Quantum vortices leave a macroscopic signature in the thermal background

FOUCAULT

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in solidarity

Collaborators

Observatoire de la Cote d'Azur

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Peter Stasiak

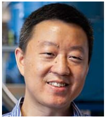
Andrew Baggaley

Collaborators

Università La Sapienza

Carlo Casciola

Florida Natl. Lab



Wei Guo

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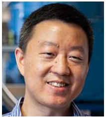

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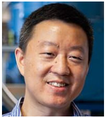

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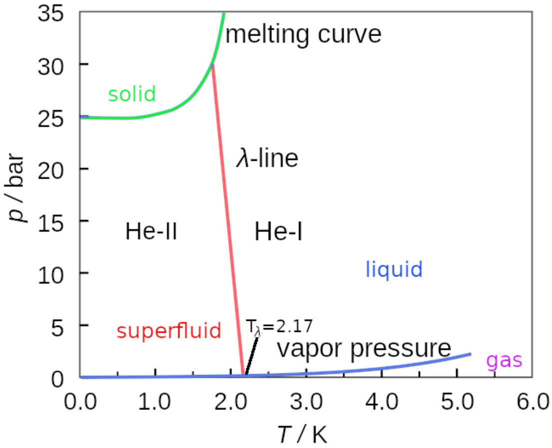
Overview

Overview

Helium-4


35
30
25
20
15
10
5
0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
p / bar
T / K
melting curve
solid
λ-line
He-II
He-I
liquid
superfluid
Tλ=2.17
vapor pressure
gas

Helium II - TWO FLUID MODEL

Tisza (1938), Landau(1941)

SUPERFLUID

- ~ condensate
  - related to BEC
- $\rho_s, \mathbf{v}_s$
- no entropy
- inviscid $\nu_s = 0$
- ~ Euler fluid

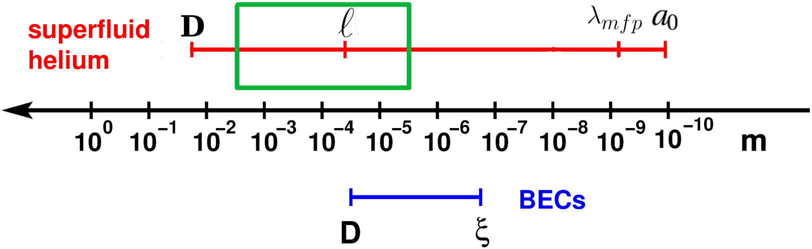
NORMAL FLUID

- thermal excitations
  - phonons
  - rotons $(1.5K < T < 2.1K)$
- $\rho_n, \mathbf{v}_n$
- entropy $s \neq 0$
- viscosity $\nu_n \sim 10^{-8} m^2/s$
- ~ Navier-Stokes fluid

Helium II *vs* BECs: lenghtscales

LENGHTSCALES	He II	BECs
vortex core	$a_0 \sim 10^{-10}$m	$\xi \sim \mu$m
mfp excitations	$\lambda_{mfp} \sim 10^{-10}$m $\div$ 10^{-9}m	$\lambda_{mfp} \lesssim D$
intervortex distance	$\ell \sim 10^{-4}$m $\div$ 10^{-5}m	$\ell \lesssim D$
system size	$D \sim 10^{-1}$m $\div$ 10^{0}m	$D \sim 100\mu$m

Helium II *vs* BECs: lenghtscales



Helium II

- $D/a_0 \sim 10^8$
- $D/\ell \sim 10^3$

BECs

- $D/\xi \lesssim 10^2$
- $\ell \sim D$

Classical Turbulence

- $D/\eta \sim 10^6$

Helium II *vs* BECs: lenghtscales

- continuum mechanics

  $$Kn = \frac{\lambda_{mfp}}{d_{prob}} < 0.01$$

  - $T = 1.5\text{K}, \;\; d_{prob} > 0.2 \;\; \mu\text{m}$
  - $T = 2.1\text{K}, \;\; d_{prob} > 0.02 \;\; \mu\text{m}$

- $\ell \gg d_{prob}$

- $r_p \gg d_{prob}$

  solid H/D_2 tracking particles

Helium II - Quantised Vortices

- topological defects of the superfluid
- one–dimensional structures $a_0 \sim 1\text{Å}$
  - $\ell \sim 10^{-4}$ m $\div$ 10^{-5} m
  - $D \sim 10^{-2}$ m $\div$ 10^{0} m
- $\omega_s = \nabla \times \mathbf{v}_s$ confined to vortex lines $\mathbf{s}(\zeta, t)$

$$\omega_s(\mathbf{x}, t) = \kappa \oint_{\mathcal{L}} \mathbf{s}'(\zeta, t)\delta^{(3)}(\mathbf{x} - \mathbf{s}(\zeta, t))\, d\zeta$$

- circulation quantized, $\kappa = h/m = 10^{-7}\text{m}^2/\text{s}$
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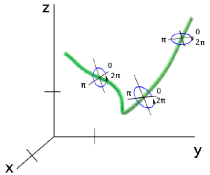

vortex-lines scattering centres for thermal-excitations

$\Downarrow$

mutual friction force $\mathbf{F}_{ns} \propto (\bar{\mathbf{v}}_n - \bar{\mathbf{v}}_s)$, $(\dot{\mathbf{s}} - \mathbf{v}_n)$

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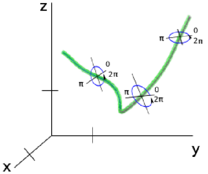

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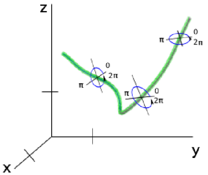

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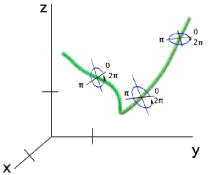

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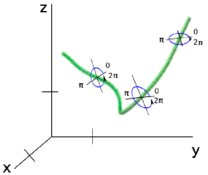

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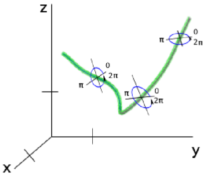

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Overview

Mutual Friction Force $\mathbf{F}_{ns}$: pioneerings and a review

The rotation of liquid helium II

II. The theory of mutual friction in uniformly rotating helium II

By H. E. Hall and W. F. Vinen

The Royal Society Mond Laboratory, University of Cambridge

Proc. R. Soc. London A **215**, 215 (1956)

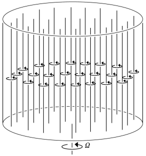
Mutual Friction in Bosonic Superfluids: A Review

Yuri A. Sergeev[1]

J. Low Temp Phys **212**, 251 (2023)

Helium II - Mutual friction force $\mathbf{F}_{ns}$

- pioneering work Hall & Vinen (1956)
  - probed lengthscales $\Delta \gg \ell$
  - friction coefficients calculation
  - HVBK Eqs.



- Schwarz (1978)
  - probed lengthscales $\delta \lesssim \ell$
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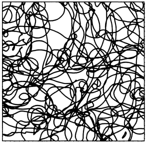

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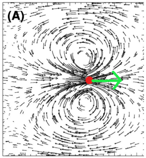

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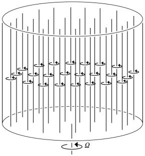

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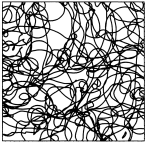

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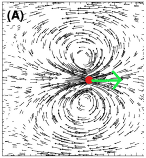

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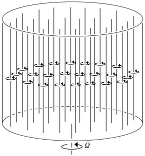

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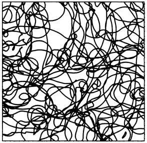

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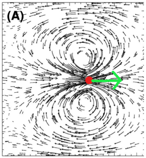
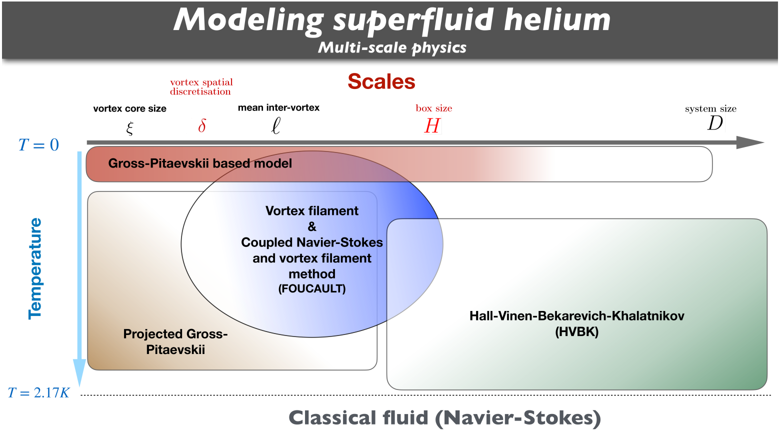


Helium-II: models



credits: Giorgio Krstulovic

Overview

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Fully cOUpled loCAl model of sUperfLuid Turbulence

1. more *realistic* classical model of $\mathbf{F}_{ns}$
2. distribution of $\mathbf{F}_{ns}$ on $\mathbf{v}_n$ grid points
   physically motivated
3. higher parallelisation
   solve wider range of scales

USE tools from Classical Turbulence

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$\mathbf{F}_{ns}$: *fully coupled* local model

- classical, low-Reynolds fluid dynamics
  [Kivotides, *Phys. Rev. Fl.* **3**, 104701 (2018)]
  - $\mathbf{f}_D \propto (\mathbf{v}_n - \dot{\mathbf{s}}) \quad \sim \quad$ Stokes drag
  - vortex locally $\sim$ cylinder
  - $R_c \gg \delta \gg a_0$
  - $\mathrm{Re} = \dfrac{|\mathbf{v}_n - \dot{\mathbf{s}}| a_0}{\nu_n} \sim 10^{-5} \div 10^{-4}$
- $\mathbf{f}_D = D\,(\mathbf{v}_n - \dot{\mathbf{s}}) \quad , \quad D = \dfrac{4\pi\rho_n\nu_n}{\left[\frac{1}{2} - \gamma - \log\left(\frac{|\mathbf{v}_{n_\perp} - \dot{\mathbf{s}}| a_0}{4\nu_n}\right)\right]}$
- $\mathbf{f}_D + \mathbf{f}_I + \mathbf{f}_M \quad = \quad 0$
- $\dot{\mathbf{s}} = \mathbf{v}_s + \beta\mathbf{s}' \times (\mathbf{v}_n - \mathbf{v}_s) + \beta'\mathbf{s}' \times \mathbf{s}' \times (\mathbf{v}_n - \mathbf{v}_s)$
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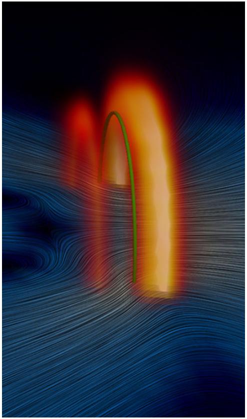

LG, CFB, AWB, GK ,
Eur. Phys. J. Plus **135**, 547 (2020)

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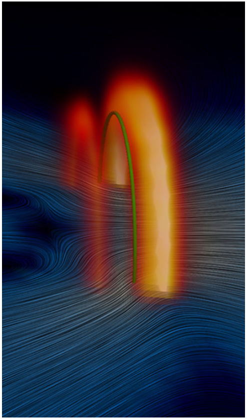

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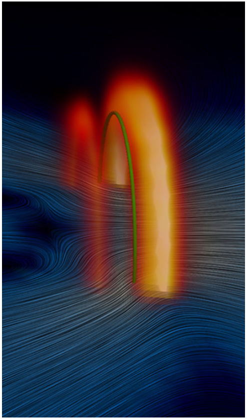

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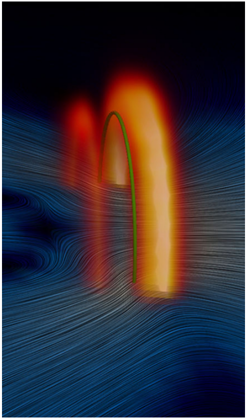

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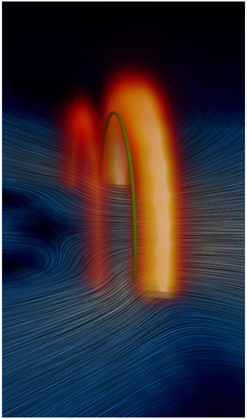

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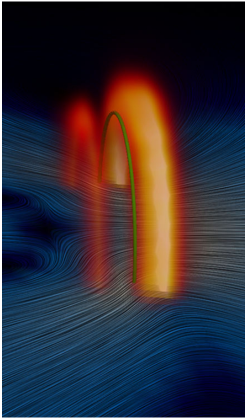

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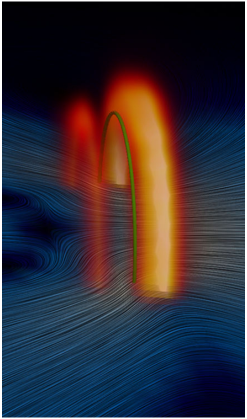

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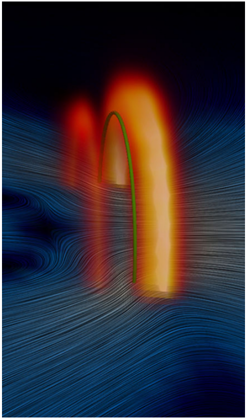

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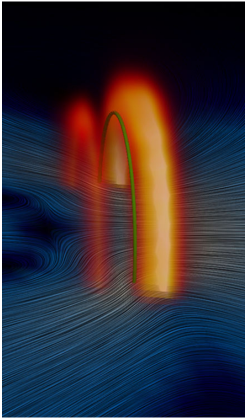

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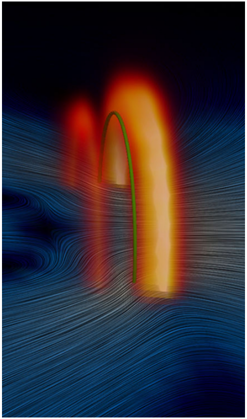

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$\mathbf{F}_{ns}$: *fully coupled* local model

- $\mathbf{v}_n(\mathbf{x},t)$ self-consistently with NS Eqs. + tangle $\{\mathbf{s}_i(t)\}_{i=1,\dots,N_p}$

$$
\begin{aligned}
\rho_n\left[\frac{\partial \mathbf{v}_n}{\partial t}+(\mathbf{v}_n\cdot\nabla)\,\mathbf{v}_n\right] &= -\frac{\rho_n}{\rho}\nabla p+\eta\nabla^2\mathbf{v}_n+\oint_{\mathscr{L}}\delta(\mathbf{x}-\mathbf{s})\mathbf{f}_{ns}(\mathbf{s})\mathrm{d}\xi\ , \\
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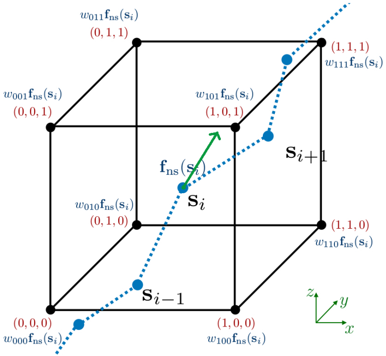
\end{aligned}
$$



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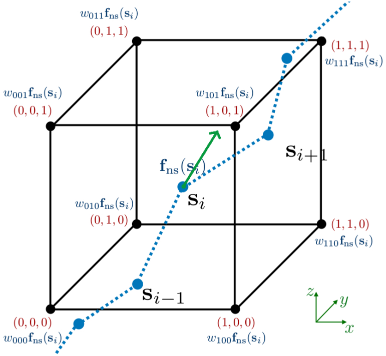
$$\nabla\cdot\mathbf{v}_n = 0$$

- $w_{\zeta,\mu,\chi}$
- $\sum_{\zeta,\mu,\chi=0}^{1} w_{\zeta,\mu,\chi}=1$
- nearest neighbours
  tri-linear extrapolation
- Filtering
  - moving avg N_{filter} points
  - Gaussian kernel
    $\sigma = N_{filter}\Delta x$



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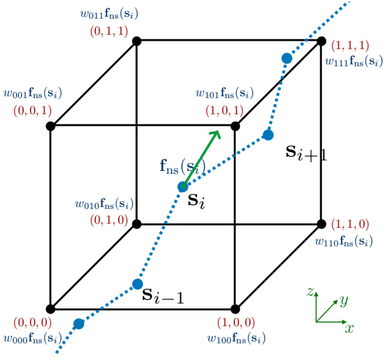


$\mathbf{F}_{ns}$: *fully coupled* local model

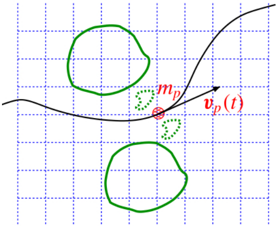
- physically consistent regularisation
- active matter systems
  strongly localised response of
  point-like agents
  - particles (PIV, PTV)
  - bacteria
  - swimmers

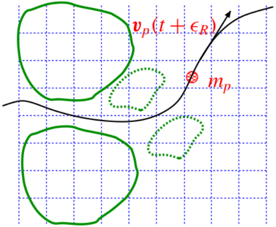
  [Gualtieri *et al.* , *J Fluid Mech* **773**, 520 (2015)]

  [Gualtieri *et al.* , *Phys Rev Fl* **2**, 034304 (2017)]

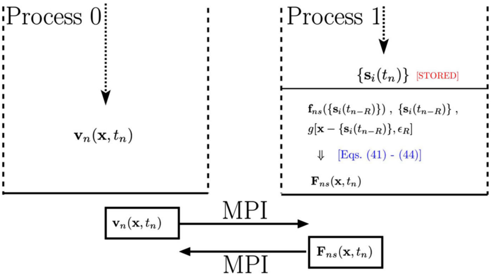
- Re$\sim 10^{-4} \div 10^{-5}$
- generation localised vorticity $\boldsymbol{\omega}_n$
- diffused by viscosity ν_n

  [LG *et al.*, *Eur. Phys. J. Plus* **135**, 547 (2020)]

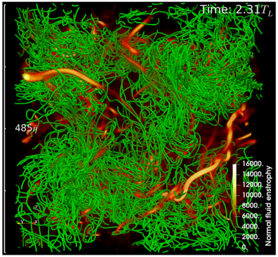




Numerical Architecture



- $\mathbf{v}_n(\mathbf{x}, t)$ on 512^3
- past: 128^3 (40^3)
- $N_p \sim 2 \times 10^5$
- wider range of scales



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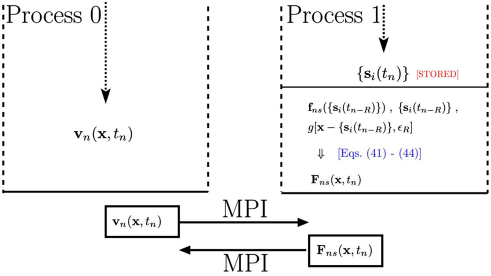

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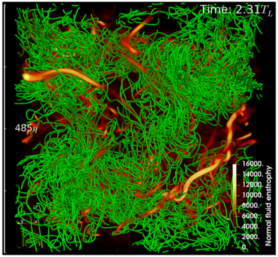

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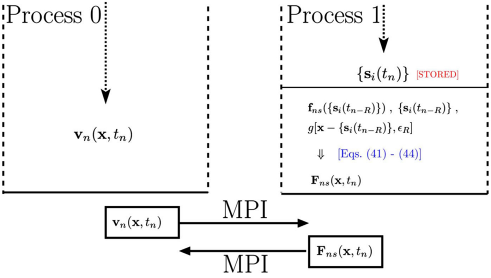

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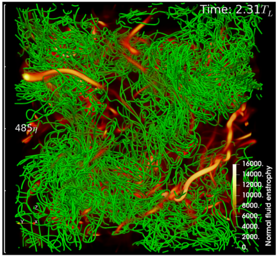
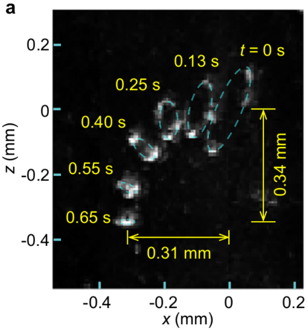
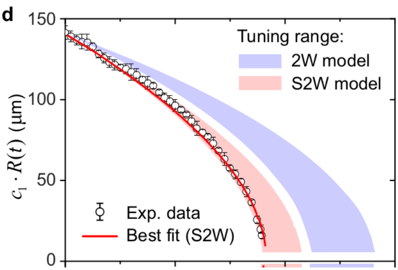


Shrinking vortex ring in quiescent normal fluid

Imaging quantized vortex rings in superfluid helium to evaluate quantum dissipation

[Tang *et al*, *Nat Comms* **14**, 2941 (2023)]





Overview

Helium II - COUNTERFLOWS

- He II thermically driven
  - heater placed in a channel
  - $\Delta T \;\Rightarrow\; \Delta p = \rho s \Delta T$
  - normal fluid transports heat , $\bar{\mathbf{v}}_n \rightleftarrows \bar{\mathbf{v}}_s$
  - $\bar{\mathbf{v}}_{ns} = \bar{\mathbf{v}}_n - \bar{\mathbf{v}}_s = \mathbf{q}/(T\rho_s s)$
  - Laminar - T I - T II
- $q_{lam} \propto \nabla T \quad , \quad q_{tur} \propto \nabla T^{1/m}$
- Dissipation Excess
  $E_n(k) \sim k^{-m}$, $m > 5/3$
  [Gao *et al.* , *Phys Rev B* (2017)]
- T I - T II transition
  [Tough, *Superfluid Turbulence* (1982)]
- large turbulent intensity
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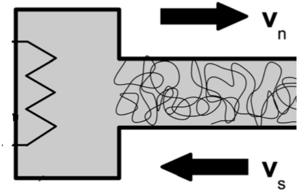

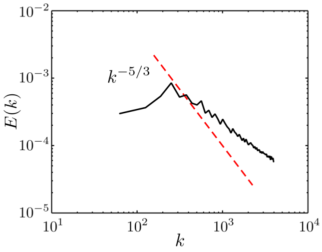

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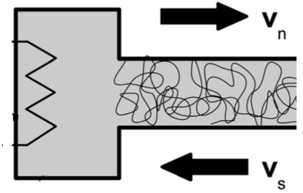

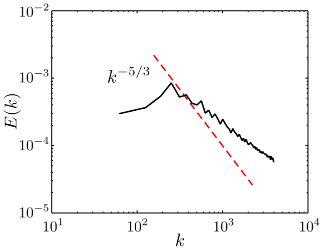

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Helium II - COUNTERFLOWS

Motivations

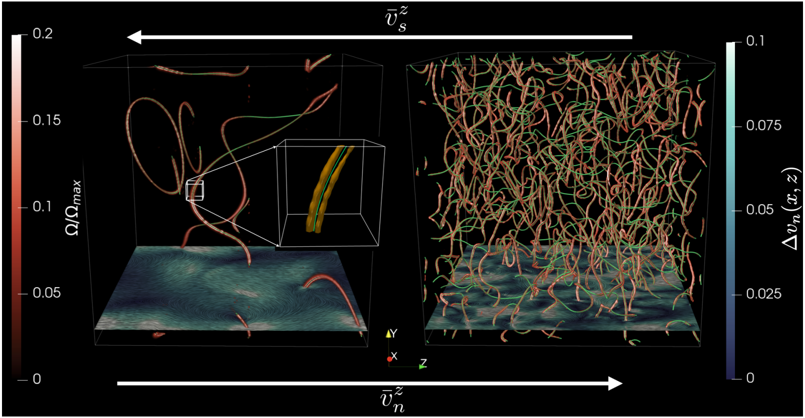
Helium II counterflows studied since 1957, nonetheless:

1. statistics of $\mathbf{v}_n$ fluctuations
2. more information needed on $\mathbf{v}_s$ (profiles)
3. only coarse-grained information on $L(\mathbf{x})$
4. more insight is needed to interpret Particle Tracking Velocimetry Exps.

[LG *et al*, *arXiv:2501.08309* (2025)]

COUNTERFLOWS: normal fluid structures

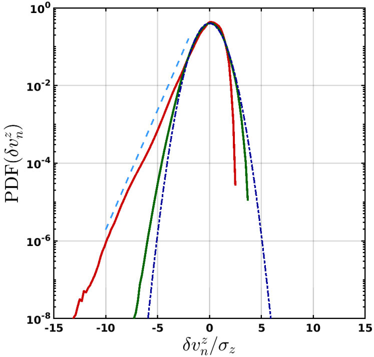
T=1.5 K

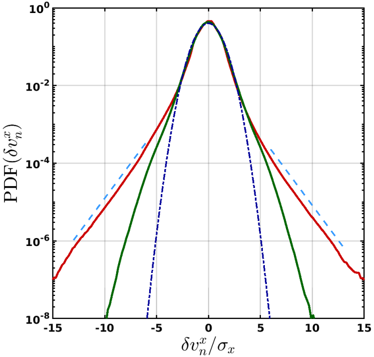


- $v_{ns}^{(1)} = 0.27\text{cm/s}$
- $L^{(1)} = 7.2 \times 10^2 \text{cm}^{-2}$
- $\Omega(\mathbf{x}) = \frac{|\boldsymbol{\omega}_n(\mathbf{x})|^2}{2}$
- $\Delta v_n(x,z) = \frac{||\delta \tilde{\mathbf{v}}_n||}{|\bar{v}_n^z|}$
- $v_{ns}^{(2)} = 0.94\text{cm/s}$
- $L^{(2)} = 1.1 \times 10^4 \text{cm}^{-2}$

COUNTERFLOWS: PDF(δv_n)

T=1.5 K $\quad v_{ns}^{(1)} = 0.27\text{cm/s} \quad v_{ns}^{(2)} = 0.94\text{cm/s}$

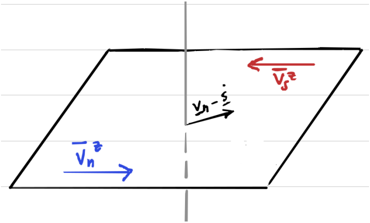




- $\sigma_z \approx 2\sigma_x$
- $\delta v_n^z = v_n^z - \bar{v}_n^z$
- $\sigma_z > \sigma_z$
- $\delta v_n^x = v_n^x$
- $\sigma_x > \sigma_x$

SINGLE VORTEX EXPERIMENT

T=1.5 K $\qquad v_{ns}^{(1)} = 0.27\mathrm{cm/s}$



- $\tau_r \approx 6\pi / |\kappa L \ln(L^{1/2} a_0)|$
- $\tau^* \approx \tau_r / 10$

SINGLE VORTEX EXPERIMENT

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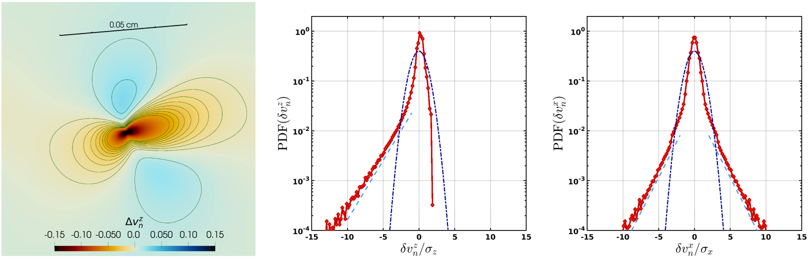
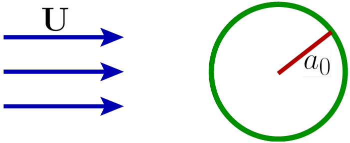


- $$w = \sqrt{\frac{\int_{\mathscr{S}} |\mathbf{x}-\mathbf{s}_0|^2 |\delta\tilde{\mathbf{v}}_n|^2 \, dxdz}{\int_{\mathscr{S}} |\delta\tilde{\mathbf{v}}_n|^2 \, dxdz}}$$

- $w^{(1)} = 190\mu\text{m} < \ell^{(1)} = 370\mu\text{m}$
- $w^{(2)} = 140\mu\text{m} < \ell^{(2)} = 94\mu\text{m}$

Oseen's classical solution



[Lamb, *Hydrodynamics* (1932)]

- straight vortex $a_0 \ll R_c$
- $Re \sim 10^{-5}$
- $\Delta x \lesssim a_0 / Re$
- $\mathbf{V} = \mathbf{U} + \mathbf{v}$
- $U \frac{\partial}{\partial z} \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v}$
- far field correction in 3D Stokes
- 2D necessary !
- for $r \rightarrow a$ error $\sim -\log(Re)^{-1}$

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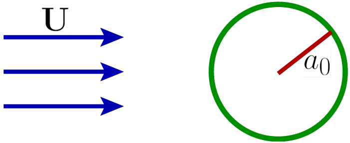

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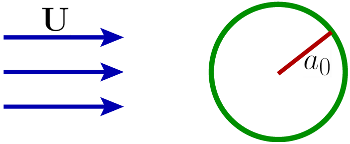

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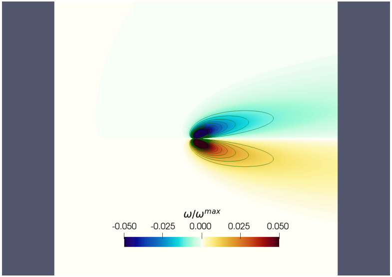
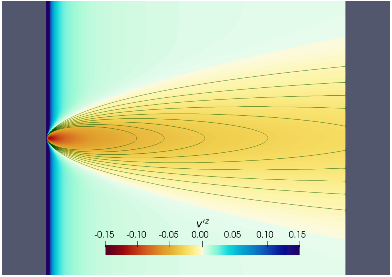
Oseen's classical solution: far field solution

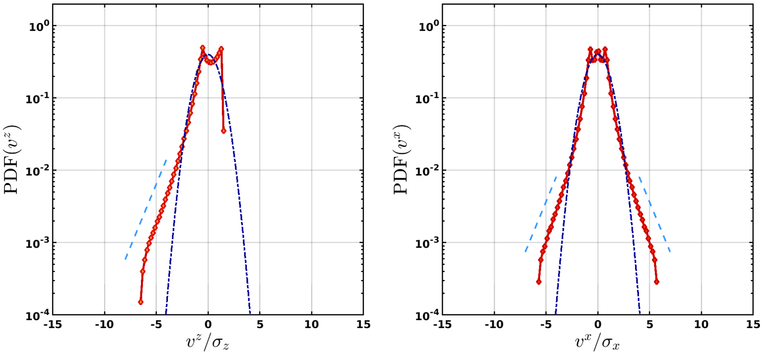
$$r \gg a_0 / Re$$



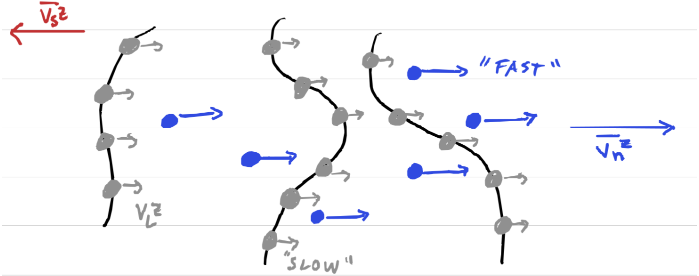


- $\omega = \frac{C_0 Re}{2} \sin\theta \, e^{-\frac{\tilde{r}}{2}(1-\cos\theta)} \sqrt{\frac{\pi}{\tilde{r}}}$
- $C_0 = -\frac{2}{0.5 - \gamma - \log(\epsilon/2)}$
- $v'_r = C_0 \sqrt{\frac{\pi}{\tilde{r}}} \, e^{-\frac{\tilde{r}\theta^2}{4}} - \frac{C_0}{\tilde{r}},$
- $v'_\theta = -\frac{C_0}{2} \sqrt{\frac{\pi}{\tilde{r}}} \theta \, e^{-\frac{\tilde{r}\theta^2}{4}}$

Oseen's classical solution: far field solution, PDFs



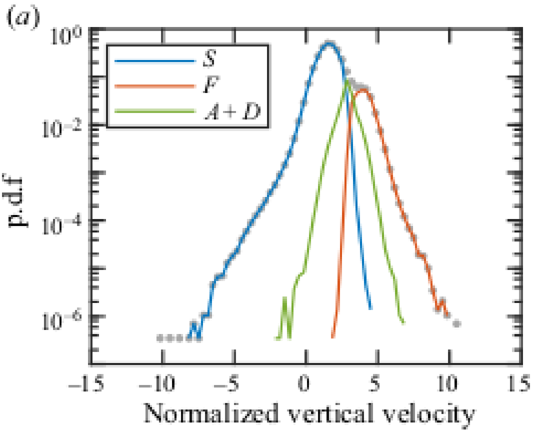
Experimental signature of wakes



- 'fast particles' $\approx v_n^z$
- 'slow particles' $\approx v_L^z$

  [Yui *et al.*, *Phys Rev Lett* **129**, 025301 (2022)]

Experimental signature of wakes



[Svancara *et al*, *JFM* **911**, A8 (2021)]

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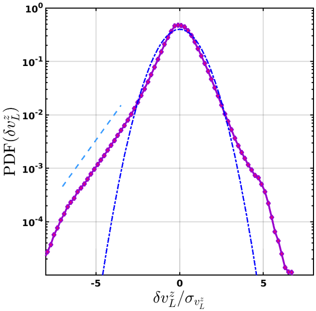
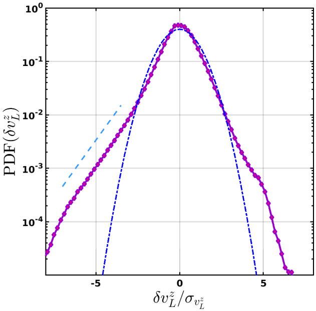


- $v_L^z(\mathbf{s}) = \varepsilon_1(\bar{v}_s^z + v_{BS}^z(\mathbf{s})) + \varepsilon_2 v_n^z(\mathbf{s})$
- $\varepsilon_1 + \varepsilon_2 = 1$, $\varepsilon_1(T)$, $\varepsilon_2(T)$
- $\delta v_L^z(\mathbf{s}) = \varepsilon_1 \delta v_{BS}^z(\mathbf{s}) + \varepsilon_2 \delta v_n^z(\mathbf{s})$
- $\mathscr{S}(v_L^z) = \overline{[\delta v_L^z(\mathbf{s})]^3} = -2.3 \times 10^{-4}$
- $\mathscr{S}(v_{BS}^z) = -1.8 \times 10^{-4}$
- $\mathscr{S}(v_n^z) = -8 \times 10^{-4}$

WAKE SIGNATURE!

Experimental signature of wakes



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WAKE SIGNATURE!

CONCLUSIONS

- large scale structures (wakes) in sea of thermal excitations
- probe the Iordanskii force $\mathbf{f}_i$?
- $\mathbf{v}_n$ can be described as
  - uniform flow

    +
  - 2D flow structures

    - vortex dipoles

    - parabolic wakes

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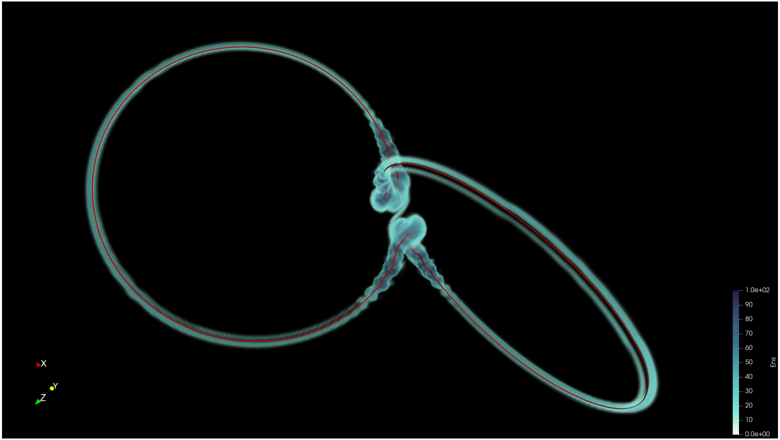
Application to fermionic systems and Neutron stars

- fermions: hydrondynamic description at core level?
- neutron stars:
  - dissipative motion of vortices
  - intrinsic spin down by viscosity
  - add a pinning mechanism
    [Antonelli & Haskell, *MNRAS* **499**, 3690 (2020)]
  - how to relate macrosopic hydrodynamics to mesocale quantised vortex dynamics

Helium II - Vortex Reconnections



Peter Stasiak