When polarons discover their statistics in two dimensions

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MUR PRIN

What is a polaron?

• In the field of ultracold gases the term polaron refers to a **single mobile impurity interacting with the particles of a many-body system** (bath).

• **Fermi polaron**: the bath is an ideal gas of identical fermions.

• **Bose polaron**: the bath is a weakly-interacting Bose gas.

• Physically realized by having few atoms of one kind interacting with atoms of a different kind. They could be atoms of a different atomic species or atoms of the same atomic species but in a different internal state (different hyperfine level).

• Clearly, the **statistics of single impurity** (i.e., whether it is a bosonic or fermionic atom) is **irrelevant.** It becomes **relevant** when the **concentration** of impurities becomes **finite**.

• Main quantities: polaron energy, quasiparticle residue Z, effective mass, lifetime (when it is not an eigenstate: i.e., at finite momentum or for the "repulsive branch").

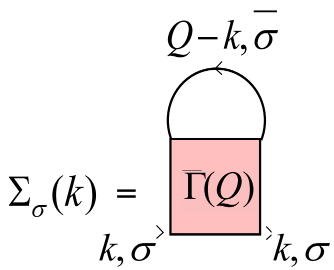
• Focus of this talk: **fate of the Fermi polaron** when the **density of impurities becomes finite**, particularly in 2D, and role of their statistics.

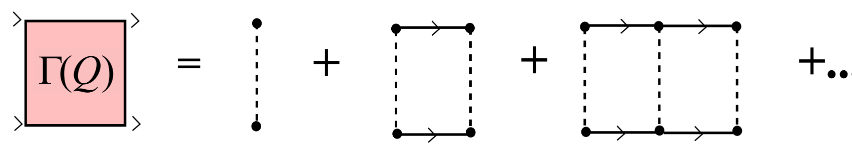
Chevy ansatz and T-matrix

• Simple variational ansatz (Chevy, 2006) describes surprisingly well several properties of the polaron.

$$|\psi\rangle = \sqrt{Z} a^{\dagger}_{\mathbf{0}\downarrow}|\mathrm{FS_N}\rangle + \sum_{q<k_F<k} \phi_{q,k} a^{\dagger}_{q-k\downarrow} a^{\dagger}_{k\uparrow} a_{q\uparrow}|\mathrm{FS_N}\rangle \,.$$

• In terms of Feynman diagrams it corresponds to a resummation of ladder diagrams (T-matrix) to construct the self-energy of a single impurity.





• **Quasiparticle residue**: square modulus of overlap between interacting ground state $|\psi\rangle$ (of system with N fermions + 1 impurity) with noninteracting ground state $a^{\dagger}_{\mathbf{0}\downarrow}|\mathrm{FS_N}\rangle$.

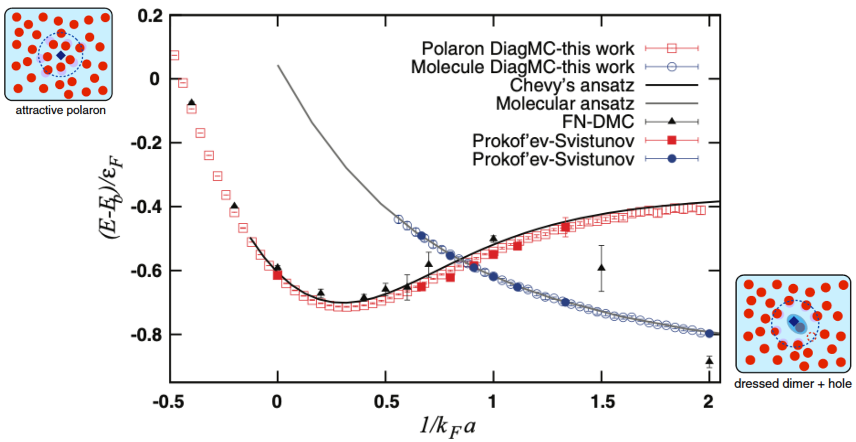
$$Z = |\langle\psi|a^{\dagger}_{\mathbf{0}\downarrow}|\mathrm{FS_N}\rangle|^2 = \frac{1}{\left|1 - \frac{\partial \mathrm{Re}\Sigma^{\mathrm{R}}_{\mathrm{p}}(\mathbf{0},\omega)}{\partial\omega}\right|_{\omega=0}}$$

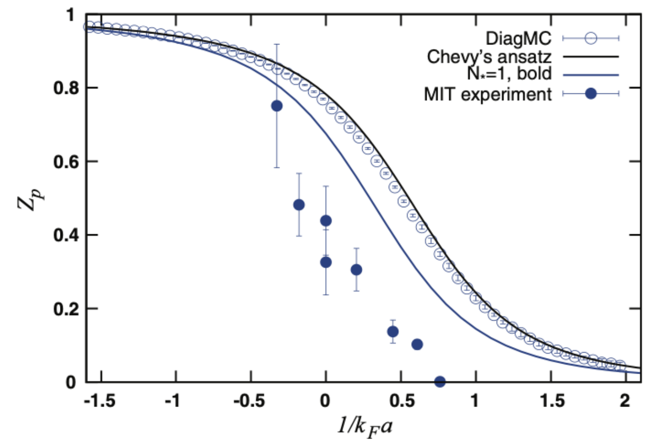
• **Polaron energy**: energy change due to adding a single impurity

$$E_{\mathrm{p}} = \mu_{\mathrm{p}} = \Sigma_{\mathrm{p}}(\mathbf{0}, 0)$$

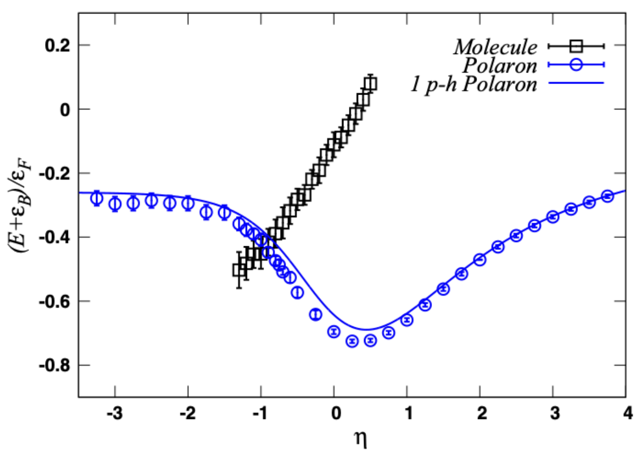
Energy and quasiparticle residue

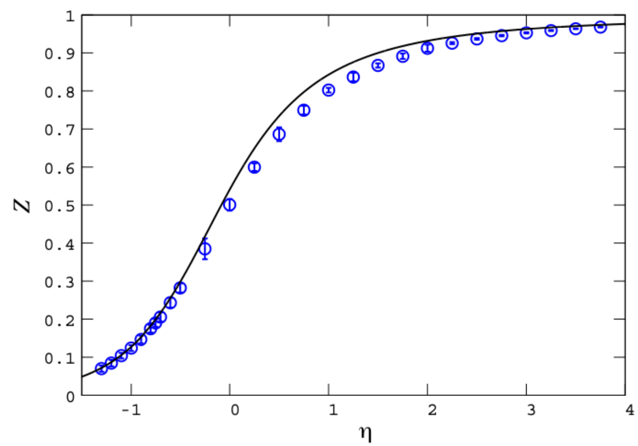
3D:Vlietinck,Ryckebusch, Van Houcke (2013)





2D:Vlietinck,Ryckebusch, Van Houcke (2014)





- **Good agreement** between Chevy ansatz(=T-matrix) and diagrammatic MC in both 3D and 2D for both E_p and Z. Overestimate of dressed-molecule energy by T-matrix: polaron molecule transition shifted from $1/k_F a = 0.9$ to 1.3 in 3D, missed in 2D.

Repulsive and attractive polaron (3D)

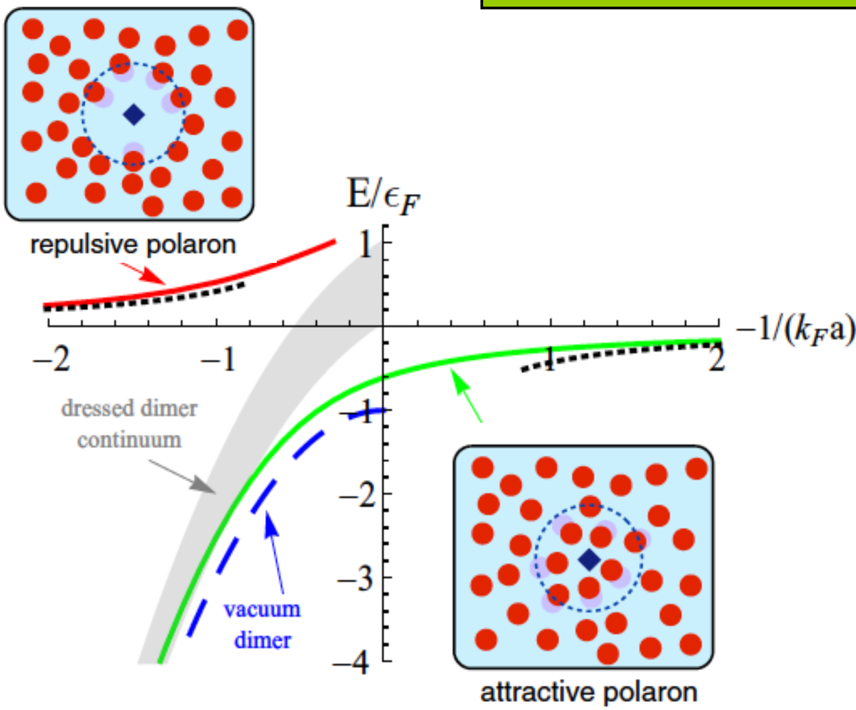
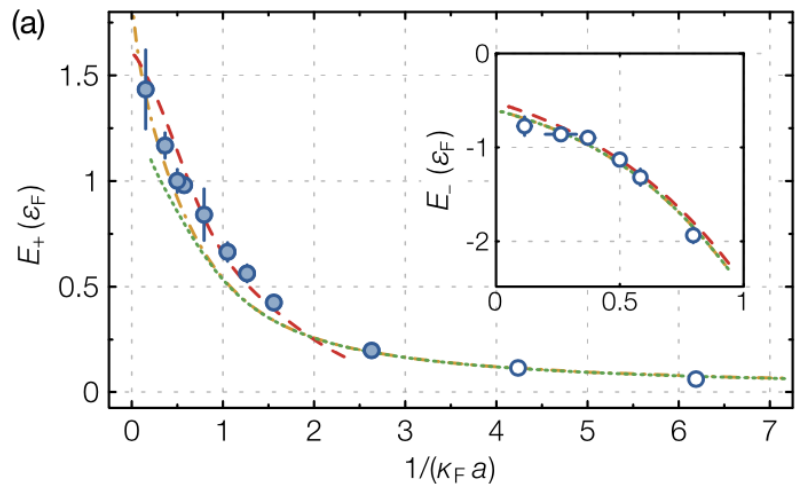


Figure adapted from Massignan, Zaccanti, Bruun, Rep. Progr. Phys. (2014)

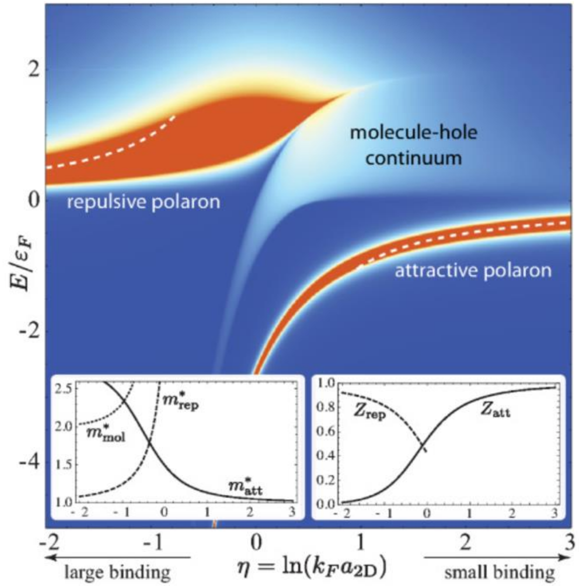


Scazza et al., PRL (2017)

• **Repulsive polaron**: quasi-particle excitation formed by the impurity repelling the surrounding Fermi atoms for positive scattering length. It is not an exact eigenstate.

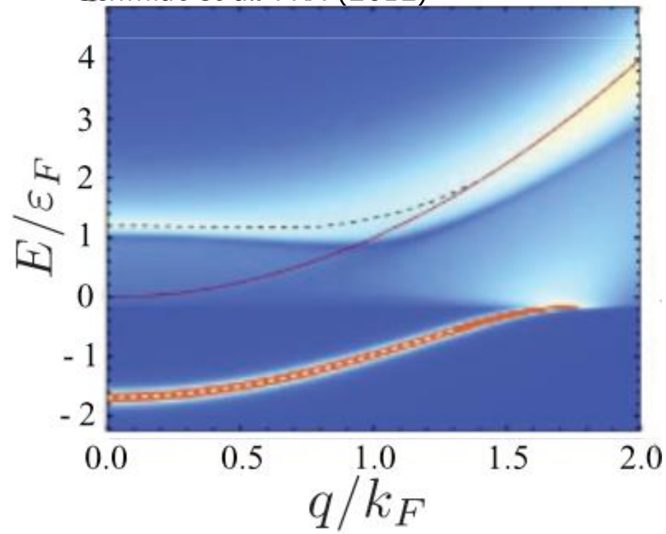
• T-matrix self-energy also allows for the calculation of the properties of the repulsive polaron (Chevy ansatz limited to the attractive one). It works quite well also in this case.

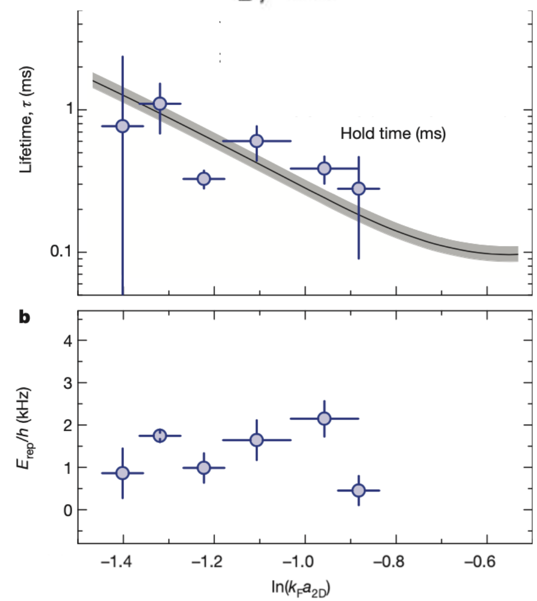
Repulsive and attractive polaron (2D)



Schmidt et al. PRA (2012),
T-matrix calculation

Schmidt et al. PRA (2012)





Repulsive polaron lifetime

Repulsive polaron energy

Koschorreck et al., Nature (2012) (RF spectroscopy on a nearly 2D higly polarized Fermi gas).

Finite concentration of impurities

Polaron energy

• Fermionic case: **Fermi-Fermi mixture**. The energy of the attractive polaron can be obtained in the zero concentration limit $x = n_\downarrow / n_\uparrow \to 0$ from the chemical potential $\mu_\downarrow$ of the minority species.
Luttinger theorem for a Fermi liquid:

$$\mu_\downarrow = E_{\mathrm{F}\downarrow} + \Sigma_\downarrow^{\mathrm{R}}(k_{\mathrm{F}\downarrow}, 0)$$

When $x \to 0$: $E_{\mathrm{F}\downarrow}, k_{\mathrm{F}\downarrow} \to 0$ and

$$\mu_\downarrow \to \Sigma_\downarrow^{\mathrm{R}}(0, 0; x = 0) \equiv \Sigma_{\mathrm{p}}(0, 0) = E_{\mathrm{p}}$$

• Bosonic case: **Bose-Fermi mixture**. Chemical potential of minority species (bosons) determined by **Hugenholtz-Pines** theorem:

$$\mu_{\mathrm{B}} = \Sigma_{11}(0, 0) - \Sigma_{12}(0, 0)$$

When $x \to 0$: $\Sigma_{12} \to 0$ (the condensate density vanishes) and

$$\mu_{\mathrm{B}} \to \Sigma_{11}(0, 0; x = 0) \equiv \Sigma_{\mathrm{p}}(0, 0) = E_{\mathrm{p}}$$

Luttinger theorem for a Fermi liquid and **Hugenholtz-Pines** theorem for bosons are **unified in the single impurity limit** into the condition determining the polaron energy.

Finite concentration of impurities

Quasiparticle residue

• Fermionic case: **Fermi-Fermi** mixture. In a Fermi liquid the quasiparticle residue

$$Z_{\downarrow} = \frac{1}{\left|1 - \frac{\partial \mathrm{Re}\Sigma^{\mathrm{R}}_{\downarrow}(k_{\mathrm{F}\downarrow},\omega)}{\partial \omega}\right|_{\omega=0}}$$

gives the **height of the Fermi step**:

$$n_{\downarrow}(k_{\mathrm{F}}^{-}) - n_{\downarrow}(k_{\mathrm{F}}^{+})$$

When $x \to 0$: $k_{\mathrm{F}\downarrow} \to 0$ and

$$Z_{\downarrow} \to \frac{1}{\left|1 - \frac{\partial \mathrm{Re}\Sigma^{\mathrm{R}}_{\downarrow}(0,\omega;x=0)}{\partial \omega}\right|_{\omega=0}} \equiv \frac{1}{\left|1 - \frac{\partial \mathrm{Re}\Sigma^{\mathrm{R}}_{\mathrm{p}}(\mathbf{0},\omega)}{\partial \omega}\right|_{\omega=0}} = Z_{\mathrm{p}}$$

• Can Z_{p} be similarly connected to some thermodynamic quantity also in the bosonic case (Bose-Fermi mixture)?

• More generally, how do the spectral properties of the polaron get modified at finite *x*?

When the Fermi polarons are bosons: Bose-Fermi mixtures (focus on 2D)

L. Pisani, P. Bovini, F. Pavan, P. Pieri, SciPost Physics **18**, 076 (2025)
P. Bovini, L. Pisani, P. Pieri, work in progress

Motivation for 2D resonant Bose-Fermi mixtures

- Bose-Fermi dimers could be a **platform** to realize a **p-wave superfluid** according to a proposal by Bazak & Petrov:

PHYSICAL REVIEW LETTERS **121,** 263001 (2018)

Stable *p*-Wave Resonant Two-Dimensional Fermi-Bose Dimers

B. Bazak[1] and D. S. Petrov[2]

We consider two-dimensional weakly bound heterospecies molecules formed in a Fermi-Bose mixture with attractive Fermi-Bose and repulsive Bose-Bose interactions. Bosonic exchanges lead to an intermolecular attraction, which can be controlled and tuned to a *p*-wave resonance. Such attractive fermionic molecules can be realized in quasi-two-dimensional ultracold isotopic mixtures. We show that they are stable with respect to the recombination to deeply bound molecular states and with respect to the formation of higher-order clusters (trimers, tetramers, etc.)

- ($p_x + ip_y$) superfluid in 2D allows for zero-energy Caroli-Matricon-Saint James states in the vortex cores. These states provide the building blocks to construct topologically protected qubits.

The model (2D)

- **Two-component Hamiltonian** with attractive contact interaction between bosons and fermions.

$$H_{\mathrm{BF}} = \sum_{s=\mathrm{B,F}} \int d\mathbf{r}\, \psi_s^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m_s} - \mu_s\right) \psi_s(\mathbf{r}) + v_0^{\mathrm{BF}} \int d\mathbf{r}\, \psi_{\mathrm{B}}^\dagger(\mathbf{r})\psi_{\mathrm{F}}^\dagger(\mathbf{r})\psi_{\mathrm{F}}(\mathbf{r})\psi_{\mathrm{B}}(\mathbf{r})$$

- Bare contact-interaction strength between bosons and fermions expressed in terms of 2D **boson-fermion** scattering length a_{BF} .

$$\frac{1}{v_0^{\mathrm{BF}}} = -\int \frac{d\mathbf{k}}{(2\pi)^2} \frac{1}{\varepsilon_0 + \frac{k^2}{2m_r}} \qquad \varepsilon_0 = 1/(2m_r a_{\mathrm{BF}}^2)$$

- Boson-boson short-range (weak) repulsion:

$$H = H_{\mathrm{BF}} + \frac{1}{2}\int d\mathbf{r} d\mathbf{r}' V_{\mathrm{BB}}(\mathbf{r}-\mathbf{r}')\psi_B^\dagger(\mathbf{r})\psi_B^\dagger(\mathbf{r}')\psi_B(\mathbf{r}')\psi_B(\mathbf{r})$$

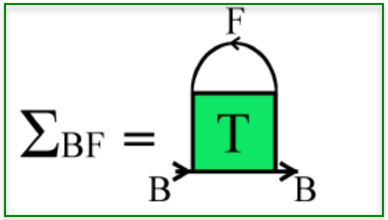
- We focus on equal masses $m_{\mathrm{F}}=m_{\mathrm{B}}$.

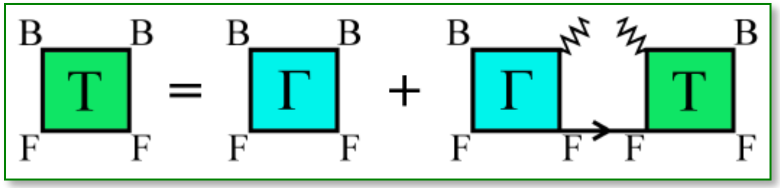
Dimensionless coupling strengths in 2D:

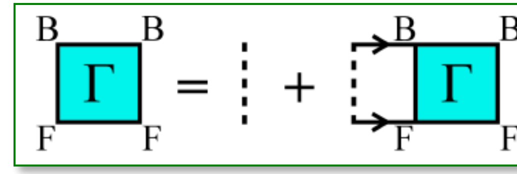
$g_{\mathrm{BF}} = -\ln(k_{\mathrm{F}} a_{\mathrm{BF}})$ for the (resonant) BF attraction

$1/\left|\ln\left(n_{\mathrm{B}} a_{\mathrm{BB}}^2\right)\right|$ for the (weak) BB repulsion.

Bosonic and fermionic self-energy diagrams for the **condensed** phase







Boson self-energy

$$\Sigma_{\mathrm{B}}^{11}\left(\bar{k}\right) = \frac{8\pi n_0}{m_{\mathrm{B}}\left|\ln\left(n_{\mathrm{B}} a_{\mathrm{BB}}^2\right)\right|} + \Sigma_{\mathrm{BF}}\left(\bar{k}\right)$$

$$\Sigma_{\mathrm{B}}^{12} = \frac{4\pi n_0}{m_{\mathrm{B}}\left|\ln\left(n_{\mathrm{B}} a_{\mathrm{BB}}^2\right)\right|}$$

$$\Sigma_{\mathrm{BF}}\left(\bar{k}\right) = \int \frac{d\mathbf{P}}{(2\pi)^2} \int \frac{d\Omega}{2\pi} T(\bar{P}) G_{\mathrm{F}}^0\left(\bar{P} - \bar{k}\right) e^{i\Omega 0^+}$$

$$\bar{P} \equiv (\mathbf{P}, i\Omega),\ \bar{k} \equiv (\mathbf{k}, i\omega) \qquad G_s^0(\bar{k})^{-1} = i\omega - \xi_{\mathbf{k}}^s$$

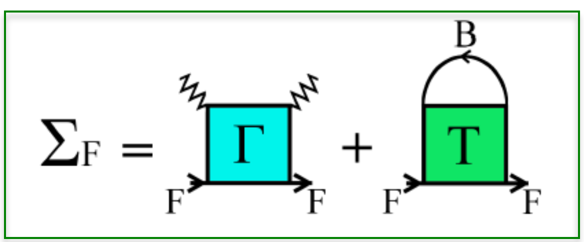
Boson-fermion T-matrix

$$T(\bar{P})^{-1} = \Gamma(\bar{P})^{-1} - n_0 G_{\mathrm{F}}^0(\bar{P})$$

$$\Gamma(\bar{P})^{-1} = -\frac{m_r}{2\pi} \ln\left(\frac{\frac{P^2}{2M} - \mu_{\mathrm{F}} - \mu_{\mathrm{B}} - i\Omega}{\varepsilon_0}\right) - I_{\mathrm{F}}(\bar{P})$$

$$I_{\mathrm{F}}\left(\bar{P}\right) = \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{\Theta\left(-\xi_{\mathbf{P-k}}^{\mathrm{F}}\right)}{\xi_{\mathbf{P-k}}^{\mathrm{F}} + \xi_{\mathbf{k}}^{\mathrm{B}} - i\Omega}$$

$$\xi_{\mathbf{p}}^s = p^2/2m_s - \mu_s \qquad \varepsilon_0 = 1/(2m_r a_{\mathrm{BF}}^2)$$



Fermion self-energy

$$\Sigma_{\mathrm{F}}(\bar{k}) = n_0 \Gamma(\bar{k}) - \int \frac{d\mathbf{P}}{(2\pi)^3} \int \frac{d\Omega}{2\pi} T(\bar{P}) G_{\mathrm{B}}^0(\bar{P} - \bar{k})$$

Coupled equations for chemical potentials and condensate density n_0

Green's functions obtained from the self-energies through Dyson's equations:

$$G_{\rm F}(\bar{k})^{-1} = G^0_{\rm F}(\bar{k})^{-1} - \Sigma_{\rm F}(\bar{k})$$

$$G'_{\rm B}(\bar{k}) = \frac{i\omega + \xi^{\rm B}_{\mathbf{k}} + \Sigma^{11}_{\rm B}(-\bar{k})}{[i\omega + \xi^{\rm B}_{\mathbf{k}} + \Sigma^{11}_{\rm B}(-\bar{k})][i\omega - \xi^{\rm B}_{\mathbf{k}} - \Sigma^{11}_{\rm B}(\bar{k})] + \Sigma^{12}_{\rm B}(\bar{k})^2}$$

Momentum distributions obtained from the Green's functions:

$$n_{\rm F}(\mathbf{k}) = \int \frac{d\omega}{2\pi} G_{\rm F}(\bar{k})\, e^{i\omega 0^+}$$

$$n_{\rm B}(\mathbf{k}) = -\int \frac{d\omega}{2\pi} G'_{\rm B}(\bar{k})\, e^{i\omega 0^+}$$

Integration over **k** + Hugenholtz-Pines relation ⇨ coupled eqs. for $\mu_B,\ \mu_F, n_0$:

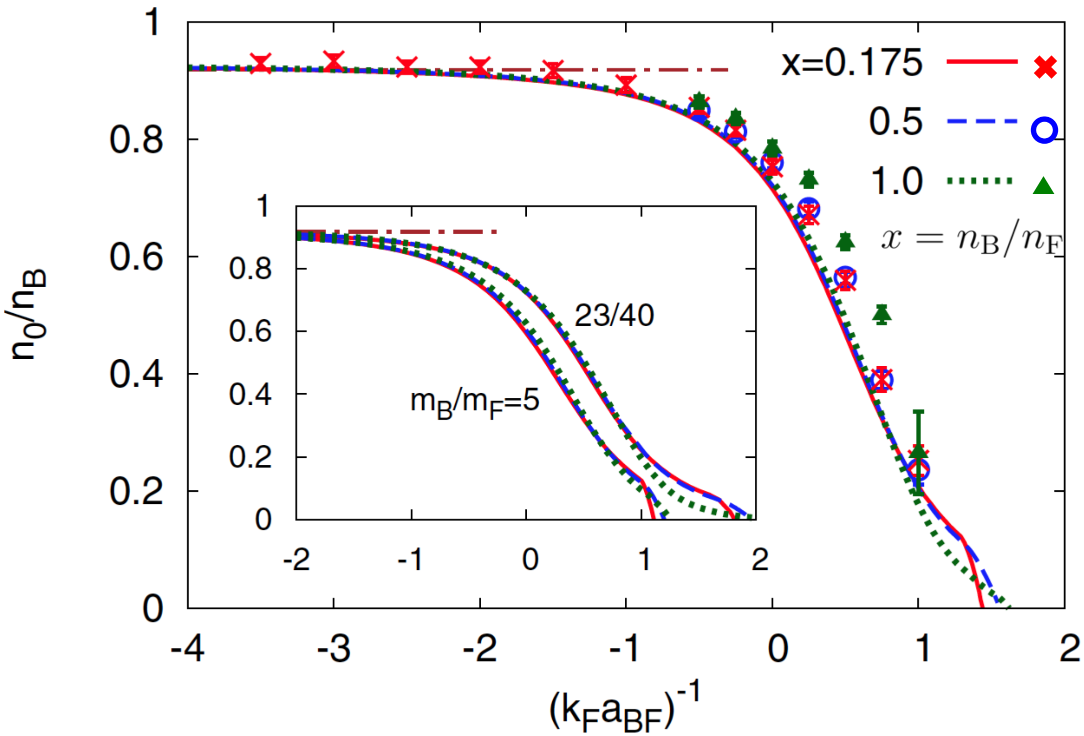
$$n_{\rm F} = \int \frac{d\mathbf{k}}{(2\pi)^2} n_{\rm F}(\mathbf{k})$$

$$n_{\rm B} = n_0 + \int \frac{d\mathbf{k}}{(2\pi)^2} n_{\rm B}(\mathbf{k})$$

$$\mu_{\rm B} = \Sigma^{11}_{\rm B}(0) - \Sigma^{12}_{\rm B}(0)$$

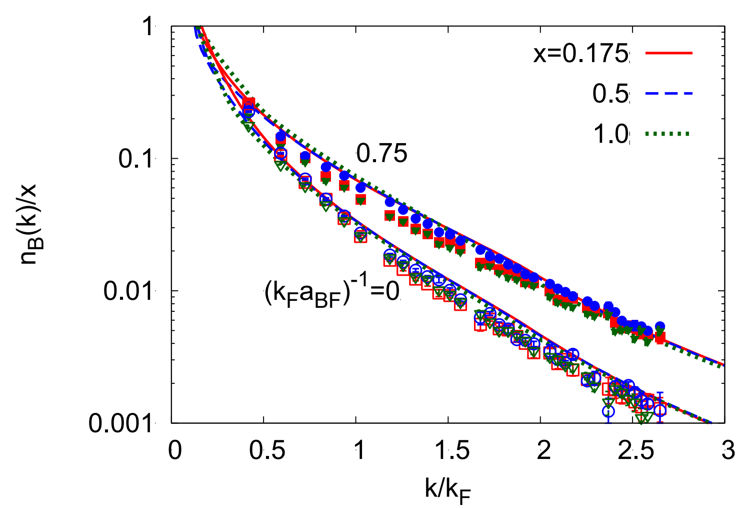
Universality of condensate fraction and boson momentum distribution (3D)

$$n_B a_{BB}^3 = 3 \times 10^{-3} \leftrightarrow k_F a_{BB} \gtrsim 0.5$$



A. Guidini, G. Bertaina, D. Galli, P. Pieri (2015)

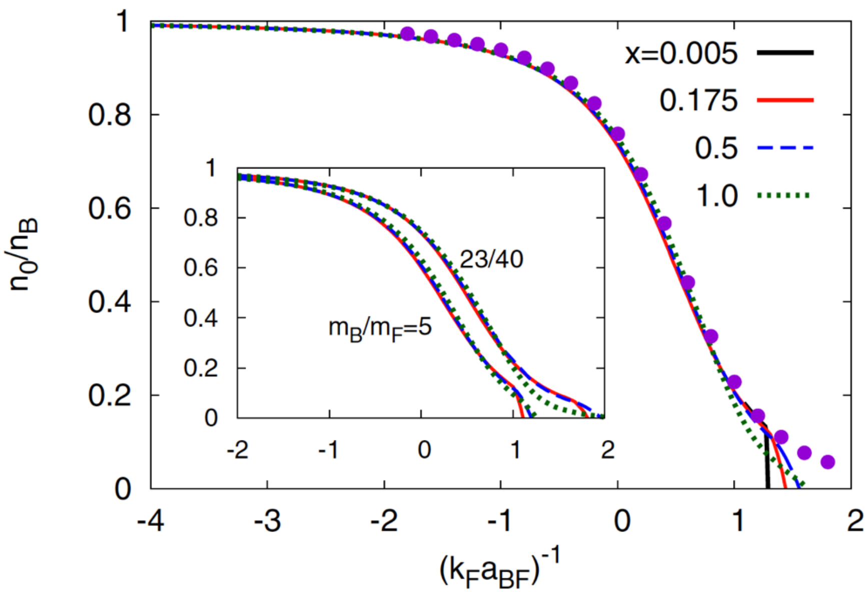
Lines: T-matrix self-energy
Symbols: FN-DMC



- Condensate fraction vanishes at a critical coupling: **quantum phase transition**.
- Condensate fraction almost **independent from the boson concentration**
- Universality also for the momentum distribution (once normalized by n_B). It suggests:

$$n_B(k) = n_B \times V n_{\text{pol}}(k) = N_B \times n_{\text{pol}}(k)$$

Comparison with polaron quasiparticle residue Z (3D)



A. Guidini, G. Bertaina, D. Galli, P. Pieri (2015)

Lines: T-matrix self-energy calculations at four different concentrations for **zero Bose repulsion**.

Circles: Diagrammatic MC results for Z_p [J. Vlietinck, J. Ryckebusch, K. Van Houcke, PRB **87**, 115133 (2013)]

Surprising agreement between 'universal condensate fraction' and Fermi polaron quasiparticle residue. Naïve explanation:

$$\frac{n_0}{n_B} = \frac{n_B(k=0)}{N_B} \to \frac{n_{pol}(k=0)}{1}$$

$$n_{pol}(k \neq 0) \propto \frac{1}{V} \to 0$$

in thermodynamic limit

$$Z_p = \lim_{x \to 0} Z_\downarrow = \lim_{k_{F\downarrow} \to 0} n_\downarrow(k_F^-) - n_\downarrow(k_F^+)$$

$$= n_{pol}(k=0) - n_{pol}(k=0^+) = n_{pol}(k=0)$$

Experimental validation of theoretical approach in 3D

nature physics

Article

https://doi.org/10.1038/s41567-023-01948-1

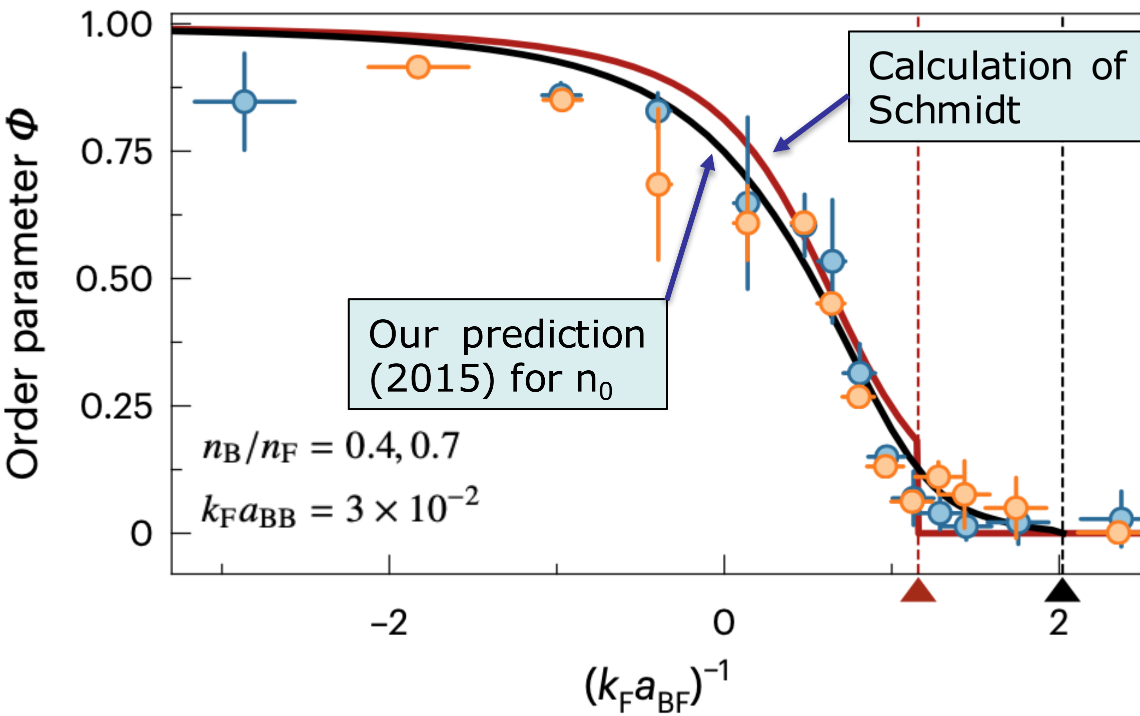
Transition from a polaronic condensate to a degenerate Fermi gas of heteronuclear molecules

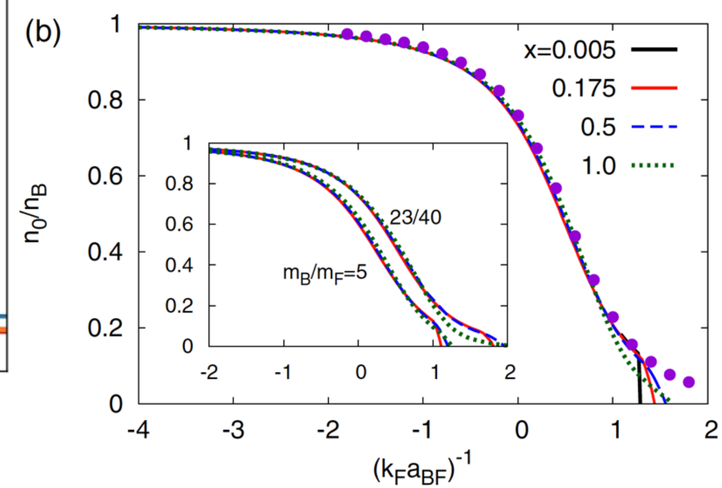
Received: 28 July 2022

Accepted: 6 January 2023

Marcel Duda[1,2], Xing-Yan Chen[1,2], Andreas Schindewolf[1,2], Roman Bause[1,2], Jonas von Milczewski[1,2], Richard Schmidt[1,2,3,4], Immanuel Bloch[1,2,5] & Xin-Yu Luo[1,2]

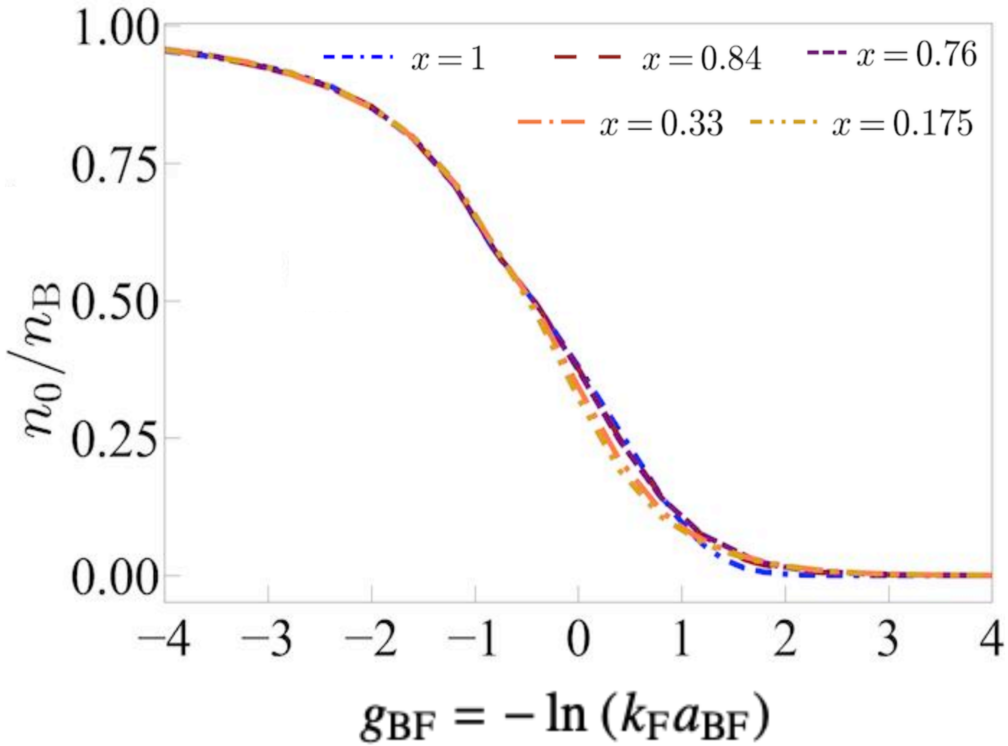
^{23}Na-^{40}K Bose-Fermi mixture with broad Feshbach resonance ($k_F R^* = 0.08$).

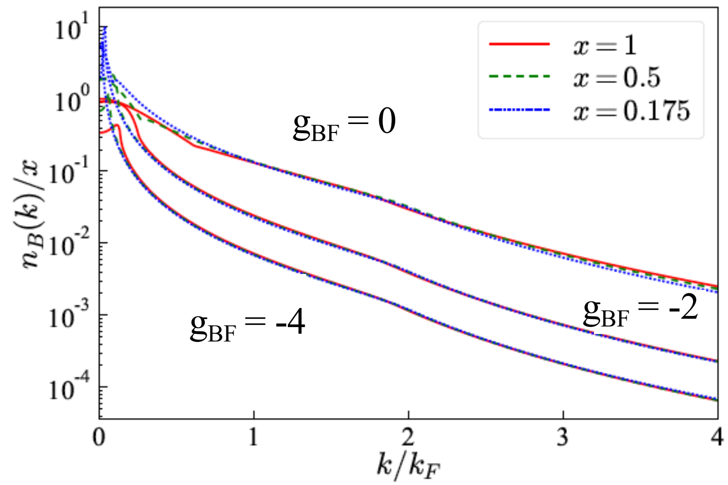




A. Guidini, G. Bertaina, D. Galli, P. Pieri (2015)

2D: condensate fraction and boson momentum distribution



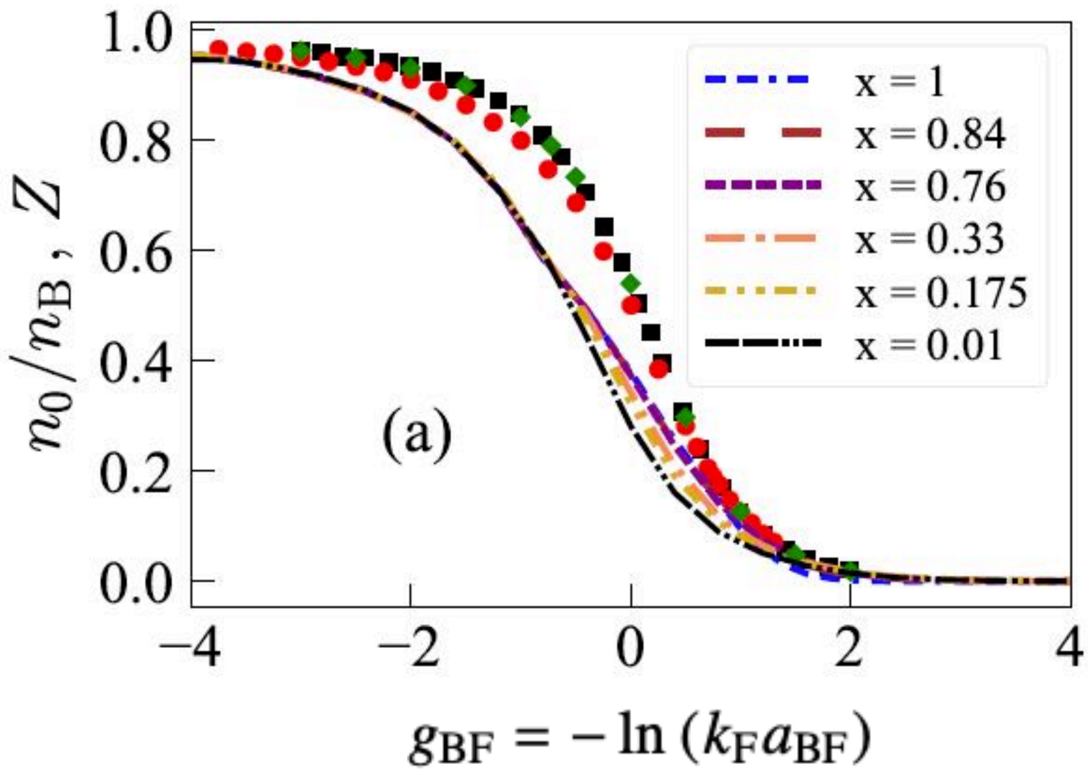


L. Pisani, P. Bovini, F. Pavan, P. Pieri (submitted to SciPost Physics)

- Like in 3D, condensate fraction and momentum distribution display nearly **universal behavior** when the concentration *x* is varied.

- However, in contrast with 3D, the condensate **does not identically vanish** beyond a critical coupling. It remains finite (**albeit exponentially small**) at large BF coupling strength).

2D: Comparison with polaron quasiparticle residue



■ T-matrix results for Z [R. Schmidt, T. Enss, V. Pietilä, E. Demler, PRA **85**, 021602 (2012)]

● Diagrammatic MC results for Z [J. Vlietinck, J. Ryckebusch, K. Van Houcke PRB **89**, 085119 (2014)]

- The universal condensate fraction **does not agree** with the polaron residue Z in 2D.
- The T-matrix results for Z (■) are based on the same self-energy as ours when restricted to the polaron limit → difference between condensate fraction and Z is **not due to different levels of approximation**.
- The 'near degeneracy' between condensate fraction and Z found in 3D was just accidental.

Back to the explanation for the connection between n_0/n_B and Z

$$\frac{n_0}{n_{\mathrm{B}}} = \frac{n_{\mathrm{B}}(k=0)}{N_{\mathrm{B}}} \to \frac{n_{\mathrm{pol}}(k=0)}{1} = n_{\mathrm{pol}}(k=0) - n_{\mathrm{pol}}(k=0^+) = \lim_{k_{\mathrm{F}\downarrow}\to 0} n_\downarrow(k_{\mathrm{F}\downarrow}^-) - n_\downarrow(k_{\mathrm{F}\downarrow}^+) = Z$$

This is the weak step.

n_0/n_{B} is defined in the thermodynamic limit: $V \to \infty$ with $n_{\mathrm{B}}/n_{\mathrm{F}}$ fixed (no matter how small we take it). Condensate fraction in $x \to 0$ limit:

First take $V \to \infty$ and then $x \to 0$. So $N_{\mathrm{B}} = n_{\mathrm{B}} V$ is always infinite.

For a single impurity instead, $N_{\mathrm{B}} = 1$ from the outset and only then $V \to \infty$.

$$\lim_{x\to 0}\lim_{V\to\infty} \frac{n_{\mathrm{B}}(k=0; N_{\mathrm{B}} = xN_{\mathrm{F}}; N_{\mathrm{F}}/V = \mathrm{const})}{N_{\mathrm{B}}} \neq \lim_{V\to\infty} \frac{n_{\mathrm{B}}(k=0; N_{\mathrm{B}} = 1; N_{\mathrm{F}}/V = \mathrm{const})}{1}$$

There is no reason for the condensate fraction to coincide with the polaron residue Z, even in the $x \to 0$ limit of vanishing boson concentration.

Spectral functions: analytic continuation to real frequencies

- Focus on the **boson spectral function** which, for a BF mixture, extends the Fermi polaron spectral function at a finite concentration *x* of the impurities.

- Neglecting BB interaction, start from the boson self-energy due to interaction with fermions on the imaginary frequency axis

$$\Sigma_{\mathrm{BF}}(\bar{k}) = \int \frac{d\mathbf{P}}{(2\pi)^2} \int \frac{d\Omega}{2\pi} T(\bar{P}) G^0_{\mathrm{F}}(\bar{P}-\bar{k}) e^{i\Omega 0^+}$$

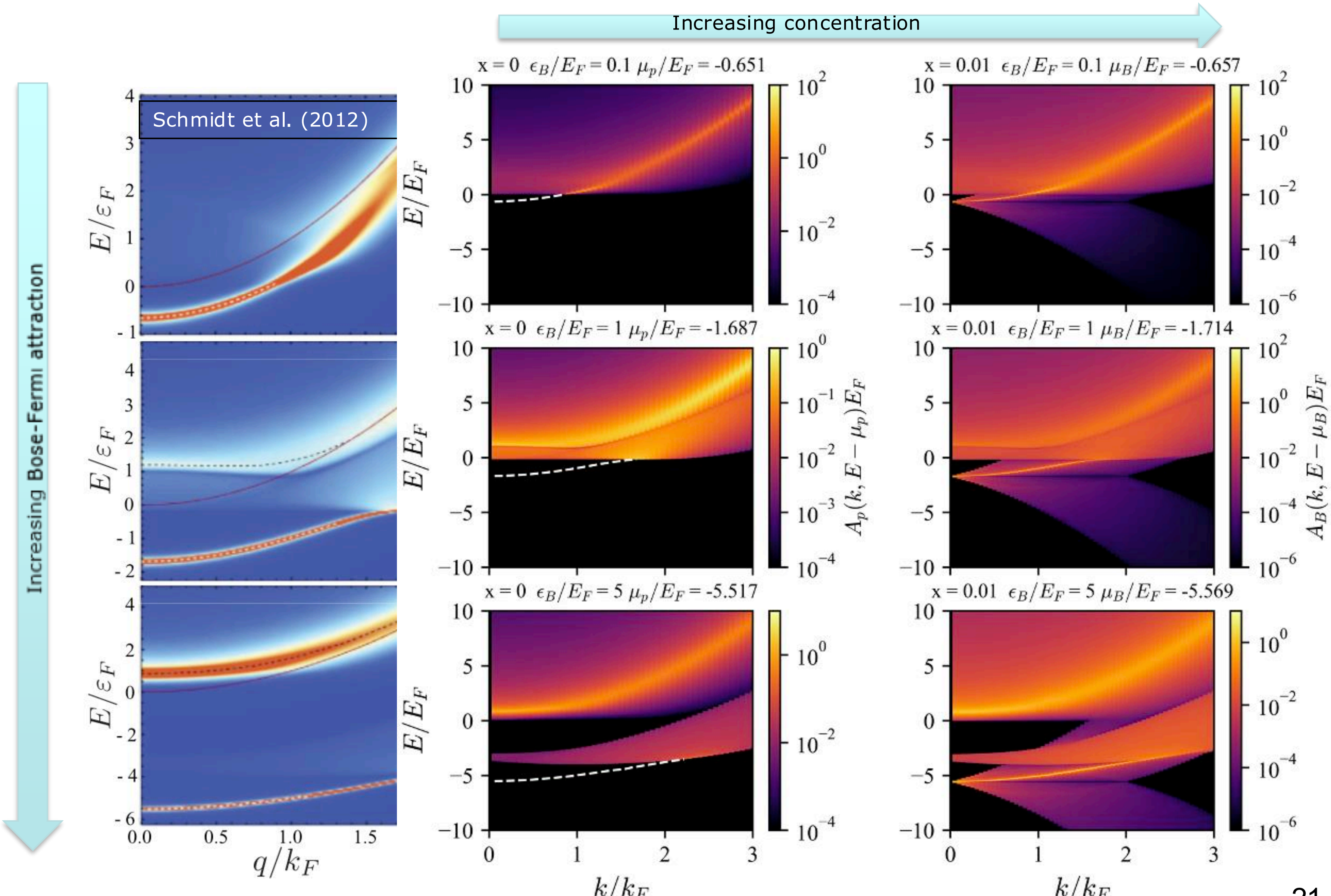
and **perform analytic continuation** to real frequencies: $i\omega \to \omega + i0^+$ to obtain

$$\Sigma^{\mathrm{R}}_{\mathrm{BF}}(\mathbf{k},\omega) = \int \frac{d\mathbf{P}}{(2\pi)^2} \int \frac{d\omega'}{\pi} \mathrm{Im} T^{\mathrm{R}}(\mathbf{P},\omega') \frac{\Theta(-\omega') - \Theta\left(-\xi^{\mathrm{F}}_{\mathbf{P}-\mathbf{k}}\right)}{\omega - \omega' + \xi^{\mathrm{F}}_{\mathbf{P}-\mathbf{k}} + i0^+}$$ with $$T^{\mathrm{R}}(\mathbf{P},\Omega) = T(\mathbf{P}, i\Omega \to \Omega + i0^+)$$

- The spectral function is then obtained as

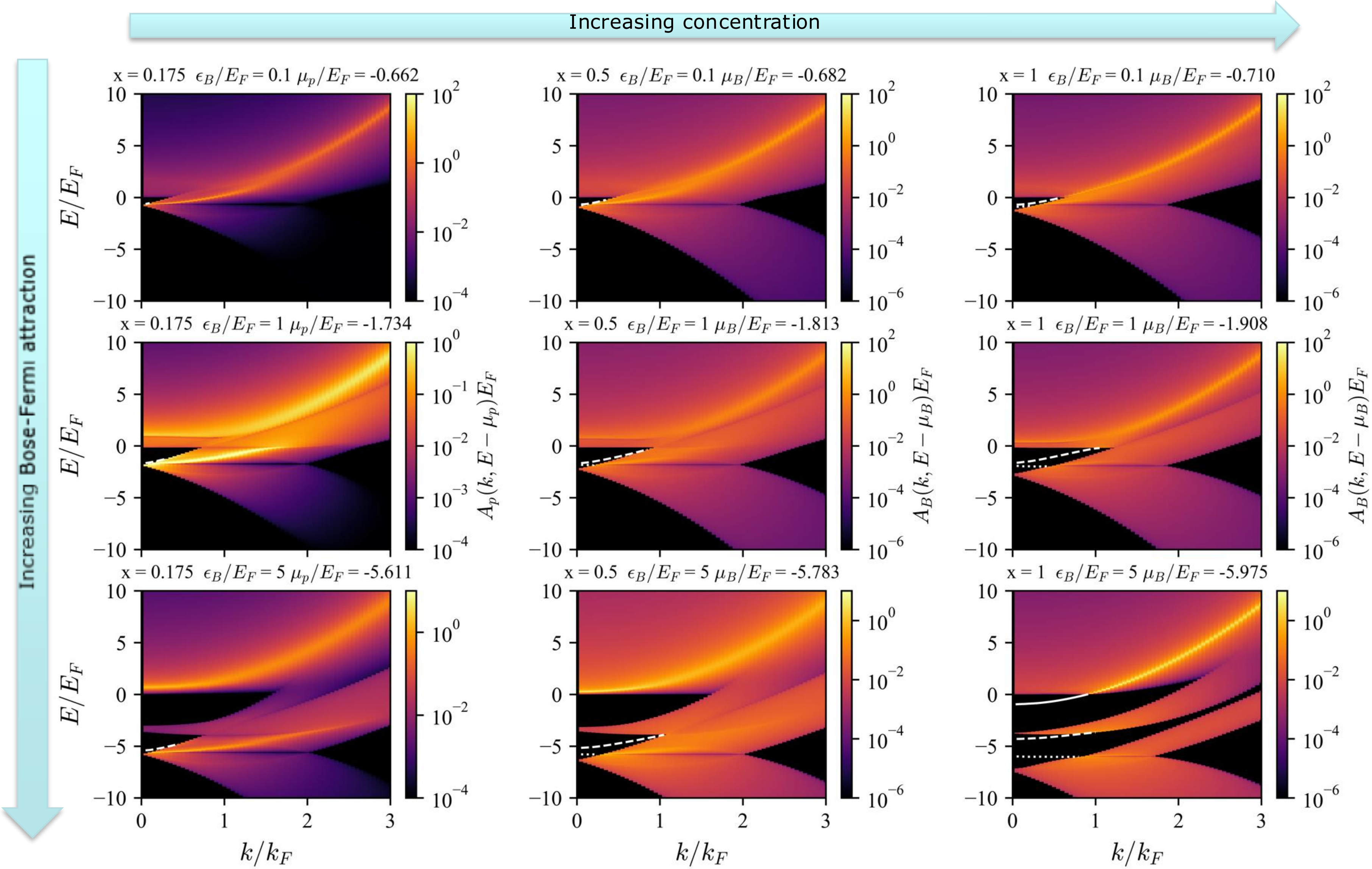
$$A_{\mathrm{B}}(\mathbf{k},\omega) = -\frac{1}{\pi}\mathrm{Im} G^{\mathrm{R}}_{\mathrm{B}}(\mathbf{k},\omega) = -\frac{1}{\pi}\mathrm{Im}\frac{1}{\omega + i0^+ - k^2/(2m_{\mathrm{B}}) + \mu_{\mathrm{B}} - \Sigma^{\mathrm{R}}_{\mathrm{BF}}(\mathbf{k},\omega)}$$

Boson (minority species) spectral weight function



P. Bovini, L. Pisani, P. Pieri, work in progress

Boson spectral weight function (larger concentrations)



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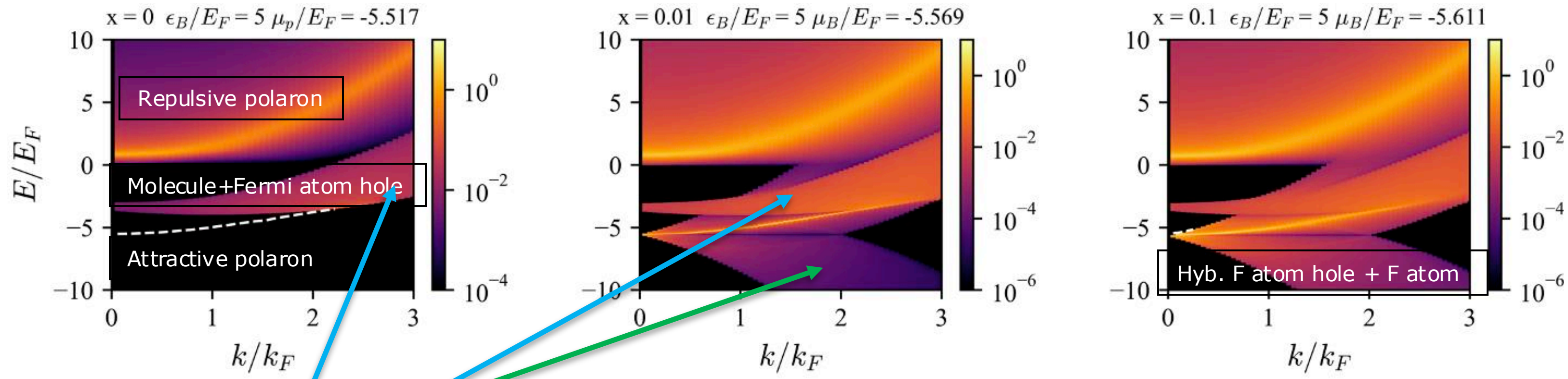
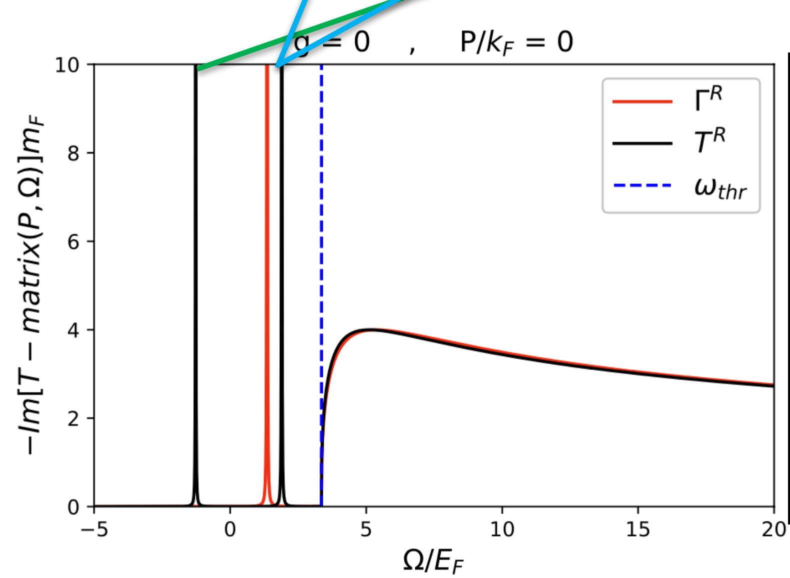
Origin of different spectral features





Two poles in the T-matrix due to **hybridization**, mediated by the boson condensate, between bare BF molecules and bare atomic fermions.

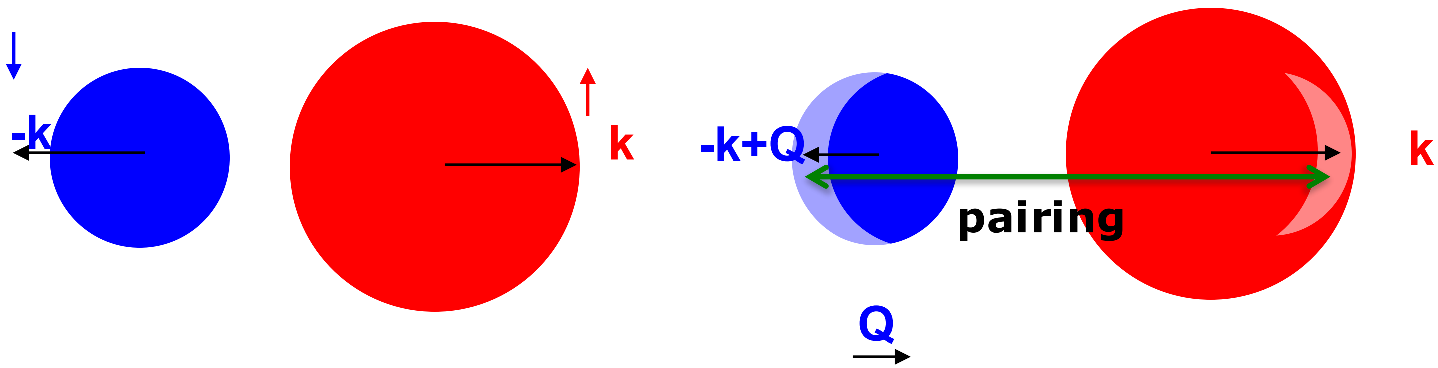
Pole in T **at lower frequency** is mostly atomic in nature. It is **absent in** the pure **polaronic case** (it requires a condensate).

It produces a **downward band** which could be observed with RF spectroscopy.

When the FERMI polarons are fermions: the case of polarized Fermi gases

Polarized Fermi gases: search for FFLO phase

- Search for FFLO phase was one of the main motivations driving experiments with polarized Fermi gases
- Phase first proposed theoretically by Fulde & Ferrell and independently by Larkin & Ovchinnikov (1964)
- Pairing between **k** and **−k+Q** to compensate mismatch of Fermi surfaces: pairs acquire a finite center of mass momentum **Q**



$$|\Psi_{\rm FF}\rangle = \prod_{\mathbf{k}\in\mathcal{R}} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}}{2}\uparrow} \prod_{\mathbf{k}\notin\mathcal{R}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}+\frac{\mathbf{Q}}{2}\uparrow} c^\dagger_{-\mathbf{k}+\frac{\mathbf{Q}}{2}\downarrow})|0\rangle$$

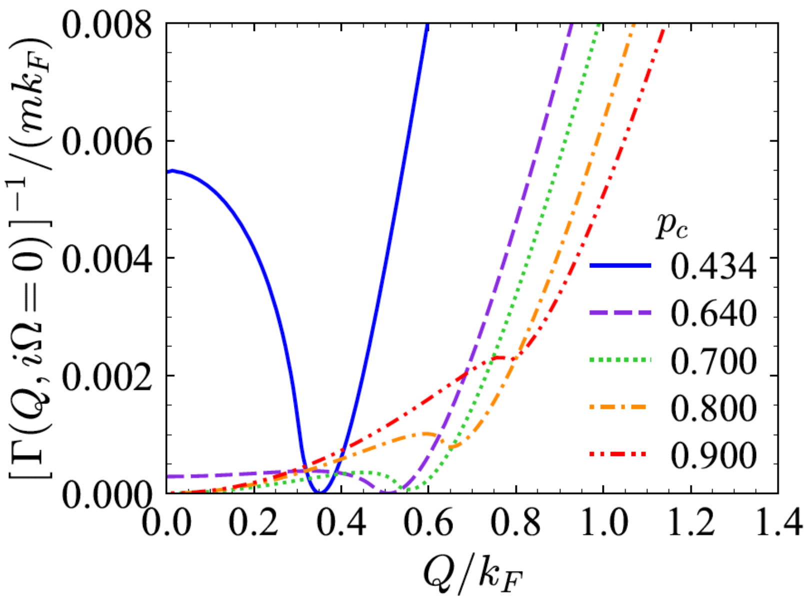
Phase diagram at T=0 with self-consistent T-matrix approach (3D)

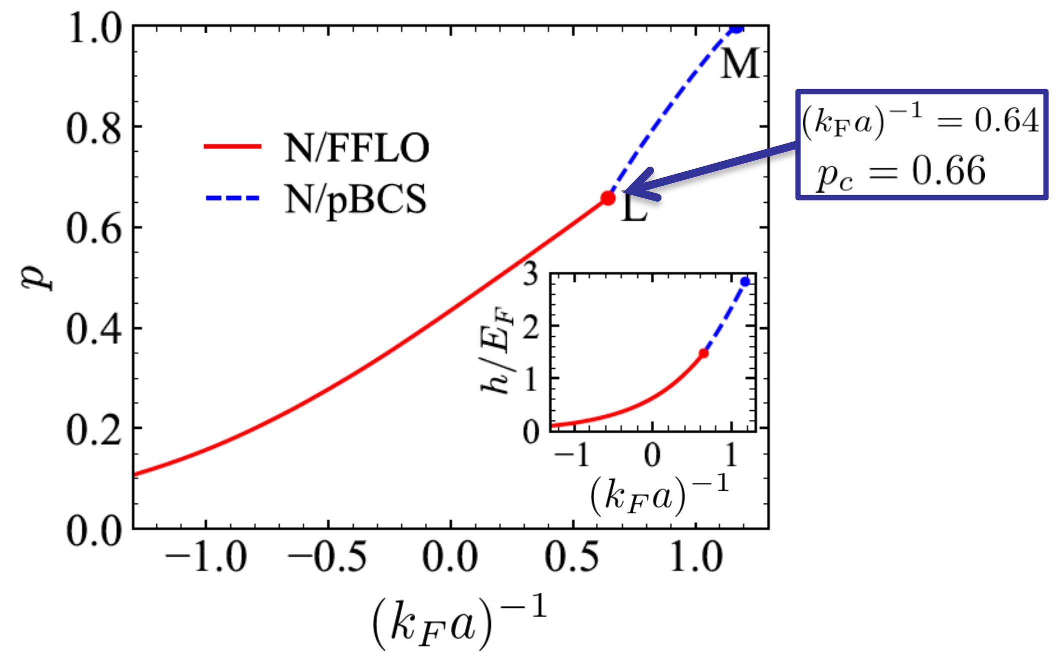
We vary dimensionless coupling $(k_F a)^{-1}$(with $k_\mathrm{F} \equiv (3\pi^2 n)^{1/3}$, $n = n_\uparrow + n_\downarrow$) and determine critical polarization p_c[with $p = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$].

Second order phase transition determined by:

$$\left[\Gamma(|\mathbf{Q}| = Q_0, i\Omega = 0)|_{p=p_c}\right]^{-1} = 0 \longleftrightarrow \text{diverging pairing susceptibility } \chi_\mathrm{pair}(Q_0),$$

where Q_0 is the value of $|\mathbf{Q}|$ minimizing $\Gamma(|\mathbf{Q}|, i\Omega = 0)^{-1}$





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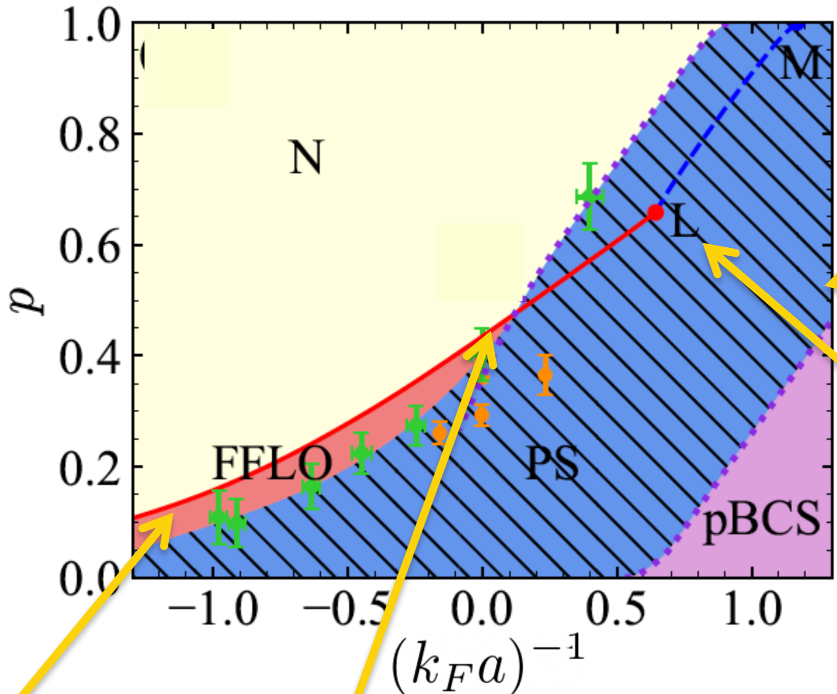
At the Lifshitz point (L) the transition changes from N/FFLO to N/pBCS where pBCS is a polarized SF with standard BCS pairing $(Q_0 = 0)$.

Phase diagram at T=0: including phase separation (3D)



PS region combines exp. data data from Shin et al., 2008 and Olsen et al., 2015 (symbols with error bars) with QMC results from Pilati et al., 2008 (dotted lines)

Lifshitz point is hidden by phase separation region. However, the L point could be reached in principle following a metastable N phase (we found a positive-definite compressibility matrix).

Relatively narrow region for observability of FFLO, still larger than within mean-field. It includes a polarization range at unitarity. Need boxlike trap for observability.

At unitarity, critical polarization to FFLO: ~0.435. This value agrees well with recent work (Tüzemen et al., 2023) by Warsaw group within a DFT approach.

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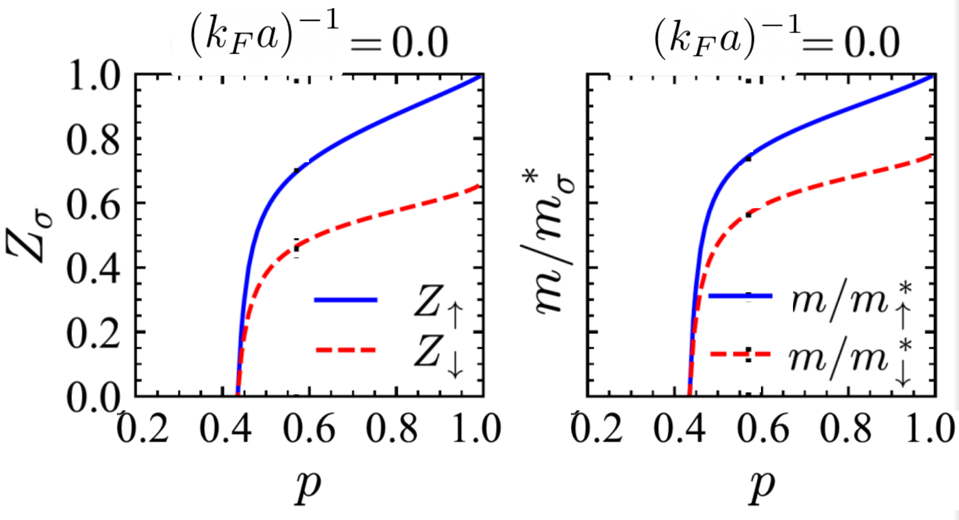
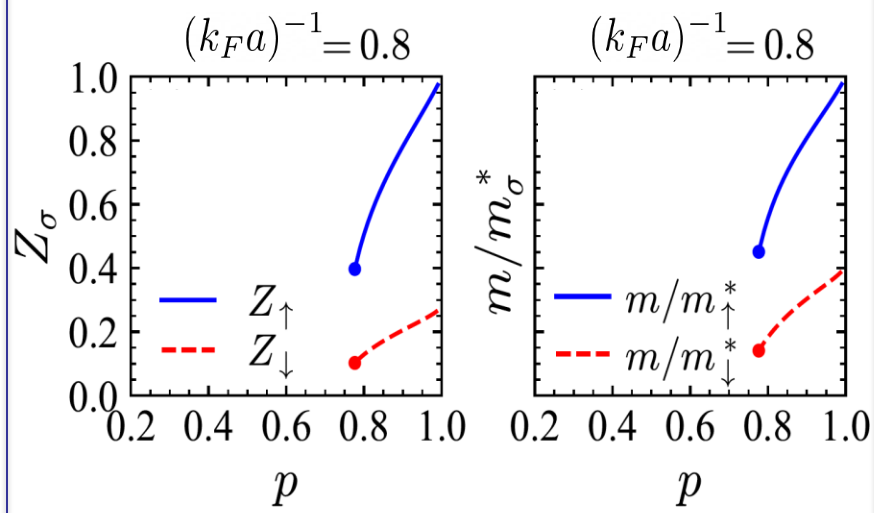
Quasi-particle residue and effective mass

Quasi-particle residue and effective mass at $|\mathbf{k}| = k_{F\sigma}$ can be calculated directly on the imaginary frequency axis:

$$Z_\sigma = \left[1 - \frac{\partial \, \mathrm{Im}\Sigma_\sigma(k_{F\sigma}, i\omega)}{\partial \omega}\bigg|_{\omega=0}\right]^{-1} \qquad \frac{m}{m^*_\sigma} = Z_\sigma \left[1 + \frac{m}{k_{F\sigma}} \frac{\partial \Sigma_\sigma(\mathbf{k}, i\omega = 0)}{\partial |\mathbf{k}|}\bigg|_{|\mathbf{k}|=k_{F\sigma}}\right]$$

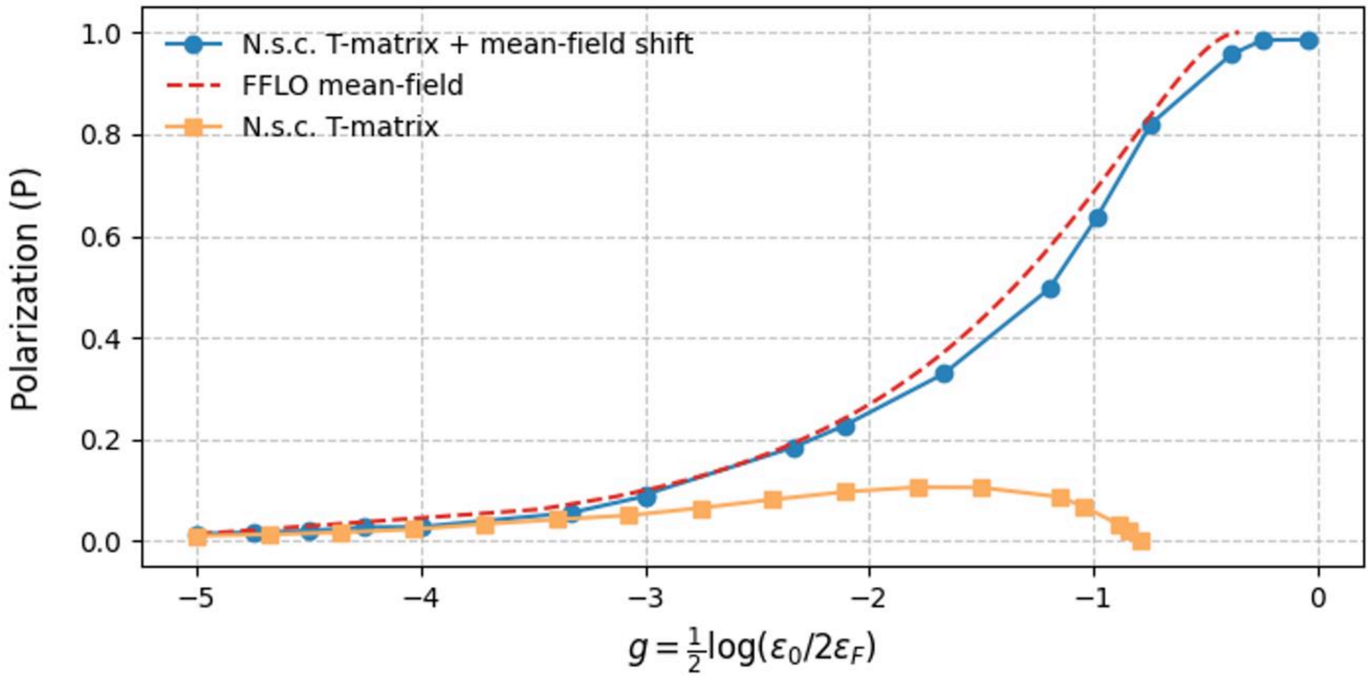


Vanishing quasi-particle residue and diverging effective mass at FFLO QCP: breakdown of FL properties analogous to what is found in heavy-fermions in the proximity of an AFM QCP.



When QCP is to standard BCS pairing, no breakdown of FL properties at QCP.

2D: Phase diagram



Away from the polaronic limit, the non self-consistent T-matrix performs poorly.

We are now in the process of implementing fully-self-consistent T-matrix in 2D.

First rough way of implementing self-consistency: insert a «mean-field shift» in the bare Green's functions entering the T-matrix self-energy:

$$\mu \to \mu'_\sigma \equiv \mu_\sigma - \Sigma_\sigma(k_{\mu'_\sigma}, 0)$$

More sensible results for the critical are obtained in this way.

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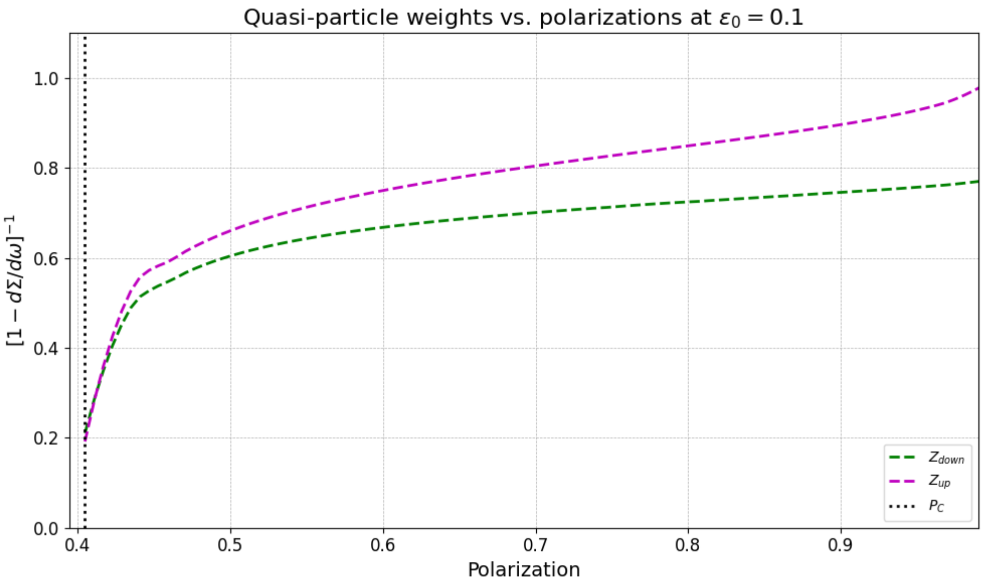
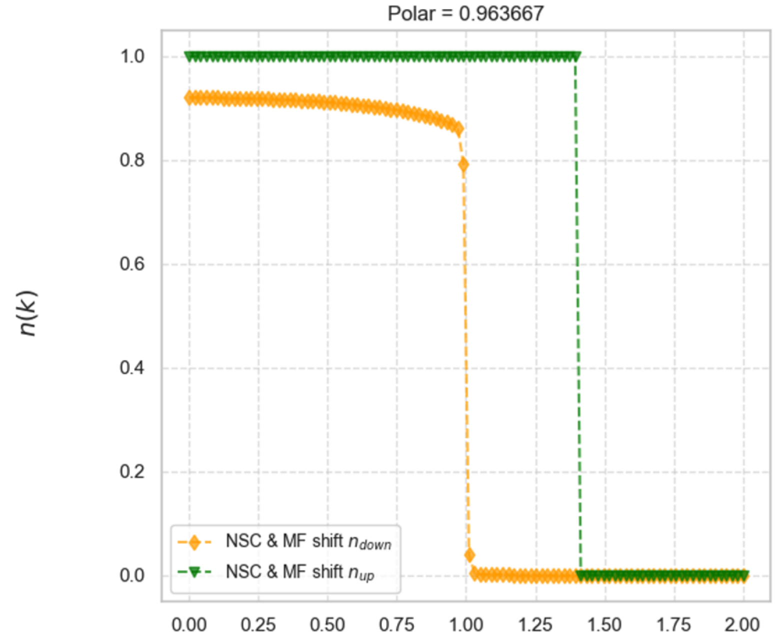
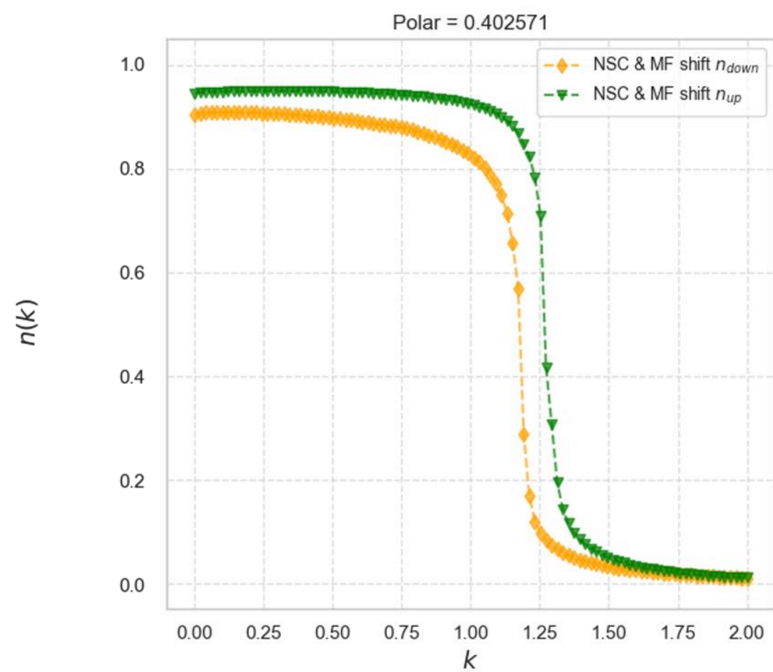
2D: Quasiparticle residue Z







Quasiparticle residue Z rapidly decreases when approaching the QCP to FFLO phase.

However, it does not exactly vanish because the absence of self-consistency hinders the validity of Luttinger theorem.

F. Pirolo, L. Pisani, P. Pieri, work in progress

Conclusions

- Luttinger theorem in Fermi liquids and Hugenholtz-Pines theorem for condensed bosons are unified under the polaron perspective.

- Calculation of the condensate fraction in 2D Bose-Fermi mixtures reveals that the identification of the polaron residue with the condensate fraction, suggested by the 3D behavior, is not valid.

- Interesting features in the spectral weight function at finite concentration when the impurities are bosons in a Fermi gas.

- When the polarons are fermions, the quasiparticle residue evolves from the polaronic limit to the QCP to the superfluid phase phase. At FFLO QCP: vanishing quasi-particle residue and diverging effective mass: breakdown of FL properties analogous to what is found in heavy-fermions at AFM QCP.

- Work in progress for 2D polarized Fermi gases: Implementing self-consistency is crucial.

Thank you!