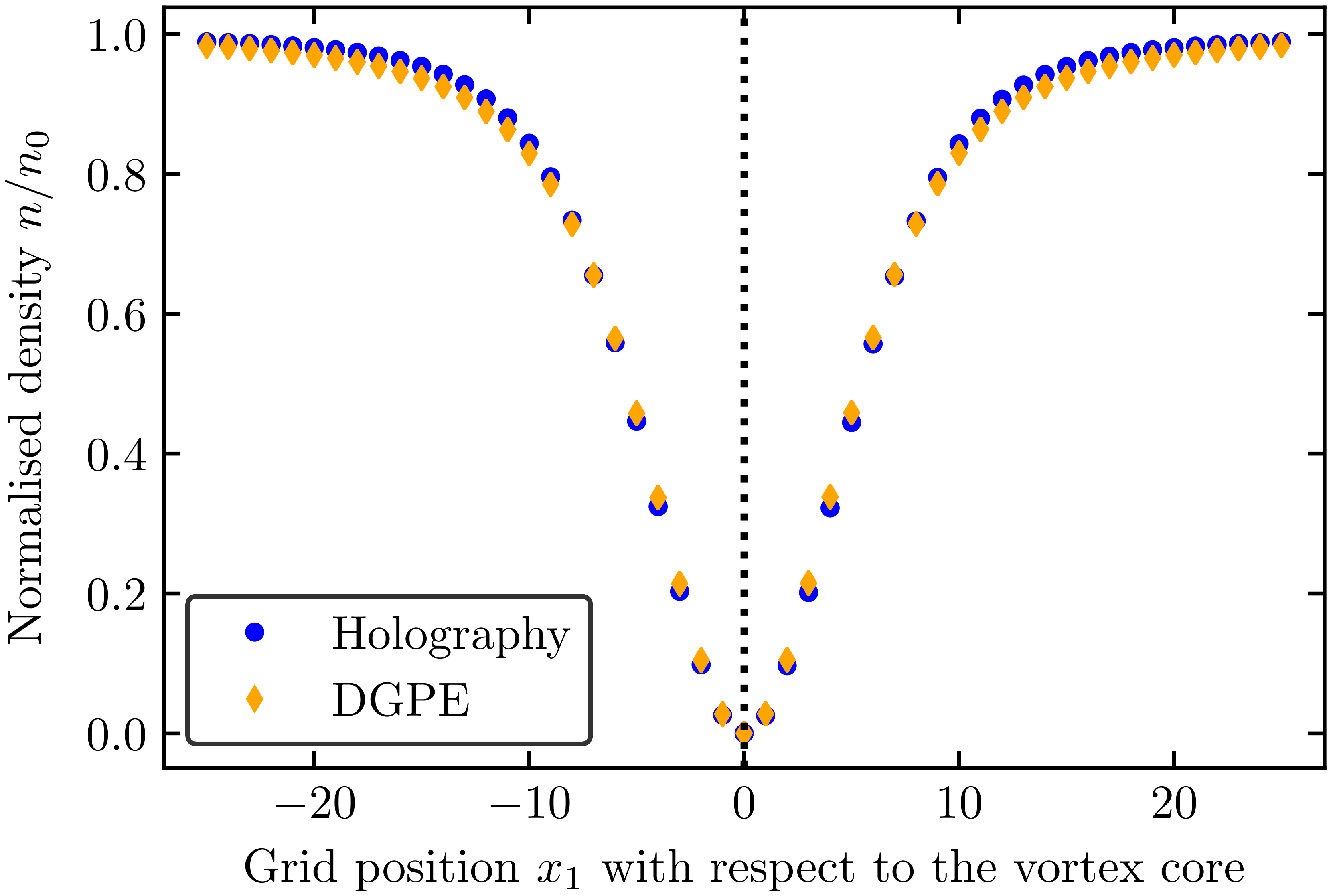
  $\xi = (2\mu)^{-1/2}, \tau, \gamma$

  ξ: healing length, controls vortex size
  τ: time rescaling parameter
  γ: quantifies dissipation

- We tune these parameters *independently* to match holographic vortex trajectories.

Quantifying Dissipation

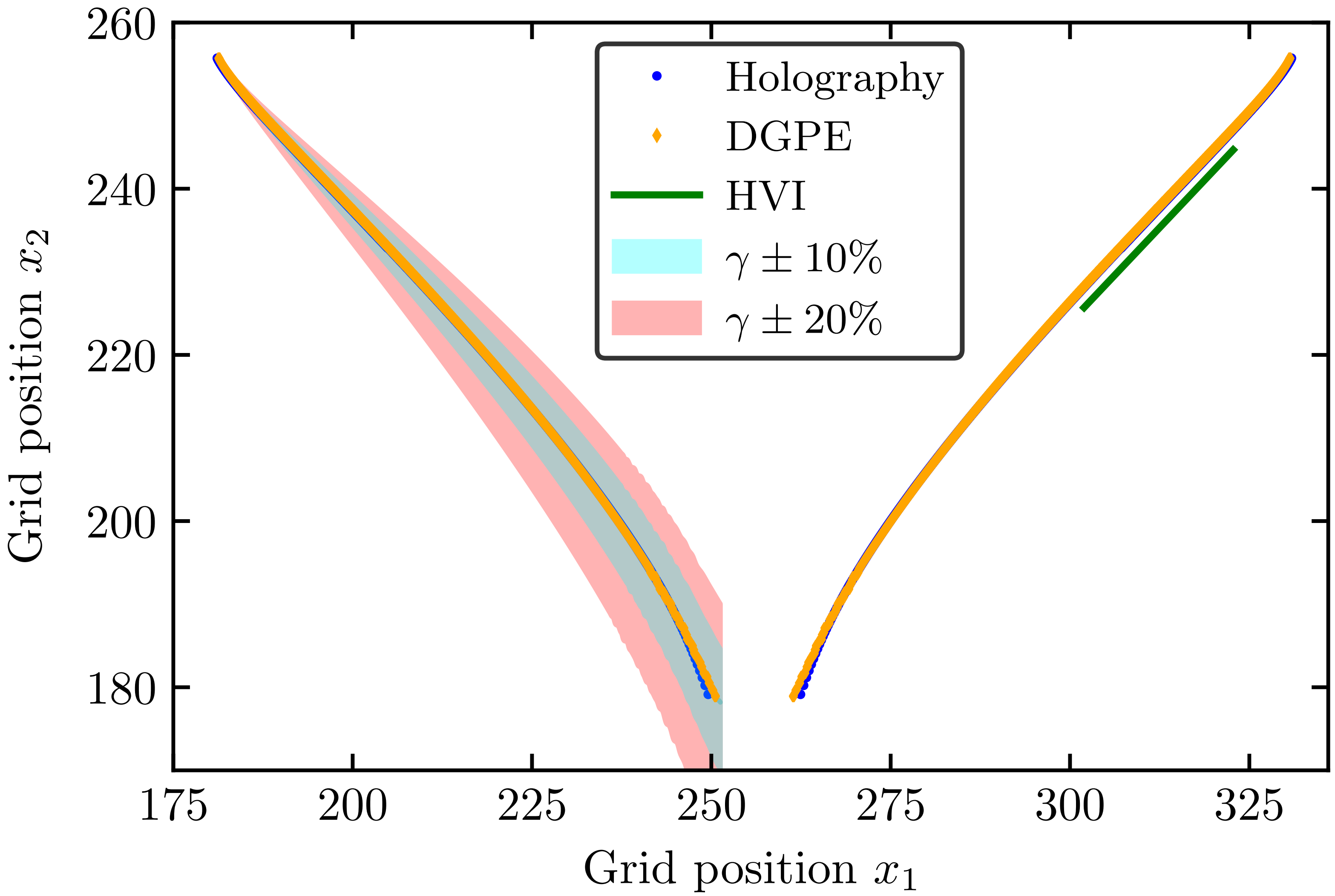
- First match vortex size to fix ξ:



- After matching size, shapes are in excellent agreement.

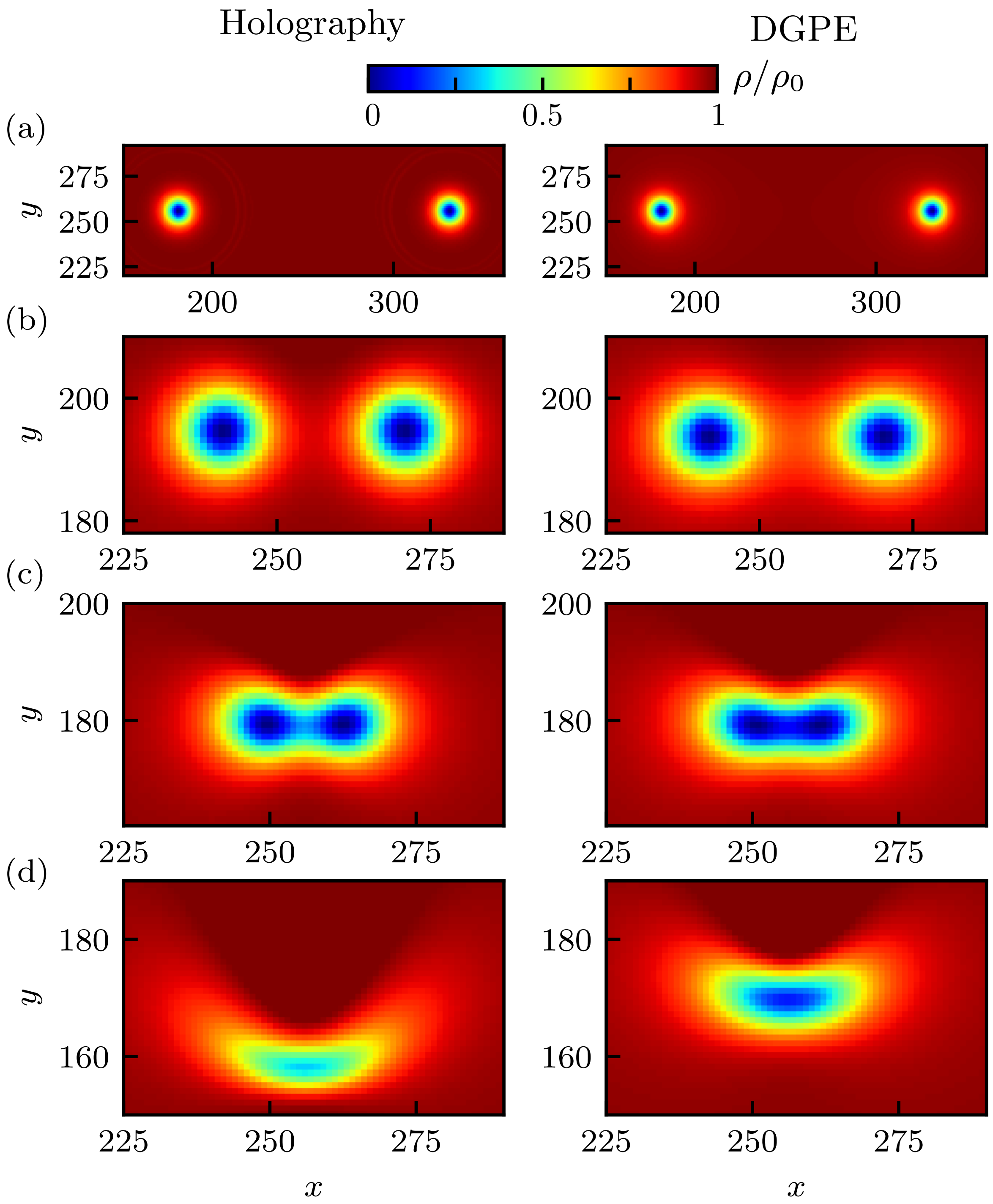
Quantifying Dissipation

- Then match trajectories in space (fixes γ) and time (fixes τ)



- Excellent agreement except shortly before annihilation

Quantifying Dissipation



- Late-time discrepancy probably due to different UV dynamics.

Quantifying Dissipation

- From matching: $\gamma \simeq 0.3$ for holographic superfluid

- typical BECs with Alkali atoms at $T \sim 100\,$nK:

  $\gamma \simeq 10^{-4} \ldots 10^{-3}$

  Holographic superfluid strongly dissipative!

Quantifying Dissipation: Friction (HVI)

- Compare trajectories to Hall-Vinen-Iordanskii (HVI) equations describing mechanical motion of
point vortices:
vortices move in flow field of resp. other vortex:

$$\mathbf{v}_i = \frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = (1 - C)\,\mathbf{v}_\mathrm{s}^i - w_i\,C'\,\hat{\mathbf{e}}_\perp \times \mathbf{v}_\mathrm{s}^i$$

with friction coefficients *C*, *C’*.

- Determine *C’* by matching, then compare to real-world values.

Quantifying Dissipation

- Holographic vortex dynamics: $C' \simeq 0.03 \ldots 0.1$

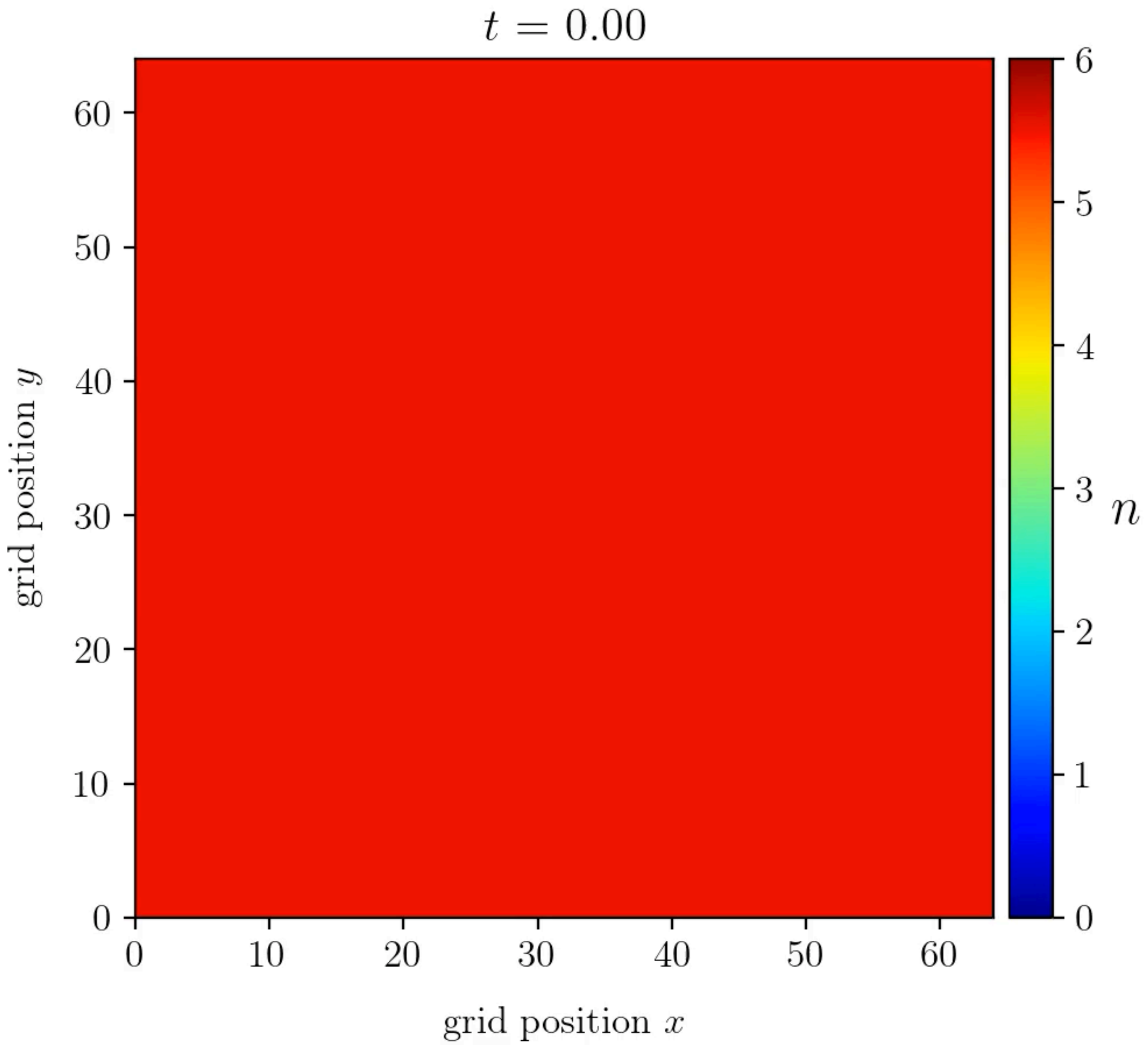
- Experiment:
  superfluid helium films: $C' \simeq 10^{-2} \ldots 1$ (at about 1K)
  thermally excited BECs: $C' \simeq 0.01 \ldots 0.03$

- Holographic vortex dynamics might be applicable to superfluid helium films and thermal BECs.

Landau Instability

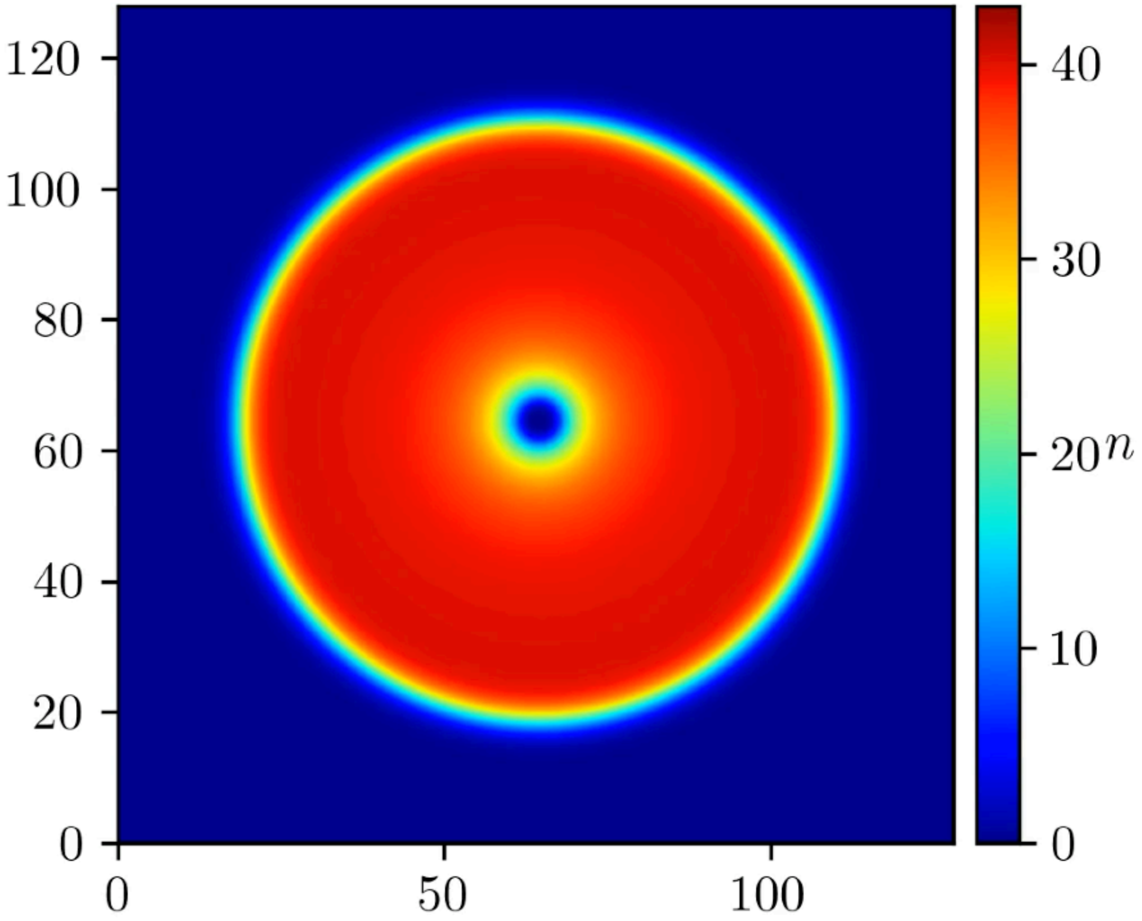
- Induce superfluid velocity by boundary value for A_x

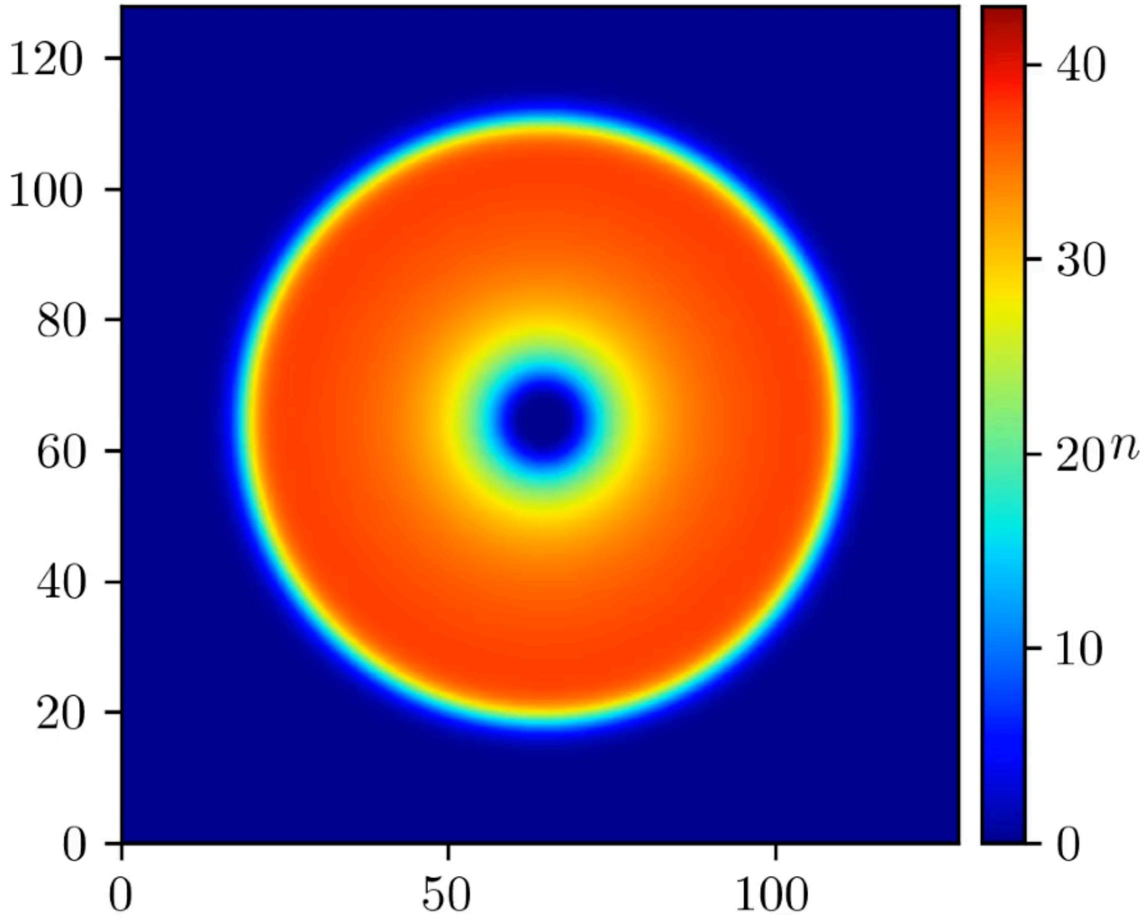
- Instability above critical velocity



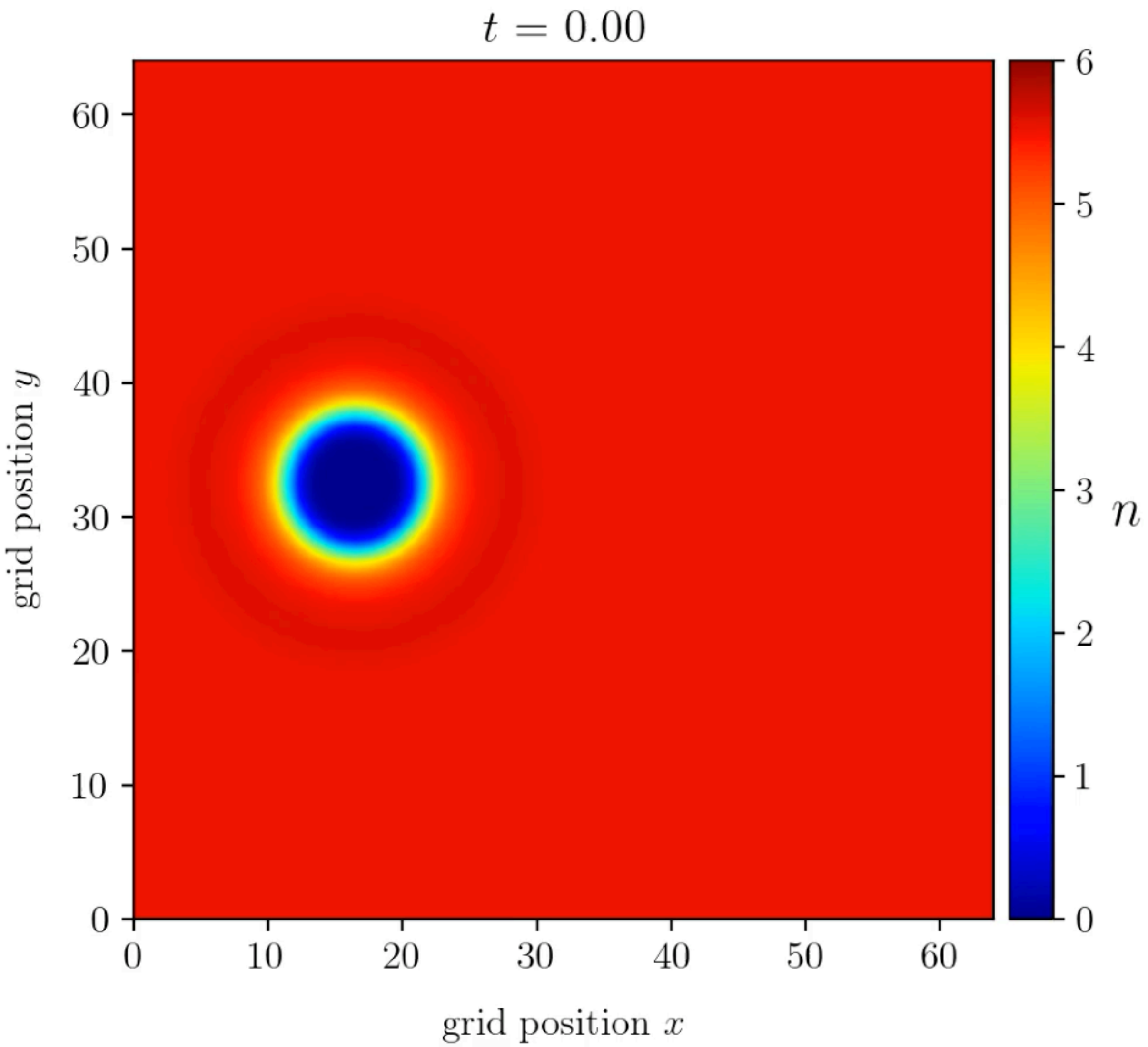
Multi-Vortex Decay

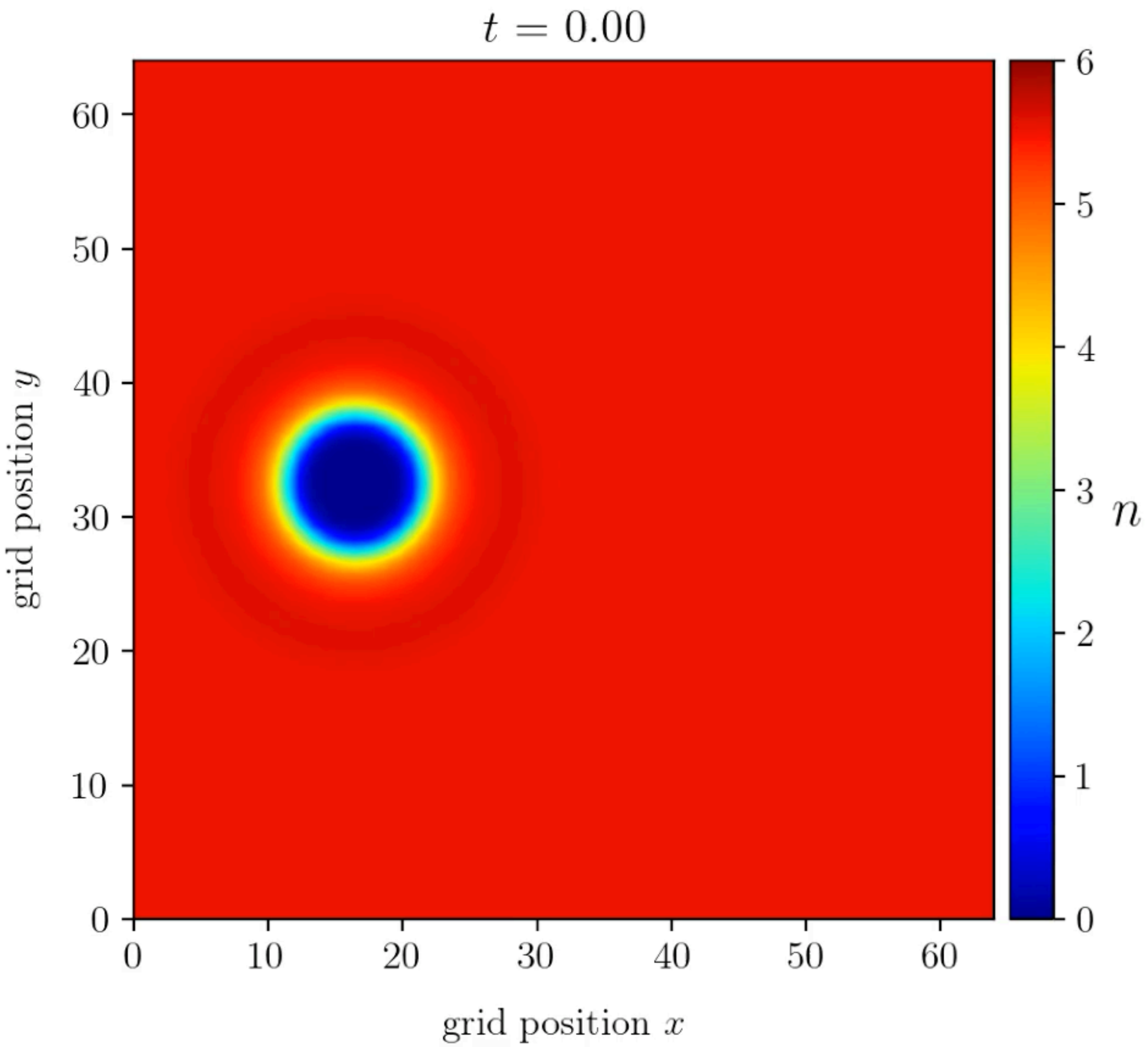
- Use flat trap potential to observe decay of vortex of multiple winding



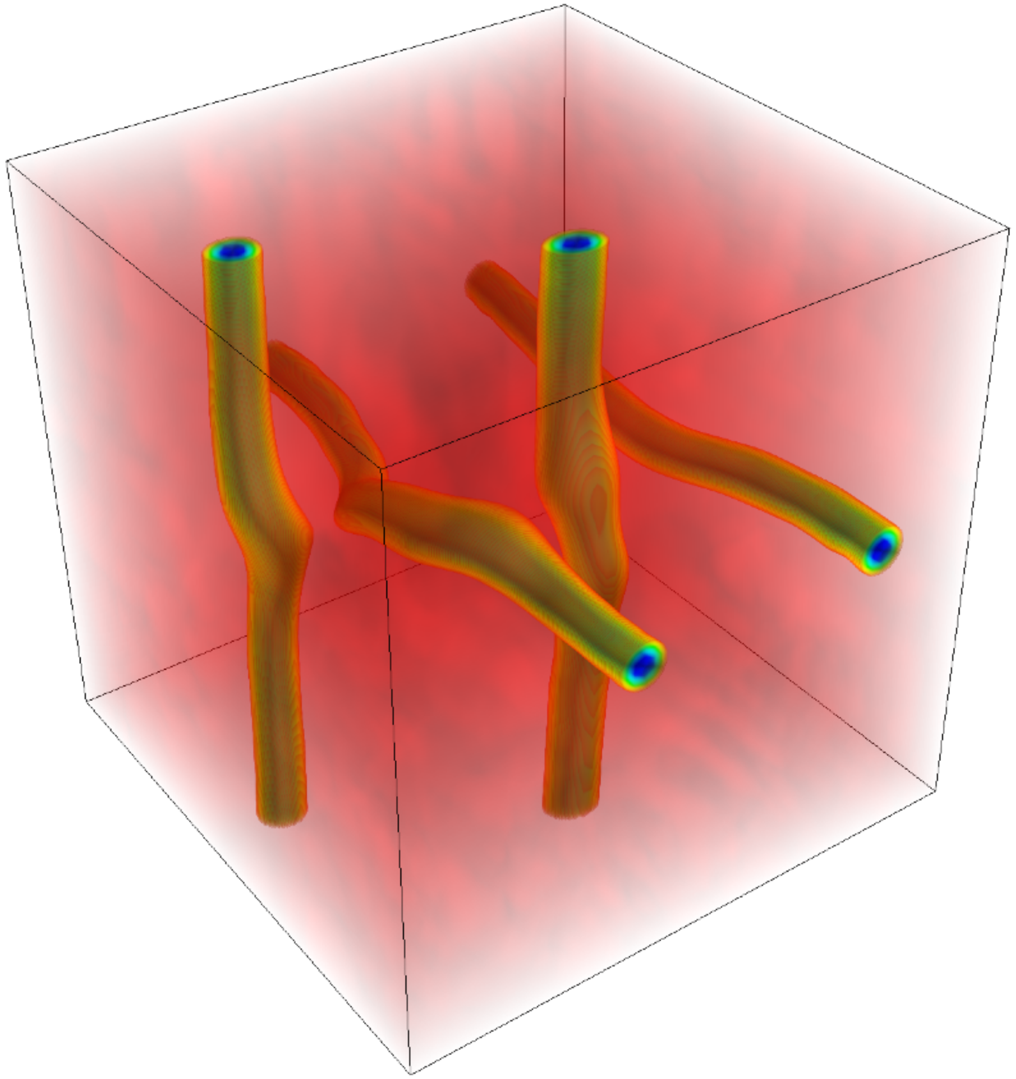


Vortex Shedding: Moving Obstacle





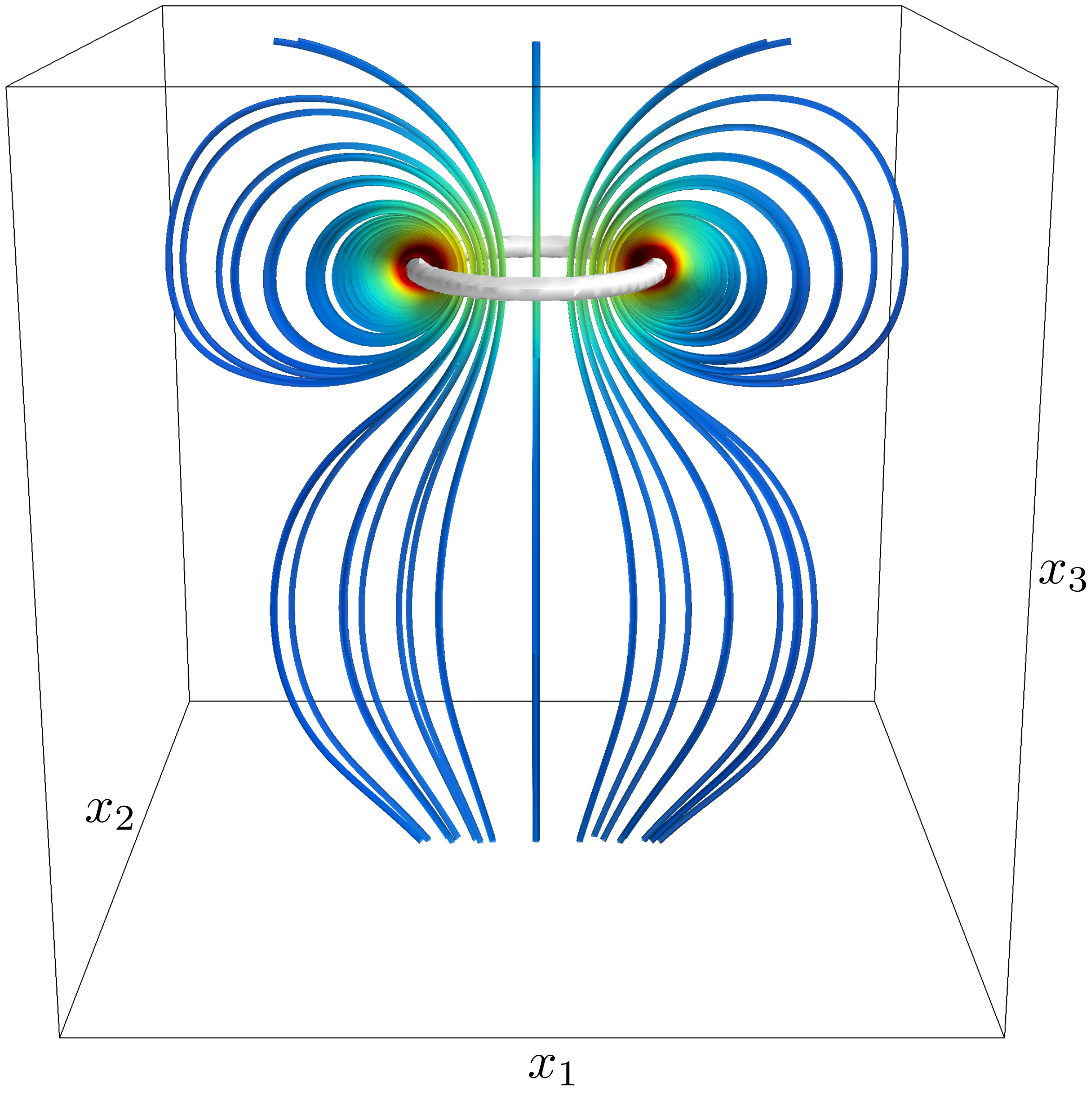
Holographic Superfluid in 3 Dimensions



P. Wittmer, CE

Vortex Rings

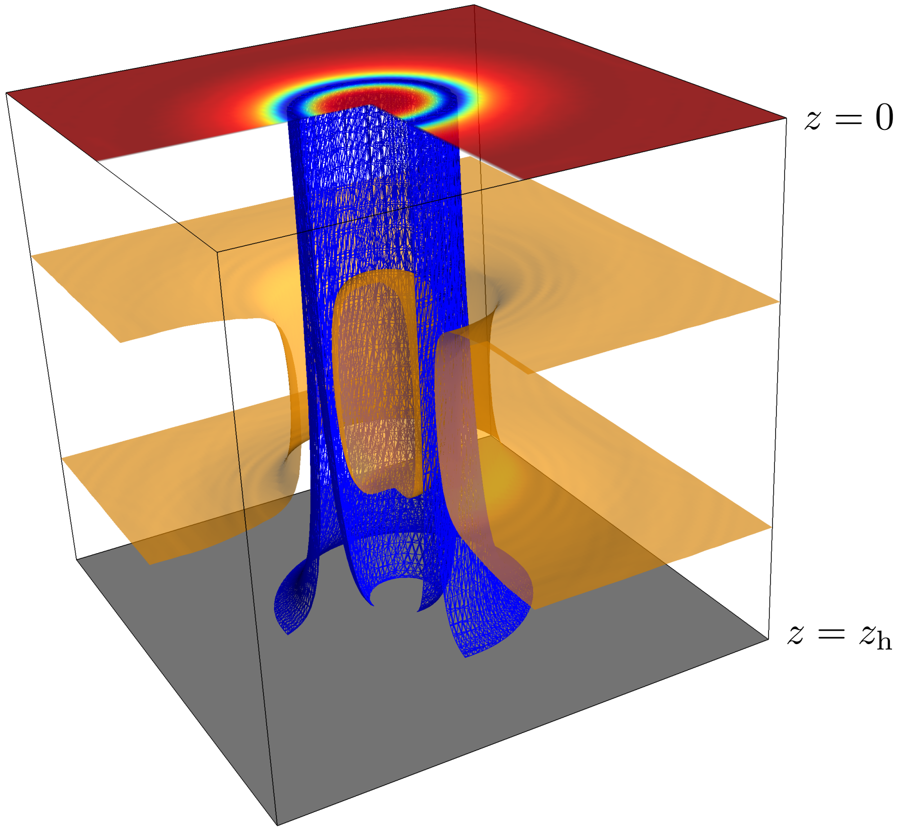
density and flow

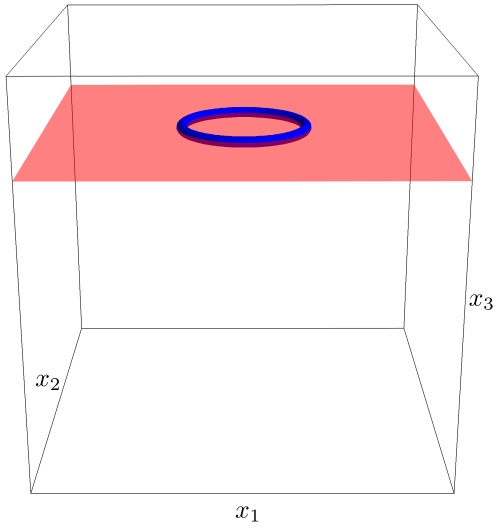


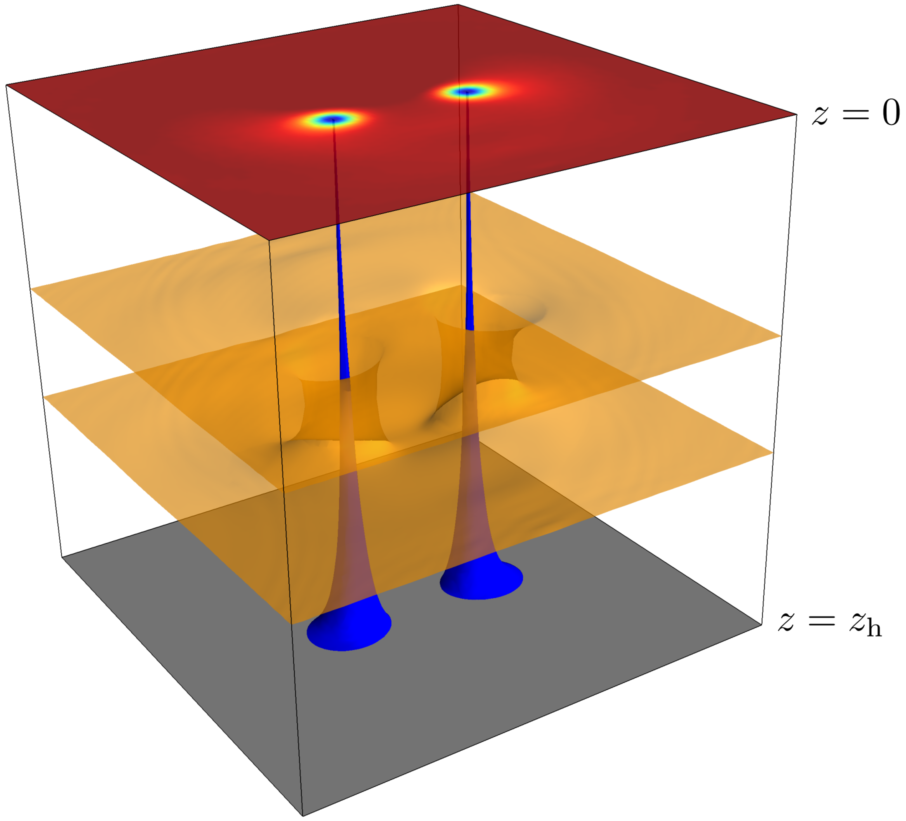
self-induced velocity depending on radius

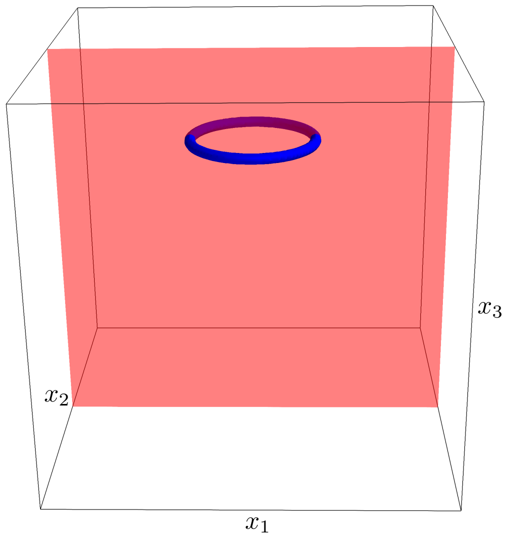
Vortex Rings

bulk view

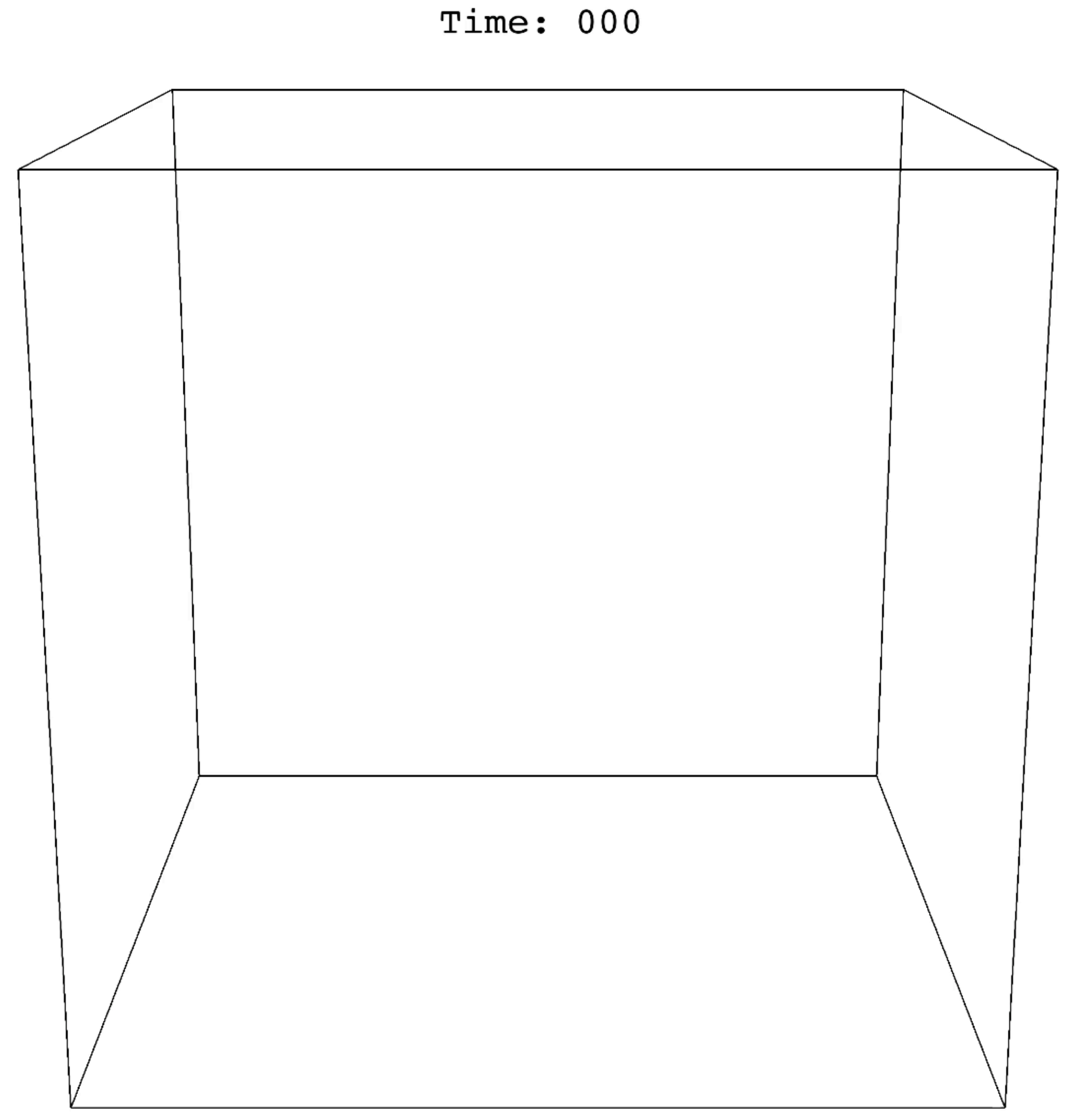








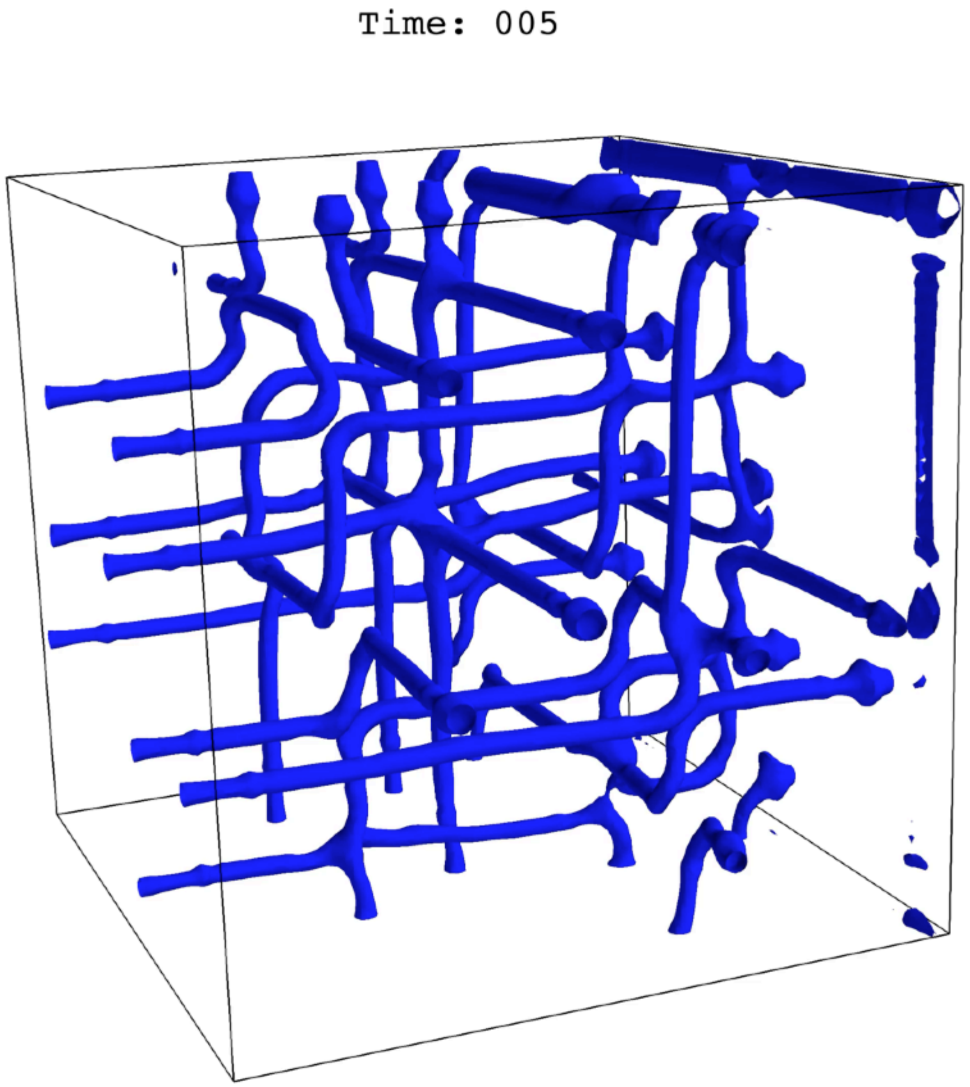
Vortex Ring Motion



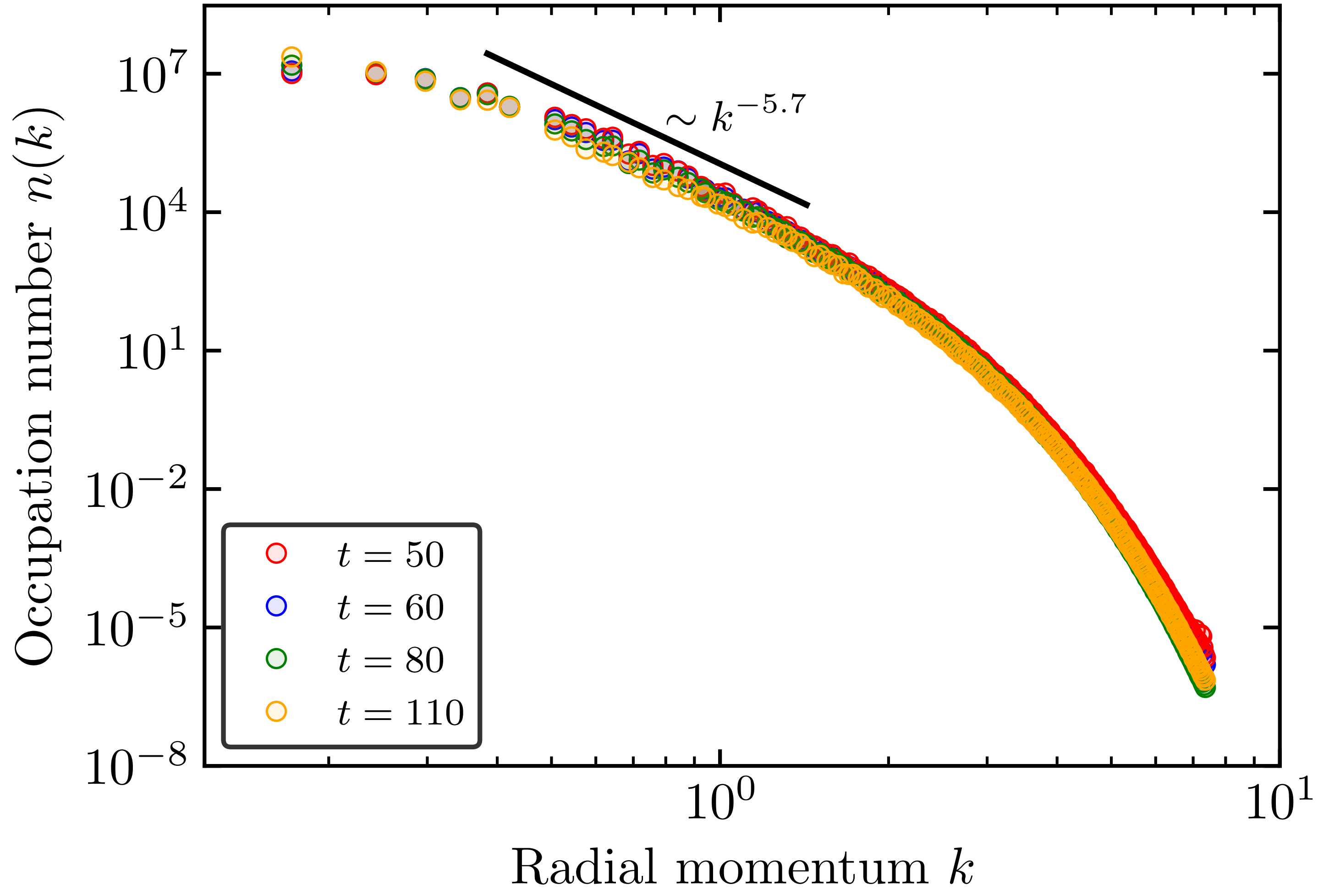
Non-Equilibrium Dynamics

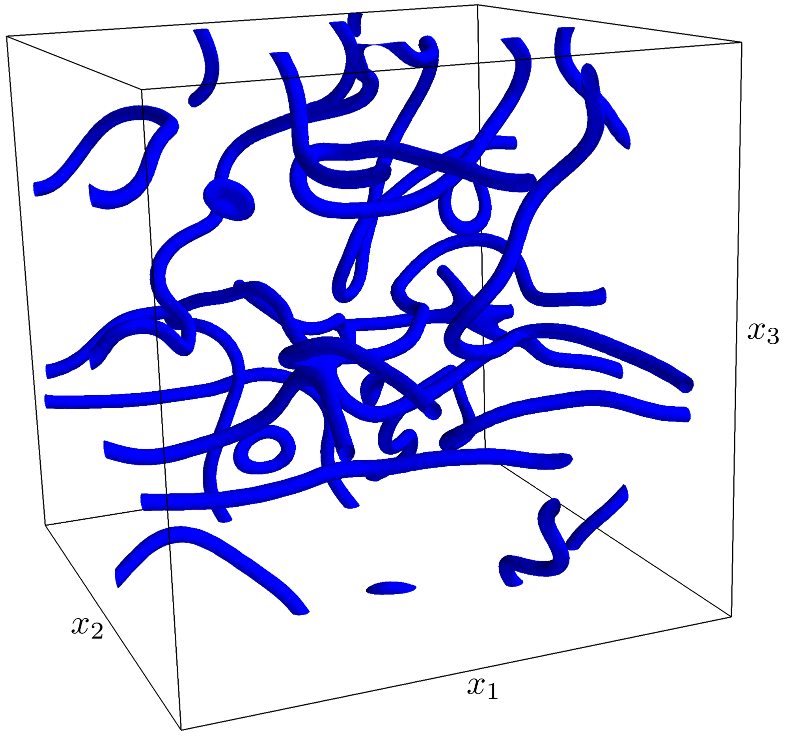
P. Wittmer, CE
[arXiv: 2410.22410]

- grid size: 128 x 128 x 128,
  32 points in holographic direction



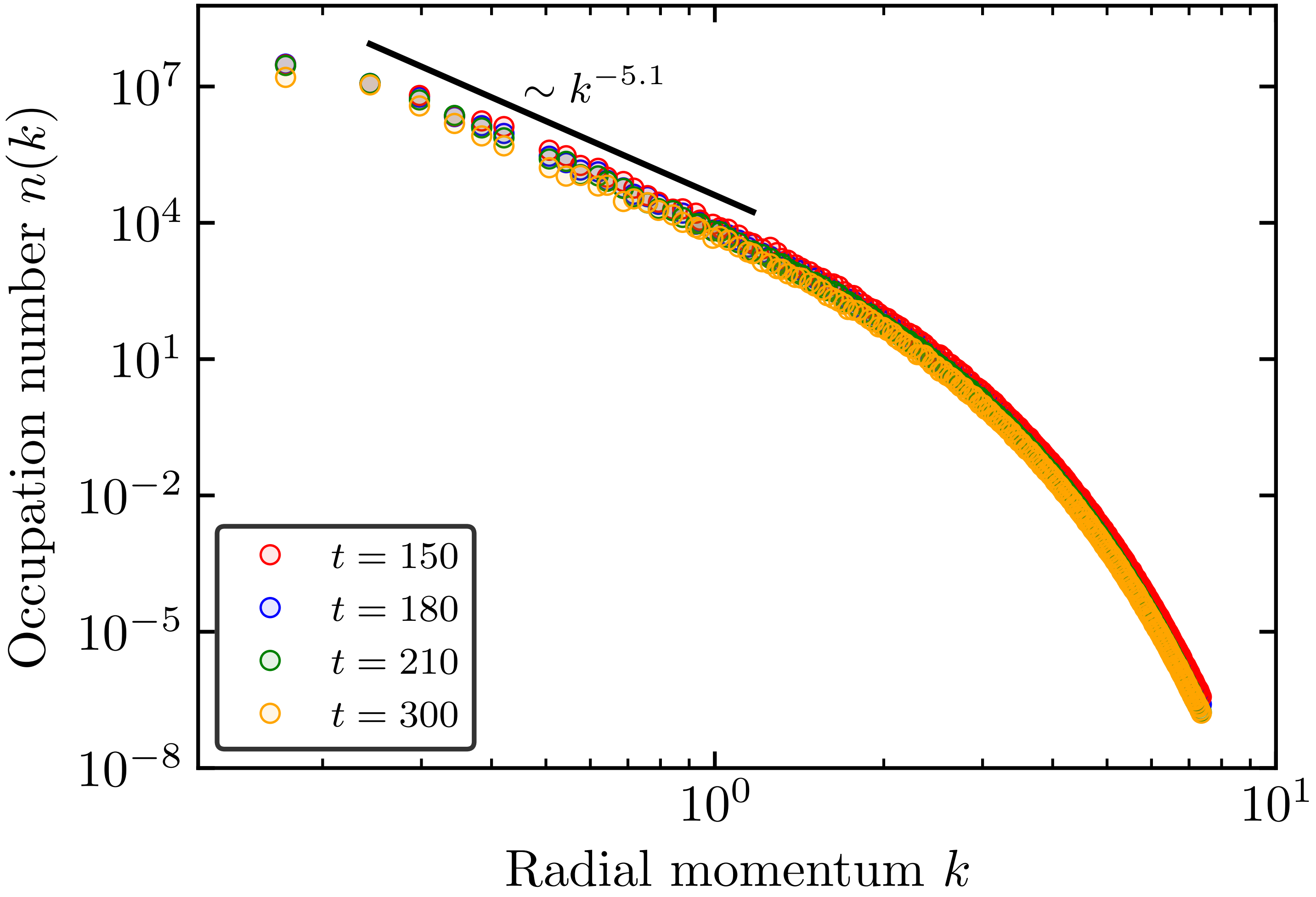
Turbulence: First Scaling Regime

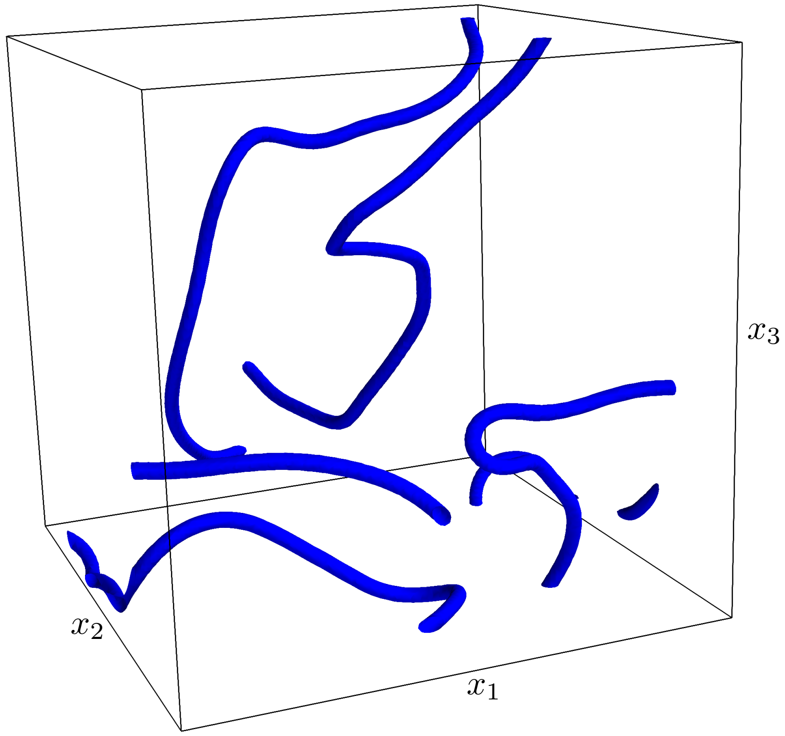




- times $50 \lesssim t \lesssim 110$: scaling $E(k) \sim k^4 n(k) \sim k^{-1.7}$
- Kolmogorov-like
- universal (for all initial conditions)

Turbulence: Second Scaling Regime

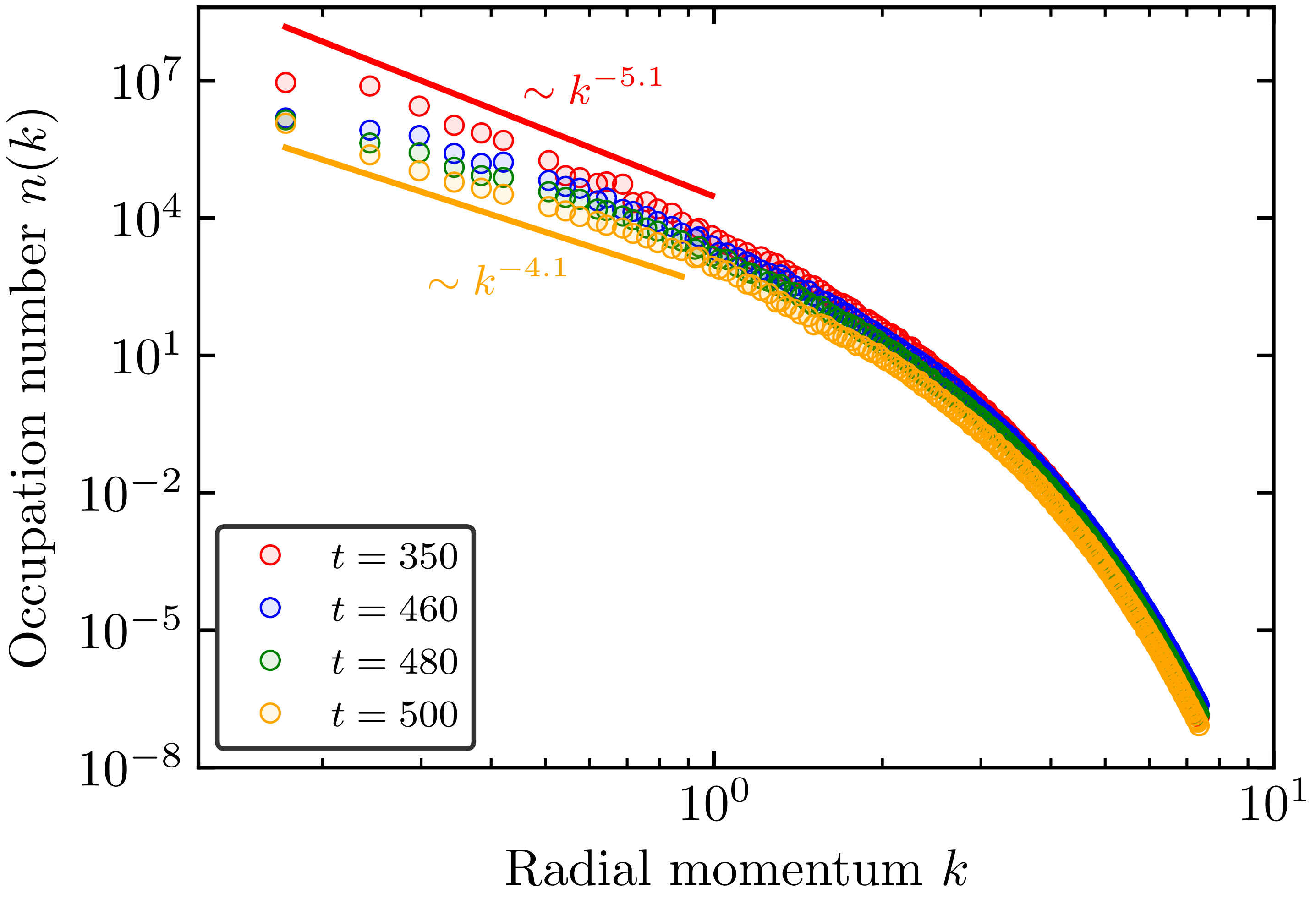


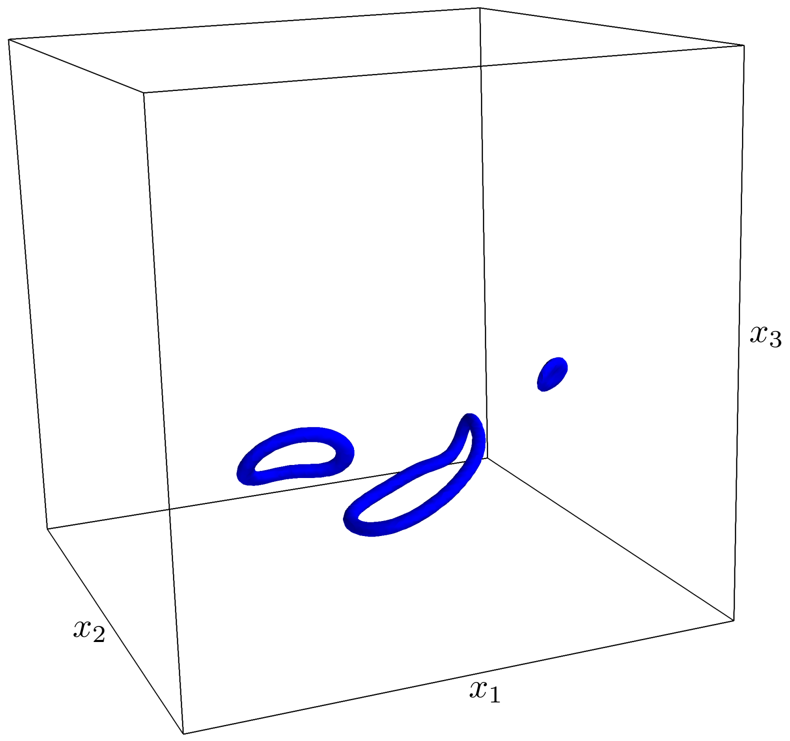


- times $150 \lesssim t \lesssim 300$: scaling $n(k) \sim k^{-5.1}$

- universal (for all initial conditions)

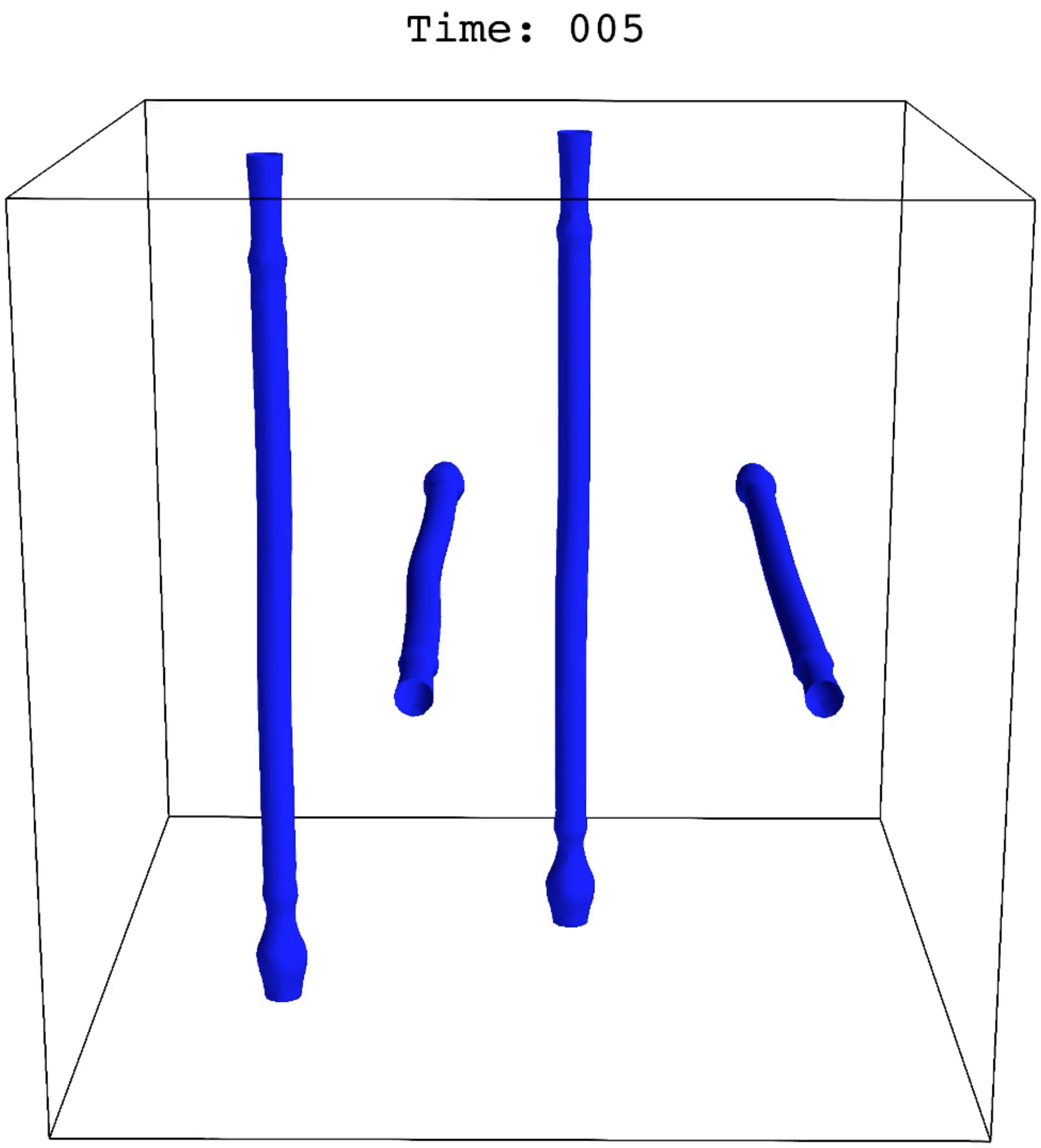
Turbulence: Late Times



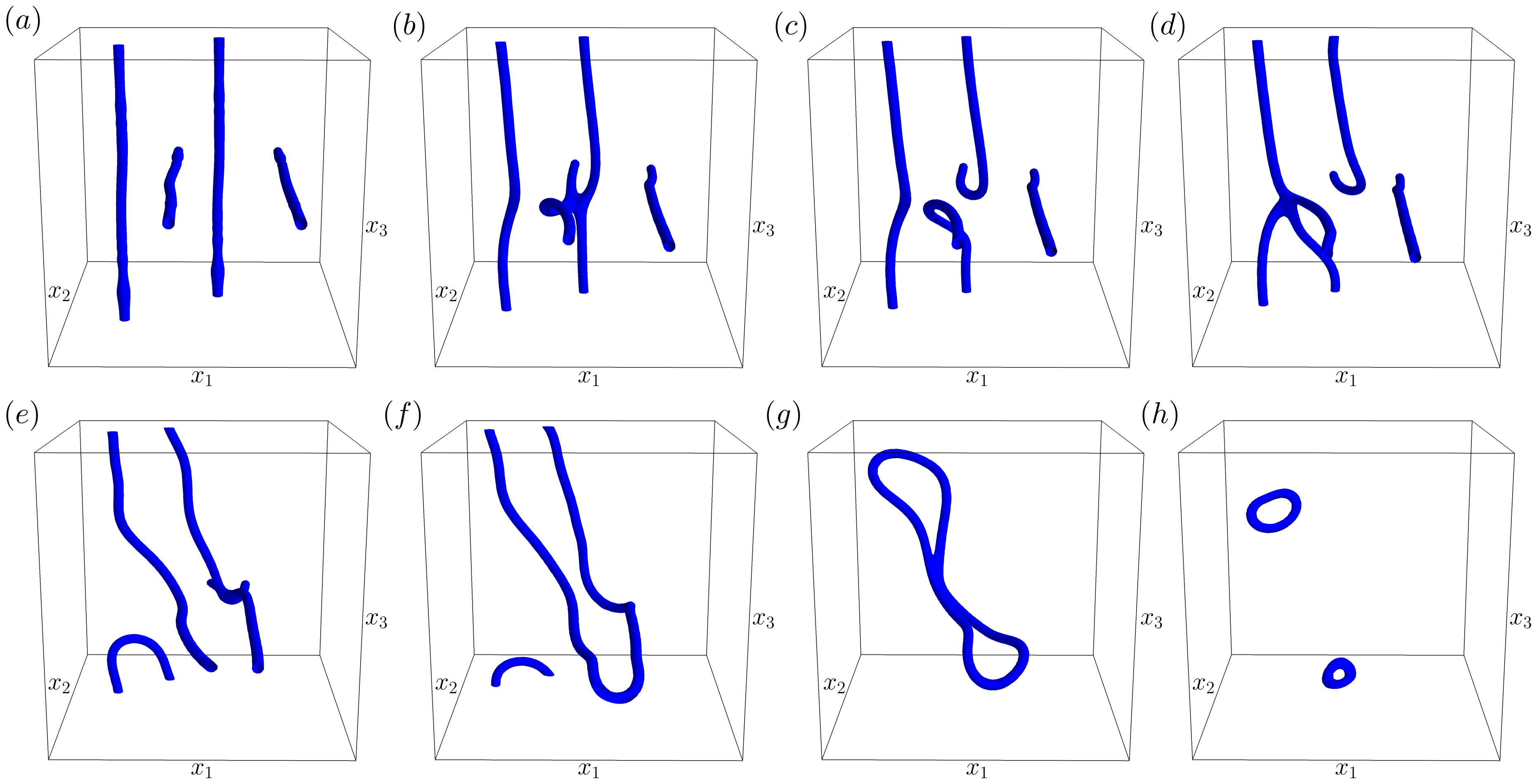


- times $t \gtrsim 350$: no (quasi-)stationary scaling
- spectrum gradually flattens
- universal (for all initial conditions)
- flattening agrees with spectrum of single vortex ring

Dynamics of Vortex Lines and Rings



Dynamics of Vortex Lines and Rings



- Fast formation of closed rings due to reconnections.
- Quick shrinking and disappearing of vortex rings indicates strong dissipation.
- Also rarefaction and sound waves observed.

Summary

Summary

- Holography relates weakly coupled gravity in higher dimension to strongly coupled quantum system.

- Matching of DGPE to 2d vortex dynamics in holographic superfluid shows strong dissipation.

- Holographic vortex dynamics applicable to superfluid helium films and to thermal BECs.

- First direct determination of parameters from dynamical process for any holographic system.

Summary

- 2d:
  turbulence, Landau instability, vortex decays, …
- 3d:
  turbulence, vortex line reconnections, …

Thank you for your interest!